Superdirective Dipole Arrays

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***Abstract*—In this paper we develop and utilize theory for finding excitation currents that maximize the directivity of arbitrary dipole arrays. Such excitation is known as superdirective. It is shown that for arrays above perfectly electric ground, the corresponding optimal currents are purely real. It is also observed that for horizontally oriented dipoles, there exist optimum for spacing and height producing maximum obtainable directivity.**

***Index Terms*—Antenna arrays, Superdirectivity, Dipole antennas, Optimal currents.**

I. Introduction

It is well known that the array can show an increase in directivity (termed superdirectivity) [1]–[6] when the feeding currents are designed to be optimal in this respect.

In this paper, we treat arrays of thin-wire dipoles and assume a current distribution on the dipoles to be of a given form (three-term King approximation [7]) and, therefore, contrary to the fully populated solution, the required matrices of interest have dimension $N \times N$ where N is number of elements of the array. Therefore, the proposed approach (coded in MATLAB [8]) is very fast. The quadratic form of the excitation currents allows finding the optimal currents by solving the associated generalized eigenvalue problem.

First, a five-element circular array is studied. Then we focus on two and three horizontal $\lambda/2$ dipole elements backed by an electric ground plane. It is known [9] that one horizontal dipole above ground show maximum directivity when its height goes to zero. However, for two and three horizontal dipoles, it is shown that the directivity reaches its maxima for unusual height around 0.7λ. This observation is also supported by analysis of two isotropic radiators above the ground plane.

The results are validated by a full-wave simulator FEKO [10].

II. Theory

A directivity of a radiating source in an angular direction (θ, ϕ) is defined as [7]

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0} = 4\pi\frac{U(\theta,\phi)}{P_\mathrm{r}} \tag{1}$$

where U is radiation intensity in the direction (θ, ϕ), $U_0 = P_\mathrm{r}/4\pi$ is average radiation intensity and P_r is radiated power. In the case of antenna arrays, it is possible to extract the vector of feeding currents and write the directivity in a compact matrix form as [11]–[13]

$$D(\theta,\phi) = 4\pi\frac{\mathbf{I}^\mathrm{H}\mathbf{u}(\theta,\phi)\mathbf{I}}{\mathbf{I}^\mathrm{H}\mathbf{p}\mathbf{I}} \tag{2}$$

where H stands for Hermitian transpose, $\mathbf{I} = [I_1 \cdots I_N]^\mathrm{T}$ is the vector of excitation currents and $\mathbf{u}$ and $\mathbf{p}$ are matrices of the normalized radiation intensities u_{mn} and powers p_{mn} respectively. Those matrices contain the self and mutual interactions in the array structure with regard to total directivity D and their entries are given as

$$u_{mn}(\theta,\phi) = \frac{15k^2}{4\pi}\int\limits_{d_m}\int\limits_{d_n}\Lambda(\mathbf{r},\mathbf{r}')\mathrm{e}^{\mathrm{j}k(\Delta-\Delta')}\,\mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{r}' \tag{3}$$

where $k = 2\pi/\lambda$ is a wavenumber, integration is performed over the dipoles and

$$\begin{aligned}\Delta-\Delta' = \mathbf{r_0}\cdot(\mathbf{r}-\mathbf{r}') =& (x-x')\sin\theta\cos\phi \\ &+ (y-y')\sin\theta\sin\phi \\ &+ (z-z')\cos\theta,\end{aligned} \tag{4}$$

$$\Lambda(\mathbf{r},\mathbf{r}') = \mathbf{j}_m(\mathbf{r})\cdot\mathbf{j}_n^*(\mathbf{r}') - \mathbf{r}_0\cdot\mathbf{j}_m(\mathbf{r})\mathbf{r}_0\cdot\mathbf{j}_n^*(\mathbf{r}'). \tag{5}$$

In the above, $\mathbf{j}_m$ and $\mathbf{j}_n$ are normalized current densities (as obtained by the King's 3-term solution) in cartesian coordinates and $\mathbf{r}_0$ is a unit vector in spherical coordinates. The normalized radiated power is obtained by integrating the radiation intensity over the complete solid angle

$$p_{mn} = \int\limits_0^{2\pi}\int\limits_0^{\pi} u_{mn}(\theta,\phi)\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi. \tag{6}$$

For maximization of directivity (2) the eigenvalue equation is employed [14]

$$4\pi\mathbf{u}(\theta,\phi)\mathbf{I} = D\mathbf{p}\mathbf{I}. \tag{7}$$

The resulting feeding current vector $\mathbf{I}$ associated with the maximum eigenvalue D will be used to feed the given array.

III. Results and Discussion

A. Circular array

The circular array in this example is composed of five z - oriented dipoles uniformly spaced with distance s. The length of each the dipole is $\mathrm{L} = \lambda/2$ and radius $a = 0.005\lambda$, see Fig. 1.

The maximum directivity in direction of x-axis ($\phi = 0$) as a function of separation s is depicted in Fig. 2. For each separation, decomposition (7) is performed. The calculated

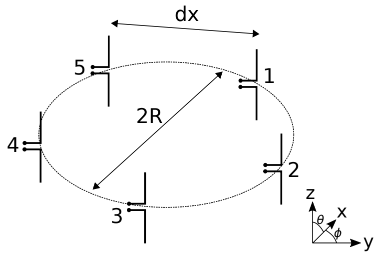


Fig. 1: A five-element circular dipole array

directivity by our code and FEKO for spacing $s = 0.1\ \lambda$ agree very well and is shown in Fig. 3.

Due to the rotational symmetry, we are able to obtain the same directivity in any ϕ direction. It is notable that for the maximization of directivity in direction $\phi = 0, \pi/2, \pi, 3\pi/2$, the amplitudes of the feeding current are exactly the same, only the phases switch.

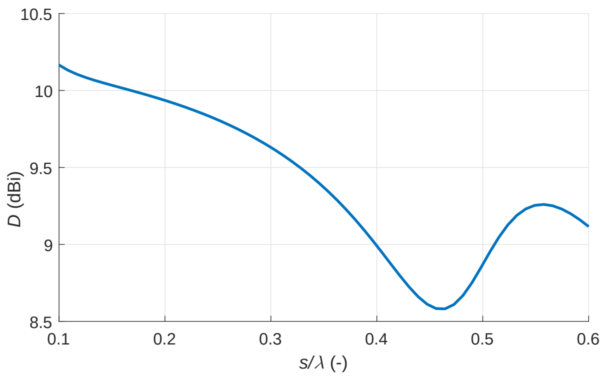
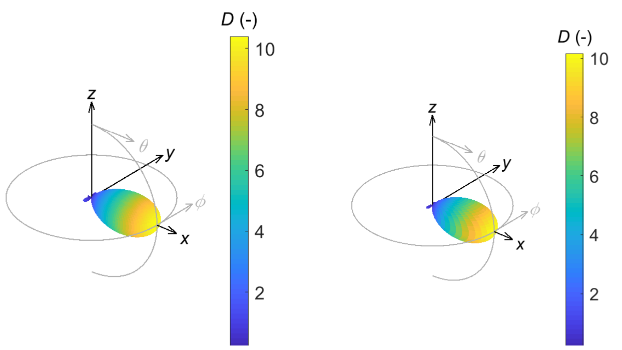


Fig. 2: Maximum directivity in x ($\phi = 0°$) axis for different distance between 5 dipoles.



(a) MATLAB $D = 10.16$ dBi (b) FEKO $D = 10.07$ dBi

Fig. 3: Radiation pattern for circular array of 5 dipoles fed by optimal currents.

This method is able to find the optimum current distribution in any direction, which is fixed in the $\mathbf{u}(\theta, \phi)$ matrix. Using

$$\mathbf{V} = \mathbf{Z} \cdot \mathbf{I}, \tag{8}$$

where $\mathbf{Z}$ is the impedance matrix of the array, corresponding voltage excitation vector $\mathbf{V}$ is obtained. These voltages were actually used as inputs for FEKO in Fig. 3.

Tab. I shows current and voltage distribution for maximization of directivity for $\phi = 0$ direction.

dipole no.	$\lvert I \rvert$ (A)	phase I (deg)	$\lvert V \rvert$	phase V (deg)
1	1	-81.07	16.42	167.8
2	0.84	90.15	17.01	-11.7
3	0.32	-104.23	13.31	-132.2
4	0.32	-104.23	13.31	-132.3
5	0.84	90.15	17.01	-11.7

TABLE I: Feed currents and voltages for maximizing directivity of circular array with $s = 0.1\lambda$

B. Dipole arrays above PEC ground

As a second example we calculate the maximum achievable broadside directivity for an array of two and three horizontal $\lambda/2$ dipoles above PEC ground. These dipoles have the same length and radius as in the previous example and are oriented in z direction. The spacing between dipoles and height above PEC ground is dx and h, respectively.

It is noted that thanks to the presence of the ground plane, the intensity matrix $\mathbf{u}$ is symmetric and therefore the superdirective currents are real.

The maximum obtainable broadside directivity for the array of two dipoles is depicted in Fig. 4. Its maximum $D = 12.82$ dBi occurs for $dx = 0.633\ \lambda$ and $h = 0.677\ \lambda$.

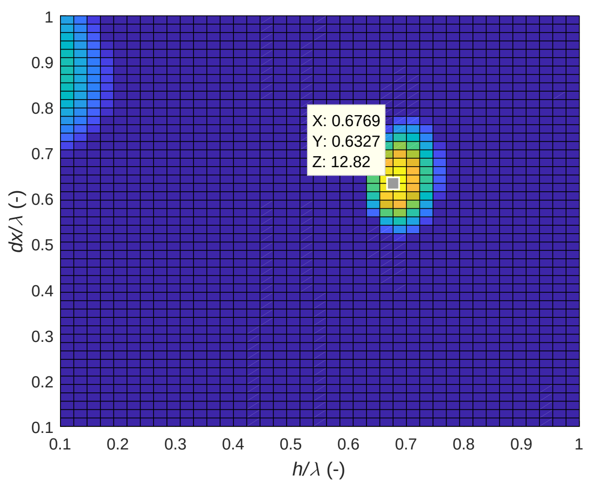


Fig. 4: Maximum directivity of 2 horizontal $\lambda/2$ dipole array above PEC ground

The same method was implemented to an array of three dipoles above PEC ground. The result is depicted in Fig. 5 with maximum directivity of $D = 14.9$ dBi and it is notable that the optimal height and mutual spacing are very similar to the previous case. This finding suggests that the height of linear array above PEC ground presents optimum for any array with a different number of elements.

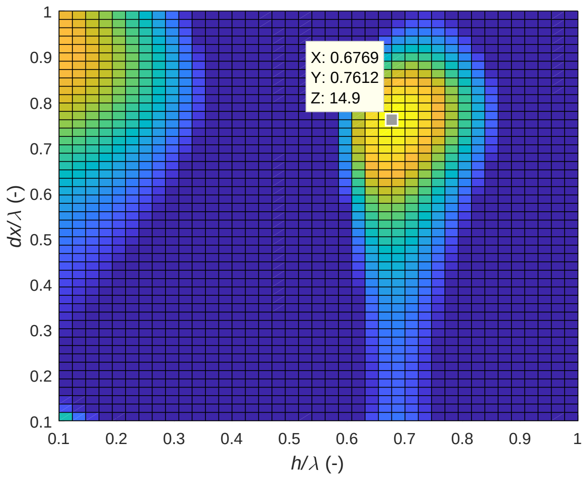


Fig. 5: Maximum directivity of 3 element dipole array above PEC ground

This idea is supported by an analytical evaluation of optimal current for two isotropic radiators and inserting them into (2). The result is depicted in Fig. 6. Contrary to the dipole case, there are two maxima. The first one is for zero height (probably due to the fact that isotropic radiators lack any polarization), but for the second we obtain similar numbers as for dipoles.

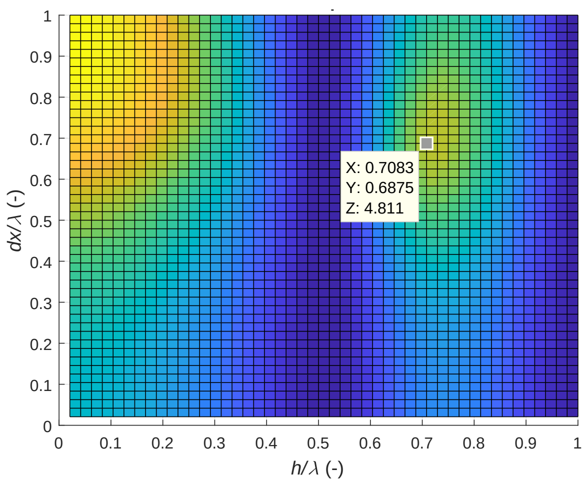
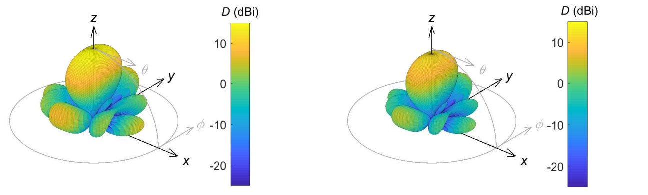


Fig. 6: maximum directivity of 2 isotropic element array above PEC ground

Finally in Fig. 7 we show the radiation pattern of the array with three dipoles placed at optimal height and spacing and fed by optimal current to achieve the maximum directivity. Agreement with FEKO is excellent.

IV. Conclusion

In this article we have used prescribed current distribution on dipoles to simulate the superdirective arrays. Two examples of array were used, circular array and array of dipoles above



(a) FEKO3x1 (b) MATLAB3x1

Fig. 7: Radiation pattern of 3x1 array optimized for maximum directivity.

PEC ground. It is notable that for linear arrays of horizontal dipoles above ground, there is evidently a general optimum for its height and spacing. Since the proposed method is very fast, further work will be aimed on arrays with many elements.

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References

[1] A. Bloch, R. Medhurst, and S. Pool, "A new approach to the design of superdirective aerial arrays," *Proc. IEE*, vol. 100, pp. 303–314, 1953.
[2] A. D. Yaghjian, T. H. O'Donnell, E. E. Altshuler, and S. R. Best, "Electrically small supergain end-fire arrays," *Radio Science*, vol. 43, no. 3, pp. 1–13, June 2008.
[3] A. Haskou, A. Sharaiha, and S. Collardey, "Design of small parasitic loaded superdirective end-fire antenna arrays," *IEEE Trans. Antennas Propag.*, vol. 63, no. 12, pp. 5456–5464, Dec. 2015.
[4] E. E. Altshuler, T. H. O'Donnell, A. D. Yaghjian, and S. R. Best, "A monopole superdirective array," *IEEE Transactions on Antennas and Propagation*, vol. 53, no. 8, pp. 2653–2661, Aug. 2005.
[5] A. Noguchi and H. Arai, "3-element super-directive endfire array with decoupling network," in *2014 International Symposium on Antennas and Propagation Conference Proceedings*, Dec. 2014, pp. 455–456.
[6] S. R. Best, E. E. Altshuler, A. D. Yaghjian, J. M. McGinthy, and T. H. O'Donnell, "An impedance-matched 2-element superdirective array," *IEEE Antennas and Wireless Propagation Letters*, vol. 7, pp. 302–305, 2008.
[7] S. J. Orfanidis. Electromagnetic waves & antennas. [Online]. Available: www.ece.rutgers.edu/ orfanidi/ewa
[8] (2016) The Matlab. The MathWorks. [Online]. Available: www.mathworks.com
[9] J. D. Kraus, *Antennas*. McGraw-Hill, 1988.
[10] (2016) FEKO. Altair. [Online]. Available: www.feko.info
[11] M. Uzsoky and L. Solymár, "Theory of super-directive linear arrays," *Acta Physica Academiae Scientiarum Hungaricae*, vol. 6, no. 2, pp. 185–205, Dec. 1956.
[12] Y. T. Lo, S. W. Lee, and Q. H. Lee, "Optimization of directivity and signal-to-noise ratio of an arbitrary antenna array," *Proceedings of the IEEE*, vol. 54, no. 8, pp. 1033–1045, Aug. 1966.
[13] E. Shamonina and L. Solymar, "Maximum directivity of arbitrary dipole arrays," *IET Microw. Antenna P.*, vol. 9, pp. 101–107, 2015.
[14] R. F. Harrington, *Field Computation by Moment Methods*. Wiley – IEEE Press, 1993.