$\underset{(N\times N)}{Var(\varepsilon|X)} = \sigma_\varepsilon^2 I_N$

$Var(\hat{\beta}_{OLS}|X)$

$= \sigma_\varepsilon^2 (X'X)^{-1}$

what if we relax the assumption of homoskedasticity, eteroskedastick setting

before it was a matrix with all same variances on the diagonal

(meaning the matrix is PD)

symmetric matrix

fully unconstrained setting

D

$\sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \sigma_N$

CHOLESKY DECOMPOSITION OF A PD MATRIX

Good assumption when data are cross-sectional

if the cov matrix is diagonal the cholesky is like a square root

↓

the results work both ways

Chol command in Matlab

related to square roots

↓

they are similar operations

RESULTS

- $\hat{\beta}_{OLS}$ remains consistent and unbiased also under eteroskedastic settings
- before $Var(\hat{\beta}_{OLS}|X) = \sigma^2_\varepsilon (X'X)^{-1}$

  now $Var(\hat{\beta}_{OLS}|X) = (X'X)^{-1} X' \Sigma X (X'X)^{-1}$

(1) unbiasedness

(2) consistency --> the mean of the error remains zero, we are not relaxing exogeneity of the error

(3) what happens to the variance?

(k×k)

now we have N of those

-> can call it also D in this case because it is diagonal

before we were estimating just one parameter with s^2

the problem is that we do not know Σ and it is difficult to estimate

$\neq \sigma^2_\varepsilon (X'X)^{-1}$

How to tackle eteroskedasticity

$$Var(C^{-1}\varepsilon)$$
$$= C^{-1}\Sigma C^{-1\prime}$$
$$= \Sigma\,\Sigma^{-1} = I_N$$

$\hat{\beta}_{GLS}$ $\underbrace{\qquad}_{\Sigma^{-1}}$ $\underbrace{\qquad}_{\Sigma^{-1}}$

Generalized least square estimator

$\to (X' \Sigma' X)^{-1}$

Σ

But in reality it is difficult to know Σ, how can we estimate it.

h. function

coefficients

this is what we regress

What w looks like depends on the heteroskedastic function
(w is the heteroskedastic function)

$$\mathrm{Var}(\hat\beta_{GLS}) = (X'\Omega^{-1}X)^{-1} \qquad \mathrm{Var}(\tilde\beta) = L\Omega L'$$

$$\hat\beta_{GLS} = \underbrace{(X'\Omega^{-1}X)^{-1}X'\Omega^{-1}}\,y$$

$$(\tilde\beta - \beta_0)(\tilde\beta - \beta_0)'$$

$$\tilde\beta = \beta_0 + L\varepsilon \qquad L\underbrace{\varepsilon\varepsilon'}L' \qquad L\Omega L'$$

$$L\Omega L'X'\Omega^{-1} - \underbrace{(X'\Omega^{-1}X)^{-1}X'\Omega^{-1}}$$

$$\begin{tikzcd} & \Sigma \text{ CHOC} \arrow[dl] \arrow[dr] & \\ \boxed{C'C} & & \boxed{CC'} \end{tikzcd}$$

$$\begin{tikzcd} & \Sigma^{-1} \arrow[dl] \arrow[dr] & \\ D'D & & DD' \end{tikzcd}$$

Given $y = X\beta + \varepsilon$ in an [illegible]