MA 374: FE-Assignment #04

Due on Monday, February 15, 2016

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PROBLEM - 1

Consider three assets with the following mean return vector and covariance matrix:

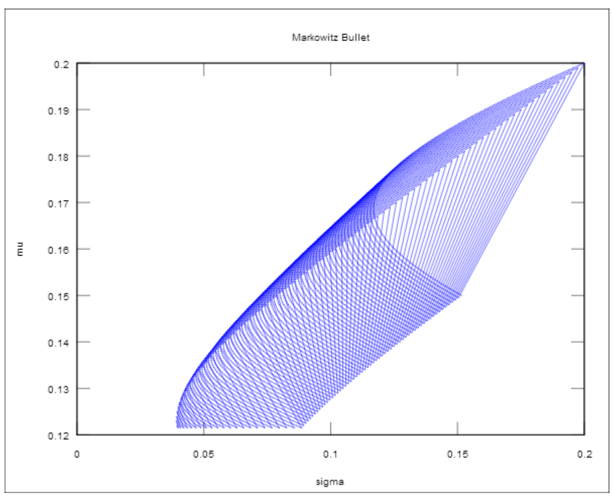
$$M = [0.1, 0.2, 0.15]$$

$$C = \begin{bmatrix} 0.005 & -0.010 & 0.004 \\ -0.010 & 0.040 & -0.002 \\ 0.004 & -0.002 & 0.023 \end{bmatrix}$$

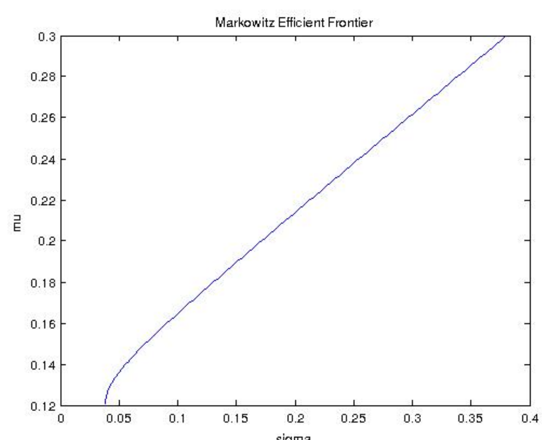
(a) Construct and plot the Markowitz efficient frontier using the above data.
(b) Tabulate the weights, return and risk of the portfolios for 10 different values on the efficient frontier.
(c) For a 15 % risk, what is the maximum and minimum return and the corresponding portfolios ?
(d) For a 18 % return, what is the minimum risk portfolio ?
(e) Assuming the risk free return rf = 10%, compute the market portfolio. Also determine and plot the Capital Market Line.
(f) Find two portfolios (consisting of both risky and risk free assets) with the risk at 10% and 25%.

SOLUTION

Part a



The above figure gives the feasible set of the portfolio weights i.e when $w_1 + w_2 + w_3 = 1$ where w_i is the weight of the i_{th} stock in the portfolio.



Markowitz Efficient Frontier

Derivation of Markowitz Efficient Frontier :
Markowitz Efficient Frontier is the pair of (σ , μ) values such the the risk σ involved with a given value of μ is minimum.

To minimize :

$$wCw^{'}$$

Subject to :

$$wm^{'} = \mu \quad wu^{'} = 1$$

where w : weight vector of the risky assets in the portfolio , C : Covariance matrix , m : vector of the expected return of the assets and u : row vector with n 1's where n is the number of assets.

$$G(w, \lambda_1, \lambda_2) = wCw' - \lambda_1(wm' - \mu) - \lambda_2(wu' - 1)$$

To minimize the variance : $\frac{dG}{dw} = 0$, $\frac{dG}{d\lambda_1} = 0$, $\frac{dG}{d\lambda_2} = 0$

$$w = \frac{\lambda_1}{2} mC^{-1} + \frac{\lambda_2}{2} uC^{-1}$$

After all the calculations the weight vector obtained is of the form :

$$w = \mu a + b$$

where a and b are n dimentional vectors.

Part b

The following are the weight vectors, expected rate of returns and the risk for 10 different portfolios on the efficient frontier.

No.	Weight Vector	Return	Risk
1	[0.818704 , 0.238720 , -0.057424]	0.121001	0.038427
2	[0.804392 , 0.244408 , -0.048800]	0.122001	0.038485
3	[0.790080 , 0.250096 , -0.040176]	0.123001	0.038659
4	[0.775768 , 0.255784 , -0.031553]	0.124001	0.038947
5	[0.761456 , 0.261473 , -0.022929]	0.125001	0.039347
6	[0.747144 , 0.267161 , -0.014305]	0.126001	0.039855
7	[0.732832 , 0.272849 , -0.005681]	0.127001	0.040468
8	[0.718520 , 0.278537 , 0.002943]	0.128001	0.041180
9	[0.704208 , 0.284225 , 0.011567]	0.129001	0.041987
10	[0.689897 , 0.289913 , 0.020191]	0.130001	0.042883

Part c

Maximum-Minimum return for 15% Risk

Part d

Minimum Risk for 18% Return

Part e

Derivation of Market Portfolio :

To maximize :

$$\frac{wm' - R}{\sqrt{wCw'}}$$

Subject to :

$$wu' = 1$$

where w : weight vector of the risky assets in the portfolio , C : Covariance matrix , m : vector of the expected return of the assets , R : risk free return and u : row vector with n 1's where n is the number of assets.

$$G(w, \lambda_1, \lambda_2) = wCw' - \lambda(wu' - 1)$$

To maximize the slope : $\frac{dG}{dw} = 0$, $\frac{dG}{d\lambda} = 0$

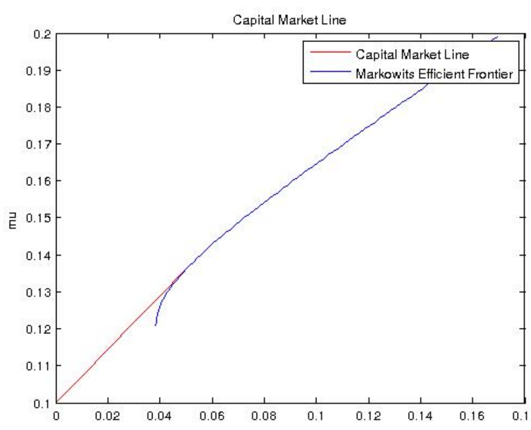
$$w = \frac{(m - Ru)C^{-1}}{(m - Ru)C^{-1}u'}$$

A portfolio with weight given by the above expression is called the market portfolio.

Derivation of Capital Market Line :

Capital Market line is the line joining $(0, R)$ and (σ_m, μ_m) given by :

$$\mu = R + \frac{(\mu_m - R)}{\sigma_m}\sigma$$



The market portfolio is :

weight vector : [0.593750 , 0.328125 , 0.078125]

Mean return : 0.13672

Standard Deviation : 0.050811

Part f

- Market Portfolio : Risk : σ_m , Mean Return : μ_m , Weight : w_1
- Risk Free Assets : Risk : σ_2 , Mean Return : μ_2 , Weight : w_2

Risk associated with Risk Free Assets will be zero, hence $\sigma_2 = 0$.
$w_1 + w_2 = 1$

$$\sigma^2 = w_1^2\sigma_m^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_m\sigma_2$$
$$\sigma^2 = w_1^2\sigma_m^2$$
$$w_1 = \frac{\sigma}{\sigma_m}$$

When $\sigma = 0.10, w_2 = 1.403, w_1 = -0.403$
When $\sigma = 0.15, w_2 = 1.718, w_1 = -0.718$

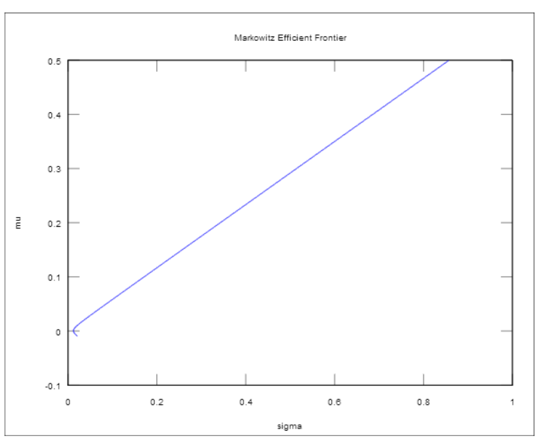
PROBLEM - 2

Obtain data (from online resources) for 10 stocks each with 50 data points all taken at the same dates (preferably spread over a year at equal intervals). Put this data and its details in a single Excel/CSV file. Using the data and assuming 7% risk free return:
(a) Construct and plot the Markowitz efficient frontier.
(b) Determine the market portfolio.
(c) Determine and plot the Capital Market Line.
(d) Determine and plot the Security Market Line for all the 10 assets.

SOLUTION

The stocks chosen are that of :
ADITYA_BIRLA , ALPHABET (GOOGLE) , AMAZON , FACEBOOK , HYATT , MTNL , ORACLE , TAJ , TATA_MOTOR , YAHOO
The data has been taken from 'Yahoo Finance' and the dates are from 1st January 2015 to 1st January 2016 (taken weekly).The expected return and variance have been calculated by taking the mean of all the returns obtained in every weekly interval.It has been assumed that the returns are independent i.e the covariance is 0 for two different stocks.

Part a



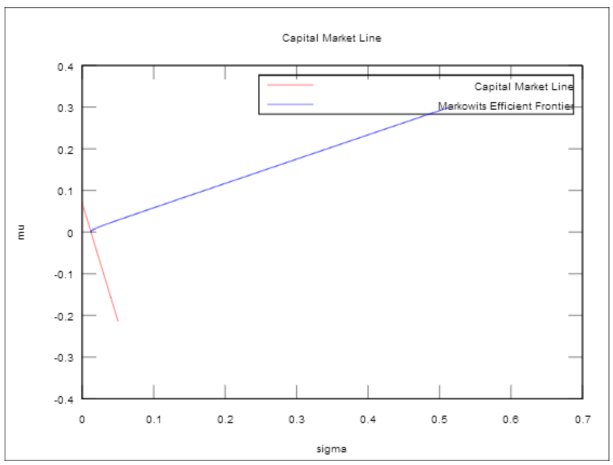
Markowitz Efficient Frontier

Part b

The market portfolio :

```
octave:1> source("Capital_Market_Line.m")
wMarket =

 Columns 1 through 6:

   3.4587e-04   9.9869e-02   1.0837e-01   1.6148e-01   2.1233e-01

 Columns 7 through 10:

   8.9725e-02   5.3477e-02   5.0896e-02   1.9733e-01

muMarket =   -2.6493e-04
sigmaMarket =  0.012399
```

Part c



Capital Market Line