MA 374: FE-Assignment #07

Due on Monday, March 14, 2016

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PROBLEM 1

Assume $T = 1, K = 1, r = 0.05, \sigma = 0.6$. Plot, in a single graph, $C(t, s)$ as a function of s alone for $t = 0, 0.2, 0.4, 0.6, 0.8, 1$. Do a similar plot for $P(t, s)$ as a function of s. Now, show the same information in a 3-dimensional form, i.e., as a function both t and s.

SOLUTION

Under the BSM framework the price of European put and call options as a function of the time and the stock price is given by :

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right]$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

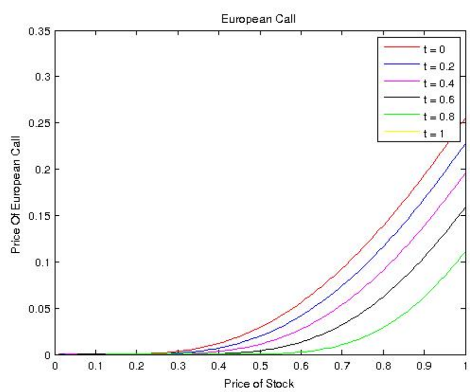
The price of a corresponding put option based on put–call parity is:

$$P(S,t) = Ke^{-r(T-t)} - S + C(S,t)$$

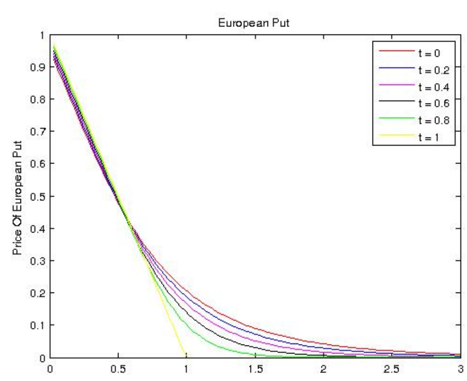
$$= N(-d_2)Ke^{-r(T-t)} - N(-d_1)S$$

In the above expression the respective variables represent :
(1) N(x) is the cumulative distribution function of the standard normal distribution
(2) T - t is the time to maturity
(3) S is the spot price of the underlying asset
(4) K is the strike price
(5) r is the risk free rate (annual rate, expressed in terms of continuous compounding)
(6) σ is the volatility of returns of the underlying asset

(1) European Call

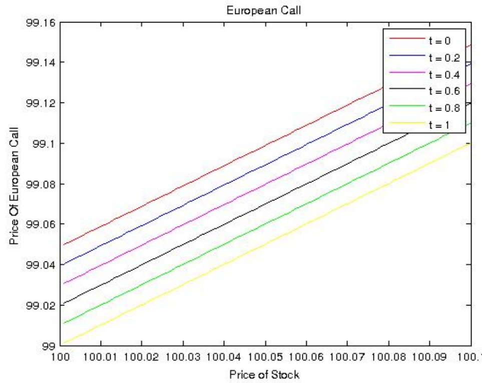


(2) European Put

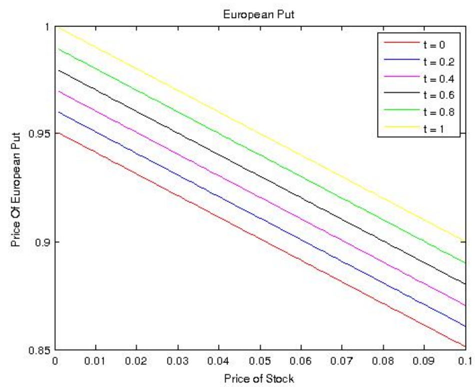


When the values for the spot price of the stock is higher, the curve almost becomes a straight line as observed in the following plots.

(1) European Call



(2) European Put



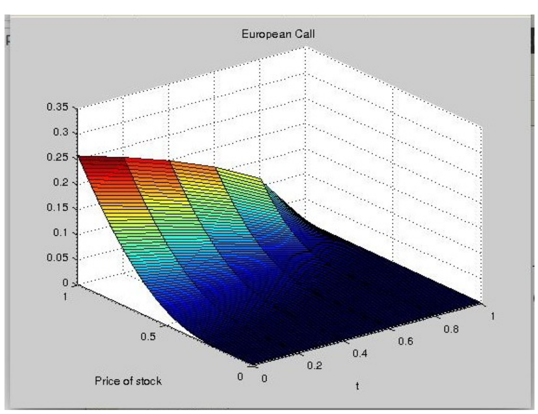
PROBLEM 2

Plot $C(t, s)$ and $P(t, s)$ as a smooth surface above the (t, s)-plane.

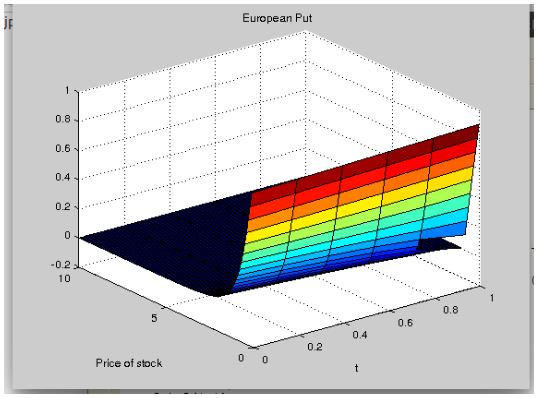
SOLUTION

The following plots were obtained :

(1) 3D - European Call



(2) 3D - European Put



PROBLEM 3

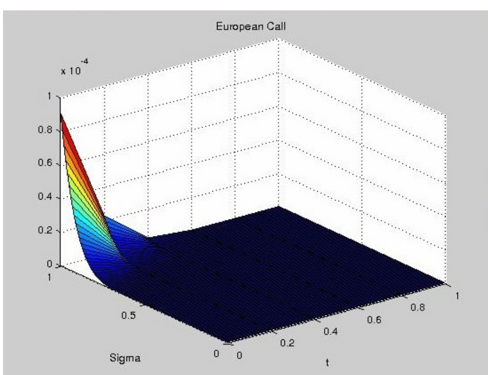
Study the sensitivity of both the functions C and P as a function of model parameters. If required, you may assume different parameter values as opposed to the one given above. Present your results in the form of tables and graphs (both in two and three dimensional).

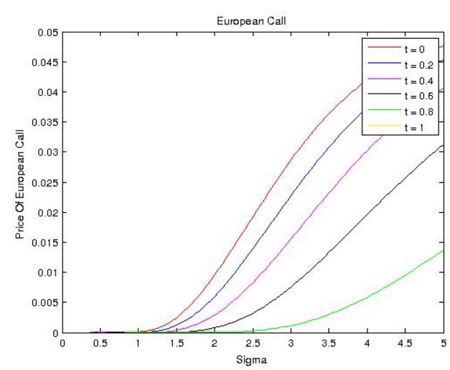
SOLUTION

The model parameters are σ , K and r. In order to do sensitivity analysis on all of them , the spot price was fixed to 0.05 and the individual parameters were varied with respect to the values of time given in the problem.The following plots were obtained by varying each parameter with time for both call and put options.

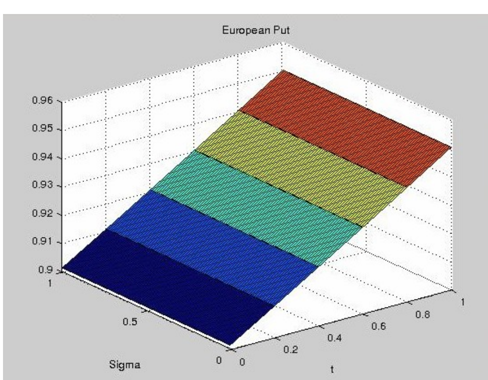
(1) Varying σ

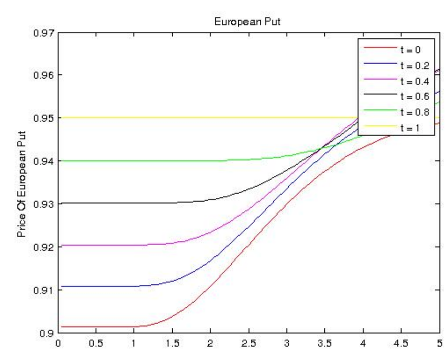
European Call Option :





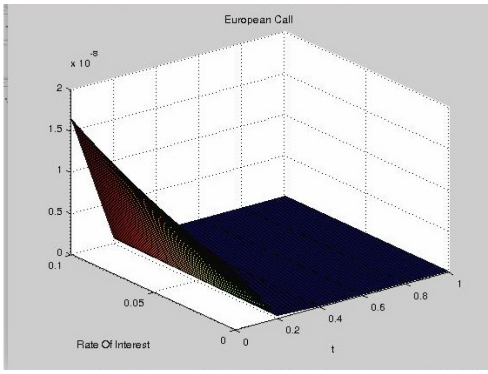
European Put Option :

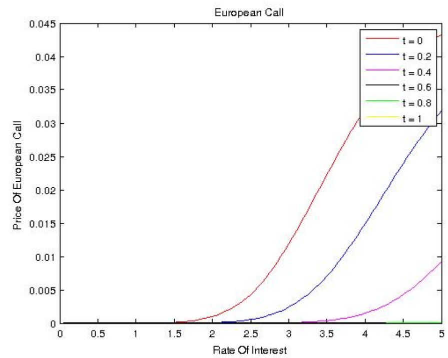




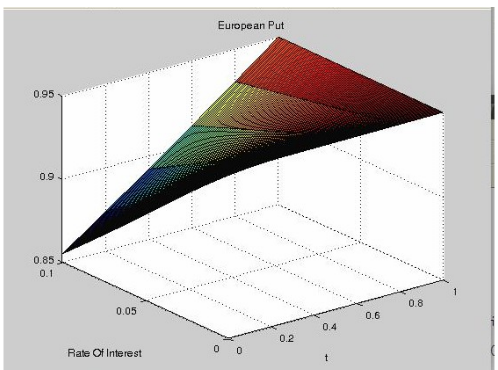
(2) Varying r

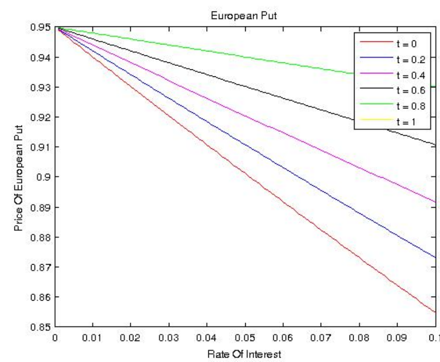
European Call Option :





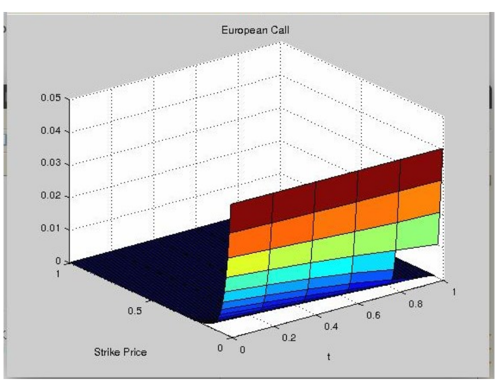
European Put Option :

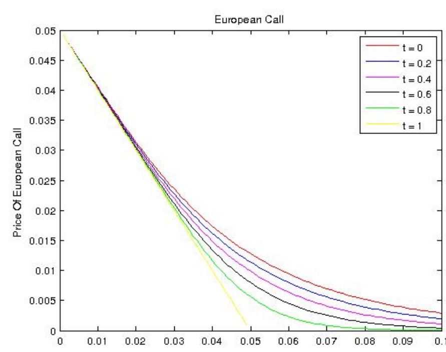




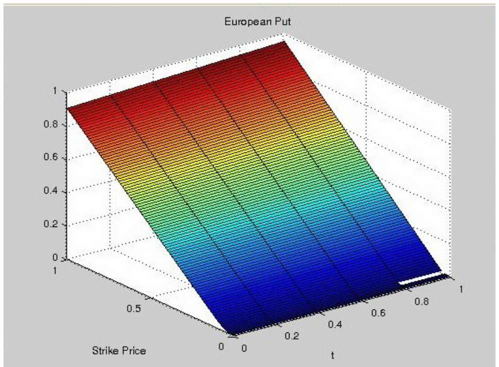
(3) Varying K

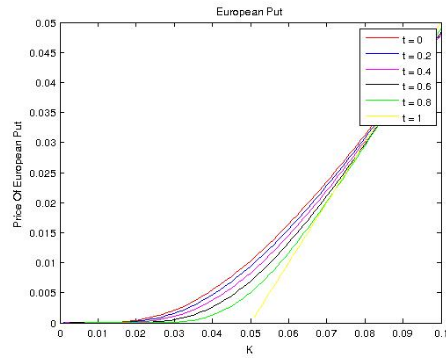
European Call Option :





European Put Option :





Observations

(1) The price Of European Call option increases with the increase in the spot price of the stock. With increase in the stock price, the holder of the call option (long the call) will have more profit, as the holder can buy the stock at a comparatively lesser price than the the actual market price of the stock by exercising the option. The price of the European Put option decreases with the increase in the spot price because of analogous reason.

(2) The prices of both European Call and put options increase with increase in the volatility of the stock

(3) The price of European Call option increases with the increase in the risk free rate because with increase in r, the spot price increases as the stock price process is a GBM and increase with μ (r).Analogous reason for decrease in the price of European Put with increase in r.

(4) The price of European Call option decreases with increase in K because with increase in K, the holder of call (long the call) will have less profit and analogously for European Put option.