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Python与机器学习

——机器学习经典案例1

华算科技 黄老师 2022年1月19日



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重现文献案例

利用机器学习方法预测金属与双金属的d带中心



RSC Advances

PAPER

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Machine-learning prediction of the d-band center for metals and bimetals

Ichigaku Takigawa,*ab Ken-ichi Shimizu,cd Koji Tsudaefg and Satoru Takakusagic

The d-band center for metals has been widely used in order to understand activity trends in metal-surface-catalyzed reactions in terms of the linear Brønsted–Evans–Polanyi relation and Hammer–Nørskov d-band model. In this paper, the d-band centers for eleven metals (Fe, Co, Ni, Cu, Ru, Rh, Pd, Ag, Ir, Pt, Au) and their pairwise bimetals for two different structures (1% metal doped- or overlayer-covered metal surfaces) are statistically predicted using machine learning methods from readily available values as descriptors for the target metals (such as the density and the enthalpy of fusion of each metal). The predictive accuracy of four regression methods with different numbers of descriptors and different test-set/training-set ratios are quantitatively evaluated using statistical cross validations. It is shown that the d-band centers are reasonably well predicted by the gradient boosting regression (GBR) method with only six descriptors, even when we predict 75% of the data from only 25% given for training (average root mean square error (RMSE) < 0.5 eV). This demonstrates a potential use of machine learning methods for predicting the activity trends of metal surfaces with a negligible CPU time compared to first-principles methods.

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www.rsc.org/advances

预测模型

输入: 主客体金属描述符

原子数

价电子数



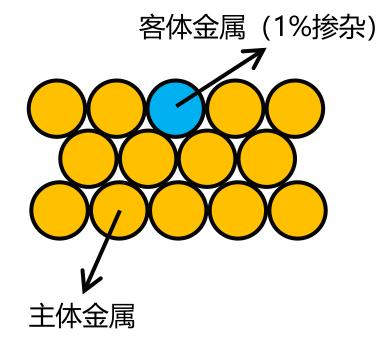
周期

密度

• • • • •



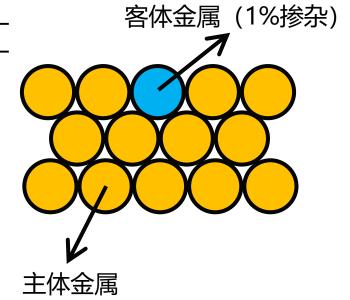
输出:双金属d带中心数值



d带中心数据集

Table 1
Shifts in d-band centers of surface impurities and overlayers relative to the clean metal values (italic)

	Fe	Со	Ni	Cu	Ru	Rh	Pd	Ag	Ir	Pt	Au
Fe	-0.92	0.05	-0.20	-0.13	-0.29	-0.54	-1.24	-0.83	-0.36	-1.09	-1.42
		0.14	-0.04	-0.05	-0.73	-0.72	-1.32	-1.25	-0.95	-1.48	-2.19
Co	0.01	-1.17	-0.28	-0.16	-0.24	-0.58	-1.37	-0.91	-0.36	-1.19	-1.56
	-0.01		-0.20	-0.06	-0.70	-0.95	-1.65	-1.36	-1.09	-1.89	-2.39
Ni	0.09	0.19	-1.29	0.19	-0.14	-0.31	-0.97	-0.53	-0.14	-0.80	-1.13
	0.96	0.11		0.12	-0.63	-0.74	-1.32	-1.14	-0.86	-1.53	-2.10
Cu	0.56	0.60	0.27	-2.67	0.58	0.32	-0.64	-0.70	0.58	-0.33	-1.09
	0.25	0.38	0.18		-0.22	-0.27	-1.04	-1.21	-0.32	-1.15	-1.96
Ru	0.21	0.26	0.01	0.12	-1.41	-0.17	-0.82	-0.27	0.02	-0.62	-0.84
	0.30	0.37	0.29	0.30		-0.12	-0.47	-0.40	-0.13	-0.61	-0.86
Rh	0.24	0.34	0.16	0.44	0.04	-1.73	-0.54	0.07	0.17	-0.35	-0.49
	0.31	0.41	0.34	0.22	0.03		-0.39	-0.08	0.03	-0.45	-0.57
Pd	0.37	0.54	0.50	0.94	0.24	0.36	-1.83	0.59	0.53	0.19	0.17
	0.36	0.54	0.54	0.80	-0.11	0.25		0.15	0.31	0.04	-0.14
Ag	0.72	0.84	0.67	0.47	0.84	0.86	0.14	-4.30	1.14	0.50	-0.15
8	0.55	0.74	0.68	0.62	0.50	0.67	0.27		0.80	0.37	-0.21
Ir	0.21	0.27	0.05	0.21	0.09	-0.15	-0.73	-0.13	-2.11	-0.56	-0.74
	0.33	0.40	0.33	0.56	-0.01	-0.03	-0.42	-0.09		-0.49	-0.59
Pt	0.33	0.48	0.40	0.72	0.14	0.23	-0.17	0.44	0.38	-2.25	-0.05
11	0.35	0.43	0.54	0.72	0.14	0.24	0.02	0.19	0.30	2.23	-0.08
Au	0.63	0.77	0.63	0.75	0.70	0.75	0.02	0.21	0.29	0.46	-3.56
Au	0.63										- 3.30
	0.33	0.74	0.71	0.70	0.47	0.67	0.35	0.12	0.79	0.43	





d带中心数据集

Table 3 Input features (descriptors) used for prediction of d-band centers from ref. 34^a

Metal	G	$R/ {A}$	AN	AM/g mol ⁻¹	P	EN	IE/ eV	$rac{\Delta_{ m fus} H/J}{ m g}^{-1}$	$ ho/\mathrm{g}$ cm ⁻³
Fe	8	2.66	26	55.85	4	1.83	7.90	247.3	7 . 87
Co	9	2.62	27	58.93	4	1.88	7 . 88	272.5	8.86
Ni	10	2.60	28	58.69	4	1.91	7.64	290.3	8.90
Cu	11	2.67	29	63.55	4	1.90	7.73	203.5	8.96
Ru	8	2.79	44	101.07	5	2.20	7.36	381.8	12.10
Rh	9	2.81	45	102.91	5	2.28	7.46	258.4	12.40
Pd	10	2.87	46	106.42	5	2.20	8.34	157.3	12.00
Ag	11	3.01	47	107.87	5	1.93	7.58	104.6	10.50
Ir	9	2.84	77	192.22	6	2.20	8.97	213.9	22.50
Pt	10	2.90	78	195.08	6	2.20	8.96	113.6	21.50
Au	11	3.00	79	196.97	6	2.40	9.23	64.6	19.30

^a Group (G), bulk Wigner–Seitz radius (R) in Å, atomic number (AN), atomic mass (AM) in g mol⁻¹, period (P) electronegativity (EN), ionization energy (IE) in eV, enthalpy of fusion ($\Delta_{\rm fus}H$) in J g⁻¹, density at 25 °C (ρ) in g cm⁻³.



模型预测

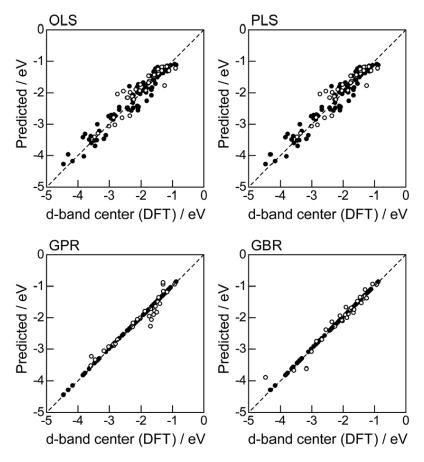


Fig. 1 DFT calculated local d-band center for metals and 1% guest metal-doped metals (Table 1) and the values predicted by linear (OLS, PLS) and nonlinear regression (GPR, GBR): (\bullet) training set = 75%, (\bigcirc) test set = 25%.

训练集: 75%

测试集: 25%

模型方法:

OLS: 普通最小二乘回归

PLS:偏最小二乘回归

GPR: 高斯过程回归

GBR: 梯度提升回归

Ichigaku. T, Satoru. T, et al. RSC Adv., 2016, 6, 52587.

主要步骤

- 1. 导入待回归数据(data_impurities.csv)
 - 1.1. 导入数据
- 1.2. 将d带中心数据从相对值改为绝对值
- 2. 导入描述符数据(data_impurities_features.csv)
- 3. 设置输入输出
- 4. 线性回归
 - 4.1. 回归寻找规律
- 4.2. 可视化
- 4.3. 交叉验证

- 5. 梯度提升回归
 - 5.1. 回归寻找规律
- 5.2. 可视化
- 5.3. 交叉验证

6. 查看数据规律(可选)

导入数据

```
[1]:
        import numpy as np
        import pandas as pd
   [2]: f = open('data impurities.csv', encoding = 'UTF-8')
        df = pd. read_csv(f, index_col = 0)
Out[2]:
               Fe
                          Ni
                    Co
                               Cu
                                    Ru
                                          Rh
                                               Pd
                                                     Αg
                                                                Pt Au
         Fe -0.92 0.05 -0.20 -0.13 -0.29 -0.54 -1.24 -0.83 -0.36 -1.09 -1.42
             0.01 -1.17 -0.28 -0.16 -0.24 -0.58 -1.37 -0.91 -0.36 -1.19 -1.56
                  0.19 -1.29 0.19 -0.14 -0.31 -0.97 -0.53 -0.14 -0.80 -1.13
                        0.27 -2.67 0.58 0.32 -0.64 -0.70 0.58 -0.33 -1.09
             0.56
                   0.60
                        0.01  0.12  -1.41  -0.17  -0.82  -0.27  0.02  -0.62  -0.84
                  0.26
                   0.34
                        0.16
                             0.44 0.04 -1.73 -0.54 0.07 0.17 -0.35 -0.49
             0.37
                   0.54
                        0.50
                             0.94
                                   0.24 0.36 -1.83
                                                   0.59
                                                         0.53
                                                              0.19
                                                                   0.17
                        0.67
                             0.84
                  0.27
                        0.05
                             0.21
                                   0.09 -0.15 -0.73 -0.13 -2.11 -0.56 -0.74
```

0.72 0.14 0.23 -0.17 0.44

0.55 0.70 0.75 0.17 0.21 0.98 0.46 -3.56

0.38 -2.25 -0.05

0.33

0.63

0.48

0.40

0.63

```
In [3]: df. loc['Co', 'Ni']
Out[3]: -0.28
In [4]: df. loc['Co']
Out[4]: Fe 0.01
        Co -1. 17
        Ni -0.28
        Cu -0.16
         Ru -0.24
        Rh -0.58
         Pd -1.37
         Ag -0.91
        Ir -0.36
        Pt -1.19
         Au -1.56
        Name: Co, dtype: float64
In [5]: for i in df. index:
            for j in df. columns:
                if(i == 'Fe' and j == 'Fe'):
                    print('host ' + i + ', guest ' + j + ', val ' + str(df.loc[i, j]))
        host Fe, guest Fe, val -0.92
```

转换数据

Out[7]:

	Fe	Co	Ni	Cu	Ru	Rh	Pd	Ag	lr	Pt	Au
Fe	-0.92	-0.87	-1.12	-1.05	-1.21	-1.46	-2.16	-1.75	-1.28	-2.01	-2.34
Со	-1.16	-1.17	-1.45	-1.33	-1.41	-1.75	-2.54	-2.08	-1.53	-2.36	-2.73
Ni	-1.20	-1.10	-1.29	-1.10	-1.43	-1.60	-2.26	-1.82	-1.43	-2.09	-2.42
Cu	-2.11	-2.07	-2.40	-2.67	-2.09	-2.35	-3.31	-3.37	-2.09	-3.00	-3.76
Ru	-1.20	-1.15	-1.40	-1.29	-1.41	-1.58	-2.23	-1.68	-1.39	-2.03	-2.25
Rh	-1.49	-1.39	-1.57	-1.29	-1.69	-1.73	-2.27	-1.66	-1.56	-2.08	-2.22
Pd	-1.46	-1.29	-1.33	-0.89	-1.59	-1.47	-1.83	-1.24	-1.30	-1.64	-1.66
Ag	-3.58	-3.46	-3.63	-3.83	-3.46	-3.44	-4.16	-4.30	-3.16	-3.80	-4.45
lr	-1.90	-1.84	-2.06	-1.90	-2.02	-2.26	-2.84	-2.24	-2.11	-2.67	-2.85
Pt	-1.92	-1.77	-1.85	-1.53	-2.11	-2.02	-2.42	-1.81	-1.87	-2.25	-2.30
Au	-2.93	-2.79	-2.93	-3.01	-2.86	-2.81	-3.39	-3.35	-2.58	-3.10	-3.56

添加描述符

```
In [9]: f = open('data_impurities_features.csv', encoding = 'UTF-8')
    feat = pd.read_csv(f, index_col = 0)
    feat
```

Out[9]:

name	group	bulk wigner-seitz radius	atomic number	atomic mass	period	electronegativity	lionization energy(eV)	H_fus	density
Iron	8	2.66	26	55.84500	4	1.83	7.9024	247.3	7.87
Cobalt	9	2.62	27	58.93320	4	1.88	7.8810	272.5	8.86
Nickel	10	2.60	28	58.69340	4	1.91	7.6398	290.3	8.90
Copper	11	2.67	29	63.54600	4	1.90	7.7264	203.5	8.96
thenium	8	2.79	44	101.07000	5	2.20	7.3605	381.8	12.10
Rhodium	9	2.81	45	102.90550	5	2.28	7.4589	258.4	12.40
alladium	10	2.87	46	106.42000	5	2.20	8.3369	157.3	12.00
Silver	11	3.01	47	107.86820	5	1.93	7.5762	104.6	10.50
Iridium	9	2.84	77	192.21700	6	2.20	8.9670	213.9	22.50
Platinum	10	2.90	78	195.07800	6	2.20	8.9588	113.6	21.50
Gold	11	3.00	79	196.96655	6	2.40	9.2255	64.6	19.30
th Rh	Iron Cobalt Nickel Copper nenium nodium Iladium Silver Iridium	Iron 8 Cobalt 9 Nickel 10 Copper 11 nenium 8 nodium 9 Iladium 10 Silver 11 Iridium 9 Iatinum 10	Iron 8 2.66 Cobalt 9 2.62 Nickel 10 2.60 Copper 11 2.67 nenium 8 2.79 nodium 9 2.81 Iladium 10 2.87 Silver 11 3.01 Iridium 9 2.84 Iatinum 10 2.90	Iron 8 2.66 26 Cobalt 9 2.62 27 Nickel 10 2.60 28 Copper 11 2.67 29 nenium 8 2.79 44 nodium 9 2.81 45 Iladium 10 2.87 46 Silver 11 3.01 47 Iridium 9 2.84 77 Iatinum 10 2.90 78	Iron 8 2.66 26 55.84500 Cobalt 9 2.62 27 58.93320 Nickel 10 2.60 28 58.69340 Copper 11 2.67 29 63.54600 nenium 8 2.79 44 101.07000 nodium 9 2.81 45 102.90550 Iladium 10 2.87 46 106.42000 Silver 11 3.01 47 107.86820 Iridium 9 2.84 77 192.21700 atinum 10 2.90 78 195.07800	Iron 8 2.66 26 55.84500 4 Cobalt 9 2.62 27 58.93320 4 Nickel 10 2.60 28 58.69340 4 Copper 11 2.67 29 63.54600 4 nenium 8 2.79 44 101.07000 5 nodium 9 2.81 45 102.90550 5 Iladium 10 2.87 46 106.42000 5 Silver 11 3.01 47 107.86820 5 Iridium 9 2.84 77 192.21700 6 Iatinum 10 2.90 78 195.07800 6	Iron 8 2.66 26 55.84500 4 1.83 Cobalt 9 2.62 27 58.93320 4 1.88 Nickel 10 2.60 28 58.69340 4 1.91 Copper 11 2.67 29 63.54600 4 1.90 nenium 8 2.79 44 101.07000 5 2.20 nodium 9 2.81 45 102.90550 5 2.28 Iladium 10 2.87 46 106.42000 5 2.20 Silver 11 3.01 47 107.86820 5 1.93 Iridium 9 2.84 77 192.21700 6 2.20 atinum 10 2.90 78 195.07800 6 2.20	Iron 8 2.66 26 55.84500 4 1.83 7.9024 Cobalt 9 2.62 27 58.93320 4 1.88 7.8810 Nickel 10 2.60 28 58.69340 4 1.91 7.6398 Copper 11 2.67 29 63.54600 4 1.90 7.7264 nenium 8 2.79 44 101.07000 5 2.20 7.3605 nodium 9 2.81 45 102.90550 5 2.28 7.4589 Iladium 10 2.87 46 106.42000 5 2.20 8.3369 Silver 11 3.01 47 107.86820 5 1.93 7.5762 Iridium 9 2.84 77 192.21700 6 2.20 8.9588 atinum 10 2.90 78 195.07800 6 2.20 8.9588	Iron 8 2.66 26 55.84500 4 1.83 7.9024 247.3 Cobalt 9 2.62 27 58.93320 4 1.88 7.8810 272.5 Nickel 10 2.60 28 58.69340 4 1.91 7.6398 290.3 Copper 11 2.67 29 63.54600 4 1.90 7.7264 203.5 nenium 8 2.79 44 101.07000 5 2.20 7.3605 381.8 nodium 9 2.81 45 102.90550 5 2.28 7.4589 258.4 Iladium 10 2.87 46 106.42000 5 2.20 8.3369 157.3 Silver 11 3.01 47 107.86820 5 1.93 7.5762 104.6 Iridium 9 2.84 77 192.21700 6 2.20 8.9670 213.9 atinum 10 2.90

查看描述符

In [10]:	feat	. head (4)									
Out[10]:		name	group	bulk wigner-seitz radius	atomic number	atomic mass	period	electronegativity	lionization energy(eV)	H_fus	density	
	Fe	Iron	8	2.66	26	55.8450	4	1.83	7.9024	247.3	7.87	
	Со	Cobalt	9	2.62	27	58.9332	4	1.88	7.8810	272.5	8.86	
	Ni	Nickel	10	2.60	28	58.6934	4	1.91	7.6398	290.3	8.90	
	Cu	Copper	11	2.67	29	63.5460	4	1.90	7.7264	203.5	8.96	
In [11]:	feat	. loc['C	o']									
Out[11]:	name group bulk wigner-seitz radius atomic number atomic mass period electronegativity Iionization energy(eV) H_fus density Name: Co, dtype: object			27 58. 9332 4 1. 88 y (eV) 7. 881 272. 5 8. 86								

设置输入输出

```
In [14]: x = 1ist()
         y = list()
         for i in df. index:
             for j in df. columns:
                vec_i = feat.loc[i].to_numpy()
                vec_j = feat.loc[j].to_numpy()
                x_val = np. concatenate((vec_i, vec_j))
                y_val = df. loc[i][j]
                x. append (x_val)
                y. append (y_va1)
                if i = 'Fe' and j = 'Co':
                    print('host' + i + ', guest' + j + ', input = ' + str(x_val) + ', output = ' + str(y_val))
         host Fe, guest Co, input = [ 8. 2.66 26.
                                                            55.845
                                                                             1.83
                                                                                     7. 9024 247. 3
                                                                    4.
           7. 87 9. 2. 62 27.
                                           58. 9332 4. 1. 88 7. 881
          272. 5 8. 86 ], output = -0.87
```

随机

```
In [18]: from sklearn.utils import shuffle
    X_r, y_r = shuffle(X, y)
    X_train, y_train = X_r[:-30, :], y_r[:-30]
    X_test, y_test = X_r[-30:, :], y_r[-30:]
```

```
In [19]: from sklearn.linear_model import LinearRegression
lr = LinearRegression()
lr.fit(X_train, y_train)
y_pred_train_lr = lr.predict(X_train)
y_pred_test_lr = lr.predict(X_test)
```

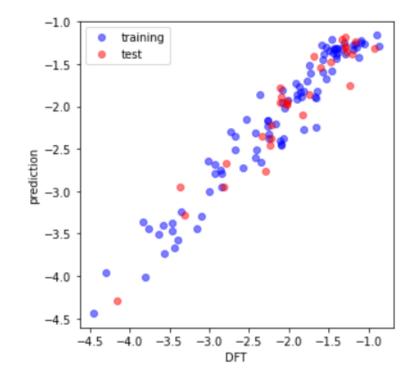
shuffle() 打乱函数

线性回归

```
In [21]: import matplotlib.pyplot as plt

plt.figure(figsize = (5, 5))
plt.scatter(y_train, y_pred_train_lr, alpha=0.5, color = 'blue', label = 'training')
plt.scatter(y_test, y_pred_test_lr, alpha=0.5, color = 'red', label = 'test')
plt.legend()
plt.xlabel('DFT')
plt.ylabel('prediction')
```

Out[21]: Text(0, 0.5, 'prediction')



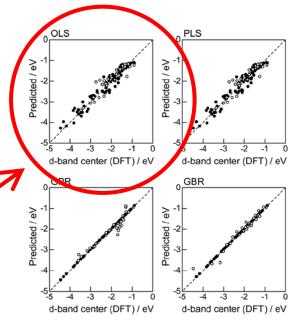
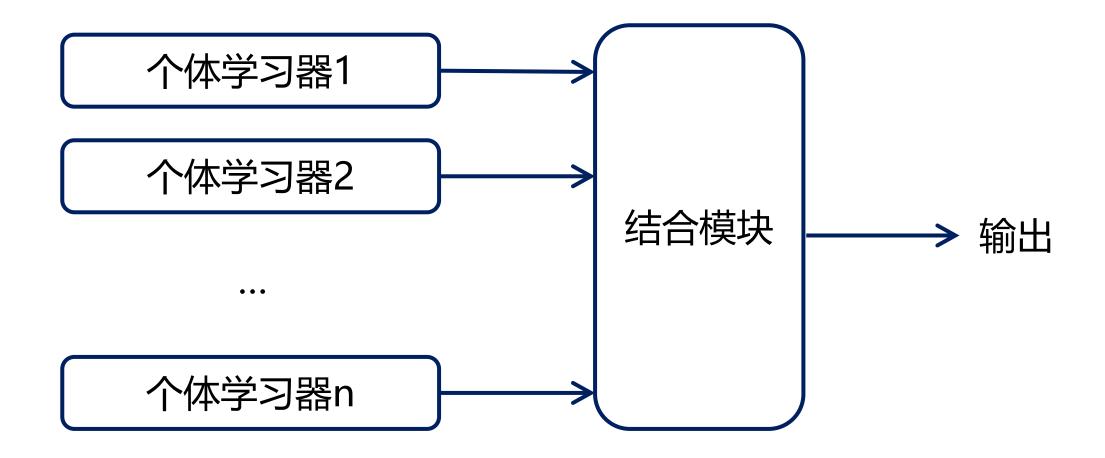


Fig. 1 DFT calculated local d-band center for metals and 1% guest metal-doped metals (Table 1) and the values predicted by linear (OLS, PLS) and nonlinear regression (GPR, GBR): (\bigcirc) training set = 75%, (\bigcirc) test set = 25%.

交叉验证

```
In [22]: from sklearn.metrics import mean_squared_error
          rmse tr 1r = mean squared error(y train, y pred train 1r, squared=Fa1se)
           rmse_te_lr = mean_squared_error(y_test, y_pred_test_lr, squared=False)
           print('RMSE(training)%.3f' % rmse tr lr)
           print('RMSE(test)%.3f' % rmse te 1r)
           RMSE (training) 0.214
           RMSE(test) 0. 213
In [23]: from sklearn.model_selection import cross_val_score, ShuffleSplit
          cvf = ShuffleSplit(n_splits=100, test_size=0.25)
          r2_lr = cross_val_score(lr, X, y, cv=cvf)
          scores_lr = cross_val_score(lr, X, y, cv=cvf, scoring = 'neg_root_mean_squared_error')
          print('100 times mean r2: %.3f' % r2_lr.mean())
          print('100 times mean RMSE: %.3f' % -scores_1r.mean())
          100 times mean r2: 0.897
          100 times mean RMSE: 0.250
```

集成学习



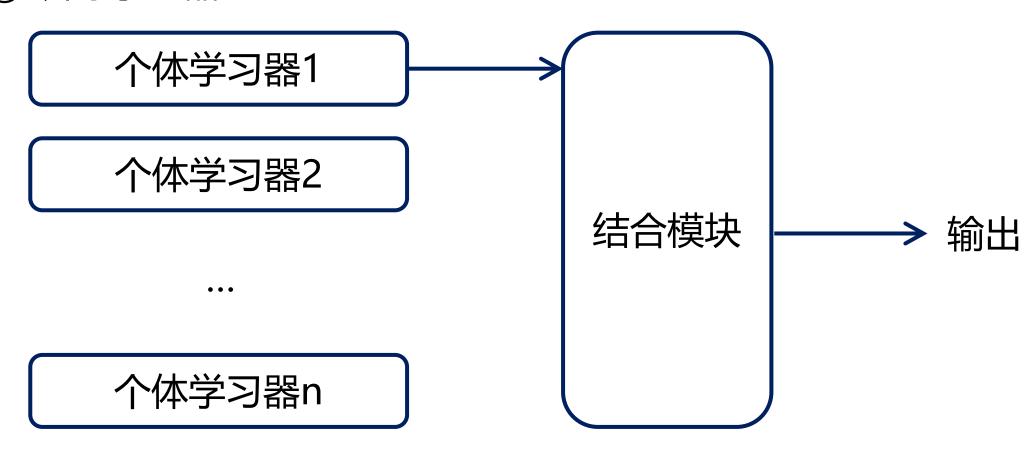
集成学习

考虑二分类任务

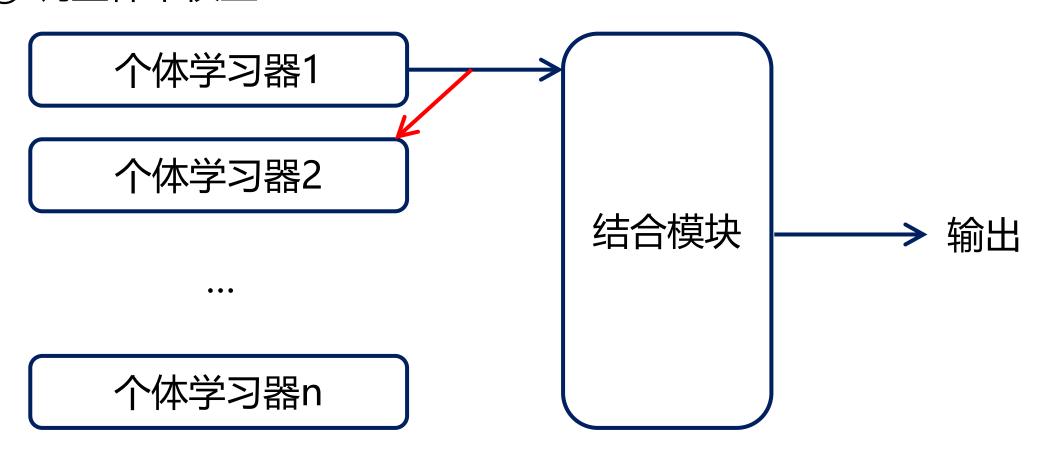
	测试例1	测试例2	测试例3
h1	\checkmark	\checkmark	×
h2	\checkmark	×	\checkmark
h3	×	\checkmark	\checkmark
集成	√	√	√
	测试例1	测试例2	测试例3
h1	√	√	×
h2	\checkmark	\checkmark	×
h3	\checkmark	\checkmark	×
集成	√	√	×
	测试例1	测试例2	测试例3
h1	√	×	×
h2	×	\checkmark	×
h3	×	×	\checkmark
集成	×	×	×

集成个体应"好而不同"

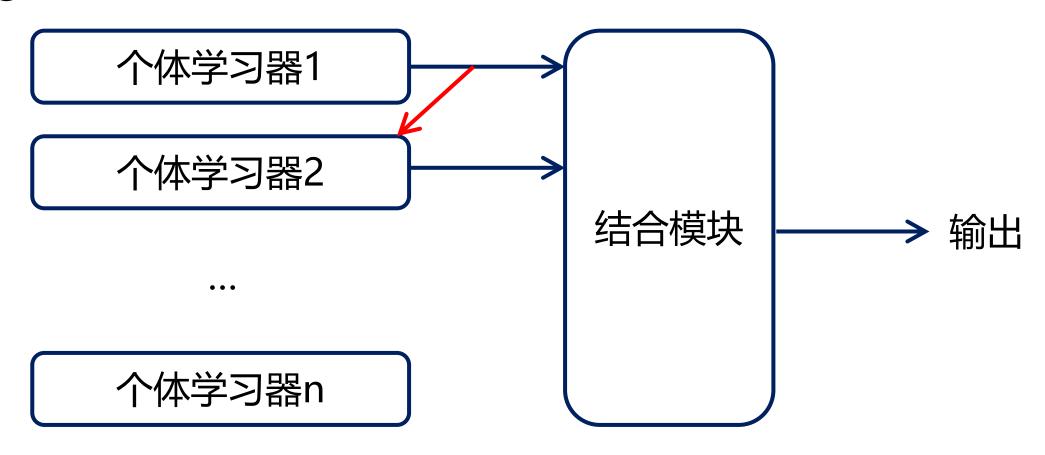
① 训练学习器1



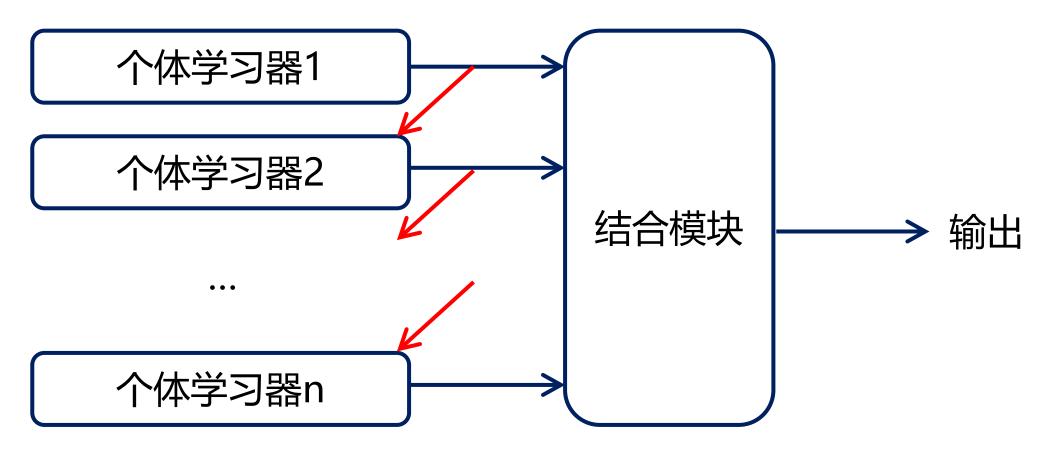
②调整样本权重



③ 训练学习器2



得到完整模型



梯度提升回归

-4.5

-4.0 -3.5 -3.0

DFT

```
In [29]: from sklearn.ensemble import GradientBoostingRegressor
          gbr = GradientBoostingRegressor()
          gbr. fit(X_train, y_train)
          y_pred_train_gbr = gbr.predict(X_train)
          y_pred_test_gbr = gbr.predict(X_test)
In [30]: plt. figure (figsize = (5,5))
          plt.scatter(y_train, y_pred_train_gbr, alpha=0.5, color = 'blue', label = 'training')
          plt.scatter(y_test, y_pred_test_gbr, alpha=0.5, color = 'red', label = 'test')
          plt.legend()
          plt.xlabel('DFT')
          plt.ylabel('prediction')
Out[30]: Text(0, 0.5, 'prediction')
                       training
              -1.0
                       test
              -1.5
              -2.0
           -2.5
-3.0
              -3.5
              -4.0
```

-2.5 -2.0 -1.5 -1.0

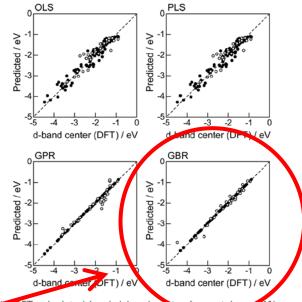


Fig. 1 DFT calculated local d-band center for metals and 1% guest metal-doped metals (Table 1) and the values predicted by linear (OLS, PLS) and nonlinear regression (GPR, GBR): (\bigcirc) training set = 75%, (\bigcirc) test set = 25%.

Ichigaku. T, Satoru. T, et al. RSC Adv., 2016, 6, 52587.

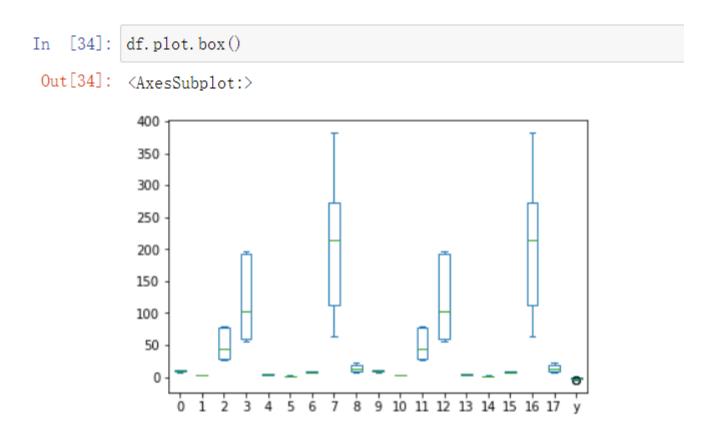
交叉验证

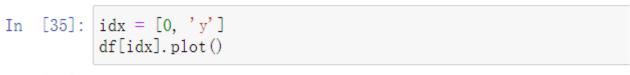
```
In [31]: rmse_tr_gbr = mean_squared_error(y_train, y_pred_train_gbr, squared=False)
    rmse_te_gbr = mean_squared_error(y_test, y_pred_test_gbr, squared=False)
    print('RMSE(training)%.3f'%rmse_tr_gbr)
    print('RMSE(test)%.3f'%rmse_te_gbr)

RMSE(training)0.032
    RMSE(test)0.126

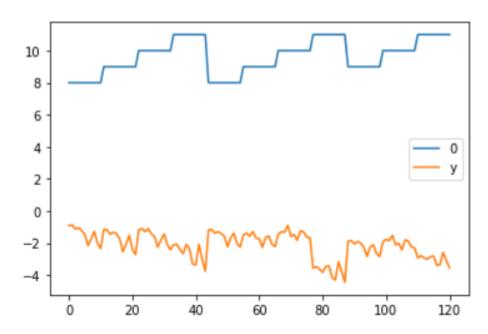
In [32]: r2_gbr = cross_val_score(gbr, X, y, cv=cvf, scoring = 'r2')
    scores_gbr = cross_val_score(gbr, X, y, cv=cvf, scoring = 'neg_root_mean_squared_error')
    print('100 times mean r2: %.3f' % r2_gbr.mean())
    print('100 times mean RMSE: %.3f' % -scores_gbr.mean())

100 times mean r2: 0.965
    100 times mean RMSE: 0.152
```





Out[35]: <AxesSubplot:>

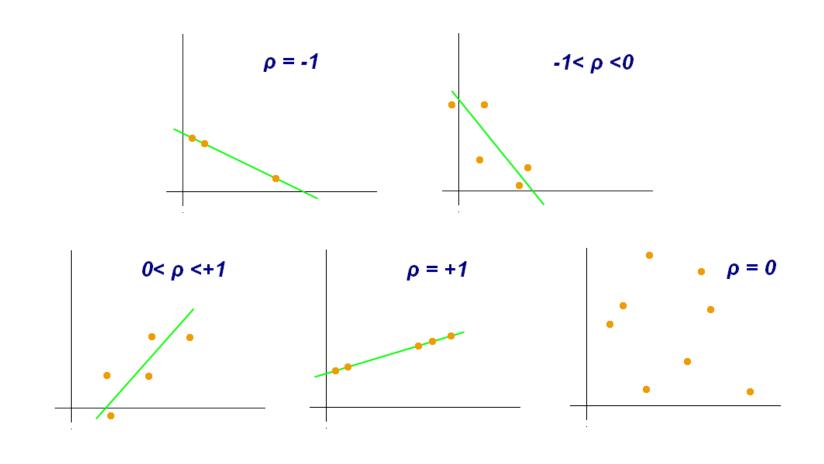


```
[36]: df. plot. scatter(x = 0, y = 'y')
Out[36]: <AxesSubplot:xlabel='0', ylabel='y'>
              -1.0
              -1.5
              -2.0
              -2.5
              -3.0
              -3.5
              -4.0
              -4.5
                                                   10.0
                                           9.5
0
                                                          10.5
                    8.0
                            8.5
                                   9.0
                                                                  11.0
```

```
[39]: df. corr()['y']
Out[39]: 0
              -0.628485
              -0.528149
              -0.291100
              -0.286463
              -0.262104
              -0.024741
                                         In [40]: import matplotlib.pyplot as plt
                                                 import seaborn as sns
              -0.152404
                                                 plt. figure (figsize=(15, 1))
              0.564631
                                                 sns. heatmap(df. corr()['y']. to_frame().T, cmap='RdY1Gn', annot=True)
              -0.169864
                                                 plt. show()
         9
              -0.236039
              -0.347411
                                                  -0.264994
         11
              -0.260405
         12
                                                                                        10
                                                                                            11
                                                                                               12
                                                                                                   13
         13
              -0.274515
         14
             -0.279717
         15
              -0.240431
              0.329626
         16
             -0.196517
         17
              1.000000
         Name: y, dtype: float64
```

Pearson相关系数

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}$$



In [44]: plt.figure(figsize=(20, 20))

sns. heatmap(df. corr(), cmap='RdY1Gn', annot=True)

Out[44]: <AxesSubplot:>



```
In [46]: import plotly.express as px
fig = px.scatter_3d(df, x=0, y=17, z='y', color = y)
fig.show()
```

