Retarded potentials, example for electric vector potential $\boldsymbol{A}$

General time variation of sources

$$\boldsymbol{A}(\boldsymbol{r},t)=\frac{\mu}{4\pi}\int_{V'}\frac{\boldsymbol{J}(\boldsymbol{r}',t-\frac{R}{c})}{R}\mathrm{d}V'$$

where $t-\frac{R}{c}$ is the retardation (c being velocity of the wave) – delay between the EM wave emission and the moment at which it reaches the observer

Now assume sinusioidal dependence so we can extract the time variation into the $e^{j\omega t}$ term and work with phasors:

$\boldsymbol{J}(\boldsymbol{r}',t)\rightarrow\boldsymbol{J}(\boldsymbol{r}')e^{j\omega t}$

$\boldsymbol{A}(\boldsymbol{r},t)\rightarrow\boldsymbol{A}(\boldsymbol{r})e^{j\omega t}$

$$\boldsymbol{A}(\boldsymbol{r})e^{j\omega t}=\frac{\mu}{4\pi}\int_{V'}\frac{\boldsymbol{J}(\boldsymbol{r}')e^{j\omega(t-\frac{R}{c})}}{R}\mathrm{d}V'=\frac{\mu}{4\pi}\int_{V'}\frac{\boldsymbol{J}(\boldsymbol{r}')e^{j\omega t}e^{-\frac{j\omega R}{c}}}{R}\mathrm{d}V'$$

cancel $e^{j\omega t}$

$$\boldsymbol{A}(\boldsymbol{r})=\frac{\mu}{4\pi}\int_{V'}\frac{\boldsymbol{J}(\boldsymbol{r}')e^{-jkR}}{R}\mathrm{d}V'$$

Since $k=\frac{2\pi}{\lambda}=\frac{2\pi}{\frac{c}{f}}=\frac{2\pi f}{c}=\frac{\omega}{c}$