State Pooling and Belief Polarization

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Columbia University - Cognition and Decision Lab Meeting

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Today's Presentation

- **Motivating Example**
- **Research Question**
- Related Literature
- State Pooling Model
- **Laboratory Experiment**

Motivating Example

Motivating Example

Setting

- Alice and Bob face a choice: go to the Theater or stay Home
  - Theater: uncertainty about the quality of the movie [state s]
  - Home: "safe" choice [status quo]

	Theater	**Home**	
s	$v_i^T(s)$	$v_A^H(s)$	$v_B^H(s)$
bad	0	0.45	0.55
medium	0.5	0.45	0.55
good	1	0.45	0.55

- Assume uniform prior $p_s = \frac{1}{3}$ and risk neutrality

Motivating Example

Scenario 1

- Alice and Bob have the same beliefs over s and $EV(\text{Theater})$
  - $EV(\text{Theater}) = 0.5$
- Alice and Bob make different choices
  - A chooses Theater as $0.5 = EV(\text{Theater}) > EV(\text{Home}) = 0.45$
  - B chooses Home as $0.5 = EV(\text{Theater}) < EV(\text{Home}) = 0.55$

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Motivating Example

Scenario 2

- Same problem as before, but now A and B can collect “some” information about the movie quality
- Note that we have 2 actions (T/H) and 3 states (b/m/g)
  - For Alice it is *sufficient* to know if the movie is b or (m/g)

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Motivating Example

Scenario 2

- If the movie is good (bad) they agree about the action Theater (Home)
- But they do not agree about the expected quality of the movie
  - Good movie: $EV_A(T|g) = 0.75 < EV_B(T|g) = 1$
  - Bad movie: $EV_A(T|b) = 0 < EV_B(T|b) = 0.25$

- If the movie is medium they still disagree about the action
- But they also disagree about the expected quality of the movie
  - Alice chooses Theater: $EV_A(T|m) = 0.75$
  - Bob chooses Home: $EV_B(T|m) = 0.25$

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Motivating Example

Summary

- Alice and Bob have the same prior beliefs
- The introduction of **endogenous information collection** created disagreement about movie quality

- **State pooling**: agents avoid redundant information when the action space is smaller than the state space
- **Belief polarization**: posterior beliefs are more distant (extreme) than prior beliefs

Puzzle

- Society today is more polarized (McCarty et al. 2006)
- Information is more easily accessible (lower cost)
- If a "true state" exists, beliefs should converge, right?
- Not necessarily true if information collection is endogenous

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Research Question

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Can endogenous information acquisition provide an explanation for belief polarization?

Broad question that includes prior heterogeneity, update heterogeneity, confirmatory/contradictory strategies, etc.

How do DMs evaluate and choose information sources?

Test whether agents:

- Seek information based on the impact on their action
- Ignore information without instrumental value (state pooling)

Jump to the Experimental design

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Jump to the Experimental design

Related Literature

Related Literature

- **Polarization** is widely studied phenomenon
- **Information and belief polarization**: McCarty, Poole and Rosenthal (2006), Boxell, Gentzkow and Shapiro (2017)
- Explanations for polarization based on **exogenous** information and/or exogenously imposed biases: Rabin, Schrag (1999); Fryer, Harms, Jackson (2017), Wilson (2014), Lord, Ross, Lepper (1979) [confirmation bias], Ortoleva and Snowberg (2015) [overconfidence and correlation neglect], Klayman and Ha (1987), Nickerson (1998) [positive test strategy]
- Confirmation bias and **rational inattention**: Su (2014), Nimark and Sundaresan (2018), Dixit and Weibull (2007) [prior heterogeneity]

▸ Related Literature

Experimental Literature

1. **Ambuehl and Li (2018)** Design AL18
   - Systematic analysis of belief updating and demand for info.
   - Compression effect: subjective valuation of useful information underreacts to increased informativeness
   - Biases mainly due to non-standard belief updating rather than risk preferences
2. **Charness, Oprea, Yuksel (2018)** Design COY 18
   - Study how people choose between biased information sources
   - Evidence of confirmation-seeking rule
   - Mistakes driven by errors in reasoning about informativeness
3. Vast experimental literature about belief updating
   - Heterogeneity in belief updating: El-Gamal and Grether 1995, Fehr-Duda and Epper 2012, Augenblick and Rabin 2015, Buser et al 2016, Antoniou et al 2017.
   - Biases in demand for information: Eli and Rao 2011, Mobius et al 2011, Bursks et al 2013, Oster et al 2013, Sicherman et al 2015

"Simplified" State Pooling Model

Model

- Simplified RI model (Matveenko and Novak)
- Stage 1: collect information, Stage 2: choose an action

- $N > 2$ possible states of the world $s \in \{1, ..., N\}$
- Binary action $a \in \{1, 2\}$
- Risky action ("stock/reform"), safe action ("bond/status quo")

- Risky action
  - value v_s, where $s \in 1, \ldots, N$
  - $v_i < v_j$ for $i < j$
- Safe action
  - value B independent from s
  - Assumption: $v_1 < B < v_N$

Model

- p_s - correct prior belief state s realized, with $\sum_{s=1}^{n} p_s = 1$
- **Stage 1: collect information**
  - Choose one "advisor" $(\pi_e, c_e) \in \{(\pi_e, c_e)\}_e$ [experiment-cost]
  - Pay the cost c_e to observe the experiment π_e
- **Observe signal realization and update beliefs**
- **Stage 2: choose one action**
  - Choose action $a \in \{1, 2\}$
  - Safe action (return B) and risky action (return v_s)

Model - 3 states, 2 signal realizations

- The experiment π_e can generate only two signals $\sigma \in \{1, 2\}$ and is defined by the triplet $\pi(\sigma = 1|s)$
- The instrumental value of a signal structure π_e is

$$U(\pi_e) = \underbrace{\sum_{\sigma} v^*(\{p(s|\sigma)\}_s)\pi(\sigma)}_{\text{EV with } \pi_e} - \underbrace{v^*(\{p(s)\}_s)}_{\text{EV w/o } \pi_e}$$

where v^* is the expected value of the optimal action (conditional on available information)

- **Stage 1: collect information**
  - A rational agent chooses the signal structure

$$e^* = \text{argmax}_e U(\pi_e) - c_e$$

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- We can simplify further the calculation of the value

- *Irrelevant* experiments have $a^*(\varnothing) = a^*(\sigma = 1) = a^*(\sigma = 2)$

$$U(\pi_e) = 0$$

- *Relevant* experiments have wlog $a^*(\varnothing) = a^*(\sigma = 1) \neq a^*(\sigma = 2)$

$$U(\pi_e) = Pr(\sigma = 2) \cdot \Big(E[v(a^*(\sigma = 2))|\sigma = 2] - E[v(a^*(\sigma = 1))|\sigma = 2] \Big)$$

▸ Appendix - Full model and main results

Simplified Environments

- Consider only pairs of advisors $\{(\pi_1, c_1), (\pi_2, c_2)\}$

- In our experiment we focus on two simple cases:

- $c_1 = c_2 = 0$ both signal structures are free
  - **The DM selects the most informative advisor**

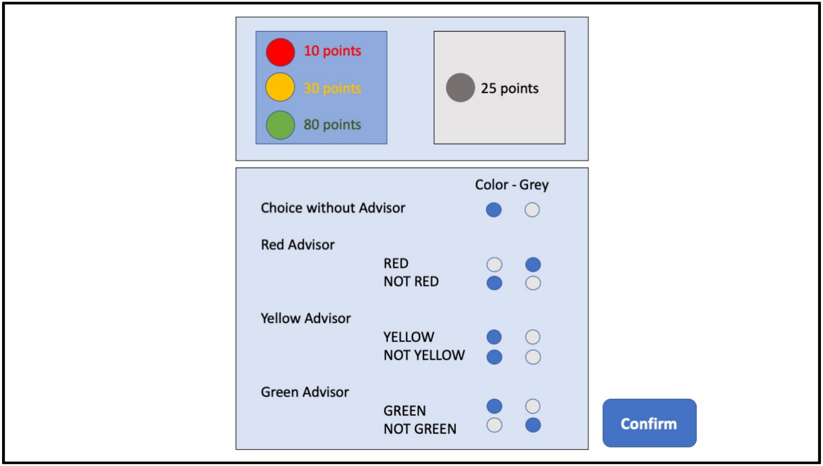
- $c_1 > c_2 = 0$ only one signal structure is costly, but $\pi_2(\sigma = 1|s) = 1$, i.e. the free signal is not informative
  - **The DM selects the informative advisor only if $U(\pi_1) \geq c_1$**

Laboratory Experiment

Laboratory Experiment

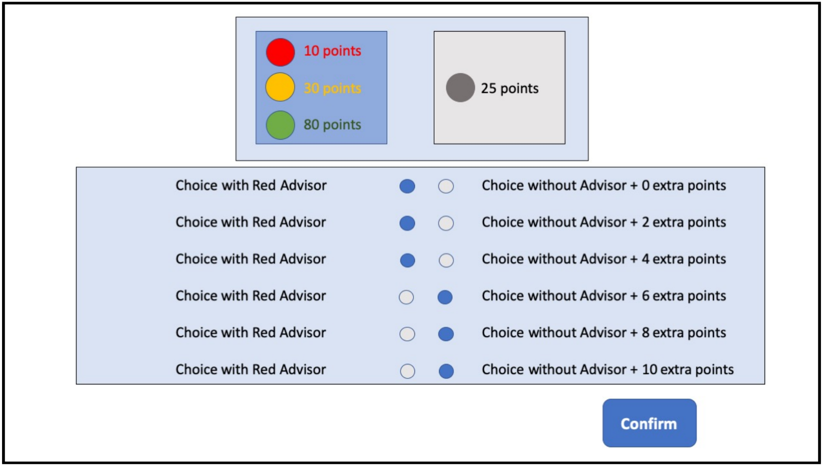
- **How do agents evaluate and choose information sources?**
- Stage 1: choose or “hire” an advisor
- Observe signal realization
- Stage 2: select an action [risky/safe]
- We want to collect separately
  - Action (conditional on posterior beliefs)
  - WTP for advisor / preferences over advisors
  - Posterior beliefs [guessing task]
- Deviations from optimality can enhance or reduce the effect predicted by the state pooling model
- A controlled lab setting allows to analyze individually all the components of the decision process

Task 1 - Colorblind Advisor Game - Action choice



Signal realization contingent choices - Collect actions $a_i(\sigma)$.

Task 1 - Colorblind Advisor Game - Hiring screen



Signal structure value elicitation - Collect subjective $U_i(\pi)$.

Task 1 - Colorblind Advisor Game

We can test:

1. whether agents choose optimally in the binary choice stage, conditional on the available information
2. how they evaluate the additional information represented by the signal
3. whether the status quo affects choice and signal valuation (if subjects' reaction is qualitatively and quantitatively coherent with the optimal one)

Theoretical predictions:

- choose the lottery with the highest expected value
- the highest price paid in order to receive the signal is $U(\pi_e)$ (instrumental value)

Task 2 - Imprecise Advisor Game

Are the results robust to noisy signal structures?

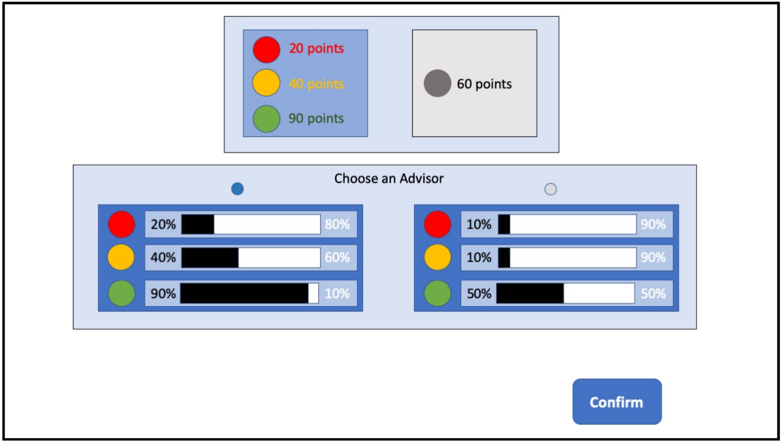
We can test:

1. if agents choose signal structures that are more informative in instrumental way
2. if agents correctly update own beliefs
3. if agents correctly estimate the probability of each realization

$$EV_e = E[v(\sigma)|\sigma = 0] \cdot P(\sigma = 0) + E[v(\sigma)|\sigma = 1] \cdot P(\sigma = 1)$$

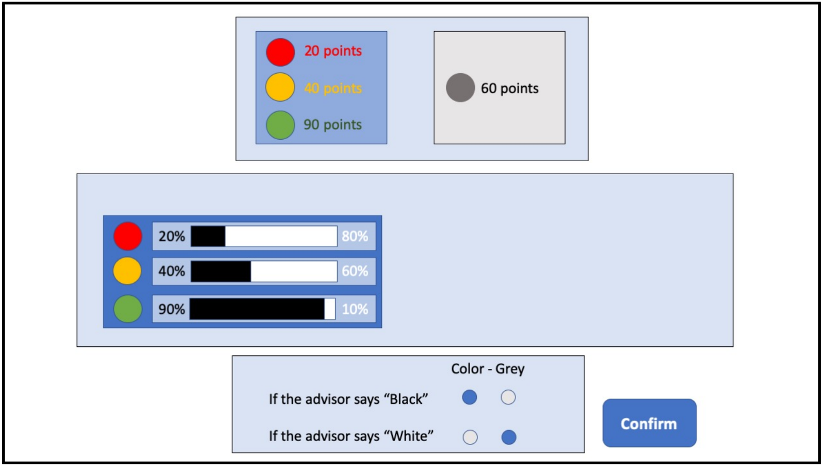
The EV given a signal structure e is a function of the strategy $v(\sigma)$ conditional on signal realization σ. We record separately subjective estimates of $P(s|\sigma)$ and $P(\sigma = 0)$

Task 2 - Imprecise Advisor Game - Advisor choice



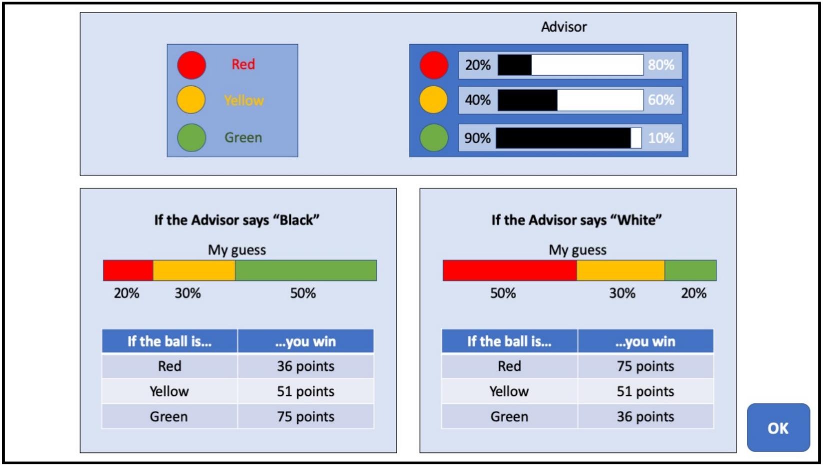
Binary advisor choice - Collect preference over π_e ($c = 0$).

Task 2 - Imprecise Advisor Game - Action choice



Signal realization contingent choice - Collect actions $a_i(\sigma)$.

Task 3 - Color Prediction Game



Posterior beliefs elicitation (exogenous signal structure) - Collect $\hat{p}_i(s|\sigma)$.

Task 4 - Message Prediction Game

Signal probability elicitation (exogenous signal structure) - Collect $\hat{p}_i(\sigma)$.

Tasks 2-4 - Advisor Choice and Control Tasks

We are mostly interested in Task 2 (advisor choice), but we need 3 and 4 (guessing tasks) for robustness.

Choose pairs of signal structures $\{\pi_1, \pi_2\}$ such that:

1. they have the same information about the states (Shannon entropy reduction)
2. π_1 should be chosen if $B < \overline{B}$
3. π_2 should be chosen if $B > \overline{B}$

The same pair appears in two separate trials, with different status quo B.

Summary

Motivation: Empirical evidence of belief polarization

Information valuation

$\Downarrow$

Endogenous information acquisition

$\Downarrow$

Belief polarization

- RI model with N=3 states and binary action choice
- State pooling depends on status quo (safe action's value)
- **Sharp predictions** about optimal information acquisition
- **Alternative hypothesis** include confirmatory strategy, biased updating, preference for non-instrumental information
- Lab experiment to test separately the model's **assumptions**

List of Potential Confounding Factors

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Appendix Slides

Appendix A - The Model

Appendix A

Appendix B - Related Literature

Appendix B

Appendix C - Confounding Factors

Appendix C

Appendix A - The Model

Full Model (Matveenko and Novak)

- DM is rationally inattentive (Sims, 2003, 2006)
  - Information costly - Shannon cost
  - λ - marginal cost of information
  - $\kappa(P, G)$ - **expected reduction in entropy**
  - $G(v)$ prior distribution
  - $P(i|v)$ probability of choosing action i conditional on v

- Main result: possible "wrong direction" updating of beliefs dependent on respective position of prior beliefs and safe option. It leads to polarization (more extreme posterior beliefs).

Agent's problem

Denote: $\mathbf{v} = (v_1, \ldots, v_n)$, $G(\mathbf{v})$ - prior joint distribution
Find an information strategy maximizing:

$$\max_{P(i|v)} \left\{ \sum_{i=1}^{2} \int_{\mathbf{v}} v_i P(i|\mathbf{v}) G(d\mathbf{v}) - \lambda \kappa(P, G) \right\},$$

where

$$\kappa(P, G) = -\sum_{i=1}^{2} P_i^0 \ln P_i^0 + \int_{\mathbf{v}} \left(\sum_{i=1}^{2} P(i|\mathbf{v}) \ln P(i|\mathbf{v}) \right) G(d\mathbf{v}).$$

$P(i|v)$ is the conditional on the realized value of v, the probability of choosing option i and

$$P_i^0 = \int_{\mathbf{v}} P(i|\mathbf{v}) G(d\mathbf{v}), \; i = 1, 2$$

where P_i^0 is the unconditional probability of option i to be chosen.

Lemma 1 (Matějka, McKay, 2015)

Conditional on the realized state of the world s^* probability of choosing risky option is

$$P(\text{picking risky}|\text{state is } s^*) = \frac{P_1^0 e^{\frac{v_s^*}{\lambda}}}{P_1^0 e^{\frac{v_s^*}{\lambda}} + (1 - P_1^0) e^{\frac{R}{\lambda}}}$$

of choosing safe option is:

$$P(\text{picking safe}|\text{state is } s^*) = \frac{(1 - P_1^0) e^{\frac{R}{\lambda}}}{P_1^0 e^{\frac{v_s^*}{\lambda}} + (1 - P_1^0) e^{\frac{R}{\lambda}}}$$

here P_1^0 is unconditional probability of choosing risky option.

Beliefs

- Agent's prior expected value of the risky option is:

$$\mathbb{E}v = \sum_{s=1}^{n} v_s g_s$$

we **fix the state** of the nature: it is s^*

- Observer sees agent's updated belief about the average of v:

$$\mathbb{E}_i[\mathbb{E}(v|i)|s^*] = P(i=1|s^*)\mathbb{E}(v|\text{picking option 1})+$$
$$+(1-P(i=1|s^*))\mathbb{E}(v|\text{picking option 2})$$

where for option $i \in \{1,2\}$

$$\mathbb{E}(v|\text{picking option i}) = \sum_{j=1}^{n} v_i P(\text{state is j}|\text{picking option i})$$

Beliefs

Theorem

Expected posterior value of the risky option for a rationally inattentive decision maker is

$$\mathbb{E}_i[\mathbb{E}(v|i)|s^*] = \sum_{i=1}^{n} v_i g_i \frac{\alpha_{s^*} e^{\frac{v_i}{\lambda}} + (1 - \alpha_{s^*})e^{\frac{R}{\lambda}}}{P_1^0 e^{\frac{v_i}{\lambda}} + (1 - P_1^0)e^{\frac{R}{\lambda}}} \tag{1}$$

where

$$\alpha_{s^*} = \frac{P_1^0 e^{\frac{v_{s^*}}{\lambda}}}{P_1^0 e^{\frac{v_{s^*}}{\lambda}} + (1 - P_1^0)e^{\frac{R}{\lambda}}}$$

Updating of beliefs

We are interested in

$$\Delta = \mathbb{E}_i[\mathbb{E}(v|i)|s^*] - \mathbb{E}v$$

Theorem

The sign of Δ is the same as the sign of $(v_{s^*} - R)$.

Proof.

Straightforward and we use:

Lemma 2

Relations $\alpha_{s^*} \gtrless P_1^0$ under $P_1^0 > 0$ are equivalent to $v_{s^*} \gtrless R$

$\square$

Example 3 states, 2 actions

- 3 possible states of the world - indexed by s
- 2 options/actions - indexed by a
  - Option 1 - Risky with values: $v_1 < v_2 < v_3$
  - Option 2 - Safe option with value R in all states
- Prior belief about the states: g_1, g_2, g_3
- Marginal cost of information: λ

Assumption 1: to rule out uninteresting cases

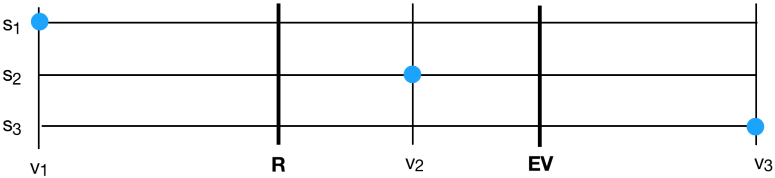
$$v_1 < R < v_3$$

Updating in "wrong" direction

We are interested when the conditional expectation moves in the **"wrong" direction**

Example for $s^* = 1$ the expectation "should" go down, so the agent is biased when

$$\mathbb{E}_a[\mathbb{E}(v|a)|s^*] > \mathbb{E}v > 0$$



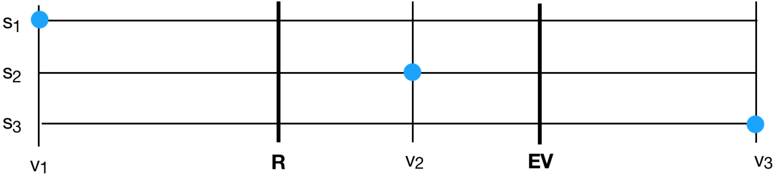
Updating in "wrong" direction

Let's denote $\Delta = \mathbb{E}_a[\mathbb{E}(v|a)|s^*] - \mathbb{E}v$.

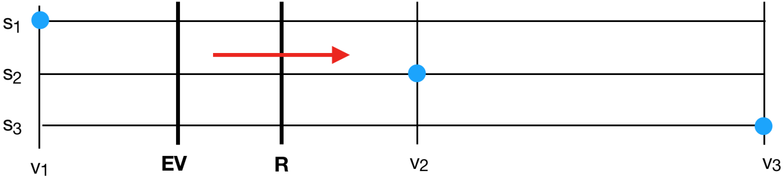
If

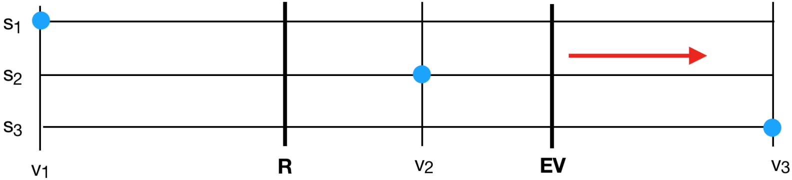
$$(\mathbb{E}v - v_{s^*}) \cdot \Delta > 0$$

then the agent is updating belief in the wrong direction



Result





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Appendix B - Literature

Information and Belief Polarization

- Polarization is an ubiquitous phenomenon
- Mixed evidence of how information contributes to polarization
  - Politicians and voters more polarized despite increased availability of information
    McCarty, Poole and Rosenthal (2006)
  - Greater Internet use is not associated with faster growth in political polarization among US demographic groups
    Boxell, Gentzkow and Shapiro (2017)

Back to Literature slides

Multiple Explanations for Polarization

1. **Confirmation bias**
   - Misreading ambiguous signals: Rabin, Schrag (1999); Fryer, Harms, Jackson (2017)
   - Limited memory: Wilson (2014)
   - Experiments: Lord, Ross, Lepper (1979)
2. **Overconfidence and correlation neglect**
   - Ortoleva and Snowberg (2015)
3. **Positive test strategy**
   - Klayman and Ha (1987), Nickerson (1998)

Results mostly based on exogeneous information and/or exogeneously imposed biases.

Back to Literature slides

Confirmation Bias and Rational Inattention

1. **Su (2014)**
    - Gaussian signal + quadratic loss function
    - Attention proportional to observation window
    - Results: conformism in learning

2. **Nimark and Sundaresan (2018)**
    - Mainly focus on polarization persistence
    - Agent pays more attention to the states which are more likely

3. **Dixit and Weibull (2007)** - not RI
    - Learning about policy in place (signal bimodal)
    - Agents agree on loss function, disagree on probabilities of states
    - Status quo vs. new reform - Divergence of opinions

Back to Literature slides

Experimental Literature

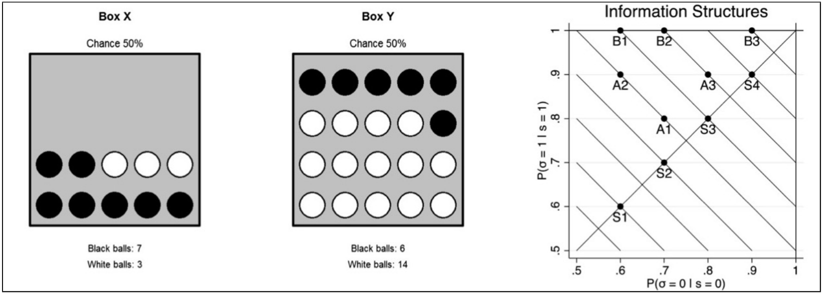
1. **Ambuehl and Li (2018)**
    - Systematic analysis of belief updating and demand for information
    - Compression effect: subjective valuation of useful information underreacts to increased informativeness
    - Biases mainly due to non-standard belief updating rather than risk preferences

2. **Charness, Oprea, and Yuksel (2018)**
    - Study how people choose between biased information sources
    - Evidence of confirmation-seeking rule
    - Mistakes are driven by errors in reasoning about informativeness

Back to Literature slides

Ambuehl and Li (2018)

- Prediction game
- **Information valuation task**
- Belief updating task
- Eliciting signal probabilities
- Gradual information task



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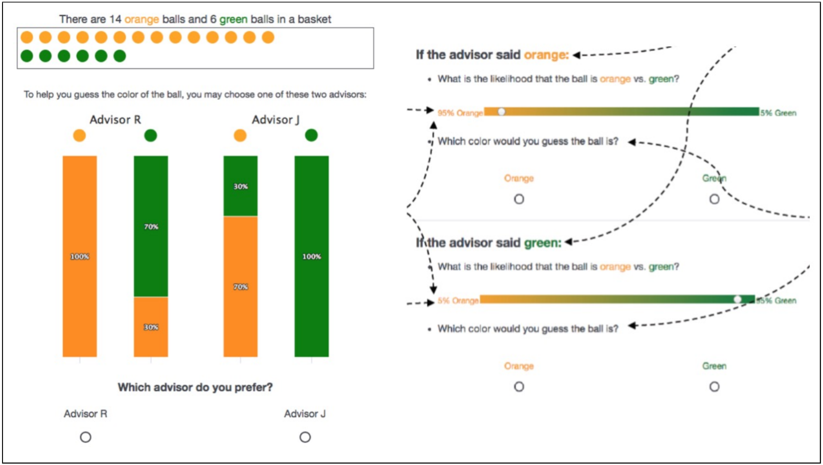
Charness, Oprea, and Yuksel (2018)



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Appendix C
Confounding Factors

Who killed RI in the lab? A list of usual suspects

- Risk attitude
- Noise/randomness
- Inertia
- Status quo effect
- Wrong updating (base-rate neglect, conservatism)
- Updating strength affected by irrelevant variables
- Confirmatory strategy (positive test)
- Preference over non-instrumental information
- Signal avoidance (ostrich effect)
- Biased information cost/value function (compression)

Back to Experiment slides