

$$\text{Var}(\epsilon|x) = \sigma^2 \mathbb{I}_N$$

(N x N)

$$\text{Var}(\beta_{OLS}|x) = \sigma^2 (x'x)^{-1}$$

What if we relax the assumption of homoskedasticity, heteroskedastic setting



→ before it was a matrix with all same variances all the diagonal  
(necessity the matrix is PD)

→ symmetric matrix

→ fully unstructured setting



CHOLVESKY DECOMPOSITION  
OF A PD MATRIX

Good assumption when data are  
cross-sectional

→ if the cov matrix is diagonal  
the chol matrix is like a  
square root

↓  
the results work both ways  
Chol command in R

related to  
square roots  
↓  
they are  
numerical  
operations

## RESULTS

- $\beta_{OLS}$  remains consistent and unbiased also under heteroskedastic settings
- before  $\text{var}(\beta_{OLS}) = \sigma_e^2 (X'X)^{-1}$   
now  $\text{var}(\beta_{OLS}) = (X'X)^{-1} X' \Sigma X (X'X)^{-1}$

① unbiasedness

② consistency

→ the mean of the error term is zero, we are not relating exposure to the error

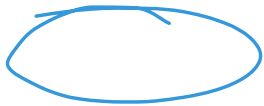
③ what happens to the variance?

$(X'X)$

now we have  
N of those

→ can call it also D in this case because it is diagonal before we were estimating just one parameter with  $s^2$   
the problem is that we do not know  $\sigma_e^2$  and it is difficult to estimate  $\neq \sigma_e^2 (X'X)^{-1}$

How to solve heteroskedasticity



$$\begin{aligned} \text{Var}(C^{-1}e) \\ &= C^{-1} \Sigma C^{-1'} \\ &= \Sigma \varepsilon^{-1} = I_N \end{aligned}$$

$\beta_{GLS}$

$\varepsilon^{-1}$

$\varepsilon^{-1}$

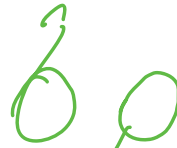
Generalized least  
square estimator

$$\checkmark \textcircled{\varnothing} \checkmark \rightarrow (X' \Sigma^{-1} X)^{-1}$$

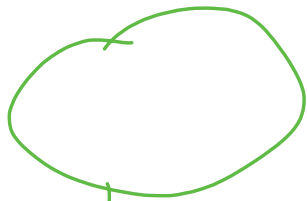
$\Sigma$

But in reality it is difficult to know  $\Sigma$ , how can we estimate it.

h. function



coefficients



this is what we  
reject

What  $w$  looks like depends on  
the stochastic function  
( $w$  is the stochastic function)

$$\text{Var}(\beta_{OLS}) = (X' \Omega^{-1} X)^{-1}$$

$$\text{Var}(\tilde{\beta}) = L \Omega L'$$

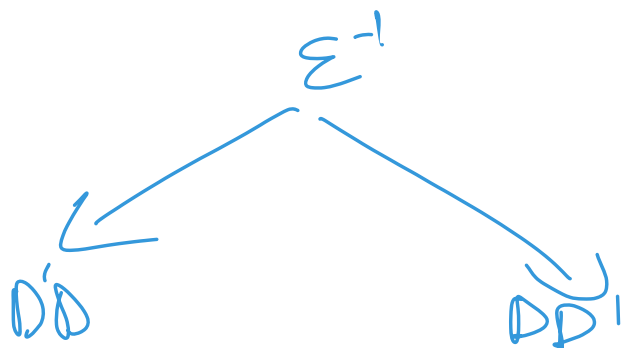
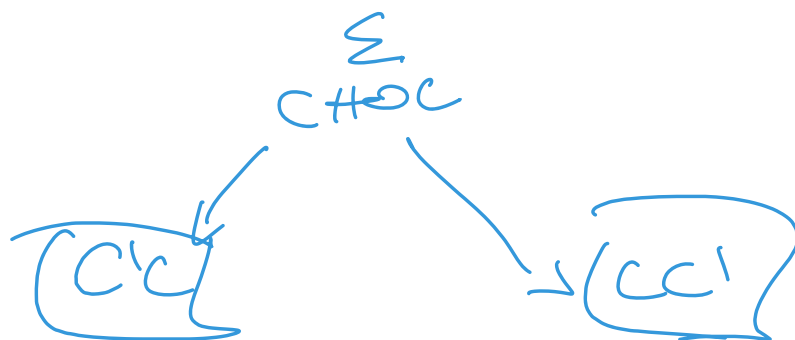
$$\hat{\beta}_{GLS} = \underbrace{(X' \Omega^{-1} X)^{-1} X' \Omega^{-1} y}$$

$$(\tilde{\beta} - \beta_0)(\tilde{\beta} - \beta_0)'$$

$$\tilde{\beta} = \beta_0 + L \varepsilon$$

$$L \underbrace{\varepsilon \varepsilon' L'}_{L \Omega L'}$$

$$L \Omega L' X' \Omega^{-1} \underbrace{(X' \Omega^{-1} X)^{-1} X' \Omega^{-1}}_{\text{GLS}}$$



Given  $y = x\beta + \varepsilon$  in an etwork