

Project 2

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<https://github.com/sim-hal/FYS3150-Project-2>

INTRODUCTION

In this article we will mainly look at ways of scaling equations, some eigenvalue problems and unit testing.

Problem 1)

We want to show that the second-order differential equation

$$\gamma \frac{d^2 u(x)}{dx^2} = -Fu(x) \quad (1)$$

can be written as

$$\frac{d^2 u(\hat{x})}{d\hat{x}^2} = -\lambda u(\hat{x}) \quad (2)$$

where $\hat{x} \equiv x/L \iff x$ is a dimensionless variable and $\lambda = \frac{FL^2}{\gamma}$. This also means that $x = \hat{x}L$. We know that

$$\frac{du}{d\hat{x}} = \frac{dx}{d\hat{x}} \frac{du}{dx} = L \frac{du}{dx} \iff \frac{du}{dx} = \frac{1}{L} \frac{du}{d\hat{x}}$$

we define $v = \frac{1}{L} \frac{du}{d\hat{x}}$ and get

$$\begin{aligned} \frac{d^2 u}{dx^2} &= \frac{d}{dx} \left(\frac{du}{dx} \right) = \frac{d}{dx} \left(\frac{1}{L} \frac{du}{d\hat{x}} \right) = \frac{d}{dx} (v) = \frac{dv}{dx} \\ &= \frac{d\hat{x}}{dx} \frac{dv}{d\hat{x}} = \frac{1}{L} \frac{dv}{d\hat{x}} = \frac{1}{L} \frac{d}{d\hat{x}} \left(\frac{1}{L} \frac{du}{d\hat{x}} \right) = \frac{1}{L^2} \frac{d^2 u}{d\hat{x}^2} \end{aligned}$$

This means that

$$\gamma \frac{d^2 u}{dx^2} = \gamma \frac{1}{L^2} \frac{d^2 u}{d\hat{x}^2} = -Fu$$

by moving terms we see that

$$\frac{d^2 u}{d\hat{x}^2} = -FL^2 \frac{1}{\gamma} u = -\lambda u$$

if we evaluate u on \hat{x} we see that

$$\frac{d^2 u(\hat{x})}{d\hat{x}^2} = -\lambda u(\hat{x})$$

□

Problem 2)

We know that \vec{v}_i is a set of orthonormal basis vectors, meaning $\vec{v}_i^T \cdot \vec{v}_j = \delta_{ij}$. We also know that \mathbf{U} is an orthonormal transformation matrix. This means that $\mathbf{U}^T = \mathbf{U}^{-1}$ and that $\mathbf{U}\mathbf{U}^T = \mathbf{U}\mathbf{U}^{-1} = I$. We want to show that transformations with \mathbf{U} preserves orthonormality, and that the set of vectors $\vec{w}_i = \mathbf{U}\vec{v}_i$ is also an orthonormal set. We start by looking at $\vec{w}_i = \mathbf{U}\vec{v}_i$

$$\vec{w}_i = \mathbf{U}\vec{v}_i$$

we transpose this equation and get

$$\vec{w}_i^T = (\mathbf{U}\vec{v}_i)^T = \vec{v}_i^T \mathbf{U}^T$$

this means that

$$\vec{w}_i^T \vec{w}_j = \vec{v}_i^T \mathbf{U}^T \mathbf{U} \vec{v}_j = \vec{v}_i^T I \vec{v}_j = \vec{v}_i^T \vec{v}_j = \delta_{ij}$$

□

Problem 3)

See implementation

Problem 4)

See implementation

Problem 5)

See implementation

Problem 6)

We suspect from reading Fig 1 that the scaling behavior has quadratic complexity. From the plot it seems that the dense matrix has a similar complexity, but with a somewhat bigger constant.

When we tried finding the scaling behaviour of A when A was a dense matrix, we saw that the size of the entries were impactful for the number of iterations. We decided to scale the matrix, so the order of magnitude of the entries was comparable to those in the tridiagonal matrix.

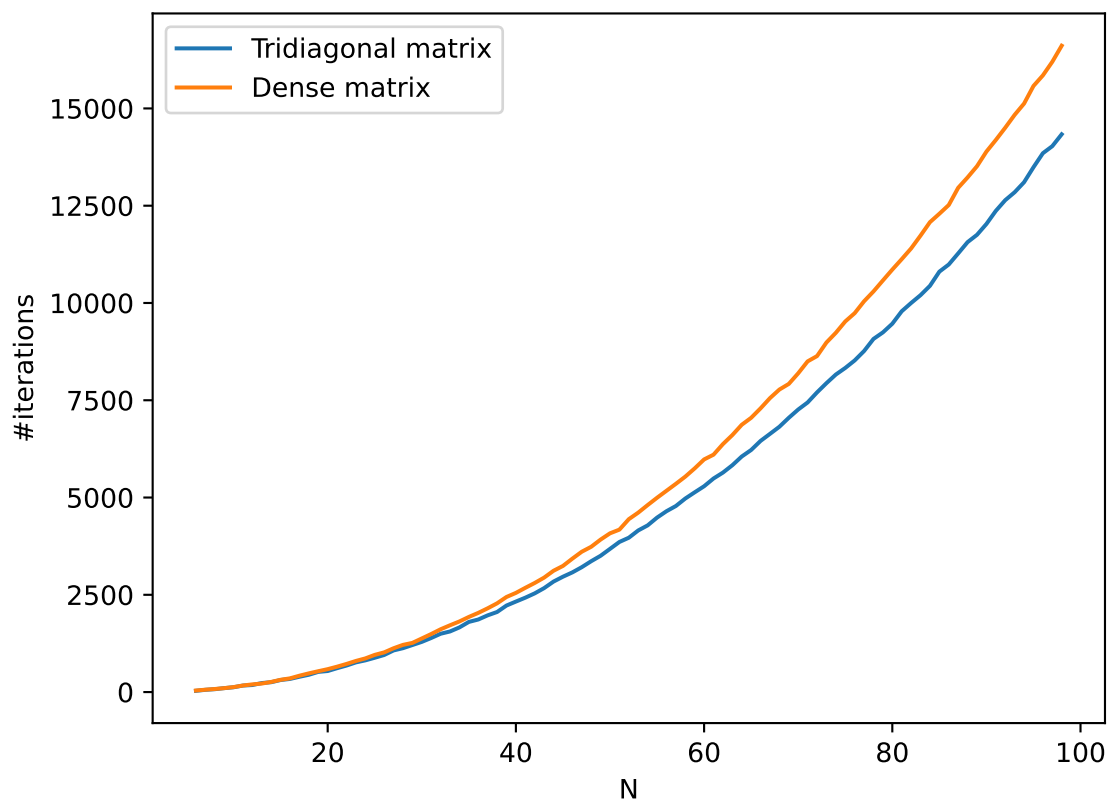


FIG. 1. A plot with the number of required transformation scales