Universität Potsdam

Master Thesis

Forecasting Macroseismic Intensities: A Sensitivity Study of a Bayesian Approach

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Institute of Earth and Environmental Science

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Declaration of Authorship

I, Silvio Schwarz, declare that this thesis titled, 'Forecasting Macroseismic Intensities: A Sensitivity Study of a Bayesian Approach' and the work presented in it are my own. I confirm that:

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“ If I have seen further it is by standing on the shoulders of Giants.”

Sir Isaac Newton

UNIVERSITÄT POTSDAM

Abstract

Faculty of Science
Institute of Earth and Environmental Science

Master of Science

**Forecasting Macroseismic Intensities:
A Sensitivity Study of a Bayesian Approach**

by Silvio SCHWARZ

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too...

Acknowledgements

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Abbreviations

I_0	Intensity at the epicentre
I_s	Intensity at site
PGA	**P**eak **G**round **A**cceleration
PSA	**P**eak **S**pectral **A**cceleration
PGV	**P**eak **G**round **V**elocity

For/Dedicated to/To my…

1 Introduction

Despite their qualitative and subjective nature macroseismic intensities are still a key subject of seismology and especially of earthquake hazard analysis and engineering. For regions with no or just a sparse network of seismic stations they pose a way of generating local attenuation relationships making use of large historical records of felt effects of earthquakes.

Since the pioneering work of Koveslighety (1906) many different models have been developed in trying to find a relation between macroseismic intensity and distance from the source of an earthquake. In the recent past this lead to an increased interest in macroseismic intensities mainly in Italy and the development of new attenuation relationships (Albarello and D'Amico (2004), Carletti and Gasperini (2003), Gasperini (2001), Pasolini et al. (2008)). The vast majority tackles this problem from a regression approach with different functional forms and parameters like local site condition fitted to a dataset of observed macroseismic intensities or by using conversion relationships between instrumental ground motion values.

Rotondi and Zonno (2004) proposed a probabilistic model that respects the categorical nature of intensities. It models the intensity decay by using the discrete-valued binomial distribution to represent the intensities. Through the Bayesian theorem prior distributions are updated by observed values of macroseismic intensity.

This thesis is concerned with evaluating the performance of the approach by Rotondi and Zonno (2004). A sensitivity study is carried out concerning the influence of different parameters on the prediction performance. This is first applied to a synthetic data set in order to be able to generate an arbitrary amount of data and to be sure no other effects influence the result of the performance test. Than this procedure is performed on macroseismic data from Central Asia.

The main aim of this thesis is to gather an understanding of the model of Rotondi and Zonno (2004) and to give a guidance how to use it in addition to knowledge of past earthquakes to draw conclusions and forecast probabilities for future scenarios. All computations where performed using the R programming language (R Core Team, 2015) and the integrated development environment RStudio (RStudio Team, 2012). The

Pakage caret (Max Kuhn et al., 2016) was used to perform the cross-validation. Foreach(Analytics and Weston, 2015b) and doParallel (Analytics and Weston, 2015a) for parallel computation. NLS2 (Grothendieck, 2013) for gridsearch.
All is available at Github: https://github.com/silvioschwarz/master-thesis.

2 Methodology

Given the epicentral intensity (I_0) a set of observation of macroseismic intensity is first divided into bins of distance. Then it is assumed that the attenuation behavior in these bins is homogeneous and can be described by the following binomial distribution:

$$P(I_s = i \mid I_0 = i_0, p) = \binom{i_0}{i} p^i (1-p)^{i_0-i} \tag{1}$$

Since the governing parameter p is also unknown it is treated, according to the Bayesian paradigm, as a random variable and a Beta distribution is defined.

$$P(p) = \frac{1}{B(\alpha, \beta)} p^{\alpha-1}(1-p)^{\beta-1} \tag{2}$$

This means that for each distance bin a Bayesian model with a likelihood of a binomial distribution and a prior of a Beta distribution is constructed and subsequently the observations for each distance bin are used to estimate the p parameter.

$$\begin{aligned} P(p \mid I_s) &= \frac{P(I_s \mid p)}{P(I_s)} P(p) && \text{(3a)} \\ &\propto p^i (1-p)^{i_0-i} * p^{\alpha-1}(1-p)^{\beta-1} && \text{(3b)} \\ &\propto p^{\alpha+i-1}(1-p)^{\beta+i_0-i-1} && \text{(3c)} \\ &\propto p^{\alpha'-1}(1-p)^{\beta'-1} && \text{(3d)} \end{aligned}$$

Choosing a Beta distribution as prior has the advantage of a conjugate model, so the calculations are performed more easily because the posterior distribution is again a beta distribution. To compute the posterior distribution $P(p \mid I_s)$ only the parameters α and

β have to be updated:

$$\alpha' = \alpha + \sum_{n=1}^{N} i_s \qquad\qquad \beta' = \beta + \sum_{n=1}^{N} (I_0 - i_s) \tag{4}$$

Because the distance is discretized it is just possible to calculate the probability distribution for each distance bin. To overcome this obstacle and to have a continuous model the mode for each distance bin is used to construct a smoothing function using an inverse power function.

$$g(d) = \left(\frac{\gamma_1}{\gamma_1 + d}\right)^{\gamma_2} \tag{5}$$

The smoothing function is subsequently applied to the mean of the posterior distribution $\hat{p}$:

$$\hat{p} = E\left[P(p \mid I_s)\right] = \frac{\alpha'}{\alpha' + \beta'} \tag{6}$$

In order to forecast macroseismic intensities at any given distance the fitted smoothing function g(d) is used as an estimate of the p parameter and plugged in to the binomial distribution:

$$P(I_s = i \mid I_0, g(d)) = \binom{I_0}{i} g(d)^i (1 - g(d))^{i_0 - i} \tag{7}$$

This approach has some special properties and assumption. Since the binomial distribution is a discrete probability distribution the categorical nature of intensities is respected. In contrast to a multinomial-Dirichlet model it has the advantage of needing only two parameters (α and β) to be estimated. Because the binomial distribution approaches the Gaussian distribution in the limits it has also a kind of neighboring effect, meaning that intensities next to the intensity with highest probability are assigned the next lowest probability. This models actually some reasonable physical behavior because for example in a scenario were the highest probability is given to intensity VII it wouldn't be logical that the second highest probability would be intensity II. At this point the model assumes that the earthquake process is linked to a point source, e.g. the isoseismals are circles.

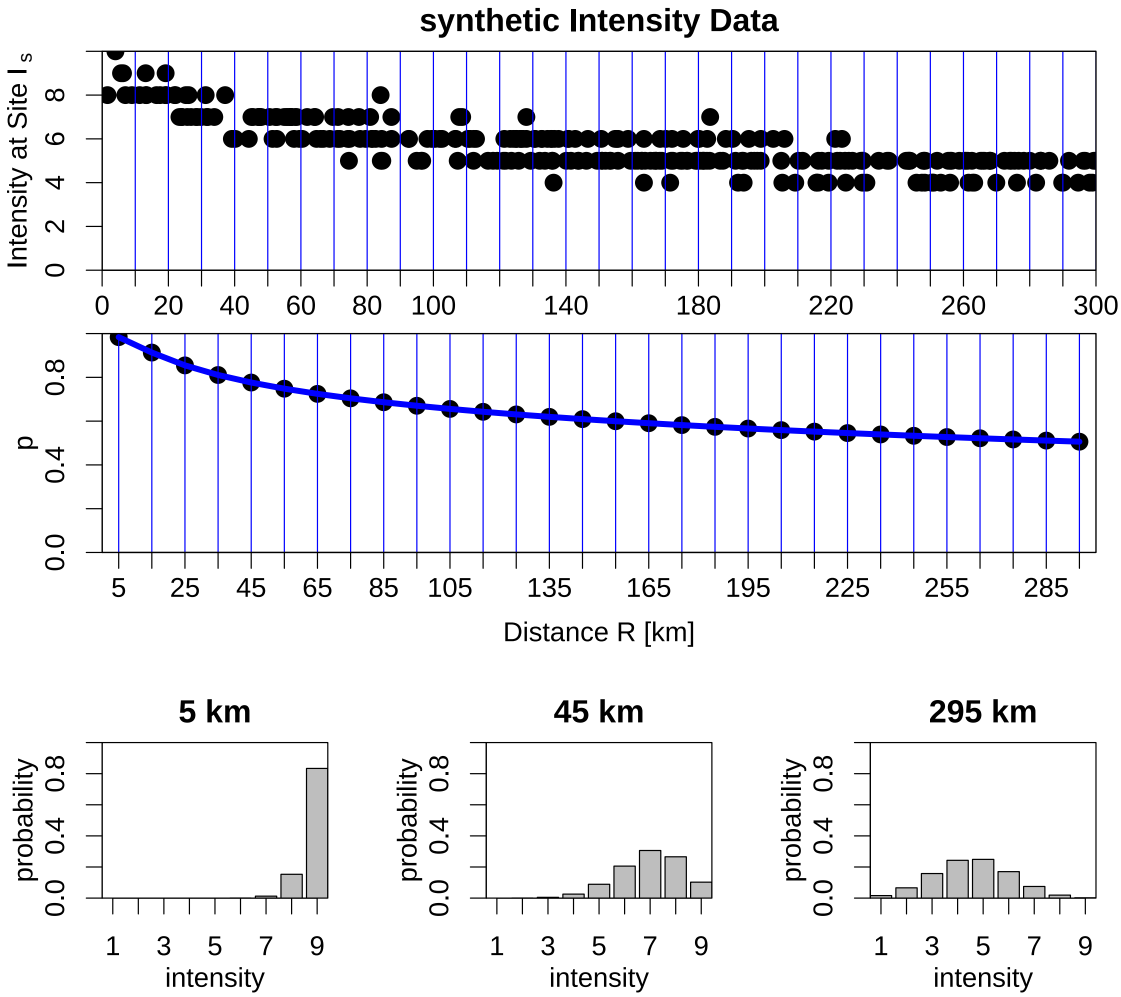

FIGURE 1: Example of the process of developing groundmotion models. The top figure shows a data set of macroseismic observations. These are discretized into distance bins with a width of 10 km. In each distance bin the observations are used to update the prior parameter of the Beta distribution. In the middle figure the mean of the posterior Beta distributions is assigned to the middle of the distance bins a smoothing function is used to get values of the p parameter for every distance. The bottom figure shows the probability distribution for different distances once the posterior mean of the Beta distribution is supplied to the binomial distribution.

3 Sensitivity study

3.1 Synthetic Data

Testing the performance of a model is a crucial step in defining new ground motion relationships and should always accompany the process of creating models. Especially in seismic risk evaluation one is interested how different parameters influence the reliability of the result. A sensitivity study evaluates the performance and gives the user a guide how to use a model in real life applications, how to quantify possible errors and which parameter should be given a higher priority in order to minimize uncertainties efficiently. To test the performance of the model from Rotondi and Zonno (2004) first, a series of tests is performed on a set of synthetic intensity data. The synthetic data is constructed by sampling 700 distance values uniformly over a range of 300 kilometres. In order to calculate corresponding intensity values the model of Koveslighety (1906) is used together with an epicentral intensity(I_0) of IX:

$$\Delta I = a * log_{10}\left(\frac{d_h}{h}\right) + a * b * log(d_h - h) \tag{8}$$

where $d_h = \sqrt{R^2 + h^2}$ is the hypocentral distance of the earthquake, h = 10km is the depth of the earthquake hypocentre, $a = 3$ is a factor for the geometrical spreading of seismic waves and $b = 0.002km^{-1}$ is a parameter that accounts for the anelastic dissipation (Stromeyer and Grünthal, 2009). To attain whole numbers the results of equation 8 are rounded. As prior distribution a uniform distribution over the range between 0 and 1 is used corresponding to the hyperparameters of the Beta distribution $\alpha = \beta = 1$.

To estimate the prediction performance the mode of the binomial distribution for each distance bin is used in order to forecast intensities. The mean absolute error between predicted and synthetic intensities is the measure for the prediction ability.

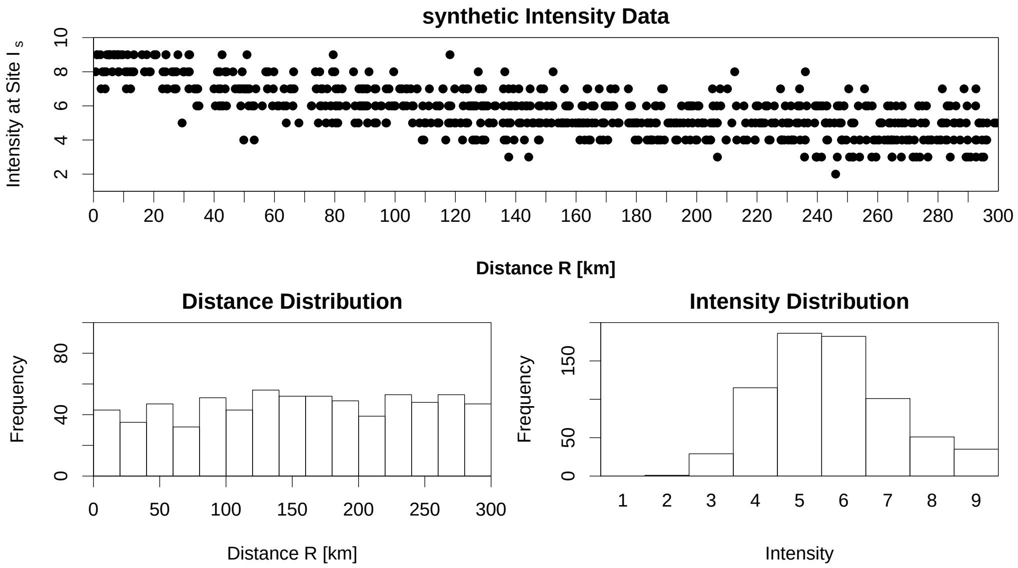

FIGURE 2: Synthetic data according to equation 8. Distance values where uniformly sampled over a distance range up to 300 kilometers.

3.2 Data Size

One important and very limiting factor for generating new ground motion models is the amount of data that is available. Depending on the region one might encounter scenarios where rich catalogues of past seismicity are available or just a sparse set of observations. To test the behaviour of the model in these different environments the synthetic data has been randomly partitioned into subsets of stepwise increasing number of observation, thus simulating the effect of different catalogue sizes. To estimate the model's performance a 10-fold cross-validation procedure is adopted to estimate the training error (in-sample error) as a measure of how well the model can approximate the data and the cross-validation error (out-of-sample error) which examines the performance on data that was not used in estimating the parameters of the model. Also it is a very easy way to qualitatively diagnose whether the model is suffering from bias or variance. Bias means the model's ability to approximate the target function that has generated the data. Variance quantifies how sensitive this estimate is to a specific subset of the data. In a high bias case fit and cross-validation error would rapidly approach each other and remain equal on a high level. This means that adding more data does not improve the models performance since the model that is used to fit the target function can only approximate it poorly. It is a case of underfitting. Conversely, in a high variance scenario there is a gap between fit and cross-validation error but the cross-validation error still decreases with more data. This situation corresponds to overfitting.

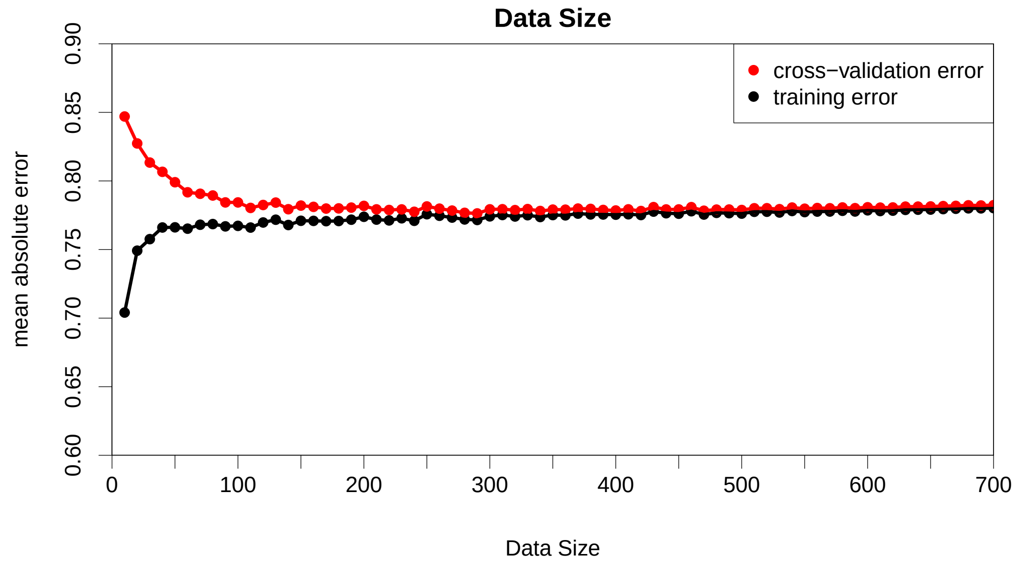

FIGURE 3: Data Size

Figure 3 shows the results of this analysis. With a small data set size the cross-validation error is larger than the training error. This means that the trained model does not well generalize to unseen data. Increasing the data set size results in decreasing cross-validation error and increasing training error. Though it becomes harder to replicate all of the training data the model performs better in predicting. At around 500 data samples there is no real difference between cross-validation and training error.

3.3 Quality of the Data

Another parameter that can heavily influence the computation of new ground motion relationships is the quality of the data. The best algorithm can only perform as well as the used data. To simulate the effect of poor data a Gaussian error with a mean equal to zero and a stepwise increasing standard deviation is added to synthetic data. Since only the isolated effect of poor data should be studied the whole 1000 synthetic observations are used. Because at this data size there is no difference in training error and cross-validation error only the cross-validation error is shown in Figure 4.

The mean absolute error increases in a logarithmically fashion with greater noise that is added to the data. This can be explained with the fact that as the noise in the data increases more individual data points take the intensity values of I or IX which are the extremes of the intensity range. This placed an upper limit to the mean absolute error that can be achieved by adding random noise.

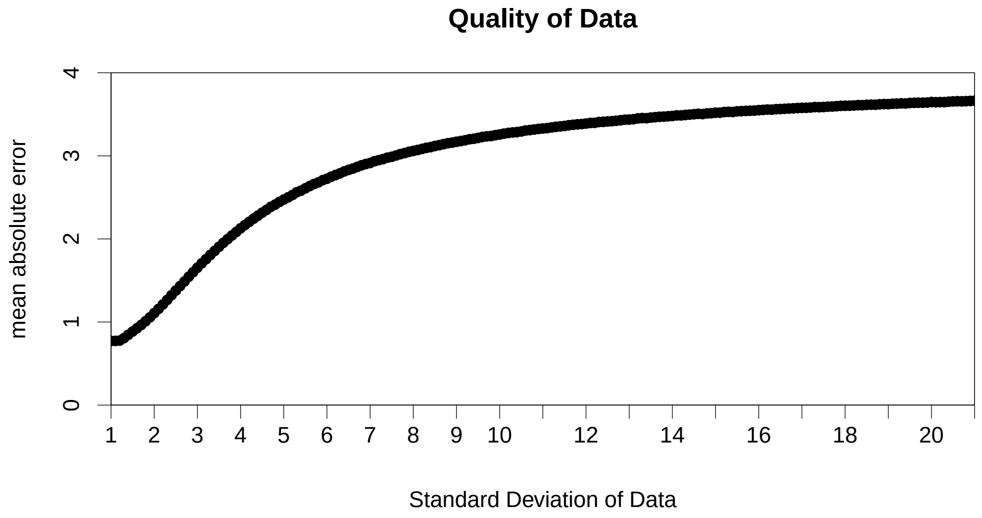

FIGURE 4: Quality of Data

Figure 5 shows the analysis of changing data size and noise level in the data simultaneously. The advantage of more data for the algorithm to learn from seems just relevant for scenarios where there is only a small data set to learn from. Beyond 300 data samples the mean absolute error depends mainly on the level of added noise. This behaviour is exactly what one would expect given that the out-of sample error can be divided into a bias, variance and noise part(Abu-Mostafa et al., 2012). From the analysis of the influence of data size on the models performance it is known that it suffers from bias. So once enough data samples have been used so that the bias does not change any more the noise level is the only contributing factor.

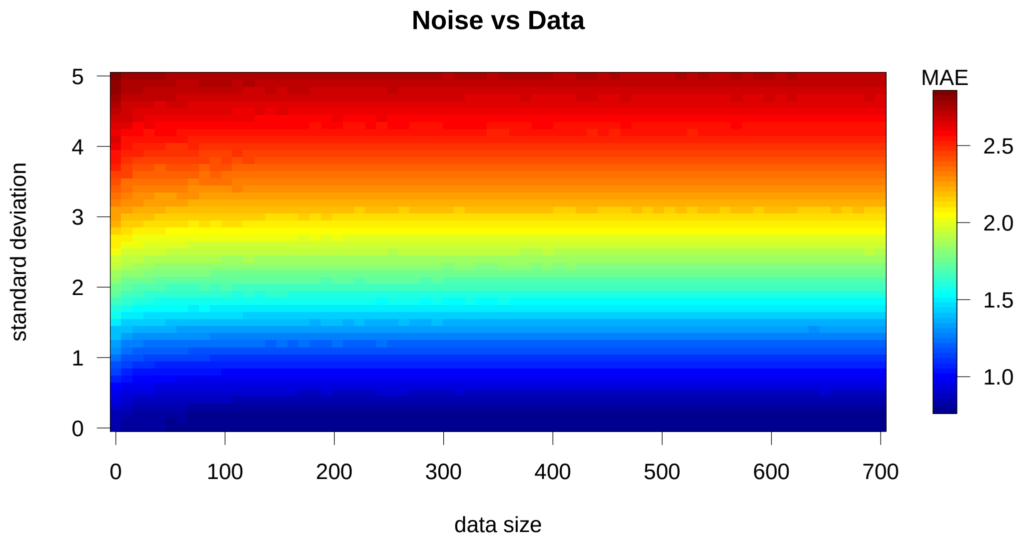

FIGURE 5: Data Size vs Noise

3.4 Discretization

One special property of the model by Rotondi and Zonno (2004) is the discretization of the data into bins of distance. This is necessary since the approach of modelling the intensity distribution by a binomial distribution can only approximate the intensity decay for a certain distance range. To jointly model the intensity distribution an it's dependence on distance one would have to use more advanced techniques like copulas. This subdivision in discrete distance bins is therefore a compromise between enough data per bin to estimate the parameter p of the binomial distribution with great accuracy and enough bins spanning the whole range of site to source distance in order to have a thorough support for the smoothing function. The whole data set of 700 synthetic observations is used and the cross-validation error is chosen as measure of prediction performance.

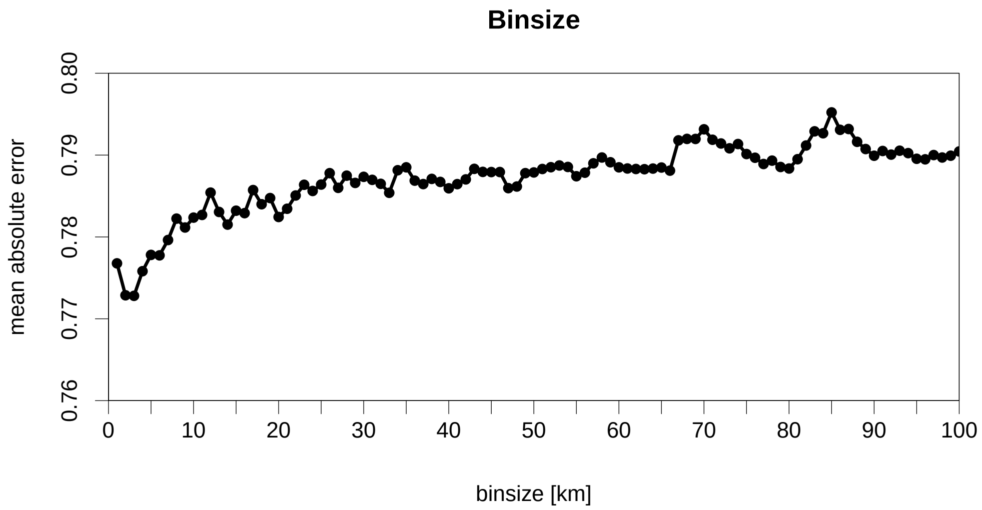

FIGURE 6: distance bin size

Figure 6 shows the result of a stepwise increase in the width of the distance bins. The lowest mean absolute errors are achieved with bin sizes of 2-3 km. After this point the error increases logarithmically with larger bin widths. It should also be noted that the maximum differences between different bin sizes, and thus the influence of the choice of one particular bin size on the overall error, are relatively small.
Figure 7 shows the results of changing bin size and data size simultaneously. Larger bin sizes affect the mean absolute error on small data sets more heavily than on larger data sets. A general trend of lower errors for smaller bin sizes can be found although this influence is decreased with larger data set size.

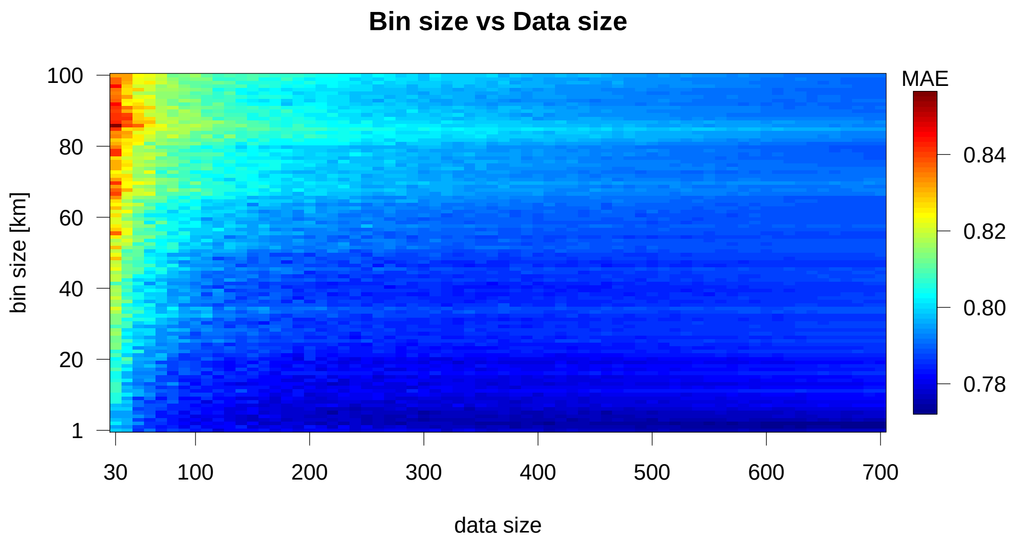

FIGURE 7: Data Size vs binsize

3.5 Sample Bias

The distribution of samples in distance has up until now be modelled by a uniform distribution. Following the effect of a biased coverage of the distance range on the performance of the model is studied. Therefore, additional to the uniform distance distribution, three distance distributions corresponding to scenarios where the majority of data is available in a middle, a near and a far distance range, respectively, are used to generate synthetic intensity data. The cross-validation error is used as an estimate of the prediction performance.
Figure 8 shows the corresponding distance distributions and the result of this analysis. The lowest mean absolute errors are produced by a uniform distance distribution. The near and middle range distributions differ for a small data size with the near range distribution performing slightly better. But both approach similar error values with larger data sets. The far range distance distribution performs the worst. This is due to the non-linear nature of the intensity decay where most of the information is in the near distance range in which the intensities attenuate more strongly.

3.6 Choice of Prior Distribution

A special advantage of a Bayesian approach to ground motion modelling is the possibility to incorporate expert knowledge through the prior distribution. This can be thought of as a start value in a grid-search algorithm where a good start value decreases the

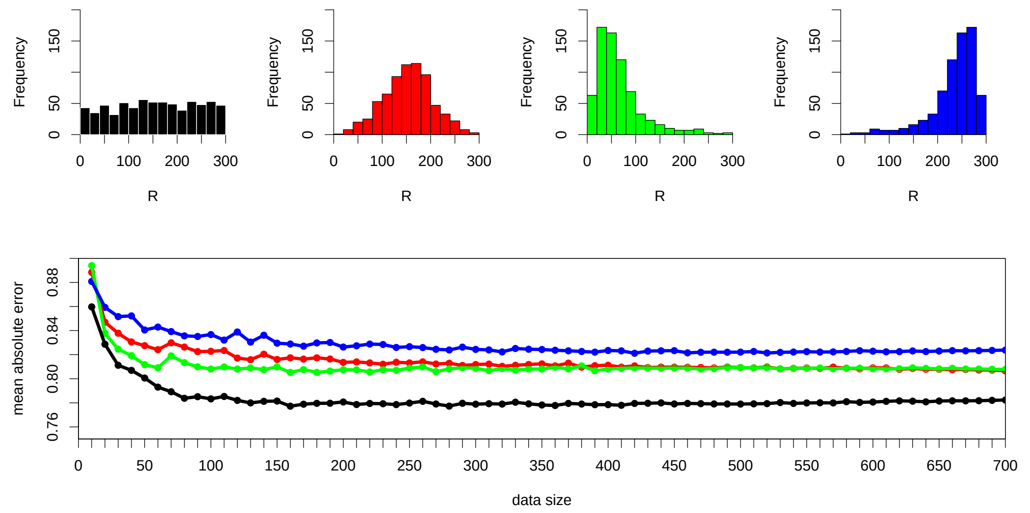

FIGURE 8: Sample Bias

number of iterations to find a local and possibly global minimum in the error function. The influence of different prior distribution (table 1) on the modelling process should be shown. These are a modified version of a prior that Tsapanos et al. (2002) use to compute the hazard for Japanese cities, a uniform distribution, two distributions where the mean decreases linearly and exponentially with distance, respectively, and a distribution that uses the prior knowledge of the already existing IPE by Koveslighety (1906).
The challenge here is that a prior distribution in term of the p parameter and not as intensities given a specific distance is needed. Given that some prior knowledge is available in form of an IPE this can be constructed with the help of the update rules for the hyper parameter α and β (Eqn:4). Since no prior values of α and β can be updated these values become zero so that:

$$\alpha = \sum_{n=1}^{N} i_s \qquad \beta = \sum_{n=1}^{N} (I_0 - i_s) \tag{9}$$

Letting N be equal to 1 this becomes:

$$\alpha = i_s \qquad \beta = (I_0 - i_s) = \Delta I \tag{10}$$

It should be noted that it is not in the scope of this thesis to give a full hands-on tutorial in how to choose a prior distribution for the algorithm of Rotondi and Zonno (2004). The aim is to show the influence that a prior can have on the computation. It should

also be stressed that a prior distribution in a Bayesian framework acts as a kind of regularization term and it is therefore not advisable to tune the prior distribution so that a minimal error is achieved. A prior should quantify expert or domain knowledge and has to be "naive".
Figure 9 shows the attenuation of the prior distributions' means with increasing distance and three visualizations of the full distribution for distances of 10, 150 and 300 kilometres, respectively. An animation can be found in the electronic supplement: Animation.
Figure 10 shows the influence that the different prior distributions have on the mean absolute error with increasing data set size. The difference between the prior distributions is the amount of data it is needed in order for the error to converge to a point where it does not decrease any more. This comes from the fact that the influence of the prior on the learning process is decreased with more data samples. Thus, even a prior that is far of as a starting point of the distribution that is in the data can be overcome by more data samples.

prior	α	β	μ	σ
Tsapanos et al. (2002)	$\left(\frac{1}{1+\frac{R}{60}}\right)^{1/I_0}$	$1-\alpha$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Koveslighety (1906)	$I_s^{Koveslighety}$	$\Delta I^{Koveslighety}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Uniform	1	1	0.5	$0.08\bar{3}$
Linear	$(\frac{1-\mu}{\sigma}-\frac{1}{\mu})*\mu^2$	$\alpha*\frac{1}{\mu-1}$	$-\frac{1}{300}*R$	0.007
Exponential	$(\frac{1-\mu}{\sigma}-\frac{1}{\mu})*\mu^2$	$\alpha*\frac{1}{\mu-1}$	$e^{-0.009*R}$	0.007

TABLE 1: prior

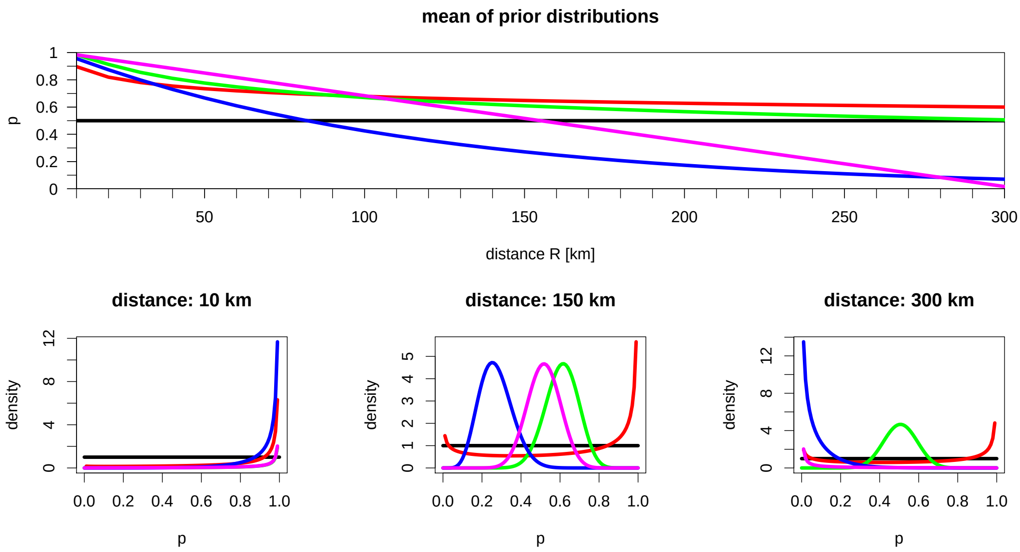

FIGURE 9: priorMeans

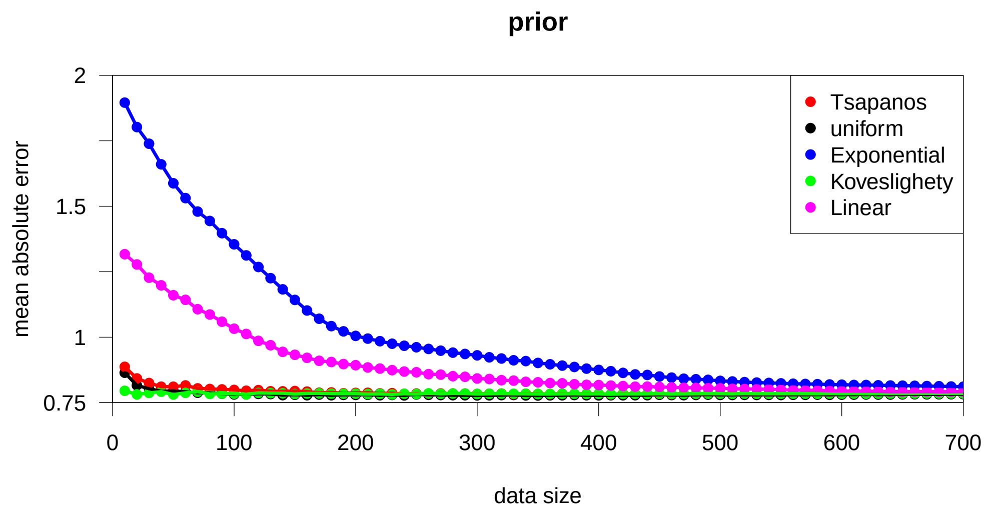

FIGURE 10: prior

3.7 Extended Source

$$\begin{aligned} x_1 &= x \\ y_1 &= y \end{aligned} \tag{11a}$$

$$\begin{aligned} x_2 &= cos(-\theta) * x_1 - sin(-\theta) * y_1 \\ y_2 &= sin(-\theta) * x_1 + cos(-\theta) * y_1 \end{aligned} \tag{11b}$$

$$\begin{aligned} x_3 &= x_2 * \frac{b}{a} \\ y_3 &= y_2 \end{aligned} \tag{11c}$$

$$\begin{aligned} \gamma &= atan(\frac{y_3}{x_3}) - atan(\frac{y_2}{x_2}) + abs(\theta) \\ x_4 &= cos(\gamma) * x_3 - sin(\gamma) * y_3 \\ y_4 &= sin(\gamma) * x_3 + cos(\gamma) * y_3 \end{aligned} \tag{11d}$$

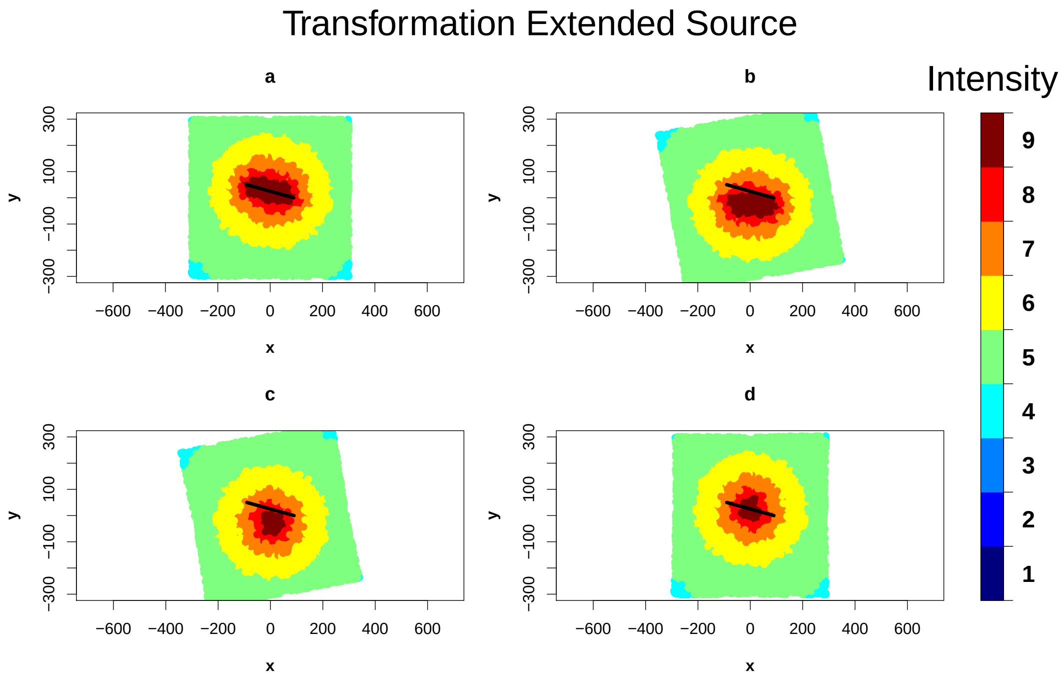

Figure 11: extended Source

Results: 0.1551224 0.2623517

4 Case Study

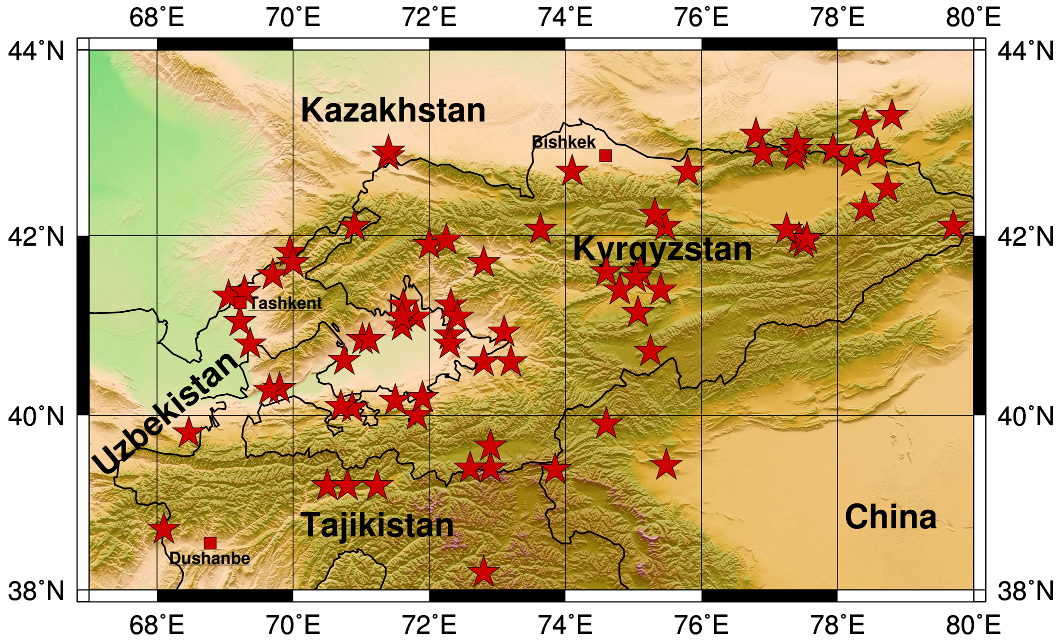

Figure 12: map

4.1 Data

I_0/I_s	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9	total
9	0	0	0	0	13	4	25	23	34	15	23	13	17	4	13	3	187
8.5	0	0	11	10	62	13	142	33	147	25	153	90	98	14	110	0	908
8	0	0	24	9	24	8	48	18	47	28	26	10	9	5	0	0	256
7.5	3	16	33	68	198	186	204	133	116	76	91	23	24	0	0	0	1171
7	4	24	69	75	149	132	139	67	79	29	29	9	0	0	0	0	805
6.5	1	15	104	98	219	146	129	60	118	71	56	0	0	0	0	0	1017
6	17	26	103	131	189	86	105	64	90	34	0	0	0	0	0	0	845
5.5	0	22	155	116	233	166	147	46	70	0	0	0	0	0	0	0	955
5	8	1	9	23	19	7	5	5	0	0	0	0	0	0	0	0	77
total	33	104	508	530	1106	748	944	449	701	278	378	145	148	23	123	3	6221

Table 2: Data

4.2 Handling uncertain data

sdavasgas

I_0	round	floor	ceiling	Certain	Inflated	Inflated Uncertain
5	-	0.493506	-	-	0.571429	0.584416
6	0.553543	0.541667	0.496124	0.489256	0.566914	0.584479
7	0.660767	0.656970	0.677229	0.681021	0.695033	0.679183
8	0.556452	0.504555	0.601721	0.521674	0.593300	0.544509
9	0.748899	0.743379	0.816151	0.697095	0.807437	0.787307
10	0.620321	-	0.582888	-	0.604278	0.614973

TABLE 3: Data

4.3 Comparison to other studies

I_0	Rotondi	Ullah
5	0.493506	0.934634
6	0.541667	0.613935
7	0.656970	0.570863
8	0.504555	0.562258
9	0.743379	0.743313
10	-	-

TABLE 4: Data

4.4 Extended Source

5 Conclusions

Conclusions: what did you find out? general advises outlook

A Data

	year	month	day	λ	θ	ϕ	I_{max}	depth	K	MLH	M_W	Length	F1.1	F1.2	F2.1	F2.2
1	1883	11	14	72.80	40.60	200	7	12	13.9	5.383	5.68	4.98	72.790	40.579	72.823	40.586
2	1885	8	2	74.10	42.70	250	9	15	15.6	6.182	6.20	11.43	74.034	42.682	74.155	42.668
3	1887	6	8	76.80	43.10	250	9	20	16.9	6.793	6.81	29.89	76.627	43.054	76.945	43.017
4	1889	7	11	78.40	43.20	250	9	40	18.5	7.545	7.55	97.54	77.837	43.049	78.872	42.929
5	1897	9	17	68.47	39.80	270	8	25	15.4	6.088	6.15	10.54	68.405	39.800	68.515	39.771
6	1902	12	16	72.30	40.80	270	9	9	15.6	6.182	6.20	11.43	72.232	40.800	72.353	40.768
7	1907	10	21	68.10	38.70	270	9	24	17.0	6.840	6.85	32.18	67.915	38.700	68.246	38.611
8	1911	1	3	76.90	42.90	250	10	25	17.8	7.216	7.22	58.14	76.566	42.810	77.180	42.739
9	1911	2	18	72.80	38.20	200	9	26	17.3	6.981	6.99	40.17	72.722	38.030	72.981	38.089
10	1924	7	12	73.20	40.60	270	8	14	15.6	6.182	6.20	11.43	73.132	40.600	73.253	40.568
11	1927	8	12	71.60	41.00	270	8	14	14.8	5.806	5.96	7.81	71.554	41.000	71.637	40.978
12	1932	12	24	78.20	42.80	250	6	23	14.0	5.430	5.71	5.23	78.170	42.792	78.225	42.786
13	1933	9	9	70.70	40.10	270	6	26	13.6	5.242	5.58	4.28	70.675	40.100	70.720	40.088
14	1937	12	18	70.90	42.10	270	7	25	15.6	6.182	6.20	11.43	70.831	42.100	70.955	42.068
15	1938	6	20	75.80	42.70	250	8	21	16.0	6.370	6.39	15.37	75.712	42.676	75.874	42.657
16	1941	4	20	70.50	39.20	270	9	8	15.6	6.182	6.20	11.43	70.434	39.200	70.552	39.168
17	1942	1	18	71.60	41.10	270	7	21	14.0	5.430	5.71	5.23	71.569	41.100	71.625	41.086
18	1946	11	2	72.00	41.90	270	9	25	17.0	6.840	6.85	32.18	71.806	41.900	72.153	41.811
19	1947	6	2	72.30	40.90	270	8	13	14.5	5.665	5.87	6.72	72.260	40.900	72.331	40.881
20	1948	7	28	75.40	41.40	250	7	6	13.6	5.242	5.58	4.28	75.376	41.393	75.420	41.388
21	1949	7	10	70.80	39.20	200	9	16	17.0	6.840	6.85	32.18	70.736	39.064	70.947	39.111
22	1954	12	3	74.80	41.40	250	7	15	14.0	5.430	5.71	5.23	74.771	41.392	74.825	41.386
23	1955	4	15	74.60	39.90	200	9	25	16.4	6.558	6.57	20.65	74.559	39.813	74.695	39.843
24	1957	5	8	74.60	41.60	250	6	7	13.0	4.960	5.39	3.17	74.582	41.595	74.615	41.591
25	1958	10	13	75.10	41.60	250	6	12	13.0	4.960	5.39	3.17	75.082	41.595	75.115	41.591
26	1959	7	12	72.80	41.70	270	6	14	12.9	4.913	5.36	3.02	72.782	41.700	72.814	41.692
27	1959	10	24	70.00	41.70	270	7	13	14.0	5.430	5.71	5.23	69.969	41.700	70.025	41.686
28	1960	12	18	78.40	42.30	250	6	17	12.8	4.866	5.33	2.87	78.384	42.296	78.414	42.292
29	1961	4	27	72.90	39.65	200	6	26	14.2	5.524	5.77	5.78	72.888	39.626	72.927	39.634
30	1962	9	3	73.10	40.93	270	7	20	14.0	5.430	5.71	5.23	73.069	40.933	73.125	40.919
31	1963	10	19	71.62	41.23	270	6	8	12.5	4.725	5.24	2.47	71.602	41.233	71.628	41.227
32	1965	3	17	69.37	40.80	270	7	12	13.0	4.960	5.39	3.17	69.348	40.800	69.381	40.791

	year	month	day	λ	θ	ϕ	I_{max}	depth	K	MLH	M_W	Length	F1.1	F1.2	F2.1	F2.2
33	1965	9	25	75.03	41.53	250	6	25	13.0	4.960	5.39	3.17	75.015	41.528	75.048	41.525
34	1965	10	18	77.55	41.97	250	6	15	13.0	4.960	5.39	3.17	77.532	41.962	77.565	41.958
35	1966	4	25	69.28	41.38	270	6	8	13.3	5.101	5.49	3.69	69.261	41.383	69.301	41.373
36	1966	4	30	71.80	41.10	270	6	20	13.6	5.242	5.58	4.28	71.774	41.100	71.820	41.088
37	1967	5	18	70.75	40.62	270	6	25	12.0	4.490	5.08	1.92	70.739	40.617	70.759	40.611
38	1967	9	28	79.70	42.10	250	6	18	13.5	5.195	5.55	4.07	79.677	42.094	79.719	42.089
39	1967	11	30	77.40	43.00	250	6	10	12.0	4.490	5.08	1.92	77.389	42.997	77.409	42.995
40	1968	3	20	75.07	41.15	250	6	17	12.6	4.772	5.27	2.60	75.052	41.146	75.079	41.143
41	1970	1	19	69.22	41.05	270	7	25	12.1	4.537	5.11	2.02	69.205	41.050	69.226	41.044
42	1970	6	5	78.73	42.52	250	8	15	15.6	6.182	6.20	11.43	78.668	42.499	78.788	42.485
43	1971	5	10	71.40	42.92	250	7	20	14.0	5.430	5.71	5.23	71.370	42.909	71.425	42.902
44	1971	10	28	72.25	41.95	270	6	17	14.0	5.430	5.71	5.23	72.218	41.950	72.275	41.936
45	1972	3	17	69.65	40.28	270	6	20	13.5	5.195	5.55	4.07	69.626	40.283	69.669	40.272
46	1974	1	22	71.90	40.20	270	7	24	12.7	4.819	5.30	2.73	71.884	40.200	71.913	40.192
47	1974	2	20	75.25	40.72	250	6	15	13.2	5.054	5.46	3.51	75.230	40.711	75.266	40.707
48	1974	7	2	75.32	42.23	250	6	15	12.9	4.913	5.36	3.02	75.299	42.229	75.331	42.225
49	1974	8	11	73.85	39.38	200	6	15	16.6	6.652	6.67	23.94	73.802	39.282	73.960	39.317
50	1975	2	12	78.80	43.30	250	6	10	13.0	4.960	5.39	3.17	78.782	43.295	78.815	43.291
51	1977	1	31	70.87	40.08	296	8	20	15.5	6.135	6.15	10.62	70.811	40.104	70.916	40.054
52	1977	6	3	71.82	40.00	128	6	15	14.2	5.524	5.77	5.78	71.843	39.984	71.843	39.984
53	1977	12	6	69.70	41.57	270	7	15	14.0	5.430	5.71	5.23	69.669	41.567	69.725	41.552
54	1978	3	24	78.58	42.88	270	8	22	15.6	6.182	6.20	11.43	78.513	42.883	78.639	42.852
55	1978	11	1	72.60	39.40	200	8	30	16.2	6.464	6.48	17.81	72.565	39.325	72.682	39.351
56	1979	4	6	77.43	41.97	121	6	25	13.5	5.195	5.55	4.07	77.454	41.957	77.453	41.955
57	1980	7	5	77.50	41.92	250	6	20	13.8	5.336	5.65	4.73	77.473	41.909	77.523	41.904
58	1980	12	11	69.05	41.33	270	7	10	13.5	5.195	5.55	4.07	69.026	41.333	69.069	41.322
59	1982	5	6	71.50	40.17	112	8	20	14.4	5.618	5.83	6.39	71.535	40.156	71.530	40.149
60	1982	12	31	77.37	42.87	274	6	15	13.6	5.242	5.58	4.28	77.340	42.868	77.387	42.855
61	1983	12	16	72.90	39.40	228	7	15	14.6	5.712	5.90	7.06	72.870	39.379	72.932	39.380
62	1983	12	21	77.25	42.07	250	6	20	12.5	4.725	5.24	2.47	77.236	42.063	77.262	42.060
63	1984	2	2	71.40	42.87	250	6	15	12.6	4.772	5.27	2.60	71.385	42.863	71.413	42.859
64	1984	2	17	71.02	40.85	240	8	10	14.1	5.477	5.74	5.50	70.988	40.838	71.042	40.835
65	1984	10	26	71.23	39.20	37	8	15	14.8	5.806	5.96	7.81	71.261	39.228	71.269	39.178
66	1985	4	27	71.12	40.85	103	8	15	12.8	4.866	5.33	2.87	71.133	40.847	71.130	40.842
67	1985	8	23	75.48	39.43	308	7	20	16.5	6.605	6.62	22.24	75.381	39.495	75.585	39.372
68	1985	10	13	69.80	40.30	48	8	10	14.8	5.806	5.96	7.81	69.834	40.323	69.836	40.278
69	1987	3	26	69.95	41.82	203	6	5	13.1	5.007	5.42	3.33	69.942	41.803	69.966	41.807
70	1988	3	13	75.47	42.10	250	6	7	12.6	4.772	5.27	2.60	75.452	42.096	75.479	42.093
71	1988	6	17	77.40	42.93	29	6	21	12.9	4.913	5.36	3.02	77.409	42.945	77.415	42.925
72	1988	12	21	72.32	41.23	270	6	10	12.9	4.913	5.36	3.02	72.299	41.233	72.331	41.225
73	1990	11	12	77.93	42.93	211	8	15	15.3	6.041	6.12	10.02	77.902	42.895	77.982	42.906
74	1992	5	15	72.42	41.10	270	8	10	15.3	6.041	6.12	10.02	72.357	41.100	72.464	41.072

	year	month	day	λ	θ	ϕ	I_{max}	depth	K	MLH	M_W	Length	F1.1	F1.2	F2.1	F2.2
75	1992	8	19	73.63	42.07	250	10	25	17.0	6.840	6.85	32.18	73.451	42.017	73.787	41.978

TABLE 5: Data (Dziewonski et al., 1981) (Ekström et al., 2012)

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