

1 Linear (wire) antennas

1. *Units of all quantities*

$[E] = \text{V/m}$, $[H] = \text{A/m}$, $[Z] = \Omega$, directivity and gain $[D] = [G] = \text{dB}$, $[S] = \text{W/m}^2$

2. *What is the directivity of isotropic antenna (in linear scale and dBi)?*

$D = 0 \text{ dBi} = 1$

3. *Describe antenna as a filter (in which domains does it filter) and a transformer of waves from guided to waves in free space.*

Antenna behaves like a passive filter in both frequency and spatial domain. It transforms guided waves from a waveguide into radiates waves propagating in free space.

4. *When an antenna is electrically small and large (in terms of ka)? Give examples of such antennas.*

An electrically small antenna is an antenna with $ka \in (0.5, 1)$, where $k = 2\pi/\lambda$ is the wave number. En example of an electrically small antenna is the antenna of Titanic: physical length of 50 m with $f = 500 \text{ kHz}$.

5. *What characterizes a TEM transmission line (impedance and operation with frequency)? Sketch distribution of E and H in (a) coaxial line, (b) rectangular waveguide with TE_{10} mode, (c) microstrip line.*

A truly TEM transmission line is non-dispersive meaning that its parameters such as characteristic impedance and phase velocity don't change with frequency. Such TEM mode exists in a coaxial line on all frequencies or in a microstrip line (quasi-TEM) under a specified *cut-off frequency*. In metallic waveguides such as a rectangular waveguide, TE/TM modes propagate above respective cut-off frequencies.

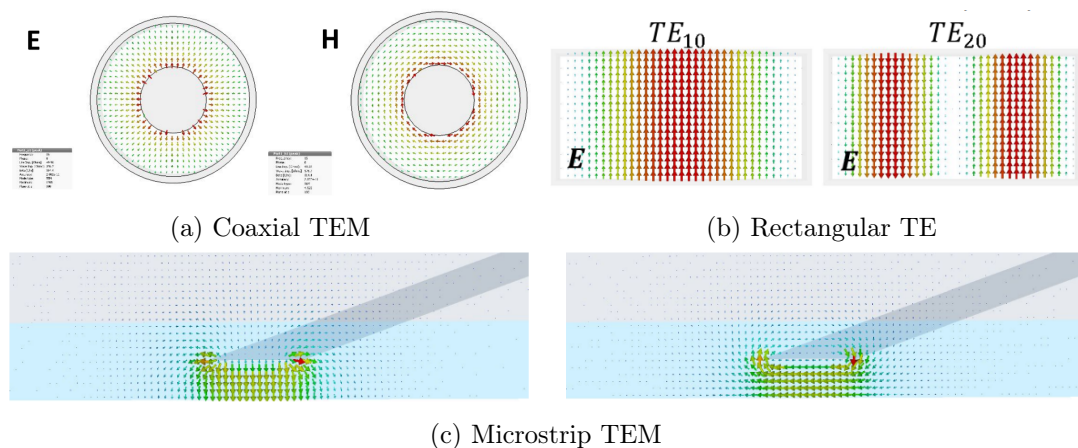


Figure 1: Propagation modes in different transmission lines

6. *Sketch circuit model of a transmitting and receiving antenna.*

See figure 2.

7. *What is radiation resistance and radiation efficiency? Radiation efficiency of electrically small antennas.*

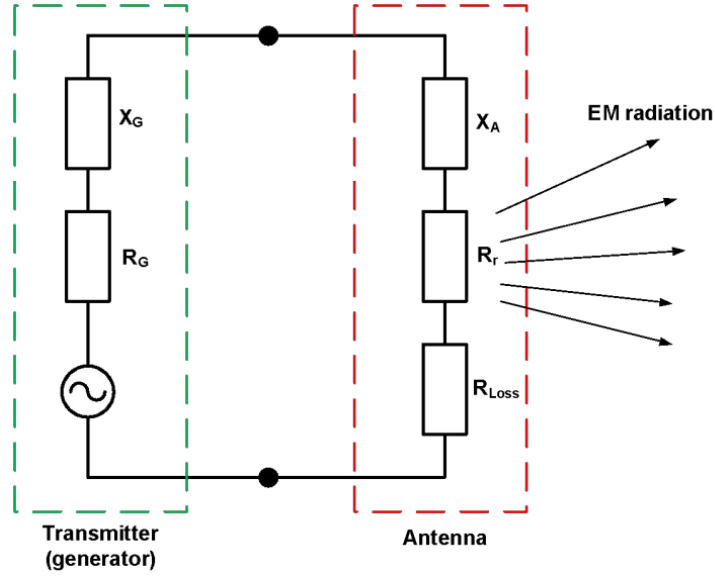


Figure 2: Circuit model of a transmitting antenna

Radiation resistance is the equivalent resistance accounting for the signal loss due to radiation.

Radiation efficiency ($\eta = R_r / (R_r + R_{\text{Loss}})$) is the ratio of the radiated power to the total power supplied to the antenna. Electrically small antennas tend to have small radiation efficiency.

8. Define VSWR, Return Loss and relative bandwidth.

Voltage Standing Wave Ratio: $\text{VSWR} = (1 + |\Gamma|) / (1 - |\Gamma|)$, where $\Gamma = (Z_L - Z_0) / (Z_L + Z_0)$ is the reflection coefficient.

Return Loss: $\text{RL} = -20 \log_{10} |\Gamma|$ in dB.

Relative bandwidth: $\text{BW} = (f_2 - f_1) / f_0 = (f_2 - f_1) / \sqrt{f_1 f_2}$.

9. What is the physical meaning of the free space Green's function?

The free space Green's function corresponds to the one of a spherical wave.

10. Elementary electric dipole – what field components it does have (if z-oriented) in near and far field? What is its farfield pattern and why it is not isotropic even for dipole length approaching 0.

- Near (reactive) field region $kr \ll 1$: Fields are similar to those of a static electric dipole and to the of a static current element. E_r and E_θ are out-of-phase with H_φ . There is no time-average power flow nor radiated power; energy is stored in the near-zone.
- Intermediate field region $kr > 1$: Field are similar to those of a static electric dipole and to the of a static current element (quasistationary fields). E_θ and H_φ approach time-phase which means the beginning of time-average power flow in the outward (radial) direction.
- Farfield region $kr \gg 1$: Most important region of an antenna. E_r vanishes and only transversal (to r) field components (E_θ and H_φ) remain.

It can't be isotropic even for dipole length approaching 0 due to its farfield pattern $f(\theta, \varphi) = \sin(\theta)$. Magnetic dipole has farfield components E_φ and H_θ .

11. *Define radiation intensity and antenna directivity. Directivity vs electrical size of antenna. Radiation pattern properties (sidelobe level, front-to-back ratio, half power beamwidth, polarization).*

Radiation intensity U is defined as the power radiated from an antenna per unit space angle (in steradians) and is related to the farfield E of the antenna:

$$U(\theta, \varphi) = r^2 S(\theta, \varphi) = \frac{r^2}{2Z_0} \|E(\theta, \varphi)\|^2.$$

Directivity is the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions, i.e., isotropic source:

$$D(\theta, \varphi) = \frac{U(\theta, \varphi)}{U_0} = \frac{4\pi U(\theta, \varphi)}{P_r}.$$

Maximum directivity is then given by

$$D_{\max} = \frac{U_{\max}}{U_0} = \frac{4\pi}{\oint_S f_n^2(\theta, \varphi) dS}.$$

For example for the elementary electric dipole, $f_n(\theta, \varphi) = \sin(\theta)$ and $D_{\max} = 3/2$ in linear scale which corresponds to $10 \log(3/2)$ dBi ≈ 1.76 dBi.

Sidelobe level: radiation intensity level of the most significant sidelobes.

Front-to-back ratio: power ratio of the main lobe to the back lobe.

HPBW: bandwidth across which half of the power is radiated.

Polarization is defined as the property of an EM wave describing the time-varying direction and relative magnitude of \mathbf{E} . Most common types are linear and circular (LHC/RHC).

12. *Can we speak about farfield if an antenna is embedded in an lossy space of infinite background?*

We cannot since the radiation pattern is defined on a sphere with the radius approaching infinity. Thus it does not make sense to speak about it in a lossy environment.

13. *How to evaluate radiation resistance from radiation pattern and are there other possibilities?*

One possibility is to integrate the Poynting vector \mathbf{S} over a closed surface around the antenna to obtain the radiated power $P_r = I^2 R/2$ from which we can obtain the radiation resistance R .

An alternate way is to calculate the radiated power P_r using source currents instead of fields.

14. *What is EIRP?*

EIRP stands for effective isotropic radiated power and it is the total power which must be radiated by an isotropic antenna in order for it to yield the same radiation intensity in a given direction. The units of EIRP are watts (W).

15. *Explain the physical meaning of the following integral:*

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int_{V'} \mathbf{J}(\mathbf{r}') \frac{e^{-ikR}}{R} dV'$$

The integral states that the field vector potential is a sum of contributions in the form of source currents \mathbf{J} multiplied by the Green's function of a spherical wave.

16. Mark r terms which contribute to near and far field. Which antenna does produce these fields? What is its orientation in cartesian coordinates?

$$\begin{aligned}
 H_\varphi(r, \theta) &= \frac{ikIL}{4\pi} \sin(\theta) \left[\underbrace{\frac{1}{r}}_{\text{farfield}} + \underbrace{\frac{1}{ikr^2}}_{\text{nearfield}} \right] e^{-ikr}, \\
 E_r(r, \theta) &= \frac{Z_0 IL}{2\pi} \cos(\theta) \left[\underbrace{\frac{1}{r^2}}_{\text{nearfield}} + \underbrace{\frac{1}{ikr^3}}_{\text{nearfield}} \right] e^{-ikr}, \\
 E_\theta(r, \theta) &= \frac{ikZ_0 IL}{4\pi} \sin(\theta) \left[\underbrace{\frac{1}{r}}_{\text{farfield}} + \underbrace{\frac{1}{ikr^2} - \frac{1}{k^2 r^3}}_{\text{nearfield}} \right] e^{-ikr}, \\
 H_r &= H_\theta = 0, \\
 E_\varphi &= 0.
 \end{aligned}$$

From simple vanishing properties of polynomials: only the terms of order -1 don't vanish in farfield. The above is the solution for the elementary electric dipole oriented in the z -axis and its length approaches 0.

17. Current distribution on linear (wire) antenna, what approximations are used (constant, triangular and sinusoidal current)?

Assuming z -axis orientation (thin antenna), the distributions take form of

$$I(z) = I_0 \left(1 - \frac{2|z|}{L} \right), \quad (\text{Triangular})$$

$$I(z) = I_0 \sin \left(k \left(\frac{L}{2} - |z| \right) \right). \quad (\text{Sinusoidal})$$

For length $L = 0.5\lambda$, we obtain the sinusoidal distribution, the triangular distribution arises for smaller dipoles, and the constant distribution is the case of the elemental dipole.

18. Sketch current distribution at dipoles of lengths 0.1λ , 0.5λ , 1λ , 1.25λ , 2λ , \dots . What effect do out-of-phase currents have on farfield? Out-of-phase currents (caused by $L > \lambda$) produce

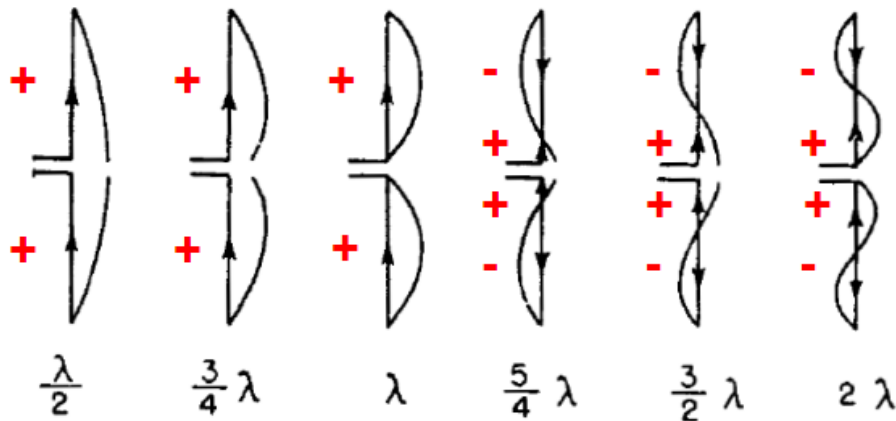


Figure 3: Current distributions for different dipole lengths

sidelobes in the farfield.

Missing distribution of $L = 0.1\lambda$: triangular distribution.

19. *Evaluation of near and far field of linear antennas – far field approximation, Fourier transform between current and far field, polarization projections, ...*

The farfield approximation revolves around the expression

$$R \approx r - \Delta = r - z' \cos(\theta) = r - \mathbf{r}' \cdot \mathbf{e}_r.$$

The Δ term's contribution can then be neglected in amplitude but not in phase. This approximation leads to the useful conclusion that the radiated field is directly proportional to the Fourier transform of the source currents.

20. *Directivity of linear antennas (1.25λ dipole)*

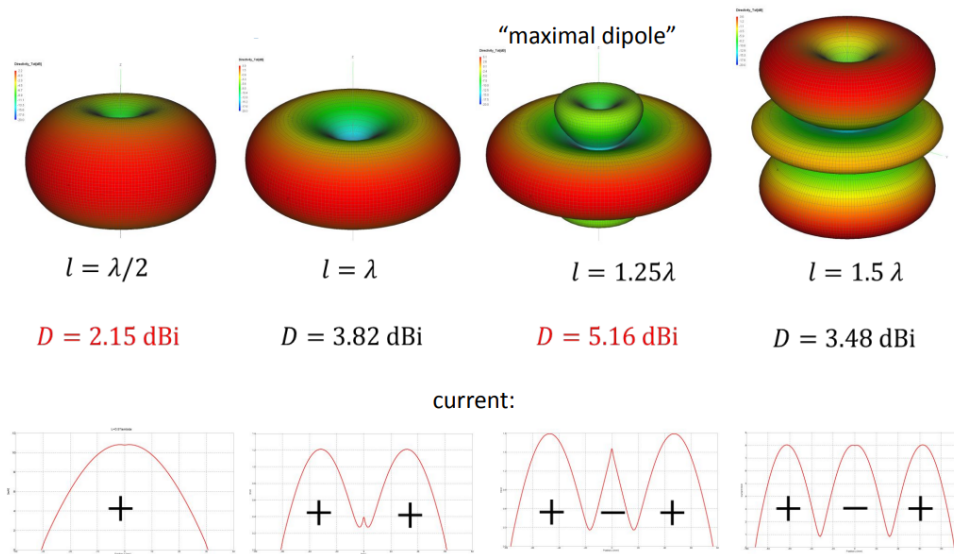


Figure 4: Radiation patterns of dipoles

21. *$\lambda/2$ dipole properties (input impedance, pattern), shortening to resonance, bandwidth*

Radiation pattern: omnidirectional in the H -plane which is a useful property for many applications including mobile communications.

Directivity: reasonable value (2.15 dBi), larger than of short dipoles.

Input impedance: 73Ω which is pretty much well matched with a standard transmission line of characteristic impedance of 75Ω ; not sensitive to changes in the radius of the dipole.

Shortening: the dipole itself is not exactly resonant and hence should be shortened by a little amount depending on the radius.

Bandwidth: 5 to 15 % from the central frequency depending on the input impedance.

22. *Folded $\lambda/2$ dipole – impedance, pattern*

Its impedance is about 4 times of the normal dipole which significantly increases its bandwidth. The pattern is similar to the normal dipole I think?

23. *Symmetrization, baluns*

Symmetrization problem arises when connecting an unbalanced transmission line to the antenna. The balanced mode is when there are equal and opposite currents.

Balun transforms the balanced input impedance of the dipole to the unbalanced impedance of the coaxial line such that there is no net current on the outer conductor of the coax.

Further comments: non-symmetrical feeders are cheaper and simpler but require a symmetrization element (balun) for connection to the symmetrical antenna.

24. *Monopole antennas (impedance, pattern compared to dipoles), method of images*
 Impedance: roughly half the dipole version
 Gain: roughly double (+3 dBi)
 Pattern: radiates only above ground
 Method of images: mathematical replacement of the ideally infinite ground plane by a virtual opposite electrode creating dipole structure for radiation. In practice, we use radial lines (pieces of wires) instead of the ideal infinite ground plane.
25. *Horizontal dipole above ground – method of images*
 By superposition, the ground plane above which the horizontal dipole sits can be modelled by a virtual image dipole with opposite current feed in the halfspace below ground plane.
26. *Explain what the terms in braces physically represent:*

$$E_{\theta}(r, \theta, \varphi) = \underbrace{\frac{ikZ_0}{4\pi} \frac{e^{-ikr}}{r} \sin(\theta)}_A \underbrace{\int_{-l/2}^{l/2} I_z(z') e^{ikz' \cos(\theta)} dz'}_B$$

A: element factor (elementary dipole contribution)

B: space factor (Fourier transform of the sources)

2 Aperture antennas

1. *Aperture antennas – equivalent source approach*
 First, we envelop the antenna with a closed surface because we are interested in the field far beyond the antenna, not the field nearby. Then we substitute the sources distributed inside the closed surface ('volume of the antenna') by sources only on the surface. These currents must be so that they produce the same field around the antenna. Lastly we set the fields inside equal to zero which alters the field inside where we don't care.
2. *Aperture in infinite ground plane and in free space – what equivalent sources to use?*
 Infinite ground plane: $\mathbf{M} = -2\mathbf{n} \times \mathbf{E}$.
 Free space: $\mathbf{M} = -\mathbf{n} \times \mathbf{E}$ and $\mathbf{J} = \mathbf{n} \times \mathbf{H}$. Additionally we demand that the feeding waveguide contains a TEM wave.
3. *Physical meaning of radiation integrals in near and far field. Far field distance, conditions for far field (1/r dependence, E/H ratio, transversal fields)*
 Aperture radiation can be imagined as the radiation of an infinite number of point sources placed in the aperture. The radiation integrals are a superposition of these sources' contributions.
 For nearfield, we need a numerical solution using original sources. Farfield described as distance greater than $2D^2/\lambda$, where D is the maximal dimension of the antenna, allows for approximation which yields analytical solution. In farfield, impedance is equal to the impedance of the free space $Z = E/H = 120\pi \approx 377 \Omega$.
4. *Structure of the far field (Fourier transform \times obliquity factors). Relation to array.*
 In farfield, the field can be expressed as the Fourier transform of the sources multiplied by obliquity factors, which include projection from Cartesian to spherical coordinates and a relation between E and H .
5. *Huygens source properties, element factor.*
 Aperture antennas which can be considered Huygens sources have the virtue of perpendicular electric and magnetic fields. Furthermore:

- $E/H = Z_0 = 120\pi$,
- locally a plane wave,
- radiation pattern is a cardioid,
- element factor $(1 + \cos(\theta))/2$.

6. *Farfield of aperture with constant (amplitude, phase) source field*

It can be expressed as a product of Fourier transforms of the source fields. Since constant source field means rectangular distribution, the far field is a product of sinc functions.

7. *Directivity of aperture antennas (effective area)*

Directivity of aperture antennas can be easily computed using the effective area:

$$D_{\max} = \frac{4\pi U_{\max}}{P_r} = \frac{4\pi}{\lambda^2} \frac{\left| \int_{S_A} \mathbf{E}_a(x', y') dx' dy' \right|^2}{\int_{S_A} \|\mathbf{E}_a(x', y')\|^2 dx' dy'}$$

$$= \frac{4\pi}{\lambda^2} A_{\text{eff}} = \frac{4\pi}{\lambda^2} \underbrace{A_{\text{phys}}}_{A \cdot B} \underbrace{\eta_{\text{amp}}}_{0.81} \underbrace{\eta_{\text{phase}}^E(s)}_{0.8 @ s_{\text{opt}}} \underbrace{\eta_{\text{phase}}^H(t)}_{0.79 @ t_{\text{opt}}}$$

8. *Sketch the field distribution in a rectangular waveguide with the TE₁₀ mode. What is the amplitude and phase at its aperture?*

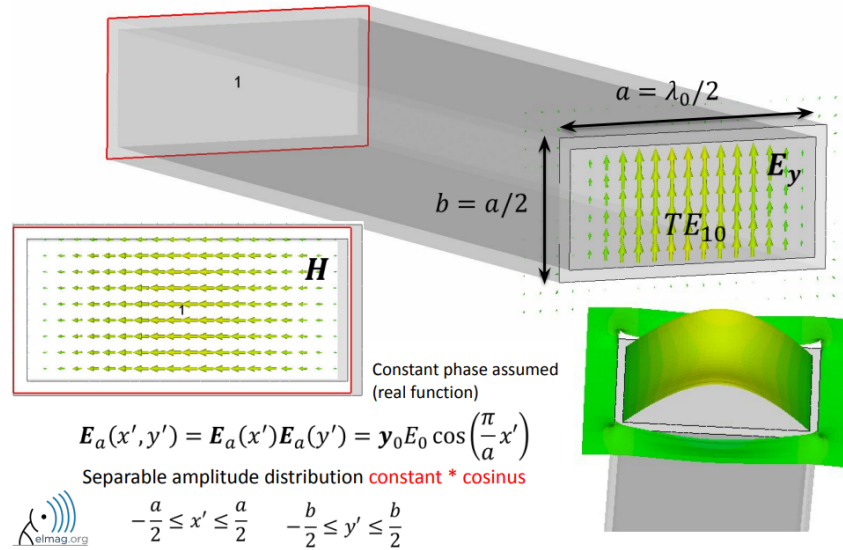


Figure 5: Rectangular waveguide in free space

9. *Why does its radiation pattern differ in its E and H plane?*

Because the field distribution in the aperture is different in each plane. One is constant (thus a sinc function field) and the other is a cosine (narrow field).

10. *1D aperture with linear and quadratic phase – effects on pattern. Quadratic phase error – where does it appear?*

(a) linear phase variation: $\phi(x) \sim \beta x$

- HPBW increase
- directivity decrease

(b) quadratic phase variation: $\phi(x) \sim \beta x^2$

- side-lobe levels rise
- minimums rise (filling zeros)
- loss in gain (widening of the main lobe)
- due to displacement of the reflector feed from focus, distortion of the reflector or lens, or a non-ideally-spherical wavefront of the feed, curved field in the aperture of a radiator

11. *Polarization of aperture antennas*

Decomposition of the radiated fields into co-polarization (intended to radiate) and cross-polarization (orthogonal to it). Highly dependent of the coordinate system: Ludwig-3 is the best because it gives the impression of polarization for aperture antennas and because it yields zero cross-polarization for Huygens sources.

12. *Horn antennas – basic properties, why we use them and for what*

- Widening of a waveguide aperture, hence increase in gain
- low VSWR, fairly wide BW
- easy to manufacture/construct
- aperture efficiency of 50 to 80 %
- primary feed for reflector antennas, radar, satellite, etc.

13. *Phase effects in the aperture of a horn*

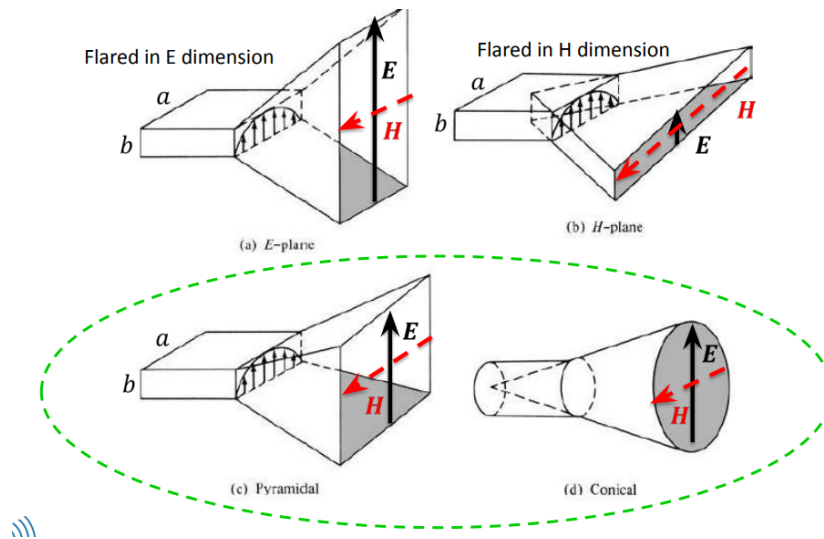


Figure 6: Types of horn antennas

14. *Phase effects in the aperture of a horn*

Due to the boundary conditions, electric field must be perpendicular to the slanted metal walls of the aperture, causing the field in the aperture to be slightly curved. This introduces a quadratic phase error, thus lower gain.

15. *Directivity vs aperture size (phase distortion), optimal horn, aperture efficiency*

For a fixed axial length, the directivity increases by virtue of the increased aperture area. Optimum performance is reached for $t_{\text{opt}} = 3/8$ which corresponds to a phase lag at the aperture edges of $\delta = 135^\circ$. Further increase beyond the optimum results in cancellations in the far field, thus a decrease in directivity.

16. What is the phase center of a horn antenna

Apparent center of the spherical waves that emanate from the horn at a given radial distance, usually farfield. It is important for measurement and for reflector antennas – phase center should always be aligned with the reflector focal point. For horns, the phase center is located inside the horn.

17. Horns with mixed modes in aperture, why we do that. Polarization of horn antennas

Usually, we mix modes to obtain Huygens source, i.e., a source with no field curvature. Such a source produces a rotationally symmetrical radiation pattern which implies no cross-polarization.

18. Explain the farfield approximation of the Green's function $\exp(-ikR)/R$ using figure 7:

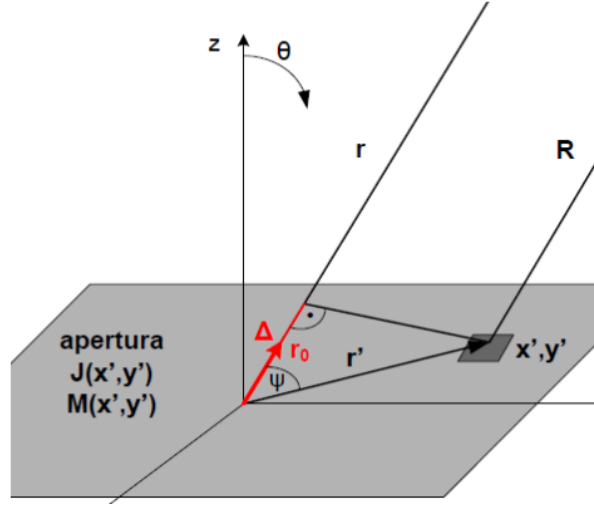


Figure 7: Farfield approximation illustration

Using $R = r - \Delta = r - z' \cos(\theta)$, we can put

$$\frac{e^{-ikR}}{R} = \frac{e^{-ik(r-\Delta)}}{r-\Delta} \approx \frac{e^{-ikr}}{r} e^{ik\Delta}.$$

The approximation is done assuming that $r \gg \Delta$ which means the resulting amplitude doesn't change much when we neglect the term in the denominator. However, doing this in the numerator would result in a huge error because phase is much more sensitive to small changes due to the behaviour of $\exp(ix)$. This approximation is the reason why we can use Fourier transform of the source currents to calculate farfield.

19. What does the following integral describe and where is it used?

$$\mathbf{P}(\theta, \varphi) = \int_S \mathbf{E}_a(x', y') e^{ik(x' \sin(\theta) \cos(\varphi) + y' \sin(\theta) \sin(\varphi))} dx' dy'$$

It is called the radiation vector and it is basically a Fourier transform of the aperture field. We can also regard it as a Huygens-like superposition of plane waves thanks to the exponential term.

20. Explain all terms in the following equation. What does this equation represent?

$$E_\theta(\theta, \varphi, r) = \underbrace{\frac{iE_0 ab}{\lambda}}_{\text{const.}} \underbrace{\frac{e^{-ikr}}{r}}_{\text{spherical wave}} \underbrace{\frac{1 + \cos(\theta)}{2}}_{\text{obliquity factor}} \underbrace{\sin(\varphi)}_{\text{polarization projection}} \underbrace{\frac{\sin(k_x \frac{a}{2})}{k_x \frac{a}{2}} \frac{\sin(k_y \frac{b}{2})}{k_y \frac{b}{2}}}_{\text{F.T of the source field}}$$

This equation represents far field from an aperture with constant field in both dimensions (constant illumination).

21. What does the graph in figure 8 describe?

It describes the impact of the quadratic phase error $\phi(x) = \beta(2/a)^2 x^2$. In the figure, $\beta = 0$ represents constant phase and $\beta = \phi/2$ represents a path length deviation of $\lambda/4$ from constant phase at the edges of the aperture.

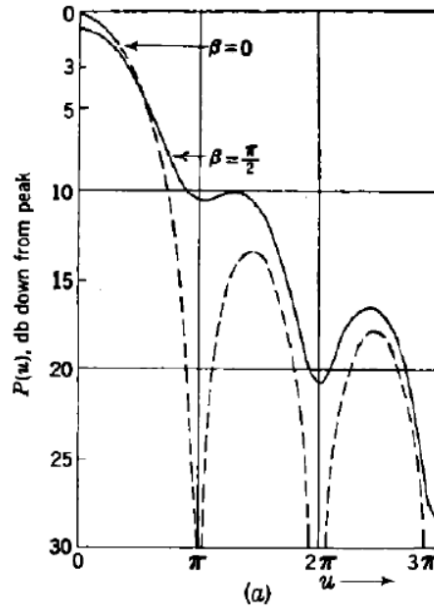


Figure 8: Constant aperture illumination

3 Reflector antennas

1. Why do we use the parabolic reflector antenna? What is its physical principle?

There are two big reasons to use parabolic reflector antennas: very high gain and a narrowly-directed beam with low sidelobe levels. The physical principle is that it transforms spherical wave into plane waves and vice versa.

2. What happens if the feed is off the focus of a parabolic reflector antenna?

An offset causes phase errors which can slightly break the radiation pattern.

3. Explain/sketch illumination and spillover loss in a parabolic reflector antenna.

See figure 9 for illustration. Illumination loss occurs because the feed's radiation pattern doesn't have a parabolic shape, hence the amplitude is not constant. Only a constant amplitude on the reflector would mean 100% amplitude efficiency. Spillover loss is due to the fact that a part of the pattern 'spills over' the reflector. An ideal feed (no illumination or spillover loss) cannot be fabricated because the goals contradict each other.

4. What value of aperture efficiency is typical for the parabolic reflector antenna? What edge taper corresponds to the maximum efficiency?

Aperture efficiency 75 to 82 %; optimal edge taper -11 dB.

5. What are the properties of an 'ideal' feed for a parabolic reflector antenna?

An ideal feed produces uniform amplitude and phase distribution which compensates for

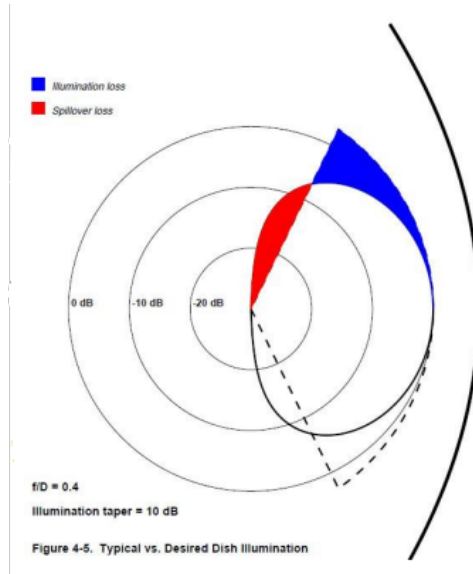


Figure 9: Illumination and spillover loss

spherical spreading loss and doesn't have spillover (cannot be made in practice). The feed should aim to accomplish the following goals:

- its pattern should be rotationally symmetrical (balanced feed);
- its pattern should be such that the reflector edge taper is -11 dB;
- have a point phase center located at the focal point of the reflector;
- be small in order to reduce blockage – usually its diameter of the same order as wavelength;
- have low cross-polarization, usually below -30 dB.

6. What effects mostly contribute to an antenna's noise temperature?

Elevation angle, spillover.

4 Antenna arrays

1. Canonical arrays based on isotropic radiators, basic configurations, wavefront canceling, endfire/broadside arrays

The arrays are depicted in 10, 11 and 12.

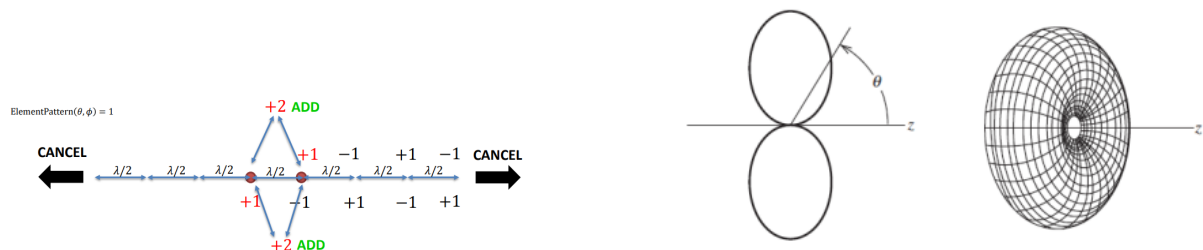


Figure 10: Canonical array with in-phase excitation

2. What is a canonical minimum scattering antenna, its importance in arrays. Element pattern, embedded pattern, array factor. Mutual coupling, mutual impedance.



Figure 11: Canonical array with out-of-phase excitation

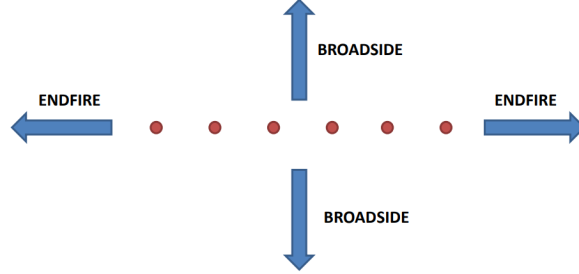


Figure 12: Broadside and endfire arrays

Antennas with identical radiation patterns can differ in the manner and extent to which they modify an incident wave, i.e., in the way they scatter. A canonical minimum-scattering antenna (CSMA) is defined as one which becomes ‘invisible’ when the accessible waveguide terminals are open-circuited. The scattering matrix of such an antenna is shown to be unique once N arbitrary orthogonal radiation patterns have been specified.

Element pattern is the contribution of a single antenna in the array. It can be measured either isolated or embedded. Embedded means the antenna is placed in the array configuration with just one port excited and others terminated by 50Ω , i.e. isolated element pattern with scattering from the surrounding array.

Array factor is the other part of the total array radiation pattern. It is dependent on the geometry of the configuration. If we use minimum scattering antennas, the array radiation pattern is given as a product of the element pattern and the array factor.

Mutual coupling represents the interaction of antennas in an array. Due to this effect, we define impedances

$$Z_{ij} = \frac{U_i}{I_j} \bigg|_{\forall k \neq j : I_k = 0}$$

for $i, j \in \{1, 2, \dots, N\}$ in an N -antenna array. Cases of $i = j$ correspond to so-called mutual impedances, whereas for $i \neq j$, we speak of mutual impedances. These make up the impedance matrix $[Z_{ij}]$. Additionally, it holds that $Z_{ij} = Z_{ji}$.

3. Equally-spaced isotropic array – properties, linear phasing

For general equally-spaced isotropic arrays, it holds that

$$AF = \sum_{n=0}^{N-1} I_n e^{iknd \cos(\theta)}.$$

If the current has linear phase progression $I_n = A_n e^{in\alpha}$, we obtain

$$AF = \sum_{n=0}^{N-1} A_n e^{in\psi},$$

where $\phi = kd \cos(\theta) + \alpha$. Furthermore, for a uniform array (same amplitudes A_0), we get

$$AF = A_0 e^{i(N-1)\psi/2} \frac{\sin(N\psi/2)}{\sin(\psi/2)}.$$

Using this, we can tune the so-called scan angle θ_0 and optimize α to obtain a broadside ($\theta_0 = \pm 90^\circ$, $\alpha = 0$) or an endfire ($\theta_0 \in \{0^\circ, 180^\circ\}$, $\alpha = \pm kd$). Other options include the Hansen-Woodyard endfire array (increased directivity) or the Superdirective endfire array.

4. *Amplitude taper in arrays, its effect on pattern*

More tapering towards the edges means more side lobe level reduction.

5. *Can be maximal directivity provided by a broadside or an endfire array?*

Endfire arrays can reach more interesting values of directivity.

6. *Horizontal dipole above ground – directivity, efficiency and impedance properties*

- Directivity: depends on the height of the dipole above ground,
- efficiency: bad,
- impedances: $Z_{11} = Z_{22}$, $Z_{12} = Z_{21}$, $Z_{in} = Z_{11} - Z_{12}$.

7. *Array directivity optimization, superdirectivity, sensitivity*

Directivity optimization is done via tuning of the phase and amplitude distribution. Superdirectivity is very sensitive, thus it is imperative to have a precise feed.

8. *What does the following equation describe?*

$$AF = I_0 + I_1 e^{ikd \cos(\theta)} + I_2 e^{ik2d \cos(\theta)} + \dots = \sum_{n=0}^{N-1} I_n e^{iknd \cos(\theta)}$$

It describes the array factor of a general equally-spaced array of isotropic radiators.

9. *Sketch the element pattern, array factor and total pattern in the xz -plane for the following array:*

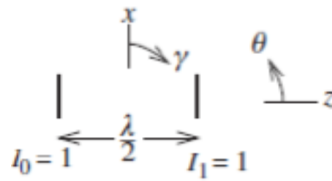


Figure 13: Array configuration

See figure 14

10. *Receiving antenna, circuit model*

See figure 15

11. *Effective length, effective aperture*

Effective length: $U_{load} = l_{eff} E$.

Effective area: $P_{load} = A_{eff} S_{inc}$, boundary $A_{eff,max} = (\lambda^2/4\pi) D_{max}$.

Both are virtual constructs for transforming incident quantity (electric field/power density) to load quantity (voltage/power).

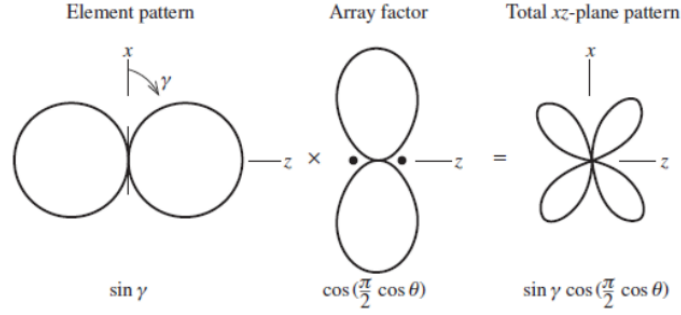


Figure 14: Array configuration patterns

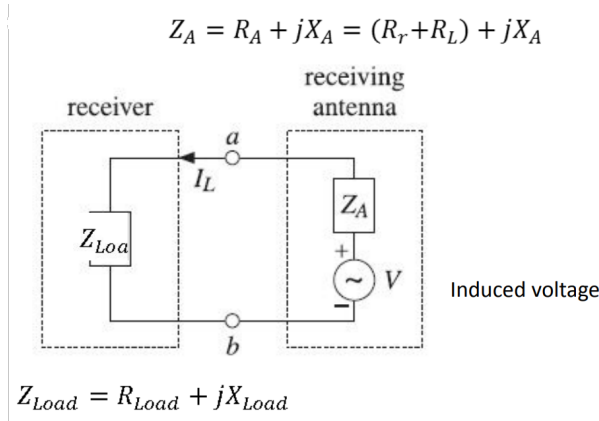


Figure 15: Circuit model of a receiving antenna

12. *Effective aperture of the isotropic radiator and an antenna of arbitrary directivity D*
Effective aperture of an antenna with arbitrary directivity is

$$A_{\text{eff}} = \frac{\lambda^2}{4\pi} D$$

which means that an isotropic radiator ($D = 1$) has the effective aperture of $\lambda^2/(4\pi)$.

13. *Friis' transmission equation*

$$P_{\text{RX}} = P_{\text{TX}} G_{\text{TX}} G_{\text{RX}} \left(\frac{\lambda}{4\pi R} \right)^2$$

14. *Explain what the following equation describes:*

$$S_{\text{RX}} = \frac{P_{\text{TX}} G_{\text{TX}}}{4\pi R^2} \frac{\sigma}{4\pi R^2}.$$

This is the radar equation and it expresses the power density of an echo signal received at the radar, reflected back from a target. Parameter σ ($[\sigma] = m^2$) is the Radar Cross-Section (RCS) and it's a characteristic of the target as a measure of its size as seen by the radar.

15. *What is the difference between monostatic and bistatic radar?*

See figure 16.

16. *Reciprocity theorem for antennas and its consequence for mutual impedances*

Let us assume we apply voltage V_a on antenna A and measure induced current I_b on antenna B. Then we do the same vice versa: apply voltage V_b on antenna B and measure induced current I_a on antenna A. The reciprocity theorem for antenna states that if $V_a = V_b$, then $I_a = I_b$. As a consequence, $Z_{12} = Z_{21}$.

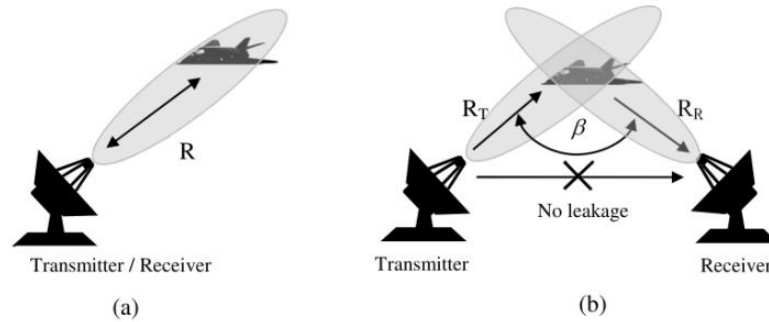


Figure 16: Monostatic and bistatic radar

17. *Babinet's principle*

An antenna has the exact same radiation pattern as its complementary structure which is the same configuration with 'air and metal flipped', i.e. a slot in an infinite metal sheet of the same dimensions as the original antenna. An example of this duality is a dipole strip antenna and a slot antenna.

18. *Slot antenna vs strip dipole*

Following from the Babinet's principle, their radiation pattern are ideally the same (practically not due to a finite metal sheet). However, the slot antenna has higher impedance. Furthermore, the equivalent circuit of a dipole is a serial RLC circuit, whereas for the slot antenna, its equivalent RLC circuit is parallel. They also differ in polarization.

19. *Slot antenna array based on rectangular waveguide*

It is an array of fairly decent radiation efficiency. In a rectangular waveguide,¹ we create a standing wave which we then allow to radiate outside. The only source of radiation is the field in the slots which we can tune to set desired amplitude and reduce sidelobe levels. The slots are created in a zig-zag fashion in order to achieve in-phase pattern.

20. *Show the polarization and the radiation pattern in both planes of the antenna in figure 17:*



Figure 17: Slot antenna array

See figure 18.

21. *Rectangular microstrip antenna – basic structure, field distribution, physical principle of radiation and parameters*

Basic structure of rectangular microstrip antennas is identical to microstrip resonators except they are designed to radiate. Typical characteristics:

- incredible ease of manufacturing,
- moderate gain (7-9 dBi single element),

¹The waveguide can be either fed from one side and shorted on the other or it can have both sides shorted and be fed from a input hole in the rear wall.

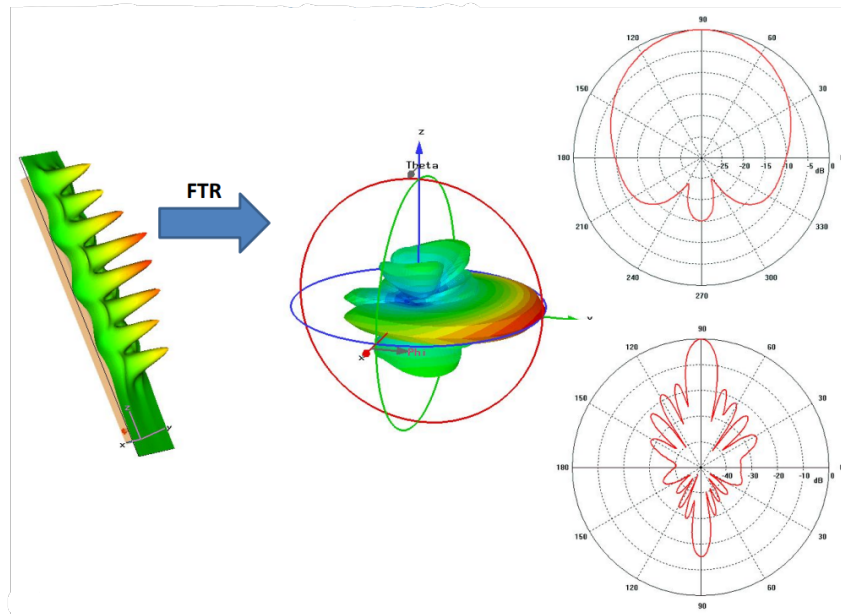


Figure 18: Slot antenna array patterns

- usually low bandwidth due to its resonant nature,
- very good integration with a microwave circuit.

Radiation from a rectangular patch antenna can be modelled as of two slots in a metal plate which is equivalent to two parallel RLC circuits connected in parallel. The field distribution is illustrated in figure 19.

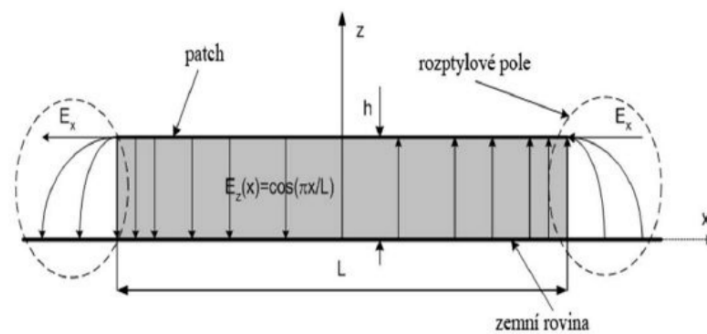


Figure 19: Patch antenna field distribution

22. Feeding of microstrip antennas for linear and circular polarization

The most common way to feed a microstrip antenna is by a coaxial line. This can be easily done by 'sticking out' the central electrode and using it to excite a mode of electrical field in the microstrip. At the same time, we shouldn't stick the central electrode in the middle of the microstrip because that's the point of zero electric field in a microstrip. Furthermore, sticking out the central electrode introduces a parasitic inductance to the transmission so bending it is a good idea to counterweigh it with a parasitic capacitance.

Ways of feeding a microstrip antenna in order to radiate circularly-polarized waves is illustrated in figure 20.

23. Frequency independent antennas – self complementary antennas. By which dimensions are they specified?

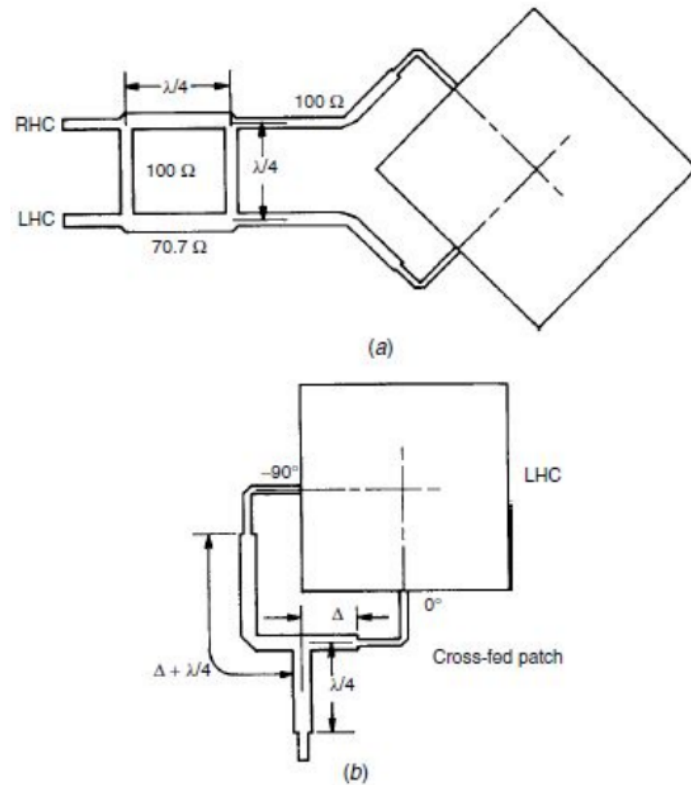


Figure 20: Feeding a microstrip antenna to produce a circular polarization

24. *Log-per antenna properties (directivity, bandwidth, phase center), compare to Yagi-Uda*
25. *Helix antenna modes and their radiation and impedance properties*

5 Characteristic modes

1. *Antenna as an external resonator – Characteristic modes (CM). Impedance matrix decomposition. Does CM depend on feeding? What happen with modes when you feed the antenna?*
2. *Physical properties of CM – characteristic currents and eigenvalues, orthogonality, resonance*
3. *Are CM important for electrically small or large antennas?*
4. *Sketch first few modes on a electric dipole*
5. *Excitation of modes, importance of CM for design of mobile antennas*

6 Q factor

1. *Definition of quality factor Q*
2. *Relation between Q and bandwidth of antenna.*
3. *Q factor and size of the antenna*

4. *Stored energy for antenna in frequency domain – why is it infinite by definition?*
5. *Q factor from input impedance*
6. *Q factor for coupled structures, will it be higher for in- or out-of phase currents and why?*