Appendix 2: Graphs of Trigonometric Functions

A2.1 The Radian Measure

Besides degrees, the **radian** is also a common unit of measurement for angles.

In *Figure A2.1*, the radius of the circle is r as shown.

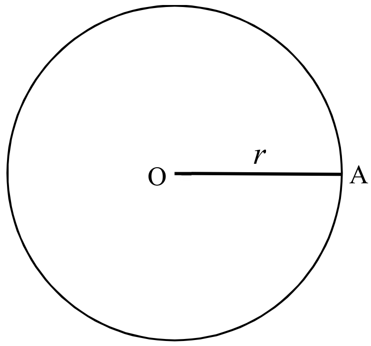


Figure A2.1

Take the radius r and lay it out on the circumference (*Figure A2.2*).

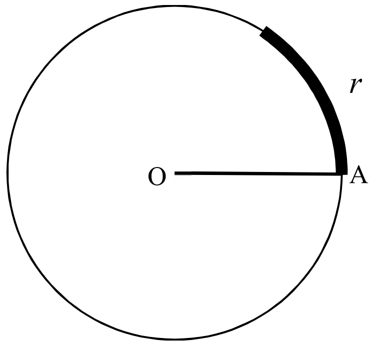


Figure A2.2

Join point O to point B.
The angle $\measuredangle AOB$ in the centre of the circle is **1 radian** (*Figure A2.3*).

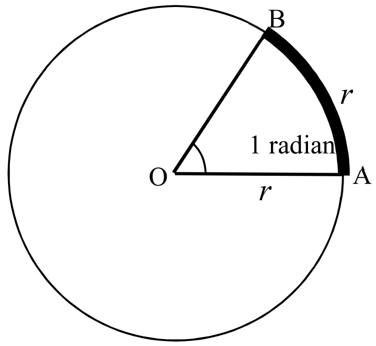


Figure A2.3

In *Figure A2.4* , the angle $\measuredangle AOC$ in the centre of the circle is **2 radians**.

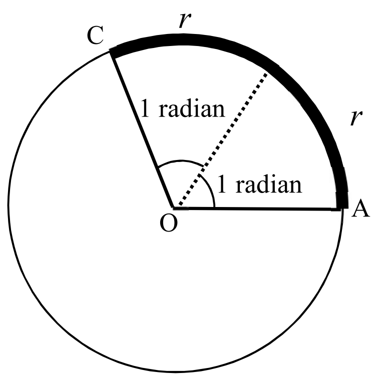
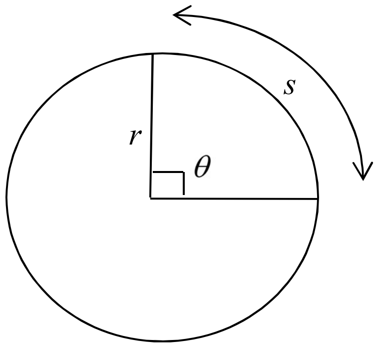


Figure A2.4

Definition of a Radian

One radian is defined as the angle made at the centre of a circle by an arc whose length is equal to the radius of the circle.

Let's now measure a right angle in radians:



$$\theta = \frac{s}{r} = \frac{\frac{1}{4}(circumference)}{r} = \frac{\frac{1}{4}(2\pi r)}{r} = \frac{\pi}{2}$$

In radian measure, a right angle is $\frac{\pi}{2}$ radians. $(\frac{\pi}{2} = 1.57079...)$

From the fact $90^\circ = \frac{\pi}{2}$ radians, it follows directly that:

$180^\circ = \pi$ radians and $360^\circ = 2\pi$ radians.

A2.2 Graphs of the three Basic Trigonometric Functions

The two basic trigonometric functions are $y = \sin x$ and $y = \cos x$.
In this section we will look at their graphs.

Graph of $y = \sin x$

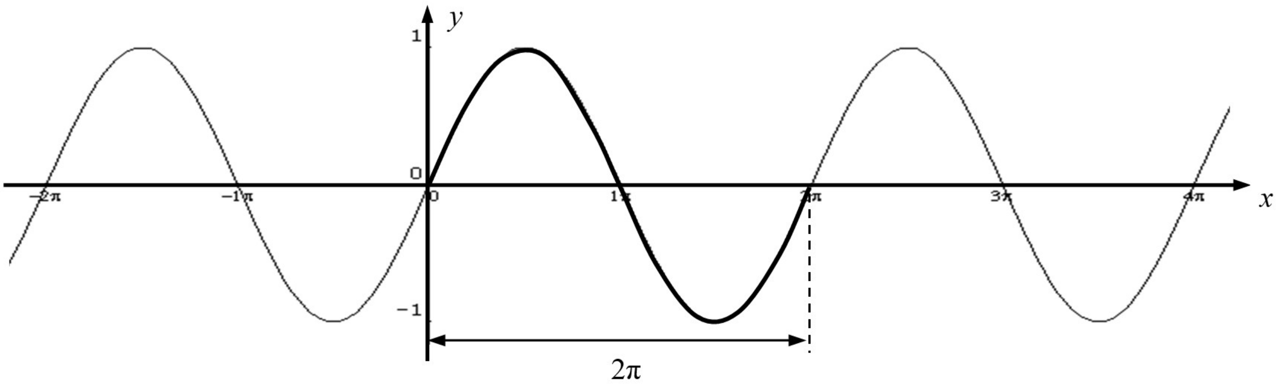


Figure A2.5

Characteristics of the curve of $y = \sin x$:

1. It is a regular wave oscillating between $y = 1$ and $y = -1$. This means the sine curve has **amplitude** 1.

2. The wave repeats its basic pattern over every interval of 2π. In the graph above, it is highlighted in bold. This pattern forms one complete cycle of the sine wave. The horizontal interval covered by one cycle is called the **period**. The sine curve $y = \sin x$ in *Figure A2.5* is periodic with a period of 2π.

Graph of $y = \cos x$

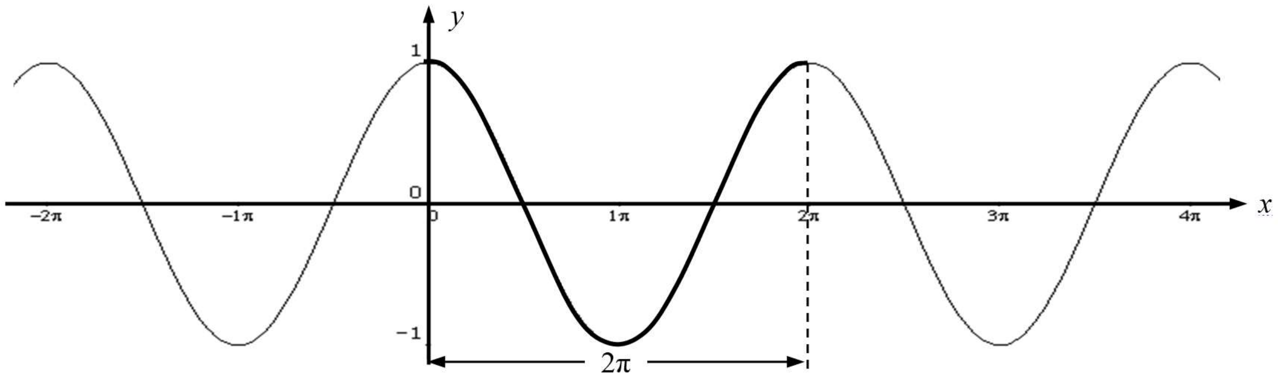


Figure A2.6

Characteristics of the curve of $y = \cos x$:

1. It closely resembles the sine curve, except for a horizontal translation of $\frac{\pi}{2}$ to the left.
2. It is a regular wave oscillating between $y = 1$ and $y = -1$, and has an amplitude of 1.
3. The cosine function is also periodic with a period of 2π (the highlighted bold wave in *Figure A2.6*).

Graph of $y = \tan x$

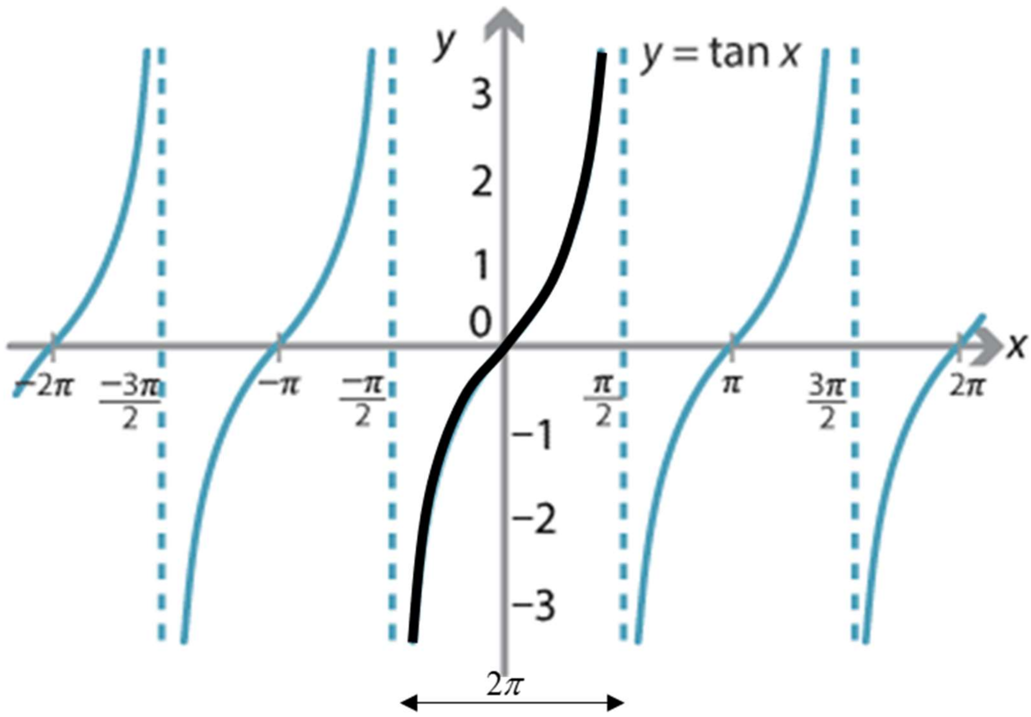


Figure A2.7

Note that $\tan x = \frac{\sin x}{\cos x}$, so $\tan x$ is not defined when $\cos x = 0$, that is when $x = \frac{\pi}{2}, \frac{3\pi}{2}$, etc.

Therefore, $\tan\frac{\pi}{2}$, $\tan\frac{3\pi}{2}$, $\tan\left(-\frac{\pi}{2}\right)$, etc, are not defined.

A2.3 Characteristics of Sinusoidal Graphs

The wavelike curves of $y = \sin x$ and $y = \cos x$ are often referred to as sinusoidal waves.
In the previous section, the basic characteristics of these waves were introduced. In this section, the general forms of the sinusoidal waves given by the equations $y = a\sin(bx)$ and $y = a\cos(bx)$, where *a*, and *b* are positive constants, will be discussed.

Amplitude

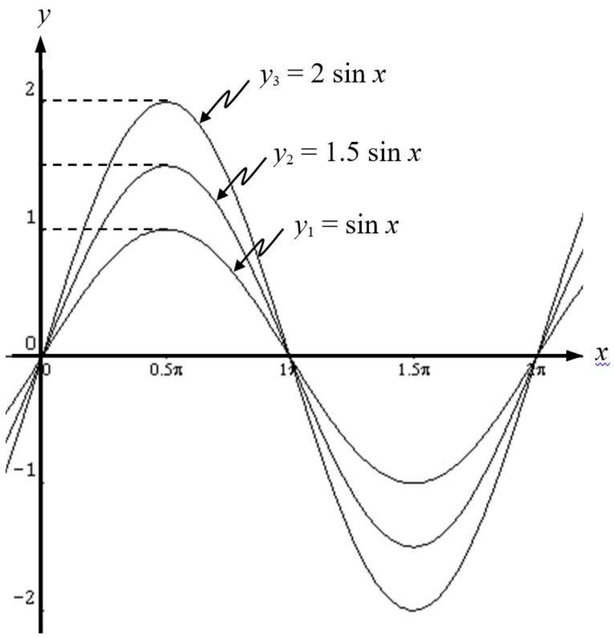


Figure A2.8

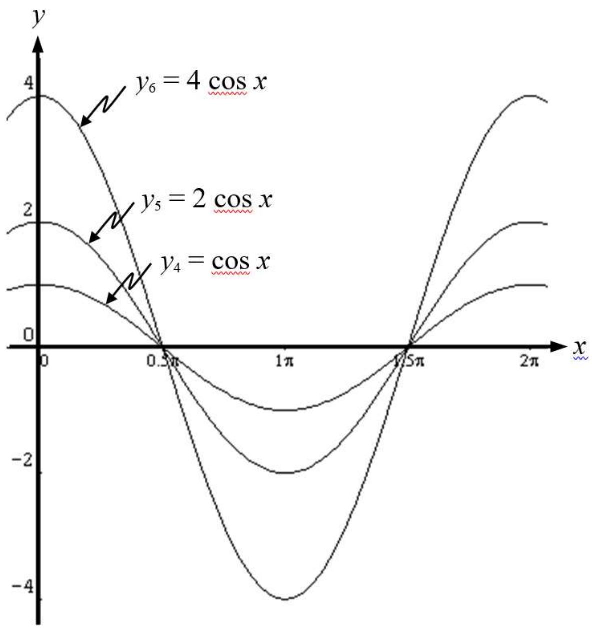


Figure A2.9

Three graphs of $y_1 = \sin x$, $y_2 = 1.5\sin x$ and $y_3 = 2\sin x$ are superimposed in *Figure A2.8*.
Similarly, the three graphs of $y_4 = \cos x$, $y_5 = 2\cos x$ and $y_6 = 4\cos x$ in *Figure A2.9*.
It is obvious that the constant in front of the "sin" and "cos" affects the amplitude of the wave.

Hence if $\boldsymbol{y = a\sin x}$ or $\boldsymbol{y = a\cos x}$, then

$$\boxed{\text{amplitude of } y = |\boldsymbol{a}|}$$

Period

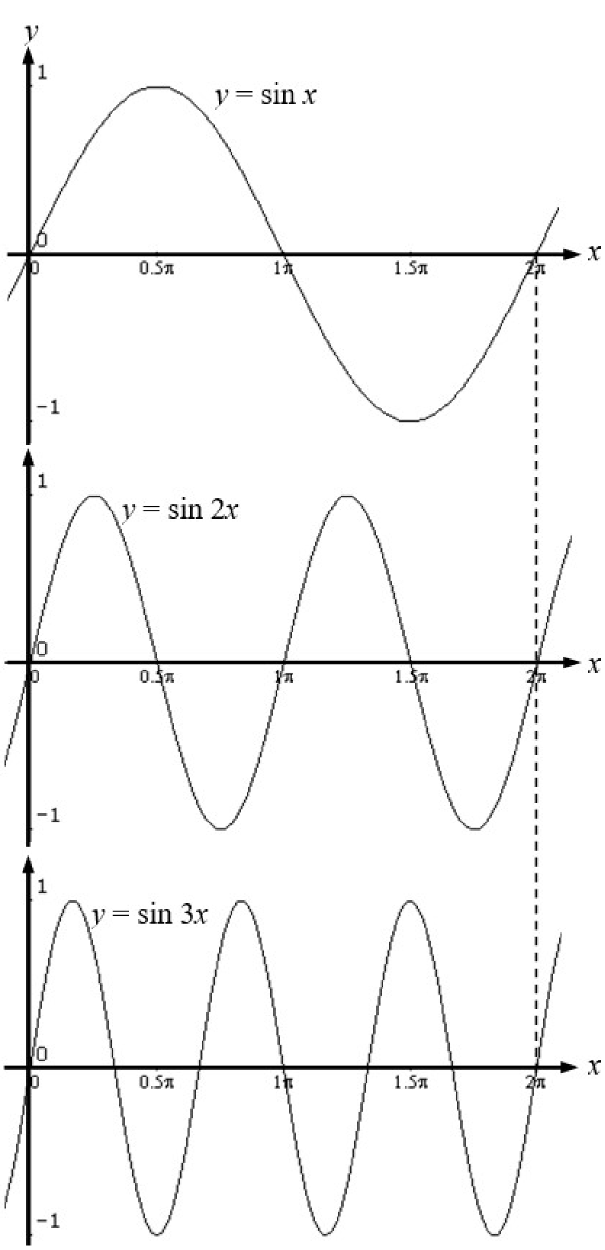


Figure A2.10

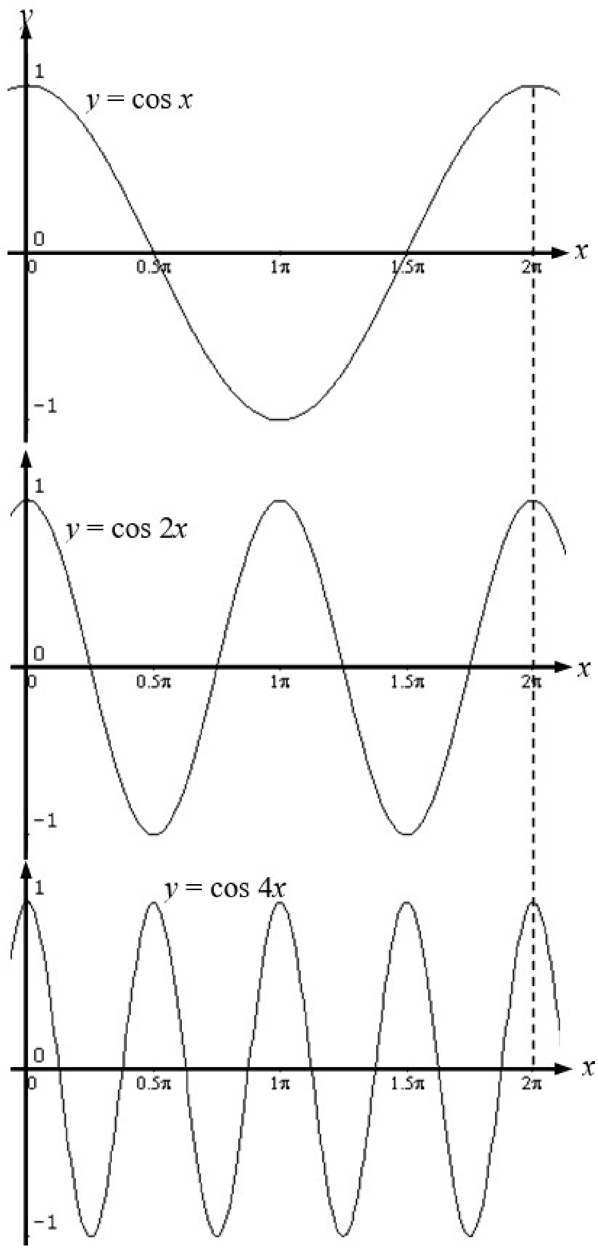


Figure A2.11

For $y = \sin x$ and $y = \cos x$, we see one cycle over an interval of 2π .
For $y = \sin 2x$ and $y = \cos 2x$, there are two cycles over 2π .

Hence if $\boldsymbol{y = \sin bx}$ or $\boldsymbol{y = \cos bx}$, then $\boldsymbol{b}$ is the number of cycles over an interval of 2π .

A2.4 Reciprocal Trigonometric Functions

We have looked at sine, cosine and tangent trigonometric functions. Each of them have its own reciprocal function.

Cosecant is the reciprocal of sine. Its abbreviation is **csc**:

$$\boxed{\csc x = \frac{1}{\sin x}}$$

Secant is the reciprocal of cosine. Its abbreviation is **sec**:

$$\boxed{\sec x = \frac{1}{\cos x}}$$

Cotangent is the reciprocal of tangent. Its abbreviation is **cot**:

$$\boxed{\cot x = \frac{1}{\tan x}}$$