Hypothesis testing → Hypothesis on the coefficients of the regression

H_0: null hypothesis : $R\beta = c$ $\quad$ R and c matrix and vector of coefficient

R: $\mu \times k$ $\quad$ β: $k \times 1$ $\quad$ c: $r \times 1$

μ: generic number of rows of r (is the number of restrictions we are testing)

k: for sure because it has to be compatible with β

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_5 \end{bmatrix}$$

$$H_0: \begin{cases} \beta_1 + \beta_2 = 5 \\ \beta_5 - \beta_4 = 2\beta_3 \end{cases}$$

→ example of linear assumption

→ $\beta_5 - \beta_4 - 2\beta_3 = 0$

R in this case is

$$R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & -1 & 1 \end{bmatrix}$$

$r \times k$

k = 5 in this case because β is 5×1

r: 2, number of equations

hypothesis are made on the true parameters of the model

c in this case is $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ (2×1)

$\boxed{\varepsilon \sim N(0, \sigma^2_\varepsilon I_N)}$

$$\hat{\beta} | X \sim N(\beta^0; \sigma^2_\varepsilon (X'X)^{-1})$$

→ this holds independently to the sample size

↓ because we are assuming normality, then, if we are assuming normality with only 10 observations, maybe the assumption is wrong

$\hat{\beta}|X \sim N$: Multivariate normal distribution

$(R\hat{\beta} - c) \sim ?$ $\quad$ Gaussian is closed under linear operations

$$R\hat{\beta} - c \sim N(\boxed{R\beta^0 - c} ; R\, \sigma^2_\varepsilon (X'X)^{-1} R')$$

$R\beta^0 - c$: LINEARITY OF EX. VALUE

$R\, \sigma^2_\varepsilon (X'X)^{-1} R'$: $r \times r$ → Quadratic operator, as R is squared

if β is normal also this linear transformation is

NOTABLE DISTRIBUTIONS

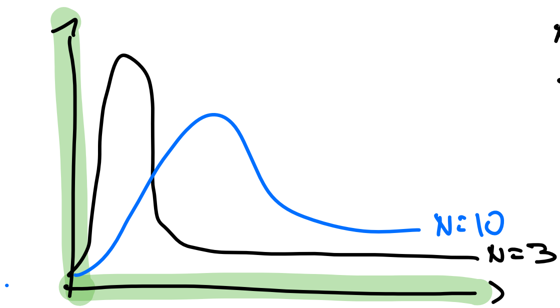
$(X_1 \dots X_n)$ iid random variables, normal with $\mu=0$ and $\sigma^2=1$

$$Y = \sum_{i=1}^{n} X_i^2$$ sum of squares of standard gaussians

(I)

$\chi^2(n)$ ↳ degrees of freedom

• we are summing squares, the distribution can only be positive



Adding degrees of freedom shifts the distribution more on the right

χ distribution is only positive!

(II) Student's t

$(X_1 \dots X_n)$ iid normal $(0,1)$

$W_1 \sim$ independent from the others and $\sim N(0,1)$

and $Y \sim$ t student

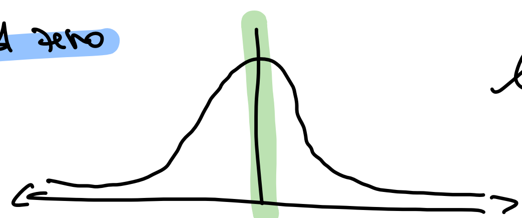
→ normal of the numerator

$$Y = \frac{W_1}{\sqrt{\frac{\sum x_i^2}{N}}} \sim t(N)$$ ↳ degrees of freedom

→ it is like a quadratic mean of normals of the denominator

t distribution takes value on the entire real line, and it is symmetric around zero

looks like this



• F distribution

W_1 is $\sim \chi^2(N)$ W_1 is independent from Z_1 $W_1 \perp Z_1$

$Z_1 \sim \chi^2(M)$

$$Y = \frac{W_1/N}{Z_1/M} \sim F(N, M) \rightarrow \text{POSITIVE}$$

two chi-squared distributions' ratio

degrees of freedom of numerator and denominator

$$R\hat{\beta} - c \mid X \sim N(R\beta^0 - c ; R\sigma^2_\varepsilon (X'X)^{-1} R')$$

Suppose we find a $\check{\beta}$ very close to the constraint, the difference $R\check{\beta} - c$ will be close to zero

we want a r.v. as close as possible to zero when the difference is very small and large when we are far from zero

we square things because we are interested in distance, both positive and negative, from the value we want

$$\underset{(1\times r)}{(R\check{\beta} - c)'}\,[R\sigma^2_\varepsilon (X'X)^{-1} R']^{-1}\,\underset{(r\times 1)}{(R\check{\beta} - c)} \quad (1)$$

r×k k×1 r×1

↳ maybe we are not subtracting because under the null hyp. the mean is also zero

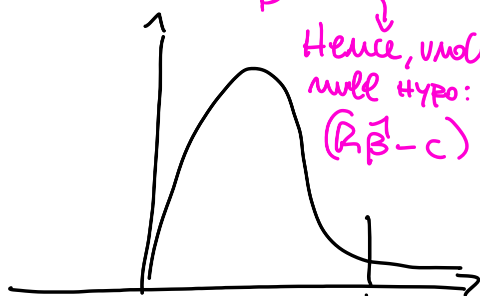
never going to be = 0 because continuous random variables

$$\sum^{r} w^2$$

under the null hypothesis (1) is zero and all the t statistics are calculated under the null hypothesis → NULL HYPO IS $R\check{\beta} = c$

Hence, under null hypo: $(R\check{\beta} - c) = 0$

dividing by the variance, not the std to standardize, because at the numerator we have squares



I obtained this value from the t statistics ex 8.74

this is $\sim \chi(r)$

↳ sum of standard normals squared

- i can choose an α% level and for example α% is 9.78 and since 8.74 is smaller i do not reject the results

- or i can compute the p-value and compare it directly with α, if the probability is larger than α we do not reject the null hypothesis

But we do not know σ^2_ε, hence we cannot do computations

we can use s^2, but we are substituting a scalar with a random variable, hence we are changing the distribution

$(R\hat{\beta}c)'[R\hat{\sigma}^2_\varepsilon(X'X)^{-1}R']^{-1}(R\hat{\beta}c)$

this is standard normal

$$(R\hat{\beta}-c)'(R\hat{S}^2_\varepsilon(X'X)^{-1}R')^{-1}(R\hat{\beta}-c)$$

$$\frac{\hat{\varepsilon}'\hat{\varepsilon}}{N-K} \to$$ it is a scalar and it is an inverse, so it is going at the denominator

$$\frac{(R\hat{\beta}c)'[R(X'X)^{-1}R']^{-1}(R\hat{\beta}c)}{\frac{\hat{\varepsilon}'\hat{\varepsilon}}{N-K}}$$

sort a normal square

$$\frac{(R\hat{\beta}-c)'(\sigma^2_\varepsilon R(X'X)^{-1}R')^{-1}(R\hat{\beta}-c)/r}{\frac{(\hat{\varepsilon}'\hat{\varepsilon})}{(N-K)\sigma^2_\varepsilon}}$$

$\to \sim \chi^2(r)$

$\to$ they are not standardize

$\to \sim \frac{\chi^2(N-K)}{N-K}$

$\to \sim F(r, N-K)$

$\to$ the p-value is $= 1 - Fcdf(\hat{F}, r, N-K)$

F test statistic for the existence of the regression

$$Y = \beta_1 + \beta_2 X_1 + \dots \beta_K X_{K-1} + \varepsilon$$

test that all the coefficients are equal to zero

$\downarrow$

by testing this we are testing if none of the Xs have explanatory power for Y

$Y = \beta_1 + \varepsilon$ is the model under the null hypothesis

$H_0: \beta_2 = 0 \cap \beta_3 = 0 \dots \cap \beta_K = 0$

the Bs are jointly zero

H_1: the opposite of the null, is that at least one is $\neq 0$ $\beta_i \neq 0$ $i = 2, \dots, k$

$R\beta = c$

we are imposing $k-1$ restrictions

$k-2$ zeros, one is β_1 one is the $\beta = 0$

$$\underset{(k-1)\times k}{R} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 1 & \dots & \\ 0 & 0 & 0 & 1 & 0 & 0 \dots \\ \vdots & & \ddots & & \\ 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix} = \begin{bmatrix} \underset{(k-1)\times 1}{0} & I_{(k-1)} \end{bmatrix}$$

always equal to the row of β

$$\underset{(k-1)\times 1}{c} = \underline{0}$$

why this is an F distribution if we not divide for the variance of the numerator?

$$\frac{\overbrace{(R\hat{\beta}-c)'(\sigma_\varepsilon^2 R(X'X)^{-1}R')^{-1}(R\hat{\beta}-c)}/k-1}{\dfrac{\hat{\varepsilon}'\hat{\varepsilon}}{N-k}\cancel{\sigma_\varepsilon^2}} \sim F(k-1, N-k)$$

CAREFUL TO NOT SWAP THESE!

we can also do univariate test, on each of the coefficients

$Y = \beta_1 + \beta_2 X + \varepsilon$

test $\beta_2 = 0 \longrightarrow$ $H_0: \beta_2 = 0$

$H_1: \beta_2 \neq 0$

I can use the F-test, it works also with a single coefficient

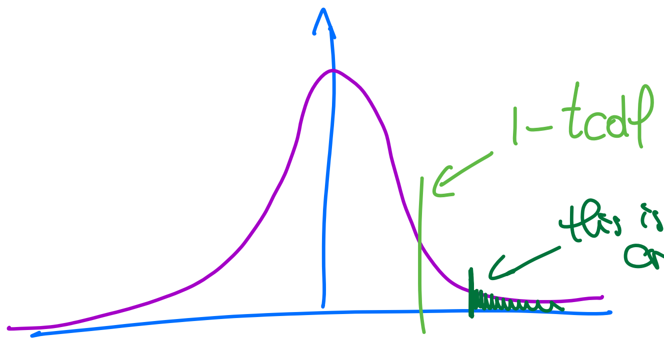
but we can use the t-test for a single restriction

$$t\text{-test} = \frac{\hat{\beta}_2}{S.E.\,\hat{\beta}_2} \sim t\text{-distributed}(N-k)$$

WHAT IS $\boxed{SE(\hat{\beta}_2) = \sqrt{s_\varepsilon^2 (X'X)^{-1}_{ii}}}$

t - test is related to signal/noise ratio

in Matlab we computed values for each β, then we have computed the CDF



t - test
↓
How likely this test i got the t-value that i have calculated if the null hypothesis is true?
↓
if $H_0 : \beta = 0$ and the t-test is low
↓
the prob. of getting that number GIVEN that the null hypothesis is true, may be is very small, hence i reject the hypothesis

t-test for an hypothesis test is not $\beta_0 = 0$ is $\dfrac{\hat{\beta} - \text{value to test}}{\hat{\beta}\ S.E}$

t test is used to test single parameters

- if i want to test multiple parameters (test of existence of the model) i use the F-test

$$H_0 = \beta_i = 0 \quad i = 1, \dots, K$$

$$H_1 = \beta_i \neq 0 \quad i = 1 \dots 4$$ → if also one parameter is significantly different from zero i reject the null hypothesis
↓
of course if i have the t-test i already know if some parameters are significantly $\neq 0$

think also to the F-test as a ratio between RSS

restricted is with $\beta_1 = \dots \beta_4 = 0$

$$\frac{RSS_{restricted} - RSS_{unrestricted} / df_1}{RSS_{unrestricted} / df_2}$$

RSS restricted is larger than unrestricted

~~Exercise~~ – test the hypothesis $\beta_4 = \beta_5 = \beta_6 = 0$

the model has 6 Betas

$$\underset{(3\times6)}{R}\ \underset{(6\times1)}{\beta} = \underset{(3\times1)}{C}$$

$$R = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

point 5 under Cobb → $\beta_{4,5,6} = 0$

test $\beta_3 = 1 - \beta_2 \Rightarrow \beta_2 + \beta_3 = 1$

$$R = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \qquad C = \begin{bmatrix} 1 \end{bmatrix}$$