Návrh a Konstrukce Antén
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Reflector antennas, formulas for design of parabolic reflector

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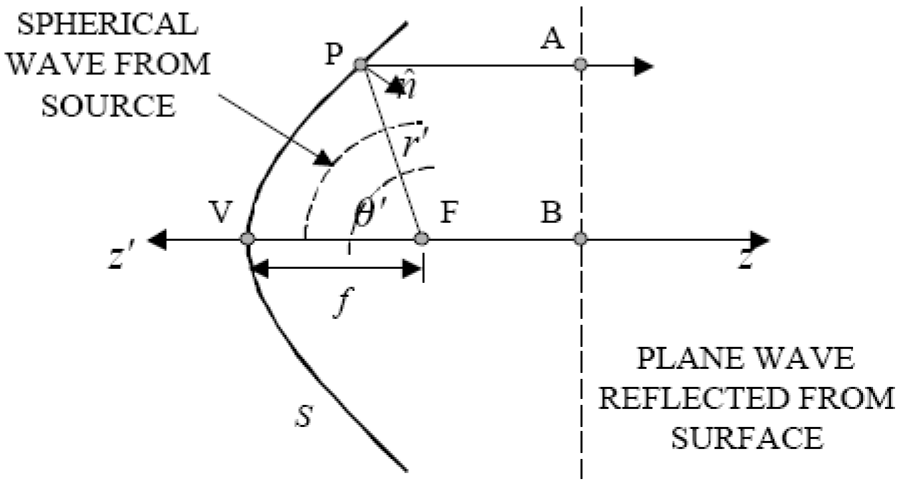
Parabolic reflectors – geometry (1)

What is the required shape of a surface so that it converts a spherical wave to a plane wave on reflection? All paths from O to the plane wave front AB must be equal:



F is the <u>focus</u>
V is the <u>vertex</u>
f is the <u>focal length</u>

$$\overline{FP} + \overline{PA} = \overline{FV} + \overline{VB}$$

$$\overline{PA} = \overline{FP}\cos\theta' + \overline{FB}$$

$$\overline{VB} = \overline{FV} + \overline{FB}$$

Plug in for $\overline{VB}$ and $\overline{PA}$

$$\overline{FP} + \left(\overline{FP}\cos\theta' + \overline{FB}\right) = \overline{FV} + \left(\overline{FV} + \overline{FB}\right)$$

$$\overline{FP}(1+\cos\theta') = 2\overline{FV}$$

$$r'(1+\cos\theta') = 2f$$

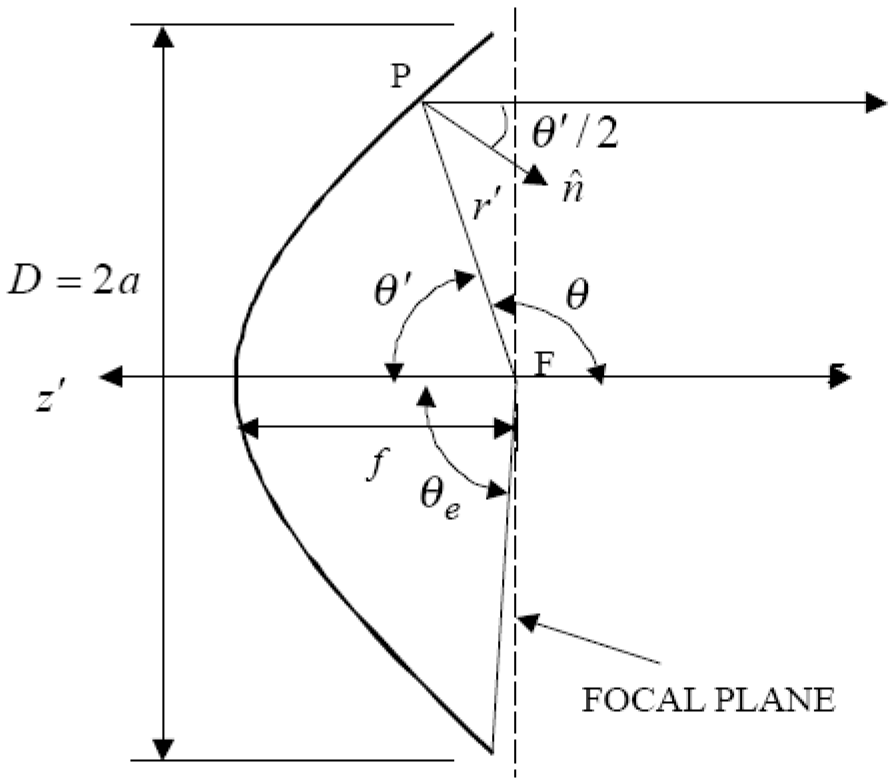
$$r' = 2f/(1+\cos\theta')$$

This is an equation for a parabola.

Parabolic reflectors – geometry (2)

The feed antenna is located at the focus. The design parameters of the parabolic reflector are the diameter D, and the ratio f / D. The edge angle is given by

$$\theta_e = 2\tan^{-1}\left[\frac{1}{4f/D}\right]$$

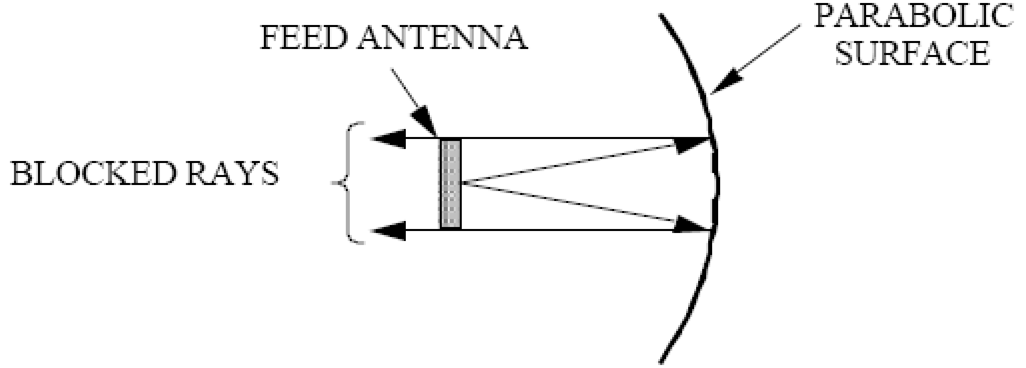


Ideally, the feed antenna should have the following characteristics:

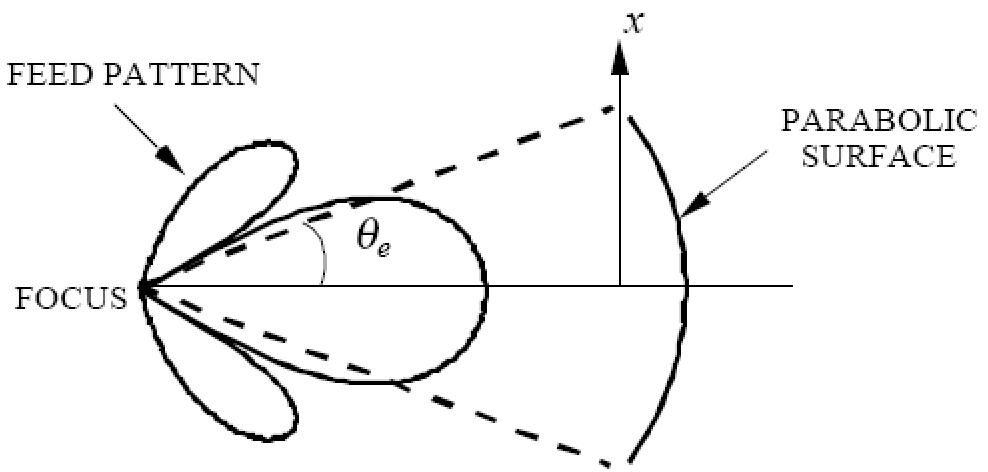
1. Maximize the feed energy intercepted by the reflector (small HPBW $\rightarrow$ large feed)
2. Provide nearly uniform illumination in the focal plane and no spillover (feed pattern abruptly goes to zero at θ_e)
3. Radiate a spherical wave (reflector must be in the feed's far field $\rightarrow$ small feed)
4. Must not significantly block waves reflected off of the surface $\rightarrow$ small feed

Reflector antenna losses (1)

1. Feed blockage reduces gain and increases sidelobe levels (efficiency factor, e_b). Support struts can also contribute to blockage loss.



2. Spillover reduces gain and increases sidelobe levels (efficiency factor, e_s)



Reflector antenna losses (2)

3. Aperture tapering reduces gain (this is the same illumination efficiency that was encountered in arrays; efficiency factor, e_i)

4. Phase error in the aperture field (i.e., due to the roughness of the reflector surface, random phase errors occur in the aperture field, efficiency factor, e_p). Note that there are also random amplitude errors in the aperture field, but they will be accounted for in the illumination efficiency factor.

5. Cross polarization loss (efficiency factor, e_x). The curvature of the reflector surface gives rise to cross polarized currents, which in turn radiate a crossed polarized field. This factor accounts for the energy lost to crossed polarized radiation.

6. Feed efficiency (efficiency factor, e_f). This is the ratio of power radiated by the feed to the power into the feed.

This gain of the reflector can be written as

$$G = \frac{4\pi A}{\lambda^2} e_a = \frac{4\pi A}{\lambda^2} \underbrace{e_i e_p e_x}_{\equiv e_A} e_f e_s e_b$$

For reflectors, the product denoted as e_A is termed the aperture efficiency.

1.

Calculation of Efficiencies (1)

Spillover loss can be computed from the feed antenna pattern. If the feed pattern can be expressed as $\vec{E}_f(r',\theta') = \sqrt{g(\theta')}\,\dfrac{e^{-jkr'}}{r'}\,\hat{e}_f$ where $g(\theta')$ gives the angular dependence and $\hat{e}_f$ denotes the electric field polarization, then the spillover efficiency is

$$e_s = \frac{\text{FEED POWER INTERCEPTED}}{\text{FEED POWER RADIATED}} = \frac{\int_0^{2\pi}\int_0^{\theta_e} |g(\theta')| \sin\theta'\, d\theta'\, d\phi'}{\int_0^{2\pi}\int_0^{\pi} |g(\theta')| \sin\theta'\, d\theta'\, d\phi'}$$

Example: What is the spillover loss when a dipole feeds a paraboloid with $f/D = 0.4$?

$$e_s = \frac{\int_0^{2\pi}\int_0^{64^\circ} |\sin^2\theta'| \sin\theta'\, d\theta'\, d\phi'}{\int_0^{2\pi}\int_0^{\pi} |\sin^2\theta'| \sin\theta'\, d\theta'\, d\phi'} = \frac{\left[\cos\theta' + \dfrac{\cos^3\theta'}{3}\right]_0^{64^\circ}}{\left[\cos\theta' + \dfrac{\cos^3\theta'}{3}\right]_0^{180^\circ}} = \frac{-1.3}{-2.667} = 0.488 = -3.1\text{ dB}$$

2.

$$e_s = \frac{\text{FEED POWER INTERCEPTED}}{\text{FEED POWER RADIATED}} = \frac{\int_0^{2\pi}\int_0^{\theta_e} \left|g(\theta')\right| \sin\theta' \, d\theta' \, d\phi'}{\int_0^{2\pi}\int_0^{\pi} \left|g(\theta')\right| \sin\theta' \, d\theta' \, d\phi'}$$

$$\vec{E}_f(r',\theta') = \sqrt{g(\theta')}\,\frac{e^{-jkr'}}{r'}\,\hat{e}_f$$

$$e_i = 32\left(\frac{f}{D}\right)^2 \frac{\left|\int_0^{2\pi}\int_0^{\theta_e} \sqrt{g(\theta')}\,\tan(\theta'/2)\, d\theta' \, d\phi'\right|^2}{\int_0^{2\pi}\int_0^{\pi} \left|g(\theta')\right| \sin\theta' \, d\theta' \, d\phi'}$$

3a.

$$\vec{E}_f(r',\theta') = \sqrt{g(\theta')}\,\frac{e^{-jkr'}}{r'}\,\hat{e}_f$$

$$e_s = \frac{\text{FEED POWER INTERCEPTED}}{\text{FEED POWER RADIATED}} = \frac{\int_0^{2\pi}\int_0^{\theta_e} |g(\theta')| \sin\theta' \, d\theta' \, d\phi'}{\int_0^{2\pi}\int_0^{\pi} |g(\theta')| \sin\theta' \, d\theta' \, d\phi'}$$

3b.

Calculation of Efficiencies (2)

The illumination efficiency (also known as tapering efficiency) depends on the feed pattern as well

$$e_i = 32\left(\frac{f}{D}\right)^2 \frac{\left|\int_0^{2\pi}\int_0^{\theta_e}\sqrt{g(\theta')}\tan(\theta'/2)\,d\theta'\,d\phi'\right|^2}{\int_0^{2\pi}\int_0^{\pi}\left|g(\theta')\right|\sin\theta'\,d\theta'\,d\phi'}$$

A general feed model is the function $g(\theta') = \begin{cases} 2(n+1)\cos^n\theta', & 0 \le \theta' \le \pi/2 \\ 0, & \text{else} \end{cases}$

The formulas presented yield the following efficiencies for this simple feed model:

$$e_s = (n+1)\int_0^{\theta_e}\cos^n\theta'\sin\theta'\,d\theta' = 1 - \left[\cos\left(\frac{\theta_e}{2}\right)\right]^{n+1}$$

$$e_i = \left(\frac{f}{D}\right)^2 2(n+1)\left[\int_0^{\theta_e/2}\cos^{n/2}\theta'\tan(\theta'/2)\,d\theta'\right]^2$$

3c.

$$g(\theta') = \begin{cases} 2(n+1)\cos^n \theta', & 0 \le \theta' \le \pi/2 \\ 0, & \text{else} \end{cases}$$

$$e_s = (n+1)\int_0^{\theta_e} \cos^n \theta' \sin\theta' \, d\theta' = 1 - \left[\cos\left(\frac{\theta_e}{2}\right)\right]^{n+1}$$

$$e_i = \left(\frac{f}{D}\right)^2 2(n+1)\left[\int_0^{\theta_e/2} \cos^{n/2} \theta' \tan(\theta'/2)\, d\theta'\right]^2$$

3d.

$$e_s = (n+1)\int_0^{\theta_e} \cos^n \theta' \sin\theta' \, d\theta' = 1 - \left[\cos\left(\frac{\theta_e}{2}\right)\right]^{n+1}$$

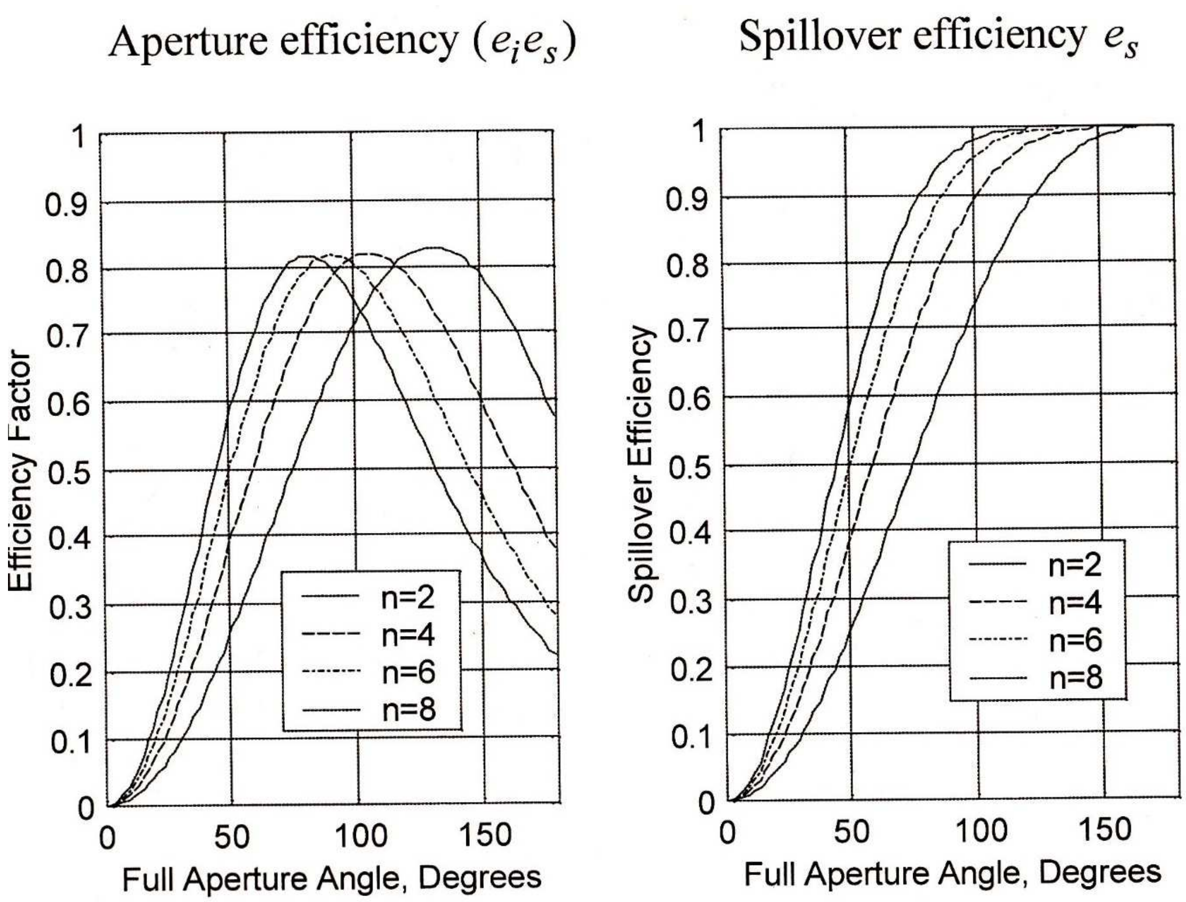
$$e_i = \left(\frac{f}{D}\right)^2 2(n+1)\left[\int_0^{\theta_e/2} \cos^{n/2}\theta' \tan(\theta'/2)\, d\theta'\right]^2$$

3e.

$$e_i = 32\left(\frac{f}{D}\right)^2 \frac{\left|\int\limits_0^{2\pi}\int\limits_0^{\theta_e}\sqrt{g(\theta')}\tan(\theta'/2)\,d\theta'\,d\phi'\right|^2}{\int\limits_0^{2\pi}\int\limits_0^{\pi}\left|g(\theta')\right|\sin\theta'\,d\theta'\,d\phi'}$$

$$g(\theta') = \begin{cases} 2(n+1)\cos^n\theta', & 0 \le \theta' \le \pi/2 \\ 0, & \text{else} \end{cases}$$

4a.



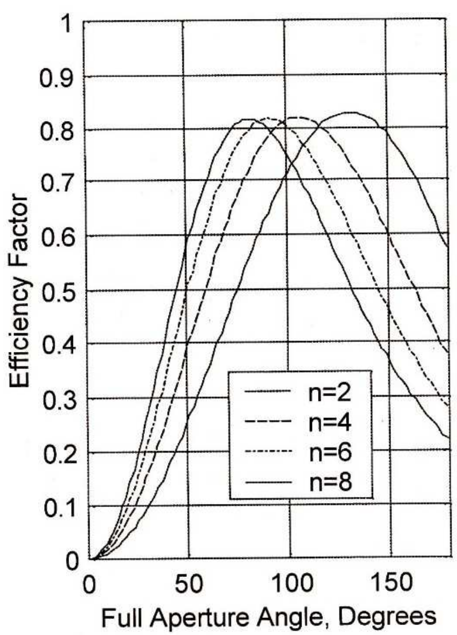
Aperture efficiency ($e_i e_s$)
Efficiency Factor
Full Aperture Angle, Degrees
n=2
n=4
n=6
n=8
Spillover efficiency e_s
Spillover Efficiency
Full Aperture Angle, Degrees
n=2
n=4
n=6
n=8

4b.

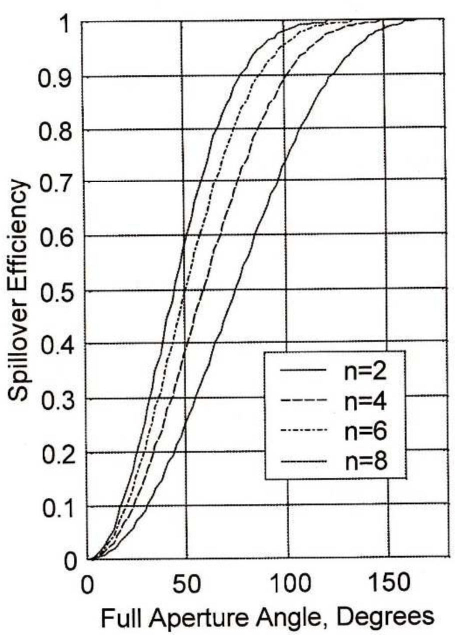
Cosine Feed Efficiency Factors

Efficiencies for a $\cos^n \theta'$ feed: (full aperture angle is $2\theta_e$)

Aperture efficiency $(e_i e_s)$



Spillover efficiency e_s



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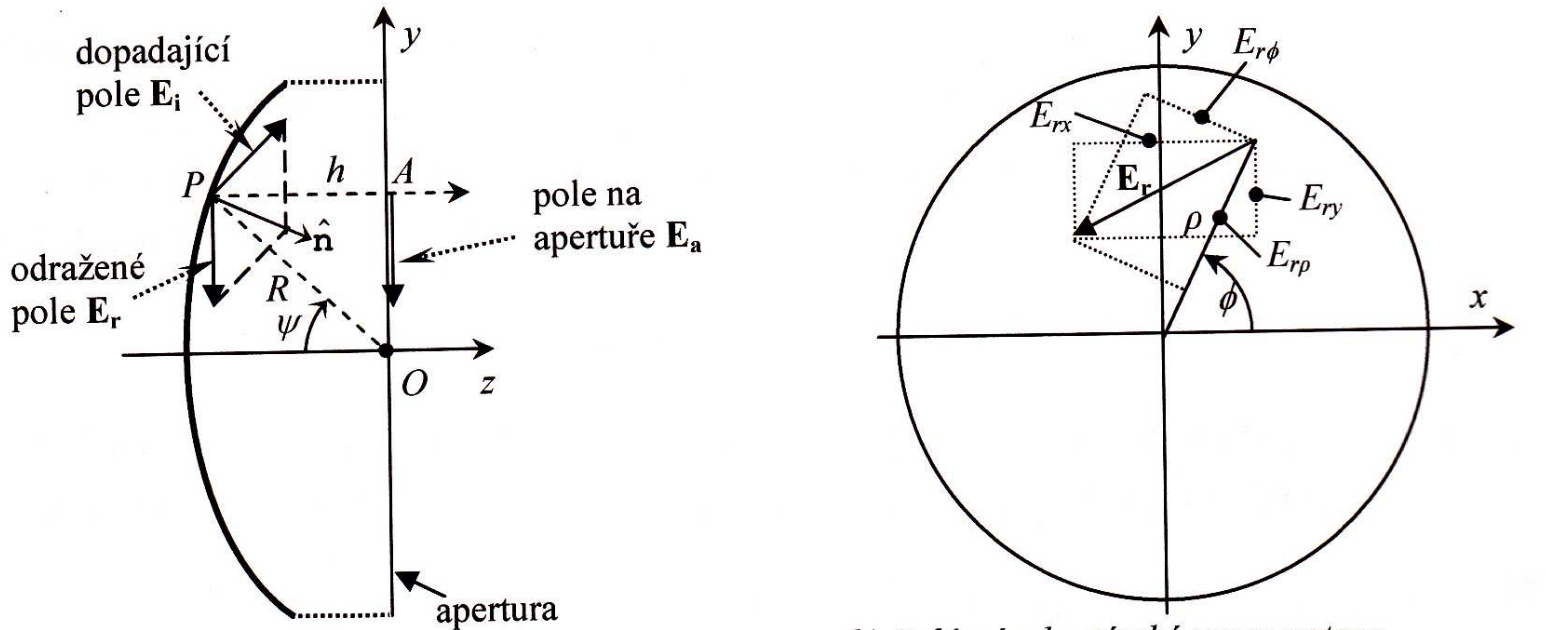
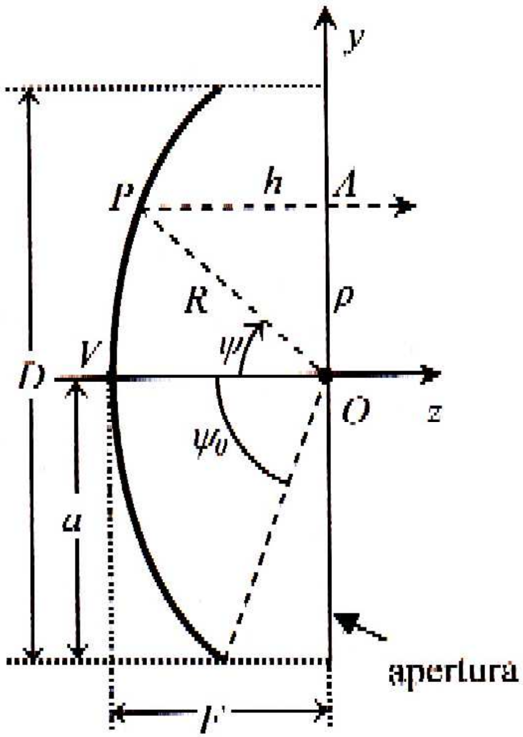
Reflections from the parabolic reflector



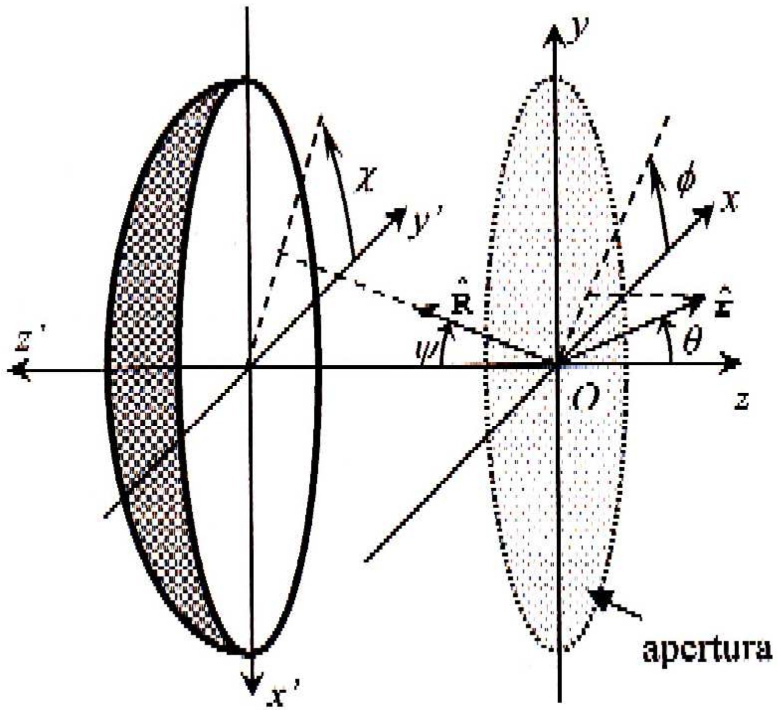
a) Vztah odraženého a dopadajícího pole

b) Polární a kartézská reprezentace odraženého pole

Parabolic reflector - geometry

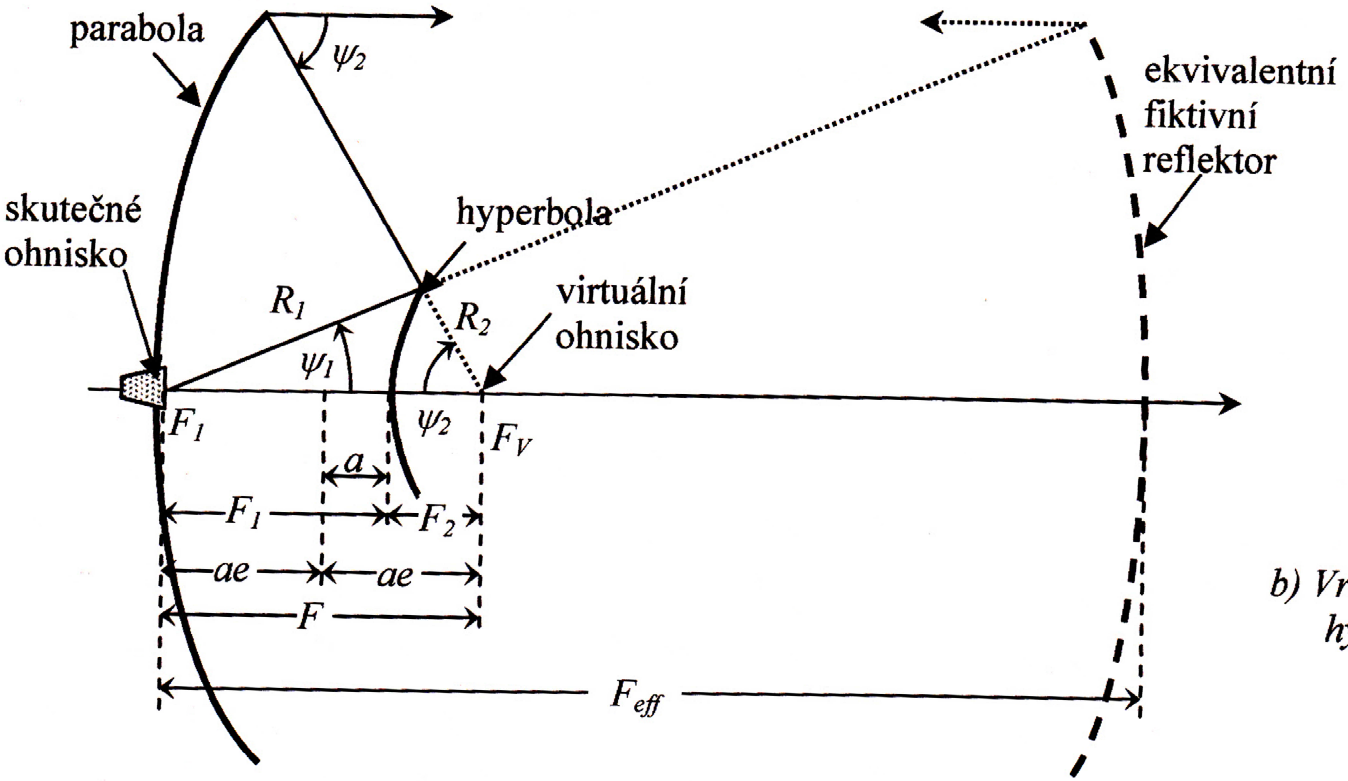


a) Geometrie

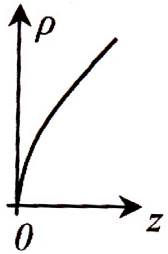


b) Souřadné systémy

Cassegrain reflector antenna



a) Geometrie sestavy, fiktivní reflektor



b) Vrcholové umístění hyperboly

Reflector design - apendix

Theory of REFLECTOR

The reflector geometry is shown in Fig. 5.3. The dish is of diameter d, with a focal length f. The feed pattern is assumed to be closely approximated by the expression

$$G_f(\theta)=2(n+1)\cos^n\theta \qquad (5.63)$$

where $n=2,4,6$, or 8. The aperture efficiency is defined as

$$\epsilon_{ap}=\cot^2\left(\frac{\theta_0}{2}\right)\left|\int_0^{\theta_0}\sqrt{G_f(\theta)}\,\tan\frac{\theta}{2}\,d\theta\right|^2 \qquad (5.64)$$

$$\theta_0=\tan^{-1}\left(\frac{d}{2z_0}\right) \qquad (5.65)$$

$$z_0=f-d^2/16f \qquad (5.66)$$

Equation (5.64) can be evaluated using (5.63) to give [2]:

$$\epsilon_{ap}(n=2)=24\left[\sin^2\frac{\theta_0}{2}+\ln\left(\cos\frac{\theta_0}{2}\right)\right]^2\cot^2\frac{\theta_0}{2} \qquad (5.67)$$

$$\epsilon_{ap}(n=4)=40\left[\sin^4\frac{\theta_0}{2}+\ln\left(\cos\frac{\theta_0}{2}\right)\right]^2\cot^2\frac{\theta_0}{2} \qquad (5.68)$$

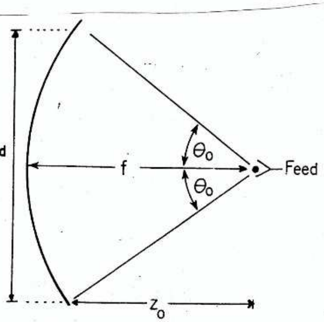


Figure 5.3 Geometry of a parabolic reflector

$$\epsilon_{ap}(n=6)=14\left[2\ln\left(\cos\frac{\theta_0}{2}\right)+\frac{(1-\cos\theta_0)^3}{3}+\frac{1}{2}\sin^2\theta_0\right]^2\cot^2\frac{\theta_0}{2} \qquad (5.69)$$

$$\epsilon_{ap}(n=8)=18\left[\frac{1-\cos^4\theta_0}{4}-2\ln\left(\cos\frac{\theta_0}{2}\right)-\frac{(1-\cos\theta_0)^3}{3}-\frac{1}{2}\sin^2\theta_0\right]^2\cot^2\frac{\theta_0}{2} \qquad (5.70)$$

The spillover efficiency is defined as

$$\epsilon_S=\frac{\int_0^{\theta_0}G_f(\theta)\sin\theta\,d\theta}{\int_0^{\pi}G_f(\theta)\sin\theta\,d\theta} \qquad (5.71)$$

and can be evaluated as

$$\epsilon_s=1-\cos^{n+1}\theta,\ \text{for all } n. \qquad (5.72)$$

The taper efficiency ϵ_t is

$$\epsilon_t=\epsilon_{ap}/\epsilon_s \qquad (5.72)$$

Then, the antenna directivity is

$$D=\left(\frac{\pi d}{\lambda}\right)^2\epsilon_{ap} \qquad (5.73)$$

which is simply the gain due to a uniformly illuminated aperture of diameter d reduced by the aperture efficiency. The 3dB beamwidth is calculated as

$$BW=\sqrt{33700/D} \qquad (5.74)$$