MA 374: FE-Assignment #02

Due on Friday, January 29, 2016

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PROBLEM 1

Write a program to determine the initial price of an European call and an European put option in the binomial model with the following data :

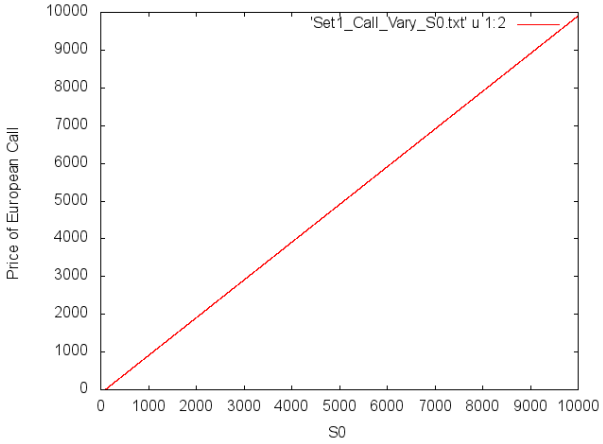
$$S(0) = 100 \quad K = 100 \quad T = 1 \quad M = 100 \quad r = 8\% \quad \sigma = 20\%$$

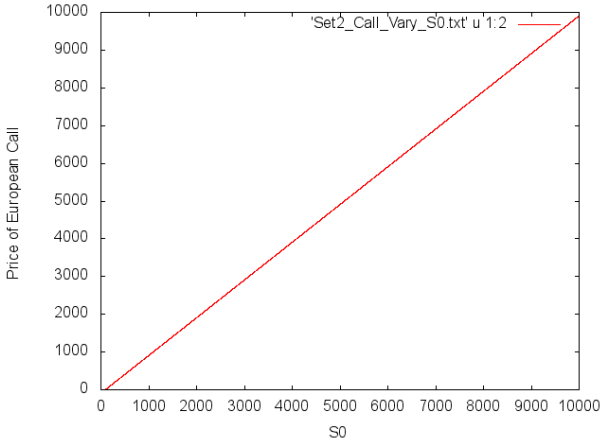
Use the following two sets of u and d for your program.
(a) Set 1 : $u = e^{\sigma\sqrt{\Delta t}}$, $d = e^{-\sigma\sqrt{\Delta t}}$
(b) Set 2 : $u = e^{\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$, $d = e^{-\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$
Here $\Delta t = \frac{T}{M}$ with M being the number of subintervals in the time interval [0,T]. Use the continuous compounding convention in your calculations (i.e., both in $\tilde{p}$ and in the pricing formula).
Now, plot the initial prices of both call and put options (for both the above sets of u and d) by varying one of the parameters at a time (as given below) while keeping the other parameters fixed (as given above) :
(a) S(0).
(b) K.
(c) r.
(d) σ
(e) M (Do this for three values of K, K = 95 , 100 , 105).

SOLUTION

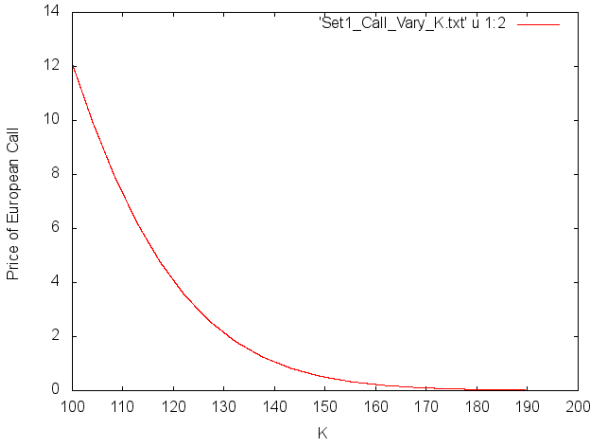
European Call Option

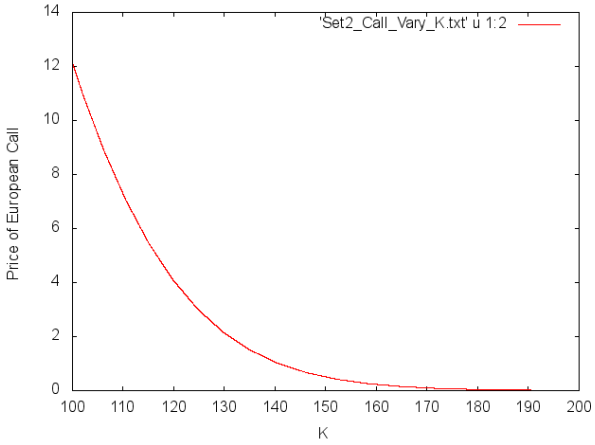
In the following plots, the left plot is for Set - 1 (u,d) and the right one corresponds to Set - 2.
(a) Varying S(0) :



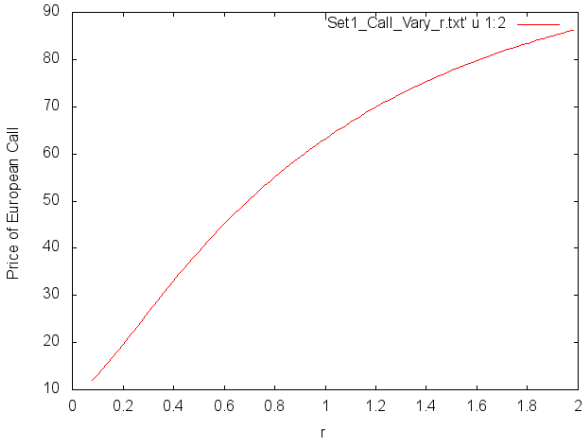


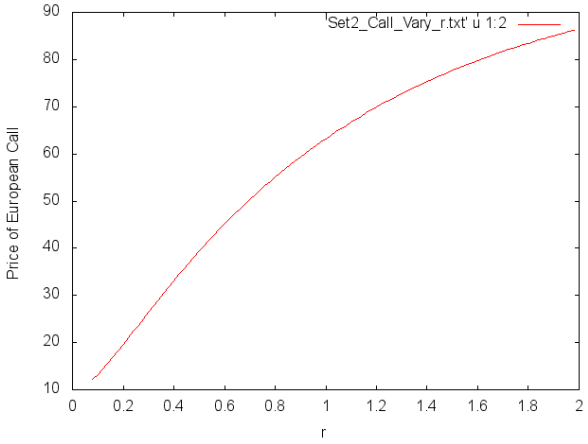
(b) Varying K :



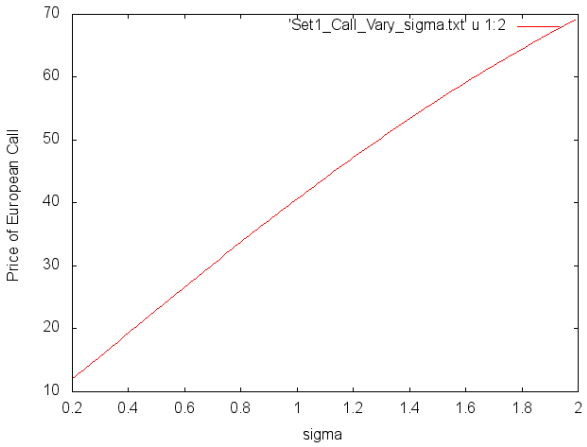


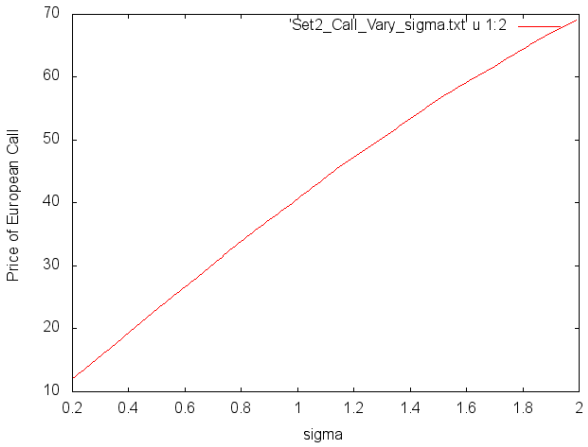
(c) Varying r :



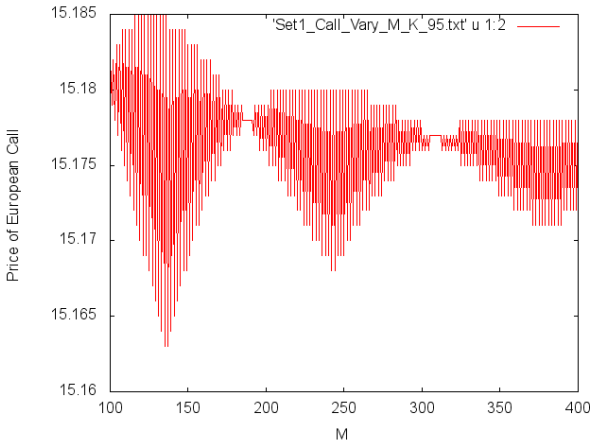


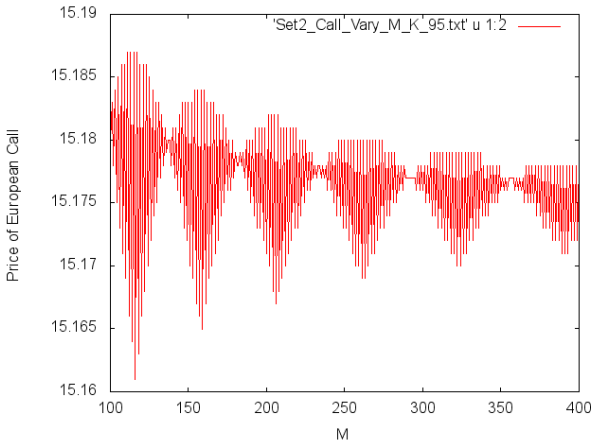
(d) Varying σ :



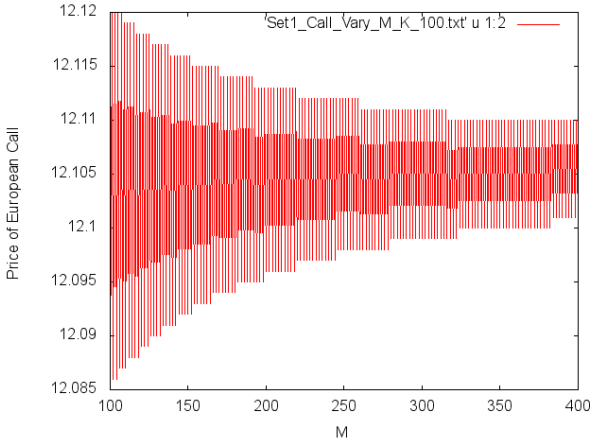


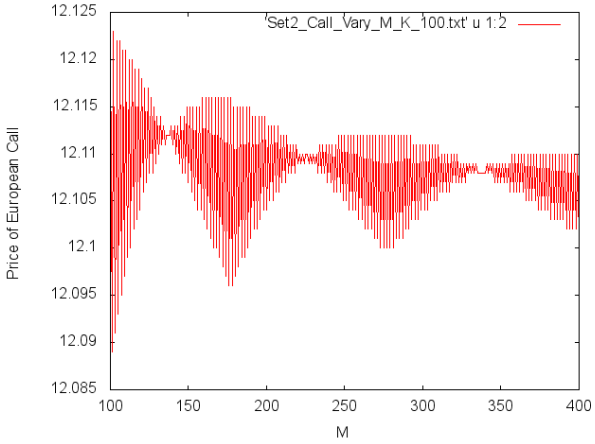
(e) Varying M (K = 95) :



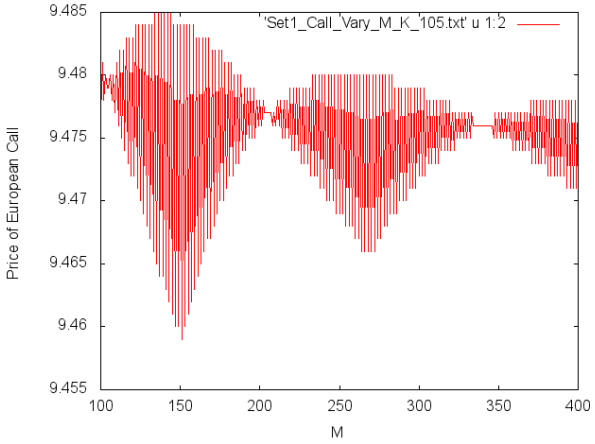


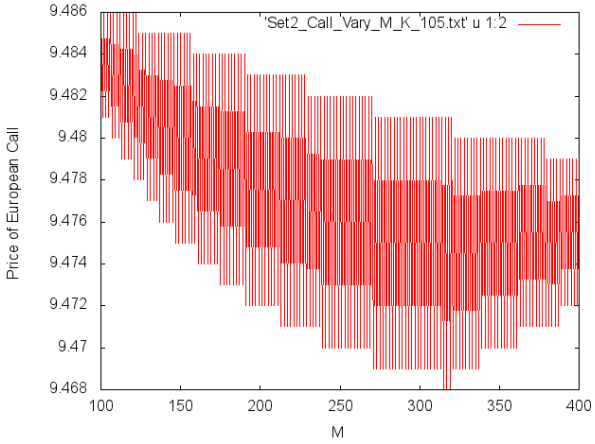
(f) Varying M (K = 100) :





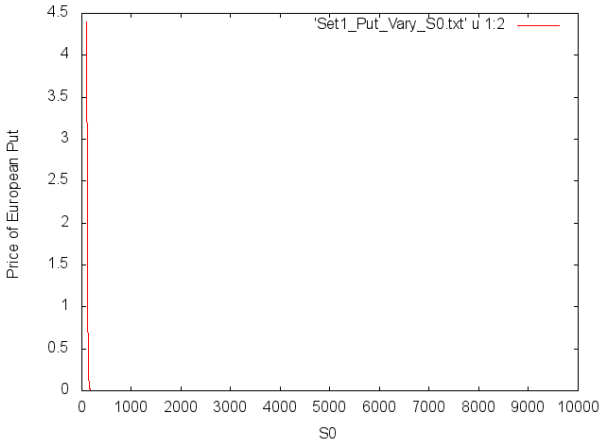
(g) Varying M (K = 105) :

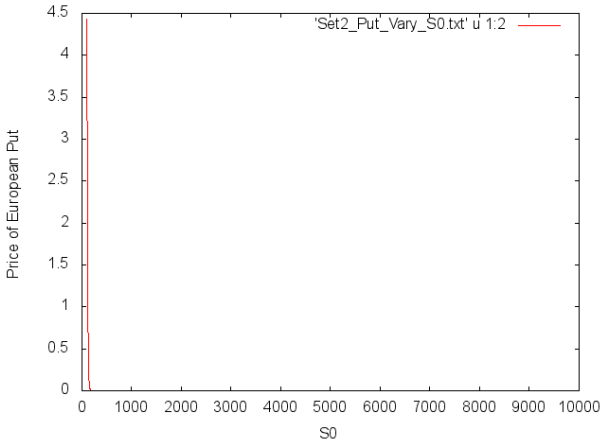




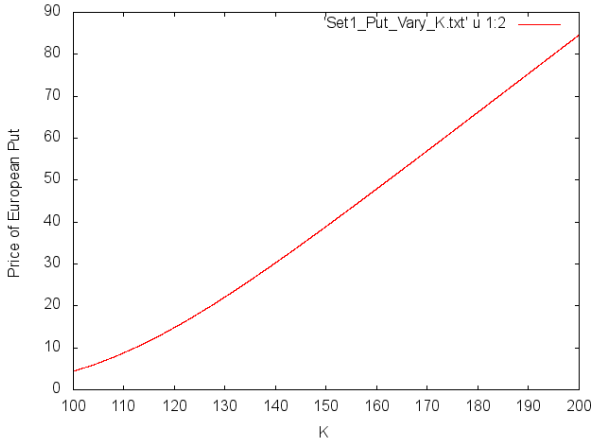
European Put Option

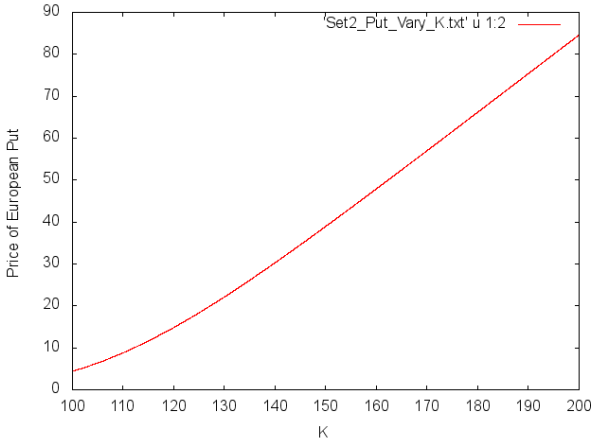
(a) Varying S(0) :



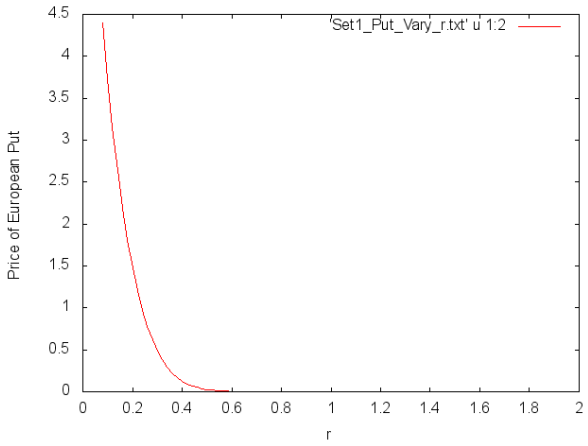


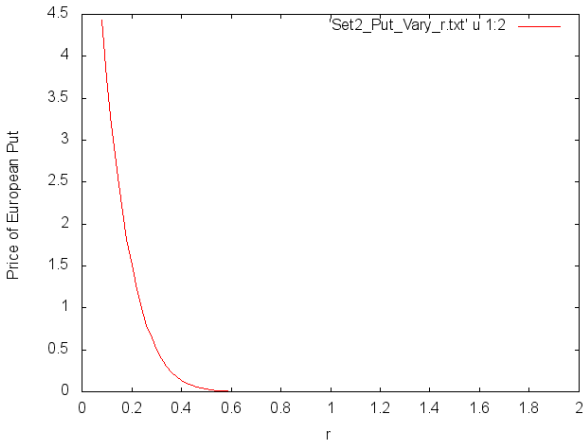
(b) Varying K :



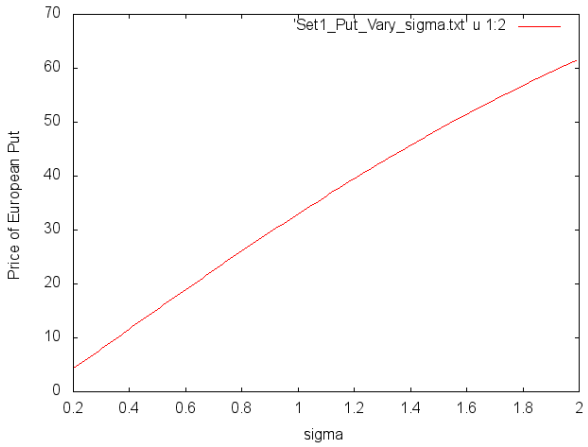


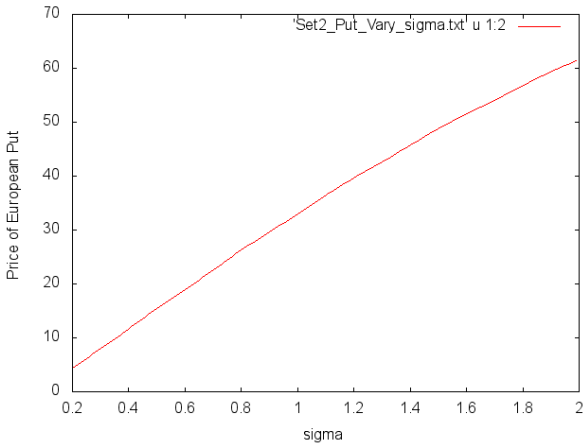
(c) Varying r :



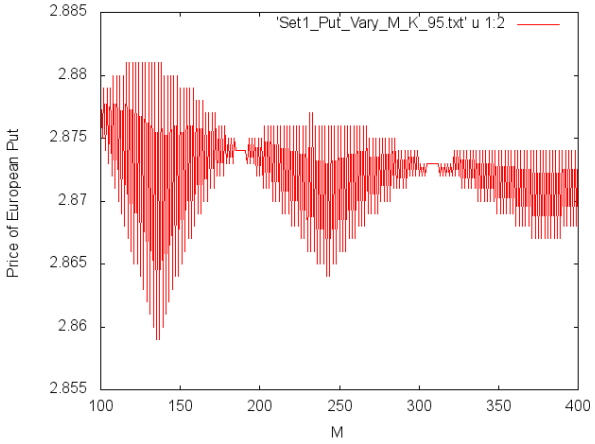


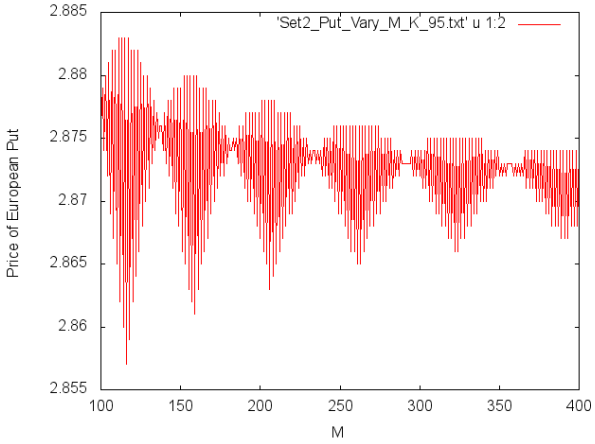
(d) Varying σ :



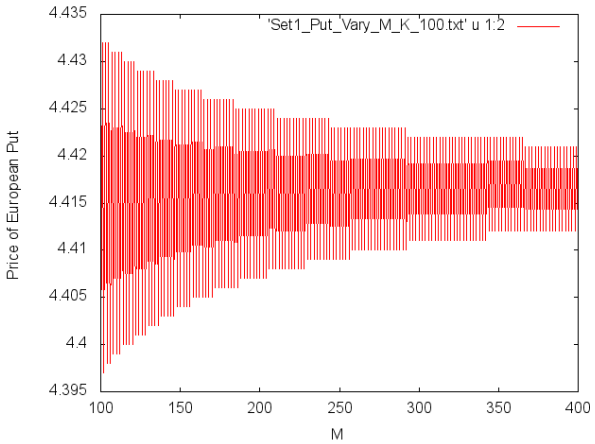


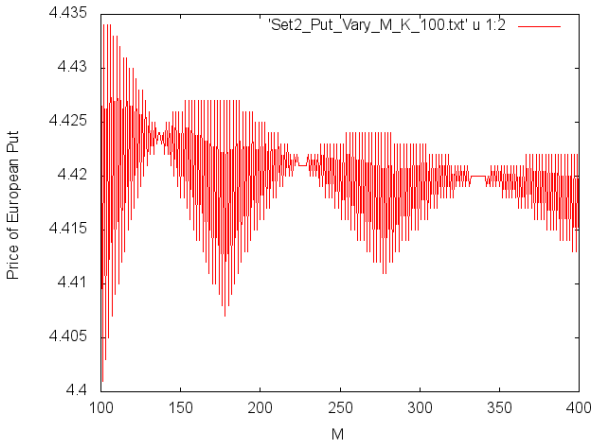
(e) Varying M (K = 95) :



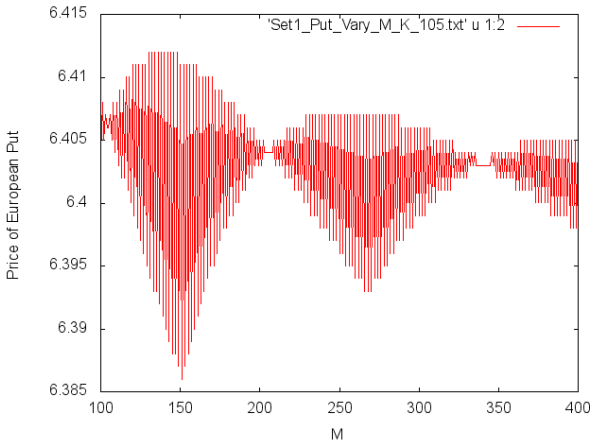


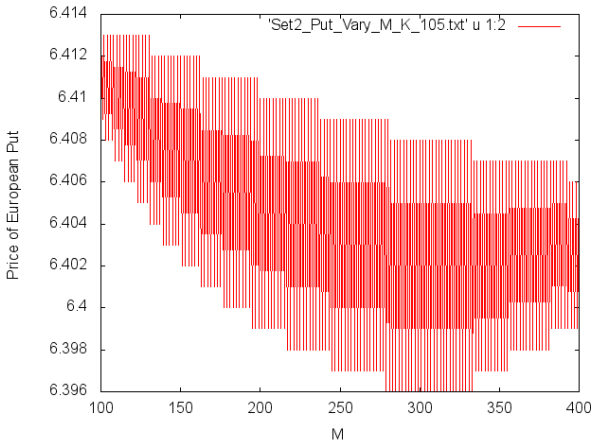
(f) Varying M (K = 100) :





(g) Varying M (K = 105) :





Observation and Explanation

(1) With increase in S(0), the price of European Call Option increases while price of put decreases. Analyzing with respect to the person long the call , increase in S(0) leads to increase in the market value of stock at maturity hence an increase in the potential payoff from the call option . This leads to an increase in the value/price of the European Call Option. An analogous argument can be given for put option as well.

(2) With increase in K ,the price of European Call Option decreases while price of put increases. Analyzing

with respect to the person long the call, increase in K leads to lesser payoff, hence the price of call decreases. An analogous argument can be given for put option as well.

(3) With increase in r, the price of European call option increases while price of put option decreases. Again analyzing with respect to the person long the call ,let's assume the person has S(0) at time t = 0, then the value of S(0) due to risk free rate will be more at maturity now, hence his/her gain is likely to increase, so the price of European Call Option should increase . An analogous argument can be given for put option as well.

(4) Increase in σ has a similar effect as increase in S(0). Increase in M is an oscillatory function of the prices of both the call and put option.

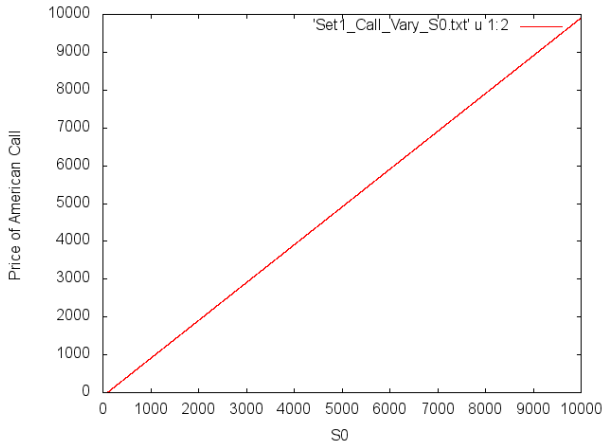
PROBLEM 2

Now take any path-dependent derivative of your choice and do the above exercise for at least one set (of u , d).

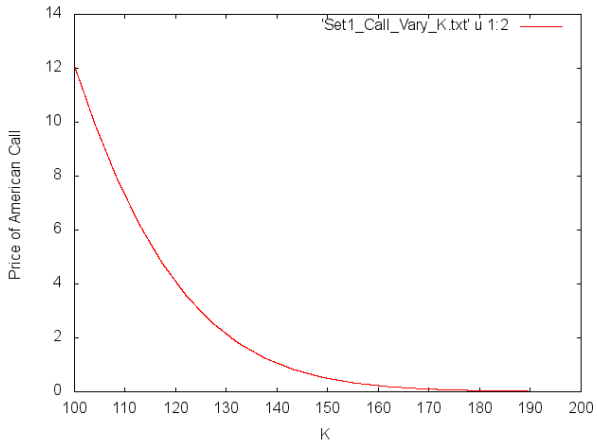
SOLUTION

American Call Option

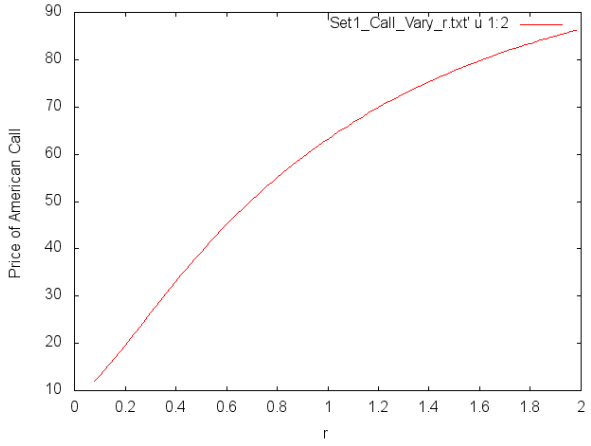
(a) Varying S(0) :



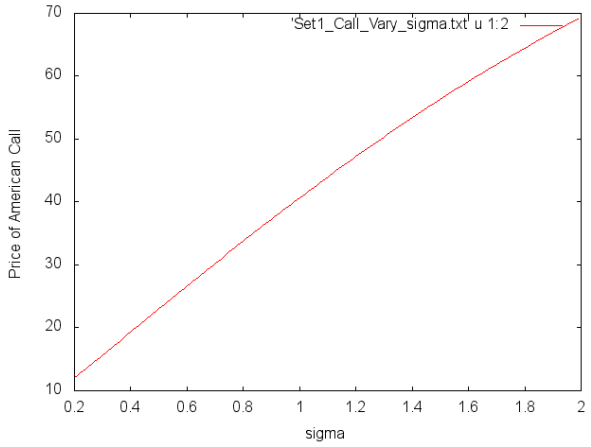
(b) Varying K :



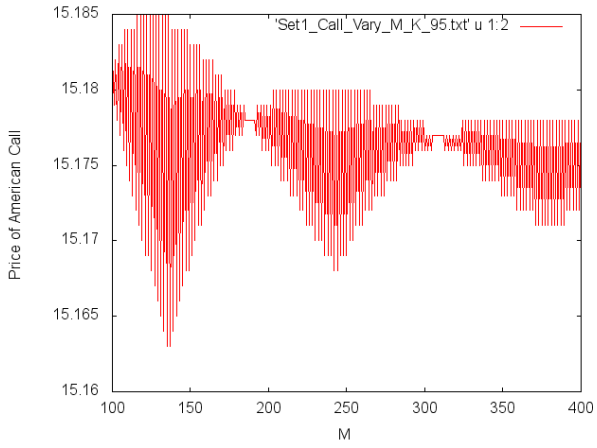
(c) Varying r :



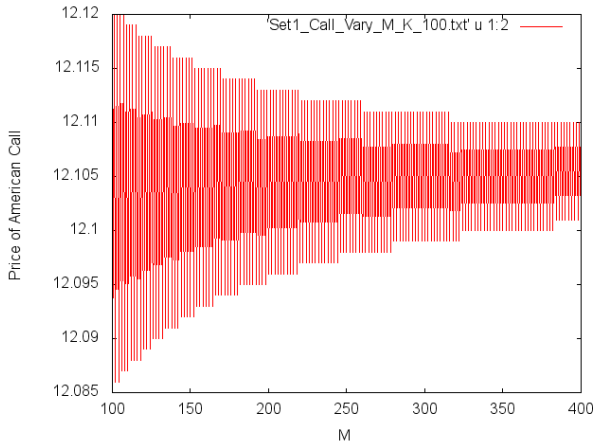
(d) Varying σ :



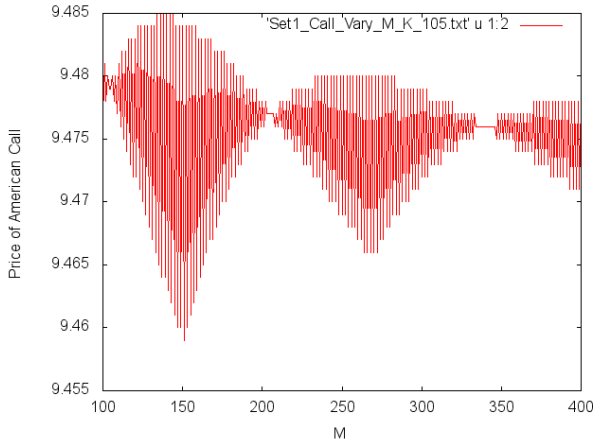
(e) Varying M (K = 95) :



(f) Varying M (K = 100) :

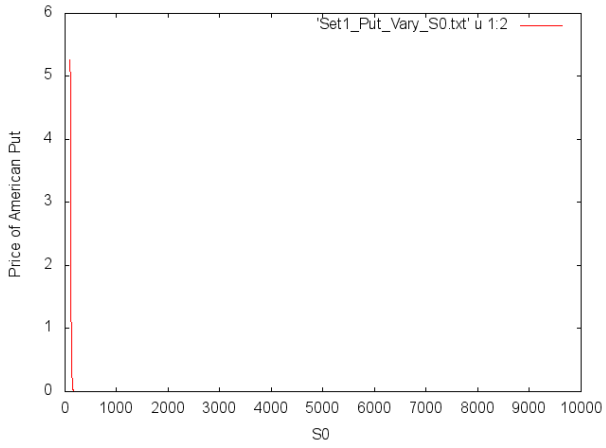


(g) Varying M (K = 105) :

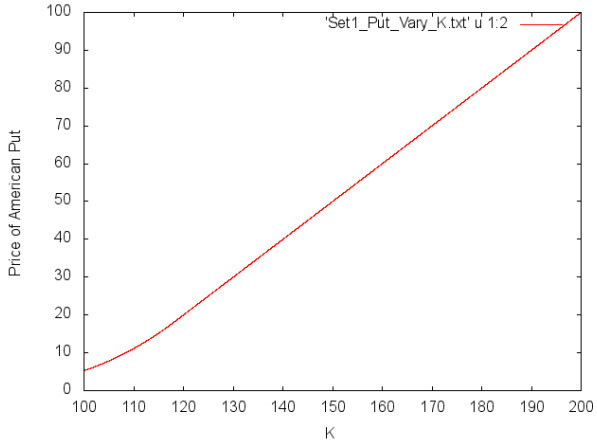


American Put Option

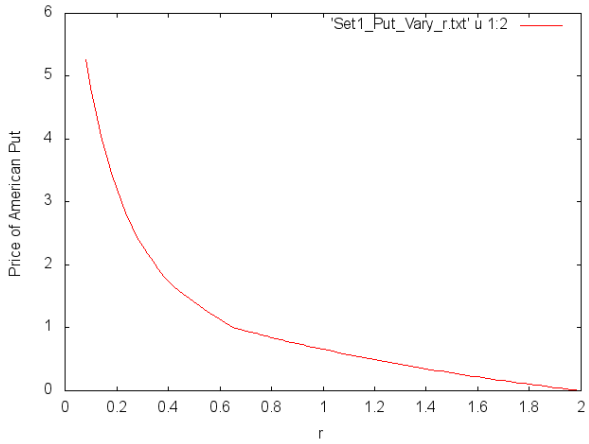
(a) Varying S(0) :



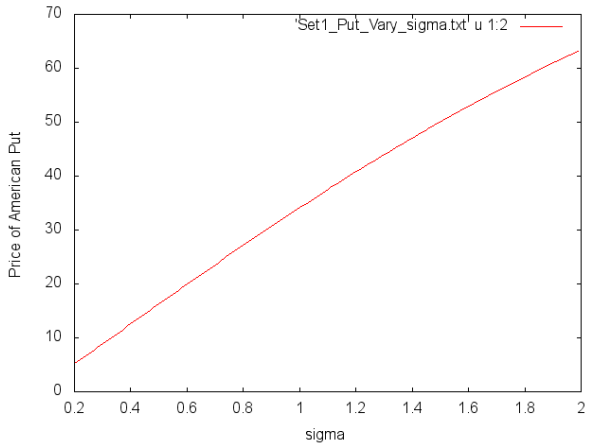
(b) Varying K :



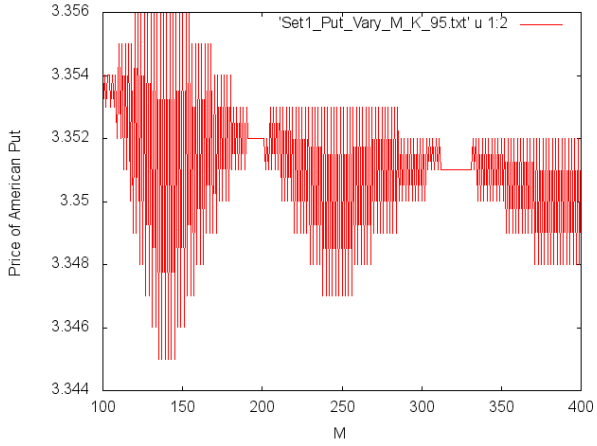
(c) Varying r :



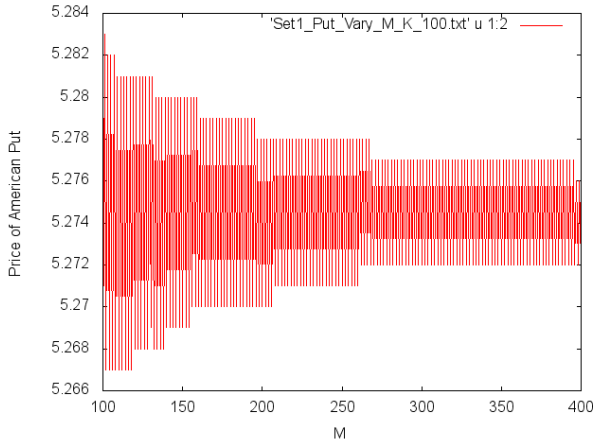
(d) Varying σ :



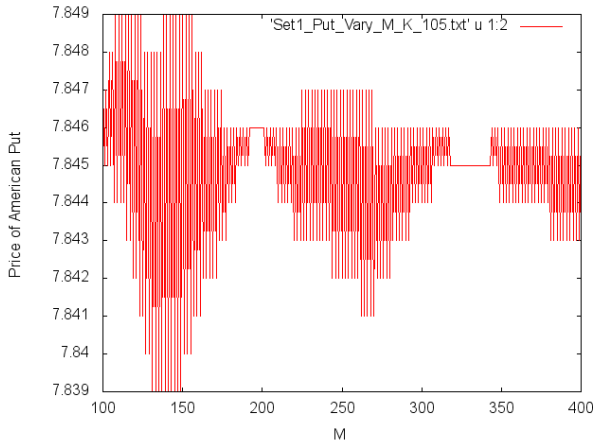
(e) Varying M (K = 95) :



(f) Varying M (K = 100) :



(g) Varying M (K = 105) :



Observation and Explanation

(1) The path dependent option taken is the American option which can be accessed at any instant of time. The value of the option is the maximum out of the risk-neutral value and the intrinsic value. The set chosen for u and d is Set - 1.

(2) The variation is similar to that of European Call and Put option, and the reason behind the trends are also similar. The only difference here is the access time which is not fixed.