

Short Notes

Validation of Intensity Attenuation Relationships

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Abstract A new approach is proposed for the empirical validation of intensity attenuation relationships to be implemented in standard procedure devoted to probabilistic seismic hazard assessment (PSHA). To this purpose, the overall number of documented intensities above a given threshold in the area of interest is compared with the one expected on the basis of the considered attenuation relationship. By using this methodology, the reliability of empirical relationships can be analyzed by also taking into account the uncertainty affecting ill-defined intensity attributions both at the epicenter and at the site. To assess the feasibility of this procedure, an attenuation relationship recently proposed for the Italian region has been evaluated by considering felt intensities documented in the area during the past two centuries. Although the macroseismic database considered for validation is the same used for the parameterization of the investigated relationship, important discrepancies have been detected between observed and computed intensities in the range of values significant for seismic hazard. This result indicates that a careful parameterization of attenuation relationships in their complete probabilistic form is mandatory when such relationships have to be implemented in PSHA procedures.

Introduction

Despite the availability of several alternative models, the procedures actually used for probabilistic seismic hazard assessment (PSHA) in common practice all over the world are quite few (McGuire, 1993). In particular, the most popular approach is the one introduced by Cornell (1968), which, in a more sophisticated implementation (Bender and Perkins, 1987), has recently been adopted as an international “standard” (Giardini and Basham, 1993).

Empirical relationships used to evaluate ground-motion decay with epicentral distance (attenuation) represent a basic element of this standard procedure. When seismic hazard maps are compiled in terms of macroseismic intensity (as about 60% of national hazard maps described by McGuire, 1993), attenuation relationships are required that estimate intensity at the site as a function of epicentral intensity (or maximum intensity) and distance from the presumed source. In the standard procedure, macroseismic attenuation relationships are supplied in the general probabilistic form

$$\text{prob}[I > I_s | I_0, R] = 1 - P(I_s | I_0, R), \quad (1)$$

where P is a cumulative distribution function that represents attenuation properties (probabilistic attenuation function), I_s is the intensity threshold of interest, I_0 is the intensity at the source, and R is the distance of the site from the source.

With the lack of a direct physical model, the form of P

is empirically assessed by a statistical analysis carried out on a set of seismic effects (hereafter, felt intensities) observed during past earthquakes at several sites. In principle, P is a discrete probabilistic function defined over the limited set of possible intensity values (twelve in most common macroseismic scales). Although alternative strategies are possible (e.g., Magri *et al.*, 1994; D'Amico and Albarello, 2003), this analysis is performed in two steps. In the first step, the average μ of the distribution P is assessed by assuming a generic functional dependence F such that

$$\mu = F(I_0, R). \quad (2)$$

(Loosely speaking, F is the commonly defined “attenuation law.”) Examples of different parameterizations of the function F for several parts of the world are given by Blake (1941), Karnik (1969), Howell and Schultz (1975), Gupta and Nuttli (1976), Chandra *et al.* (1979), Ambraseys (1985), López Casado *et al.* (2000) and, for Italy, by Peruzza (1996) and Gasperini (2001). As a second step, residuals (i.e., observed minus predicted intensities) are examined to assess the form of P .

In general, most attention is devoted to defining and parameterizing F , whereas relatively fewer efforts are dedicated to establishing a realistic form for P , which in most cases is implicitly assumed to have a normal distribution.

However, seismic hazard depends on the whole P distribution and not only on the average (2). This implies that the estimate of the average, however accurate, cannot warrant by itself a correct hazard assessment. Furthermore, the probability distribution P is characterized by quite a large variance (variance explained by empirical relationships [equation 2] seldom exceeds 60% of the total), which deeply affects hazard estimates (Albarelli *et al.*, 2002). In particular, it can be easily seen that low variances result in lower hazard and vice versa. Thus, a realistic parameterization of the general features of P plays a major role in PSHA.

A key problem is to evaluate how accurate the determination of P (as a whole) should be to warrant reliable hazard estimates. In common practice, the effectiveness of empirical attenuation relationships is evaluated by using standard goodness-of-fit estimators, which are mainly sensitive to the pattern of the F function in equation (2). In general, such estimators are not fully reliable when applied to discrete variables defined over finite intervals, such as in the case of macroseismic intensity. In addition, uncertainty that affects felt intensities used for the parameterization of attenuation is generally discarded. Last but not least, these estimators are not effective in evaluating the real feasibility of the resulting attenuation relationship in its general probabilistic form (1).

To overcome such drawbacks, a new empirical approach is presented aiming at the direct evaluation of effectiveness of the attenuation relationship to be used for PSHA. In this approach, both the probabilistic nature of the attenuation relationship and eventual uncertainties on documented intensities are taken into account. To illustrate how this procedure actually works, an intensity attenuation relationship recently proposed for the Italian region (Gasperini, 2001; Carletti and Gasperini, 2003) has been analyzed.

Validation Procedure

Given a set of Q earthquakes, observations are constituted by seismic effects documented, for each of these events, at M localities. By the use of the cumulative distribution function P (1), it is possible to estimate the expected number of these sites \hat{N}_{I_s} that experienced felt intensities at least equal to a fixed threshold I_s . Because such occurrences can be considered as realizations of a Bernoullian stochastic process, \hat{N}_{I_s} can be computed in the form

$$\hat{N}_{I_s} = \sum_{j=1}^Q \sum_{i=1}^{M_j} \text{prob}[I \geq I_s | I_j, R_{ij}] = \sum_{j=1}^Q \sum_{i=1}^{M_j} H_{ij}(I_s), \quad (3)$$

where the j th event is characterized by the epicentral intensity I_j and the i th site is located at a distance R_{ij} from the source of the j th earthquake. H is given by

$$H_{ij}(I_s) = \sum_{l=I_s}^{12} p(l|I_j, R_{ij}), \quad (4)$$

with p being the density related to the cumulative distribution function P in (1).

The associated standard deviation is

$$\sigma_{N_{I_s}} = \sqrt{\sum_{j=1}^Q \sum_{i=1}^{M_j} \{H_{ij}(I_s)[1 - H_{ij}(I_s)]\}}. \quad (5)$$

If epicentral intensities are not unequivocally defined, these formulas have to be modified to take into account such additional uncertainty. Uncertainty in the intensity attribution can be described (Magri *et al.*, 1994; Albarelli and Mucciarelli 2002) by a probability density distribution $k(I)$. Thus, the probability $G_{ij}(I_s)$ that the j th event at the i th site is characterized by felt intensity $\geq I_s$ is given by

$$G_{ij}(I_s) = \sum_{l=I_s}^{12} [k_j(l)H_{ij}(l)]. \quad (6)$$

Accordingly, (3) and (5) have to be modified in the form

$$\hat{N}_{I_s} = \sum_{j=1}^Q \sum_{i=1}^{M_j} G_{ij}(I_s) \quad (7)$$

and

$$\sigma_{N_{I_s}} = \sqrt{\sum_{j=1}^Q \sum_{i=1}^{M_j} \{G_{ij}(I_s)[1 - G_{ij}(I_s)]\}}, \quad (8)$$

respectively.

The value \hat{N}_{I_s} can be compared with the observed number $N_{I_s}^{\text{obs}}$ of sites where felt effects were actually $\geq I_s$. If no uncertainty affects documented intensities, it simply holds that

$$N_{I_s}^{\text{obs}} = \sum_{j=1}^Q \sum_{i=1}^{M_j} C_{ij}(I_s), \quad (9)$$

with $C_{ij} = 1$ if at the i th site felt effects are $\geq I_s$ and $C_{ij} = 0$ otherwise. When uncertain intensity attributions exist, $N_{I_s}^{\text{obs}}$ becomes an aleatory variable with expectation $\hat{N}_{I_s}^{\text{obs}}$ and variance $\sigma_{N_{I_s}^{\text{obs}}}^2$. To compute these values, one can consider C_{ij} a Bernoullian variable with an associated probability $K(I_s)$ given by

$$K(I_s) = \sum_{l=I_s}^{12} k(l). \quad (10)$$

In this case we have

$$\hat{N}_{I_s}^{\text{obs}} = \sum_{j=1}^Q \sum_{i=1}^{M_j} K_{ij}(I_s) \quad (11)$$

with an associated standard deviation

$$\sigma_{N_{I_s}^{\text{obs}}} = \sqrt{\sum_{j=1}^Q \sum_{i=1}^{M_j} \{K_{ij}(I_s)[1 - K_{ij}(I_s)]\}} \quad (12)$$

By the Central Limit Theorem, the parent population of both $\hat{N}_{I_s}^{\text{obs}}$ and \hat{N}_{I_s} can be considered Gaussian. This implies that the statistics

$$Z_{I_s} = \frac{\hat{N}_{I_s}^{\text{obs}} - \hat{N}_{I_s}}{\sqrt{\sigma_{N_{I_s}^{\text{obs}}}^2 + \sigma_{N_{I_s}}^2}} \quad (13)$$

follow the standardized Gauss distribution. Equation (13) can be used to evaluate the statistical significance of the observed discrepancy between $\hat{N}_{I_s}^{\text{obs}}$ and \hat{N}_{I_s} . In particular, when $Z_{I_s} \geq 2$ resulting discrepancy can be considered statistically significant (prob < 0.05).

An Application to the Italian Area

To explore the possibility of applying this procedure to real cases, results of a recent study of macroseismic intensity attenuation in Italy (Gasperini, 2001; Carletti and Gasperini, 2003) have been analyzed. This study was based on the most recent data set of Italian documented intensities (CPTI Working Group, 1999). The database includes more than 50,000 Mercalli-Cancani-Sieberg scale (MCS) intensity observations relative to about 1000 earthquakes from ancient times until the present. To ensure the homogeneity of the intensity assessment and derived-source parameters, only data relative to earthquakes occurring from 1801 until 1990, including intensity estimates in at least 20 localities, were considered in that study. A further selection was performed to reduce possible biases induced by incomplete reporting of lowest intensities and eventual source directionality effects (see Gasperini, 2001, for details). The statistical analysis of the resulting data set (constituted by 19,912 felt intensities) allowed the identification of a bilinear pattern for the average attenuation, in the form (Carletti and Gasperini, 2003)

$$\mu(I_0, R) = \begin{cases} I_0 - 0.445 - 0.059 R & R \leq 45 \text{ km} \\ I_0 - 0.445 - 0.059(45) - 0.0207(R - 45) & R > 45 \text{ km} \end{cases} \quad (14)$$

where I_0 is the epicentral intensity (MCS) and

$$R = \sqrt{D^2 + 10^2} \text{ ,}$$

with D being the epicentral distance in kilometers. The percentage of explained variance associated with the preceding attenuation law for the considered data set is 56%. In terms of common criteria, this parameterization of intensity attenuation is physically plausible and “optimal,” from the statis-

tical point of view, with respect to alternative formulations (Gasperini, 2001). These features make this attenuation law appealing for future seismic hazard estimates in Italy. Residuals analysis (Gasperini, 2001) indicated that the cumulative probability distribution P in equation (1) can be assumed as approximately Gaussian. This implies that P can be written as

$$P(I_s | I_0, R) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{I_s + 0.5} e^{-\frac{[\mu(I_0, R) - x]^2}{\sigma^2}} dx \text{ ,} \quad (15)$$

where a correction of 0.5 units has been applied to I_s to take into account the discrete character of intensity. The standard deviation can be assumed independent on both R and I_0 (Gasperini, personal comm.) and equal to 1.04 (Carletti and Gasperini, 2003). To evaluate the reliability of such a parameterization, the validation procedure delineated in the previous section has been applied.

By using the parameterization in equations (14) and (15) and by considering intensity threshold values I_s ranging from VI to XI MCS, the probabilities $P(I_s | I_j, R_{ij})$ have been computed for each j th earthquake and i th locality in the same database considered for the parameterization of the attenuation pattern. Equations (3) and (5) have been used to calculate \hat{N}_{I_s} and $\sigma_{N_{I_s}}$, respectively, by using the probability H in (4) in the form

$$H_{ij}(I_s) = \frac{1}{\sigma \sqrt{2\pi}} \int_{I_s - 0.5}^{\infty} e^{-\frac{[\mu(I_j, R_{ij}) - x]^2}{\sigma^2}} dx \quad (16)$$

derived from (15). To compute \hat{N}_{I_s} and $\sigma_{N_{I_s}}$, eventual uncertainty on the relevant epicentral intensity has to be taken into account. In the considered database, epicentral intensities are coded in the form of a real number v . When unequivocal intensity attributions are available (e.g., VI MCS), the real number corresponds to the ordinal value of intensity (e.g., VI MCS becomes 6.0). In the case of ill-defined intensity attributions, only uncertainty between two contiguous in-

tensity values is admitted (e.g., VI–VII MCS), which implies that both intensity attributions are equally probable on the basis of the available documentation. In these cases, intensity reported in the database is coded in terms of a real value intermediate between the two contiguous integer values (e.g., VI–VII MCS becomes 6.5).

Thus, if v^{int} is the integer part of v , the parameterization of the probabilities $k(I)$ in equation (6) has been obtained in the form

$$\left\{ \begin{array}{ll} v^{\text{int}} = v & \begin{cases} I < v & \rightarrow k(I) = 0 \\ I = v & \rightarrow k(I) = 1 \\ I > v & \rightarrow k(I) = 0 \end{cases} \\ v^{\text{int}} \neq v & \begin{cases} I < v^{\text{int}} & \rightarrow k(I) = 0 \\ I = v^{\text{int}} & \rightarrow k(I) = 0.5 \\ I = v^{\text{int}} + 1 & \rightarrow k(I) = 0.5 \\ I > v^{\text{int}} + 1 & \rightarrow k(I) = 0 \end{cases} \end{array} \right. \quad (17)$$

(Magri *et al.*, 1994; Albarello and Mucciarelli, 2002). The values of \hat{N}_{I_s} and $\sigma_{N_{I_s}}$ resulting from the application of such parameterization in equations (6)–(8) are given in Figure 1 for each intensity threshold.

To compute $\hat{N}_{I_s}^{\text{obs}}$ and $\sigma_{N_{I_s}^{\text{obs}}}$, the 19,912 felt intensities reported in the data set have been considered. This set also includes uncertain intensity attributions coded in the same way as epicentral intensities. In accordance with the parameterization (17), the position

$$\left\{ \begin{array}{ll} v^{\text{int}} = v & \begin{cases} I \leq v & \rightarrow K(I) = 1 \\ I > v & \rightarrow K(I) = 0 \end{cases} \\ v^{\text{int}} \neq v & \begin{cases} I \leq v^{\text{int}} & \rightarrow K(I) = 1 \\ I = v^{\text{int}} + 1 & \rightarrow K(I) = 0.5 \\ I > v^{\text{int}} + 1 & \rightarrow K(I) = 0 \end{cases} \end{array} \right. \quad (18)$$

has been adopted to compute the probabilities K in (10) to

be introduced in equations (11) and (12). Figure 1 reports the values of $\hat{N}_{I_s}^{\text{obs}}$ and $\sigma_{N_{I_s}^{\text{obs}}}$ obtained in this way.

The results in Figure 1 clearly indicate that significant discrepancies exist between estimates obtained from observations and from the examined attenuation relationship. In general, values obtained from the latter slightly underestimate (6%) observed values for $I_s = \text{VI MCS}$, whereas a good agreement exists relative to $I_s = \text{VII MCS}$. However, when one considers intensity thresholds $\geq \text{VIII MCS}$, the attenuation relationship highly overestimates (i.e., 45% for $I_s = \text{VIII MCS}$ and 204% and 333% for $I_s = \text{IX}$ and X MCS , respectively) the number of felt intensities that actually exceeded the relevant threshold. This result could appear quite surprising, because the validation has been performed by using the same data set adopted for the parameterization of the attenuation relationship.

The overestimates are related to the most important seismic effects, and thus, they are particularly significant for PSHA. In particular, in regard to the highest threshold values (IX or X MCS), seismicity rates deduced from the attenuation relationship would result in rates three times greater than those actually observed, thus implying erroneously high hazard.

Though the interpretation of such discrepancies is beyond the scope of this article, some preliminary hypotheses can be advanced. Because the major goal of the article by Gasperini (2001) was the seismological interpretation of the intensity attenuation pattern in Italy, most attention was ded-

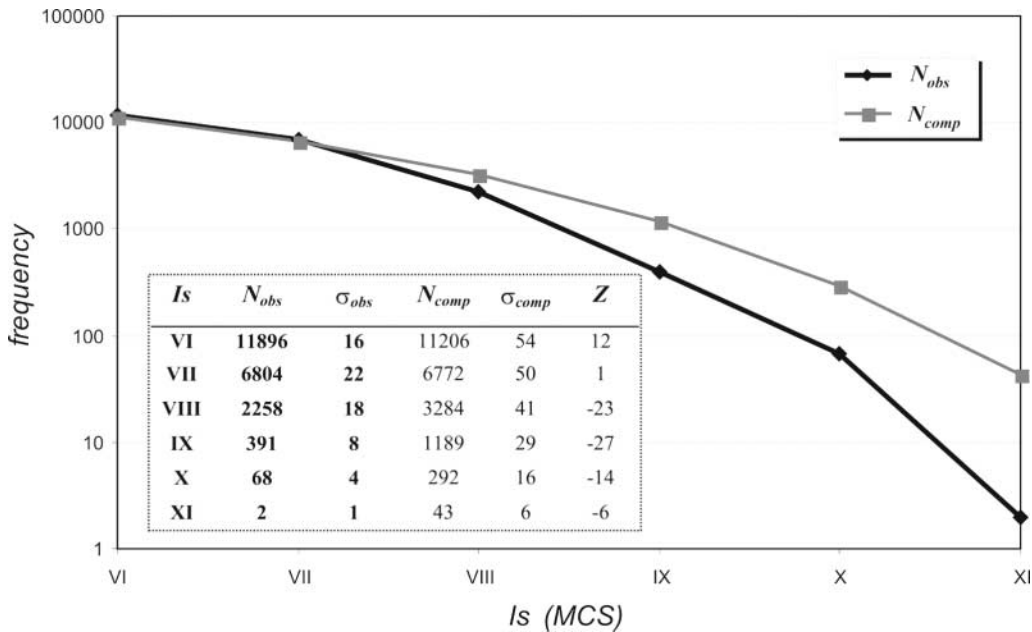


Figure 1. Comparison, for each choice of the intensity threshold I_s in the range VI–XI MCS, between the number N_{obs} of documented felt intensities (11) and the number N_{comp} of expected felt intensities (7) with the assumption that the probabilistic attenuation function is in the form provided by Carletti and Gasperini (2003). The columns σ_{obs} and σ_{comp} indicate the relevant standard deviations (equations 12 and 8, respectively). In the last column, the value of Z relative to the test in equation (13) is displayed.

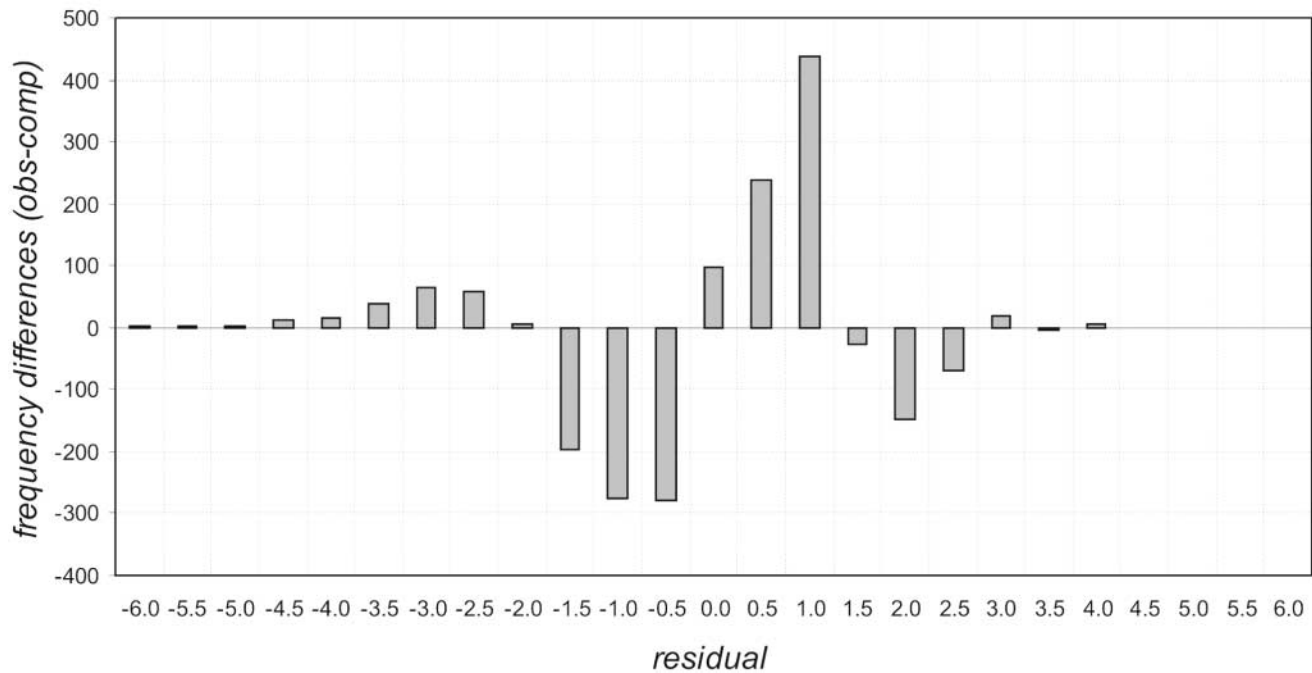


Figure 2. Differences between the frequency distribution of residuals from (14) (obs) and the frequency distribution of residuals computed in the assumption of normal distribution (comp).

icated in that study to verify the physical plausibility of the “attenuation law” (14). Much less attention, instead, was devoted to the accurate characterization of the probability distribution to be used in equation (1). Actually, the assumption of a normal distribution is, in the considered case, quite approximate because the distribution of residuals is significantly leptokurtic and more skewed than the normal distribution (see Gasperini, 2001, for details). These deviations from the normal distribution (Fig. 2) are apparently small but could be sufficient to explain the results in Figure 1. In general, the analysis described previously suggests that relatively small deviations from normality (discarded in the common practice) could result in “macroscopic” discrepancies and would produce significant hazard overestimates in the case of an acritical application of the attenuation relationship in the form (14) and (15) for PSHA in Italy.

Conclusions

The setup of effective validation procedures is becoming an essential tool for the definition of feasible methodologies devoted to PSHA. Major efforts have so far been devoted to the validation of estimated seismicity rates (McGuire, 1979; McGuire and Barnhard, 1981; Grandori *et al.*, 1998; Kagan and Jackson, 2000; Mucciarelli *et al.*, 2000; Petersen *et al.*, 2000). A contribution in this direction has been presented in this article, which allows the empirical validation of another essential aspect of seismic hazard assessment, that is, intensity attenuation relationships.

The validation is performed by taking into account the complete probabilistic formulation of intensity attenuation relationship, that is, the one actually implemented in common PSHA studies. The procedure can be summarized as follows:

1. an intensity threshold I_s is defined;
2. several earthquakes are selected such that, for each of them, a set of localities exists that experienced documented seismic effects;
3. for each j th earthquake and i th site, a probability K_{ij} is defined on the basis of available documentation: if this documentation unequivocally testifies that effects were at least of degree I_s or surely lower, then K_{ij} is set 1 or 0, respectively; otherwise, a number between 0 and 1 is attributed to K_{ij} , depending on the degree of belief in the hypothesis that the seismic effects actually were not less than I_s ;
4. the sum \hat{N}^{obs} of all the K_{ij} values is computed; the relevant variance $\sigma_{\hat{N}^{\text{obs}}}^2$ is computed by summing up the products $K_{ij}(1 - K_{ij})$;
5. using epicentral information relative to the j th earthquake and the attenuation relationship in its probabilistic formulation (e.g., equations 14 and 15), the conditional probability H_{ij} is computed (eventually by taking into account uncertainty on the epicentral intensity by equation 6) that felt effects at the i th site, shaken by the j th event, were not less than I_s ;
6. the expected value of \hat{N} is computed by summing up H_{ij} values from each j th event and i th site; the relevant var-

- iance σ_N^2 is computed by summing up the products $H_{ij}(1 - H_{ij})$;
7. the Z test in equation (13) is used to compare \hat{N}^{obs} and \hat{N} ;
 8. eventually the whole procedure is iterated by changing the intensity threshold I_s .

The proposed approach is insensitive to the specific probability distribution used to parameterize uncertainty on estimated intensities and allows one to take into account the uncertainty of both epicentral and site intensities. In principle, the same procedure could be applied for the validation of attenuation relationships relative to other relevant ground-motion parameters (peak ground acceleration, pseudo-spectral velocity, etc.). The basic limitation in this sense comes from the lack of extensive databases of experimental measurements of these quantities to be used on purpose.

An application of the proposed methodology to a state-of-the-art determination of macroseismic intensity attenuation in Italy has been presented. The results indicate that, whereas in the common practice major efforts are devoted to the parameterization of the average attenuation pattern (attenuation law), the definition of reliable and unbiased attenuation relationships requires an accurate and reliable modeling of their complete probabilistic form.

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