Halbert White (1980 Econometrica)

- sandwich estimator of $VAR(\hat{\beta}_{OLS}) \to$ HC0 estimator, also called this way (zero) or robust covariance matrix estimator

PAPER ROMANO - WOLF (JOE) (Journal of economics)

- White test for eteroskedasticity

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starting point $y = X\beta + \varepsilon$ $\quad E[\varepsilon\varepsilon' | X] = \Sigma$ or D (Cross sectional setting)

$$\hat{\beta}_{GLS} = (X'\hat{\Sigma}^{-1}X)^{-1}(X'\hat{\Sigma}^{-1}y)$$

we have to estimate Σ

the question is how to estimate Σ:

WHITE (1980): estimate $\hat{\beta}$ using the OLS $\hat{\beta}_{OLS} = (X'X)^{-1}X'y$ which is unbiased and consistent (also under eteroskedasticity)

in our homoskedastic setting $Var(\hat{\beta}|X) = \sigma_\varepsilon^2 (X'X)^{-1}$ but this is wrong, would lead to wrong inferences, to systematic mistakes (like accepting a coefficient too often or rejecting it over/under rejecting the null hypothesis)

$$HC0: Var(\hat{\beta}|X) = \underbrace{(X'X)^{-1}}(X'\hat{\Sigma}X)\underbrace{(X'X)^{-1}}$$

Quasi-maximum likelihood

$$\hat{\Sigma} = \underset{(N\times N)}{\begin{bmatrix} \hat{\varepsilon}_1^2 & & & 0 \\ 0 & \hat{\varepsilon}_2^2 & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & \hat{\varepsilon}_N^2 \end{bmatrix}}$$

N has to be large for this theorem to work

$y = X\beta + \varepsilon$ is there a way to test for eteroskedasticity? to know if i can use GLS or OLS

- WHITE TEST (based on auxiliary regression)
- BREUSH-PAGAN TEST
- ENGLE TEST (ARCH TEST)

1) estimate $\hat{\beta}_{OLS} = (X'X)^{-1}X'y$

2) $\hat{\varepsilon} = y - X\hat{\beta}$

3) $$\hat{\varepsilon}_i^2 = \gamma_0 + \gamma_1 x_{1i} + \dots + \gamma_k x_{ki} + \gamma_{k+1} x_{1i}^2 \dots + \gamma_{2k} x_{ki}^2$$

$$\boxed{\underset{(N\times 1)}{\hat{\varepsilon}^2} = \underset{N\times(2k+1)}{W}\ \underset{(2k+1)\times 1}{\gamma} + \underset{(N\times 1)}{\mu}}$$

with the xs and the squares of them

$H_0: E[\varepsilon^2] = \sigma_i^2 = \sigma^2 = \gamma_0$ homoskedasticity

all the γ except the intercept one $= 0$

$\gamma_1 = \gamma_2 = \dots \gamma_{2k} = 0$

$H_1: E[\hat{\varepsilon}_i^2] = \sigma_i^2 \rightarrow$ at least one of γ_i is different from zero

White test $\rightarrow N \cdot R^2 \xrightarrow{d} \chi^2(2k)$

sample size

the R^2 that refers to this $\varepsilon^2 = W\gamma + \mu$

then in Matlab $1 - \text{chi2cdf}(NR^2, 2k)$

is the x

we are imposing $2k$ restrictions

we want all the par. to be zero

SUR: seemingly uncorrelated regressions

$i = 1 \dots M \qquad$ CAPM: $E[r_i - r_f] = \beta_i E[r_m - r_f]$

excess return on asset i

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{m,t} - r_{f,t}) + \varepsilon_{i,t}$$

↳ double index

$t = 1 \dots T$

$i = 1 \dots M$

in a simple regression i have n observation k parameters ↓ here i have MT observations for every parameter

$$\Theta = [\alpha_1 \beta_1 \ \alpha_2 \beta_2 \ \dots \ \alpha_M \beta_M]' ; \quad 2M \times 1$$

$$\begin{cases} \underbrace{r_{1,t} - r_{f,t}}_{z_{1,t}} = \alpha_1 + \beta_1 \underbrace{[r_{m,t} - r_{f,t}]}_{z_{m,t}} + \varepsilon_{1,t} & t = 1 \dots T \\ r_{2,t} - r_{f,t} = \alpha_2 + \beta_2 [r_{m,t} - r_{f,t}] + \varepsilon_{2,t} & t = 1 \dots T \\ \vdots \\ r_{M,t} - r_{f,t} = \alpha_M + \beta_M [r_{m,t} - r_{f,t}] + \varepsilon_{M,t} & t = 1 \dots T \end{cases}$$

$$\begin{cases} \underset{(T\times 1)}{z_1} = \alpha_1 + \beta_1 \underset{T\times 1}{\underline{z_m}} + \varepsilon_1 \\ z_2 = \alpha_2 + \beta_2 z_m + \varepsilon_2 \\ \vdots \\ z_M = \alpha_M + \beta_M z_m + \varepsilon_M \end{cases}$$

↳ excess return on the market at each point in time

$\underset{(T\times 2)}{x_1} = [i, z_m]$

$$X = \begin{bmatrix} X_1 & 0 & 0 & 0 \\ 0 & X_2 & 0 & 0 \\ & & \ddots & \\ & & & X_M \end{bmatrix}$$

block of zeros $(T\times 2)$

NOT DIAGONAL MATRIX but BLOCK DIAGONAL, because elements are matrices

$$\underset{(TM\times 1)}{Z} = [z_1 | z_2 | z_3 | \dots | z_M]$$

↓ vertical concatenation

$x_2 = [i, z_m]$ it is $= x_1$

$$\boxed{\underset{(TM\times 1)}{Z} = \underset{(TM\times 2M)}{X} \underset{(2M\times 1)}{\Theta} + \underset{(TM\times 1)}{\varepsilon}}$$

↳ now this is only a linear model

$x_M = [i, z_m]$

$$\begin{cases} \underset{(T\times 1)}{y_1} = \underset{(T\times k_1)}{x_1}\underset{(k_1\times 1)}{\theta_1} + \underset{(T\times 1)}{\varepsilon_1} & \theta_1 \to \alpha_1\beta_1 \\ y_2 = \underset{(T\times k_2)}{x_2}\theta_2 + \varepsilon_2 & \theta_2 \to \alpha_2\beta_2 \\ y_M = \underset{(T\times k_M)}{x_M}\theta_M + \varepsilon_M \end{cases}$$

$(T\times k_2)$ → can be different from those on top

$$\underset{(TM\times 1)}{y} = [y_1 | y_2 | y_3 | \dots | y_M]$$

$$\varepsilon = [\varepsilon_1 | \varepsilon_2 | \dots | \varepsilon_M]$$

$$\theta = [\theta_1 | \theta_2 | \dots | \theta_3]$$

$$\underset{TM\times(k_1+k_2\dots+k_M)}{X} = \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_M \end{bmatrix}$$

$$y = X\theta + \varepsilon$$
