Calculation with decibels

Which parameters are expressed in following units?

dBi	…	antenna gain
dBd	…	antenna gain
dBm	…	power
dBW	…	power
dBμV	…	voltage
$dB\mu V.m^{-1}$	…	intensity of electric field

Remember:

0 dBd = 2.15 dBi   0 dBi = -2.15 dBd

$X_{dB} = 10.\log X$ for power functions (P, S, gain) $X = 10^{X_{dB}/10}$

$X_{dB} = 20.\log X$ for voltage functions (U, I, E, H, AF) $X = 10^{X_{dB}/20}$

Determine:

10 dBi	=	7.85 dBd
7 dBd	=	9.15 dB
0 dBm	=	0.001 W
3 dBm	=	2 mW
10 dBm	=	0.01 W
16 dBW	=	40 W
-3 dBW	=	0.5 W
-4 dBW	=	26 dBm
40 dBμV	=	0.1 mV
$-6\ dB\mu V.m^{-1}$	=	$0.5\ \mu V.m^{-1}$
33 dBi	=	2000 [-]

Calculation of electromagnetic field, EIRP and ERP

Example: Transmitting antenna is fed by a power P_v = 10 W, antenna's gain is G_v = 6 dBi. Determine effective value of intensity of electric field E_{10m} and E_{100m}, intensity of magnetic field H_{10m} and H_{100m}, and power density S_{10m} and S_{100m}. Frequency is f = 3 GHz and an impedance is equal to the impedance of free space. Calculate Effective Isotropic Radiated Power (EIRP) and Effective Radiated Power (ERP) and of transmitter.

$$S_{10m} = \frac{P_V G_V}{4\pi R^2} = \frac{10 \cdot 10^{\frac{6}{10}}}{4\pi 10^2} = ... = 31.67 mW \cdot m^{-2}$$

$$S_{100m} = \frac{P_V G_V}{4\pi R^2} = ... = 0.3167 mW \cdot m^{-2}$$

$$E_{10m} = \frac{\sqrt{30 P_V G_V}}{R} = \frac{\sqrt{30 \cdot 10 \cdot 10^{\frac{6}{10}}}}{10} = ... = 3.46 V \cdot m^{-1}$$

$$H_{10m} = \frac{E_{10m}}{Z_0} = \frac{3.46}{377} = ... = 9.18.10^{-3} A \cdot m^{-1}$$

where $Z_0 = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon + \sigma}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi.10^{-7}}{8{,}85.10^{-12}}} = 120\pi = 377\Omega$

$$S_{10m} = E_{10m} \cdot H_{10m} = \frac{E_{10m}^{\;2}}{Z_0} \Rightarrow E_{10m} = \sqrt{S_{10m} \cdot Z_0} = \sqrt{0.03167 \cdot 377} = ... = 3.46 V \cdot m^{-1}$$

$$E_{100m} = \frac{\sqrt{30 P_V G_V}}{R} = ... = 0.346 V \cdot m^{-1} \quad H_{100m} = \frac{E_{100m}}{Z_0} = \frac{0.346}{377} = ... = 9.18 \cdot 10^{-4} A \cdot m^{-1}$$

$$EIRP = P_V G_V = 10 \cdot 10^{\frac{6}{10}} = ... = 40W = 16dBW$$

$$ERP = EIRP - 2{,}15dB = 13{,}85dBdW = 24{,}3W$$

Example: Determine a distance where magnitude (effective value) of intensity of electric field radiated by dipole antenna (gain G_v = 0 dBd) is equal to 1V.m^{-1}. Antenna is fed with a power 1 kW.

$$E_{\max} = \frac{\sqrt{60 P_V G_V}}{R} \Rightarrow R = \frac{\sqrt{60 P_V G_V}}{E_{\max}} = \frac{\sqrt{60.1000.10^{\frac{2,15}{10}}}}{1} = ... = 314m$$

$$(E_{ef} = \frac{\sqrt{30 P_V G_V}}{R} \Rightarrow R = \frac{\sqrt{30 P_V G_V}}{E_{ef}} = \frac{\sqrt{30.1000.10^{\frac{2,15}{10}}}}{1} = ... = 222m)$$

Impedance and matching

Dipole antenna is fed from generator with impedance $Z_g = 50\ \Omega$. We require antenna matching better than 20 dB (RL>=20dB). (reflection coefficient |s11|<-20dB).

a) Determine the interval of real antenna impedances Z_a for RL better than 20 dB. (Use real impedances only – for simplification.)

b) Determine VSWR for RL=20dB?

c) Determine power which is transmitted to the antenna.

d) Determine reflected power.

For RL = 20dB ………. Reflecting coefficient is $|\Gamma| = 0.1$

$$|\Gamma| = 10^{\frac{RL}{-20}} = 0.1$$

a)

$$|\Gamma| = \left|\frac{Za - Zo}{Za + Zo}\right|$$

$$0{,}1 = \left|\frac{Za - 50}{Za + 50}\right|$$

$$|Za + 50| = 10|Za - 50|$$

$$Za > 50 \Rightarrow Za + 50 = 10(Za - 50) \Rightarrow Za = 61\Omega$$

$$Za < 50 \Rightarrow Za + 50 = 10(50 - Za) \Rightarrow Za = 40.9\Omega$$

$$Za \in \langle 40.9;61 \rangle \Omega$$

b)

$$PSV = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.1}{1-0.1} = 1.2$$

c)

$1-|\Gamma|^2 = 1 - 0.01 = 0.99$ …….. 99% of power is transmitted to the antenna

d)

$|\Gamma|^2 = 0.01$……..1% of power is reflected

Friis transmission equation

Example: Transmission link on frequency 10 GHz consists of 2 parabolic antennas.

a) Determine power received by receiving antenna.
b) Determine effective value of voltage in receiving antenna connector with impedance 50 Ohm.
c) Determine effective aperture of receiving antenna.

P_g = 10 W	power of generator
Z_g = 50 Ohm	impedance of generator
Z_v = 70 Ohm	impedance of transmitting antenna
D_v = 38 dBi	directivity of transmitting antenna (diameter 1 m)
R = 5 km	range of link
a = 0,5 dB.km^{-1}	additional atmospheric loss caused by hydrometeors (rain, snow, hail …)
α = 10°	polarisation imperfection
D_p = 33 dBi	directivity of receiving antenna (diameter 0.6 m)
RL_p = 20 dB	return loss of receiving antenna
Z_p = 50 Ohm	impedance of receiver
L_{dv} = 5 %	dielectrical and conducting loss of each antenna

a)
$$P_p = \frac{P_g.G_v.G_p}{L} = \frac{P_g.(1-|\Gamma_v|^2).D_v.(1-L_{dv}).(1-|\Gamma_p|^2).D_p.(1-L_{dv})}{L_{FLS}.L_{pol}.L_a}$$

$$|\Gamma_v| = \frac{Z_v - Z_g}{Z_v + Z_g} = \frac{70-50}{70+50} = \frac{20}{120} = 0.167 \Rightarrow \left(1-|\Gamma_v|^2\right) = 0.972$$

$$|\Gamma_p| = 10^{\frac{RL_p}{-20}} = 10^{\frac{20}{-20}} = 10^{-1} = 0{,}1 \Rightarrow \left(1-|\Gamma_p|^2\right) = 0.99$$

$$P_p = \frac{P_v.G_v.G_p}{L} = \frac{P_g.(1-|\Gamma_v|^2).D_v.(1-0.05).(1-|\Gamma_p|^2).D_p.(1-0.05)}{L_{FLS}.L_{pol}.L_a}$$

$$P_p = \frac{10\cdot 0.972\cdot 6310\cdot 0.95\cdot 0.99\cdot 1995\cdot 0.95}{4.3865.10^{12}\cdot 1.031\cdot 1.778} = 1.36\cdot 10^{-5} W$$

$$D_v = 10^{\frac{38}{10}} = 6310 \qquad D_p = 10^{\frac{33}{10}} = 1995$$

$$L_{FSP} = \left(\frac{4\pi R}{\lambda}\right)^2 = \left(\frac{4.\pi.5000}{\frac{3.10^8}{10.10^9}}\right)^2 = 4.3865.10^{12} = 126.42 dB$$

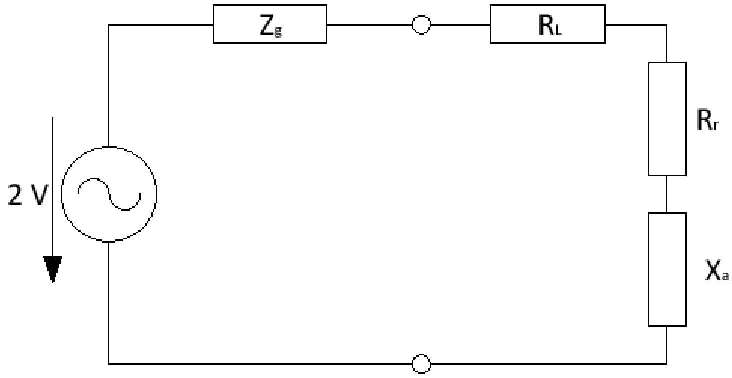
$$L_{pol} = \frac{1}{\cos^2\alpha} = \frac{1}{\cos^2 10^\circ} = 1.031 \qquad L_a = R.a = 5\cdot 0.5 = 2.5 dB = 10^{\frac{2.5}{10}} = 1.778$$

b)
$$P_p = \frac{U_{ef}^2}{Z_p} \Rightarrow U_{ef} = \sqrt{P_p.Z_p} = \sqrt{1.36\cdot 10^{-5}.50} = 0.026V$$

c)
$$A_{ef} = G_p\frac{\lambda^2}{4\pi} = D_p.\left(1-|\Gamma_p|^2\right)(1-L_{dv})\frac{\lambda^2}{4\pi} = 1995\cdot 0.99\cdot 0.95\frac{0{,}03^2}{4\pi} = 0.134 m^2$$

Equivalent circuit of antenna

A dipole antenna is connected to generator with inner impedance Zg=50+j25 Ω. The generator output voltage amplitude is 2V. The dipole antenna has loss resistance of 1 Ω plus its own impedance 73+j42.5 Ω.
Determine power delivered to the antenna, radiated and loss power and radiation efficiency.



$$I_g = \frac{U_g}{Z_g + R_L + Z_r} = \frac{2}{50 + j25 + 1 + 73 + j42.5} = \frac{2}{124 + j67{,}5} = (12.442 - j6.7724)\cdot 10^{-3}\,A$$

$$= 14.166.e^{-j28.56^\circ}\,mA$$

$$P_S = \frac{1}{2}\mathrm{Re}\{U_g . I_g^*\} = \frac{1}{2}\mathrm{Re}\{2\cdot(12.442 + j6.7724)\cdot 10^{-3}\} = 12.442\cdot 10^{-3}\,W$$

$$P_r = \frac{1}{2}|I_g|^2 R_r = \frac{1}{2}\cdot 0.014166^2 \cdot 73 = 7.325\cdot 10^{-3}\,W$$

$$P_L = \frac{1}{2}|I_g|^2 R_L = \frac{1}{2}\cdot 0.014166^2 \cdot 1 = 0.1003\cdot 10^{-3}\,W$$

$$\eta_r = \frac{R_r}{R_r + R_L} = \frac{73}{73+1} = 0.986$$