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Estimates of Site Seismicity Rates Using Ill-defined Macroseismic Data

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Abstract—A new approach to the problem of site seismic hazard analysis is proposed, based on intensity data affected by uncertainties. This approach takes into account the ordinal and discrete character of intensities, trying to avoid misleading results due to the assumption that intensity can be treated as a real number (continuous distribution estimators, attenuation relationships, etc.). The proposed formulation is based on the use of a distribution function describing, for each earthquake, the probability that site seismic effects can be described by each possible intensity value. In order to obtain site hazard estimates where local data are lacking, the dependence of this distribution function with the distance from the macroseismic epicenter and with epicentral intensity is examined. A methodology has been developed for the purpose of combining such probabilities and estimating site seismicity rates which takes into account the effect of uncertainties involved in this kind of analysis. An application of this approach is described and discussed.

Key words: Seismic hazard, macroseismic intensity.

Introduction

In recent years, our knowledge of seismogenetic processes has been greatly impoved, but not enough to allow for an effective estimate of seismic hazard based only on deterministic approaches. Thus, a possible estimate of hazard can be obtained by the analysis of site seismic history. Following this approach, it is necessary to use historical macroseismic data, particularly for intensities characterized by long return periods.

Using this kind of data three main problems arise:

— historical data are qualitative and they can be summarized by the use of macroseismic scales, which are ordinal and discrete;
— when historical earthquakes are considered, the attribution of the severity level to the earthquake effects is affected by uncertainties due to the difficult

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interpretation of documentary data; it is well-known, for example, that many problems are connected with the translation of old sparse documents in a single intensity value, related to the historical context existing when the source text was written;

— even in the case of seismic effects directly observed, due to the semi-qualitative character of macroseismic scales, the subjective judgement of the researcher who attributes the value of intensity contains an intrinsic uncertainty that should be taken into account.

Discussion of these topics can be found, for example, in VOGT (1991) and MONACHESI and MORONI (1993).

In general, for a given site, three situations are possible:

— data are sufficient for an unequivocal attribution of intensity value. This is particularly the case for recent earthquakes or for the sites where effects have been the most intense (macroseismic epicenters);
— data allow only the determination of a range of acceptable intensity values, possibly assigning a probability value to each intensity (this is particularly the case of documentary data);
— data are not available at the site but only, for instance, at the epicenter. This is a frequent situation, above all when it is necessary to estimate seismic hazard in sites with few inhabitants, where historical information is generally lacking (dam sites, power plants, etc.).

Considering these problems, a generalized characterization of observed site intensity can be performed by expressing a probability level for each class of possible felt intensity. As a consequence, even the measure of the simplest parameters of seismic activity should be substituted by a probabilistic estimate of the same parameters.

Probability Levels of Felt Intensity

For each earthquake felt at the site, a probability density function $p(I)$ is defined. This function assigns to each intensity value I the probability that this value represents the true estimate of the felt intensity. In other words, $p(I)$ represents the confidence attributed to the statement: "The value I is significant of the true severity of shaking."

To assure the probabilistic character of the function $p(I)$, the following assumption must be fulfilled

$$\sum_{I=I_{\min}}^{I_{\max}} p(I) = 1 \tag{1}$$

where I_{min} and I_{max} are respectively the minimum and the maximum values allowed by the adopted macroseismic scale.

If data are sufficient for an unequivocal attribution of intensity equal to I_0, the function $p(I)$ will be 0 for each value of I different from I_0 and 1 if I is equal to I_0.

For each probability density function $p(I)$ it is possible to define the exceedance probability $P(I)$ defined as

$$P(I) = \sum_{j=I}^{I_{max}} p(j). \tag{2}$$

The function $p(I)$ can be defined using formalized or qualitative procedures. As an example, for a given earthquake an intensity VI–VII (often uncorrectly written as 6.5) is assessed. In this case, the probabilistic distribution vectors $p(I)$ and $P(I)$ can be respectively given in the form

$$p(I) = \{0, 0, 0, 0, 0, 0.5, 0.5, 0, 0, 0, 0\}: I \in [1, 11]$$

and

$$P(I) = \{1, 1, 1, 1, 1, 1, 0.5, 0, 0, 0, 0\}: I \in [1, 11]$$

where it has been assumed that intensities VI and VII have the same reliability. As observed above, data at site are not always available for all past earthquakes. In many cases, information only exists about those sites where the effects have been the strongest (epicentral sites). In this case, site intensity has to be deduced on the basis of empirical considerations. In general, the experimental evidence that the severity level tends to monotonically decrease with distance from the epicentral site is widely accepted. On this basis it is possible to define a probability density function $R(I \mid f, I_e)$ representing the probability that the earthquake has been felt at the site with intensity greater or equal to I, conditioned by the epicentral intensity I_e (maximum observed intensity) and by the relative position of the site and the area of maximum felt intensity. Due to simplicity, f is assumed to be a function of the epicentral distance r only and thus

$$f = f(r).$$

The function R can be determined on an empirical basis. Its shape is presumably related to the geological features of the region under study and thus R must be considered as a characteristic of the region where the hazard estimates are performed.

Assuming that the intensity at the epicenter e is characterized by a probability density function $p_e(I)$, the probability $P_s(I)$ that the earthquake has been felt at the site s with intensity greater or equal to I, will be given by the combination of $R(I \mid f(r), I_e)$ and $p_e(I)$. These two probabilities can be considered independent after which the exceedance probability $P_s(I)$ will be given by

$$P_s(I) = \sum_{J=1}^{I_{max}} p_e(J) R(I \mid f(r), J). \tag{3}$$

It is worth noting that, in the framework of the proposed formalism, the probability density function R represents the equivalent of the attenuation laws largely described in the literature (e.g., VON KOVESLIGETHY, 1906; BLAKE, 1941; HOWELL and SHULTZ, 1975). In a different way with respect to the usual attenuation laws, the function R has a probabilistic character which explicitly takes into account the uncertainties involved when this kind of law is used.

Empirical Determination of Probability Density Function R for a Sample Region

In order to show the features of the density function R, an empirical determination has been performed for a sample area: the Italian region.

For this study, 166 earthquakes that occurred from 1348 to 1984 are considered, totalling 12,206 site intensity values. Only directly reported values are taken into account, and no data have been obtained from isoseismals or other interpolations. Moreover, the maximum distance considered between the epicenter and the site is 600 km.

Figure 1 shows the frequency distribution of data used in this study with respect to the epicentral distance. The majority of the observations concern sites within 100 km of the macroseismic epicenter. Figure 2 shows the distribution of sampled points with respect to the natural logarithm of the epicentral distance. This

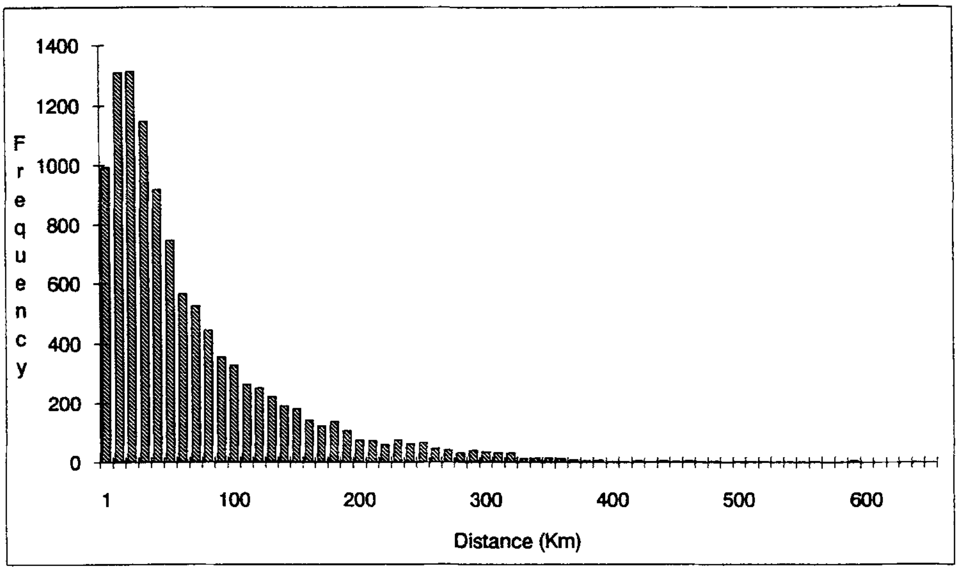


Figure 1
Frequency distribution of available attenuation data as a function of epicentral distance in km. Only attenuation data with epicentral distance less than 600 km have been taken into account.

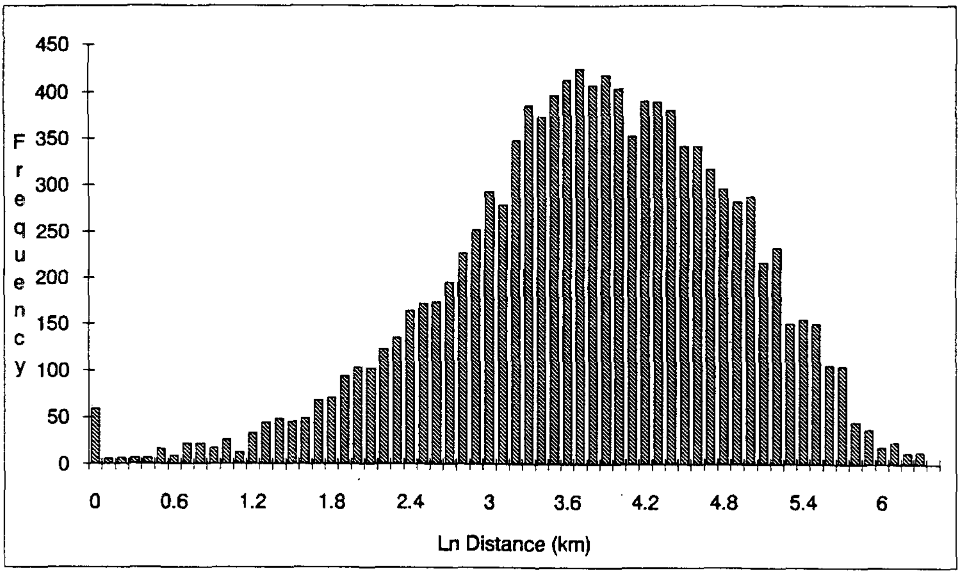


Figure 2
Frequency distribution of available attenuation data as a function of the natural logarithm of epicentral distance in km.

demonstrates how the probability distribution function R may have a more homogeneous distribution of data along its determination field if expressed as

$$R = R(I \,|\, \log(r), I_e). \tag{4}$$

As a first step, the distribution considered was

$$R' = R'(A_0/\log(r)) \tag{5}$$

where A_0 represents the attenuation threshold, i.e., the maximum difference expected between the intensities at the epicenter and at the site.

Figure 3 shows the percentages of sites for which at a given distance the attenuation is less or equal to I, II, III, IV degrees of intensity. The pattern of the data suggests that a suitable shape for the probability function shown in (5) can be provided by logistic models (see, e.g., COX, 1970). These models are widely adopted in the study of binary output or stimulus-response phenomena. Actually, they are used in order to describe the relationship between the probability associated with one of the possible outcomes of a binary variable (success/failure, true/false, etc.) and a continuous variable. In general, these models are expressed as follows

$$P(x) = \frac{e^{a+bx}}{1 + e^{a+bx}} \tag{6}$$

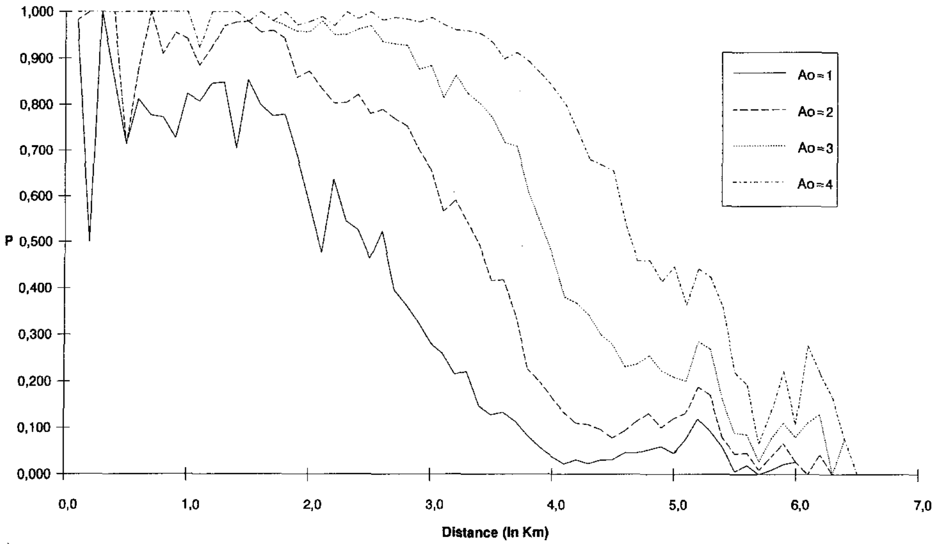


Figure 3
Percentage (P) of the sites for which at a given distance the attenuation is less or equal than the threshold A_0 as a function of natural logarithm of epicentral distance in km. Each curve represents the relative frequency distribution corresponding to a different value of the attenuation threshold A_0 from 1 to 4.

where $P(x)$ is the probability of observing one of the two possible values of the binary variable when the value of the continuous variable is x. The parameters a and b have to be determined empirically. In our case, the model can be interpreted as the estimate of the probability that the attenuation is greater or not than a threshold value (binary variable) given a certain distance from the epicenter (continuous variable).

The distribution function (5) can be rewritten as (6), substituting the natural logarithm of the epicentral distance for x. The parameters a and b will be generally different depending on A_0 and they can be estimated from available data for different attenuation values. The statistical techniques to be used for a regression analysis of a logistic model have been extensively studied and can be considered as standard procedures (for a detailed description of these techniques see the previously mentioned text by Cox, 1979). In our case a and b have been obtained through maximizing the likelihood function by a numerical algorithm based on a quasi-Newtonian method (FLETCHER, 1972) available in the IMSL numeric library. The estimated values of the parameters a and b for various values of A_0 are reported in Table 1, along with their 95% confidence intervals. It can be noted that for different values of A_0 the estimates of a and b are significantly different. The fact that both parameters are monotonically increasing with an increase of A_0, suggests the

Table 1

Values of parameters a and b of the logistic attenuation curves in the form (6) obtained on the basis of attenuation data available in the Italian region. For each value of threshold attenuation value the regression parameters are given along with the corresponding 95% confidence interval

	Attenuation			
	I	II	III	IV
a	3.0 (0.4)	4.9 (0.4)	6.7 (0.6)	8.9 (0.8)
b	−1.3 (0.1)	−1.5 (0.1)	−1.6 (0.1)	−1.8 (0.2)

possibility of modelling the dependence of (6) from A_0 by a linear approximation. Using the values of Table 1 and performing a simple linear regression analysis, the following two relationships are obtained

$$a(A_0) = 1.00 + 1.95A_0 \tag{7}$$

$$b(A_0) = -1.15 - 0.16A_0 \tag{8}$$

whose correlation coefficients are higher than 0.99. Formulae (7) and (8) can be substituted in eq. (6) which can be rewritten as

$$R' = \frac{e^{a(A_0) + b(A_0)\log(r)}}{1 + e^{a(A_0) + b(A_0)\log(r)}} \tag{9}$$

and its pattern is shown in Figure 4. The slope of the ramp between levels 0 and 1 in function (9) is a direct measure of the dispersion in the attenuation data. If we have had a single, perfectly isotropic intensity field with no local site effects, this ramp would have approached a step function. Moreover, in Figure 4, it is possible to note that the trend for different attenuations appears to be parallel: but since the X axis is logarithmic, the dispersion is larger for higher attenuations and corresponding greater distances. This is a clear advantage of the proposed formulation, which is able to take into account the increase of uncertainty due to distance and different attenuation levels by the use of a single expression.

In order to compute the probability P_s in equation (3), the knowledge of the distribution R (eq. (4)) is required. In general, this distribution function will be different from the distribution R' given in (9) because the latter has been obtained assuming only a dependence on A_0 and r. For instance, it seems reasonable to assume that the behavior of R' is influenced also by the epicentral intensity I_e. To examine this kind of dependence, the regression procedure has been applied to data subsets having different epicentral intensities. Three intensity classes have been taken into account: events with epicentral intensity equal to X or XI, equal to VIII or IX, or $\leq$VII. The results of this analysis are reported in Table 2. It is worth

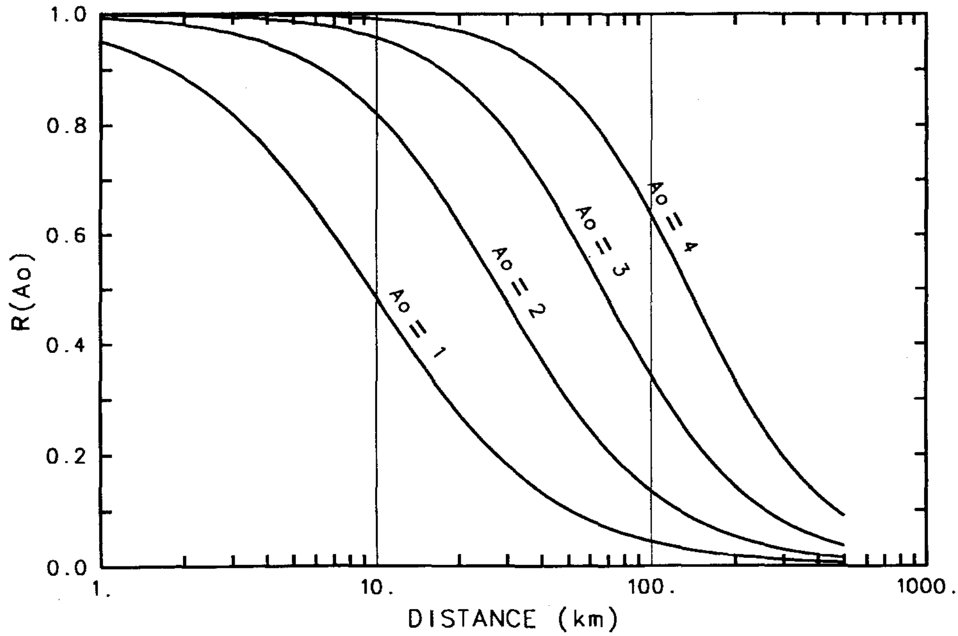


Figure 4

Logistic curves representing the probabilities R to observe attenuations lower than a given threshold A_0 varying the epicentral distance corresponding to the regression curves obtained for the Italian region. Each curve represents the relative frequency distribution corresponding to a different value of the attenuation threshold A_0 from 1 to 4.

Table 2

Values of parameters a and b of the logistic attenuation curves in the form (6) obtained on the basis of attenuation data available in the Italian region for different values of epicentral intensity I_e. For each value of threshold attenuation value the regression parameters are given along with the corresponding 95% confidence interval

	Attenuation				
	I	II	III	IV	I_e
a	2.8 (0.4)	4.9 (0.5)	7.0 (0.6)	10.1 (0.9)	X, XI
b	−1.3 (0.1)	−1.6 (0.1)	−1.8 (0.1)	−2.2 (0.2)	
a	2.9 (0.4)	5.2 (0.5)	7.5 (0.6)	8.7 (0.8)	VIII, IX
b	−1.3 (0.1)	−1.6 (0.1)	−1.9 (0.2)	−1.8 (0.2)	
a	3.5 (0.4)	4.5 (0.4)	6.2 (0.6)	8.3 (1.0)	≤VIII
b	−1.3 (0.1)	−1.2 (0.1)	−1.3 (0.1)	−1.4 (0.2)	

noting that for all the events with epicentral intensity $\geq$ VIII and for $A_0 \leq$ intensity III, the values obtained with the general relationship given in Table 1 fall within the confidence interval given in Table 2. Thus, in this range, the distribution R can be obtained from eqs. (7), (8) and (9) by the substitution of A_0 with $(I_e - I)$.

A final remark concerns the empirical distribution R estimated for the Italian region. Equation (9) gives a low but not null probability value for very long distances (e.g., attenuations less than one degree at 100 km). These results may seem unrealistic if we think about normal "well-behaved" earthquakes. Regardless, they do not depend on an unsuitable functional shape of the logistic model, but on the data themselves (see Fig. 3). In fact, in the applied sample, some earthquakes present anomalous characteristics. For example, the large earthquake that struck central-southern Italy in 1456 is likely to be considered as a multiple event. It produced many different zones with intensity X MCS along the Italian Apennines for over 170 km (POSTPISCHL, 1985a), and thus the definition of the epicenter in this case makes little sense. Other well studied (presumably deep) events show many intensity peaks throughout the Po Plain, Northern Italy, which is more than 200 km wide. We want to emphasize that we did not remove any earthquake from the sample because our model is based on statistical properties, and not on *a priori* physical explanations. If strange attenuation values do exist, we cannot remove them simply on the basis of the fact that they do no fit a physical model: this would result in underconservative seismic hazard estimation. Somebody might question whether these anomalous events are overrepresented in the sample studied, just because of their peculiar behavior, but this is beyond the aim of this study.

Estimate of Site Occurrence Rates

Using the probability for each earthquake to be felt with a given intensity, it will be possible to compute statistics useful for hazard assessment such as the seismicity rate associated with each intensity level.

The following formulae represent an example of possible applications of the method, and can be reduced to the usual statistics on the number of events if the probability levels are set to two possible values only: 0 and 1.

The simplest estimators for the mean return period $\tau(I)$ and the occurrence rate $\lambda(I)$ of earthquakes with intensity greater or equal to I are:

$$\hat{\tau}(I) = \frac{T}{N(I)} \tag{10}$$

$$\hat{\lambda}(I) = \frac{1}{\tau(I)} = \frac{N(I)}{T} \tag{11}$$

where T is the sampling period (i.e., the duration of the seismic catalog) and $N(I)$ is the number of earthquakes whose site intensity is expected to be greater or equal to I during the time span T.

The computation of the expected number of events $\hat{N}(I)$ may be performed as follows. Let x_i be the Bernoullian random variable describing the exceedance of intensity I at a site for the ith event in a seismic catalog, containing n events recorded during the span T. The value of the random variable x_i is respectively equal to 1 or to 0 if the site intensity due to that earthquake is greater or not greater than intensity I. The probability density function f of x_i is

$$f(x_i) = P_i(I)^{x_i}(1 - P_i(I))^{1-x_i}. \tag{12}$$

The probability $P_i(I)$ represents the probability that the ith earthquake is felt with intensity greater or equal to I. If available information allows us to assign a precise intensity value at the site, $P_i(I)$ assumes a value equal to 0 or 1. In the event that uncertainty occurs, $P_i(I)$ may be assigned by expert subjective judgment or other more or less formalized procedures. If no observation is available for a given earthquake at the site, $P_i(I)$ coincides with the probability function P_s (eq. (3)) evaluated for the ith event in the seismic catalogue. The expected number $\hat{N}(I)$ is now

$$\hat{N}(I) = E\left(\sum_{i=1}^{n} x_i\right) = \sum_{i=1}^{n} E(x_i). \tag{13}$$

Considering that the average of the Bernoullian random variable x_i is

$$E(x_i) = P_i(I) \tag{14}$$

at last the formulae (10) and (11) may be written as

$$\hat{\tau}(I) = \frac{T}{\sum_{i=1}^{n} P_i(I)} \tag{15}$$

$$\hat{\lambda}(I) = \frac{\sum_{i=1}^{n} P_i(I)}{T} \tag{16}$$

where $\hat{\tau}$ and $\hat{\lambda}$ represent the estimators of τ and λ, respectively. An approximate evaluation of the standard deviations σ_τ and σ_λ of the two estimators $\tau(I)$ and $\lambda(I)$ is provided by the following formulas (see Appendix):

$$\sigma_\tau^2 \simeq \frac{(n-1)\sigma_t^2}{\hat{N}(I)^2} + \frac{T^2\sigma_N^2}{\hat{N}(I)^4} \tag{17}$$

$$\sigma_\lambda^2 \simeq \frac{\sigma_N^2}{T^2} + \frac{\hat{N}(I)^2\sigma_t^2}{T^4} \tag{18}$$

where σ_t is the standard deviation of the frequency distribution of interevent time intervals $\{t_i\}$ and σ_N is the standard deviation of the estimator $\hat{N}(I)$ of the expected number of events with the site intensity greater or equal to I

$$\sigma_t^2 = \frac{(n-1)\sum_{i=1}^{(n-1)} t_i^2 - \left(\sum_{i=1}^{(n-1)} t_i\right)^2}{(n-1)(n-2)} \tag{19}$$

$$\sigma_N^2 = \sum_{i=1}^{n} P_i(I)(1 - P_i(I)). \tag{20}$$

It is worth noting that no assumptions have been made on the probability density function of the intensity.

Demonstrative Example

To illustrate the proposed methodology the seismicity felt at the site of Porretta Terme in Northern Italy (Fig. 5) has been analyzed. There are no known active major faults near the site. There is a diffuse background seismicity and stronger events ($I_e \geq$ IX) occur in the seismogenetic zones located within 150 km off the site (POSTPISCHL, 1985a,b). In this case, great importance for hazard estimates is assumed by intensity attenuation patterns associated with distant earthquakes. In the following, the mean return times are computed for each class of expected site intensity using the statistics proposed above. The data used are those reported in the Catalogue of the Italian Earthquakes from 1000 to 1980 (POSTPISCHL, 1985b) and in the instrumental bulletins of the National Institute of Geophysics (ING) from 1981 to 1988.

In order to supply significant estimates of the mean return time, the seismic history considered should be representative of the true seismicity rate in the time span considered ("completeness" of the seismic catalogue). In other terms, the time interval over which the seismic history of the site can be considered "complete" for seismic hazard purposes should be defined. Since each intensity class is treated separately, the completeness analysis is relevant to each class. An expeditive completeness analysis for the site of Porretta Terme has been performed using the CUVI method (MULARGIA *et al.*, 1987) and the intervals assumed "complete" for our purposes result thusly: from 1300 to 1988 for $I_s \geq$ VIII, from 1400 to 1988 for $I_s =$ V and from 1750 to 1988 for $I_s =$ III and $I_s =$ IV.

In order to estimate expected seismicity rates, all earthquakes which occurred within 150 km from the site have been taken into account. No value of documented site intensity has been considered. Equation (3) has been used for estimating the distribution of expected site intensities. Uncertainty on epicentral intensity has been taken into account. When a single intensity value I_0 is associated with the

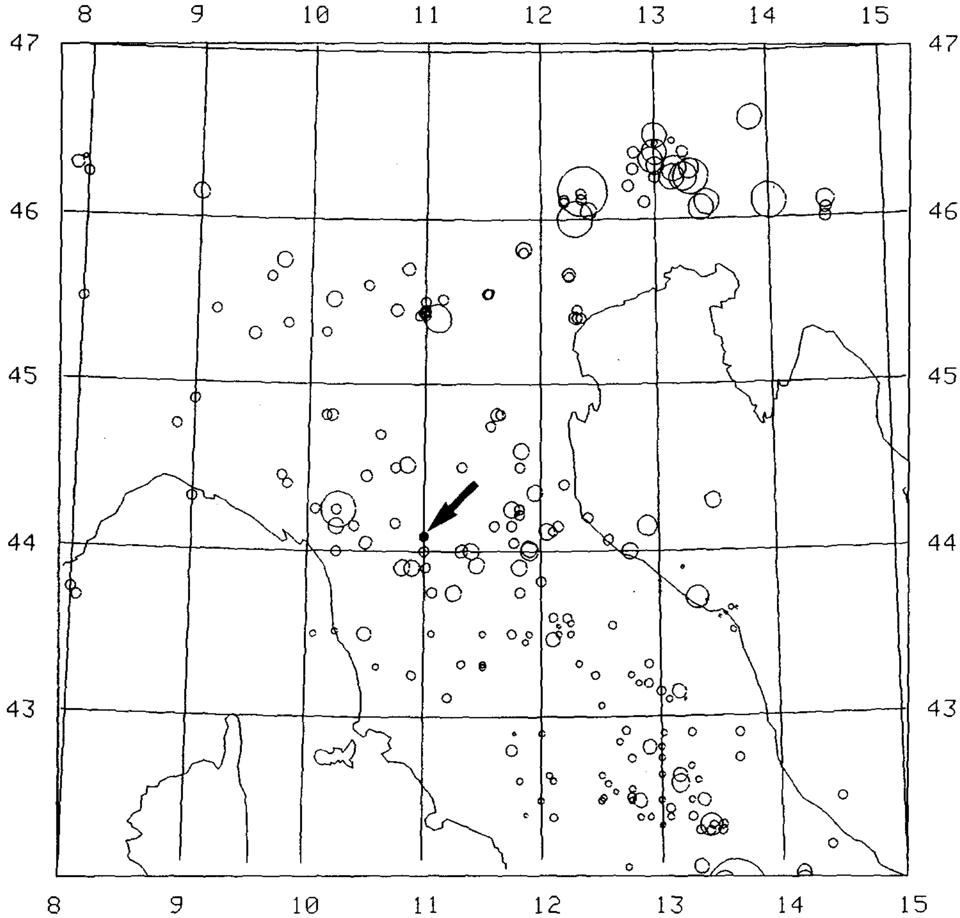


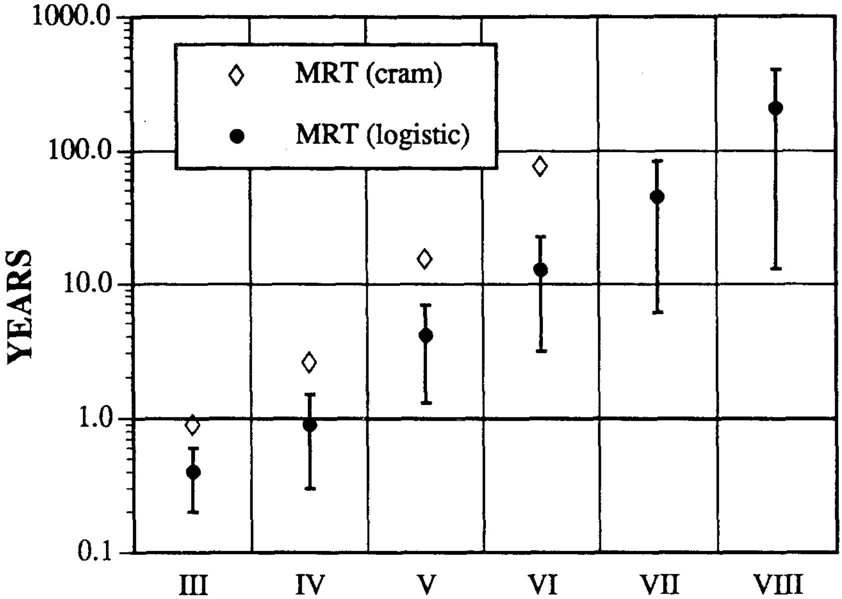
Figure 5
Distribution of earthquakes with epicentral macroseismic intensity $\geq$ VIII which occurred in northern Italy from 1000 A.D. to 1988 (POSTPISCHL, 1985b; Bulletins of National Institute of Geophysics). Circles are the earthquakes focal volumes (BÅTH and DUDA, 1964). The arrow indicates the location of the Porretta Terme test site.

earthquake, the relevant probability distribution $p_e(I)$ is equal to 1 for $I = I_0$ and 0 elsewhere. When an earthquake is characterized by an intermediate intensity value $(I_0 - I_1)$, p_e is assumed equal to 0.5 for $I = I_0$ and $I = I_1$ while it is assumed $p_e = 0$ elsewhere.

Mean return times for expected intensity $\geq$ III have been computed, assuming for a and b parameters in eq. (9) the values given in eqs. (7) and (8) which hold for Italy. These results have been compared with those obtained using a more classic approach based on deterministic attenuation laws. The deterministic attenuation model used in this example has been derived by BERARDI *et al.* (1990) from the study of more than 6000 observed site intensities (hereafter referred to as CRAM, Cubic Root Attenuation Model); it has the form

$$I_e - I_s = -0.729 + 1.122r^{1/3}$$

where I_e is the maximum observed intensity, I_s is the intensity at the site and r is the distance from the epicenter to the site. Using the CRAM the intensities estimated at the site have been rounded to the nearest half intensity degree. The results of this comparative analysis are reported in Figure 6. The values of expected mean return



	III	IV	V	VI	VII	VIII
MRT (cram)	0.9	2.6	15.5	76.6		
MRT (logistic)	0.4	0.9	4.1	12.8	45.0	211.0

MACROSEISMIC INTENSITY

Figure 6

Mean return times (MRT) for macroseismic intensities expected at the site of Porretta Terme. MRT values computed on the basis of the deterministic attenuation law (cram) are reported along with the values obtained using the probabilistic approach (logistic) proposed in this work (see text for details). The values of expected MRT are only reported for those intensity values which occurred at least twice at the site considered. Error bars represent the standard deviation interval (eq. (17) in the text) around the expected value of MRT (eq. (15) in the text). The table below the picture reports MRT values expected for each intensity class following the two approaches proposed.

times are reported for those intensity values which occurred at least twice at the site considered.

The estimated mean return times using CRAM are systematically longer than the ones estimated using the approach proposed in this work. This is due to the fact that, in the methodology proposed here, the probability is spread around the most probable site intensity value. Since the distribution of site intensities versus frequency is roughly negative exponential (as confirmed *a posteriori* by the approximately log-linear trend of the mean return times in Fig. 6), the frequency of minor events with a probability of being more intense results to be significantly greater than the frequency of major events with a probability of being less intense. When probabilistic attenuation laws are used, this lack of symmetry produces a number of expected earthquakes for a given class which is greater than the same number obtained through a deterministic attenuation law. These results suggest that if no probability distribution is taken into account for the expected felt intensity and for epicentral intensity values, the net effect could be the underestimate of the seismic hazard.

In order to take into account these problems when a deterministic attenuation law is used, it is possible to introduce a sort of "error distribution" around the expected site intensity. The main advantage of the logistic model with respect to this approach is that the probabilistic attenuation model requires no additional parameters to consider the variability of this "error distribution" with respect to distance, epicentral intensity and site intensity, and no additional assumptions are requested pertaining to the shape of this "error distribution." Lastly, it is worth noting that the estimate of standard deviation associated with mean return times (Fig. 6) suggests a strong increase of uncertainty when higher intensity classes are considered. This result seems realistic since the low number of earthquakes used to compute site hazard estimates for intense earthquakes.

Conclusions

The intensity data are often improperly used for seismic hazard estimates. In fact, the discrete and nonmetric character of the intensity scales does not permit the use of statistical techniques developed for instrumentally measured data. Furthermore, in many cases, intensity data are affected by wide uncertainties and often different intensity estimates are acceptable for the same earthquake with different degrees of reliability. In this situation, to allow a correct treatment of intensity data in seismic hazard assessment, a new approach has been proposed. In this approach, to each earthquake a probability distribution function is assigned which represents, for each intensity value I, the degree of reliability associated with the statement: "at the site the earthquake has been felt with intensity I." These values can be assigned on the basis of expert judgments or more or less formalized procedures. On this basis, it is possible to elaborate statistics which take into account the uncertainties involved when intensity data are used. For this purpose, suitable estimators of mean return times and seismicity rates are supplied on the basis of the proposed approach.

When no direct information is available, attenuation laws are described; these allow definition of probability distribution expected for site intensities as a function of epicentral intensity and of epicentral distance. These laws are logistic functions of two empirical parameters. An estimate of these parameters has been supplied for the Italian region on the basis of a wide set of macroseismic data.

The proposed approach has been applied for the estimate of the expected seismicity rate at a site in Northern Italy. The results obtained reveal that, when uncertainty on epicentral intensity and attenuation pattern is taken into account, estimated mean return times become systematically shorter than those obtained using classic approaches based on deterministic attenuation laws. This suggests that significant underestimates of seismic hazard are possible when uncertainties in site intensities are not properly considered and analyzed.

Appendix

A first-order approximation of the standard deviations σ_τ and σ_λ of the mean return period and of the occurrence rate estimators reported in (16) and (17) may be provided as follows. Let $z = \phi(x, y)$ a variable function of two random variable x and y. The first-order Taylor approximation of ϕ is expressed by

$$\phi(x, y) = \phi(\bar{x}, \bar{y}) + (x - \bar{x}) \frac{\partial}{\partial x} \phi(\bar{x}, \bar{y}) + (y - \bar{y}) \frac{\partial}{\partial y} \phi(\bar{x}, \bar{y}) + o(x, y) \qquad \text{(A1)}$$

Using the approximated formula (A1), it is possible to write

$$\sigma_\phi^2 \simeq \left[\frac{\partial}{\partial x} \phi(\bar{x}, \bar{y})\right]^2 \sigma_x^2 + \left[\frac{\partial}{\partial y} \phi(\bar{x}, \bar{y})\right]^2 \sigma_y^2. \qquad \text{(A2)}$$

This formula may be used in order to evaluate σ_τ and σ_λ where

$$\hat{\tau}(I) = \hat{\lambda}(I)^{-1} = \frac{T}{\hat{N}(I)} = \frac{\sum_{i=1}^{n-1} t_i}{\sum_{i=1}^{n} P_i} \qquad \text{(A3)}$$

where t_i represents interevent times of the n earthquakes occurred in the time span T and P_i is the probability that the intensity felt at the site in correspondence of the ith earthquake is $\geq I$.

Actually, variables x and y in formula (A2) may be alternatively substituted for T and $N(I)$. Noting that

$$\sigma_T^2 = \sigma^2\left(\sum_{i=1}^{n-1} t_i\right) = \sum_{i=1}^{n-1} \sigma_t^2 = (n-1)\sigma_t^2 \tag{A4}$$

$$\sigma_N^2 = \sigma^2\left(\sum_{i=1}^{n} P_i\right) = \sum_{i=1}^{n} P_i(1-P_i) \tag{A5}$$

at last, it is possible to compute

$$\sigma_\tau^2 \simeq \frac{\sigma_T^2}{\hat{N}(I)^2} + \frac{T^2\sigma_N^2}{\hat{N}(I)^4} \tag{A6}$$

$$\sigma_\lambda^2 \simeq \frac{\sigma_N^2}{T^2} + \frac{\hat{N}(I)^2\sigma_T^2}{T^4} \tag{A7}$$

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