Price Competition Under Limited Comparability

Michele Piccione and Ran Spiegler (2012)

Presented by Silvio Ravaioli

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Motivation

- Firms can manipulate consumers' ability compare product alternatives through choice of price "formats"
- Examples: quantities indicated in different units of measurement, price divided into a large number of contingencies (calling plan, banking services, etc.)
- Apparent incomplete preferences may emerge from inability to make comparisons
- Consumers may display inertia (low switching rates)
- The regulators recognize the importance of comparability for market competition and create standardized information (e.g. food labeling, retail financial services)

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Research Questions

- **What are the implications of limited comparability for the competitiveness of the market outcome?**
- **Do regulatory interventions aimed at enhancing comparability necessarily increase competitiveness?**
- Measures of competitiveness of the market:
- Equilibrium profits (max-min profit as "constrained" second best for competition)
- Consumers' inertia and switching probability

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Model Overview

- Extension of Varian 1980 (price dispersion) with endogenous limited comparability between firms
- A single consumer wishes to consume one unit and has a reservation value equal to one
- Two profit-maximizing firms produce perfect substitutes at zero cost: symmetric simultaneous game, each firm chooses price and format
- Consumers' ease of comparison depends on format decisions
- Consumers are randomly assigned a default option
- Conditional on comparison, choose the cheapest option
- If comparison is not possible, keep buying the default product

Preview of Results

- Firms choose the format endogenously to reduce comparability
- Limited comparability lead to constrained minimum profits bounded away from zero
- Market forces drive down the firms' profits to a constrained competitive profit level iff the comparability structure have a particular property (weighted regularity)
- Narrow regulatory interventions that aim to facilitate comparisons may have an anticompetitive effect, reduce comparability in equilibrium, and increase inertia

Model Setup

- Pure strategy for firm i is (x_i, p_i), with format $x \in X$ finite, and $p \in [0, 1]$ price
- Measure of comparability of formats $\pi : X \times X \to [0, 1]$
- Assume that $\pi(x, x) = 1$ for every format
- Comparability Structure (X, π) [CS in short]
  CSs can be represented as random directed graphs (X nodes, π directed links)
- The consumer is randomly assigned to a firm i (equal prob.)
- She makes a price comparison with probability $\pi(x_i, x_j)$
- Only if she can compare, she switches to j if $p_j < p_i$

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Main assumptions

- Order-independent comparability structure: $\pi(x_i, x_j) = \pi(x_j, x_i)$
- Simultaneity of price and format decisions [discussion section]
- Exogeneity of the comparability structure [partial equilibrium]
- Focus on symmetric Nash Equilibria. Many asymmetric equilibria are possible, some conjectures about results
- Formats are utility-irrelevant
- Comparability depends only on formats, not on actual prices
- Format and prices are chosen independently (no restrictions)
- Firms cannot charge prices $p > 1$

Pure and Mixed Strategies

- Given a strategy profile $\{(x_i, p_i)\}_i$ firms' payoffs are
  - $\frac{p_1}{2}$ if $p_1 = p_2$
  - $p_1 \frac{1+\pi(x_2,x_1)}{2}$ if $p_1 < p_2$
  - $p_1 \frac{1-\pi(x_2,x_1)}{2}$ if $p_1 > p_2$

- Mixed strategy $(\lambda, (F^x)_{x\in X})$
  - $\lambda \in \Delta(X)$ marginal format strategy
  - F^x is the pricing cdf conditional on format x

- When j plays the MS $(\lambda_j, (F_j^x)_{x\in X})$ and i plays the PS (x_i, p_i)

  E. Profit for $i = \frac{p_i}{2}\left(1+\Sigma_y \lambda_j(y)[(1-F_j^y(p_i))\cdot\pi(y, x_i)-F_j^{y-}(p_i)\cdot\pi(x_i, y)]\right)$

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Hide and Seek Game

- Introduce an auxiliary two-player, zero-sum game, associated with each CS (X, π) [generalized Matching Pennies]
- The players (hider and seeker) are NOT the firms
- Given the action profile (x_h, x_s) the payoffs are
  - Hider: $-\pi(x_h, x_s)$ Seeker: $+\pi(x_h, x_s)$
- Given a mixed strategy profile (λ_h, λ_s) the seeker's payoff is

$$\upsilon(\lambda_h, \lambda_s) = \Sigma_x \Sigma_y \lambda_h(x) \lambda_s(y) \pi(x, y)$$

- This is a finite zero-sum game, so we can use minimax theorem to find the seeker's payoff in all NE

$$\upsilon^* = max_{\lambda_s} min_{\lambda_h} v(\lambda_h, \lambda_s) = min_{\lambda_h} max_{\lambda_s} v(\lambda_h, \lambda_s)$$

- Max-min payoffs represents "constrained" competitive profits

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Equilibrium Analysis under Order Independence

- Symmetric equilibrium under order-independent CSs
- We are interested in the CS that allow a format strategy that induces a constant comparison probability (neutralize format)
  - Simplified example with $|X| = m > 1$ and $\pi(x, y) = 0$
- An order-independent comparability structure (X, π) is weighted-regular if there exist $\beta \in \Delta(X)$ and $\bar{v}$ such that $\forall x$ $\Sigma_y \beta(y)\pi(x, y) = \bar{v}$
- Weighted regularity generalizes the notion of regular graphs [graph where each vertex has the same number of neighbors]
  - Draw some examples on the blackboard
- The distribution $\lambda \in \Delta(X)$ verifies weighted regularity iff (λ, λ) is a NE in the hide-and-seek game

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Main Results - Theorem 1

- Equivalence between weighted regularity and max-min profits
- **Theorem 1**: In any symmetric equilibrium, firms earn max-min payoffs iff (X, π) is weighted-regular. Furthermore, if (X, π) is w-r, then in any symmetric Eq. each firm's marginal format strategy verifies w-r [i.e. makes the opponent indifferent].
- **Intuition**: when firms earn max-min payoffs, the marginal format strategy max-minimizes the comparison probability.
- When a firm charges an high price, it wants to hide own price (and viceversa).
- When the CS is not w-r, acting like a hider or like a seeker are distinct strategies.
- When the CS is w-r, firms are both hiders and seekers

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Main Results - Corollary 1

- **Corollary 1**: Suppose that (X, π) is weighted-regular. Then, in any symmetric equilibrium, firms play a marginal pricing strategy given by the cdf

$$F^*(p) = 1 - \frac{1-\upsilon^*}{2\upsilon} \cdot \frac{1-p}{p}$$

defined over the support $\left[\frac{1-\upsilon^*}{1+\upsilon^*}, 1\right]$

- Since every price p in the support of the equilibrium marginal pricing cdf F generates the same payoff
- Under w-r, equilibrium market share is determined as a function of prices charged given the constant comparison probability
- Same result as in Varian (1980), given v^*

Main Results - Theorem 2

- Relation between comparison probability and price realizations
- **Theorem 2**: A symmetric equilibrium exhibits a constant comparison probability [i.e. $v(\lambda^I, \lambda^J)$ is the same for every pair of closed intervals I, J] iff (X, π) is weighted-regular. Furthermore, if (X, π) is weighted-regular, the constant equilibrium comparison probability if v^*
- **Intuition**: under w-r format and prices are uncorrelated in the symmetric equilibrium
- When w-r is violated, price and formats must be correlated
- Correlation is also possible in asymmetric equilibria

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Main Results - Comparability Increase

- Effect of a change in the comparability structure
- From (X, π) to (X, π') with $\pi'(x, y) \geq \pi(x, y)$
- If the change is exogenous, it is beneficial for consumers
- If firms can adjust their strategies, it may not be
- The effect depends whether (X, π') is weighted-regular
- If the new CS is not w-r, the intervention may rise equilibrium profits
  - If formats are divided into two groups (e.g. star graph) firms may have an incentive to move to the complex formats
- Similar results for consumer switching: under w-r the switching rate is $\frac{1}{2}\upsilon^*$, when w-r is violated the outcome is ambiguous

Extra - Violation of Transitivity

- From a revealed-preference point of view, this choice model is not consistent with rational choice, as we can get intransitive revealed-preferences
- Consider the order-independent CS: $X = \{a, b, c\}$, $\pi(x, y) = 1$ for all x, y except for $\pi(a, c) = 0$. Suppose that $p < p' < p''$. We obtain the transitivity violation

$$(a, p) \succ (b, p') \succ (c, p'') \sim (a, p)$$

- The revealed strict preference relation is transitive iff the graph of the CS represents an equivalence relation over X

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