MA 374: FE-Assignment #03

Due on Monday, February 8, 2016

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PROBLEM 1

Write a program to determine the initial price of an American call and American put option in the binomial model with the following data :

$$S(0) = 100 \quad K = 100 \quad T = 1 \quad M = 100 \quad r = 8\% \quad \sigma = 20\%$$

Use the following two sets of u and d for your program.

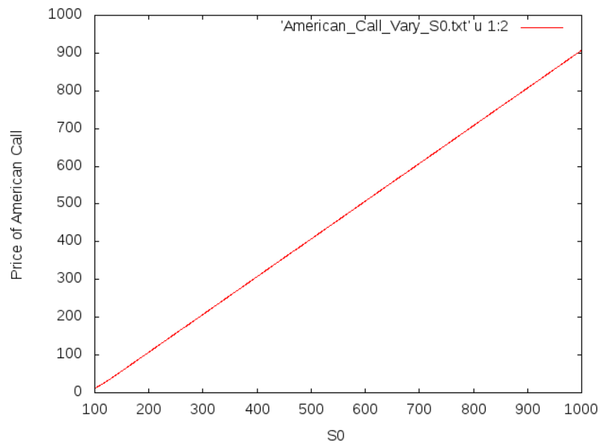
$$u = e^{\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t} \quad d = e^{-\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$$

Here $\Delta t = \frac{T}{M}$ with M being the number of subintervals in the time interval [0 , T]. Use the continuous compounding convention in your calculations (i.e., both in p̃ and in the pricing formula).
Now, plot the initial prices of both call and put options (for both the above sets of u and d) by varying one of the parameters at a time (as given below) while keeping the other parameters fixed (as given above) :
(a) S(0).
(b) K.
(c) r.
(d) σ
(e) M (Do this for three values of K, K = 95 , 100 , 105).

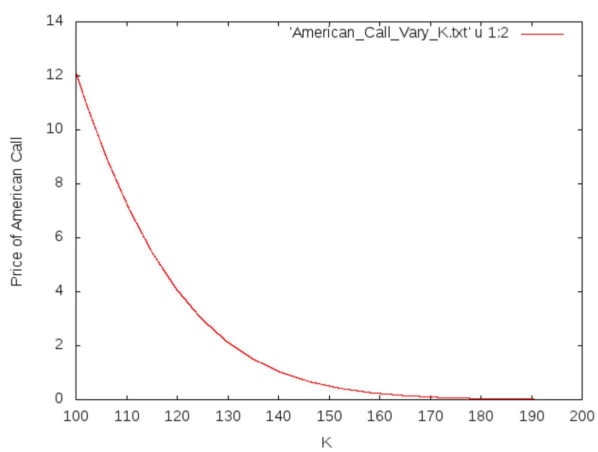
SOLUTION

American Call Option

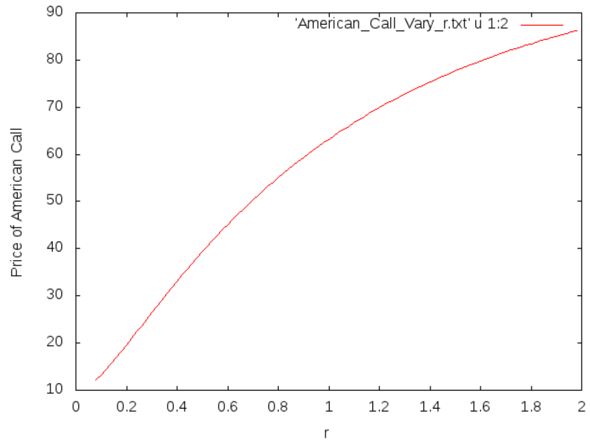
(a) Varying S(0) :



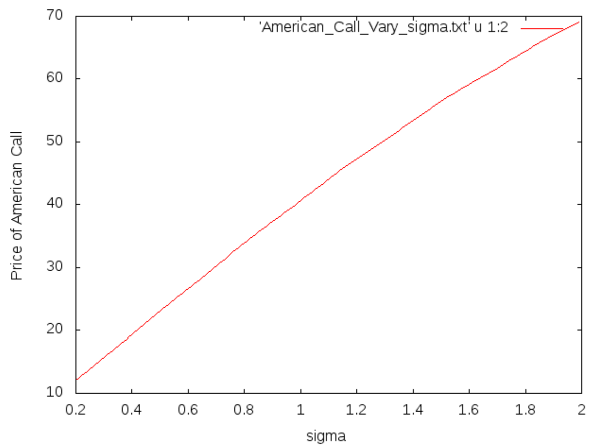
(b) Varying K :



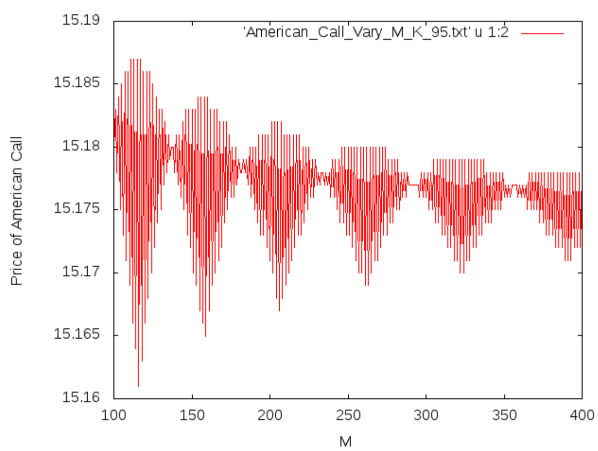
(c) Varying r :



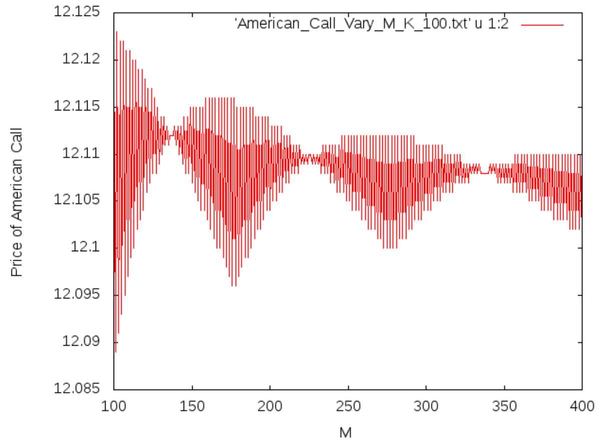
(d) Varying σ :



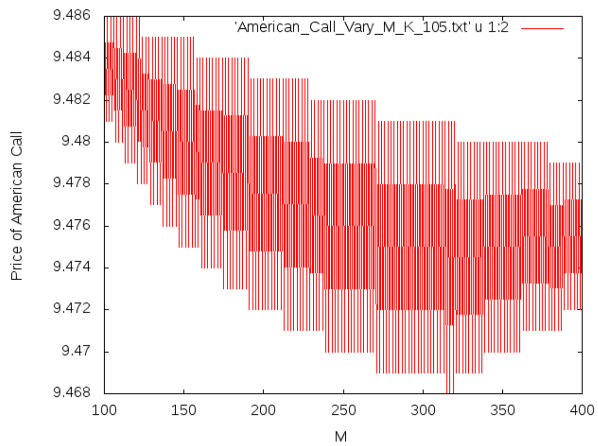
(e) Varying M (K = 95) :



(f) Varying M (K = 100) :

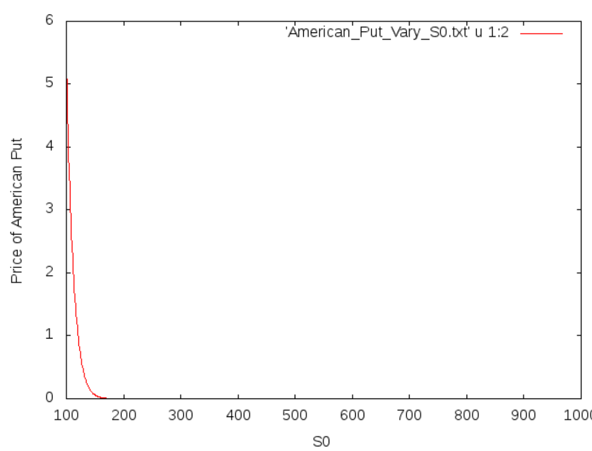


(g) Varying M (K = 105) :

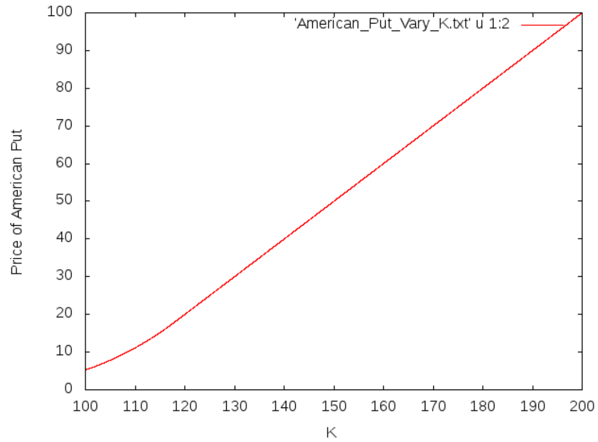


American Put Option

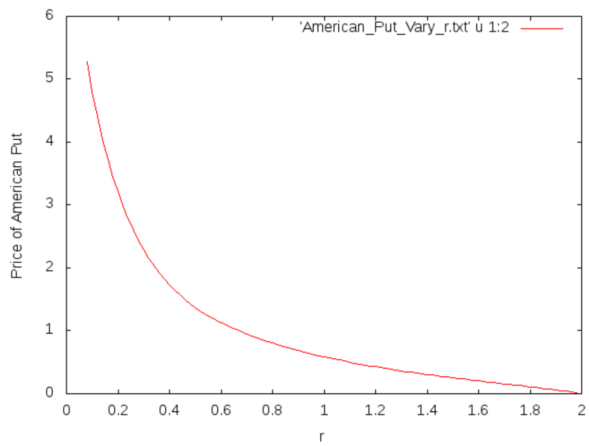
(a) Varying S(0) :



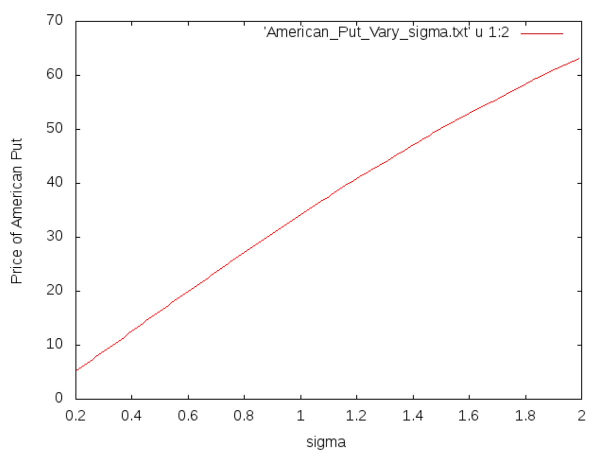
(b) Varying K :



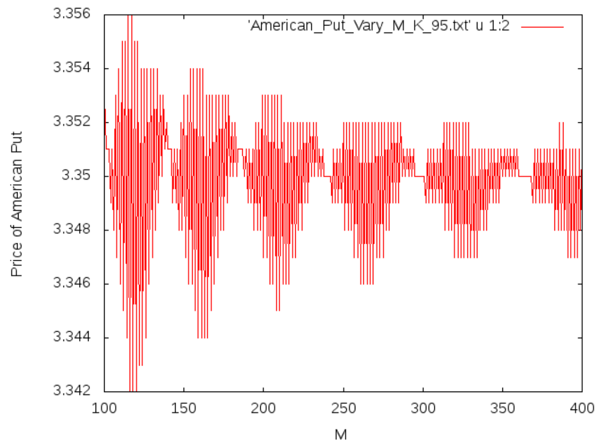
(c) Varying r :



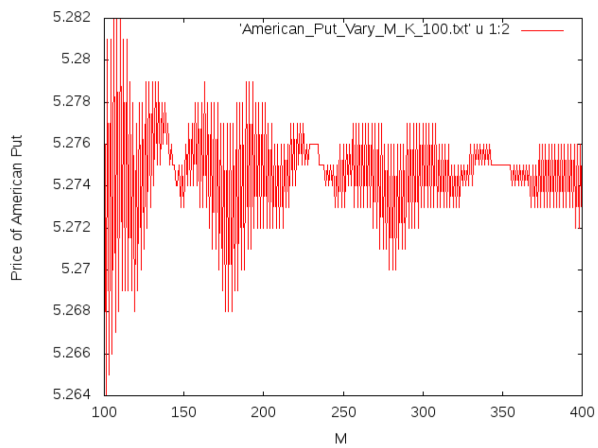
(d) Varying σ :



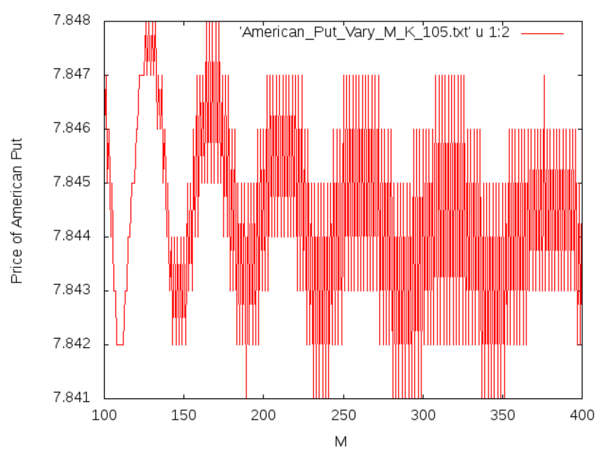
(e) Varying M (K = 95) :



(f) Varying M (K = 100) :



(g) Varying M (K = 105) :



Observation and Explanation

(1) With increase in S(0), the price of American Call Option increases while price of put decreases. Analyzing with respect to the person long the call , increase in S(0) leads to increase in the market value of stock at maturity hence an increase in the potential payoff from the call option . This leads to an increase in the value/price of the European Call Option. An analogous argument can be given for put option as well.

(2) With increase in K ,the price of American Call Option decreases while price of put increases. Analyzing with respect to the person long the call, increase in K leads to lesser payoff, hence the price of call decreases. An analogous argument can be given for put option as well.

(3) With increase in r, the price of American call option increases while price of put option decreases. Again analyzing with respect to the person long the call ,let's assume the person has S(0) at time t = 0, then the value of S(0) due to risk free rate will be more at maturity now, hence his/her gain is likely to increase, so the price of European Call Option should increase . An analogous argument can be given for put option as well.

(4) Increase in σ has a similar effect as increase in S(0). Increase in M is an oscillatory function of the prices of both the call and put option

PROBLEM 2

Write a program to determine the initial price of a lookback (European) option in the binomial model, using the basic binomial algorithm (used in earlier lab assignments), with the following data:

$$S(0) = 100 \quad T = 1 \quad r = 8\% \quad \sigma = 20\%$$

The payoff of the lookback option is given by

$$V = max \quad S(i) - S(M)$$

where S(i) = S(iΔt) with $\Delta t = \frac{T}{M}$ (M being the number of subintervals of the time interval [0, T]). Use the continuous compounding convention in your calculations (i.e., both in p̃ and in the pricing formula).
Use the following two sets of u and d for your program.

$$u = e^{\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t} \quad d = e^{-\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$$

(a) Obtain the initial price of the option for M = 5, 10, 25, 50.
(b) How do the values of options at time t = 0 compare for the above values of M that you have taken ?
(c) Tabulate the values of the options at all intermediate time points for M = 5.

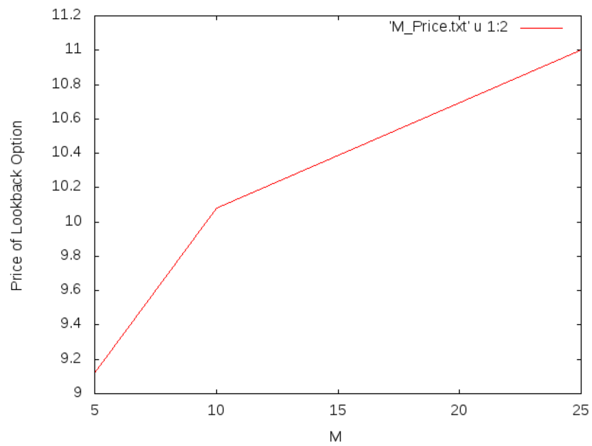
SOLUTION

Part (a)

M	Price of Option
5	9.12104
10	10.0815
25	11.0038
50	Computation not possible

The above table gives the prices of the lookback option obtained by the naive exponential time algorithm . Since 2^{50} is a very large number, the computation was not possible for M = 50.

Part (b)



The trend is generally increasing with increase in the value of M.

Part (c)

Time	Value of Option
0.8	25.052, 10.682, 10.682, 3.847, 13.072, 3.847, 8.005, 4.601, 21.189, 6.682, 8.005, 4.601, 15.633, 4.601, 9.573, 5.502
0.6	17.584, 7.150, 8.326, 6.203, 13.715, 6.203, 9.957, 7.418
0.4	12.171, 7.149, 9.801, 8.550
0.2	9.507, 9.030
0.0	9.121