UNIVERSITÀ
DEGLI STUDI
DI BERGAMO

Dipartimento
di Ingegneria Gestionale,
dell'Informazione e della Produzione

Control Automation Lab

ADAPTIVE LEARNING, ESTIMATION AND SUPERVISION OF DYNAMICAL SYSTEMS (ALES)

Case study 1: Virtual Reference Feedback Tuning (VRFT)

Master Degree in COMPUTER ENGINEERING

Data Science and Data Engineering Curriculum

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Syllabus

1. **Recursive and adaptive identification**
   1.1 Recursive ARX estimation (RLS)
   1.2 Least Mean Squares (LMS)
   1.3 Instrumental Variables (IV)

2. **Closed-loop identification**

3. **Subspace and MIMO identification**
   3.1 Singular Value Decomposition
   3.2 Impulse data: Ho-Kalman, Kung algorithms
   3.3 Generic I/O data: the MOESP algorithm

4. **Supervision of dynamical systems**
   4.1 Introduction to fault diagnosis
   4.2 Model-based fault diagnosis
   4.3 Parity space approaches
   4.4 Observer-based approaches
   4.5 Signal-based fault diagnosis
   4.6 Knowledge-based fault diagnosis

CASE STUDIES

- Virtual Reference Feedback Tuning
- Nuclear particles classification
- Leak detection in an industrial valve
- Bearing fault identification

Outline

1. Application study: Virtual Reference Feedback Tuning (VRFT)
2. Defining the control problem
3. Basic idea
4. VRFT problem solution
5. Example

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1. **Application study: Virtual Reference Feedback Tuning (VRFT)**
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Virtual Reference Feedback Tuning (VRFT)

The VRFT is a **direct method** for designing controllers. It operates using a **«batch»** of input-output (open loop) data collected from the system

Indirect control design methods

1. Perform experiments on the open-loop system
2. Identify a model of the system
3. Design the controller based on identified model

Traditional **«model-based control design»** paradigm

Direct control design methods

1. Perform experiments on the open-loop system
2. Design directly the controller («identify the controller»)

«Direct Data-driven» control design paradigm

Ingredients of the VRFT method

1. Unknown SISO linear system $y(t) = G(z)u(t)$

$G(z)$ is **not known** (and we do not want to identify it)

2. Family of linear parametric 1-DOF controllers

$$R(z;\boldsymbol{\theta}) = \underbrace{\boldsymbol{\beta}^\top(z)}_{1\times d}\underbrace{\boldsymbol{\theta}}_{d\times 1} \qquad \boldsymbol{\beta}(z) = [\beta_1(z)\ \ \beta_2(z) \dots \beta_d(z)]^\top \qquad \boldsymbol{\theta} = [\theta_1\ \theta_2\ \dots \theta_d]^\top$$

Examples:

$$\boldsymbol{\beta}(z) = [1 \quad z^{-1} \quad \dots \quad z^{-n}]^\top \quad \Rightarrow \quad R(z;\boldsymbol{\theta}) = \theta_1 + \theta_2 z^{-1} + \cdots + \theta_n z^{-n}$$

$$\boldsymbol{\beta}(z) = \left[1 \quad \frac{1}{1-z^{-1}} \quad 1-z^{-1}\right]^\top \quad \Rightarrow \quad R(z;\boldsymbol{\theta}) = \theta_1 + \theta_2 \frac{1}{1-z^{-1}} + \theta_3(1-z^{-1})$$

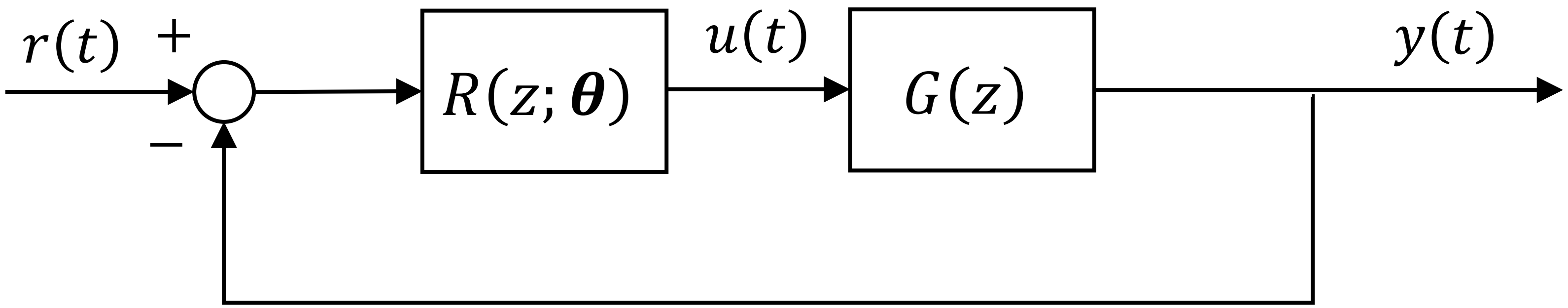
PID controller

Ingredients of the VRFT method

3. Model-reference control specification

Let $M(z)$ a reference model for the desired closed-loop behaviour. The aim is to design the controller $R(z;\boldsymbol{\theta})$ so that

$$M(z) \approx \frac{G(z)R(z;\boldsymbol{\theta})}{1+G(z)R(z;\boldsymbol{\theta})}$$



Outline

1. Application study: Virtual Reference Feedback Tuning (VRFT)

2. **Defining the control problem**

3. Basic idea

4. VRFT problem solution

5. Example

Defining the control problem

What **we would like** to minimize is the (possibly weighted) **closed-loop mismatch** between the reference model and the attained behaviour given the controller $R(z;\boldsymbol{\theta})$

$$\boxed{J_{\mathrm{MR}}(\boldsymbol{\theta}) = \left\| \left(\frac{G(z)R(z;\boldsymbol{\theta})}{1+G(z)R(z;\boldsymbol{\theta})} - M(z) \right) \cdot W(z) \right\|_2^2}$$

Model-reference cost

This cost **cannot be computed** since $G(z)$ is not known!

We have to find another cost function (different but similar) that can be minimized, using a set of **open-loop** system measurements

Outline

1. Application study: Virtual Reference Feedback Tuning (VRFT)
2. Defining the control problem
3. **Basic idea**
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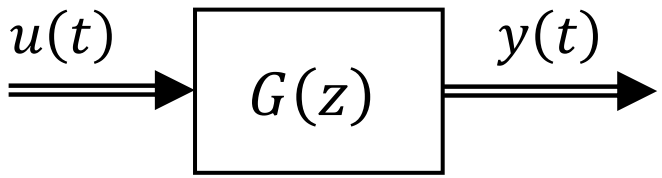
Basic idea

1. Perform an **open-loop experiment** on the system, and collect the measurements $\mathcal{D} = \{u(t), y(t)\}_{t=1}^{N}$



2. **IF** I was in closed-loop, and **IF** the **closed loop worked perfectly**, then the output $y(t)$ would have been such that

$$y(t) = M(z)\bar{r}(t)$$

where $\bar{r}(t)$ is the **reference signal** that generated the measurement $y(t)$. The signal $\bar{r}(t)$ is **«virtual»** since it does not exist, but it can be computed as

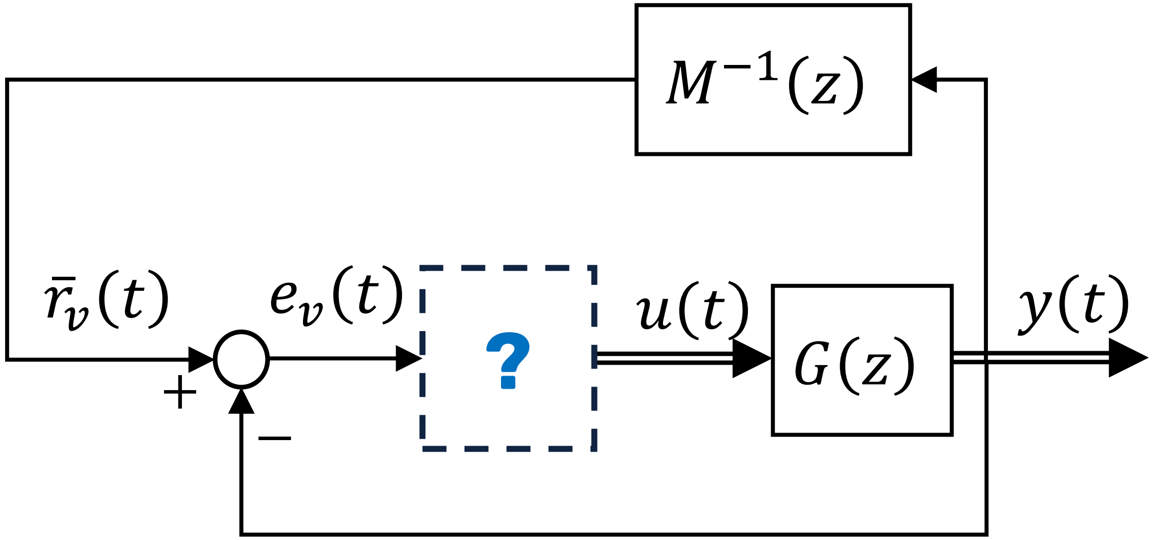
$$\bar{r}(t) = M^{-1}(z)y(t)$$

3. It is then possible to define the **virtual tracking error** $e_v(t)$ of this closed-loop (which does not exist)

$$e_v(t) = \bar{r}(t) - y(t)$$

Basic idea

4. The controller that grants this closed-loop to exist must be such that, when fed with the tracking error $e_v(t)$, would provide the input $u(t)$ that generated $y(t)$



Thus, the problem is to **identify the controller** $R(z;\boldsymbol{\theta})$ from $\mathcal{D} = \{u(t), y(t)\}_{t=1}^{N}$ by minimizing

$$\boxed{J_{\mathrm{VR}}^{N}(\boldsymbol{\theta}) = \frac{1}{N}\sum_{t=1}^{N}\big(u(t) - R(z;\boldsymbol{\theta})e_v(t)\big)^2}$$

Virtual-reference cost

Basic idea

Remark 1:

$M(z)$ is typically a strictly proper transfer function, so that $M^{-1}(z)$ will result into a **non-causal** transfer function, i.e. computing $\bar{r}(t)$ requires $y(t+1), y(t+2), \ldots$

This is not a problem, since we are working **off-line**, in a **batch** manner. So, future samples of $y(\cdot)$ are already available

The reference model should be $M(z) \neq 1$, otherwise

$$e_v(t) = \bar{r}(t) - y(t) = (M^{-1}(z) - 1)y(t) = 0$$

Basic idea

Remark 2:

Using a controller which **is linear in the parameters**, the solution can be computed using least squares. In fact, the linear in the parameter controller is described by

$$R(z;\boldsymbol{\theta}) = \boldsymbol{\beta}^\top(z)\boldsymbol{\theta}$$

Thus

$$R(z;\boldsymbol{\theta})e_v(t) = \boldsymbol{\beta}^\top(z)\boldsymbol{\theta}e_v(t) = \boldsymbol{\beta}^\top(z)e_v(t)\boldsymbol{\theta} = \boldsymbol{\varphi}^\top(t)\boldsymbol{\theta}$$

where

$$\boldsymbol{\varphi}(t) = [\beta_1(z)e_v(t) \quad \beta_2(z)e_v(t) \quad \dots \quad \beta_d(z)e_v(t)]^\top$$

Basic idea

Then, the virtual reference cost function $J_{\mathrm{VR}}(\boldsymbol{\theta})$ can be rewritten as

$$J_{\mathrm{VR}}^{N}(\boldsymbol{\theta}) = \frac{1}{N}\sum_{t=1}^{N}\big(u(t) - R(z;\boldsymbol{\theta})e_v(t)\big)^2 = \frac{1}{N}\sum_{t=1}^{N}\big(u(t) - \boldsymbol{\varphi}^\top(t)\boldsymbol{\theta}\big)^2$$

So that the solution is

$$\underset{d\times 1}{\widehat{\boldsymbol{\theta}}_{\mathrm{LS}}} = \left(\sum_{t=1}^{N} \underset{d\times 1}{\boldsymbol{\varphi}(t)}\,\underset{1\times d}{\boldsymbol{\varphi}^\top(t)}\right)^{-1} \cdot \sum_{t=1}^{N} \underset{d\times 1}{\boldsymbol{\varphi}(t)}\,\underset{1\times 1}{u(t)}$$

Basic idea

Remark 3:

We modified the original problem of minimizing $J_{\mathrm{MR}}(\boldsymbol{\theta})$ into the minimization of $J_{\mathrm{VR}}^{N}(\boldsymbol{\theta})$. The two cost functions **may not have the same minimum!**

In which measure the **two minima differ?** To answer this question, the $J_{\mathrm{VR}}^{N}(\boldsymbol{\theta})$ function is slightly modified by considering a **pre-filtered** version of input and virtual error signals

$$e_L(t) = L(z)e_v(t)$$

$$u_L(t) = L(z)u(t)$$

$$\Rightarrow$$

$$J_{\mathrm{VR}}^{N}(\boldsymbol{\theta}) = \frac{1}{N}\sum_{t=1}^{N}\big(u_L(t) - R(z;\boldsymbol{\theta})e_L(t)\big)^2 = \frac{1}{N}\sum_{t=1}^{N}\big(u_L(t) - \boldsymbol{\varphi}_L^{\top}(t)\boldsymbol{\theta}\big)^2$$

$$\boldsymbol{\varphi}_L^{\top}(t) = \boldsymbol{\beta}^{\top}(z)e_L(t)$$

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VRFT problem solution and choice of filter $L(z)$

In general, $J_{\mathrm{MR}}(\boldsymbol{\theta})$ and $J_{\mathrm{VR}}^{N}(\boldsymbol{\theta})$ are different and they have different minima. However, it is possible to **choose the pre-filter** $L(z)$ so that the **two minima coincide**

Consider the **model-reference cost** $J_{\mathrm{MR}}(\boldsymbol{\theta})$: the (squared) $\mathcal{H}_2$ norm can be written as

$$J_{\mathrm{MR}}(\boldsymbol{\theta}) = \left\| \left(\frac{G(z)R(z;\boldsymbol{\theta})}{1+G(z)R(z;\boldsymbol{\theta})} - M(z) \right) \cdot W(z) \right\|_2^2$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{G(e^{j\omega})R(e^{j\omega};\boldsymbol{\theta})}{1+G(e^{j\omega})R(e^{j\omega},\boldsymbol{\theta})} - M(e^{j\omega}) \right|^2 \cdot \left| \quad W(e^{j\omega}) \quad \right|^2 d\omega$$

VRFT problem solution and choice of filter $L(z)$

The reference model $M(z)$ can be expressed as

$$M(z) = \frac{G(z)R_0(z)}{1 + G(z)R_0(z)}$$

where $R_0(z)$ is the controller that **exactly solves** the model-matching problem. Notice that $R_0(z;\boldsymbol{\theta}^0)$ could not belong to the model set $R(z;\boldsymbol{\theta})$

Substituting this definition of $M(z)$ in $J_{\mathrm{MR}}(\boldsymbol{\theta})$, we get (dropping $e^{j\omega}$ for simplicity)

$$\boxed{J_{\mathrm{MR}}(\boldsymbol{\theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|G|^2|W|^2}{|1+GR(\boldsymbol{\theta})|^2} \cdot \frac{|R(\boldsymbol{\theta}) - R_0|^2}{|1+GR_0|^2}\, d\omega}$$

VRFT problem solution and choice of filter $L(z)$

Consider now the **virtual-reference cost** $J_{\mathrm{VR}}^N(\boldsymbol{\theta})$

$$J_{\mathrm{VR}}^N(\boldsymbol{\theta}) = \frac{1}{N}\sum_{t=1}^{N}\big(u_L(t) - R(z;\boldsymbol{\theta})e_L(t)\big)^2$$

If $u(t)$ and $y(t)$ are realizations of **stationary** and **ergodic processes**, also $\bar{r}(t), e_v(t), e_L(t), u_L(t)$ are stationary, since computed by linear filtering. Then

$$J_{\mathrm{VR}}^N(\boldsymbol{\theta}) \to J_{\mathrm{VR}}(\boldsymbol{\theta}) = \mathbb{E}\left[\big(u_L(t) - R(z;\boldsymbol{\theta})e_L(t)\big)^2\right]$$

so that $J_{\mathrm{VR}}(\boldsymbol{\theta})$ is the **variance** of the stationary process $u_L(t) - R(z;\boldsymbol{\theta})e_L(t)$

VRFT problem solution and choice of filter $L(z)$

We can also write

$$u_L(t) - R(z;\boldsymbol{\theta})e_L(t) \;= L(z)u(t) - L(z)R(z;\boldsymbol{\theta})e_v(t) \;= L(z)u(t) - L(z)R(z;\boldsymbol{\theta})[M^{-1}(z) - 1]y(t)$$

$$= L(z)u(t) - L(z)R(z;\boldsymbol{\theta})[M^{-1}(z) - 1]G(z)u(t) \;= L(z)\left[1 - R(z;\boldsymbol{\theta})G(z)\left(\frac{1}{M(z)} - 1\right)\right]u(t)$$

$$= \frac{L(z)}{M(z)}\left[M(z) - R(z;\boldsymbol{\theta})G(z)\big(1 - M(z)\big)G(z)\right]u(t)$$

$$= \frac{L(z)}{M(z)}\left[\frac{G(z)R_0(z)}{1 + G(z)R_0(z)} - R(z;\boldsymbol{\theta})G(z)\left(1 - \frac{G(z)R_0(z)}{1 + G(z)R_0(z)}\right)G(z)\right]u(t)$$

$$= \frac{L(z)}{M(z)}\frac{G(z)\big(R_0(z) - R(z;\boldsymbol{\theta})\big)}{1 + G(z)R_0(z)}u(t)$$

VRFT problem solution and choice of filter $L(z)$

The asymptotic virtual-reference cost $J_{\mathrm{VR}}(\boldsymbol{\theta})$ can thus be written as:

$$\boxed{J_{\mathrm{VR}}(\boldsymbol{\theta}) = \mathbb{E}\left[\left(u_L(t) - R(z;\boldsymbol{\theta})e_L(t)\right)^2\right] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|L|^2}{|M|^2}\frac{|G|^2}{|1+GR_0|^2}|R_0 - R(\boldsymbol{\theta})|^2 \cdot \Phi_{uu}\ d\omega}$$

where $\Phi_{uu}(z)$ is the **power spectral density** of the input $u(t)$

VRFT problem solution and choice of filter $L(z)$

$$J_{\mathrm{MR}}(\boldsymbol{\theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|G|^2|W|^2}{|1+GR(\boldsymbol{\theta})|^2} \cdot \frac{|R(\boldsymbol{\theta}) - R_0|^2}{|1+GR_0|^2}\, d\omega$$

Model-reference cost

$$J_{\mathrm{VR}}(\boldsymbol{\theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|L|^2}{|M|^2} \frac{|G|^2}{|1+GR_0|^2} |R_0 - R(\boldsymbol{\theta})|^2 \cdot \Phi_{uu}\, d\omega$$

Virtual-reference cost

Result 1: $R_0(z) \in R(z;\boldsymbol{\theta})$

If $R_0(z) \in R(z;\boldsymbol{\theta})$, then it exists $\boldsymbol{\theta}^0$ s.t. $J_{\mathrm{MR}}(\boldsymbol{\theta}^0) = J_{\mathrm{VR}}(\boldsymbol{\theta}^0) = 0$, so that the VRFT approach is able to attain the **optimal controller** $R_0(z)$ that achieves $M(z)$

VRFT problem solution and choice of filter $L(z)$

$$\boxed{\textbf{Result 2:} \quad R_0(z) \notin R(z;\boldsymbol{\theta})}$$

If $R_0(z) \notin R(z;\boldsymbol{\theta})$, it is possible to choose $L(z)$ so that $J_{\mathrm{MR}}(\boldsymbol{\theta}^0)$ and $J_{\mathrm{VR}}(\boldsymbol{\theta}^0)$ coincide. Recall

$$J_{\mathrm{MR}}(\boldsymbol{\theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|G|^2|W|^2}{|1+GR(\boldsymbol{\theta})|^2} \cdot \frac{|R(\boldsymbol{\theta})-R_0|^2}{|1+GR_0|^2} d\omega \qquad J_{\mathrm{VR}}(\boldsymbol{\theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|G|^2|L|^2}{|M|^2} \cdot \frac{|R_0-R(\boldsymbol{\theta})|^2}{|1+GR_0|^2} \cdot \Phi_{uu} \, d\omega$$

By choosing $L(z)$ so that $\boxed{L(z) = \frac{M(z)W(z)}{1+G(z)R(z;\boldsymbol{\theta})} \cdot \frac{1}{U(z)}}$ $\Rightarrow$ $|L|^2 = \frac{|M|^2|W|^2}{|1+GR(\boldsymbol{\theta})|^2} \cdot \frac{1}{\Phi_{uu}}$

it results that $J_{\mathrm{MR}}(\boldsymbol{\theta}) = J_{\mathrm{VR}}(\boldsymbol{\theta})$, where $U(z)$ is a spectral canonical factor of $\Phi_{uu}(\omega)$, s.t. $u(t) = U(z)\xi(t)$, $\xi(t) \sim \mathrm{WN}(0,1)$.

VRFT problem solution and choice of filter $L(z)$

Observation:

The **optimal filter**

$$L(z) = \frac{M(z)W(z)}{1 + G(z)R(z;\boldsymbol{\theta})} \cdot \frac{1}{U(z)}$$

cannot be computed since it depends on the unknown system $G(z)$ and $\boldsymbol{\theta}$

Idea:

Substitute $R(z;\boldsymbol{\theta})$ with $R_0(z)$ (that, near the minimum, will be «almost» equal)

$$\boxed{L(z) = \frac{M(z)}{1 + G(z)R_0(z)} \cdot \frac{W(z)}{U(z)}}$$

VRFT problem solution and choice of filter $L(z)$

The **asymptotic virtual-reference** cost now becomes

$$J_{\mathrm{VR}}(\boldsymbol{\theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|G|^2|R_0 - R(\boldsymbol{\theta})|^2}{|1+GR_0|^2} \cdot \frac{|L|^2}{|M|^2} \cdot \Phi_{uu}(z)\ d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|G|^2|R_0 - R(\boldsymbol{\theta})|^2}{|1+GR_0|^2} \cdot \frac{|M|^2}{|1+GR_0|^2} \cdot \frac{|W|^2}{\Phi_{uu}} \cdot \Phi_{uu} \cdot \frac{1}{|M|^2}\ d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|G|^2}{|1+GR_0|^2} \cdot \frac{|R_0 - R(\boldsymbol{\theta})|^2}{|1+GR_0|^2} \cdot |W|^2 d\omega$$

VRFT problem solution and choice of filter $L(z)$

$$J_{\mathrm{MR}}(\boldsymbol{\theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|G|^2|W|^2}{|1+GR_0|^2}\cdot\frac{|R(\boldsymbol{\theta})-R_0|^2}{|1+GR(\boldsymbol{\theta})|^2}d\omega$$

Model-reference cost

$$J_{\mathrm{VR}}(\boldsymbol{\theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|G|^2|W|^2}{|1+GR_0|^2}\cdot\frac{|R_0-R(\boldsymbol{\theta})|^2}{|1+GR_0|^2}d\omega$$

Virtual-reference cost

If the family of controllers $R(z;\boldsymbol{\theta})$ is **only slightly under-parameterized**, it holds that

$$R_0 \approx R(\overline{\boldsymbol{\theta}}), \qquad \overline{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, J_{\mathrm{MR}}(\boldsymbol{\theta})$$

Then, we can use the **approximation**

$$1 + G(z)R(z;\boldsymbol{\theta}) \approx 1 + G(z)R_0(z)$$

VRFT problem solution and choice of filter $L(z)$

Since

$$1 - M(z) = \frac{1}{1 + G(z)R_0(z)}$$

we have that

$$L(z) = \frac{M(z)}{1 + G(z)R_0(z)} \cdot \frac{W(z)}{U(z)} \quad = M(z)\big(1 - M(z)\big) \cdot \frac{W(z)}{U(z)}$$

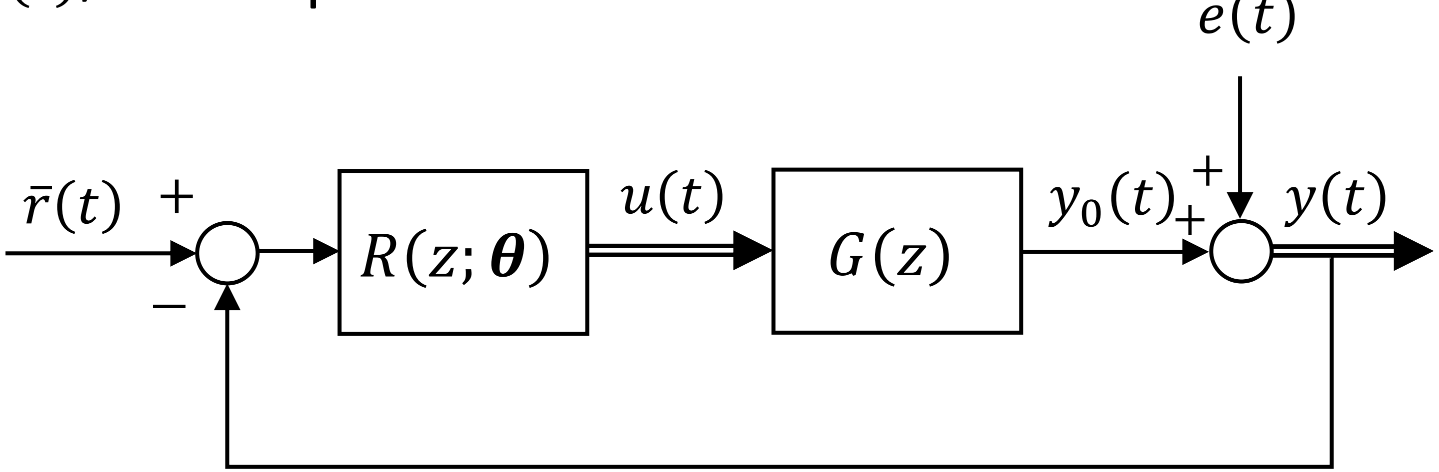
$$\Rightarrow \quad \boxed{L(z) = M(z)\big(1 - M(z)\big) \cdot \frac{W(z)}{U(z)}}$$

which is a filter perfectly **implementable in practice**

VRFT for noisy data

In case the plant is affect by **noise** $e(t)$, the output measurements will be

$$y(t) = G(z)u(t) + e(t)$$



The regressors

$$\boldsymbol{\varphi}_L^\top(t) = \boldsymbol{\beta}^\top(z;\boldsymbol{\theta})e_L(t) = \boldsymbol{\beta}^\top(z;\boldsymbol{\theta})L(z)e_v(t) = \boldsymbol{\beta}^\top(z;\boldsymbol{\theta})L(z)(M^{-1}(z) - 1)y(t)$$

are **correlated with the noise** $e(t)$ that acts on $y(t)$. Thus, the **instrumental variable** method has to be employed. The two-stage method can be employed to avoid a second experiment

VRFT for noisy data

VRFT summary

- Set $L(z) = \big(1 - M(z)\big)M(z)W(z)U(z)^{-1}$, where $\left|U\big(e^{j\omega}\big)\right|^2 = \Phi_{uu}(\omega)$
- Perform an open-loop experiment on the plant, collecting $\mathcal{D} = \{u(t), y(t)\}_{t=1}^{N}$
- Compute $u_L(t) = L(z)u(t)$
- Compute $\boldsymbol{\varphi}_L(t) = \boldsymbol{\beta}(z)L(z)(M(z)^{-1} - 1)y(t)$
- Identify a high-order model $\hat{G}(z)$ from $\mathcal{D}$
- Compute the IV $\boldsymbol{z}(t) = \boldsymbol{\beta}(z)L(z)(M(z)^{-1} - 1)\hat{G}(z)u(t)$
- Estimate the controller parameters $\hat{\boldsymbol{\theta}}_{\mathrm{IV}} = \left(\sum_{t=1}^{N} \boldsymbol{z}(t)\boldsymbol{\varphi}^\top(t)\right)^{-1} \cdot \sum_{t=1}^{N} \boldsymbol{z}(t)u_L(t)$

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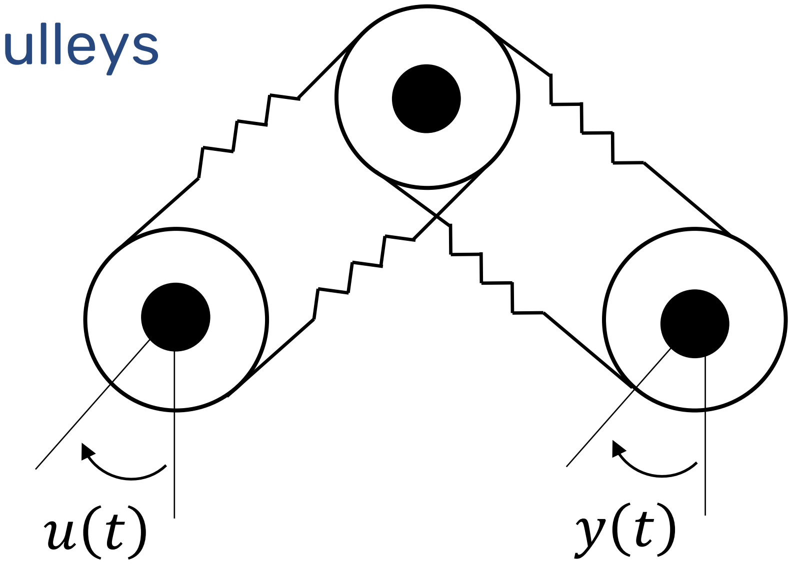
5. **Example**

Example: VRFT of a flexible transmission system

The flexible transmission consists of three horizontal pulleys connected by two elastic belts



Input: angular position of the first pulley

Output: angular position of the third pulley

The **control objective** is to make the two **angular positions** as **close as possible**. The plant input-output behaviour can be described by (with sampling time $T_s = 0.05\ s$)

$$G(z) = z^{-3} \cdot \frac{0.28261 + 0.50666z^{-1}}{1 - 1.41833z^{-1} + 1.58939z^{-2} - 1.31608z^{-3} + 0.88642z^{-4}}$$

Example: VRFT of a flexible transmission system

The control objective is expressed by the following reference model

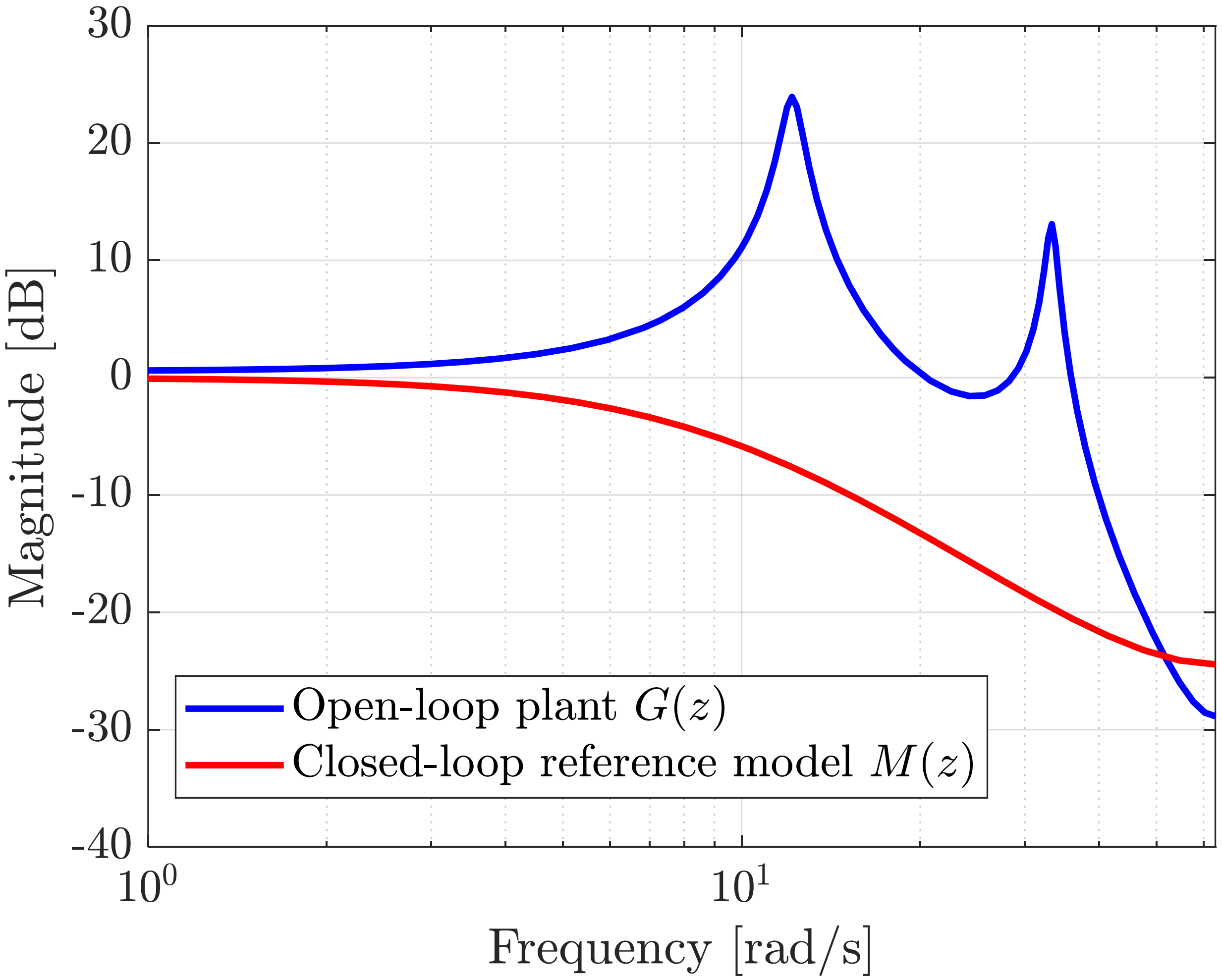
$$M(z) = z^{-3} \cdot \frac{(1-\alpha)^2}{(1-\alpha z^{-1})^2},$$

$$\alpha = e^{-T_s \bar{\omega}}, \qquad \bar{\omega} = 10$$

The frequency weight is $W(z) = 1$

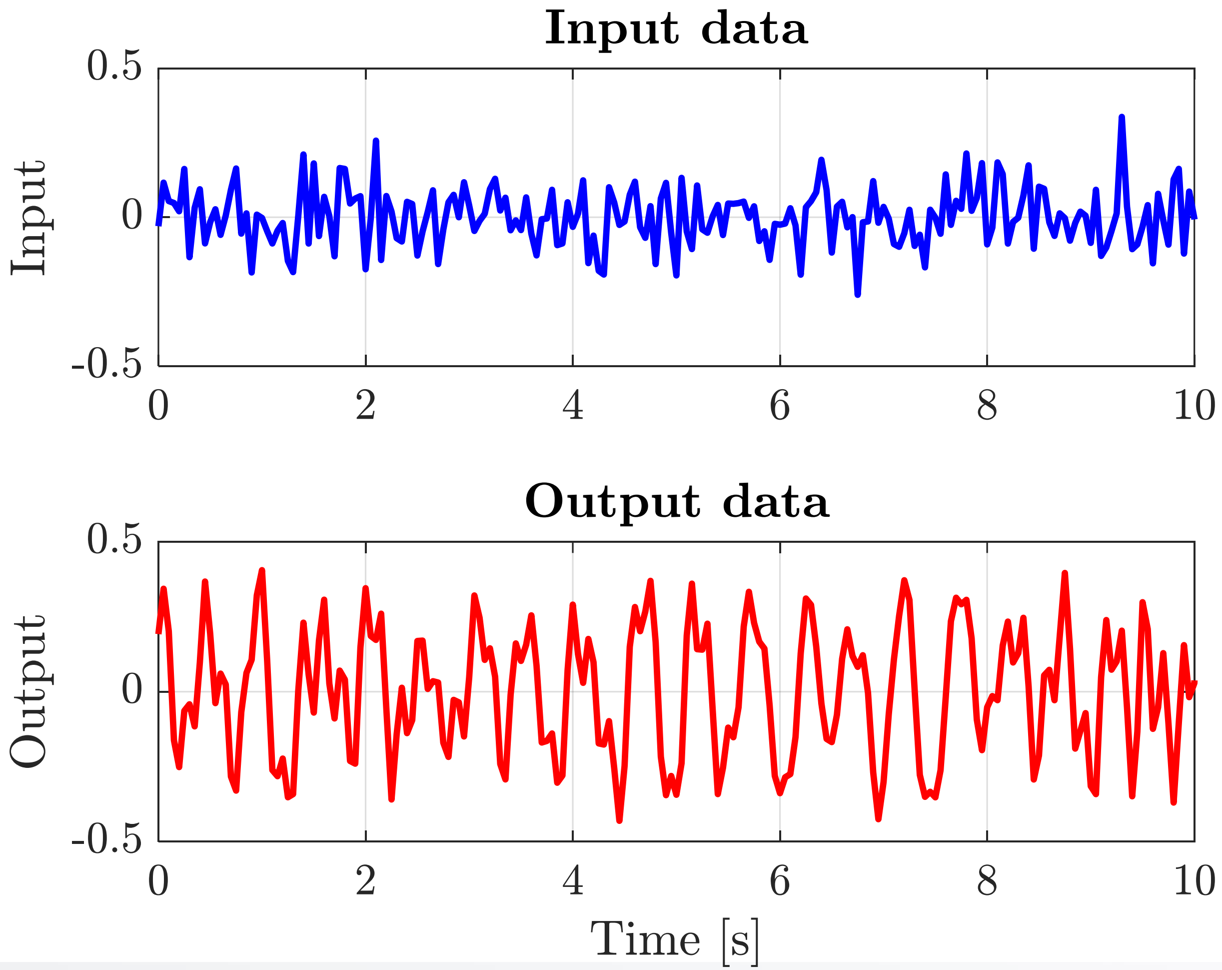
The class of controllers is

$$R(z;\boldsymbol{\theta}) = \frac{\theta_0 + \theta_1 z^{-1} + \theta_2 z^{-2} + \theta_3 z^{-3} + \theta_4 z^{-4} + \theta_5 z^{-5}}{1 - z^{-1}}$$



Example: VRFT of a flexible transmission system

The plant was excited with a white noise input $u(t) \sim \mathrm{WN}(0, 0.01)$, collecting $N = 512$ data

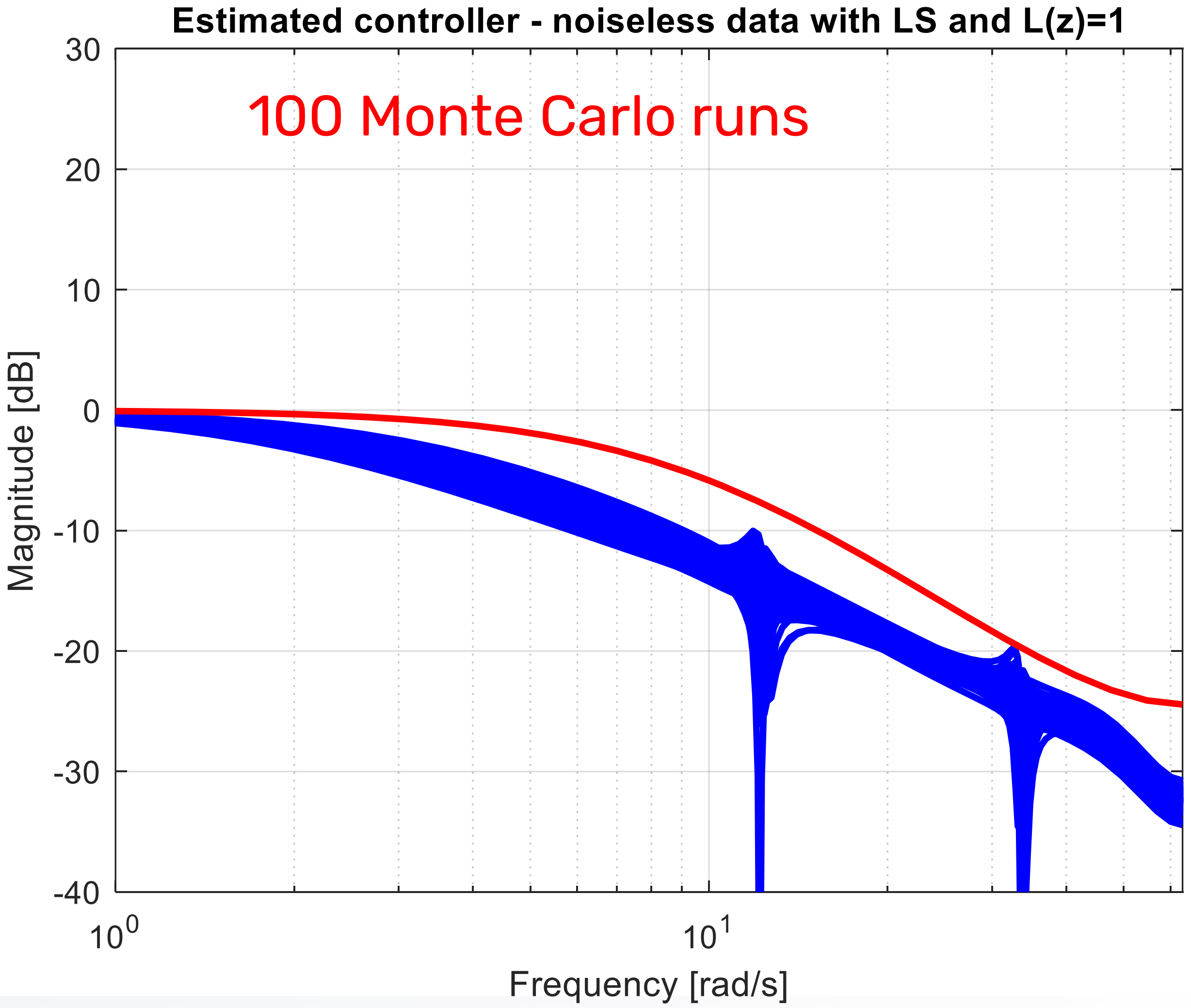


Example: VRFT of a flexible transmission system

Case 1: no prefilter, noiseless data

$W(z) = 1, L(z) = 1$

Not using the optimal prefilter leads to bad results



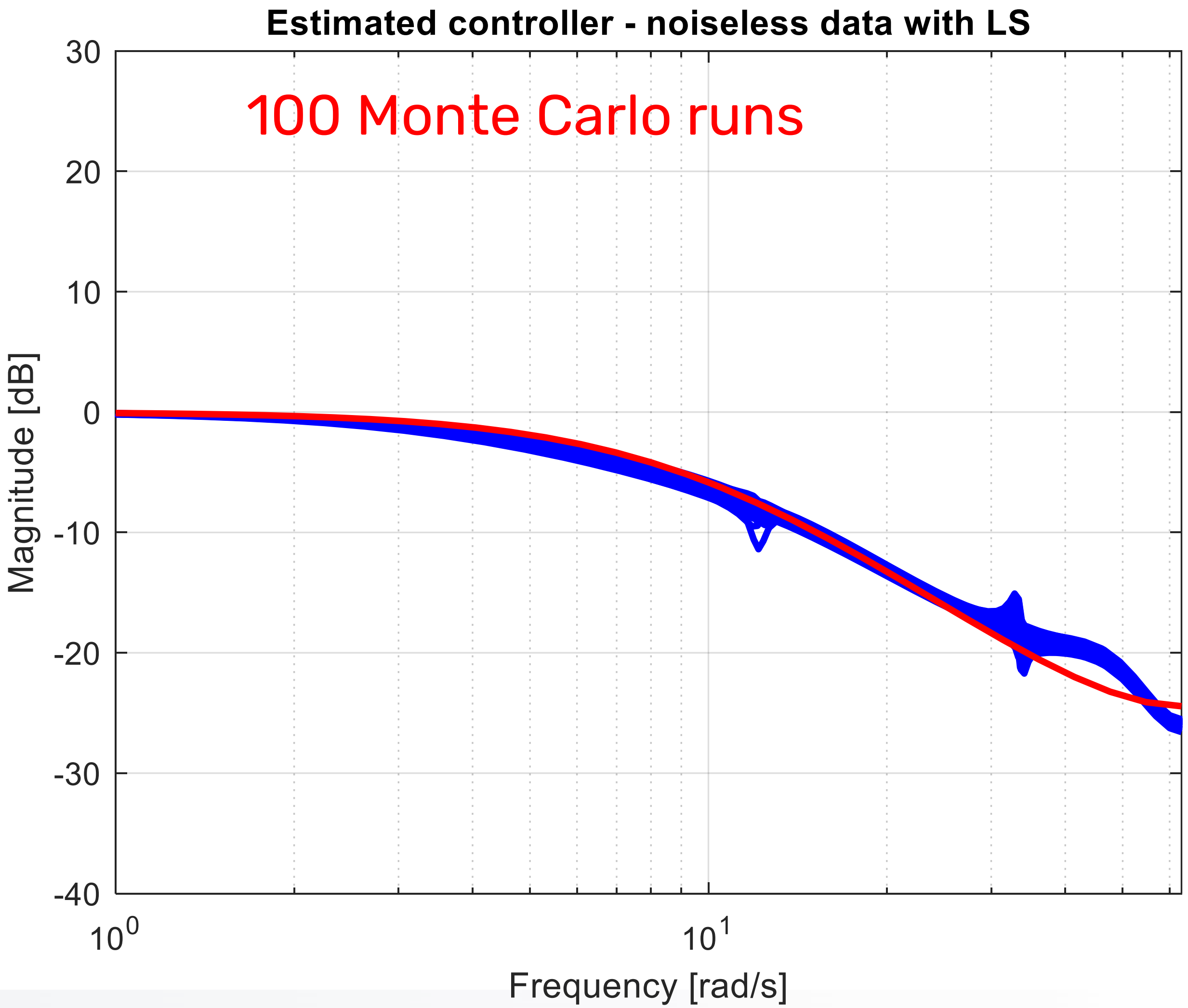
Example: VRFT of a flexible transmission system

Case 2: optimal prefilter, noiseless data

$$W(z) = 1, \qquad \Phi_{uu}(z) = 0.01$$

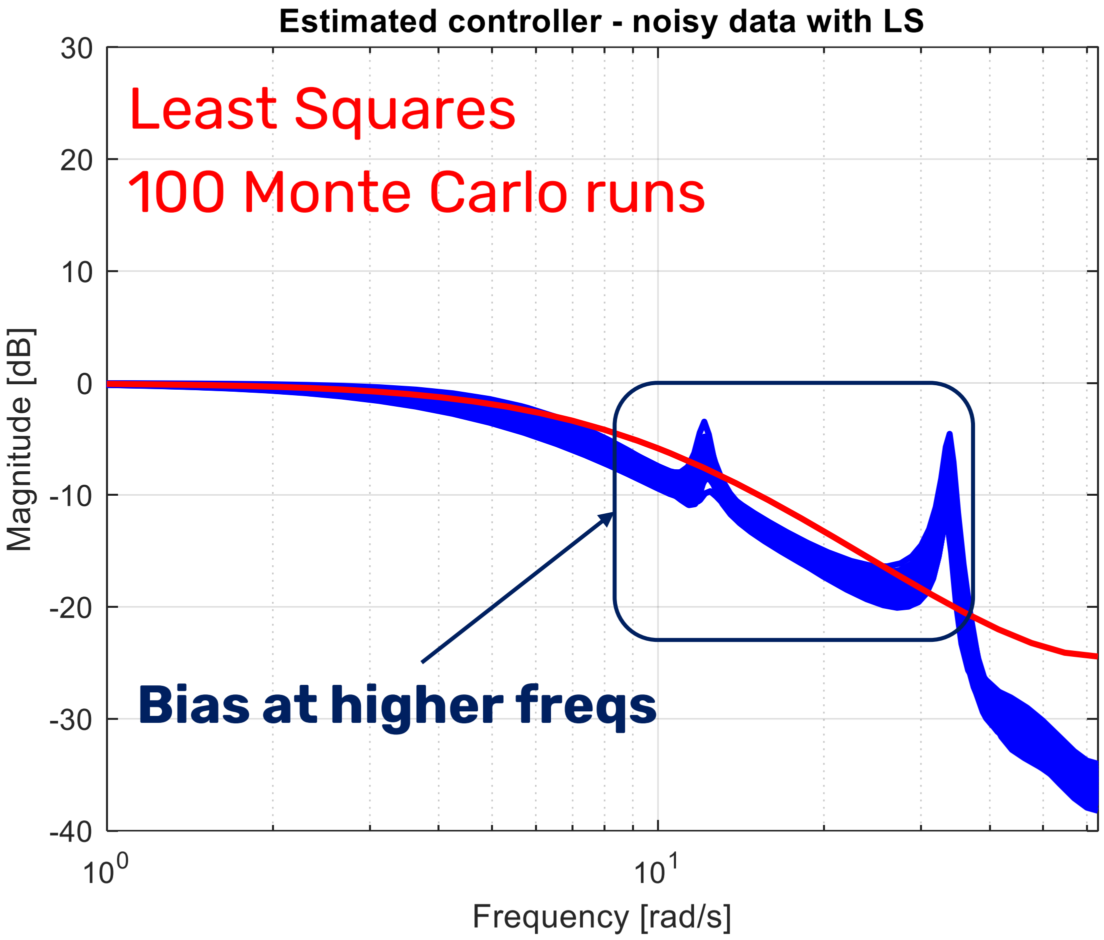
$$L(z) = W(z) \cdot \big(1 - M(z)\big) M(z) \cdot \frac{1}{\Phi_{uu}^{1/2}(z)}$$

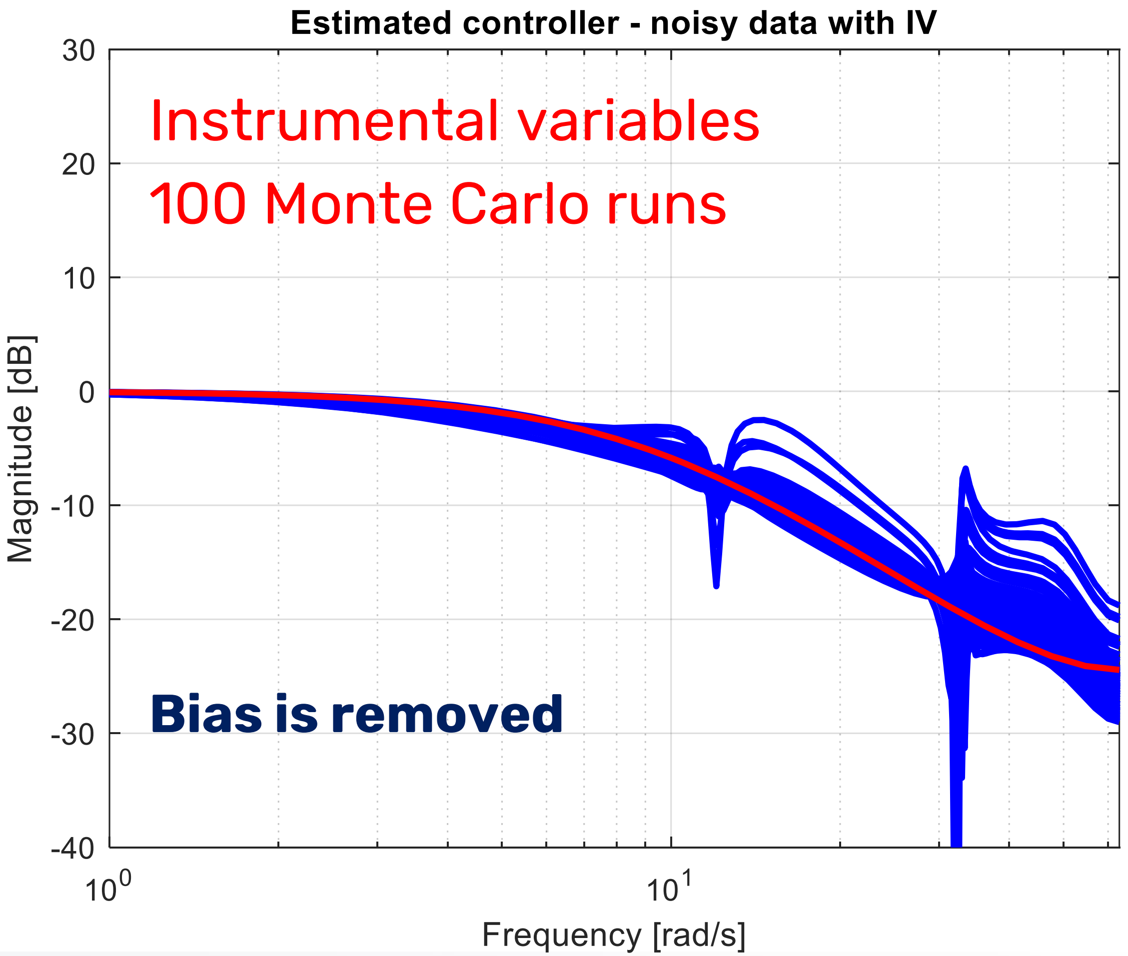
The use of the optimal prefilter leads to accurate results



Example: VRFT of a flexible transmission system

Case 3: optimal prefilter, noisy data





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