$$\hat{\beta}_{GMM} = \underset{\beta \in B \subset \mathbb{R}^k}{\operatorname{argmin}} \ \bar{m}(\beta)' W_N \bar{m}(\beta)$$

W_N: weighting matrix dependent on N

FOC

$$2 \frac{\partial \bar{m}(\beta)'}{\partial \beta} \cdot W_N \bar{m}(\beta) = 0$$

$(k \times L)$

if i can find a β that sets to zero these conditions (k equations) then i have found GMM in close form (it does not happen very often)

$$\sqrt{N}\left(\hat{\beta}_{GMM} - \beta_0\right) \xrightarrow{d} ?$$

i expand the average moment condition

$$\bar{m}(\hat{\beta}) = \bar{m}(\beta_0) + \left.\frac{\partial \bar{m}(\beta)}{\partial \beta}\right|_{\beta=\bar{\beta}} (\hat{\beta} - \beta_0)$$

also $\bar{\beta} \xrightarrow{p} \beta^0$ $\qquad \bar{\beta} = w\beta_0 + (1-w)\hat{\beta}$

$$\left.\frac{\partial \bar{m}(\beta)}{\partial \beta}\right|_{\beta=\hat{\beta}} W_N \left[\bar{m}(\beta_0) + \left.\frac{\partial \bar{m}(\beta)}{\partial \beta}\right|_{\beta=\bar{\beta}} (\hat{\beta} - \beta^0)\right] = 0$$

$$\Downarrow$$

$$\frac{\partial \bar{m}(\hat{\beta})}{\partial \beta} W_N \, \bar{m}(\beta_0) + \frac{\partial \bar{m}(\hat{\beta})}{\partial \beta} W_N \frac{\partial \bar{m}(\bar{\beta})}{\partial \beta}(\hat{\beta}-\beta_0) = 0$$

- $\sqrt{N}\,\bar{m}(\beta_0) \xrightarrow{d} N(0; V_m)$

  because it is a sample average

- $W_N \xrightarrow{p} W \quad P.D. > 0$

- $\frac{\partial \bar{m}(\hat{\beta})}{\partial \beta} \longrightarrow D = E\left[\frac{\partial m_i(\beta)}{\partial \beta}\Big|_{\beta=\beta_0}\right]$

$$\sqrt{N}(\hat{\beta}-\beta^0) = -\underbrace{\left[\frac{\partial \bar{m}(\hat{\beta})'}{\partial \beta} W_N \frac{\partial \bar{m}(\bar{\beta})}{\partial \beta}\right]^{-1}}_{\downarrow p \;\; -(D' W_N D)^{-1}} \underbrace{\frac{\partial \bar{m}(\hat{\beta})'}{\partial \beta}}_{D' W_N} W_N \underbrace{\sqrt{N}\,\bar{m}(\hat{\beta})}_{\sim N(0; V_m)}$$

$$\sqrt{N}(\hat{\beta}-\beta^0) \sim N_k\left(0;\ (D' W_N D)^{-1} D' W_N V_N W_N' D (D' W_N D)^{-1}\right)$$

$$W = (V_N)^{-1} \longrightarrow (D' V_N^{-1} D)^{-1} \cancel{D' V_N^{-1}} \cancel{V_N} \cancel{V_N^{-1}} \cancel{D (D' V_N^{-1} D)^{-1}}$$

$$\sqrt{N}(\tilde{\beta}-\beta^0) \sim N_k\left(0, (D' V_N^{-1} D)^{-1}\right)$$

$$J(\beta) = \bar{m}_\beta ' W_N \bar{m}(\beta)$$

$$\bar{m}(\beta) = \frac{1}{N}(Z'y - Z'X\beta)$$

$$E\left[(Z'y - Z'X\beta)(Z'y - Z'X\beta)'\right]$$

$$Z'yy'Z + Z'y\beta'X'Z - Z'X\beta y'Z + Z'X\beta\beta'X'Z$$

$$Z'yy'Z + 2Z'X\beta y'Z + Z'X\beta\beta'X'Z$$

$$\lambda \exp(-\lambda \cdot \text{Prod})$$

$$F(X\beta) = \frac{\exp(X\beta)}{1+\exp(X\beta)} \Rightarrow \frac{\beta \cdot \exp(X\beta)(1+\exp(X\beta)) - \exp X\beta \cdot \beta \cdot \exp(X\beta)}{(1+\exp(X\beta))^2}$$

$$= \frac{\beta \exp(X\beta)}{(1+\exp(X\beta))^2}$$

$$\frac{dF(x\beta)}{dx} = \frac{\exp(x\beta)}{(1+\exp(x\beta))^2} \cdot \beta \qquad \frac{\exp(x\beta)}{1+\exp(x\beta)}$$

CDF

$$\frac{\exp(x\beta)}{1+\exp(x\beta)} \cdot \frac{1}{\exp(x\beta)^2} \cdot \beta$$

$$\frac{\exp}{1+\exp(x\beta)} = CL$$

$$1 - CL = 1 - \frac{\exp(x\beta)}{1+\exp(x\beta)} = \frac{1}{1+\exp(x\beta)}$$

$$CL(1-CL) =$$

$$\lambda \cdot e^{-\lambda\, prod} \cdot \beta$$

$\to$ f(λ

$$f(prod) = \lambda e^{-\lambda\, prod}$$

$$f(x\beta) = \lambda e^{-\lambda(x\beta)}$$

$$F(\cdot) = \frac{\exp(\cdot)}{1+\exp(\cdot)}$$

$$f(\cdot) = F(\cdot)(1-F(\cdot)) \cdot \beta$$

$$f(F\beta) \cdot \beta$$

$$\left(\lambda e^{-\lambda \bar{x}}\right) \cdot \beta$$

$$\downarrow$$

$$f(\bar{x}) \cdot \beta$$

$\log(\text{wc}) = \beta_0 + \beta_1 \text{educ} + \varepsilon$

$\log(\text{wc}) = \beta_0 + \beta_1 \text{educ} \underbrace{+ \beta_2 A + M}_{\varepsilon}$

$E[\varepsilon \mid x] = 0$ falso since $E[\text{educ}\, A] \neq 0$

$$\hat{\beta}_1 = \beta_1 + \frac{\text{Cov}(\beta_2 A + M, \text{educ})}{\text{Var}(\text{educ})}$$

GRAZIE SILVIO ☺

$$\hat{\beta}_1 \neq \beta_1 + \frac{\beta_2 \text{Cov}(A, \text{educ})}{\text{Var}(\text{educ})}$$

$$\hat{\beta} = \frac{\text{Cov}(x, y)}{\text{Var}(x)} =$$

$$P(y=1) = \frac{\exp(x\beta)}{1+\exp(x\beta)}$$

$$\frac{\partial P(y=1)}{\partial X} = \frac{\exp(x\beta)\cdot\beta\,(1+\exp(x\beta)) - \exp(x\beta)\cdot\beta\cdot\exp(x\beta)}{(1+\exp(x\beta))^2}$$

$$= \frac{\exp(x\beta)}{(1+\exp(x\beta))^2} \cdot \beta$$

$$dur = \beta_0 + \beta_1 Prod + \varepsilon$$

$$\frac{\partial dur}{\partial Prod} = \beta_1$$

---

dur Prod

$$\lambda e^{-\lambda dur}$$

$$\lambda_i = \exp(x_i'\beta)$$

$t_i \sim$ exponential

$$\lambda_i(t_i) = \exp(x_i'\beta) \cdot \lambda^*$$

$$f(t) = \lambda e^{-\lambda t}$$

$$f(t) = \exp(x\beta) \cdot \lambda^* \, e^{-\exp(x_i'\beta) \cdot \lambda^* \cdot t}$$

$$\lambda_i(t_i) = \exp(x_i'\beta) \cdot \lambda^*$$

dur

$$P(y=1) = E(y|x)$$

$$E[dur] = \frac{1}{\lambda} = \frac{1}{\exp(x\beta) \cdot \lambda^*}$$

$$\frac{\partial E(\text{dur})}{\text{Prod}} = -\frac{\cancel{\exp(x\beta)\cdot\lambda^{5}}\cdot\beta}{(\exp(x\beta)\cdot\lambda^{*})^{2}}$$

$$-\beta\cdot\frac{1}{\exp(x\beta)\lambda^{*}}$$