Chapt 3

sample mean vector

$$\boxed{\bar{\underset{\sim}{x}} = \frac{1}{n} X' \mathbf{1}_n}$$

sample cov. matrix

$$\boxed{C = \frac{1}{n-1} (X - \mathbf{1}_n \bar{\underset{\sim}{x}}')' (X - \mathbf{1}_n \bar{\underset{\sim}{x}}')}$$

generalised sample variance

↳ $|C|$

total sample variance

↳ $tr(C)$

interpreting sample cov matrix

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

= variance of each variable

↳ explains variability

$var(X_1) = c_{11}$

$var(X_2) = c_{22}$

= covariance

↳ explains relationship between different variables

$cov(X_1, X_2) = c_{12} = c_{21}$

Correlation matrix

$$R = \begin{bmatrix} 1 & r_{12} & \cdots & r_{1p} \\ r_{12} & 1 & \cdots & r_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ r_{1p} & r_{2p} & \cdots & 1 \end{bmatrix} \quad \text{where} \quad \boxed{r_{ik} = \frac{C_{ik}}{\sqrt{C_{ii}}\sqrt{C_{kk}}}}$$

$$\boxed{R = S^{-1} C S^{-1}}$$

sample SD matrix

$$S = \begin{bmatrix} \sqrt{C_{11}} & 0 & \cdots & 0 \\ 0 & \sqrt{C_{22}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & \sqrt{C_{pp}} \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} \frac{1}{\sqrt{C_{11}}} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sqrt{C_{22}}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & \frac{1}{\sqrt{C_{pp}}} \end{bmatrix}$$

interpreting corr. matrix

$$R = \begin{bmatrix} 1 & r_{12} \\ r_{21} & 1 \end{bmatrix}$$

$$r_{12} = r_{21} = \mathrm{corr}(X_1, X_2)$$

$$\mathrm{corr}(X, Y) = \frac{\mathrm{cov}(X, Y)}{SD(X) \cdot SD(Y)}$$

1) a. I. $r_{12} = \frac{C_{12}}{\sqrt{C_{11}}\sqrt{C_{22}}} = \underline{\underline{0}}$

II. $r_{12} = \frac{C_{12}}{\sqrt{C_{11}}\sqrt{C_{22}}} = \frac{4}{\sqrt{5}\sqrt{5}} = \underline{\underline{0.8}}$

III $r_{12} = \frac{C_{12}}{\sqrt{C_{11}}\sqrt{C_{22}}} = \frac{-4}{\sqrt{5}\sqrt{5}} = \underline{\underline{-0.8}}$

b. I → B , II → A , III → C

c. I. $|C| = \begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix} = \underline{\underline{9}}$ II. $|C| = \begin{vmatrix} 5 & 4 \\ 4 & 5 \end{vmatrix} = 25 - 16 = \underline{\underline{9}}$

III. $|C| = \begin{vmatrix} 5 & -4 \\ -4 & 5 \end{vmatrix} = 25 - 16 = \underline{\underline{9}}$

2). a. $X = \begin{bmatrix} 9 & 1 \\ 5 & 3 \\ 1 & 2 \end{bmatrix}$

$\underset{\sim}{\bar{x}} = \frac{1}{n}(X^T 1_n)$

$\underset{\sim}{\bar{x}} = \frac{1}{3}\begin{bmatrix} 9 & 5 & 1 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\underset{\sim}{\bar{x}} = \frac{1}{3}\begin{bmatrix} 15 \\ 6 \end{bmatrix}$

$\underset{\sim}{\bar{x}} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

$X - 1_n \underset{\sim}{\bar{x}}^T \rightarrow \begin{bmatrix} 9 & 1 \\ 5 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\begin{bmatrix} 5 & 2 \end{bmatrix}$

$\begin{bmatrix} 9 & 1 \\ 5 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 5 & 2 \\ 5 & 2 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 1 \\ -4 & 0 \end{bmatrix}$

$C = \frac{1}{n-1}(X - 1_n \underset{\sim}{\bar{x}}')'(X - 1_n \underset{\sim}{\bar{x}}')$

$= \frac{1}{2}\begin{bmatrix} 4 & -1 \\ 0 & 1 \\ -4 & 0 \end{bmatrix}'\begin{bmatrix} 4 & -1 \\ 0 & 1 \\ -4 & 0 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 4 & 0 & -4 \\ -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 0 & 1 \\ -4 & 0 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 32 & -4 \\ -4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 16 & -2 \\ -2 & 1 \end{bmatrix}$

contd...