State Pooling and Belief Polarization

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Columbia University - Applied Micro Theory Colloquium

January 28, 2019

Today's Presentation

- Motivating Example
- Research Question
- State Pooling Model
- Related Literature
- Laboratory Experiment

Motivating Example

Motivating Example

Setting

- Alice and Bob face a choice: go to the Theater or stay Home
  - Theater: uncertainty about the quality of the movie s
  - Home: "safe" choice [status quo]

	Theater	**Home**	
	$v_i^T(s)$	$v_A^H(s)$	$v_B^H(s)$
s=bad	0	0.45	0.55
s=medium	0.5	0.45	0.55
s=good	1	0.45	0.55

- Assume uniform prior $p_s = \frac{1}{3}$ and risk neutrality

Motivating Example

Scenario 1

- Alice and Bob have the same beliefs (expected quality)
  - $EV(\text{Theater}) = 0.5$
- Alice and Bob make different choices
  - A chooses Theater as $0.5 = EV(\text{Theater}) > EV(\text{Home}) = 0.45$
  - B chooses Home as $0.5 = EV(\text{Theater}) < EV(\text{Home}) = 0.55$

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Motivating Example

Scenario 2

- Same problem as before, but now A and B can collect "some" information about the movie quality
- Note that we have 2 actions (T/H) and 3 states (b/m/g)
  - For Alice it is *sufficient* to know if the movie is b or (m/g)

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Motivating Example

Scenario 2

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- Note that we have 2 actions (T/H) and 3 states (b/m/g)
  - For Bob it is *sufficient* to know if the movie is (b/m) or g

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Motivating Example

Scenario 2

- If the movie is good (bad) they agree about the action Theater (Home)
- But they do not agree about the expected quality of the movie
  - Good movie: $EV_A(T|g) = 0.75 < EV_B(T|g) = 1$
  - Bad movie: $EV_A(T|b) = 0 < EV_B(T|b) = 0.25$

- If the movie is medium they still disagree about the action
- But they also disagree about the expected quality of the movie
  - Alice chooses T: $EV_A(T|m) = 0.75$
  - Bob chooses H: $EV_B(T|m) = 0.25$

▸ Motivating example 2

Motivating Example

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Motivating example 2

Motivating Example

Summary

- Alice and Bob are initially very similar
- The introduction of **endogenous information collection** created agreement about actions for extreme states
- and disagreement about expected quality in all states

- **State pooling**: avoid redundant information if the action space is smaller than the state space
- **Belief polarization** : posterior beliefs are more extreme because of the different relevant information

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Research Question

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Can endogenous information acquisition provide an explanation for belief polarization?

Broad question that includes prior heterogeneity, update heterogeneity, confirmatory/contradictory strategies, etc.

What is the relation between subjective and objective information valuation?

Test whether agents:

- Seek information based on the impact on their action
- Ignore information without instrumental value (state pooling)

Jump to the Experimental design

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State Pooling Model

Full Model (Matveenko and Novak)

- Rational inattention model (Matveenko and Novak)
- Stage 1: collect information, stage 2: make a choice
- N possible states of the world

- DM facing discrete binary choice problem $i \in \{1, 2\}$
- Binary actions - safe ("status quo") and risky ("reform")

- Risky option
  - value v_s, where $s \in 1, \ldots, n$
  - $v_i < v_j$ for $i < j$
- Safe option
  - value R
  - $v_k \leq R \leq v_{k+1}$ for some $k \in 1, \ldots, n-1$
  - Assumption: $v_1 < R < v_n$

Full Model (Matveenko and Novak)

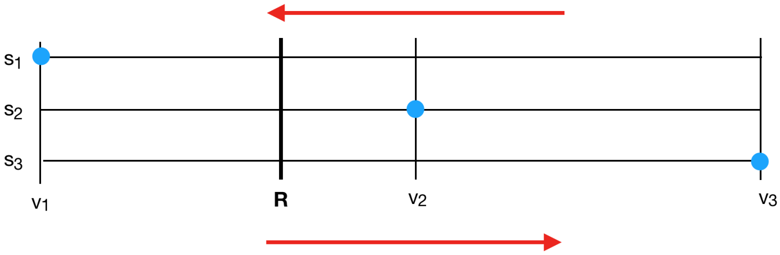
- DM uncertain about realized state of the world
- p_s - prior belief state s realized, $\sum_{s=1}^{n} p_s = 1$
- DM is rationally inattentive (Sims, 2003, 2006)
  - **DM can acquire information about the state**
  - Information costly - Shannon cost
  - λ - marginal cost of information
  - κ - expected reduction in entropy
- Main result: possible "wrong direction" updating of beliefs dependent on respective position of prior beliefs and safe option. It leads to polarization (more extreme posterior beliefs).

Main Results

Theorem

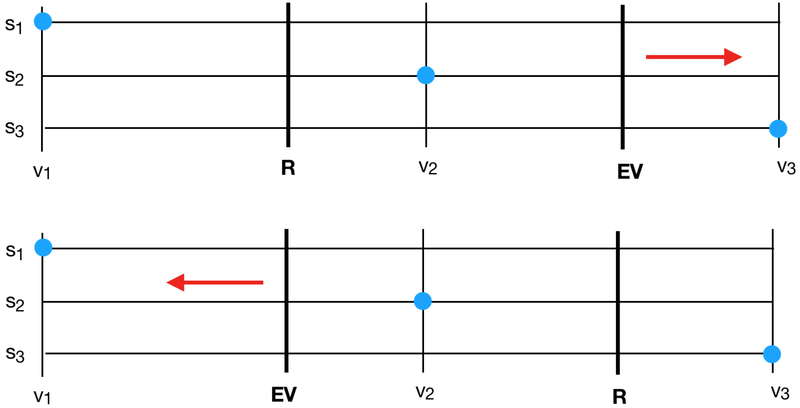
The sign of** $(v_{s^*} - R)$ **is the same as the sign of
$\Delta_{EV} = E[EV_{post}] - EV_{prior}$

- Agent cannot be biased (cannot update against the true state)
  - when the realized state is $s^* = 1$ or $s^* = 3$



Updating in the Wrong Direction

- Agent is biased when $s^* = 2$ in the following cases
  - when $v_2 < R$ and at the same time $\mathbb{E}v < v_2$
  - when $v_2 > R$ and at the same time $\mathbb{E}v > v_2$



- And unbiased otherwise.

Simplified Model

- $N = 3$ possible states of the world
- $p_s = \frac{1}{N}$ uniform prior belief state s realized
- **Stage 1: collect information**
  - Choose one pair experiment-cost $(\pi_e, c_e) \in \{(\pi_e, c_e)\}_e$
  - Pay the cost c_e to observe the experiment π_e
  - We focus on sets with two signal structures only
- **Stage 2: make a choice**
  - Choose one action $a \in \{1, 2\}$
  - Safe action (return R) and risky action (return v_s)

Simplified Model

- The experiment π_e can generate only two signals $\sigma \in \{1, 2\}$ and is defined by the triplet $\pi(\sigma = 1|s)$
- The instrumental value of a signal structure π_e is

$$U(\pi_e) = \underbrace{\sum_{\sigma} v^*(p(s|\sigma))\pi(\sigma)}_{\text{EV with } \pi_e} - \underbrace{v^*(p(s))}_{\text{EV w/o } \pi_e}$$

  where v^* is the expected value of the optimal action (conditional on available information)

- **Stage 1: collect information**
  - A rational agent chooses the signal structure

$$e^* = \text{argmax}_e U(\pi_e) - c_e$$

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Simplified Model

- Two special cases of signal structure choice:
- $c_1 = c_2 = 0$ both signal structures are free
  - **The DM selects the most informative**
- $c_1 > c_2 = 0$ only one signal structure is costly,
  but $\pi_2(\sigma = 1|s) = 1$, i.e. the free signal is not informative
  - **The DM selects the informative signal only if $U(\pi_1) \geq c_1$**
- We can simplify further the calculation of the value:

$$U(\pi_e) = Pr(\text{choice-reverting } \sigma) \cdot \Delta E[v_{choice}|\text{choice-reverting } \sigma]$$

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Simplified Model

- We design a laboratory experiment that aims to test separately the major assumptions of the (simplified) model:
  - Optimal action choice (conditional on posterior beliefs)
  - Optimal updating of beliefs
  - Optimal signal structure choice (conditional on action/beliefs)
- Deviations from optimality can enhance or reduce the predictions of the model
- A controlled lab setting allows to analyze individually all the components of the decision process

Related Literature

Information and Belief Polarization

- Polarization is an ubiquitous phenomenon
- Mixed evidence of how information contributes to polarization
  - Politicians and voters more polarized despite increased availability of information
    McCarty, Poole and Rosenthal (2006)
  - Greater Internet use is not associated with faster growth in political polarization among US demographic groups
    Boxell, Gentzkow and Shapiro (2017)

Multiple Explanations for Polarization

1. **Confirmation bias**
   - Misreading ambiguous signals: Rabin, Schrag (1999); Fryer, Harms, Jackson (2017)
   - Limited memory: Wilson (2014)
   - Experiments: Lord, Ross, Lepper (1979)

2. **Overconfidence and correlation neglect**
   - Ortoleva and Snowberg (2015)

3. **Positive test strategy**
   - Klayman and Ha (1987), Nickerson (1998)

Results mostly based on exogeneous information and/or exogeneously imposed biases.

Confirmation Bias and Rational Inattention

1. **Su (2014)**
   - Gaussian signal + quadratic loss function
   - Attention proportional to observation window
   - Results: conformism in learning

2. **Nimark and Sundaresan (2018)**
   - Mainly focus on polarization persistence
   - Agent pays more attention to the states which are more likely

3. **Dixit and Weibull (2007)** - not RI
   - Learning about policy in place (signal bimodal)
   - Agents agree on loss function, disagree on probabilities of states
   - Status quo vs. new reform - Divergence of opinions

Experimental Literature

1. **Ambuehl and Li (2018)** Design
   - Systematic analysis of belief updating and demand for information
   - Compression effect: subjective valuation of useful information underreacts to increased informativeness
   - Biases mainly due to non-standard belief updating rather than risk preferences
2. **Charness, Oprea, Yuksel (2018)** Design
   - Study how people choose between biased information sources
   - Evidence of confirmation-seeking rule
   - Mistakes are driven by errors in reasoning about informativeness
3. Vast experimental literature
   - Heterogeneity in belief updating: El-Gamal and Grether 1995, Fehr-Duda and Epper 2012, Augenblick and Rabin 2015, Buser et al 2016, Antoniou et al 2017.
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Laboratory Experiment

3-parts Experimental Design

1. **What is the relation between subjective and objective evaluation?**
   - Part 1 collects a precise measure of subjective valuation in the simplest decision problem
2. **How do agents evaluate information?**
   - Part 2 of the experimental design captures separately signal structure choice, update, and action choice
3. **Are the result robust to a more complex setting?**
   - Part 3 contains a richer task (more exploratory)

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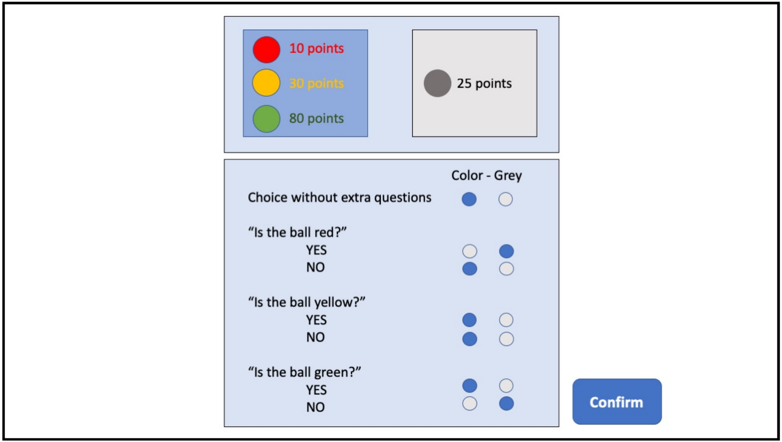
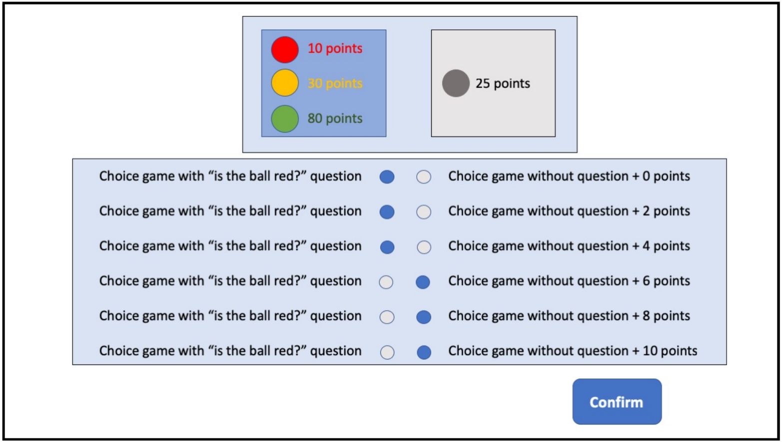
Part 1



Signal realization contingent choices - Collect actions $a_i(\sigma)$.

Part 1



Signal structure value elicitation - Collect subjective $U_i(\pi)$.

Part 1 - State Pooling and Signal Structure Valuation

We can test:

1. whether agents choose optimally in the binary choice stage, conditional on the available information
2. how they evaluate the additional information represented by the signal
3. whether the status quo affects choice and signal valuation (if subjects' reaction is qualitatively and quantitatively coherent with the optimal one)

Theoretical predictions:

- choose the lottery with the highest expected value
- the maximum v to pay in order to receive the signal is $U(\pi_e)$ (instrumental value)

Part 2 - Advisor Choice and Control Tasks

Are the results robust to noisy signal structures?

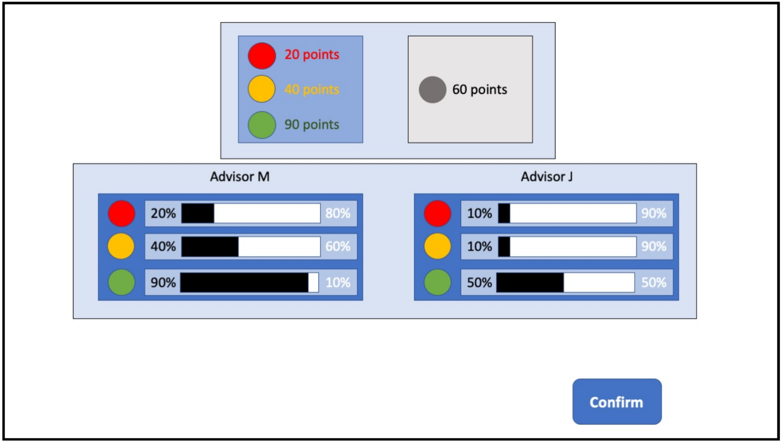
We can test:

1. if agents choose signal structures that are more informative in instrumental way
2. if agents correctly update own beliefs
3. if agents correctly estimate the probability of each realization

$$EV_e = E[v(\sigma)|\sigma = 0] \cdot P(\sigma = 0) + E[v(\sigma)|\sigma = 1] \cdot P(\sigma = 1)$$

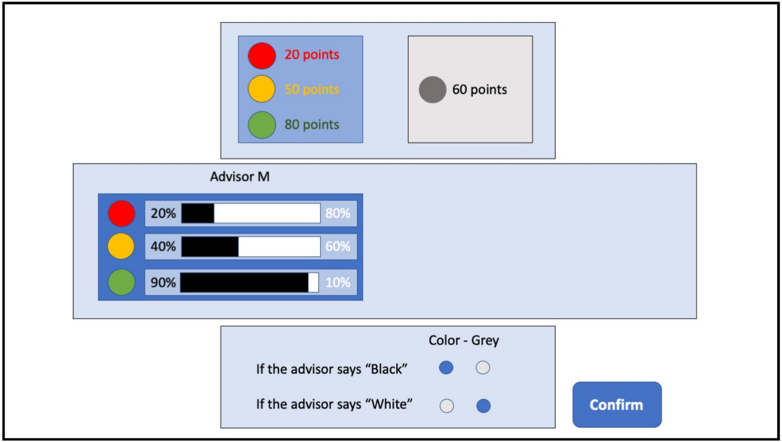
The EV given a signal structure e is a function of the strategy $v(\sigma)$ conditional signal realization σ. We record separately subjective estimates of $P(s|\sigma)$ and $P(\sigma = 0)$

Part 2 - Task A



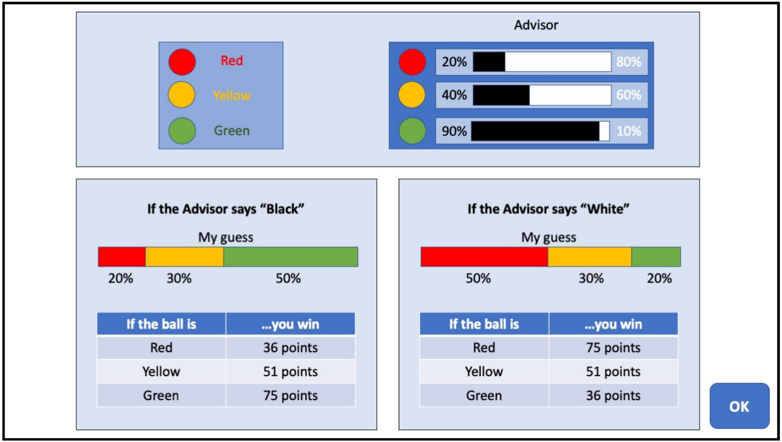
Binary advisor choice - Collect preference over π_e ($c = 0$).

Part 2 - Task A



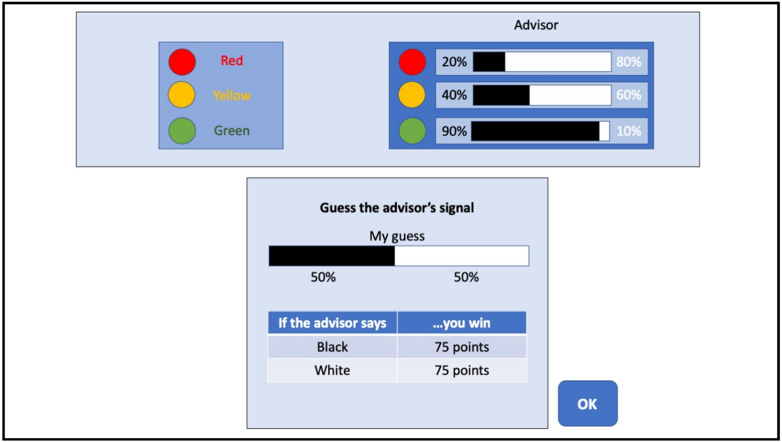
Signal realization contingent choice - Collect actions $a_i(\sigma)$.

Part 2 - Task B



Posterior beliefs elicitation (exogenous signal structure) - Collect $\hat{p}_i(s|\sigma)$.

Part 2 - Task C



Signal probability elicitation (exogenous signal structure) - Collect $\hat{p}_i(\sigma)$.

Part 2 - Advisor Choice and Control Tasks

We are mostly interested in Task A,
but we need B and C for robustness.

Choose pairs of signal structures $\{\pi_1, \pi_2\}$ such that:

1. they have the same information about the states (Shannon entropy reduction)
2. π_1 should be chosen if $R < \overline{R}$
3. π_2 should be chosen if $R > \overline{R}$

The same pairs appears in two separate trials,
with different status quo R.

Part 3 - Sequential Information Collection

Are the results robust to a more complex environment?

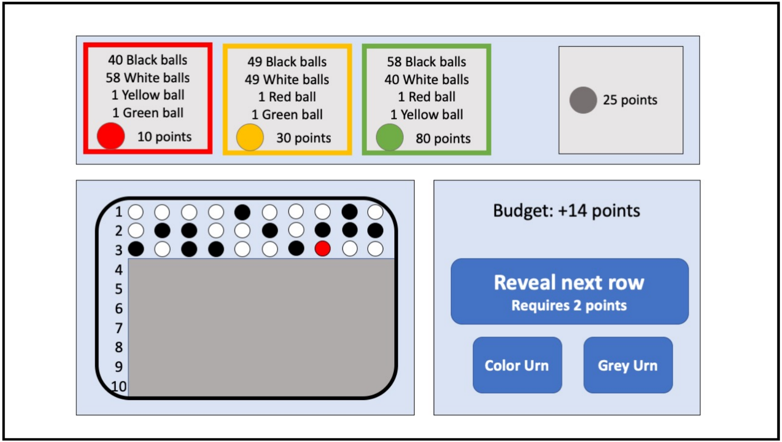
We can test:

1. whether agents adapt their strategy to the environment
2. whether “sampling” and actions follow the theoretical predictions (including state pooling)

Task with a *rational inattention* flavor:

- the true state can be exactly revealed
- cheaper signals are less informative

Part 3



Sequential information collection - Gradual resolution of uncertainty.

Summary

- **Motivation**: Empirical evidence of belief polarization
- **Can endogenous information acquisition provide an explanation for belief polarization?**
- RI model with N>2 states and binary choice
- State pooling and role of status quo (safe action)
- **Prediction about optimal information acquisition**
- Prediction about posterior beliefs

Summary

- **What is the relation between subjective and objective information valuation?**

- Lab experiment to test the **predictions** of the model and its major **assumptions**
- Contribution 1: rigorous test of the model's assumptions
- Contribution 2: extension of previous experimental designs

State Pooling and Belief Polarization

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Appendix - The Model

Agent's problem

Denote: $\mathbf{v} = (v_1, \ldots, v_n)$, $G(\mathbf{v})$ - prior joint distribution
Find an information strategy maximizing:

$$\max_{P(i|v)} \left\{ \sum_{i=1}^{2} \int_{\mathbf{v}} v_i P(i|\mathbf{v}) G(d\mathbf{v}) - \lambda \kappa(P, G) \right\},$$

where

$$\kappa(P, G) = -\sum_{i=1}^{2} P_i^0 \ln P_i^0 + \int_{\mathbf{v}} \left(\sum_{i=1}^{2} P(i|\mathbf{v}) \ln P(i|\mathbf{v}) \right) G(d\mathbf{v}).$$

$P(i|v)$ is the conditional on the realized value of v, the probability of choosing option i and

$$P_i^0 = \int_{\mathbf{v}} P(i|\mathbf{v}) G(d\mathbf{v}), \; i = 1, 2$$

where P_i^0 is the unconditional probability of option i to be chosen.

Lemma 1 (Matějka, McKay, 2015)

Conditional on the realized state of the world s^* probability of choosing risky option is

$$P(\text{picking risky}|\text{state is } s^*) = \frac{P_1^0 e^{\frac{v_s^*}{\lambda}}}{P_1^0 e^{\frac{v_s^*}{\lambda}} + (1 - P_1^0) e^{\frac{R}{\lambda}}}$$

of choosing safe option is:

$$P(\text{picking safe}|\text{state is } s^*) = \frac{(1 - P_1^0) e^{\frac{R}{\lambda}}}{P_1^0 e^{\frac{v_s^*}{\lambda}} + (1 - P_1^0) e^{\frac{R}{\lambda}}}$$

here P_1^0 is unconditional probability of choosing risky option.

Beliefs

- Agent's prior expected value of the risky option is:

$$\mathbb{E}v = \sum_{s=1}^{n} v_s g_s$$

we **fix the state** of the nature: it is s^*

- Observer sees agent's updated belief about the average of v:

$$\mathbb{E}_i[\mathbb{E}(v|i)|s^*] = P(i=1|s^*)\mathbb{E}(v|\text{picking option 1})+ \\ +(1-P(i=1|s^*))\mathbb{E}(v|\text{picking option 2})$$

where for option $i \in \{1,2\}$

$$\mathbb{E}(v|\text{picking option i}) = \sum_{j=1}^{n} v_i P(\text{state is j}|\text{picking option i})$$

Beliefs

Theorem

Expected posterior value of the risky option for a rationally inattentive decision maker is

$$\mathbb{E}_i[\mathbb{E}(v|i)|s^*] = \sum_{i=1}^{n} v_i g_i \frac{\alpha_{s^*} e^{\frac{v_i}{\lambda}} + (1-\alpha_{s^*})e^{\frac{R}{\lambda}}}{P_1^0 e^{\frac{v_i}{\lambda}} + (1-P_1^0)e^{\frac{R}{\lambda}}} \tag{1}$$

where

$$\alpha_{s^*} = \frac{P_1^0 e^{\frac{v_{s^*}}{\lambda}}}{P_1^0 e^{\frac{v_{s^*}}{\lambda}} + (1-P_1^0)e^{\frac{R}{\lambda}}}$$

Updating of beliefs

We are interested in

$$\Delta = \mathbb{E}_i[\mathbb{E}(v|i)|s^*] - \mathbb{E}v$$

Theorem

The sign of Δ is the same as the sign of $(v_{s^*} - R)$.

Proof.

Straightforward and we use:

Lemma 2

Relations $\alpha_{s^*} \gtrless P_1^0$ under $P_1^0 > 0$ are equivalent to $v_{s^*} \gtrless R$

□

Example 3 states, 2 actions

- 3 possible states of the world - indexed by s
- 2 options/actions - indexed by a
  - Option 1 - Risky with values: $v_1 < v_2 < v_3$
  - Option 2 - Safe option with value R in all states
- Prior belief about the states: g_1, g_2, g_3
- Marginal cost of information: λ

Assumption 1: to rule out uninteresting cases

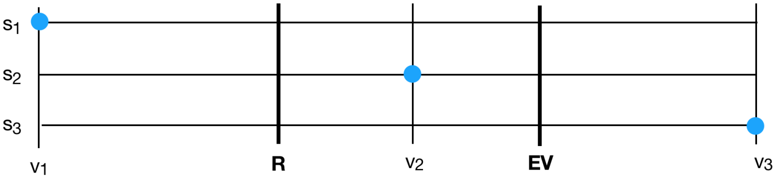
$$v_1 < R < v_3$$

Updating in "wrong" direction

We are interested when the conditional expectation moves in the **"wrong" direction**

Example for $s^* = 1$ the expectation "should" go down, so the agent is biased when

$$\mathbb{E}_a[\mathbb{E}(v|a)|s^*] > \mathbb{E}v > 0$$



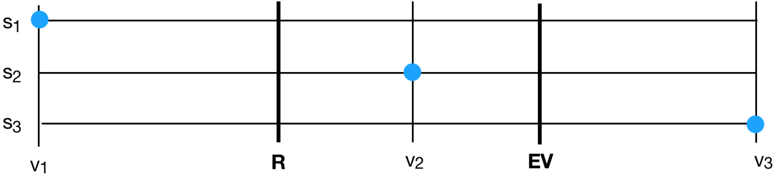
Updating in "wrong" direction

Let's denote $\Delta = \mathbb{E}_a[\mathbb{E}(v|a)|s^*] - \mathbb{E}v$.

If

$$(\mathbb{E}v - v_{s^*}) \cdot \Delta > 0$$

then the agent is updating belief in the wrong direction



Result