University of Oslo

Project 2

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https://github.com/sim-hal/FYS3150-Project-2

INTRODUCTION

In this artice we will mainly look at ways of scaling equations, some eigenvalue problems and unit testing.

Problem 1)

We want to show that the second-order differential equation

$$\gamma \frac{d^2 u(x)}{dx^2} = -F u(x) \tag{1}$$

can be written as

$$\frac{d^2 u(\hat{x})}{d\hat{x}^2} = -\lambda u(\hat{x}) \tag{2}$$

where $\hat{x} \equiv x/L \iff x$ is a dimensionless variable and $\lambda = \frac{FL^2}{\gamma}$. This also means that $x = \hat{x}L$. We know that

$$\frac{du}{d\hat{x}} = \frac{dx}{d\hat{x}}\frac{du}{dx} = L\frac{du}{dx} \iff \frac{du}{dx} = \frac{1}{L}\frac{du}{d\hat{x}}$$

we define $v = \frac{1}{L}\frac{du}{d\hat{x}}$ and get

$$\frac{d^2u}{dx^2} = \frac{d}{dx}\Big(\frac{du}{dx}\Big) = \frac{d}{dx}\Big(\frac{1}{L}\frac{du}{d\hat{x}}\Big) = \frac{d}{dx}\Big(v\Big) = \frac{dv}{dx}$$

$$= \frac{d\hat{x}}{dx}\frac{dv}{d\hat{x}} = \frac{1}{L}\frac{dv}{d\hat{x}} = \frac{1}{L}\frac{d}{d\hat{x}}\Big(\frac{1}{L}\frac{du}{d\hat{x}}\Big) = \frac{1}{L^2}\frac{d^2u}{d\hat{x}^2}$$

This means that

$$\gamma\frac{d^2u}{dx^2} = \gamma\frac{1}{L^2}\frac{d^2u}{d\hat{x}^2} = -Fu$$

by moving terms we see that

$$\frac{d^2u}{d\hat{x}^2} = -FL^2\frac{1}{\gamma}u = -\lambda u$$

if we evaluate u on $\hat{x}$ we see that

$$\frac{d^2u(\hat{x})}{d\hat{x}^2} = -\lambda u(\hat{x})$$

□

Problem 2)

We know that $\vec{v}_i$ is a set of orthonormal basis vectors, meaning $\vec{v}_i^T \cdot \vec{v}_j = \delta_{ij}$. We also know that $\mathbf{U}$ is an orthonormal transformation matrix. This means that $\mathbf{U}^T = \mathbf{U}^{-1}$ and that $\mathbf{U}\mathbf{U}^T = \mathbf{U}\mathbf{U}^{-1} = I$. We want to show that transformations with $\mathbf{U}$ preserves orthonormality, and that the set of vectors $\vec{w}_i = \mathbf{U}\vec{v}_i$ is also an orthonormal set. We start b looking at $\vec{w}_i = \mathbf{U}\vec{v}_i$

$$\vec{w}_i = \mathbf{U}\vec{v}_i$$

we transpose this equation and get

$$\vec{w}_i^T = (\mathbf{U}\vec{v}_i)^T = \vec{v}_i^T \mathbf{U}^T$$

this means that

$$\vec{w}_i^T \vec{w}_j = \vec{v}_i^T \mathbf{U}^T \mathbf{U}\vec{v}_j = \vec{v}_i^T I \vec{v}_j = \vec{v}_i^T \vec{v}_j = \delta_{ij}$$

□

Problem 3)

See implementation

Problem 4)

See implementation

Problem 5)

See implementation

Problem 6)

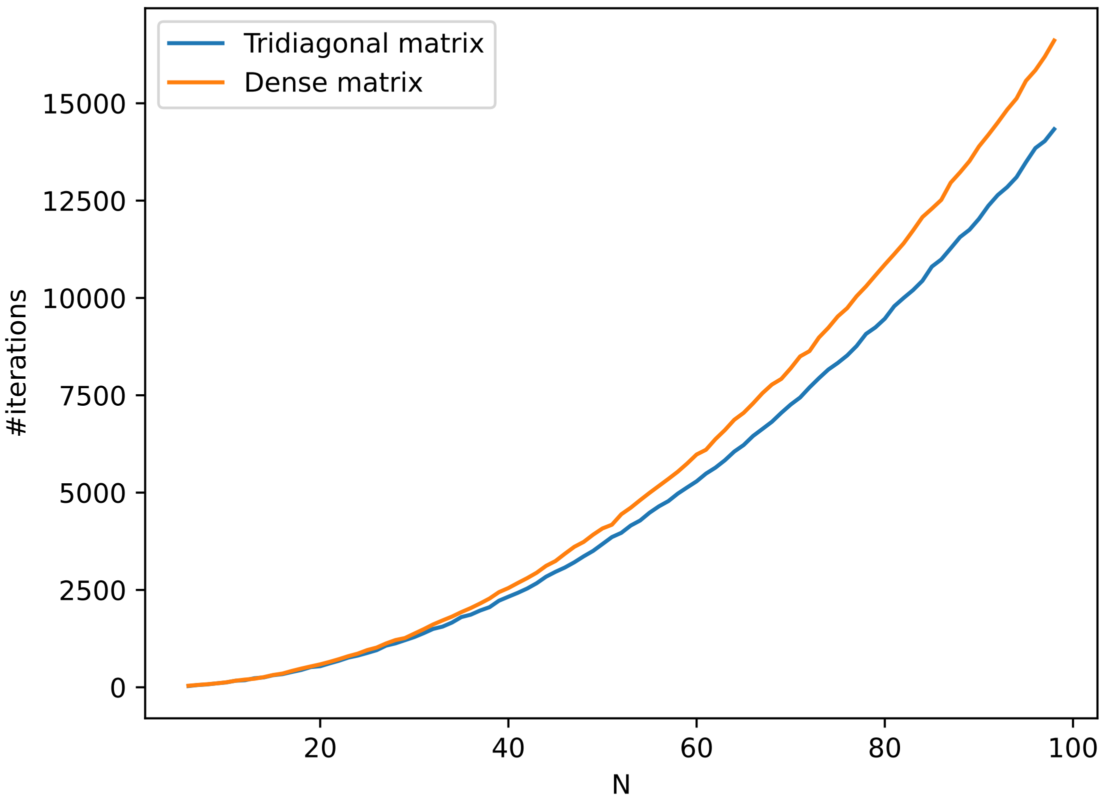

FIG. 1. A plot with the number of required transformation scales

We suspect from reading Fig 3 that the scaling behavior has quadratic complexity. From the plot it seems that the dense matrix has a similar complexity, but with a somewhat bigger constant.

When we tried finding the scaling behaviour of A when A was a dense matrix, we saw that the size of the entries were impactful for the number of iterations. We decided to scale the matrix, so the order of magnitude of the entries was comparable to those in the tridiagonal matrix.

Problem 7)

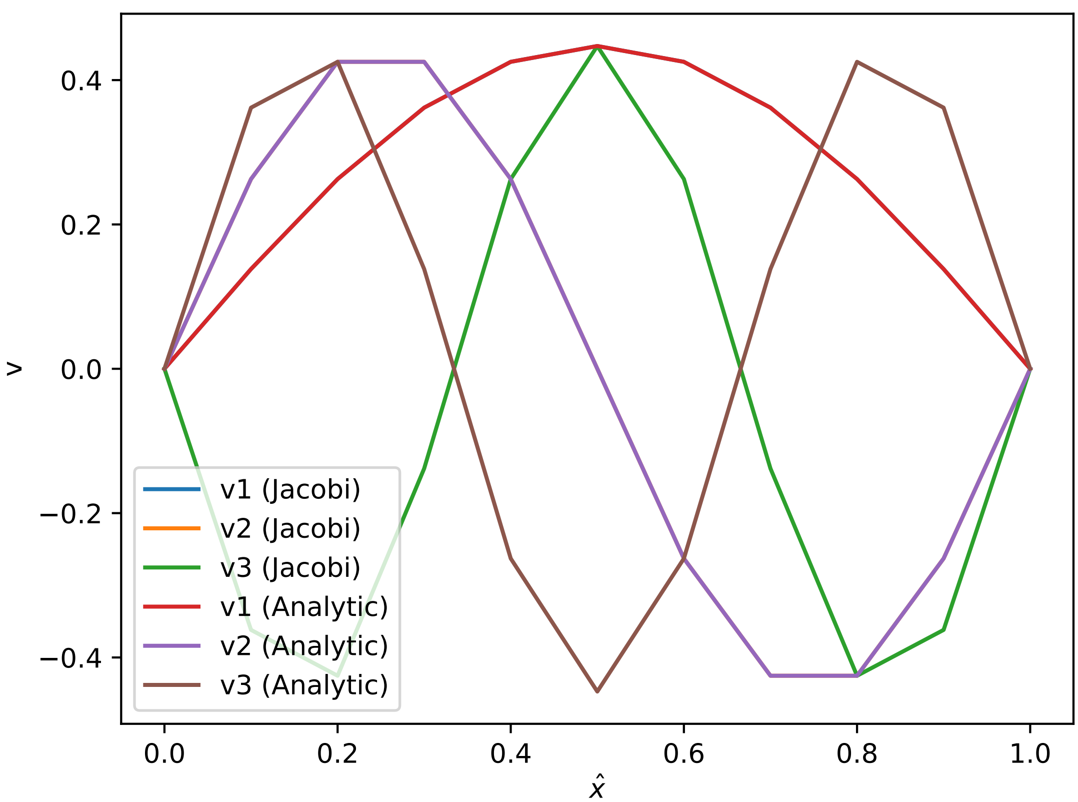

FIG. 2. A plot for n=10

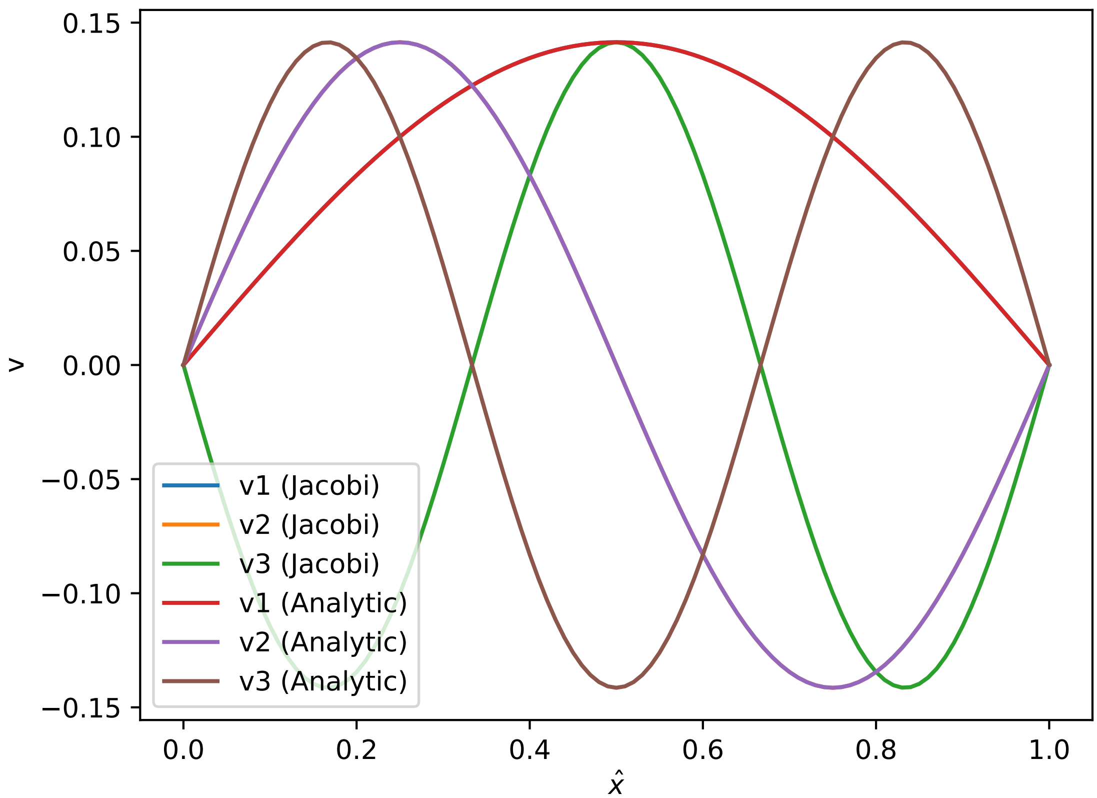

FIG. 3. A plot for n=100

We see that v_3 is not completely equal, but it is mirrored around 0, since v_i and $-v_i$ are both correct eigenvectors.