

# Microwave Circuits and Sub-Systems

## A2M17MOS

### Narrow-Band Amplifier Design – Part A

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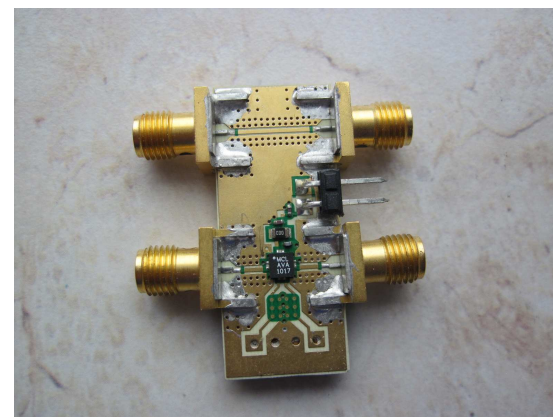
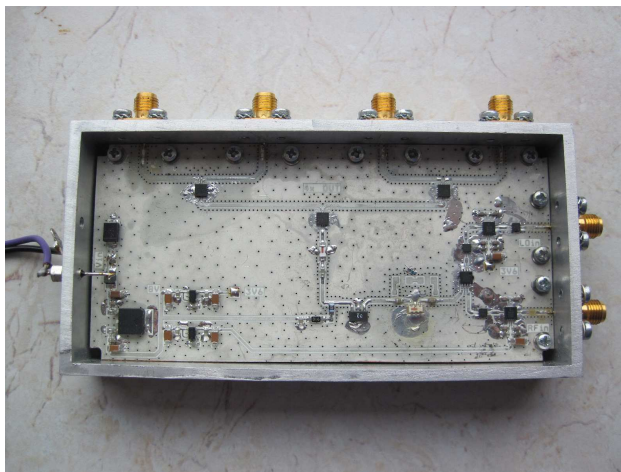
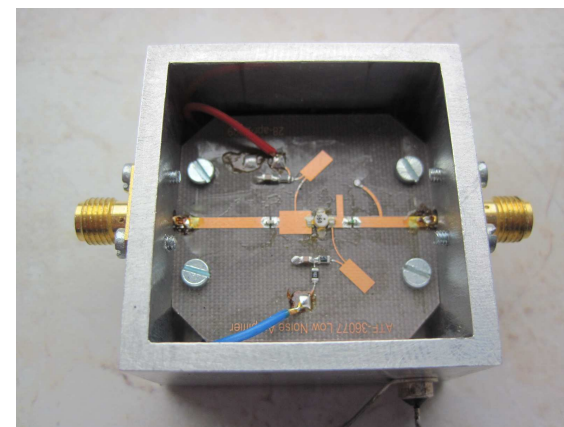


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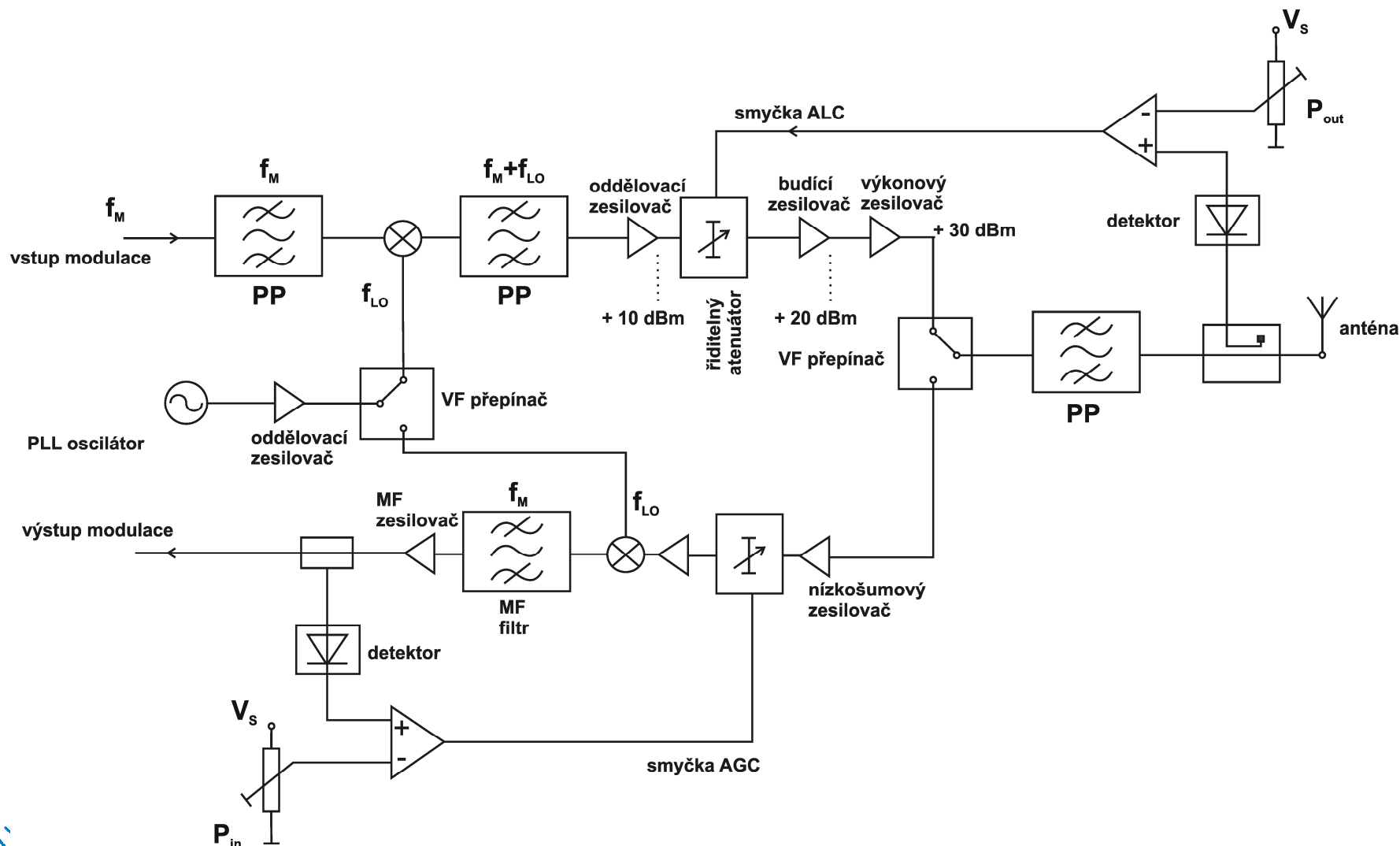
# RF and Microwave Amplifiers

- Necessary for construction of RF & microwave systems
- Frequency range → commonly up to 100GHz
- Many different types of classification
- Based on RF & microwave transistors → BJT, HBT, MESFET, HEMT, ...
- Different materials used → Si, GaAs, InP, GaN, ...
- Different manufacturing technologies used



# RF System - Example

TDMA transceiver with ALC and AGC loops



# Classification

- **According to output power:**

- **Low-power amplifiers**

- Up to  $\approx 10\text{mW}$
    - Low-power transistors used,  $I_{DS} \leq \approx 50\text{mA}$
    - Design  $\rightarrow$  based on linear s-parameters, linear simulators can be employed
    - Application  $\rightarrow$  general amplification stages
    - The most frequent amplifier type

- **Medium-power amplifiers**

- Up to  $\approx 200 - 500\text{mW}$
    - Medium-power transistors used,  $I_{DS} \leq \approx 150\text{mA}$
    - Design  $\rightarrow$  based on linear s-parameters, usually
    - Application  $\rightarrow$  medium-power output stages, driver-stages of power amplifiers

- **Power amplifiers**

- $10^0 - 10^2 \text{ W}$  (only seldom more)
    - High-power transistors
    - Design  $\rightarrow$  using non-linear simulators, large-signal models and non-linear measurement
    - Application  $\rightarrow$  transmitter (communication, radar,...) power output stages
    - Can be very expensive

# Classification

- **According to frequency bandwidth:**

- **Narrow-band amplifiers**

- $B \leq 10\%$  of the center frequency  $f_0$
- Design → relatively simple, solution exists always, direct synthesis formulas

- **Wide-band amplifiers**

- $B \geq 10\%$  of the center frequency  $f_0$ , often 2-3 octaves
- Example: 2-8 GHz, 10-18GHz
- Design → can be very complicated, no solution can exist, approximations, iterations and computer optimizations must be employed

- **Extremely wide-band amplifiers**

- B often more decades
- Examples: 0,01 – 8GHz, 1- 20GHz
- Special structures → resistive matching, feedback amplifiers, distributed amplifiers

# Classification

- **According to realization:**
  - With **absolutely stable** or **potentially unstable** transistors
  - With **lumped** (L,C) or **distributed** (microstrip, co-planar) matching elements
  - With **discrete** components or **integrated** (MMIC)
- **Special amplifiers:**
  - Low-noise amplifiers (LNA)
  - Logarithmic amplifiers
  - Variable-gain amplifiers
  - Complex amplifier structures → „Feed-Forward“, Doherty, ...

# Parameters

- Important parameters of RF & microwave amplifiers:

- **Gain** (insertion gain)

- Defined by the output/input powers ratio
- Often stated in dB

$$G = \frac{P_{out}}{P_{in}}$$

$$G_{dB} = 10 \log \frac{P_{out}}{P_{in}}$$

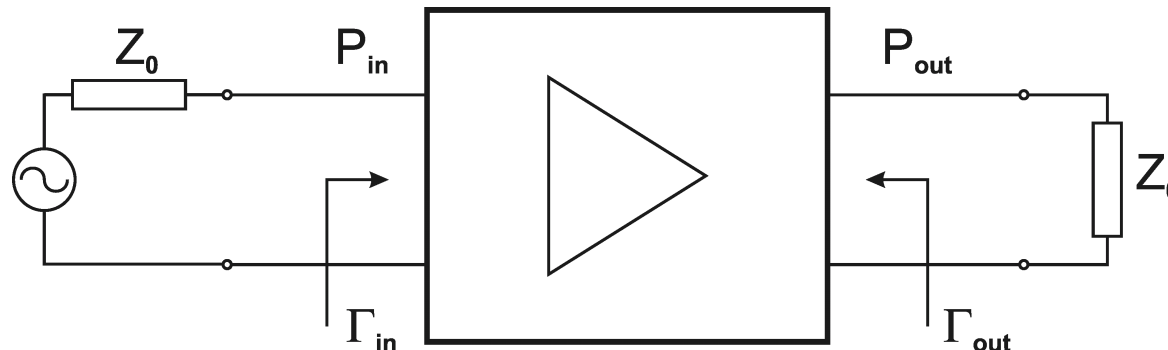
- **Input and output reflections**

- Generally described as  $\Gamma_{in}$  and  $\Gamma_{out}$
- Complex values → but often only modules are important

- Modules can be expressed as RL or SWR
- AWR MS → S11, S22 or dBS11, dBS22

$$|\Gamma|_{dB} = RL = -20 \log |\Gamma|$$

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$





# Parameters

- **Stability**

- Refers both to the transistor and the completed amplifier
- **Absolute stability** is beneficial
- But also **potential un-stability** or **conditional stability**
- Decisive for design procedures and amplifier parameters obtained
- Must be considered in the whole frequency band where the transistor is active
- Non-stability can lead to parasitic oscillations resulting in poor amplifier function or even in its destruction

- **Noise parameters**

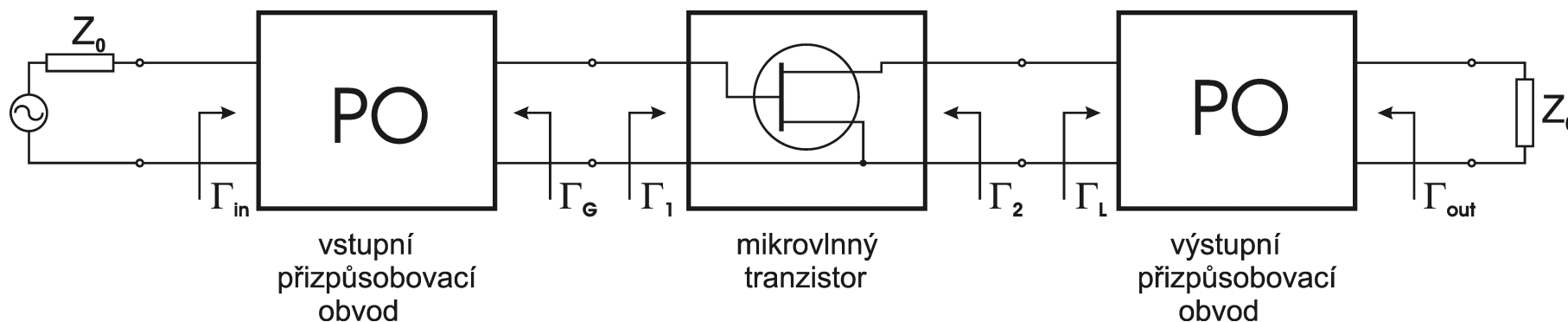
- Important in case of amplifiers processing very weak signals, typically amplifiers connected at inputs of radio receivers and processing signals from antennas
- Defined by the noise figure  $F$ , or the noise temperature  $T_e$
- For the LNA design → additional 4 design noise parameters must be known

- **Non-linear parameters**

- Important in case of amplifiers working with high-level signals
- Common non-linear parameters:  $P_{-1dB}$ , IM2, IM3, IP2, IP3, ...

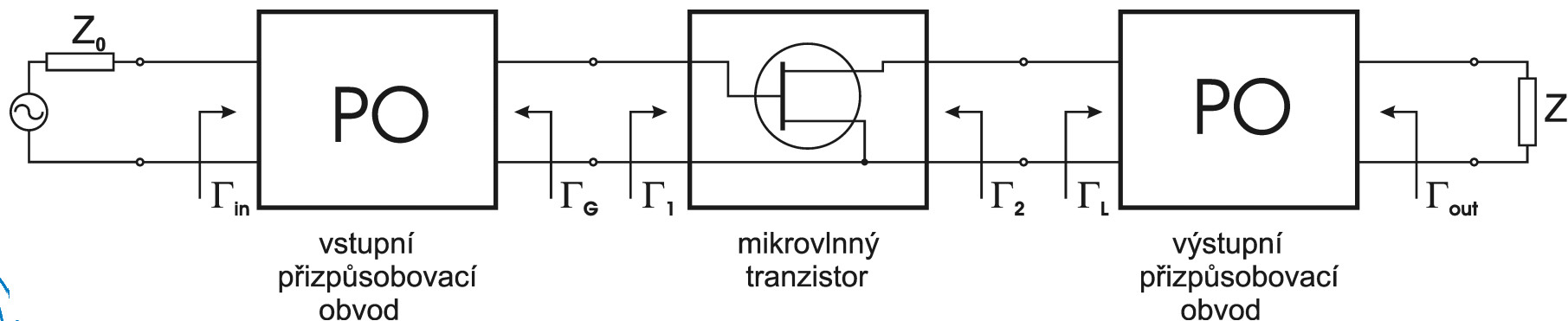
# Basic Amplifier Structure

- Important design reflection coefficients:
  - $\Gamma_1$  transistor input reflection coefficient with the  $\Gamma_L$  loading at output
  - $\Gamma_2$  transistor output reflection coefficient with the  $\Gamma_G$  loading at input
  - $\Gamma_G$  reflection coefficient „seen“ by the transistor input (gate, base)
  - $\Gamma_L$  reflection coefficient „seen“ by the transistor output (drain, collector)
  - $\Gamma_{in}$  input reflection coefficient of the whole amplifier
  - $\Gamma_{out}$  output reflection coefficient of the whole amplifier



# Basic Amplifier Structure

- Modules of  $s_{11}$  and  $s_{22}$  are too high usually
- That is why input and output **matching circuits** (MCs) are necessary
- The input MC transforms  $Z_0$  to  $\Gamma_G$
- The output MC transforms  $Z_0$  to  $\Gamma_L$
- The **optimum  $\Gamma_G$  and  $\Gamma_L$  values** can be calculated according to parameters of the transistor used and the required amplifier parameters
- For calculation of the suitable  $\Gamma_G$  and  $\Gamma_L$  values, several design formulas are necessary



# Calculation of $\Gamma_1$

- s-parameters

$$b_1 = s_{11}a_1 + s_{12}a_2$$

$$b_2 = s_{21}a_1 + s_{22}a_2$$

- Load

$$a_2 = b_2 \Gamma_L \quad b_2 = s_{21}a_1 + s_{22}b_2 \Gamma_L$$

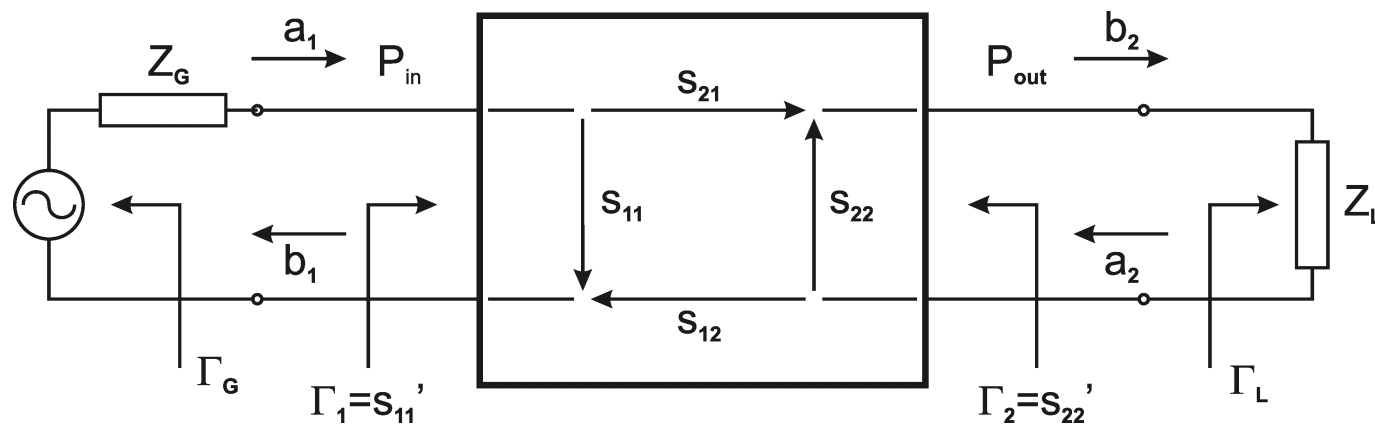
- From that

$$b_2 = \frac{s_{21}a_1}{1 - s_{22}\Gamma_L} \quad a_2 = \Gamma_L \frac{s_{21}a_1}{1 - s_{22}\Gamma_L}$$

$$b_1 = s_{11}a_1 + s_{12}\Gamma_L \frac{s_{21}a_1}{1 - s_{22}\Gamma_L}$$

- Transformation formula

$$\Gamma_1 = s_{11}' = \frac{b_1}{a_1} = s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1 - s_{22}\Gamma_L}$$



# Calculation of $\Gamma_2$

- s-parameters

$$b_1 = s_{11}a_1 + s_{12}a_2$$

$$b_2 = s_{21}a_1 + s_{22}a_2$$

- Description of generator

$$a_1 = b_1 \Gamma_G \quad b_1 = s_{11}b_1 \Gamma_G + s_{12}a_2$$

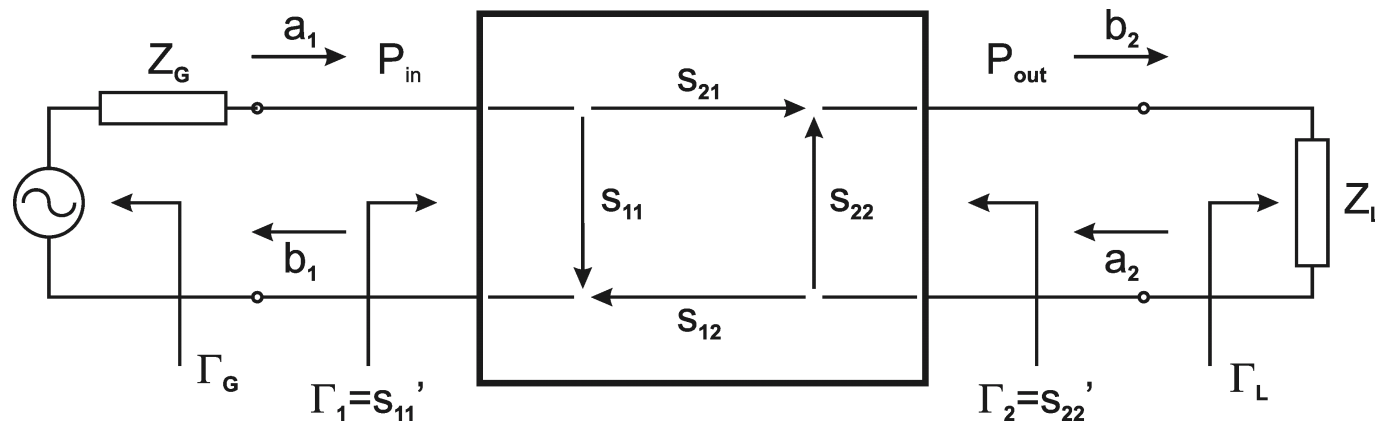
- From that

$$b_1 = \frac{s_{12}a_2}{1 - s_{11}\Gamma_G} \quad a_1 = \Gamma_G \frac{s_{12}a_2}{1 - s_{11}\Gamma_G}$$

$$b_2 = s_{22}a_2 + s_{21}\Gamma_G \frac{s_{12}a_2}{1 - s_{11}\Gamma_G}$$

- Transformation formula

$$\Gamma_2 = s_{22}' = \frac{b_2}{a_2} = s_{22} + \frac{s_{12}s_{21}\Gamma_G}{1 - s_{11}\Gamma_G}$$



# Remarks

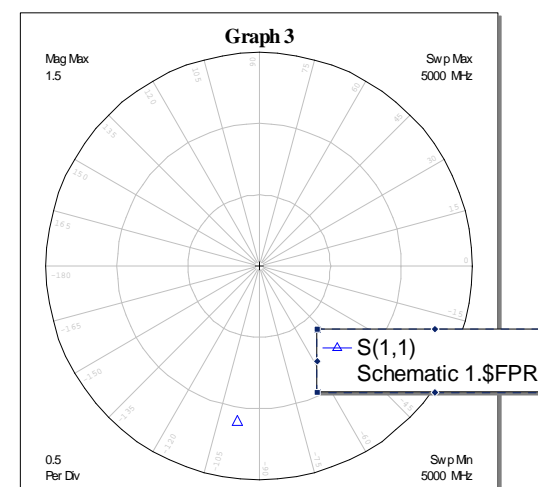
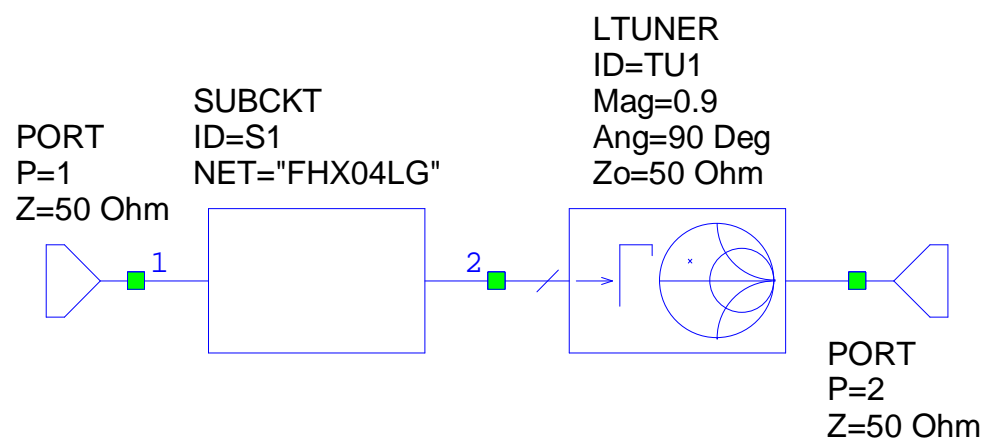
- Transformation formula

$$\Gamma_1 = s_{11}' = \frac{b_1}{a_1} = s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1 - s_{22}\Gamma_L}$$

- Considering  $\Gamma_L = 0$

$$\Gamma_1 = s_{11} + \frac{s_{12}s_{21}0}{1 - s_{22}0} = s_{11}$$

- $s_{11}$  is defined with  $\Gamma_L = 0$
- AWR-MO:** Employment of the LTUNER model can be recommended
- Example:** FHX04L  $\rightarrow$  calculation of  $\Gamma_1 = s_{11}'$  for  $\Gamma_L = 0,9 / 90^\circ$



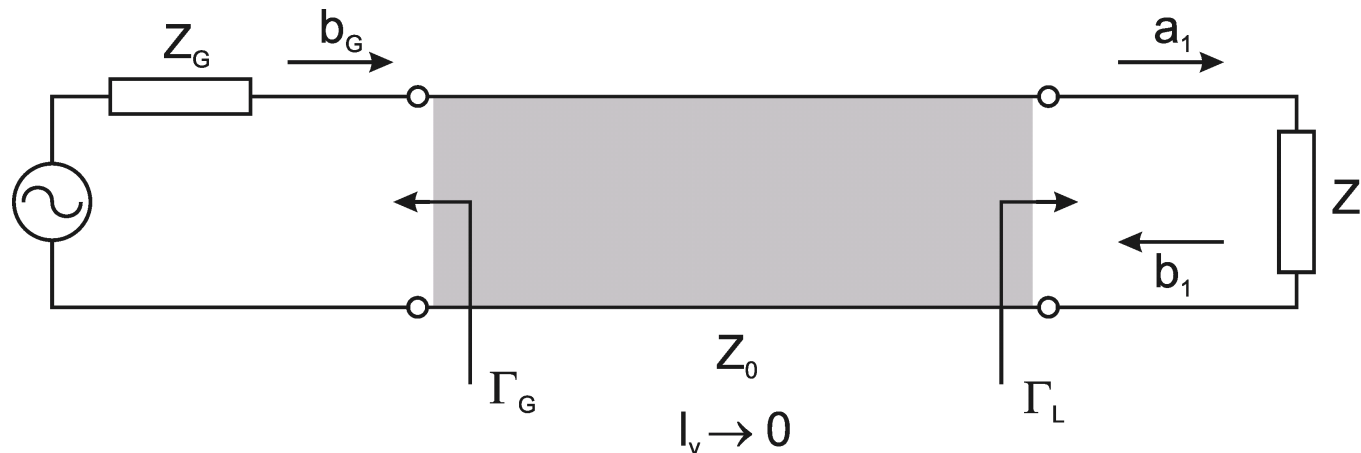
# Connection of General Source and Load

- Connection of the source with **general reflection  $\Gamma_G$  with general load  $\Gamma_L$**
- Both generator and load are **referred to the  $Z_0$  impedance line**
- Impedance line  $Z_0$  of a zero length is used for the inter-connection

• Description:  $a_1 = b_G + b_1 \Gamma_G$

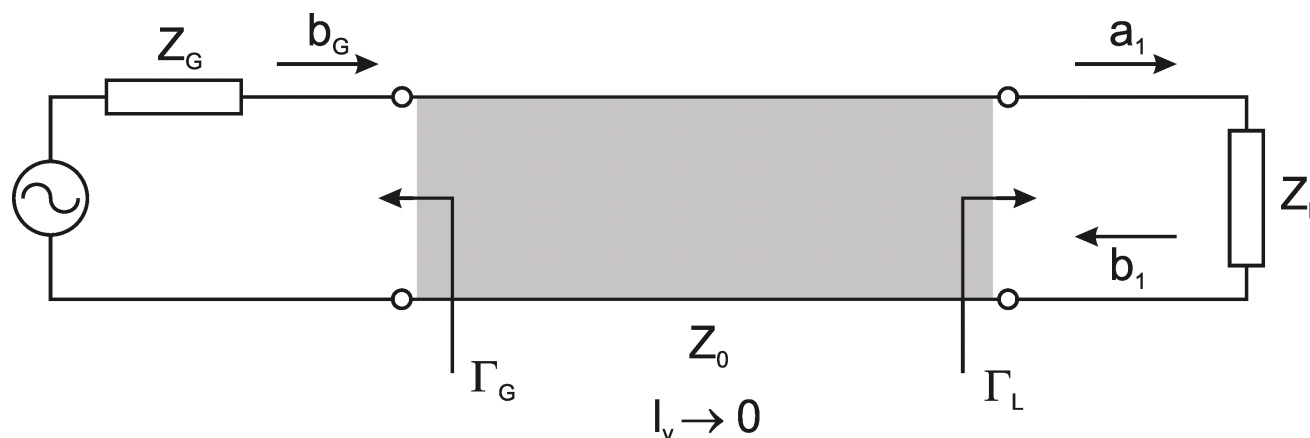
$$b_1 = a_1 \Gamma_L \quad a_1 = b_G + a_1 \Gamma_G \Gamma_L$$

- $b_G \rightarrow$  incident voltage wave from  $Z_G$  into  $Z_0$   $b_G = u_0 \frac{Z_0}{Z_G + Z_0}$



# Power Delivered to Load - $P_L$

- Voltage waves  $a_i, b_i$  are normalized  $a_i = u_i^+ / \sqrt{Z_0}$   $b_i = u_i^- / \sqrt{Z_0}$
- Power delivered to load:  $P_L = |a_1|^2 - |b_1|^2 = |a_1|^2 (1 - |\Gamma_L|^2)$
- Incident wave  $a_1 = b_G + a_1 \Gamma_G \Gamma_L$
- From that  $a_1 = \frac{b_G}{1 - \Gamma_G \Gamma_L}$   $|a_1|^2 = \frac{|b_G|^2}{|1 - \Gamma_G \Gamma_L|^2}$
- Power delivered from the  $\Gamma_G$  source to the  $\Gamma_L$  load:  $P_L = \frac{|b_G|^2 (1 - |\Gamma_L|^2)}{|1 - \Gamma_G \Gamma_L|^2}$





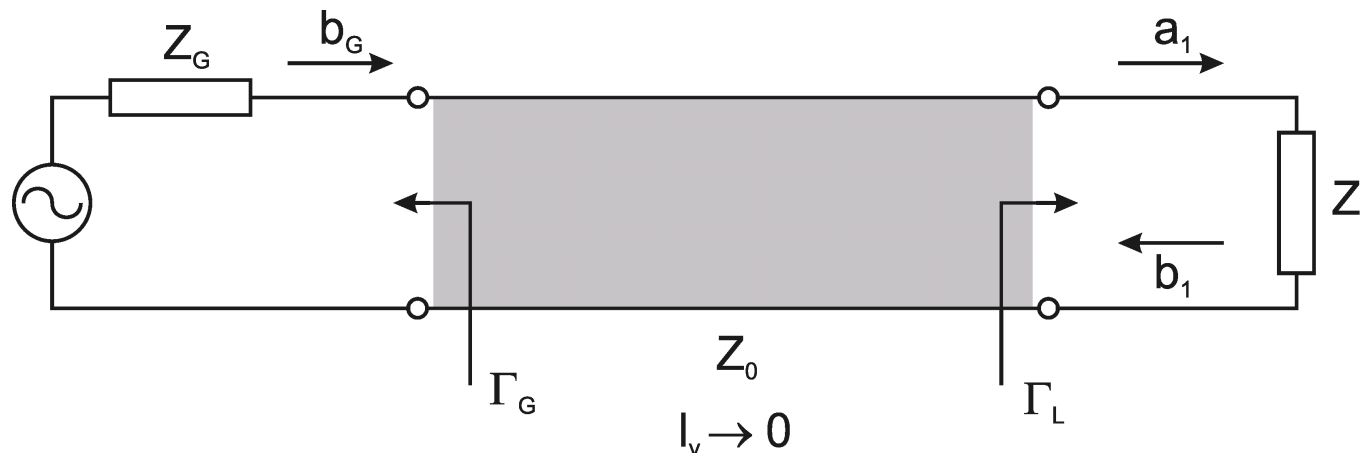
# Power Available to Load - $P_{AL}$

- Power to the given load  $\Gamma_L$  from the optimized  $\Gamma_G$
- Can be derived from the  $P_L$  using condition:  $\Gamma_G = \Gamma_L^*$

- Power delivered to load: 
$$P_L = \frac{|b_G|^2 (1 - |\Gamma_L|^2)}{|1 - \Gamma_G \Gamma_L|^2}$$

- Power delivered from the  $\Gamma_G = \Gamma_L^*$  source to the  $\Gamma_L$  load: 
$$P_{AL} = \frac{|b_G|^2 (1 - |\Gamma_L|^2)}{|1 - |\Gamma_L|^2|^2} = \frac{|b_G|^2}{1 - |\Gamma_L|^2}$$

- Condition – passive load:  $|\Gamma_L| \leq 1$



# Power Available from Generator - $P_{AG}$

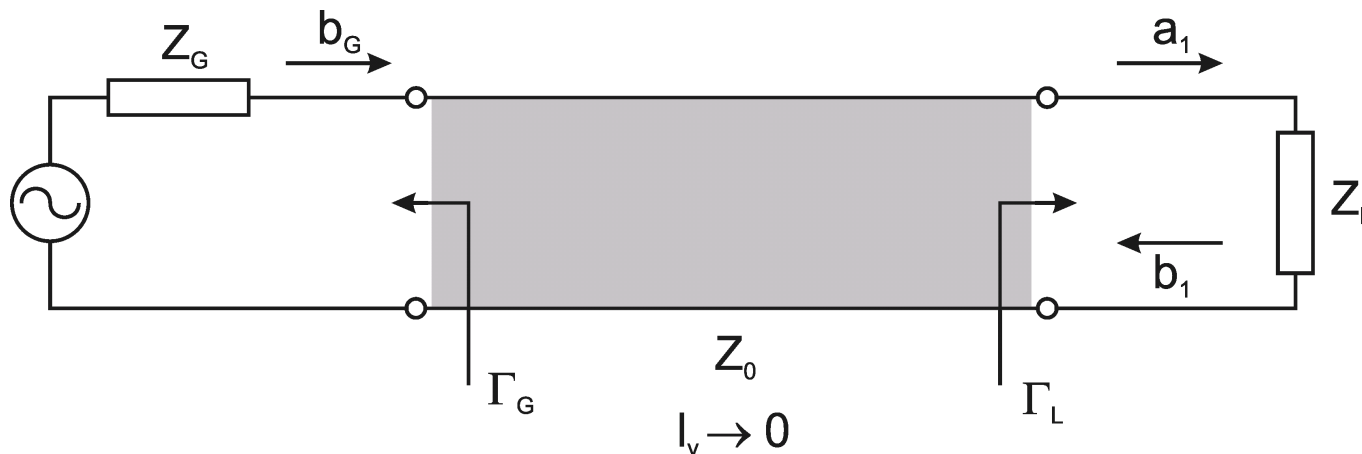
- Power from the given generator  $\Gamma_G$  to the optimized load  $\Gamma_L$

- Can be derived from  $P_L$  using condition:  $\Gamma_L = \Gamma_G^*$

- Power delivered to load: 
$$P_L = \frac{|b_G|^2 (1 - |\Gamma_L|^2)}{|1 - \Gamma_G \Gamma_L|^2}$$

- Power delivered from the  $\Gamma_G$  source to the  $\Gamma_L = \Gamma_G^*$  load: 
$$P_{AG} = \frac{|b_G|^2 (1 - |\Gamma_G^*|^2)}{|1 - \Gamma_G \Gamma_G^*|^2} = \frac{|b_G|^2}{1 - |\Gamma_G|^2}$$

- Condition – passive load:  $|\Gamma_G| \leq 1$



# Transducer Gain - $G_t$

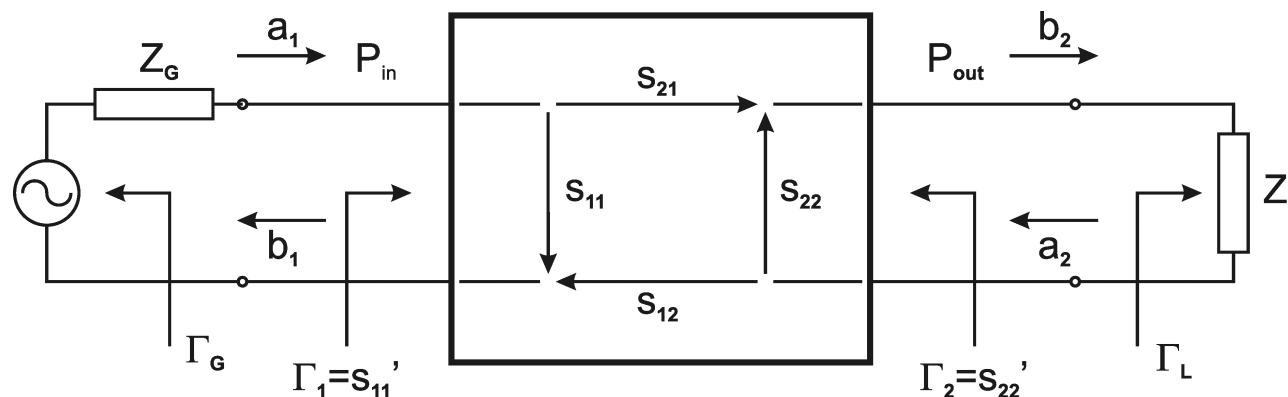
- The most general gain
- Depends upon s-parameters,  $\Gamma_G$  and  $\Gamma_L$

- **Definition:**  $G_t = f(\Gamma_G, \Gamma_L, s_{ij}) = \frac{P_L}{P_{AG}}$

- Using condition  $a_2 = b_2 \Gamma_L$ ,  $P_L$  can be expressed as:  $P_L = |b_2|^2 - |a_2|^2 = |b_2|^2 (1 - |\Gamma_L|^2)$

- Available power from the generator:  $P_{AG} = \frac{|b_G|^2}{1 - |\Gamma_G|^2}$

- From that:  $G_t = \frac{|b_2|^2}{|b_G|^2} (1 - |\Gamma_L|^2) (1 - |\Gamma_G|^2)$



# Transducer Gain - $G_t$

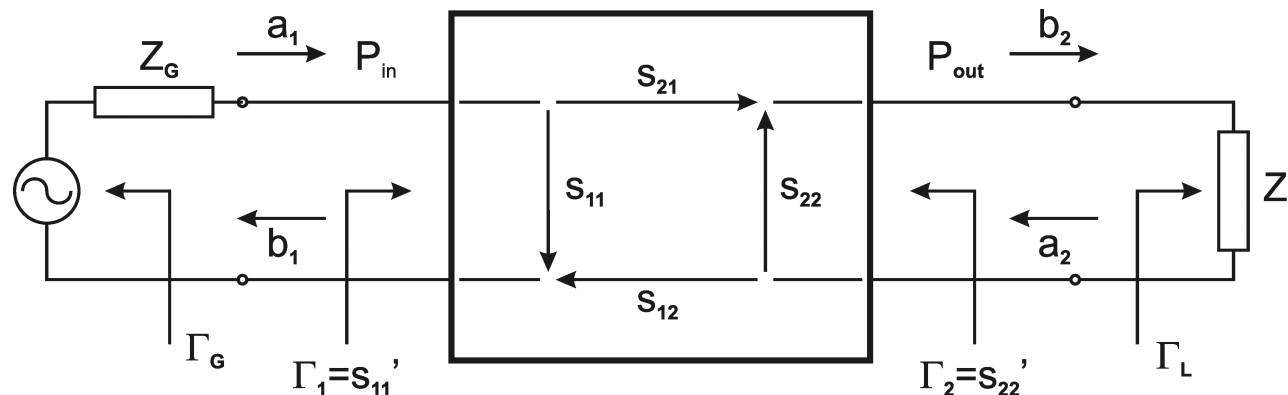
- Using  $a_2 = b_2 \Gamma_L$  and s-parameter definition:  $b_2 = s_{21}a_1 + b_2 s_{22} \Gamma_L$

- From that: 
$$b_2 = \frac{s_{21}a_1}{1 - s_{22}\Gamma_L} \quad \frac{b_2}{a_1} = \frac{s_{21}}{1 - s_{22}\Gamma_L}$$

- Connection of  $\Gamma_G$  and  $\Gamma_1$ :  $a_1 = b_G + a_1 \Gamma_G \Gamma_1$

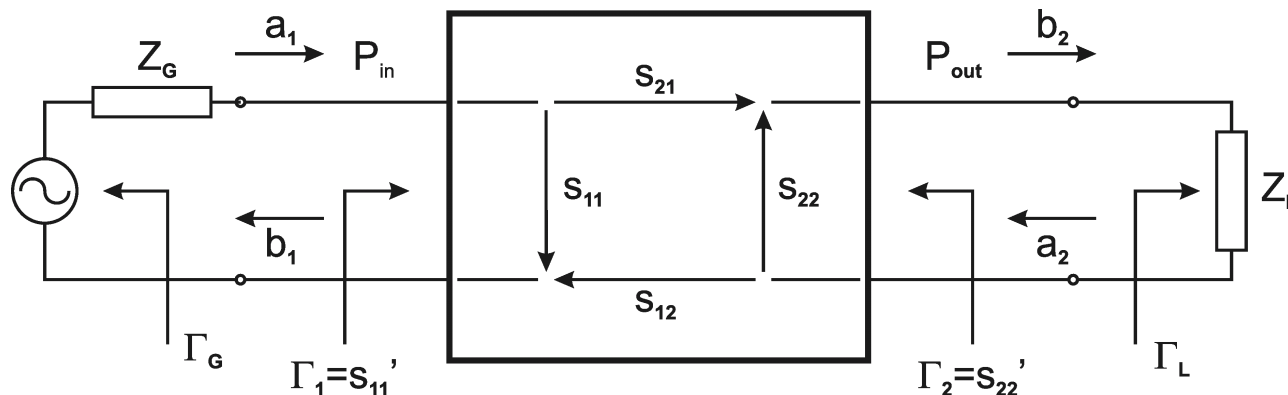
- After re-arranging: 
$$b_G = a_1(1 - \Gamma_G \Gamma_1) \quad \frac{a_1}{b_G} = \frac{1}{1 - \Gamma_G \Gamma_1}$$

- From that: 
$$\frac{b_2}{b_G} = \frac{b_2}{a_1} \frac{a_1}{b_G} = \frac{s_{21}}{1 - s_{22}\Gamma_L} \frac{1}{1 - \Gamma_G \Gamma_1} \quad \frac{|b_2|^2}{|b_G|^2} = \frac{|s_{21}|^2}{|1 - s_{22}\Gamma_L|^2 |1 - \Gamma_1 \Gamma_G|^2}$$



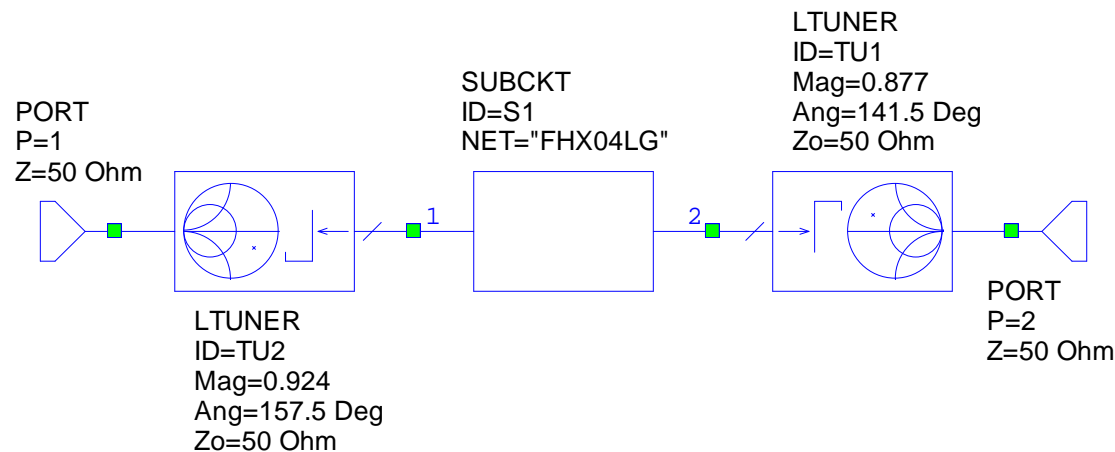
# Transducer Gain - $G_t$

- Transducer gain: 
$$G_t = \frac{|b_2|^2}{|b_G|^2} (1 - |\Gamma_L|^2) (1 - |\Gamma_G|^2)$$
- After substitution: 
$$G_t = \frac{|s_{21}|^2}{|1 - s_{22}\Gamma_L|^2 |1 - \Gamma_1\Gamma_G|^2} (1 - |\Gamma_L|^2) (1 - |\Gamma_G|^2)$$
- Input reflection coeff.: 
$$\Gamma_1 = s_{11}' = \frac{b_1}{a_1} = s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1 - s_{22}\Gamma_L}$$
- Final expression: 
$$G_t = \frac{(1 - |\Gamma_L|^2) |s_{21}|^2 (1 - |\Gamma_G|^2)}{|(1 - s_{11}\Gamma_G)(1 - s_{22}\Gamma_L) - s_{12}s_{21}\Gamma_G\Gamma_L|^2}$$



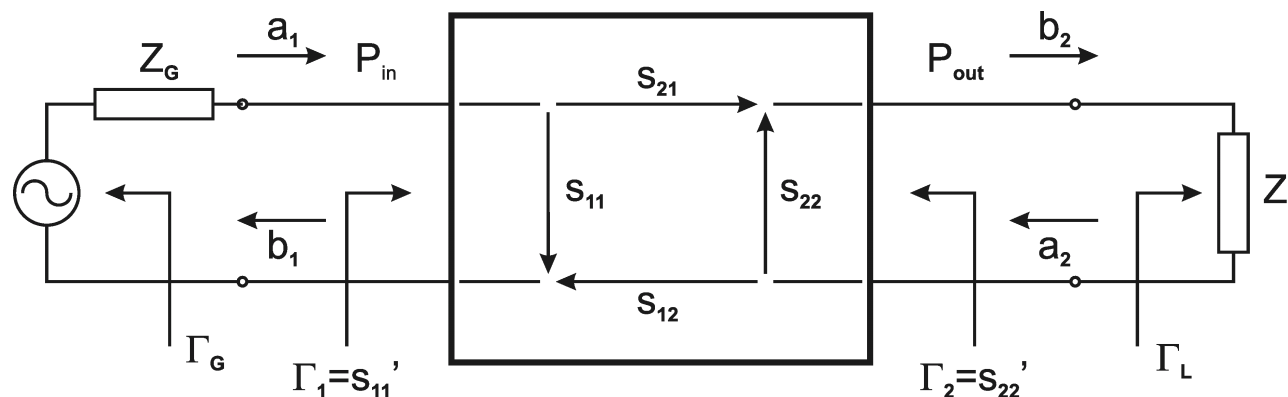
# Remarks

- Transducer gain: 
$$G_t = \frac{(1 - |\Gamma_L|^2) |s_{21}|^2 (1 - |\Gamma_G|^2)}{|(1 - s_{11}\Gamma_G)(1 - s_{22}\Gamma_L) - s_{12}s_{21}\Gamma_G\Gamma_L|^2}$$
- Depends upon s-parameters,  $\Gamma_G$  and  $\Gamma_L$   $G_t = f(\Gamma_G, \Gamma_L, s_{ij})$
- The most general gain
- From  $G_t$  – many important gains can be derived
- Simulation by **AWR-MO** → 2x **LTUNER** models
- **Example:** Calculation of  $G_t$  for  $\Gamma_G = 0,924 / 157,5^0$  and  $\Gamma_G = 0,877 / 141,5^0$



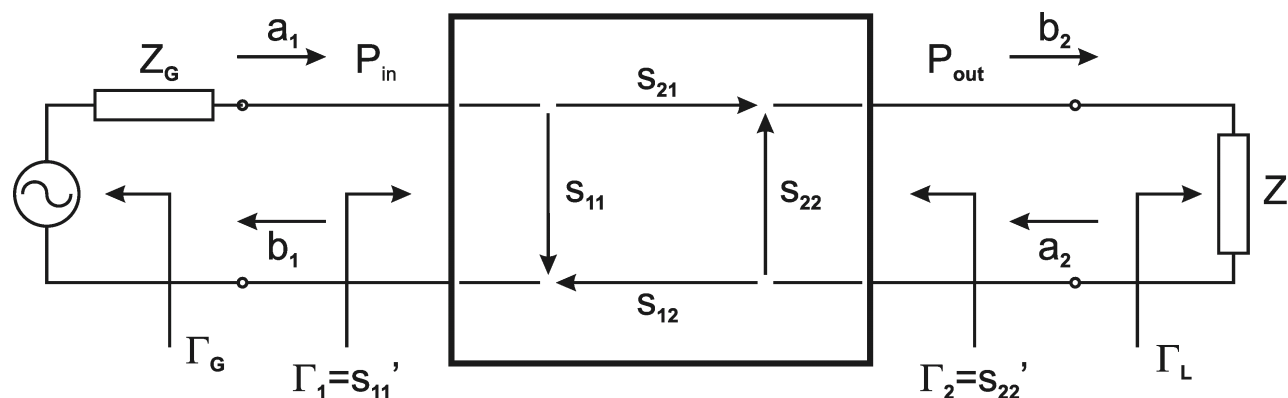
# Power Gain - $G_p$

- Can be derived from the  $G_t$  using condition:  $\Gamma_G = \Gamma_1^*$
- Final expression: 
$$G_p = \frac{(1 - |\Gamma_L|^2) |s_{21}|^2}{|1 - s_{22} \Gamma_L|^2 (1 - |\Gamma_1|^2)}$$
- Depends only on s-parameters and  $\Gamma_L$
- Can be used only if perfect input matching is ensured
- Application: Design of amplifier using potentially unstable transistor
- Design task No.3
- **AWR-MO: GP**



# Available Gain - $G_a$

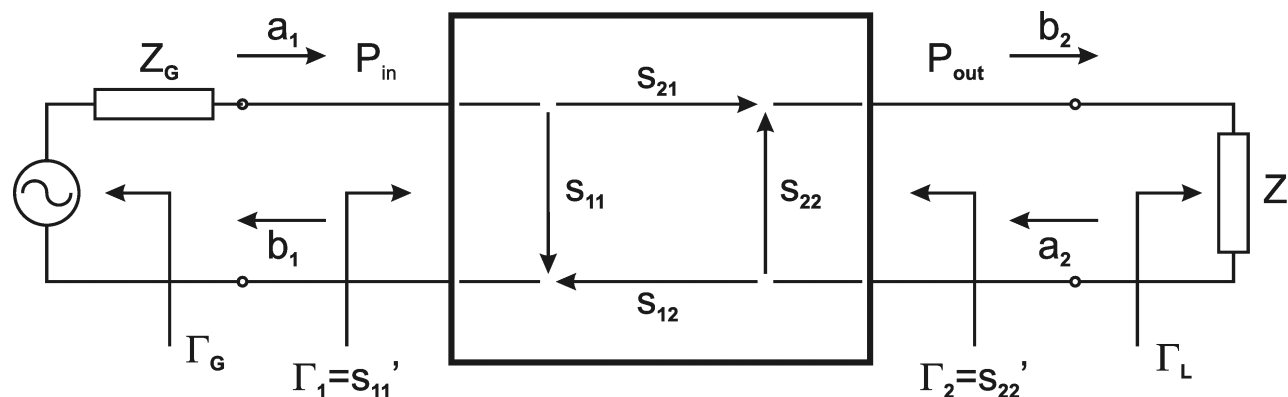
- Can be derived from the  $G_t$  using condition:  $\Gamma_L = \Gamma_2^*$
- Final expression: 
$$G_a = \frac{(1 - |\Gamma_G|^2) |s_{21}|^2}{|1 - s_{11} \Gamma_G|^2 (1 - |\Gamma_2|^2)}$$
- Depends only on s-parameters and  $\Gamma_G$
- Can be used only if perfect output matching is ensured
- Application: Design of low-noise amplifiers
- Design task No.4
- **AWR-MO: GA**





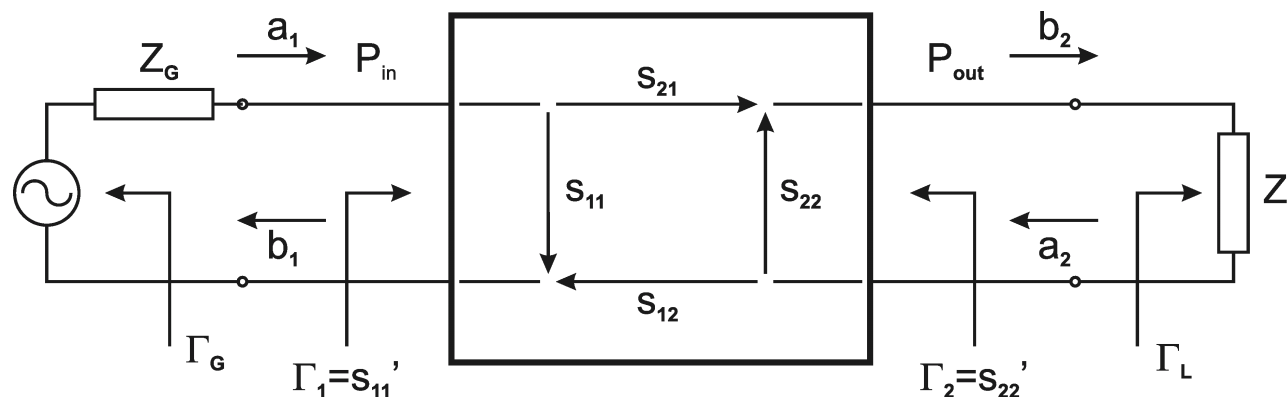
# Maximum Available Gain - $G_{\text{amax}}$

- Can be derived from the  $G_t$  using both conditions:  $\Gamma_L = \Gamma_2^*$        $\Gamma_G = \Gamma_1^*$
- Final expression: 
$$G_{a \max} = \left| \frac{s_{21}}{s_{12}} \right| (k - \sqrt{k^2 - 1})$$
- Depends only on s-parameters, valid if simultaneous input and output matching can be ensured
- Valid only for absolutely stable transistors  $k \geq 1$
- Represent the **best possible amplifier**, highest gain available from the given transistor at the given biasing point and frequency
- **AWR-MO: GMax**



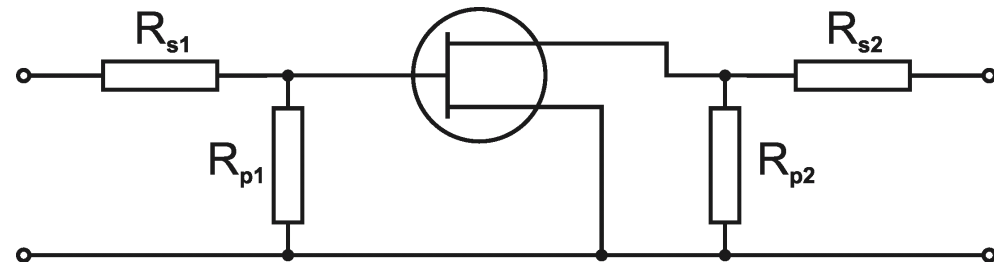
# Unilateral Transducer Gain - $G_{tu}$

- Can be derived from the  $G_t$  using condition:  $s_{12} = 0$
- Final expression: 
$$G_{tu} = \frac{(1 - |\Gamma_L|^2) |s_{21}|^2 (1 - |\Gamma_G|^2)}{|1 - s_{22}\Gamma_L|^2 |1 - s_{11}\Gamma_G|^2}$$
- $G_{tu}$  can be divided into influence of the input MC, transistor and the output MC
- ATTENTION: Can lead to **wrong results**
- Usually,  $s_{12}$  is a small value  $\rightarrow$  but in  $G_{tu}$  occurs in the  $s_{12} \cdot s_{21}$  product only !!!
- Cannot be easily neglected
- Recommendation: USE  $G_{tu}$  WITH HIGH CAUTION!!!



# Maximum Stable Gain - $G_{ms}$

- Valid only for potentially unstable transistors:  $k < 1$
- Final expression: 
$$G_{ms} = \left| \frac{s_{21}}{s_{12}} \right|$$
- Its **definition** is a bit unusual: Potentially unstable transistors with  $k < 1$  can be stabilized using one or two external resistors. If the resistors ensure just  $k = 1$ , the corresponding gain is equal to  $G_{ms}$ .
- If transistor is operated with the  $G_{ms}$  gain:
  - Either the input reflection coefficient  $|\Gamma_{in}| = 1$
  - Or the output reflection coefficient  $|\Gamma_{out}| = 1$
- That is why practical operational gain must be **lower than  $G_{ms}$**  (min. by 1-2 dB)
- Application: Design of amplifiers with potentially unstable transistors
- AWR-MO: GMS**
- Design task No.3b



# Gain Directly to $Z_0 - G_{tZ0}$

- Gain of the 2-port connected directly to the  $Z_0$  lines at input and output
- Final expression:  $G_{tZ0} = |s_{21}|^2$
- Two important cases:
  - **Transistor** →  $G_{tZ0}$  represents gain without any matching circuits connected directly to  $Z_0$  (can be high enough, but show unacceptable reflections)
  - **Amplifier** →  $G_{tZ0}$  represents real operational gain - amplifiers are operated into  $Z_0$
- **AWR-MO:**  $G_{tZ0} [\text{dB}] = \text{dBs}_{21}$

