Matching and Learning in School Choice

Silvio Ravaioli (Columbia University)
Lan Nguyen (Columbia University)
Guillaume Haeringer (Baruch College)

Columbia University - Applied Micro Theory Colloquium

March 3, 2020

Motivation - Mistakes in RoL under DA

- Empirical evidence: students make **mistakes** in ranking schools in strategyproof mechanisms
- Mistake: prefer Harvard to Columbia, and declare the opposite
- For example, when the Deferred Acceptance (DA - strategyproof) algorithm is used, we observe failures to rank truthfully the options
- Evidence from Israel medical matching (Hassidim et al. 2017): students rank the no-scholarship option before the with-scholarship option of the same program

Motivation - Mistakes in RoL under DA

- Empirical result replicated in several lab experiments (Chen and Sonmez 2006, Klijn et al. 2010, Ding and Schotter 2019)
  - Large number of deviations from truthful reporting
- Also empirical evidence of little penalization for some of these mistakes (Artemov, Che and He 2017)
  - Some systematic mistakes are payoff irrelevant
- **Can the students *learn* and avoid costly mistakes by receiving *feedback*?**

Feedback in School Choice

- Feedback and learning in the real world - 3 interpretations
  - Past self, e.g. TAing preferences (every year)
  - Advising, e.g. friends and family
  - Review process embedded in the mechanism (examples from China and France)

- Imagine that, after Ranked Ordered Lists (RoLs) are collected and the matching is generated, each student could see their own match, review the RoL, and submit it again

- What is the effect of the review process on the final matches?

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- What is the effect of the review process on the final matches?

Illustrating Example

Students				Schools			
P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{s_1}	P_{s_2}	P_{s_3}	P_{s_4}
s_3	s_1	s_2	s_3	i_3	i_1	i_2	i_4
s_4	s_2	s_3	s_1	i_2	i_4	i_3	i_3
s_1	s_3	s_1	s_2	i_1	i_3	i_1	i_1
s_2	s_4	s_4	s_4	i_4	i_2	i_4	i_2

- **First period**
  - Students i_1 and i_4 are truthful
  - Student i_2 submits $\widehat{P}_{i_2} = s_3, s_4, s_1, s_2$
  - Student i_3 submits $\widehat{P}_{i_3} = s_3, s_2, s_1, s_4$

$\Rightarrow$ i_3 is matched to $\mathbf{s_2}$. $(i_1, s_4), (i_2, s_3), (i_3, s_2), (i_4, s_1)$

- **Second period**
  - All students are truthful

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School Choice - Experimental Literature

	w/ Computer	w/ Humans
No Learn	Rees-Jones et al. (2019)	Chen and Sonmez (2006) Calsamiglia et al. (2010)
Learn	Ding and Schotter (2017)	Ding and Schotter (WP) **This Project**

- The typical experiment involves multiple rounds
- Mild increase in truthfulness during a session (from 55 to 65%)
- Ding and Schotter (WP): feedback and random re-match in every round (5 students, 3 schools, priorities known)

This Project

- Lab experiment with Deferred Acceptance algorithm
  - Eight *rounds* = different environments (preferences/priorities)
  - Each round has five *periods* = same environment
- Separate two aspects of **learning**:
  - Matching mechanism (DA is strategyproof)
  - Environment (priorities, others' preferences)
- Corresponding to two different sets of predictions for DA
  - Increase in truthful RoL over rounds
  - Decrease in "costly" deviation from truthful
- And also to different possible implications.

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Theory Overview

- Goal: we want to define when a RoL P_i'' is "better" than the previous P_i', knowing the true preferences P_i.
- Better = guarantees a weakly preferred match.

Two possible criteria:

- **Better reply** (weaker): P_i'' is better than P_i' when other students' submitted lists are the same
- **Dominating strategy** (stronger): P_i'' is better than P_i' regardless of the others' submitted lists

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Better replies in DA - Haeringer & Halaburda 2011

Take any student i
Fix P_{-i} (the submitted lists of other students + schools' priorities).

Notation:

- P_i = the true preferences of student i
- P_i' = submitted list by i ($\neq P_i$)
- $\mu'(i)$ = match of i with P_i'
- $\mathcal{P}_{-i}$ = all P_{-i} such that

$$DA(P_i', P_{-i})(i) = \mu'(i)$$

Better replies in DA - Haeringer & Halaburda 2011

P_i'' is a better reply than P_i' against $P_{-i} \in \mathcal{P}_{-i}$ if and only if P_i'' is such that only truly preferred schools are "moved up" (above the match),

$$s \; P_i'' \; \mu'(i) \quad \text{and} \quad \mu'(i) \; P_i' \; s \qquad \Longrightarrow \qquad s \; P_i \; \mu'(i)$$

In short, for a better reply a student can do:

- move above $\mu'(i)$ any school truly preferred to $\mu'(i)$
- move below $\mu'(i)$ any school
- re-order in any way above $\mu'(i)$
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and cannot do:

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Dominating Strategies and Kemeny Distance

- Consider P_i', P_i'' two possible rank order lists ($\neq P_i$ true pref.)

Define $K(P_i, P_i')$ the Kemeny set of P_i and P_i',

$$K(P_i, P_i') = \{(s, s') \ : \ P_i \text{ and } P_i' \text{ rank } s \ \& \ s' \text{ differently}\}$$

[Haeringer & Halaburda (2016)] P_i' dominates P_i'' if, and only if,

$$K(P_i, P_i') \subseteq K(P_i, P_i'')$$

That is, whenever P_i' and P_i disagree about the ranking of two schools, so do P_i and P_i''.

- The Kemeny sets provide only a partial order, so in the analysis we will use the Kemeny distance $\delta_{K(P_i,P_i')} := ||K(P_i, P_i')||$.

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Experimental Design

- Two treatments: **DA** and IA algorithms (between subjects)
- Two parts: **without zones** and with zones (within subject)

- Twenty participants (students), 5 schools (4 seats each)
- Schools' value is positively correlated across students
- Students observe own preferences (schools' values)

- Each round of the experiment contains five periods
- Each period contains two phases:
  - Submit the RoL
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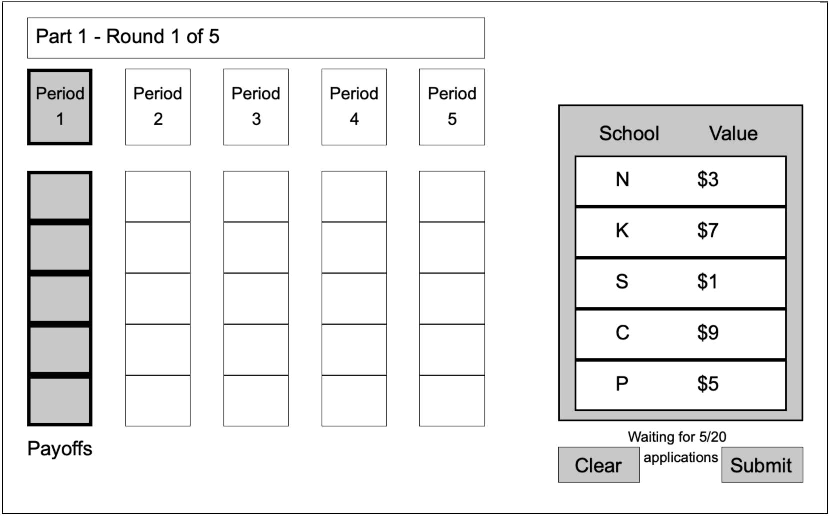

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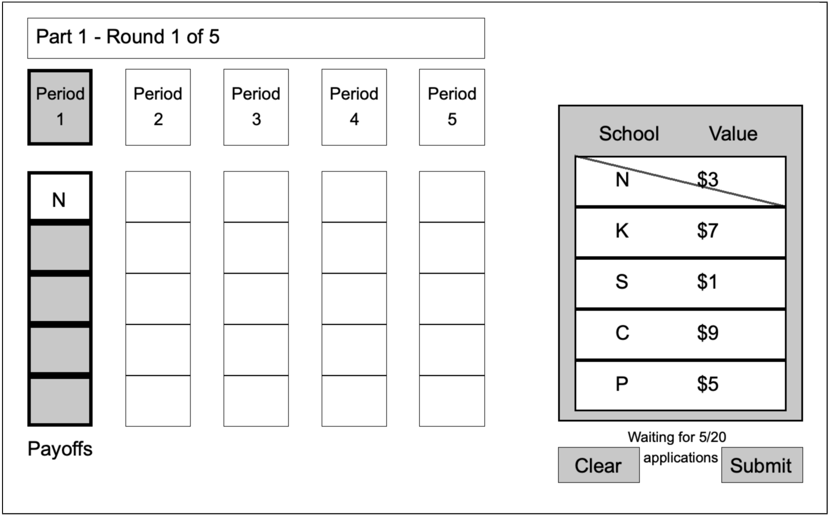

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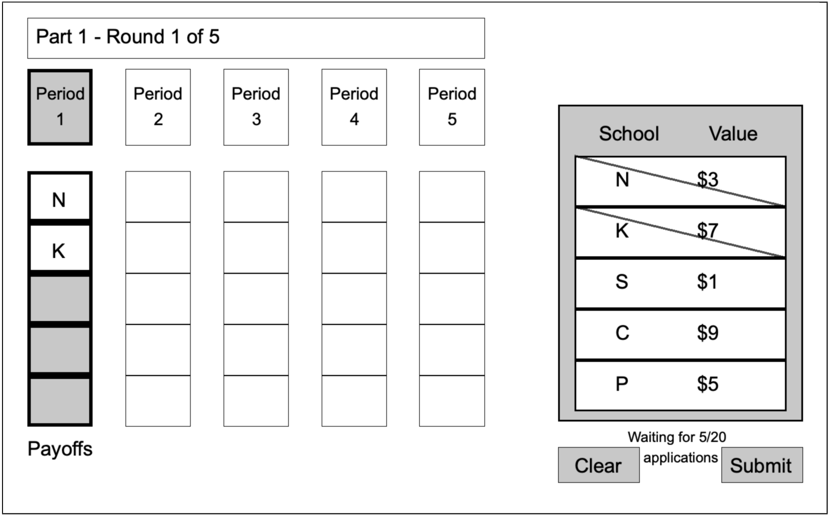

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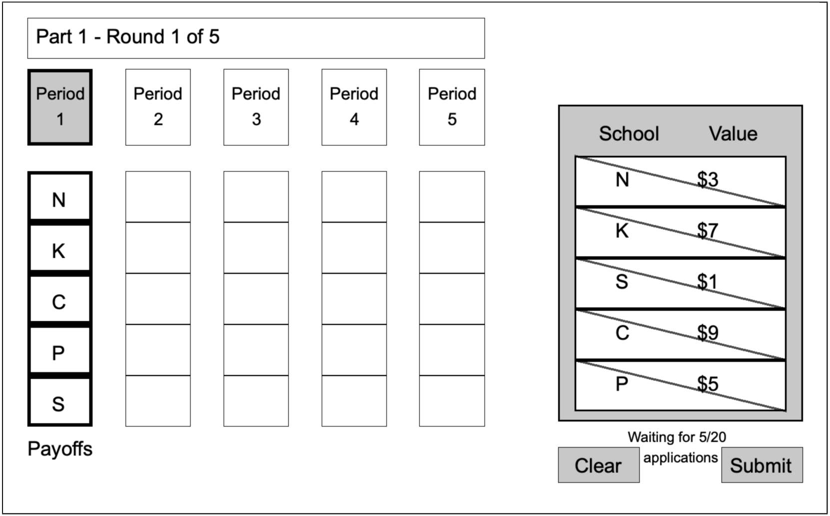

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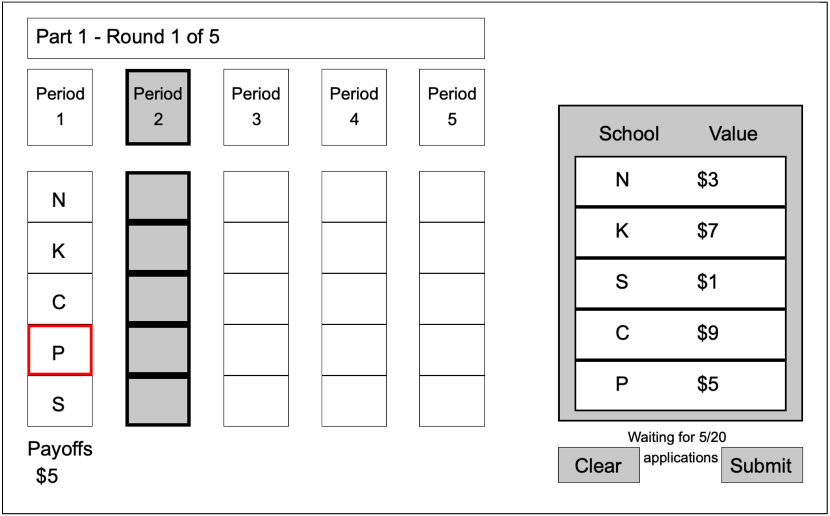

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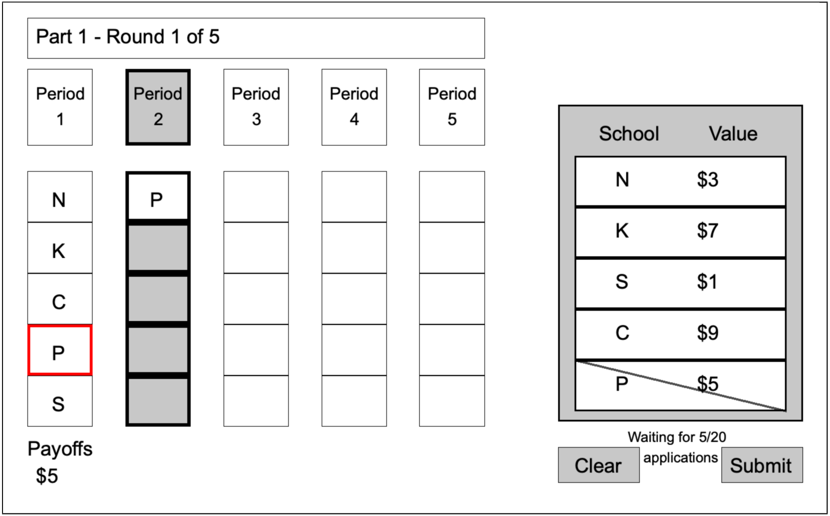

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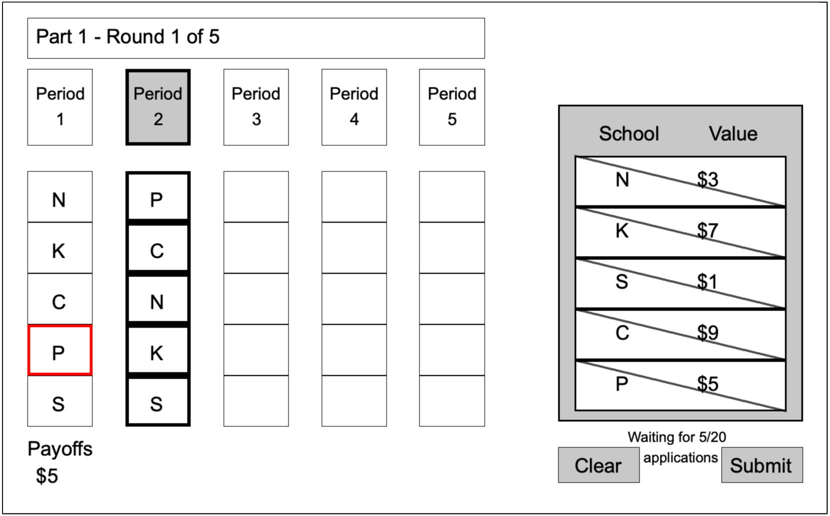

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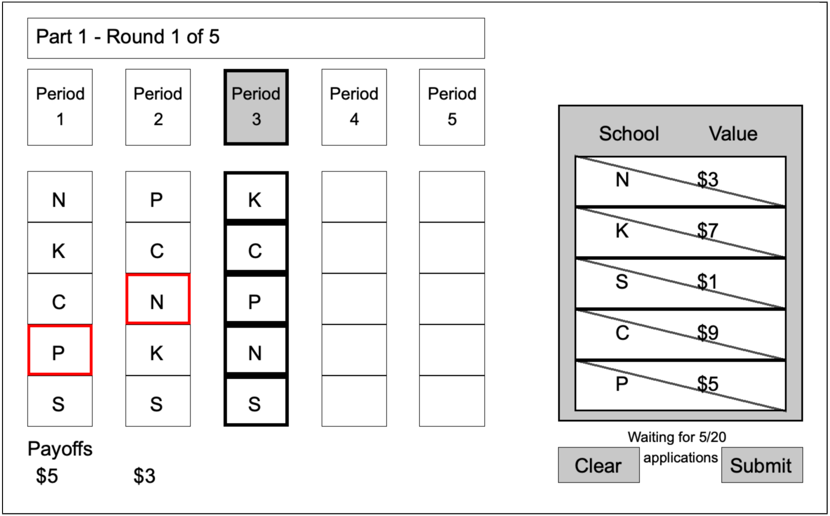

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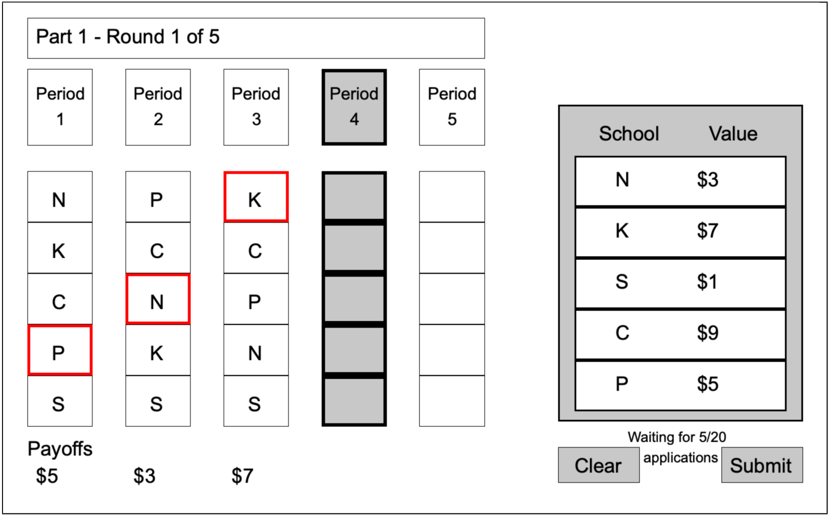

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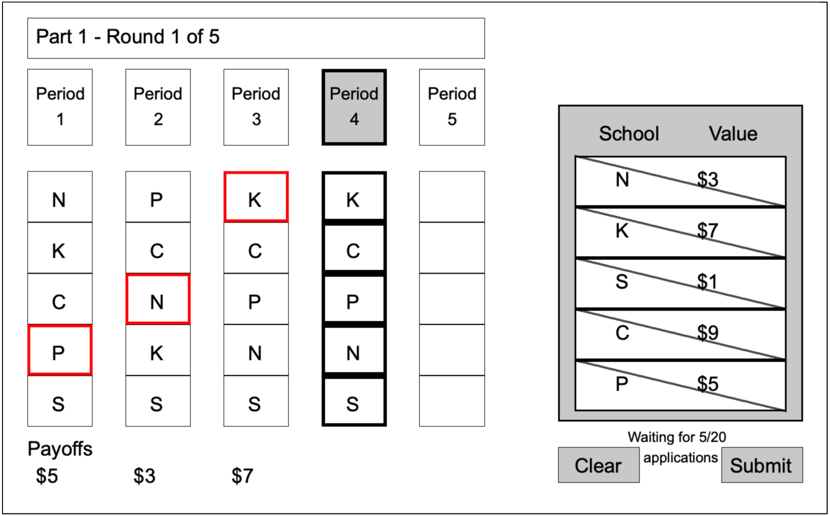

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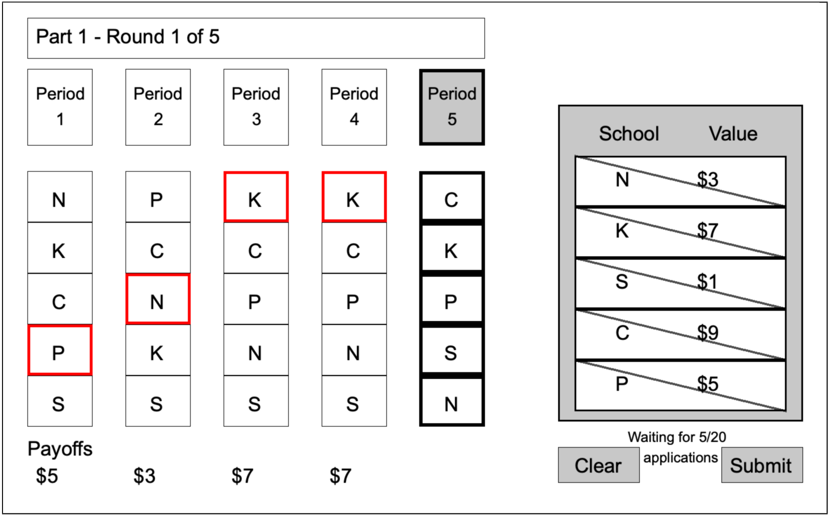

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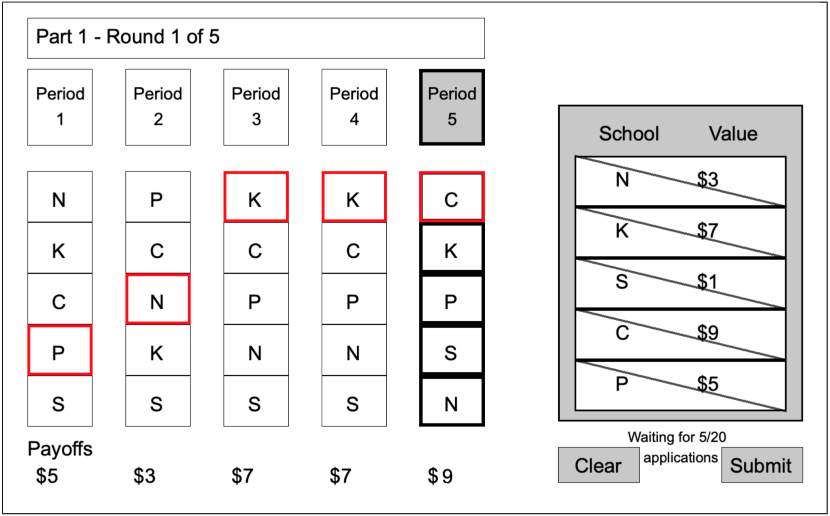
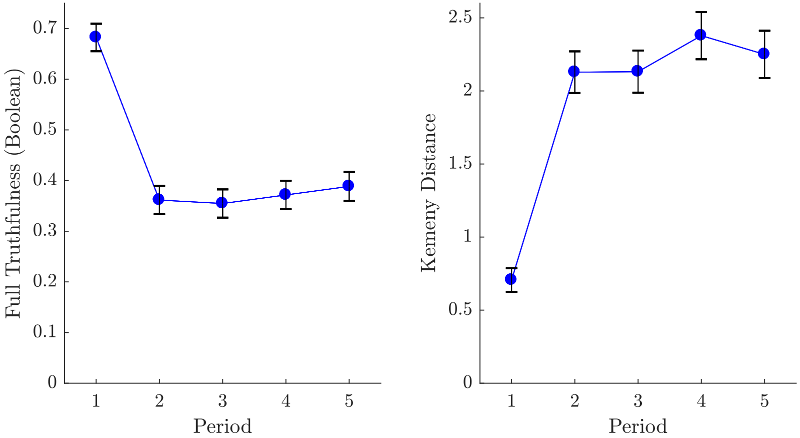


Overview of the Results

1. Truthfulness declines between periods (mostly from t_1 to t_2)
2. Little negative consequences on payoffs
3. Revisions of the RoLs are frequent in all the periods
4. Systematic patterns in the type of revisions
5. (Only) costly mistakes increase truthfulness
6. Truthfulness slowly increases between rounds

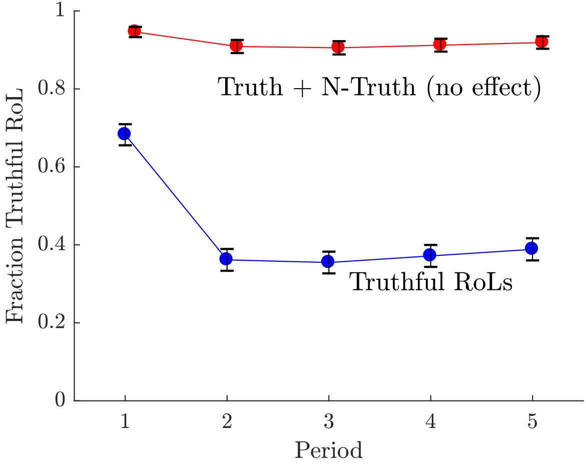
Results /1 - Measures of Truthfulness



Full Truthful: 1 if the whole RoL is truthful, 0 otherwise

Kemeny Distance: numb. of *pairwise switches* from the truthful RoL

Results /2 - Inconsequential Mistakes



Students labeled as *Not-Truthful (no effect)* are untruthful but still matched with the same school as if they were truthful.

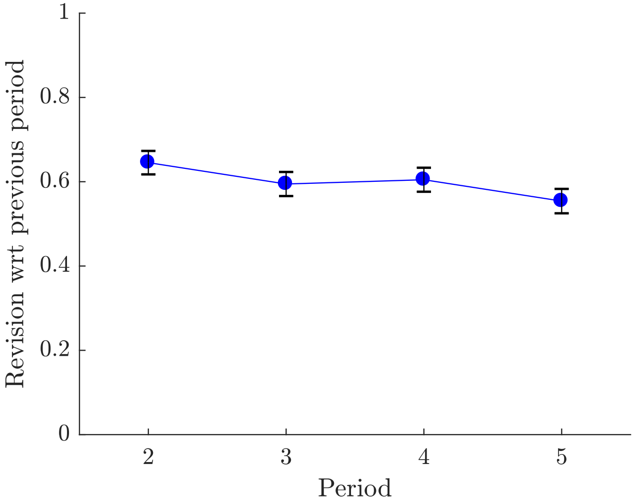
Results /2 - Inconsequential Mistakes

Period 1 \ Period 2	Truthful (36%)	N-Truthful No effect (55%)	N-Truthful W/ effect (9%)
Truthful (68%)	45%	49%	6%
N-Truthful No effect (26%)	18%	73%	9%
N-Truthful W/ effect (5%)	18%	38%	44%

Period 4 \ Period 5	Truthful (39%)	N-Truthful No effect (53%)	N-Truthful W/ effect (8%)
Truthful (37%)	75%	24%	1%
N-Truthful No effect (54%)	16%	75%	9%
N-Truthful W/ effect (9%)	23%	46%	31%

Transition matrices: periods 1 to 2 (left) and periods 4 to 5 (right).

Results /3 - RoL revisions



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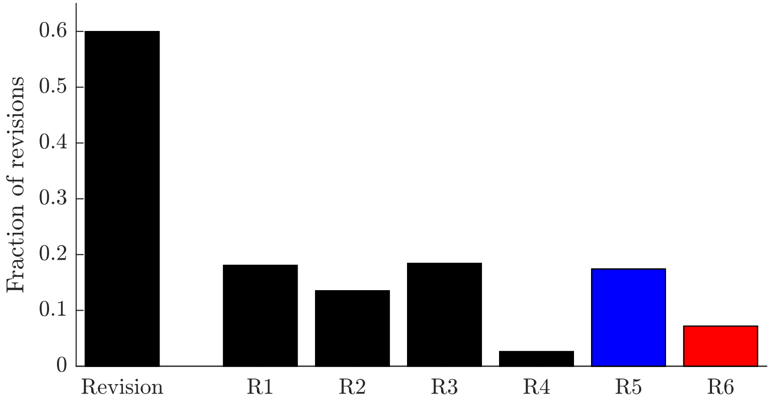
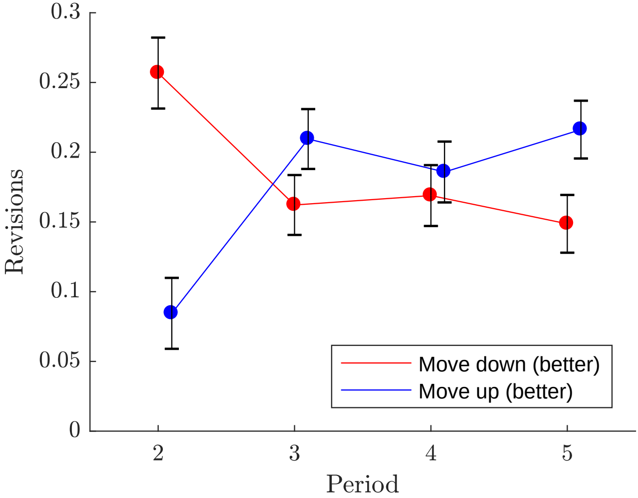
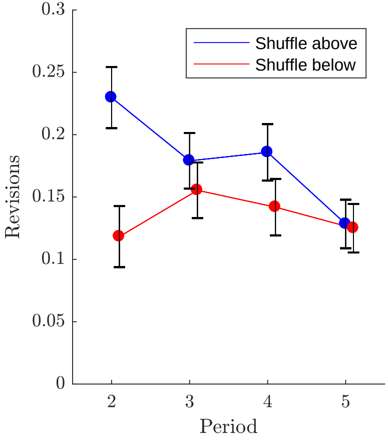


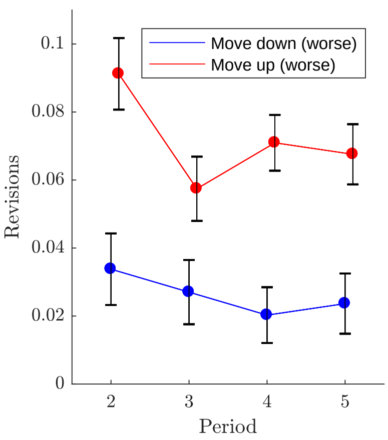
- R1 = shuffle (above), R2 = shuffle (below)
- R3 = move down (better), R4 = move down (worse)
- R5 = move up (better), R6 = move up (worse)

Results /4 - RoL revisions



Results /4 - RoL revisions





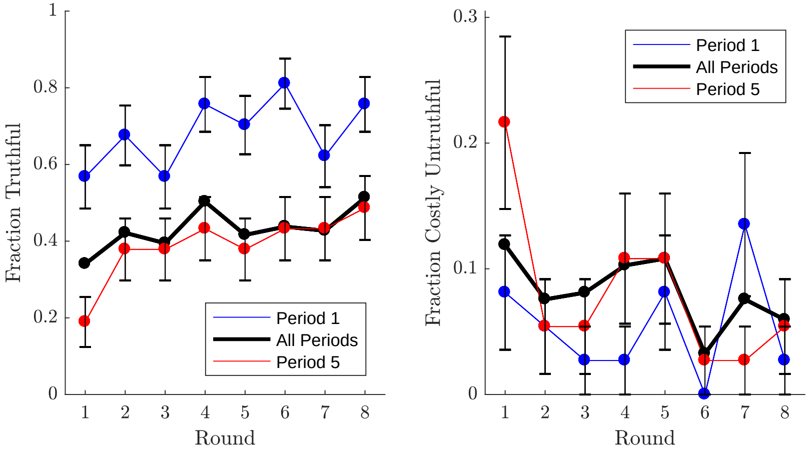
Results /5 - Revision probability

	Logistic Regression - Pr(Revision in period t)			
	(1)	(2)	(3)	(4)
Constant	0.052	-1.206***	-0.157	-1.212***
	(0.183)	(0.285)	(0.217)	(0.287)
Kemeny Distance (t-1)	0.291***	0.095**	0.374***	0.102*
	(0.046)	(0.048)	(0.069)	(0.054)
Payoff Loss (vs best reply) (t-1)	0.325***	0.225	0.131	0.164
	(0.106)	(0.149)	(0.162)	(0.165)
ΔPayoff Loss (t-2 to t-1)		0.247	0.290**	0.304
		(0.401)	(0.114)	(0.405)
Revision (t-1)		2.876***		2.856***
		(0.201)		(0.202)
ΔPayoff Loss*Revision (t-1)		0.063		0.091
		(0.398)		(0.398)
ΔKemeny Distance (t-2 to t-1)			-0.059	-0.007
			(0.055)	(0.055)
ΔPayoff Loss*ΔKemeny Distance			0.066**	0.033
			(0.032)	(0.025)
Round FEs	Y	Y	Y	Y
Periods	2-5	3-5	3-5	3-5
Pseudo R-squared	0.083	0.336	0.124	0.337
N	1184	888	888	888

Results /5 - Kemeny Distance

	Kemeny Distance (t)			
	(1)	(2)	(3)	(4)
Constant	1.179***	0.726***	0.635***	0.554***
	(0.190)	(0.202)	(0.194)	(0.199)
Kemeny Distance (t-1)	0.584***	0.622***	0.838***	0.820***
	(0.036)	(0.042)	(0.038)	(0.040)
Payoff Loss (vs best reply) (t-1)	-0.340***	-0.281***	-0.243**	-0.268**
	(0.079)	(0.107)	(0.105)	(0.109)
ΔPayoff Loss (t-2 to t-1)		0.498**	0.048	0.399**
		(0.197)	(0.071)	(0.198)
Revision (t-1)		0.303*		0.194
		(0.158)		(0.140)
ΔPayoff Loss*Revision (t-1)		-0.612***		-0.344*
		(0.194)		(0.193)
ΔKemeny Distance (t-2 to t-1)			-0.318***	-0.313***
			(0.046)	(0.046)
ΔPayoff Loss*ΔKemeny Distance			-0.043**	-0.041**
			(0.017)	(0.017)
Round FEs	Y	Y	Y	Y
Periods	2-5	3-5	3-5	3-5
R-squared	0.268	0.359	0.410	0.411
N	1184	888	888	888

Results /6 - Learning



Results /6 - Learning

	Full Truthfulness	Payoff	Payoff Loss (vs best reply)
Constant	0.542***	5.291***	0.284***
	(0.029)	(0.163)	(0.048)
Round	0.019***	-0.003	-0.023**
	(0.006)	(0.031)	(0.011)
Period	-0.047***	0.024	0.029
	(0.009)	(0.050)	(0.019)
IA Dummy	-0.057	0.089	1.712***
	(0.042)	(0.236)	(0.138)
IA * Round	-0.013*	0.055	0.010
	(0.008)	(0.044)	(0.027)
IA * Period	0.004	-0.040	-0.134***
	(0.013)	(0.072)	(0.043)
R-squared	0.029	0.0023	0.1562
N	2960	2960	2960

Conclusions

- Participants *learn* that the algorithm is strategyproof
- But they experiment making mistakes, mostly inconsequential
- Making costly mistakes accelerates learning
- Implications/applications? France stopped using the revision (also: regret, strategic manipulation in early periods)

- Major revisions to the experimental paradigm for "pilot 2"
- Manipulate the number of periods (not always 5)
- Other results for IA and for DA-with-zones

Conclusions

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Matching and Learning in School Choice

Silvio Ravaioli (Columbia University)
Lan Nguyen (Columbia University)
Guillaume Haeringer (Baruch College)

Columbia University - Applied Micro Theory Colloquium

March 3, 2020

Discussion - Design Changes for Pilot 2

- Incentive for all the periods or only the last one?
- Modify the number of periods (e.g. 1,3,5)
- Conditions (and their order): DA/IA, zones/no-zones
- Control for one unique stable matching
- Making students' preferences more correlated can increase the likelihood of costly mistakes