Noisy Integration of Value Differences

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Micro Theory Colloquium

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Today's Presentation

- **Experimental project**
- Simple setting: binary choice, multidimensional options
- Motivation and Background literature
- Experimental design
- Descriptive results
- Model fitting

Binary Choice with Multidimensional Goods

- Evaluate two options that differ across multiple dimensions

$$i \in \{L, R\} \qquad X_i \equiv \{x_{i,t}\}_{t=1}^{T} \qquad x_{i,t} \sim F(\cdot)\ \forall i, t$$

- The *true* value is linear in the dimensions' values (average)

$$v(X_i) = \frac{1}{T}\sum_{t=1}^{T} x_{i,t}$$

- The *perceived* value may be different from the true one (measurement error, perceptual noise, imperfect memory, etc.)
- Comparison by dimension: sequence of pairs $\{(x_{L,t}, x_{R,t})\}_t$

- **How do agents evaluate the vectors of dimensions?**

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Preview of Main Results

- **Laboratory experiment design to explore:**
  - Noisy integration of values under “full information”
  - Integration of available and *shrouded* values
  - Endogenous searching

- Stochastic choice and violation of stochastic transitivity
- Similarity increases accuracy
- Systematic bias in information integration and search
- Context effect consistent with *salience*

- **Biased choice pattern determined by prior value distribution and varying perceptual error**

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Motivation and Background Literature

- **Multidimensional pricing strategy**
  - Ellison (2005), Spiegler (2006), Brown, Hossain & Morgan (2010), Gabaix & Laibson (2006) [shrouded attributes]
- **Noisy integration of decision information**
  - Human bias in averaging tasks: Tsetsos et al. (2016), Spitzer, Waschke & Summerfield (2017), Li et al. (2018)
- **Context effect and Violation of stochastic transitivity**
  - Stochastic transitivity violation would not occur if information was encoded in isolation
  - Vast and discordant literature: Tversky & Simonson (1993), Kivetz et al. (2004), Soltani, de Martino & Camerer (2012), Bordalo, Gennaioli & Shleifer (2013), Koszegi & Szeidl (2013), Bushong et al. (2017), Natenzon (2018), Landry & Webb (2019)

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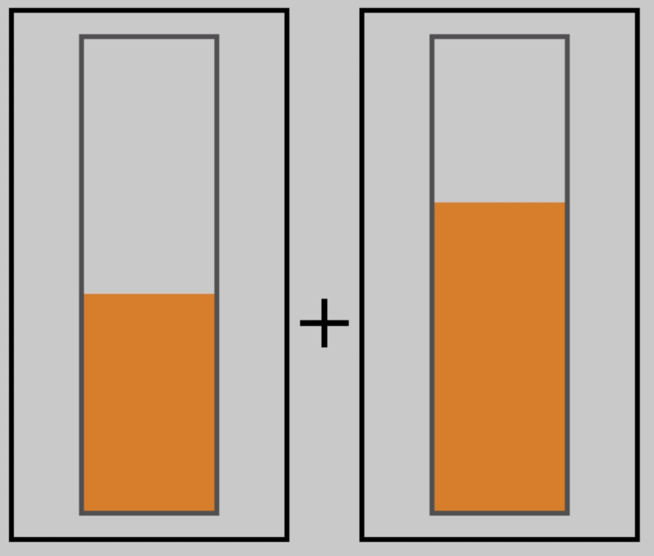
Experimental Design - Main Task

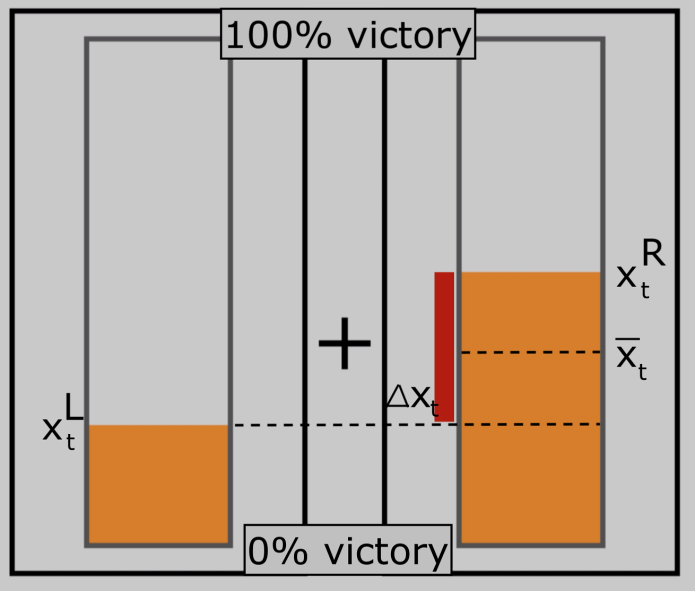


Experimental Design

- **Binary choice**: compound lottery L(eft) vs R(ight)
- Six simple lotteries (dimensions) equally likely to be selected
- Each sub-lottery is a 10-90% probability of winning one point

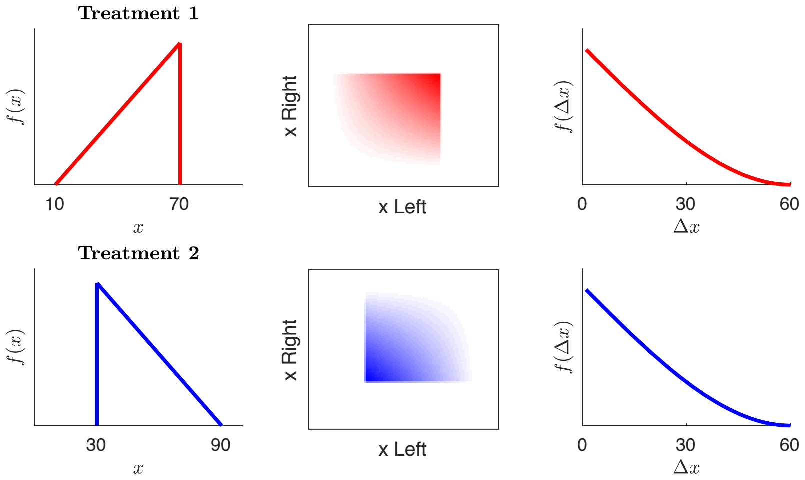
- Lab experiment at CELSS (Columbia University)
- 800 trials in a session ($\sim$ 80 min), including 2 ancillary tasks
- Incentive: collect victories (points) across the experiment
- Payment: (# points - 300) $\cdot$ 20 ¢ Avg. payment $23.60

Experimental Design



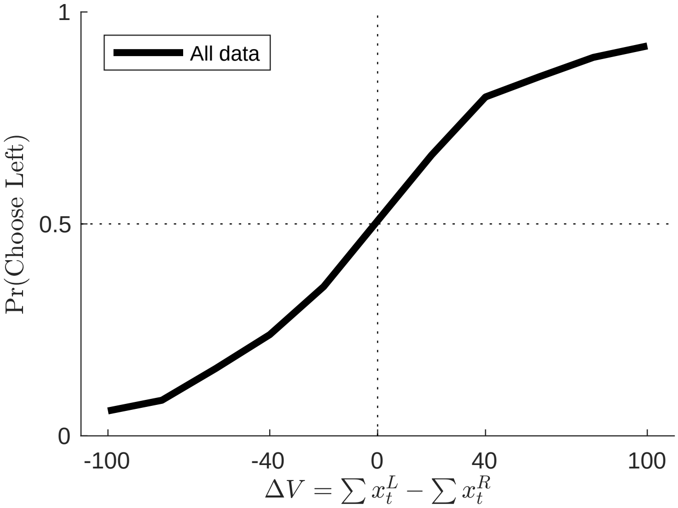
Treatments - Upward/Downward distributions

- Upward and Downward triangular distributions



Value distributions used to generate data in the two treatments

Result 0. Stochastic Choice



Choice probability in trials with different difficulty

Noisy perception models (1)

At time $t \in 1, ..., 6$ two values x_t^L and x_t^R are observed

Model 1 - Constant noise level

- Mental representation of each value $\hat{x}|x \sim N(x, s^2)$
- Leaking memory: discount previous dimensions by δ
- The perceived value of X_i is $\sum_{t=1}^{T} \delta^{T-t} \cdot \hat{x}_{i,t}$
- The agent chooses the option with the highest perceived value
- Calculate $Pr(\text{Choose } L|X_L, X_R, \delta, s^2)$

Calibrated model: BIC: 15,972 (all data), 15,969 (separate treatments)
Rescaled variance $s^2 = 0.094$, leaking memory $\delta = 0.84 < 1$

Noisy perception models (2)

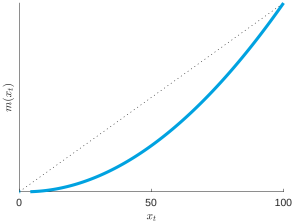
At time $t \in 1, \ldots, 6$ two values x_t^L and x_t^R are observed

Model 2 - Varying noise level

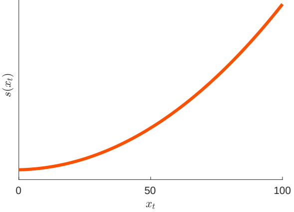
- Mental representation of each value $\hat{x}|x \sim N(m(x), s(x))$
- $m(x)$ **and** $s(x)$ **are degree 3 polynomia**
- Leaking memory: discount previous attributes by $\delta < 1$
- The perceived value of X_i is $\sum_{t=1}^{T} \delta^{T-t} \cdot \hat{x}_{i,t}$
- The agent chooses the option with the highest perceived value

Calibrated model [see next slide]
BIC: 17,929 (all data), 17,900 (separate T1/T2) [high BIC = worse fit]

Noisy Perception Models (2)



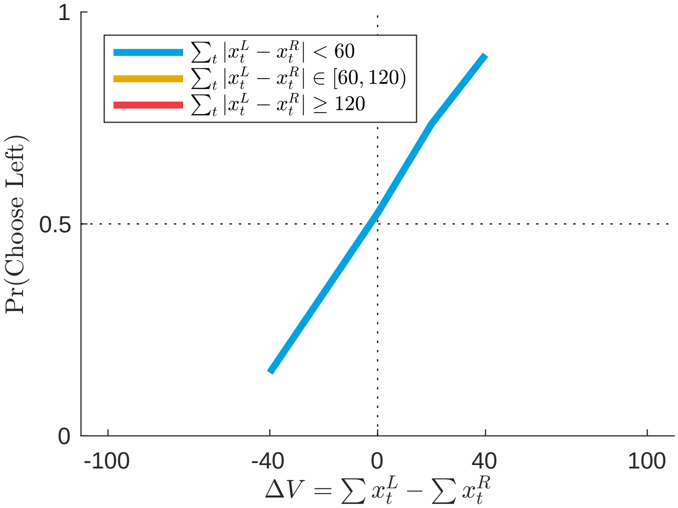
Transformation $m(x)$



Varying noise $s(x)$

- $m(x)$ transformation in the figure is a polynomial of degree 3
- Apparent *focusing* in favor of high values
- Standard deviation is increasing in x

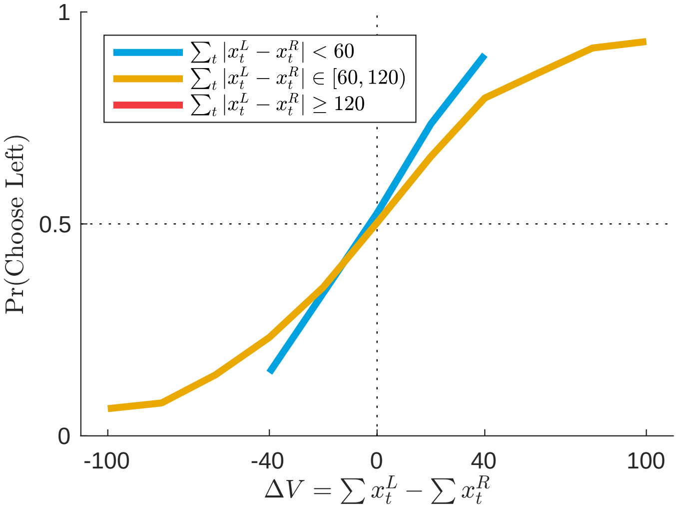
Result 1. Similarity improves Accuracy



Choice probability, after controlling for similarity

Appendix Figures

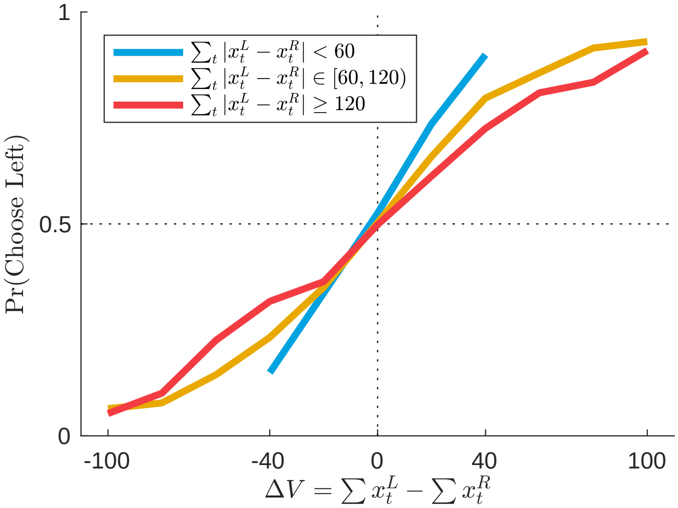
Result 1. Similarity improves Accuracy



Choice probability, after controlling for similarity

Appendix Figures

Result 1. Similarity improves Accuracy



Choice probability, after controlling for similarity

Appendix Figures

Noisy Perception Models (3)

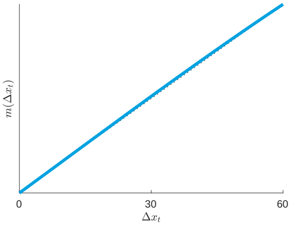
At time $t \in 1, \ldots, 6$ two values x_t^L and x_t^R are observed

Model 3 - Noisy encoding of difference $\Delta x_t := x_{L,t} - x_{R,t}$

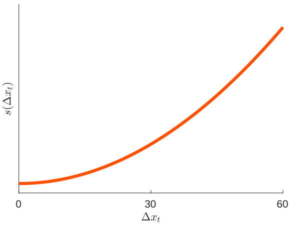
- Mental representation of value difference
  $\Delta\hat{x}|\Delta x \sim N(m(\Delta x), s(\Delta x))$
- $m(\Delta x)$ **and** $s(\Delta x)$ **are degree 3 polynomia**
- Leaking memory: discount previous attributes by $\delta < 1$
- The perceived difference for $v(X_L) - v(X_R)$ is $\sum_{t=1}^{T} \delta^{T-t} \cdot \Delta\hat{x}_t$
- The agent chooses the option with the highest perceived value

Calibrated model [see next slide]
BIC: 15,801 (all data), 15,826 (separate treatments)

Noisy Perception Models (3)



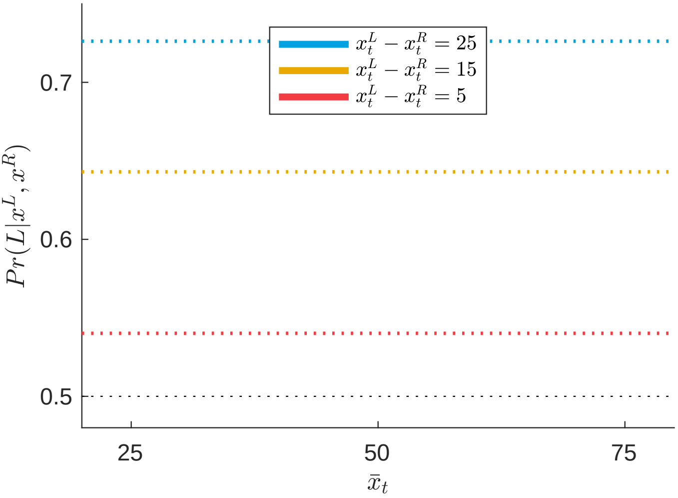
Transformation $m(\Delta x)$



Varying noise $s(\Delta x)$

- $m(\Delta x)$ transformation in the figure is a polynomial of degree 3
- Standard deviation is increasing in the Δx

Result 2. Decision Weights



Decision weight $Pr(L|x^L, x^R)$ for different magnitudes $\bar{x}$ and differences Δx

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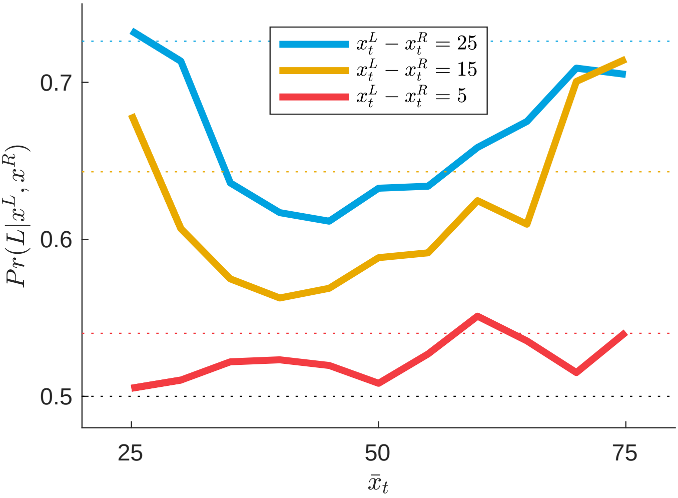

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Result 2. Decision Weights

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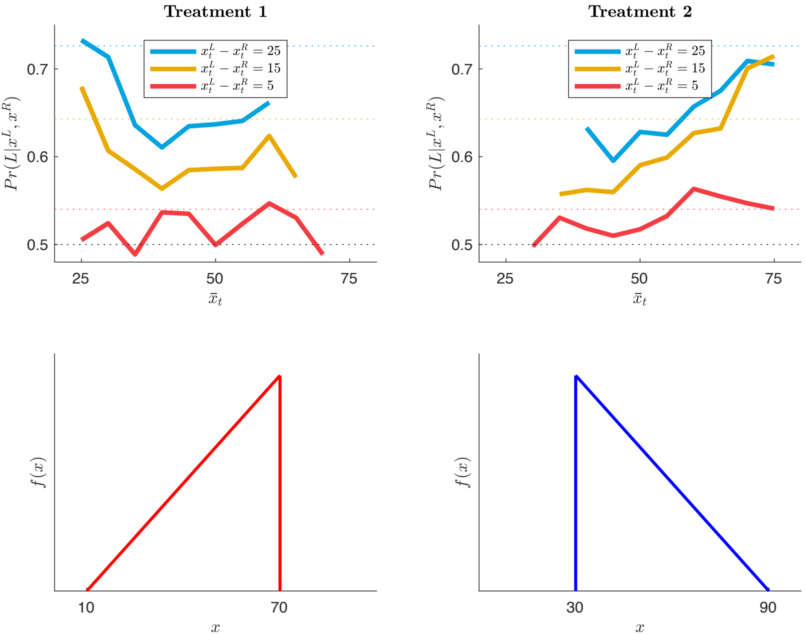
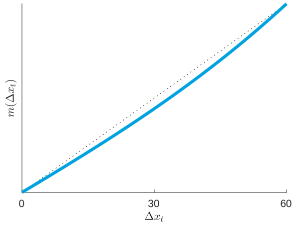


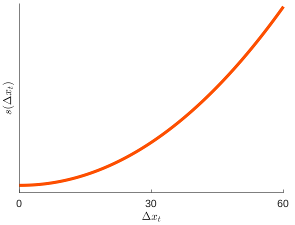
Noisy Perception Models (4)

- At time $t \in 1, \ldots, 6$ two values x_t^L and x_t^R are observed
- Mental representation of the difference $\Delta x_t := x_t^L - x_t^R$
  - Noisy representation $\Delta\hat{x}|\Delta x \sim N(m(\Delta x) \cdot \bar{x}_t^{\mu+1}, s(\Delta x) \cdot \bar{x}_t^{\sigma+1})$
  - Transformation $m(\Delta x)$, degree 3 polynomial
  - Varying noise $s(\Delta x)$, degree 3 polynomial
- Choice based on $\Delta V := \sum_{t=1}^{T} \delta^{T-t} \cdot \Delta\hat{x}_t$
- Weight/accuracy may differ between high/low values: μ and σ
- Leaking memory: $\delta < 1$

Noisy Perception Models (4)



Transformation $m(\Delta x)$



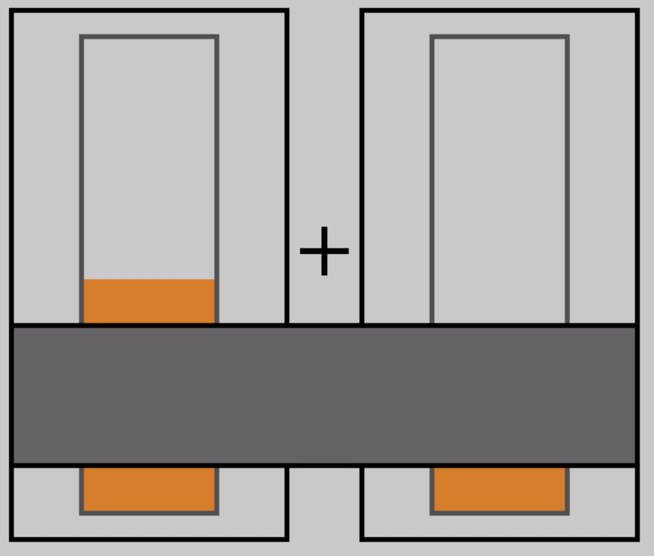
Varying noise $s(\Delta x)$

- Leaking factor $\delta = 0.82 < 1$ [0.81 in T1, 0.84 in T2]
- Focusing effect (mean) $\mu = 0.60 > 0$ [-0.01 in T1, 1.18 in T2]
- Focusing effect (variance) $\sigma = 0.74 > 0$ [-0.21 in T1, 0.95 in T2]

Noisy Perception Models - BIC summary table

Model	Merge T1+T2	Separate T1/T2
Noisy perception $N\left(x, s^2\right)$	15,972	15,969
Transformation of x $N\left(m(x), s(x)^2\right)$	17,929	17,900
Transformation of Δx $N\left(m(\Delta x), s(\Delta x)^2\right)$	15,801	15,826
Focusing based on $\bar{x}$ $N\left(m(\Delta x) \cdot \bar{x}^{\mu-1}, (s(\Delta x) \cdot \bar{x}^{\sigma-1})^2\right)$	15,525	15,520

Experimental Design - Task 2

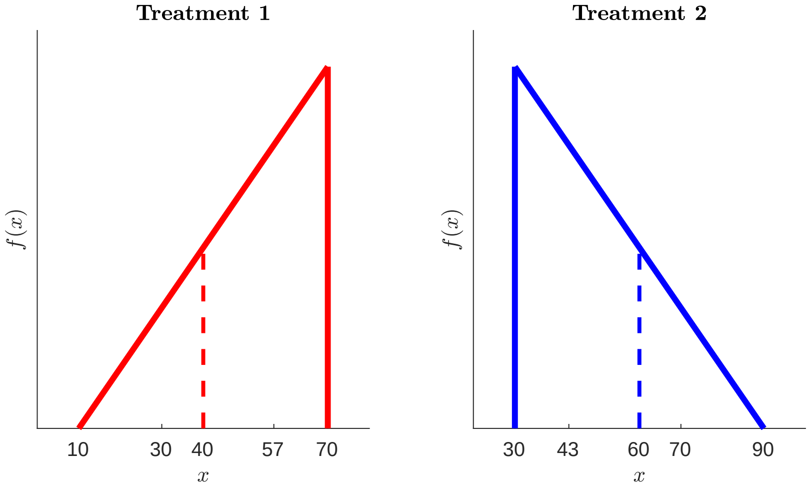


Experimental Design - Task 2

- *Exogenous* information restriction: part of the screen is obscured by a fixed rectangle
- Two conditions: upper or lower range visible
- 400 trials, divided into 20 blocks

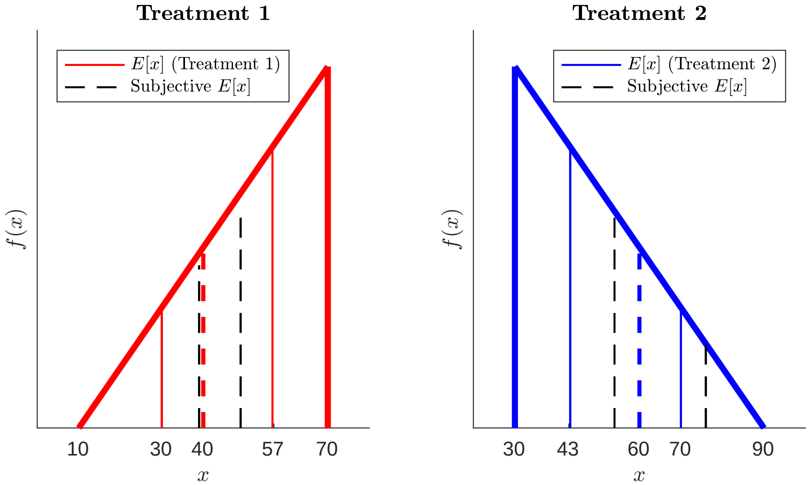
- Motivation: shrouded attributes
- Example: seller advertises low prices only

Result 3. Fitted Values for Missing Observations



Task 2, two conditions (high/low values hidden) - Optimal vs. data

Result 3. Fitted Values for Missing Observations



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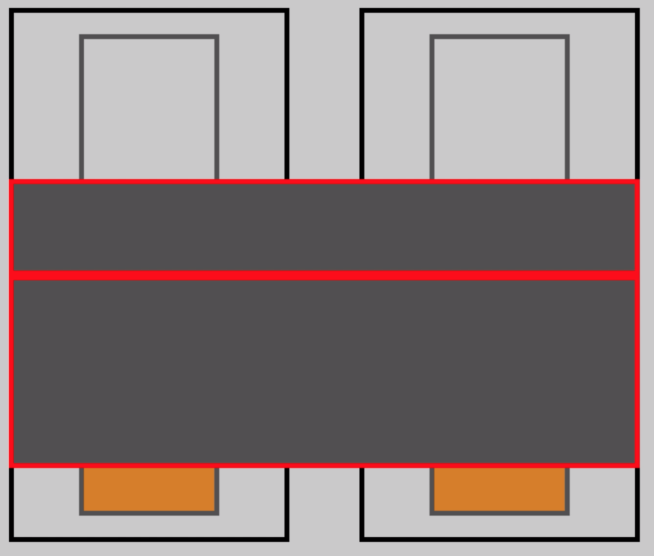
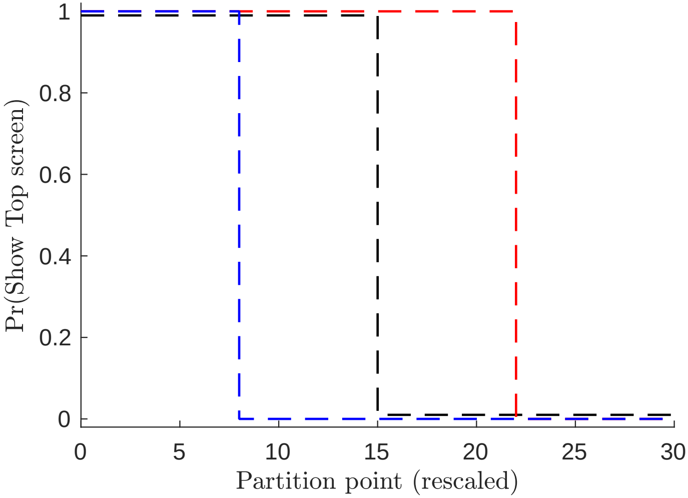


Experimental Design - Task 3

- *Endogenous* information restriction
- As in task 2, part of the screen is obscured by a fixed rectangle
- Now the participant chooses the part of the screen to observe
- Partition value $x^* \sim U[min\{x\}+10, max\{x\}-10]$
- 100 trials at the end of the session

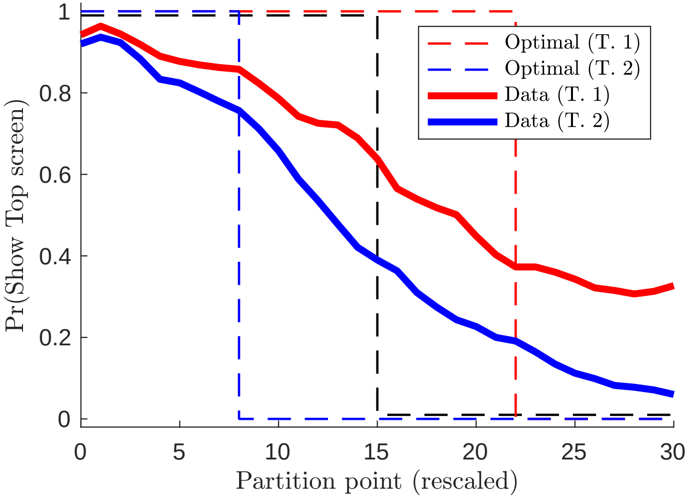
- Motivation: searching process
- Example: consumer filtering positive/negative reviews

Result 4. Endogenous Information Acquisition



Task 3 - Probability of revealing the upper part of the screen

Result 4. Endogenous Information Acquisition



Task 3 - Probability of revealing the upper part of the screen

Summary

- Laboratory experiment designed to explore choice between multidimensional options
- Controlled environment, direct comparison and noisy evaluation
- **Biased choice pattern determined by prior value distributions and varying perceptual error**
- Direct comparison: no encoding of individual values
- Context effect: focusing effect consistent with *salience*

Next Steps

Current to-do list

- Explore individual-level heterogeneity
- Connect further the results in main and ancillary tasks
- Model fitting and comparison

More ambitious applications?

- Strategic setting: sophisticated firm and biased consumers
- Extension with outside option and/or $N > 2$ options
- Empirical application

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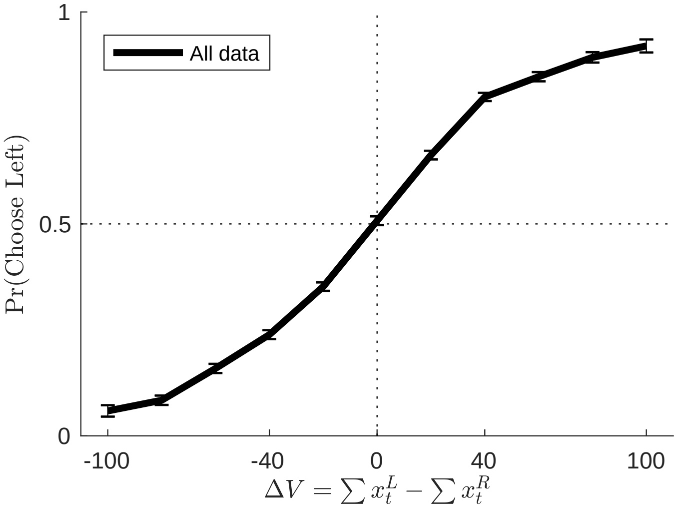
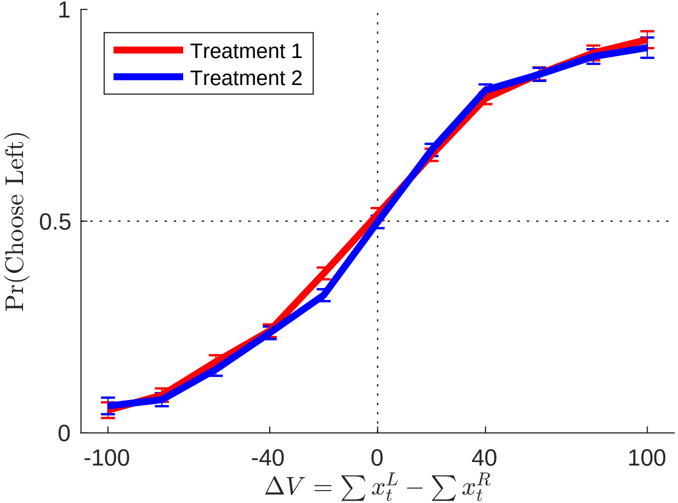
Standard Errors



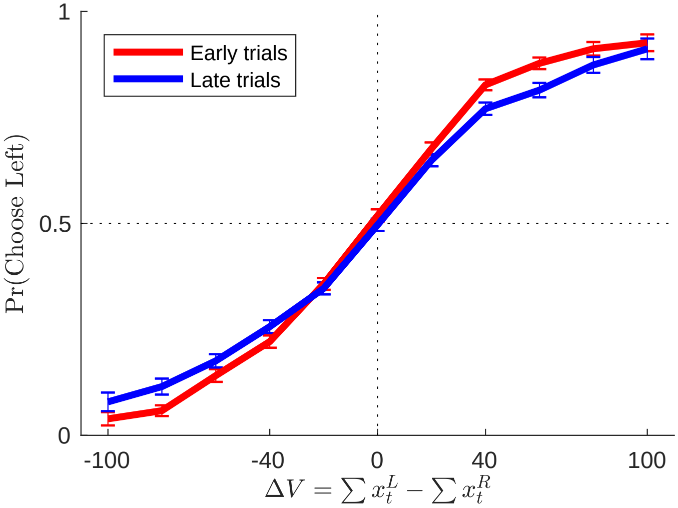
Choice probability in trials with different difficulty. Avg. accuracy 74%.

Treatment effect



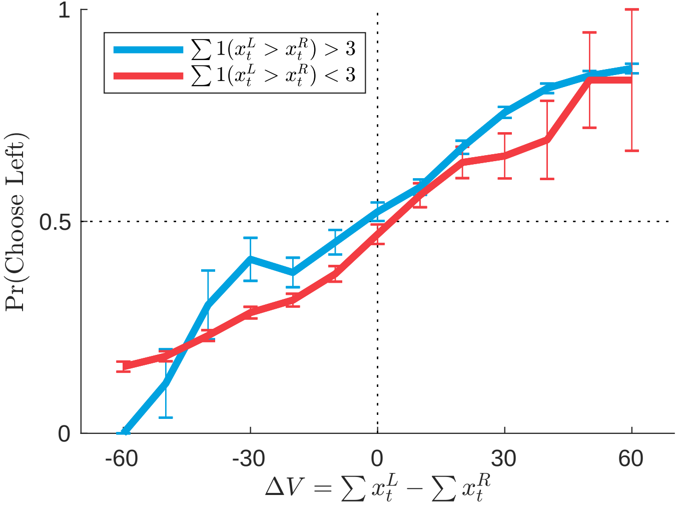
Choice probability in trials with different difficulty. Observations are grouped by treatment. Avg. accuracy 73.03% vs 74.36%.

Learning effect



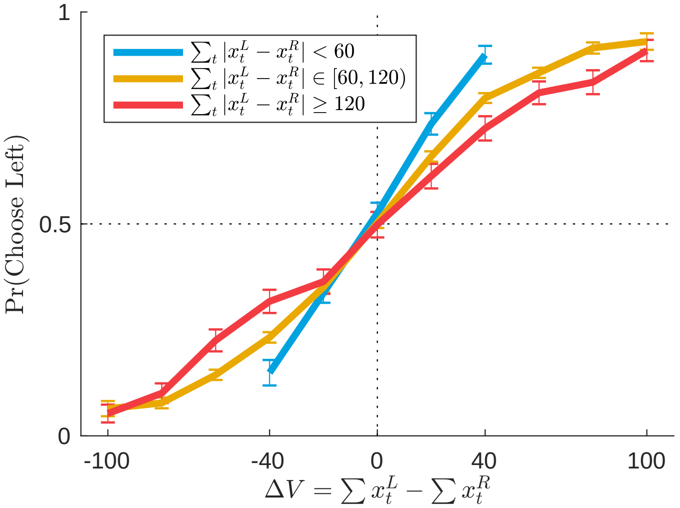
Choice probability in trials with different difficulty. Observations are divided into early (1-150) and late trials (151-300). Avg. accuracy diminishes from 74.71% to 72.65%.

Violation of stochastic transitivity



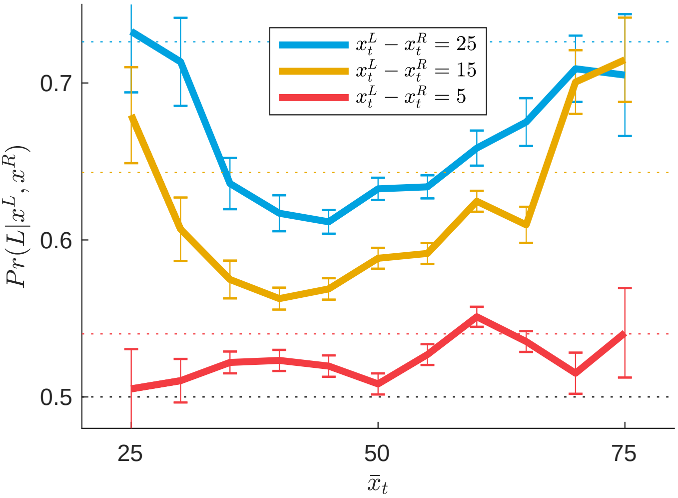
Choice probability in trials with different difficulty. Trials are grouped by number and direction of Frequent Local Winners (FLWs).

Standard Errors



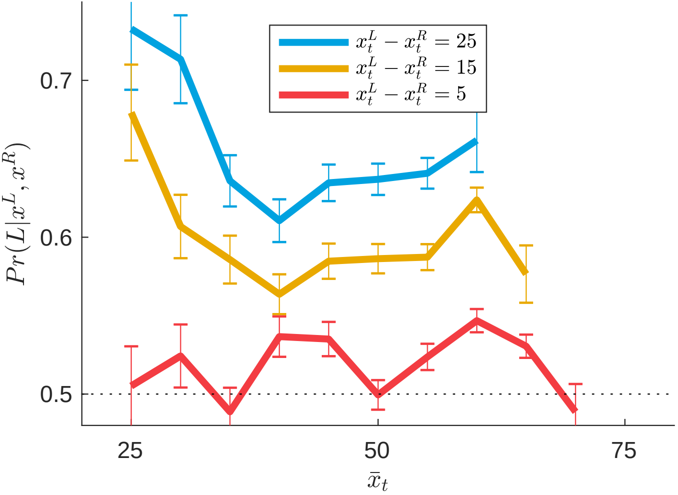
Choice probability, after controlling for similarity.

Standard Errors (all data)



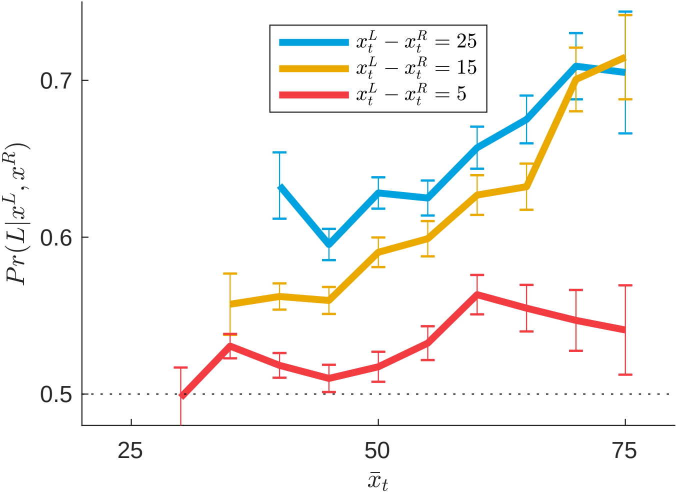
Decision weight $Pr(L|x^L, x^R)$ for different magnitudes $\bar{x}$ and differences Δx

Standard Errors (treatment 1)



Decision weight $Pr(L|x^L, x^R)$ for different magnitudes $\bar{x}$ and differences Δx

Standard Errors (treatment 2)



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