



Receiving antenna:

$$Z_{Load} = R_{Load} + jX_{Load}$$

$$Z_A = R_A + jX_A = (R_r + R_L) + jX_A$$

$$I_{Load} = \frac{U}{Z} = \frac{U}{Z_A + Z_{Load}} = \frac{U}{(R_r + R_L + R_{Load}) + j(X_A + X_{Load})}$$

$$|I_{Load}| = \frac{|U|}{\sqrt{(R_r + R_L + R_{Load})^2 + j(X_A + X_{Load})^2}}$$

The power delivered to the load

$$P_{Load} = \frac{1}{2} R_{Load} |I_{Load}|^2 = \frac{R_{Load}}{2} \frac{|U|^2}{(R_r + R_L + R_{Load})^2 + j(X_A + X_{Load})^2}$$

The *maximum power* (not the maximum efficiency of power transfer!!) delivered to the load occurs when we have conjugate matching

$$Z_A = Z_{Load}^*$$

$$R_r + R_L = R_{Load}$$

$$X_A = -X_{Load}$$

Then:

$$P_{Load} = \frac{R_{Load}}{2} \frac{|U|^2}{(R_{Load} + R_{Load})^2} = \frac{R_{Load}}{2} \frac{|U|^2}{4R_{Load}} = \frac{|U|^2}{8R_{Load}}$$

For lossless antenna $R_L = 0$ so $R_r = R_{Load}$ and

$$P_{Load} = \frac{|U|^2}{8R_r}$$

Friis formula and effective aperture

$$\begin{aligned}P_{RX} &= S_{inc} A_{eRX} = \frac{EIRP}{4\pi R^2} A_{eRX} = \frac{P_{TX} D_{TX}}{4\pi R^2} A_{eRX} \\&= \frac{P_{TX} D_{TX}}{4\pi R^2} \frac{\lambda^2}{4\pi} D_{RX} = P_{TX} D_{TX} D_{RX} \left(\frac{\lambda}{4\pi R} \right)^2\end{aligned}$$