Fast Vacuum Fluctuations and the Emergence of Quantum Mechanics

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Abstract

Fast moving classical variables can generate quantum mechanical behavior. We demonstrate how this can happen in a model. The key point is that in classically (ontologically) evolving systems one can still define a conserved quantum energy. For the fast variables, the energy levels are far separated, such that one may assume these variables to stay in their ground state. This forces them to be entangled, so that, consequently, the slow variables are entangled as well. The fast variables could be the vacuum fluctuations caused by unknown super heavy particles. The emerging quantum effects in the light particles are expressed by a Hamiltonian that can have almost any form.

The entire system is ontological, and yet allows one to generate interference effects in computer models. This seemed to lead to an unsolvable paradox, which is now resolved: exactly what happens in our models if we run a quantum interference experiment in a classical computer is explained. The restriction that very fast variables stay predominantly in their ground state appears to be due to smearing of the physical states in the time direction.

1 Formulating our problem.

It seems to be reasonable to suspect that the huge successes of the quantum mechanical machinery in describing the statistical features of atoms, molecules, and elementary particles, can be explained in terms of simple ontological descriptions of what is going on in these sub-microscopic objects. Yet there appear to be several problems standing in the way of further exploration of this idea. First, attempts to construct explicit models that explain the origin of Schrödinger's equation, invariably ended in unwieldy constructs, either requiring 'pilot waves [1]' inundating infinitely many virtual universes [2, 3], or non-linearities [4, 5] that seem to jeopardise the fundamental and perfect beauty of the Copenhagen picture.[1] It is as if the best thing we can do to-day is to be 'agnostic' about the origin of quantum mechanics.

It was felt that the ontological description of quantum mechanics requires an extra 'axiom', referred to as the collapse of the wave function. Also, the need was felt for a proof that justifies the identification of probabilities with the absolute squares of amplitudes. Why are certainties replaced by probability distributions in quantum mechanics?

Ignoring all difficulties that this picture of quantum mechanics lead to, some investigators, in particular computer scientists, speculated that the universe may be regarded as a gigantic *cellular automaton* [7, 8, 9, 10].

The author has repeatedly claimed [11, 12, 13] that models do exist, where both the full quantum machinery applies – without any deviation or approximation – and at the same time a totally classical formalism is valid. These models are fundamentally simple, and no 'collapse axiom' is needed at all. The probabilities can be naturally identified with the absolute squares of the wave functions, a fact that can be used as input for a mathematical treatment, rather than something that has to be proved or conjectured.

However, new problems gave rise to objections that are actually quite legitimate.[14] One of these was the apparent contradiction with Bell's theorems [15, 16, 17]. Bell's theorems however required the consideration of statistics, while our models are essentially formulated at a level where we can do away with statistics entirely – they only make exact statements about states, which may or may not be realised. Notions such as 'free will' [18, 19] and 'superdeterminism' [20, 21] do not mean anything here. Bell's definition of 'causality' or 'statistical independence' do not apply.[22]

More to the point is an objection that we have been struggling with more recently, which is that we should be able to construct precise models that reflect how the apparently conflicting features of quantum mechanics can come about. What is it that 'really happens' in a 'Gedanken experiment' where Bell's theorems, or the well-known CHSH inequalities [23] are violated? What happens when a quantum interference experiment is

[1] For an overview of the history on the meetings in Copenhagen and elsewhere that resulted in an important consensus, see e.g. A. Pais [6].

performed?

Constructing explicit models is technically hard, but possible in principle. We here explain how such models can be found. It will be shown that quantum mechanical correlation functions can be most easily explained if we assume the existence of very high frequency oscillations, which give rise to energy levels way beyond the regime of the Standard Model. When these fast oscillating quantities are handled as classical variables that have to be smeared over short, finite periods of time, one obtains the familiar operator structure for quantum mechanical systems. One can obtain any quantum Hamiltonian this way, although in our models the spacings of the energy levels are not completely free; they are constrained in terms of rational numbers.

We do not here perform all technical calculations all the way. We do show how realistic models can be constructed, and we we shall find that the quantum mechanical systems obtained, are subject to interesting constraints. We suspect that this result might one day be welcome to help us guess what the sub-atomic world will look like beyond the domain explored by the latest particle accelerators such as LHC, and how to connect our particle models with what is known about cosmology, a science that partly covers uncharted territory of high energy physics.

It will be important to note that our considerations do not require non-locality, but we do have problems explaining local gauge invariance, special and general relativity, and other continuous symmetries. This does not mean that these symmetries would be false but it means simply that we have not yet reached the stage that we can understand the origin of such symmetries in our classical models. This is one of many important questions that have to be left for the future.

In the present paper we show how to make models that separate fast variables from slow ones, and how energy conservation can be exploited to explain why our world seems to be perfectly quantum mechanical. A more rudimentary description of our models was also given in ref. [13].

The classical system we start with is absolutely classical, hence it does away with collapse axioms, measurement problems, pilot waves and many worlds. All we need is fast variables such as the vacuum fluctuations due to heavy particles, many of which must be unknown today.

The fast variables are moving completely classically, but too fast to be followed by the 'observers' in our model. Their dynamical rules are assumed to be very simple, and they are explained in section 2. The slow variables will be assumed to perform flip-flop movements, but before describing them, we digress to explain the ontological significance of the use of complex numbers, in Section 3, making our evolution operators unitary rather than orthogonal.

The slow variables evolve classically through switches, which are dictated by he fast variables. The equations ar described in section 4. Then comes the most essential part

of this paper: just because the fast variables are in, or close to, their energy eigenstates, the Hamiltonian for the slow variables turns into a quantum mechanical one, hiding its classical origin.

The result is discussed in section 6. Earlier models raised the question how one can account for quantum interference effects. How do the ontological variables turn into conventional operators and observables that come in wave functions that can be superimposed? How can interfering waves of particles be simulated in a classical computer? This seemed to give rise to contradictions. A flash of insight produces the correct answer to such questions. Only one set-up is treated in this section: the two-slit interference experiment. Other, related questions must be answered in a similar fashion.

2 The fast variables

We now discuss the generic structure of a class of fundamental models in more detail. If one would treat all variables, fast and slow, at the same level, one would be able to derive a Hamiltonian that at first sight indeed would look like a fully quantum mechanical one. However, there is one fundamental problem: its energy levels then have conspicuous patterns that we do not see in the real world: the energy levels should form perfect sequences that would feature exactly regular spacings everywhere.[2]

We now suggest that such sequences of energy levels will be characteristic primarily for the invisible fast fluctuating variables. Their 'exactly equal separations' are invisible to us because the actual energy spacings are much greater than a TeV, or whatever the energy is, up to which the Standard Model could be checked.

This implies that, yes, we can ascribe the quantum features of our world to classical evolution equations, but only if these classical laws act at ultra-short time scales.

And what about the slow variables? Do they contain ladder-like sequences of energy levels? It will turn out that they consist primarily of binary variables, pais of states that flipflop into one another. This gives them just pairs of energy levels, so no conspicuous sequences will emerge. The fact that the slow variables nevertheless evolve in a non-trivial way will be seen to be due to their interactions with the fast variables.

Thus, this brings us to formulating the basic equations for models that lead to quantum mechanics. The fast fluctuating variables are taken to live on periodic spaces, typically forming a torus of very small physical dimensions. One could regard these as 'extra dimensions'[3]. A few of such extra dimensions suffices: we need one fast periodic variable for each possible quantum state of the slow variables, so one can think of a (classically

[2] See the 'cogwheel models' described in Ref. [11].

[3] We put 'extra dimensions' between quotation marks as there is no need for extending the Lorentz group to these extra coordinates.

evolving) field living in space-time with the extra dimensions added. The slow variables just live in ordinary space-time (the '3-brane').

We take the number of fast variables, and the dimensionality of the Hilbert space containing the slow variables, both to be N.

Here are the equations for the fast variables of our model. The classical evolution law is:

$$x_i(t+1) = x_i(t) + 1 \mod L_i \ . \tag{2.1}$$

It can be addressed by casting its laws in the form of an ordinary Hamiltonian, such that the evolution agrees with a Schrödinger equation:

$$H_0 = \sum_i p_i \ , \qquad p_i = -i\frac{\partial}{\partial x_i} = \frac{2\pi n_i}{L_i} \ , \qquad n_i = 0, 1, \cdots , L_i - 1 \ . \tag{2.2}$$

One easily checks that, at integer values of the time t, this Hamiltonian moves all variables x_i across integer points of a lattice on the torus. The velocities are all the same. It will be important to have this lattice here, otherwise difficulties may arise in defining exactly what the classical states of the system are.

Our later considerations will require that the (large) numbers L_i be relative primes (section 4).

The spacings between the energy levels form a simple grid. Actually, our definition of the Hamiltonian was assuming continuous variables x_i , with respect to which one can differentiate. It is easy however to find the Hamiltonian when we restrict ourselves to the integer lattice points only, by deriving the eigenstates of the evolution law (2.1), which are found to agree with the Hamiltonian (2.2). On the discrete lattice, however, one finds one further restriction: since, in any direction i, there are only L_i states – a finite, integer number –, the momentum quantum numbers n_i can all be restricted to the *non-negative* integers as given in eq. (2.2).

The ground state is then found to be a function that is constant over the entire torus, so that its energy is zero. All excited states are way beyond a TeV simply because the torus is so small. Thus, if we would live in such a model, we would have no knowledge today of these extra states; we now assume them to exist.

To excite any of these higher energy states, more energy would be required than the quantum energy we can generate in any Earth bound laboratory, and therefore it sounds reasonable that we ignore them all. This is how one usually deals with 'unknown high mass particles' in 'Grand Unified Theories'. Here it is the key ingredient of the mechanism that we propose. There may well be reasons to doubt whether our mechanism will work as expected, but a comparable case is for instance the decay of a proton (a slow process), which can be ascribed to an exchange of a very high mass vector boson (a high energy excited state).

We now consider our two domains of physical states. Besides the states of the fast variables described above, we have N states that we can indeed observe. At this point, the Hamiltonian (2.2) does not depend on these states, and this ensues that all energy levels of the fast variables, discussed above, are degenerate with multiplicity N. As yet, these low energy states do not evolve. It is these N states that we consider to be observable, in the quantum mechanical sense; they are taken to be ontological also, *but they are only ontologically observable for observers who also monitor the fast variables,* otherwise, they will be ill-defined, and this will be shown to be how they get to become quantum observables.

Our next step will be the description of the domain of slowly varying states, where we claim quantum mechanics will be spontaneously generated. But first, we need to describe how ontological binary data can be made to flip in various ways.

The more experienced reader might want to skip the next section, proceeding immediately to Section 4.

3 Qubits, cbits, and the sign of the wave function

We saw that for a system that evolves classically, as in eq. (2.1), one may introduce a wave function that obeys the Schrödinger equation $\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle_t = -iH_0|\psi\rangle_t$. But actually, this Schrödinger equation may seem like overkill: the system shifts position as in eq. (2.1), but wave functions do not really mix, they are just transported as a whole. This means that, at this point, Born's rule [24] is trivially valid, when we merely define $|\psi|^2$ to represent probabilities. Whatever phase we add to these wave functions, is simply transported without anything happening to it.

Yet phase will mean something quite special in ordinary quantum mechanics. We shall attribute all of this to the mathematical procedures that will be applied, but let us first define the wave functions more precisely. Consider the phase; it is due to our addiction to complex numbers; however, the phase will stand not only for addiction, but also something real: *a complex number is a pair of real numbers.*

Thus, when we have a complex wave $\psi = \alpha + i\beta$, we can interpret this as a system that can be in two states. We may observe that conventional quantum states contain one special kind of binary object, to be called a *cbit* ('c' standing for 'classical' or 'complex-number'), which is a state that can be in position $\Re$ or in position $\Im$:

$$|\psi\rangle = (|\alpha + i\beta)|.\rangle = \alpha|\Re\rangle + \beta|\Im\rangle \ , \qquad \alpha \ \text{ and } \ \beta \ \text{ real} \ . \tag{3.1}$$

Taking this cbit into account, our wave function is now a real number. The probabilities of having state $\Re$ or state $\Im$ are α^2 and β^2, respectively.

This gives us an almost unique definition of the wave function in terms of probabilities:

the wave function is uniquely defined to be plus-or-minus the square root of the probability. But what does its sign stand for?

As far as we know, there is only one cbit in the universe, so it really serves a purely mathematical purpose. Having this cbit is necessary however, since it indicates something that should be observable in the technical sense (even if the world fluctuates strongly between its two states $\Re$ and $\Im$)[4]. The number i is actually an operator here, exchanging the states $|\Re\rangle$ and $|\Im\rangle$:

$$i\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}-\beta\\ \alpha\end{pmatrix} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} . \tag{3.2}$$

This sign is crucial if we wish to replace discrete time steps by continuous time steps. If time is discrete, then after every step the system flip-flops from $|\Re\rangle$ to $|\Im\rangle$ and back. In the continuum description, we wish to describe this as a rotation over 90°. We cannot keep plus signs everywhere. Instead of making finite, discrete, replacements, we need to make infinitesimal additions to describe the evolution, but then we need subtractions as well. If we use the correct ontological basis, the additions add the desired state and the subtractions remove the unwanted states.[5] Now, if we switch the sign of the imaginary part of the wave function, it will continue to evolve, but backwards in time. Therefore, the importance of the relative sign of $|\Re\rangle$ and $|\Im\rangle$ is that it indicates whether our evolution law will transport it forward or backwards in time. Switching the sign of *both* the real part and the imaginary part has no direct physical aspect for the ontological theory.

Thus, the relative sign of $|\Re\rangle$ and $|\Im\rangle$ has the important ontological interpretation as to whether we wish to follow the state forwards or backwards in time. The sign of the entire wave function seems only to play a role in our mathematical manipulations. It is not physical.

Consider now any other binary observable. Suppose it makes a switch, taking just a short time to do this, typically $\delta t \approx 1$ in the units used in eq. (2.1). We take a time dependent Hamiltonian. Use

$$e^{\pm\pi i/2} = \pm i \ , \qquad e^{\pm\pi i} = -1 \ , \tag{3.3}$$

to prove that for any operator σ that has all eigenvalues equal to ± 1, one has

$$e^{\frac{\pi i}{2}\sigma} = i\sigma \ , \qquad e^{\pi i\sigma} = -1 \ . \tag{3.4}$$

We find that the Hamiltonian

$$H(t) = \tfrac{1}{2}\pi\sigma\delta(t-t_1) \ , \tag{3.5}$$

[4]Only in a state with total energy zero, the cbit is conserved in time, as one can easily check.

[5]If all probabilities are limited to certainties, then the only allowed wave functions are $+1$, 0, and -1, so subtraction means replacement by 0.

yields the evolution operator U_{t_1} from $t < t_1$ to $t > t_1$ equal to

$$U_{t_1} = e^{-i\int_{t<t_1}^{t>t_1} H(t)\mathrm{d}t} \;=\; -i\sigma \;. \tag{3.6}$$

For the cbit, this tells us that a switch from $|\Re\rangle$ to $|\Im\rangle$ is generated by the (time dependent) Hamiltonian [6]

$$H = \tfrac{1}{2}\pi\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}\delta(t)\,. \tag{3.7}$$

Any other binary operator such as a spinor, consists of two states, each to be covered by a complex wave function. Thus there are 4 real numbers. This is traditionally called a qubit. As its wave function is complex, our system of operators acts on two binary states, one cbit and one other binary object describing spin or anything else. Let us assume the spin in the 3-direction to be $\pm\frac{1}{2}$. The states are now

$$\{1,\,2,\,3\,,4\} \;=\; \left\{\, |\Re,\,+\tfrac{1}{2}\rangle, \quad |\Im,\,+\tfrac{1}{2}\rangle, \quad |\Re,\,-\tfrac{1}{2}\rangle, \quad |\Im,\,-\tfrac{1}{2}\rangle \,\right\}. \tag{3.8}$$

These are the four ontological states. Ontological operations on these states consist of the 4! possible permutations, with in addition some sign switch operators. There is only one relevant sign switch, telling us whether the evolution will proceed forwards or backwards in time.[7] Thus, there are $2 \cdot 4! = 48$ distinct ontological operations.

These we can consider as built up from interchange operators. The most important ones[8] for us are generated by the Pauli matrices, σ_1, σ_2, and σ_3. If we substitute these in our Hamiltonian (3.5), we find the ontological switches realized by σ_a and i to be

$$i\sigma_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}_c \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_s, \qquad i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}_s , \tag{3.9}$$

$$i\sigma_3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}_c \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_s , \qquad i = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}_c , \tag{3.10}$$

where the subscript c stands for the cbit and s for the spinor.

This exercise was made to explain why all three Pauli matrices generate ontologically distinguishable switches, while even the spin-independent Hamiltonian (second part eq. 3.10) corresponds to repeated switches from $\Re$ to $\Im$ and back, as one could have expected from the solutions for the spin independent Hamiltonian.

[6] The continuum notation is used here. For the discrete case, the Dirac delta function may be replaced by a Kronecker delta, with some due care.

[7] The relevant sign switch is *complex conjugation*, that is, giving $\Im$ but not $\Re$ a minus sign.

[8] other members of the class of switches are obtained by multiplication with $^1/_2(1 \pm \sigma_a)$, which means that the states with $\sigma_a = +1$ or -1 are singled out.

It means that ontological models should only consist of operators of the form (3.5), where only one kind of transition is allowed at any time t_1, and the constant $\frac{1}{2}\pi$ is fixed, otherwise we produce superpositions, which was not allowed in our classical models. Note that the switches in eqs. (3.9) and (3.10) are all antisymmetric matrices.[9]

4 The slowly evolving states

In section 2, we ended with having N non evolving states in addition to our fast variables. It is these N states that will be supposed to describe our world, in terms of slow variables. Together with the fast variables, this evolution will be totally classical. The Hamiltonian we have up to now, H_0, eq. (2.2), does not directly affect the slow states $\psi_i = |i\rangle$, where i runs from 1 to N. Now, we add an interaction that changes this. It is a switch operation, as defined in section 3. Consider two states, $|i\rangle$ and $|j\rangle$, with $i < j$, and impose classically that these two states switch one into the other, at the moment t_1, which is defined to occur when both the (discrete) fast variables x_i and x_j happen to be at given positions $x_{i,1}$ and $x_{j,1}$:

$$H_{1,ij} = \pm\tfrac{1}{2}\pi\sigma_a(i,j)\,\delta x_i - x_{i,1}\,\delta x_j - x_{j,1}\ , \tag{4.1}$$

The deltas are Kronecker deltas. We may replace one of them by a Dirac delta, provided that the corresponding variable x is then treated as a continuous variable, but we cannot do this for both x_i and x_j simultaneously without running into conflicts. For sake of symmetry, we prefer to keep both variables discrete. This Hamiltonian then acts through one unit of time, and one may verify that this requires the unmodified factor $\frac{1}{2}\pi$ in front.[10]

In eq. (4.1), $\sigma_a(i,j)$ is defined to be the Pauli matrix σ_a, with $a = 1, 2$, or 3 acting on the two state world $(|i\rangle,\ |j\rangle)$. The operator (4.1) is the operator (3.5) in that space. It generates the evolution operators (3.9) and (3.10) in this two state world (plus the cbit), if the subscript s is now taken to represent the two states $|i\rangle$ and $|j\rangle$. Thus, from now on, a binary flipflop takes place whenever x_i and x_j simultaneously arrive at the pre-designed values $x_{i,1}$ and $x_{j,1}$. So the combination $H = H_0 + H_{1,ij}$ may be used to describe an ontological evolution law.

We may repeat this procedure to add more interaction terms. The evolution stays unitary provided that we use different points x_1 for every term. Classically, the system

[9] Side remark: we could rephrase the theory of quantum mechanics for only real wave functions by replacing the Hamiltonian by a real, antisymmetric matrix. This cancels out the i in eq. (3.7), so that real functions evolve into real functions only. It makes little difference in physics, actually simplifies things somewhat, but I found the formalism rather unfamiliar, requiring further explanations on the way, so we shall not hang on to it.

[10] The sign in front of eq. (4.1) has no ontological significance by itself, but it may become important in combination with other switch interactions introduced later in this section. The order in which switches take place then becomes important,

now hops from one state to another, in response to the classical, fast variables. Nevertheless, this evolution is slow, because the coincidences where both x_i and x_j are required to arrive simultaneously at given positions, do not happen often. It typically takes time L_iL_j for this to happen. Thus, our fast time scale is the largest value of the L_i, the slow time scale is the average product, L_iL_j.

5 Why is this quantum mechanical?

The freedom we have is to choose the pairs (i,j) for which this flipflop may occur, and to choose at which points they occur. The latter choice seems to be of secondary importance since the classical evolution goes very quickly for the fast variables. What matters most is the values of the pairs (i,j) and how frequently switches between given pairs occur.

Now follows an important question: how does the solution of these evolution equations behave? The behaviour is classical. If one tries to solve the equations exactly, one finds that the system stays classical. The energy levels form equally spaced sequences, and consequently, the entire theory seems to be a failure. Imagine however, applying this doctrine to any remotely realistic quantum field theory. Even classically, it will be far too complex to follow with infinite precision what goes on. What we are interested in is only the slow variables $|i\rangle$, and how their behaviour can be described while ignoring the fast variables.

To find good approximated solutions, we now must assume that the fast time scale and the slow time scale are well-separated[11], even if the pairs (i,j) featured in H_1 are fairly numerous. They must still be much fewer than the large numbers L_i permit. Then, we may employ the large L expansion.

At first sight, the evolution of the fast variables is not affected by H_1. Classically, our system is non-Newtonian: there is no reaction of the variables $x_i(t)$ to what the slow states $|i\rangle$ do, while the slow states $|i\rangle$ are dictated by the fast variables. The quantum Hamiltonian, $H_0+\sum_{i,j}H_{1,ij}$, is fully acceptable, as it is unitary and finite.

Our approximation will be that the fast variables are in their energy ground state. In spite of the absence of a classical back reaction, there will be a quantum back reaction in the higher energy states, so the classical fast variables will not stay in their ground state. This situation is quite familiar in standard quantum mechanics, and there is no reason to treat the present situation in any way differently from what we usually do: apply perturbation expansions.

This means that all energy eigenstates are shifted due to the perturbation, but the lowest states will still be most frequently used, when energy conservation forbids the

[11] We suspect that this condition can be relaxed. All that matters is that fast variables are smeared due to limits in our time resolution.

higher energy states. The N classical states, which started out completely degenerate, now will shift to form some pattern, but one may expect them to stay well separated from the excited energy values for the fast variables. Remember, our model obeys the same type of Schrödinger equation as is usual, so there is no need for alarm.

When we calculate the lowest order effect in perturbation theory, we find that, in the Hamiltonian H_1, eq. (4.1), the x-dependence for the fast variable must be replaced by its expectation value. Since there are L_iL_j sites for the fast variable, one therefore must replace the delta functions by their averages, as follows,

$$\begin{aligned} \delta x_i - x_{i,1}\,\delta x_j - x_{j,1} \;&\to\; 1/(L_iL_j)\ , \\ H_{\rm slow} \;\to\; \sum H_{1,ij} \;&=\; \sum(\pm\tfrac{1}{2}\pi)\sigma_a(i,j)/L_iL_j\ . \end{aligned} \tag{5.1}$$

The main result reported in this paper is that by adding many interactions of the form (4.1), the slow variables end up by being described by a fully quantum mechanical Hamiltonian $H_{\rm slow}$ that is a sum of the form (5.1). All switches involving the sites i and j, together generate the matrix element $H_{1,ij}$ of $H_{\rm slow}$. The diagonal parts of $H_{\rm slow}$ are taken care of by the matrices σ_3 and i, the off-diagonal ones by σ_1 and σ_2. The fixed coefficients $\pm\frac{1}{2}\pi$ are now replaced by any arbitrary coefficients, and this has the effect of changing our slow variables from classical to quantum variables. We can mimic almost any quantum Hamiltonian this way, although in the model the strengths of all on- and off diagonal terms is controlled by ratios that have to be rational numbers (they all take the form $\pi R/(2L_iL_j)$), where R, L_i and L_j are integers). One may also try to evaluate *exactly* what happens: at any of the transition points x_{i1}, x_{j1}, the wave functions typically rotate by 90° in some channels. This has no local effect, it only manifests itself when the boundary conditions on the entire torus are considered, which is exactly equal to the perturbative effect, except when we mix non-commuting contributions; their effects will depend on the order, that is, the exact locations of the transition points. Indeed one can verify that, if all terms of the switches commute, one sees that the exact quantum mechanical effect is obtained: certainty about the switch is only seen after a full period of the fast variables. In the more interesting case that many non-commuting terms are added in the effective Hamiltonian, the exact solutions are more complicated but still fully quantum mechanical, and accurately reproduced by the perturbative expression (5.1).

The reason why we fail to see longer sequences of energy eigenvalues with much smaller spacings (which is what an exact analysis will seem to give), is that energy is only defined *modulo* the length of the inverse time steps, which results in energy levels of the unperturbed case that are much farther apart then if we considered single rings of length L_iL_j instead of a torus with radii L_i and L_j. The 90° rotations are still fine in the torus, but technically more difficult to follow.

An interesting question concerns the signs of the effective Hamiltonian terms; there seems to be some freedom in choosing them. Closer inspection suggests that these signs

are actually fixed when non-commuting elements are taken into account, but the real mathematical puzzle has not yet completely been resolved.

6 Discussion

We admit that not yet all questions have been answered, but there is a thing that we are quite certain about: in spite of the fact that our theory is entirely ontological, it is also controlled by a Schrödinger equation, and with the Hamiltonians (2.2) and (3.5) all inserted, this Schrödinger equation is exactly valid; it is genuinely quantum mechanical. Therefore, whatever the higher order corrections are that will ensue from our model Hamiltonian, they will merely be small corrections that do not jeopardise the quantum nature of the system.

The real reason why we have evaded the usual no-go barriers is something more subtle: we imposed that the highest energy states must be forbidden by the law of energy conservation. It is the *total* energy that is constrained to be small. Via thermodynamics, this also generates statistical dominance of low energy states locally.

Important observations may be added concerning the question of quantum interference. Since we derived quantum mechanics, it is obvious that quantum interference must occur, but how can it be explained that quantum solutions for wave equations can be superimposed in a way that (constructive or destructive) interference takes place, even if single events are considered? We claim that interference is a direct consequence of limiting ourselves to the lowest energy states of the fast variables. If we start with a strictly statistically even distribution for all fast variables, we shall see interference patterns arising. This is because the zero energy state has maximal overlap with the completely even distribution. Thus we predict that an interference experiment can be mimicked with this model.

But then, an objection can be put forward. What puzzles people most is that closing one slit may actually enhance the probability of a particle hitting a screen at some place downstream. What happens inside my classical computer? In the computer, one could have registered which slit the particles pass through, without disturbing the outcome. Suppose we perform an ideal interference experiment. After having admired the beautiful interference pattern, we select out all cases where the particle was seen to pass through one slit, and compare that with the cases where the particle went through the other slit. Both of these selections should not show interference, because the particles went through one slit only. This obviously cannot happen in a classical computer, since recombining the two sets should give dark spots where no particles arrived at all. What did we do wrong?

Indeed, the argument that, since the particles went through a single slit, there now

should be no interference, in this case is wrong.

What really happens can de derived from quantum mechanics. Since none of the particles could have arrived at the dark spots, selecting out all particles that went through one given slit indeed will not remove the interference pattern. It does something else however: if we select one slit, and repeat this experiment many times, then we are making a selection among the initial states chosen for the fast variables. This selection will not be an even one! Therefore, the initial state of the fast variables must now be described as a superposition of different energy eigenstates. Since now the particles went through a given slit, each energy eigenstate of the fast particles should not give any interference pattern. However, we now are describing an experiment where the initial state was a superposition of at least two energy eigenstates of the fast variables! Superposition in = superposition out. Thus, the observed interference pattern is due to interference between two differently chosen initial states of the fast variables, and not due to the slits. This explains the result.[12]

In short, the answer comes from the small-print: limiting ourselves to the lowest energy eigenstate of the fast variables was imperative. The fast variables have to start in a perfectly even distribution.

It may be concluded that all that is needed to turn a classical system into a quantum mechanical one, is to define energy as it is only done in quantum mechanics, that is, by diagonalising the evolution operator. After this, one has to postulate that the fastest moving parts of the system must be limited to their lowest energy states. Effectively, this amounts to slightly smearing the amplitudes in the time direction in order to account for limitations in our time resolution.

Besides possible modifications of the Standard Model in its highest energy domains, there may be implications for investigations of cosmology. When the universe was very small, the energy density must have been very high. Maybe all energy states were equally occupied when the universe started as a single point, or almost as a point. At the indivisible instant of the Big Bang, there was no quantum mechanics yet. The universe expanded, and this forced it to cool off. Thus the laws of thermodynamics deprived our world of its highest energy states, with quantum mechanics as a result.

We also see implications for quantum black hole physics. When a black hole forms, imploding matter gets compressed against the past horizon, as seen by a distant observer at later times. Similarly, outgoing Hawking particles line up along the future event horizon, ready to spring to life much later. When matter reaches very high energy densities this way, it may well be that the *highest energy state possible* is approached near both

[12]Of course, one can continue asking questions: *Why then, does the interference pattern vanish when a physical observer checks the particles at the slits?* There again, ordinary quantum mechanics gives the answers. A physical observer cannot affect the fast variables, he does not have the energy to modify their statistical distribution, while for the computer this had been no impediment at all.

horizons. This state contrasts with the state with lowest possible energy density, the vacuum state. It is called the 'antivacuum' state. In the classical picture, a symmetry relating vacuum to antivacuum seems to be evident. When we describe stationary black holes, matter appears to be almost absent, as if the antivacuum of compressed imploding particles has been transformed into a vacuum. Since matter is the source of curvature, this replacement of antivacuum with vacuum forces the past and future horizons to change their effects on space and time. This is where the 'antipodal identification' is suspected to originate. Antipodal identification is known to be needed if one wants to restore unitarity for the evolution of a stationary black hole.[25]

Other questions are also still wide open: for instance, we wish to explain the existence of quite a lot of continuous, global and local symmetries of our world. Making classical theories that respect these symmetries (gauge symmetries, Goldstone symmetries, special and general relativity, supersymmetry perhaps, and so on), is notoriously difficult. We leave these questions for future investigations.

In our model, all slow observables $|i\rangle\,,\ |j\rangle\,,\ \ldots$ are ontological, but the fast ones are put in an energy eigenstate, which is ontological, or more precisely, the total energy of the entire universe is declared to be ontological.[13] The computational rues are as in ordinary quantum mechanics, but the entire theory is fundamentally deterministic. Statistics enters at the moment we single out the ground state for the fast variables.

We note that our model differs wildly from the Standard Model, but it does seem to be built from variables that, in a more advanced stage of this theory, may be regarded as quantum fields. Regarding our observation that models of the sort described here are perfectly guaranteed to represent pure quantum mechanics, it may well be that a strict separation between fast and slow modes is unnecessary. Even the Standard Model admits, and indeed favours, the existence of ultra heavy particles. The vacuum fluctuations of their quantised fields are perfectly suitable to play the role of fast variables, and this is why we suspect that, indeed, quantum mechanics generated in line with our description, is almost inevitable. Thus, our theory also explains *why* we have quantum mechanics.

We conclude that quantum mechanics may well be perfectly understandable if the right mathematical framework is used. An intriguing observation is furthermore that the quantum field variables appear to be constrained to lattices (the locations x_i of the fast variables in our model). This forces also the interaction constants of the resulting theories to lie on lattices; they are not continuously adjustable.

[13] We say this because the total energy is conserved, but we have to realise that total energy does not commute with the other ontological observables, and normally this would not be llowed. Here, we see no problems in constraining the total energy, but a more precise justification would be welcome. For instance, we do not impose any absolute constraint on total energy, but demand energy to lie within some domain, in such a way that time-smeared observables may be colled ontological.

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