MA 374: FE-Assignment #01

Due on Wednesday, January 27, 2016

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PROBLEM

Write a program, using the binomial pricing algorithm, to determine the price of an European call and an European put option (in the binomial model framework) with the following data :

$$S(0) = 9 \quad K = 10 \quad T = 3 \quad r = 0.06 \quad \sigma = 0.3$$
$$u = e^{\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t} \quad d = e^{-\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$$

where $\Delta t = \frac{T}{M}$, with M being the number of subintervals in the time interval [0,T]. Use the continuous compounding convention in your calculations (i.e., both in $\sim p$ and in the pricing formula).
(1) Run your program for M = 1 , 5 , 10 , 20 , 50 , 100 , 200 , 400 to get the initial option prices and tabulate them.
(2) How do the values of options at time t = 0 compare for various values of M? Compute and plot graphs (of the initial option prices) varying M in steps of 1 and in steps of 5. What do you observe about the convergence of option prices?
(3) Tabulate the values of the options at t = 0 , 0.30 , 0.75 , 1.50 , 2.70 for the case M = 20.
Note that your program should check for the no-arbitrage condition of the model before proceeding to compute the prices.

SOLUTION

No-Arbitrage Condition for continuous compounding: $0 < d < exp(r\Delta t) < u$
This condition has been checked in every calculation : There is no arbitrage possible.
$\sim p$ for continuous compounding : $\frac{e^{r\Delta t}-d}{u-d}$
$\sim q$ for continuous compounding : $\frac{u-e^{r\Delta t}}{u-d}$
$\sim p+\sim q = 1$

Part-1

European Call :

M	Option Price
1	1.987350
5	2.122623
10	2.142532
20	2.099561
50	2.123226
100	2.119768
200	2.119670
400	2.120391

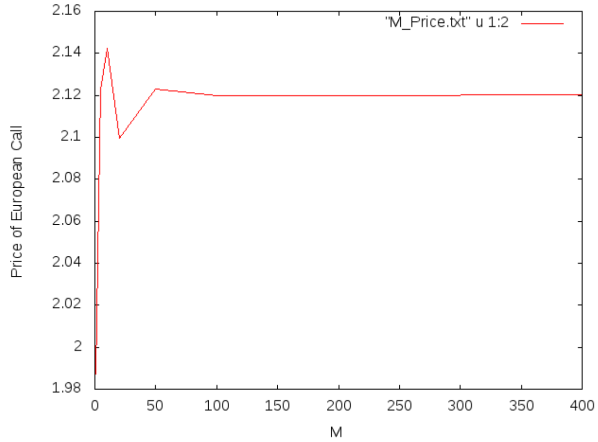
European Put :

M	Option Price
1	1.340052
5	1.475326
10	1.495234
20	1.452263
50	1.475929
100	1.472470
200	1.472372
400	1.473093

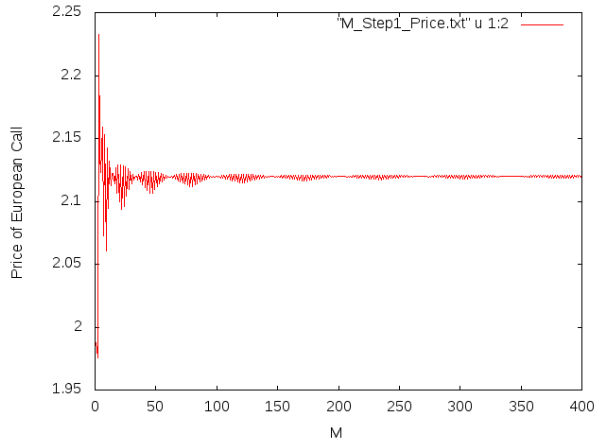
Part-2

European Call :

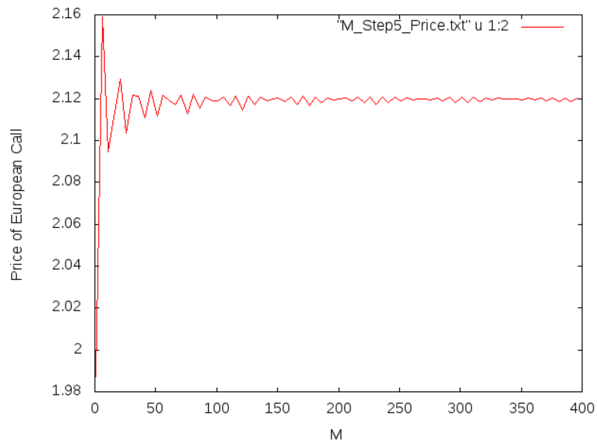
M(Given in Part-1) - Price of European call option Plot :



M(increased in steps of 1) - Price of European call option Plot :

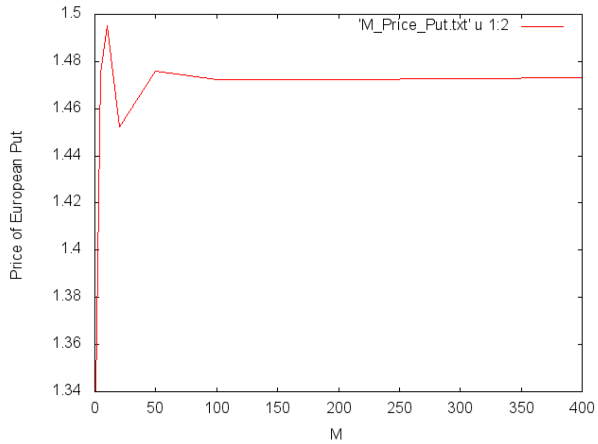


M(increased in steps of 5) - Price of European call option Plot :

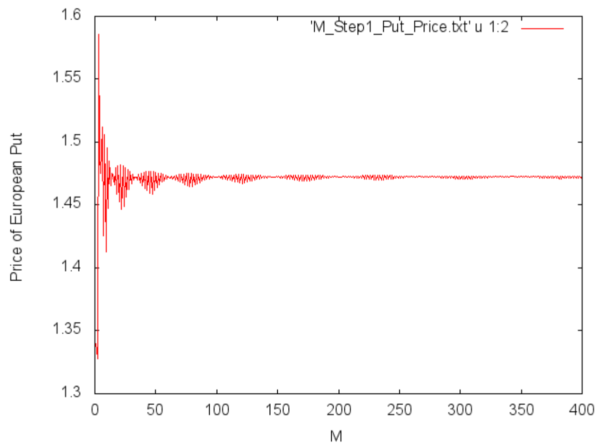


European Put :

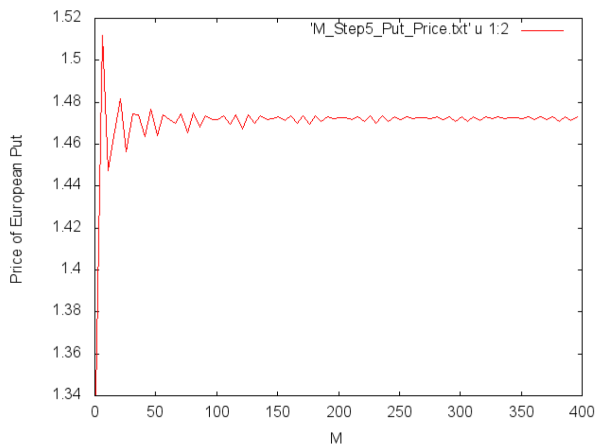
M(Given in Part-1) - Price of European put option Plot :



M(increased in steps of 1) - Price of European put option Plot :



M(increased in steps of 5) - Price of European put option Plot :



Convergence in the price of option : As observed in the plot , the points of convergence are as follows :
(1) European Call Option : M (Step of 1) = 150(approx) , M (Step of 5) = 200(approx)
(1) European Put Option : M (Step of 1) = 200(approx) , M (Step of 5) = 200(approx)

Part-3

European Call :

t	Value of Option
2.70	56.296 , 42.586 , 31.719 , 23.106 , 16.278 , 10.866 , 6.576 , 3.176 , 0.892 , 0.054 , 0.000
1.50	18.113 , 12.473 , 8.054 , 4.709 , 2.378 , 0.978 , 0.304 , 0.064 , 0.008 , 0.000
0.75	7.151 , 4.237 , 2.241 , 1.024 , 0.389 , 0.118
0.30	3.559 , 1.880 , 0.871
0.00	2.100

European Put :

t	Value of Option
2.70	0.000 , 0.411 , 1.709 , 3.349 , 4.691 , 5.755 , 6.598 , 7.266 , 7.796 , 8.216 , 8.549 , 8.813
1.50	0.001 , 0.011 , 0.071 , 0.277 , 0.760 , 1.591 , 2.685 , 3.846 , 4.901 , 5.774 , 6.471
0.75	0.227 , 0.560 , 1.139 , 1.962 , 2.944 , 3.955
0.30	0.831 , 1.481 , 2.319
0.00	1.452