Chapt 4

$$\boxed{A\vec{v} = \lambda\vec{v}}$$

v can be normalized

$$\boxed{\hat{v} = \frac{\vec{v}}{\sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}}}$$

characteristic polynomial of A

$$|A - \lambda I|$$

characteristic equation of A

$$|A - \lambda I| = 0$$

Diagonalization theorem

$$A = V \Lambda V^{-1}$$

diagonalizable: has n linearly independent eigenvectors

linear independence

$a_1 v_1 + a_2 v_2 = 0$ if $a_1 \wedge a_2 \neq 0$

↳ linearly dependent

ex $\begin{pmatrix}2\\-4\end{pmatrix}$ & $\begin{pmatrix}-1\\2\end{pmatrix}$ are ↰ because

$$-\begin{bmatrix}2\\-4\end{bmatrix} + 2\begin{bmatrix}-2\\2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$a_1 v_1 + a_2 v_2 = 0$

iff $a_1 = a_2 = 0$

↳ linearly independent

$$a\begin{bmatrix}1\\2\end{bmatrix} + b\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

if & only if $\boxed{a = b = 0}$

1) $Au = -u$ $\therefore u =$ an eigenvector of A corresponding to $\lambda = -1$

$Av \neq v$ $\therefore v \neq$ an ——— " ———

$Aw = w$ $\therefore w =$ ——— " ——— " ——— $\lambda = 1$

2) $$Av = \begin{bmatrix} 0 & -2 & -5 \\ 1 & 3 & 1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -7 \\ 5 \\ -1 \end{bmatrix}$$ v is not an evector of A

$$Aw = \begin{bmatrix} 0 & -2 & -5 \\ 1 & 3 & 1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$ w is an evector of A corresponding to $\lambda = 2$

3) a) $$cp = \left| \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = \left| \begin{bmatrix} 4-\lambda & 3 \\ 2 & 5-\lambda \end{bmatrix} \right| = (4-\lambda)(5-\lambda) - 6$$
$$= \lambda^2 - 9\lambda + 14$$

λ: $\lambda^2 - 9\lambda + 14 = 0$

$(\lambda - 7)(\lambda - 2) = 0$

$\lambda_1 = 7 \lor \lambda_2 = 2$

$\boxed{(A - \lambda I)v = 0}$ v_1: $$\begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} -3 & 3 & 0 \\ 2 & -2 & 0 \end{array}\right] \xrightarrow{2R_1 - 3R_2} \left[\begin{array}{cc|c} -3 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$-3v_1 + 3v_2 = 0$

$v_1 = v_2$

$v_1 = t$

Let $v_2 = t$

$$\vec{v}_1 = \begin{bmatrix} t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$ where $t \neq 0$

$\lambda_2 = 2$

$$\begin{bmatrix} 4-2 & 3 \\ 2 & 5-2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 2 & 3 & 0 \\ 2 & 3 & 0 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{cc|c} 2 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

let $v_2 = t$

$2v_1 + 3v_2 = 0$

$v_1 = -\frac{3}{2} t$

$$\vec{v_2} = \begin{bmatrix} -\frac{3}{2} t \\ t \end{bmatrix}$$

$$\vec{v_2} = \frac{t}{2} \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

$$\vec{v_2} = -\frac{t}{2} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\vec{v_2} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

1) c. $\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 9 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 3 & 4 & 2 & 0 \\ 5 & 9 & 1 & 0 \end{array}\right] \xrightarrow[5R_1 - R_2]{3R_1 - R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 0 & 5 & -5 & 0 \\ 0 & 6 & -6 & 0 \end{array}\right] \xrightarrow{6R_1 - 5R_3} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 0 & 5 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$5y - 5z = 0$

$y - z = 0$

independent

4) a. $\left[\begin{array}{c|c} 4 & 6 \\ 2 & \end{array}\right]$

$$= 3\begin{bmatrix} 4 \\ 2 \end{bmatrix} - 2\begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 12 \\ 6 \end{bmatrix} - \begin{bmatrix} 12 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

dependent

b. $\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

independent

4d. $\left[\begin{array}{ccc|c} 3 & 9 & 0 & 0 \\ 2 & 6 & 1 & 0 \\ 1 & 4 & 1 & 0 \end{array}\right]$

independent: $|A| \neq 0$
dependent: $|A| = 0$

$$\begin{vmatrix} 3 & 9 & 0 \\ 2 & 6 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 3\begin{vmatrix} 6 & 1 \\ 4 & 1 \end{vmatrix} - 9\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= 3 \cdot 2 - 9 \cdot 1$$

$$= 6 - 9$$

$$= -3$$

independent

$$\begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix} = 12 - 12 = 0$$

dependent

7) $A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$ $\lambda = 1$, $\lambda = 5$ diagonalize A.

1. find v_1, v_2, v_3

$$\lambda = 1 \left[\begin{array}{ccc|c} 2-1 & 2 & -1 & 0 \\ 1 & 3-1 & -1 & 0 \\ -1 & -2 & 2-1 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 1 & 2 & -1 & 0 \\ -1 & -2 & 1 & 0 \end{array}\right] \xrightarrow[R_1+R_3]{R_1-R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

let $v_2 = s$ $v_3 = t$

$v_1 + 2v_2 - v_3 = 0$

$v_1 + 2s - t = 0$

$v_1 = t - 2s$

$$\vec{v} = \begin{bmatrix} t-2s \\ s \\ t \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} t \\ 0 \\ t \end{bmatrix} + \begin{bmatrix} -2s \\ s \\ 0 \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} // \quad \vec{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} //$$

$$\lambda = 5 \left[\begin{array}{ccc|c} -2 & 2 & -1 & 0 \\ 1 & -2 & -1 & 0 \\ -1 & -2 & -3 & 0 \end{array}\right] \xrightarrow[R_1-3R_3]{R_1+3R_2}$$

$$\left[\begin{array}{ccc|c} -3 & 2 & -1 & 0 \\ 0 & -4 & -4 & 0 \\ 0 & 8 & 8 & 0 \end{array}\right] \xrightarrow{2R_2+R_3}$$

$$\left[\begin{array}{ccc|c} -3 & 2 & -1 & 0 \\ 0 & -4 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{ let } v_3 = t$$

$v_2 + v_3 = 0$ $\quad -3v_1 = v_3 - 2v_2$

$v_2 = -t$ $\quad -3v_1 = t + 2 \cdot t$

$v_1 = -t$

$$\vec{v}_2 = \begin{bmatrix} -t \\ -t \\ t \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} //$$

7). $A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$ $\lambda = 1$ $\lambda = 5$ diagonalize A.

$$V = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \quad \Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}^{-1}$$