# Belief Polarization and State Pooling

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Columbia University - Experimental Lunch

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# Today's presentation

- Motivating example
- Motivation and research question
- RI Model (Matveenko and Novak)
- **Lab experiment (Novak and Ravaioli)**

# Motivating example

- Problem: invest \$ 1
- Simple setting with only two alternatives
  - A treasury bond with a fixed return (e.g. 5%)
  - Or a stock with a state-contingent return (e.g. 0%, 3%, or 10%)
- We do not need to know the exact stock return...
- ...but only if it is above or below the bond return!
- Categorization (state pooling)

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# Motivation

- The bond return represents a “status quo” with known value

- The status quo determines the magnitude of incentives
- therefore also the amount of costly information to acquire

- It also affects the state pooling process
- Agents do not learn about the states separately, but by pooling them into groups: “good” vs “bad” states
- information acquisition aims to distinguish between the groups

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- information acquisition aims to distinguish between the groups

# Preview of model's results

- Small changes in the status quo can lead to large change in the posterior beliefs
- We can observe belief polarization and wrong updating
- **Polarization**: more extreme beliefs (even when the true state is intermediate)
- **Wrong direction of update** of the expected belief conditional on the true state
- If agents have different, yet close, status quo values, we will also observe opposite directions between agents

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# Information and belief polarization

- Polarization is an ubiquitous phenomenon
- Mixed evidence of how information contributes to polarization
  - Politicians and voters more polarized despite increased availability of information
    McCarty, Poole and Rosenthal (2006)
  - Greater Internet use is not associated with faster growth in political polarization among US demographic groups
    Boxell, Gentzkow and Shapiro (2017)

# Multiple explanations of polarization

1. **Confirmation bias**
   - Misreading ambiguous signals: Rabin, Schrag (1999); Fryer, Harms, Jackson (2017)
   - Limited memory: Wilson (2014)
   - Experiments: Lord, Ross, Lepper (1979), ... and many others
2. **Overconfidence - correlation neglect**
   - Ortoleva and Snowberg (2015)
3. **Positive test strategy**
   - Klayman and Ha (1987), Nickerson (1998)

Results mostly based on exogeneous information and/or exogeneously imposed biases.

# Research question

**What is the role of the status quo on the endogenous information acquisition of inattentive agents and furthermore on their beliefs formation?**

- The "status quo" indicates a known policy (e.g. default).
- The agent in the model selects information and updates own beliefs optimally.
- RI model with endogenous information acquisition and rationally inattentive agents

# Research question

**What is the role of the status quo on the endogenous information acquisition of inattentive agents and furthermore on their beliefs formation?**

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# Confirmation Bias and Rational Inattention

1. **Su (2014)**
   - Gaussian signal + quadratic loss function
   - Attention proportional to observation window
   - Results: conformism in learning
2. **Nimark and Sundaresan (2018)**
   - Mainly focus on polarization persistence
   - Agent pays more attention to the states which are more likely
3. **Dixit and Weibull (2007)** - not RI
   - Learning about policy in place (signal bimodal)
   - Agents agree on loss function, disagree on probabilities of states
   - Status quo vs. new reform - Divergence of opinions

# Experimental literature

1. **Shermeta and Shields (2013)**
   - Recover subjective beliefs from observed choice under uncertainty
2. **Blanco et al. (2010)**
   - Rewarded action and belief elicitation, test for possible hedging behavior
3. **Catena et al. (1998)**
   - Effect of frequency of judgment in a signal detection task
   - Participant strategy includes a type of trials hidden units layer
4. **Hollard et al. (2010)**
   - Free elicitation rule outperforms quadratic scoring rule (*"All in all, we find little support for the use of the QSR in economics."*)
   - Evidence in favor of the "lottery rule" similar to BDM
5. **Aydogan et al. (2017)**
   - Signal perception model to explain belief update
   - Evidence of conservative and confirmatory bias

# Model (Matveenko and Novak)

- $N$ possible states of the world
- DM facing discrete binary choice problem $i \in \{1, 2\}$
- Binary actions - safe ("status quo") and lottery ("reform")
- Risky option
  - value $v_s$, where $s \in 1, \ldots, n$
  - $v_i < v_j$ for $i < j$
- Safe option
  - value $R$
  - $v_k \leq R \leq v_{k+1}$ for some $k \in 1, \ldots, n-1$
  - Assumption: $v_1 < R < v_n$

# Model (Matveenko and Novak)

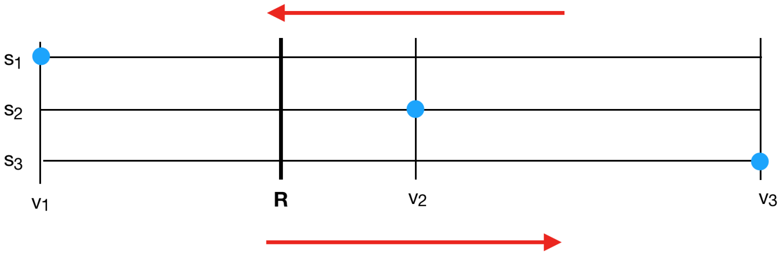
- DM uncertain about realized state of the world
- $g_s$ - prior belief state $s$ realized, $\sum_{s=1}^{n} g_s = 1$
- DM is rationally inattentive (Sims, 2003, 2006)
  - DM can acquire information about the state
  - Information costly - Shannon cost
  - $\lambda$ - marginal cost of information
  - expected reduction in entropy - $\kappa$
- Main result: possible "wrong direction" updating of beliefs dependent on respective position of prior beliefs and safe option - leads to polarization

# Main result

**Theorem**

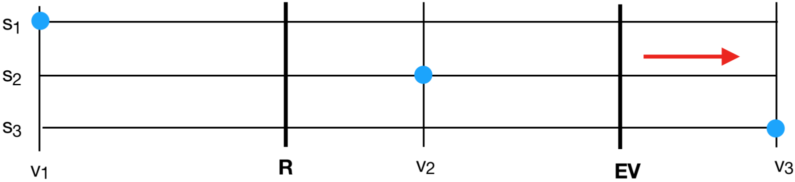
***The sign of*** $(v_{s^*} - R)$ ***is the same as the sign of***
$\Delta_{EV} = E[EV_{post}] - EV_{prior}$

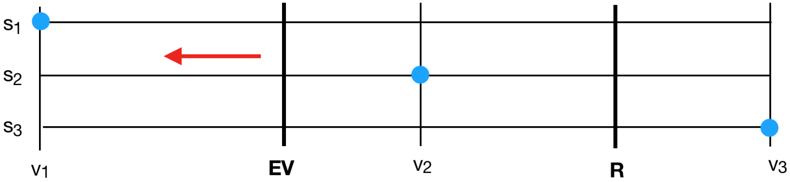
- Agent cannot be biased
  - when the realized state is $s^* = 1$ or $s^* = 3$



# Updating in the wrong direction

- Agent is biased when $s^* = 2$ in the following cases
  - when $v_2 < R$ and at the same time $\mathbb{E}v < v_2$
  - when $v_2 > R$ and at the same time $\mathbb{E}v > v_2$




- And unbiased otherwise.

# Testing the main results in the lab

Setting: Binary choice with endogenous info acquisition
Objective: Study the effect of a change in the status quo

The **state pooling model** predicts:

- the optimal signal design strategy
- the optimal belief update
  - the conditions under which we should observe expected update of beliefs in "wrong direction"
- that a small status quo change can generate belief divergence

**Alternative model** have different predictions about signal design and posterior beliefs

- Confirmatory/contradictory sampling strategy (signal design)
- Confirmation bias (update)

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# Experimental design

The experiment is comprised of four parts that gradually introduce the complexity of the scenario

1. Training 1 - Bayesian update task
2. Training 2 - Add binary choice component
3. Main task 1 - Introduce endogenous information acquisition
4. Main task 2 - Gradual information acquisition

# Experimental design

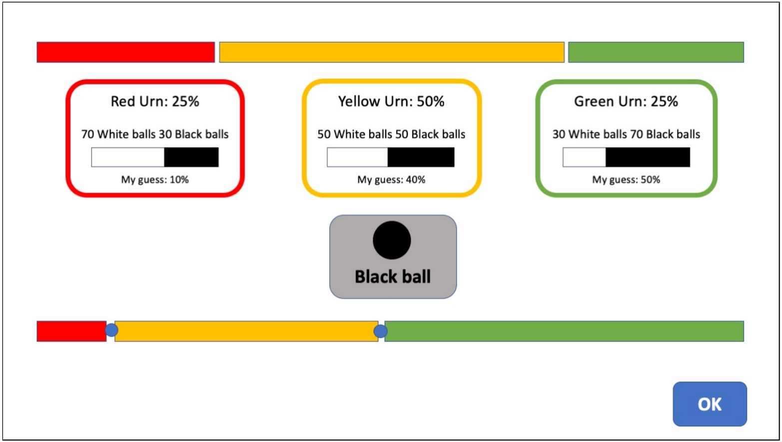
- 1-hour lab experiment
- Show up fee \$10 + variable reward based on performance
- In each trial, payoffs are expressed in probability points:
  60 points $\rightarrow$ 60 % chance to win \$5
- For each training and task block, one trial is selected and the participant plays the \$5 lottery with a number of tickets that depends on the performance during the selected trial.

# Training Task 1

# Training 1 - Updating /1

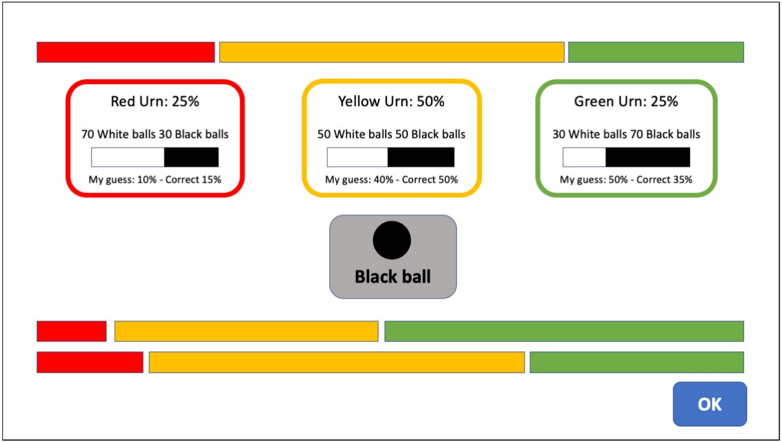

Information about prior and signal structure.

# Training 1 - Updating /2



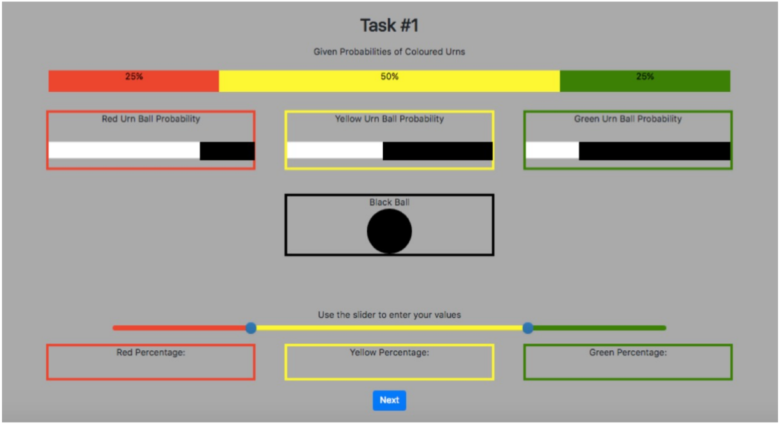

The realized signal is displayed. Elicit posterior by moving the slider.

# Training 1 - Updating /3



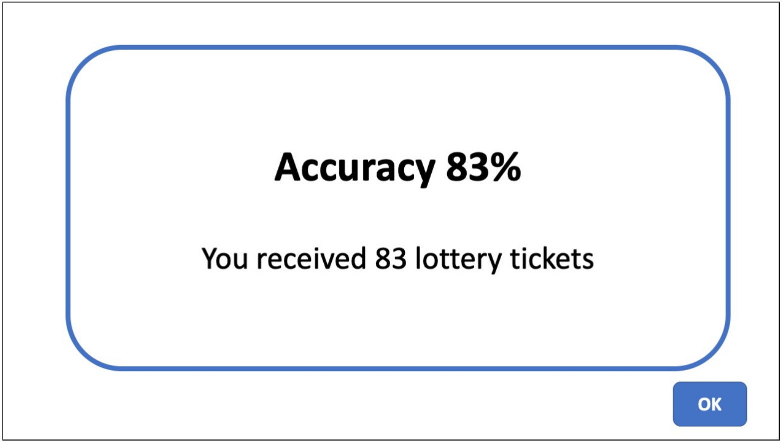

Feedback-learning screen containing the correct answer.

# Training 1 - Screenshot




Current version of the oTree interface.

# Training 1 - Updating /4




Feedback-score screen containing accuracy and payoff.

# Training task 1

- 2 conditions for the prior (urn more likely to be selected)
- 10 trials per prior with different signals (urn composition)
- 2 guided trials + 10 practice trials + 20 rewarded trials
- Provide feedback about the posterior accuracy
- Payoff is calculated based on accuracy (QSR)
- Accuracy = $4 * [0.25 - \sum_{i=1}^{3} max\{0.25, (\pi_i - \hat{\pi}_i)^2\}]$

# Training Task 2

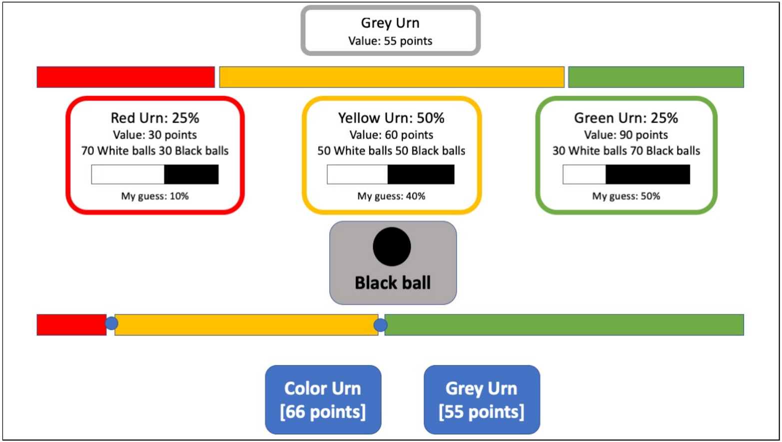
# Training 2 - Binary choice /1


Information about the value of the grey urn (safe option).

# Training 2 - Binary choice /2

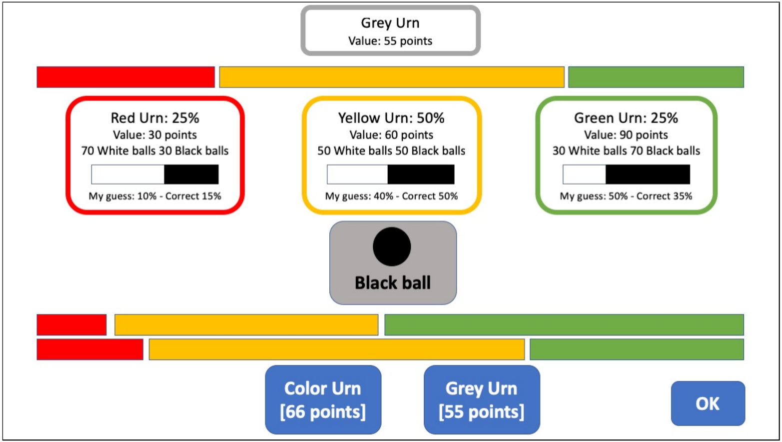

Information about prior and signal structure.

# Training 2 - Binary choice /3



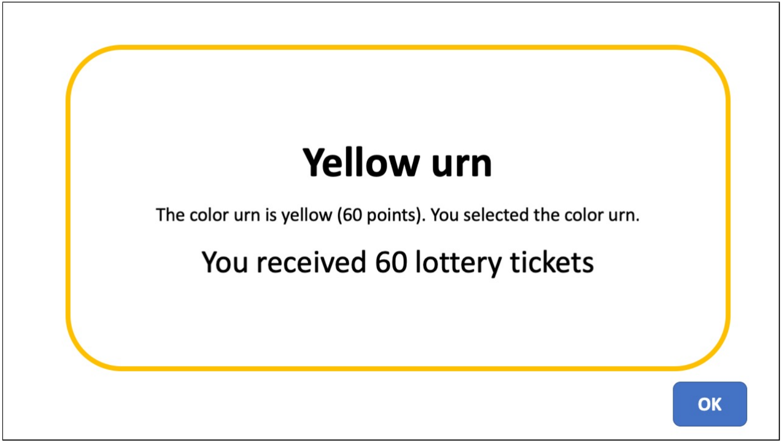

The realized signal is displayed. Elicit posterior and choice.

# Training 2 - Binary choice /4



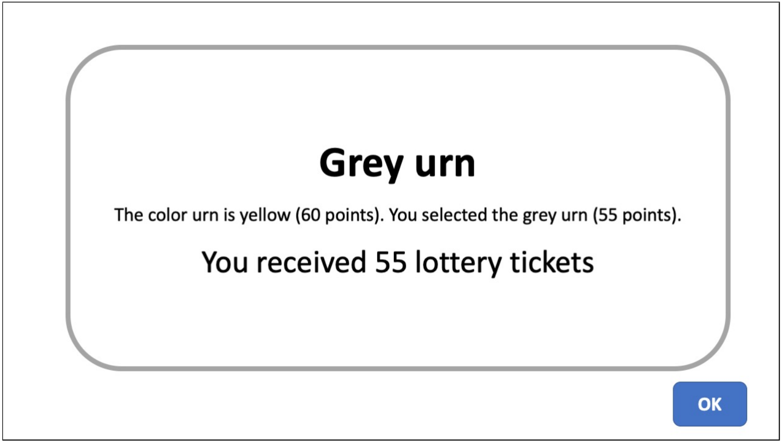

Feedback-learning screen containing the accurate posterior.

# Training 2 - Binary choice /5




Feedback-score screen containing the realized color and payoff (color).

# Training 2 - Binary choice /5



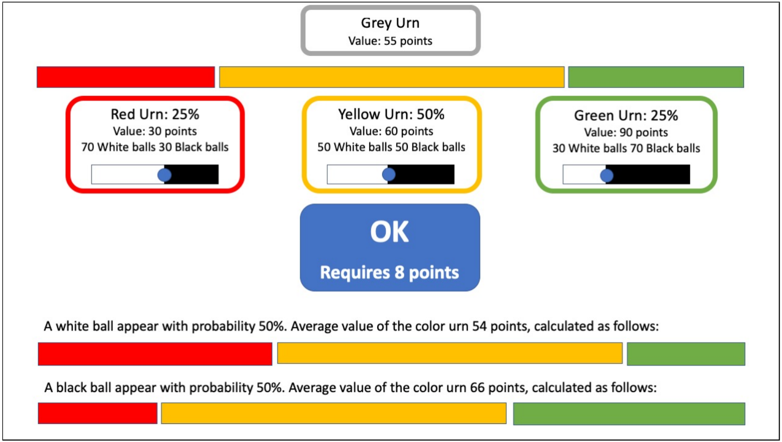

Feedback-score screen containing the realized color and payoff (grey).

# Training task 2

- Values of the color urns are fixed [30, 60, 90 points]
- 2 conditions for the prior (urn more likely to be selected)
- Different status quo [$R \gtrless EV$] and urn composition
- 2 guided practice + 3 practice trials + 20 rewarded trials
- Provide feedback about the accurate posterior and the realized state of the world
- Payoff only depends on the binary choice

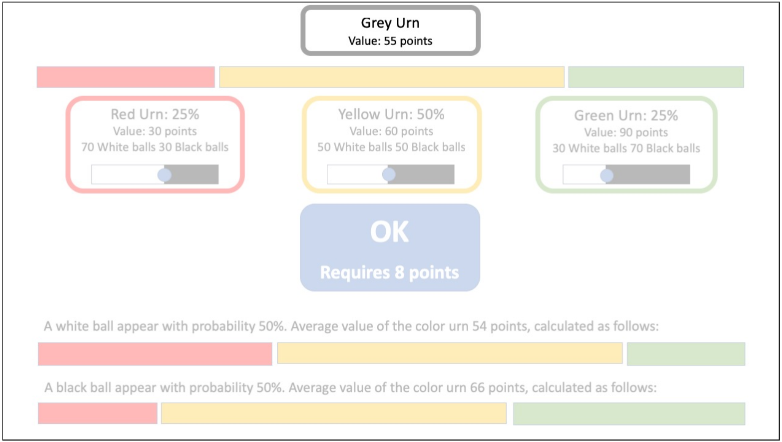
# Main Task 1

# Task 1 - Select information /1



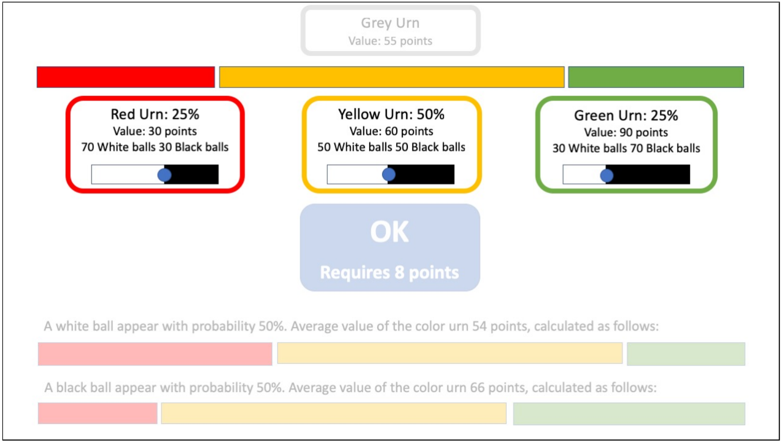

Screen 1 - Select signal structure and confirm by pressing OK.

# Task 1 - Select information /1



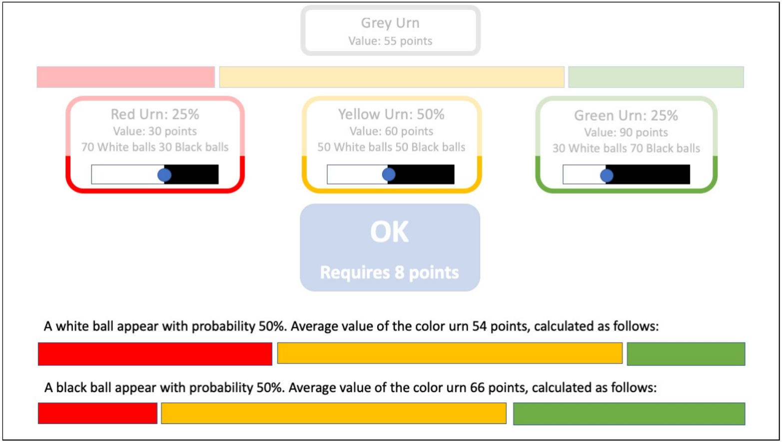

The grey urn delivers a fix amount of points and represents the safe option.

# Task 1 - Select information /1



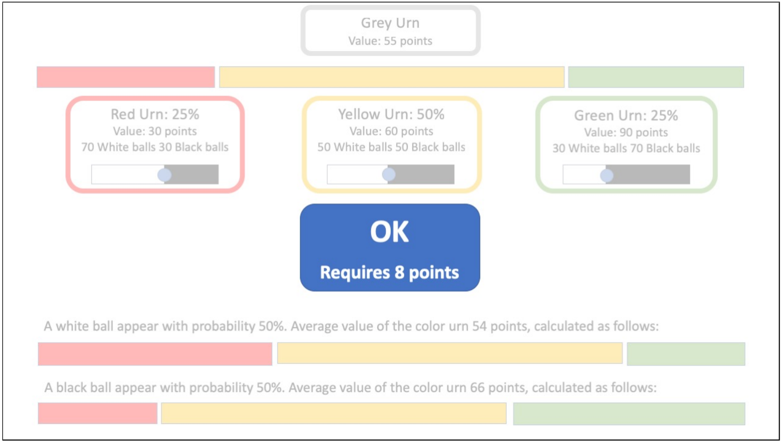

Select the signal structure by dragging the three sliders.

# Task 1 - Select information /1



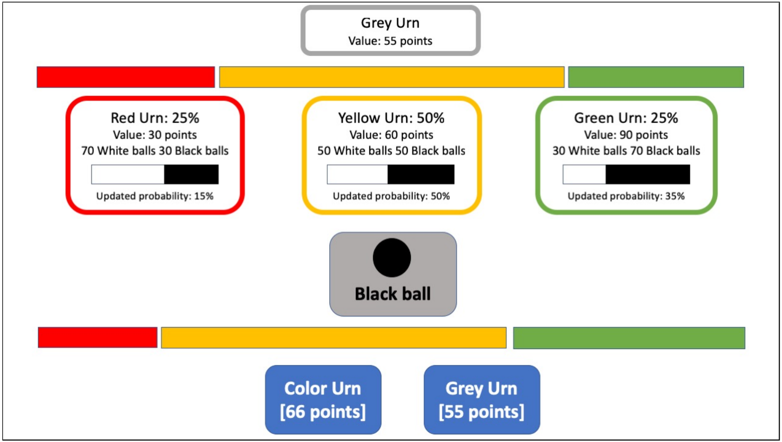

Information about the posterior are updated in real time.

# Task 1 - Select information /1



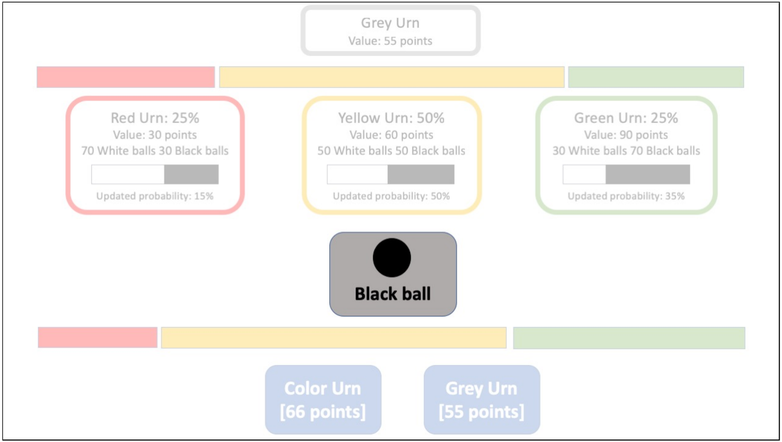

The cost is also updated based on the signal structure.

# Task 1 - Select information /2




Screen 2 - A ball is randomly drawn and revealed.

# Task 1 - Select information /2




Choose one among the two possible actions: color or grey urn.

# Main task 1

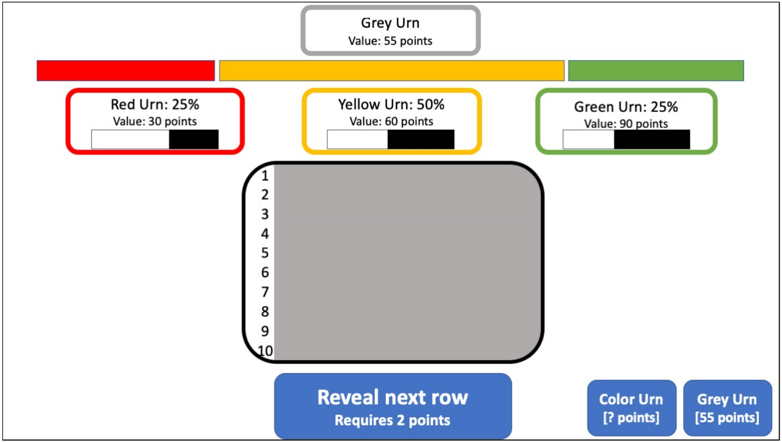
- Values of the color urns are fixed [30, 60, 90 points]
- 2 conditions for the prior (urn more likely to be selected)
- Different value of the status quo [$R \gtrless EV$]
- 2 guided practice + 3 practice trials + 20 rewarded trials
- No feedback about the realized state of the world
- Payoff depends on signal cost and binary choice

# Main Task 2

# Task 2 - Ball counting /1

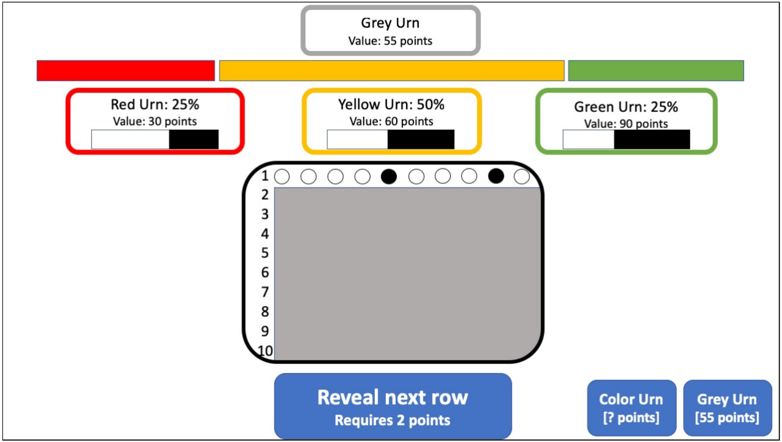

Observe the information of the trial and press OK.

# Task 2 - Ball counting /2




Click on the central button to reveal more rows.

# Task 2 - Ball counting /2

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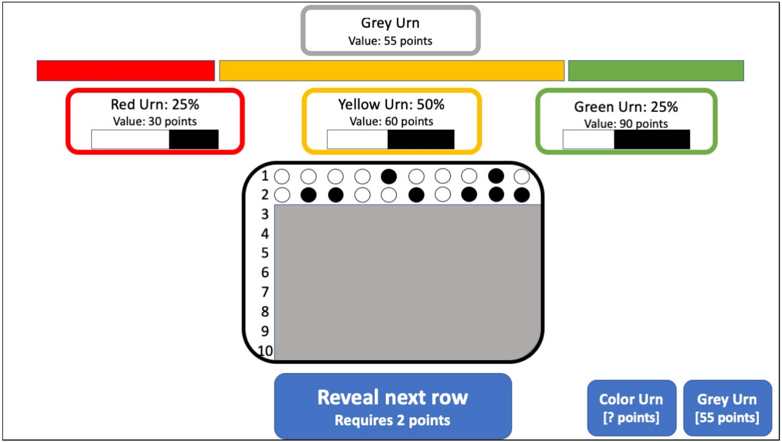


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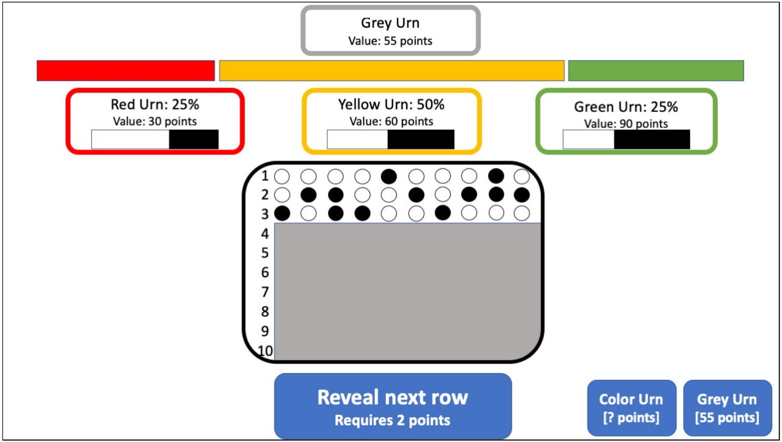
# Task 2 - Ball counting /2




Select the color urn (risky options) or grey urn (safe option).

# Main task 2

- Values of the color urns are fixed [30, 60, 90 points]
- Urn composition is fixed [30, 50, 70 black balls]
- 2 conditions for the prior (urn more likely to be selected)
- Different values of the status quo [$R \gtrless EV$]
- 2 guided practice + 3 practice trials + 20 rewarded trials
- No feedback about the realized state of the world
- Payoff depends on signal cost and binary choice

# Summary

- RI model with N>2 states and binary choice
- State pooling and role of status quo
- Prediction about optimal information acquisition
- Prediction about posterior beliefs

# Summary

- Lab experiment to test the predictions of the model vs alternative explanations (confirmatory sampling strategy)
- Extensive training in Bayesian update (but we are not testing their ability to correctly compute the posterior, at least in the main task)
- Main task follows precisely the setting of the model
- Ball counting task is less structured and more intuitive (sequential sampling)

# Belief Polarization and State Pooling

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# The Model

# Agent's problem

Denote: $\mathbf{v} = (v_1, \ldots, v_n)$, $G(\mathbf{v})$ - prior joint distribution
Find an information strategy maximizing:

$$\max_{P(i|v)} \left\{ \sum_{i=1}^{2} \int_{\mathbf{v}} v_i P(i|\mathbf{v}) G(d\mathbf{v}) - \lambda \kappa(P, G) \right\},$$

where

$$\kappa(P, G) = -\sum_{i=1}^{2} P_i^0 \ln P_i^0 + \int_{\mathbf{v}} \left( \sum_{i=1}^{2} P(i|\mathbf{v}) \ln P(i|\mathbf{v}) \right) G(d\mathbf{v}).$$

$P(i|v)$ is the conditional on the realized value of $v$, the probability of choosing option $i$ and

$$P_i^0 = \int_{\mathbf{v}} P(i|\mathbf{v}) G(d\mathbf{v}), \;\; i = 1, 2$$

where $P_i^0$ is the unconditional probability of option $i$ to be chosen.

## Lemma 1 (Matějka, McKay, 2015)

Conditional on the realized state of the world $s^*$ probability of choosing risky option is

$$P(\text{picking risky}|\text{state is } s^*) = \frac{P_1^0 e^{\frac{v_s^*}{\lambda}}}{P_1^0 e^{\frac{v_s^*}{\lambda}} + (1 - P_1^0) e^{\frac{R}{\lambda}}}$$

of choosing safe option is:

$$P(\text{picking safe}|\text{state is } s^*) = \frac{(1 - P_1^0) e^{\frac{R}{\lambda}}}{P_1^0 e^{\frac{v_s^*}{\lambda}} + (1 - P_1^0) e^{\frac{R}{\lambda}}}$$

here $P_1^0$ is unconditional probability of choosing risky option.

# Beliefs

- Agent's prior expected value of the risky option is:

$$\mathbb{E}v = \sum_{s=1}^{n} v_s g_s$$

  we **fix the state** of the nature: it is $s^*$
- Observer sees agent's updated belief about the average of $v$:

$$\mathbb{E}_i[\mathbb{E}(v|i)|s^*] = P(i = 1|s^*)\mathbb{E}(v|\text{picking option 1}) + \\ + (1 - P(i = 1|s^*))\mathbb{E}(v|\text{picking option 2})$$

  where for option $i \in \{1, 2\}$

$$\mathbb{E}(v|\text{picking option i}) = \sum_{j=1}^{n} v_i P(\text{state is j}|\text{picking option i})$$

# Beliefs

**Theorem**

Expected posterior value of the risky option for a rationally inattentive decision maker is

$$\mathbb{E}_i[\mathbb{E}(v|i)|s^*] = \sum_{i=1}^{n} v_i g_i \frac{\alpha_{s^*} e^{\frac{v_i}{\lambda}} + (1 - \alpha_{s^*}) e^{\frac{R}{\lambda}}}{P_1^0 e^{\frac{v_i}{\lambda}} + (1 - P_1^0) e^{\frac{R}{\lambda}}} \tag{1}$$

where

$$\alpha_{s^*} = \frac{P_1^0 e^{\frac{v_{s^*}}{\lambda}}}{P_1^0 e^{\frac{v_{s^*}}{\lambda}} + (1 - P_1^0) e^{\frac{R}{\lambda}}}$$

# Updating of beliefs

We are interested in

$$\Delta = \mathbb{E}_i[\mathbb{E}(v|i)|s^*] - \mathbb{E}v$$

**Theorem**
The sign of $\Delta$ is the same as the sign of $(v_{s^*} - R)$.

Proof.
Straightforward and we use:

**Lemma 2**
Relations $\alpha_{s^*} \gtrless P_1^0$ under $P_1^0 > 0$ are equivalent to $v_{s^*} \gtrless R$

$\square$

# Example 3 states, 2 actions

- 3 possible states of the world - indexed by s
- 2 options/actions - indexed by a
  - Option 1 - Risky with values: $v_1 < v_2 < v_3$
  - Option 2 - Safe option with value $R$ in all states
- Prior belief about the states: $g_1, g_2, g_3$
- Marginal cost of information: $\lambda$

**Assumption 1:** to rule out uninteresting cases

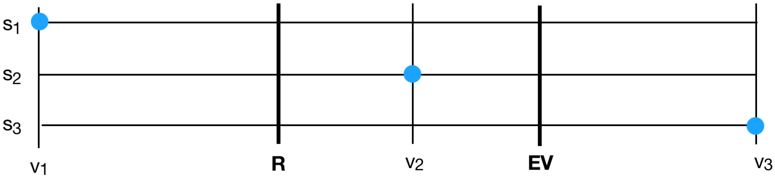
$$v_1 < R < v_3$$

# Updating in "wrong" direction

We are interested when the conditional expectation moves in the **"wrong" direction**

**Example** for $s^* = 1$ the expectation "should" go down, so the agent is biased when

$$\mathbb{E}_a[\mathbb{E}(v|a)|s^*] > \mathbb{E}v > 0$$



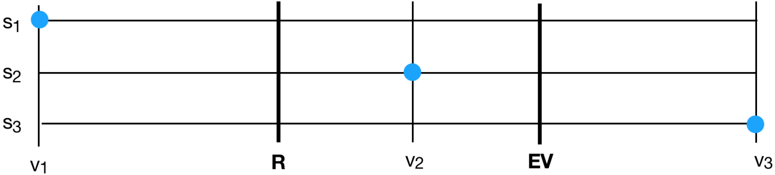
# Updating in "wrong" direction

**Let's denote** $\Delta = \mathbb{E}_a[\mathbb{E}(v|a)|s^*] - \mathbb{E}v$.

If

$$(\mathbb{E}v - v_{s^*}) \cdot \Delta > 0$$

then the agent is updating belief in the wrong direction



# Result