University of Oslo

Project 2

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https://github.com/sim-hal/FYS3150-Project-2

INTRODUCTION

In this artice we will mainly look at ways of scaling equations, some eigenvalue problems and unit testing.

Problem 1)

We want to show that the second-order differential equation

$$\gamma \frac{d^2 u(x)}{dx^2} = -Fu(x) \tag{1}$$

can be written as

$$\frac{d^2 u(\hat{x})}{d\hat{x}^2} = -\lambda u(\hat{x}) \tag{2}$$

where $\hat{x} \equiv x/L \iff x$ is a dimensionless variable and $\lambda = \frac{FL^2}{\gamma}$. This also means that $x = \hat{x}L$. We know that

$$\frac{d}{d\hat{x}} = \frac{dx}{d\hat{x}}\frac{du}{dx} = L\frac{du}{dx} \iff \frac{du}{dx} = \frac{1}{L}\frac{du}{d\hat{x}}$$

we define $v = \frac{1}{L}\frac{du}{d\hat{x}}$ and get

$$\frac{d^2u}{dx^2} = \frac{d}{dx}\left(\frac{du}{dx}\right) = \frac{d}{dx}\left(\frac{1}{L}\frac{du}{d\hat{x}}\right) = \frac{d}{dx}\left(v\right) = \frac{dv}{dx}$$

$$= \frac{d\hat{x}}{dx}\frac{dv}{d\hat{x}} = \frac{1}{L}\frac{dv}{d\hat{x}} = \frac{1}{L}\frac{d}{d\hat{x}}\left(\frac{1}{L}\frac{du}{d\hat{x}}\right) = \frac{1}{L^2}\frac{d^2u}{d\hat{x}^2}$$

This means that

$$\gamma\frac{d^2u}{dx^2} = \gamma\frac{1}{L^2}\frac{d^2u}{d\hat{x}^2} = -Fu$$

by moving terms we see that

$$\frac{d^2u}{d\hat{x}^2} = -FL^2\frac{1}{\gamma}u = -\lambda u$$

if we evaluate u on $\hat{x}$ we see that

$$\frac{d^2u(\hat{x})}{d\hat{x}^2} = -\lambda u(\hat{x})$$

$\square$

Problem 2)

We know that $\vec{v}_i$ is a set of orthonormal basis vectors, meaning $\vec{v}_i^T \cdot \vec{v}_j = \delta_{ij}$. We also know that $\mathbf{U}$ is an orthonormal transformation matrix. This means that $\mathbf{U}^T = \mathbf{U}^{-1}$ and that $\mathbf{U}\mathbf{U}^T = \mathbf{U}\mathbf{U}^{-1} = I$. We want to show that transformations with $\mathbf{U}$ preserves orthonormality, and that the set of vectors $\vec{w}_i = \mathbf{U}\vec{v}_i$ is also an orthonormal set. We start b looking at $\vec{w}_i = \mathbf{U}\vec{v}_i$

$$\vec{w}_i = \mathbf{U}\vec{v}_i$$

we transpose this equation and get

$$\vec{w}_i^T = (\mathbf{U}\vec{v}_i)^T = \vec{v}_i^T \mathbf{U}^T$$

this means that

$$\vec{w}_i^T \vec{w}_j = \vec{v}_i^T \mathbf{U}^T \mathbf{U} \vec{v}_j = \vec{v}_i^T I \vec{v}_j = \vec{v}_i^T \vec{v}_j = \delta_{ij}$$

$\square$