Chapter 9

Crosstalk

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Outline

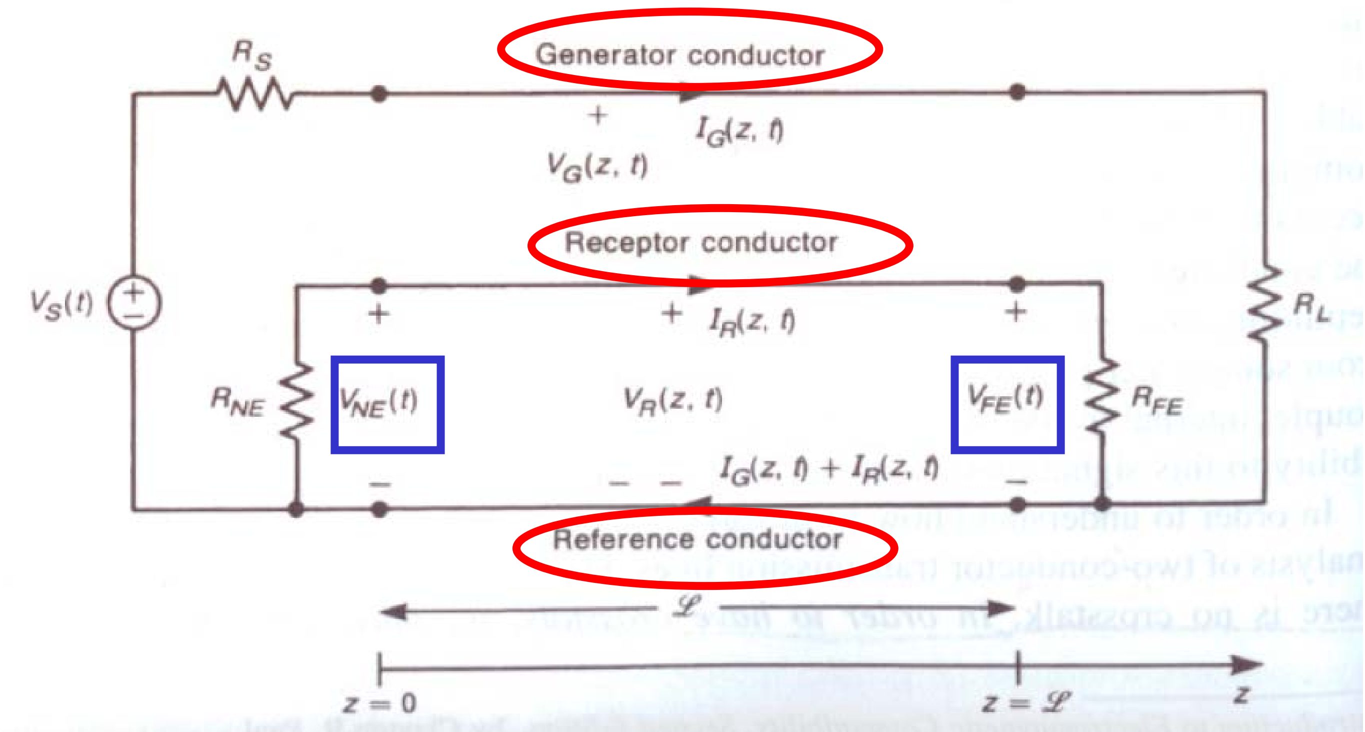
- Three-Conductor Transmission Lines and Crosstalk
- The Transmission-Line Equations for Lossless Lines
- The Per-Unit-Length Parameters
- The Inductive-Capacitive Coupling Approximate Model
- Lumped-Circuit Approximate Models
- Shielded Wires
- Twisted Wires

Preview

- Definition
  - Crosstalk is distinguished from antenna coupling in that it is a near-field coupling problem.
  - The design of the product such that it does not interfere with itself is another EMC concern.
  - Examples for this are the unintended electromagnetic coupling between wires and PCB lands that are in close proximity.

Three-Conductor Transmission Lines and Crosstalk

- Three-Conductor TX Lines Configuration
  - General Form
    - NE and FE refer to "near end" and "far end", respectively.



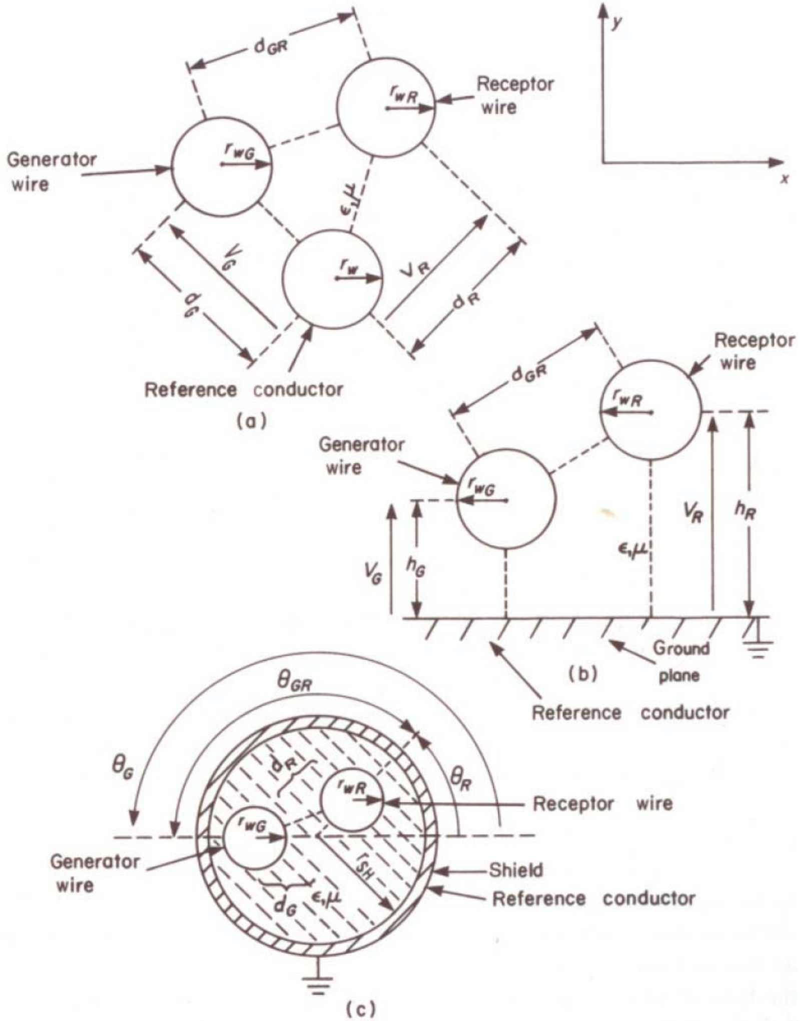
Three-Conductor Transmission Lines and Crosstalk

- Wire-Type Cross Sections

Three wires: inhomogeneous

Two wires above PEC: inhomogeneous

Two wires in a shield: homogeneous



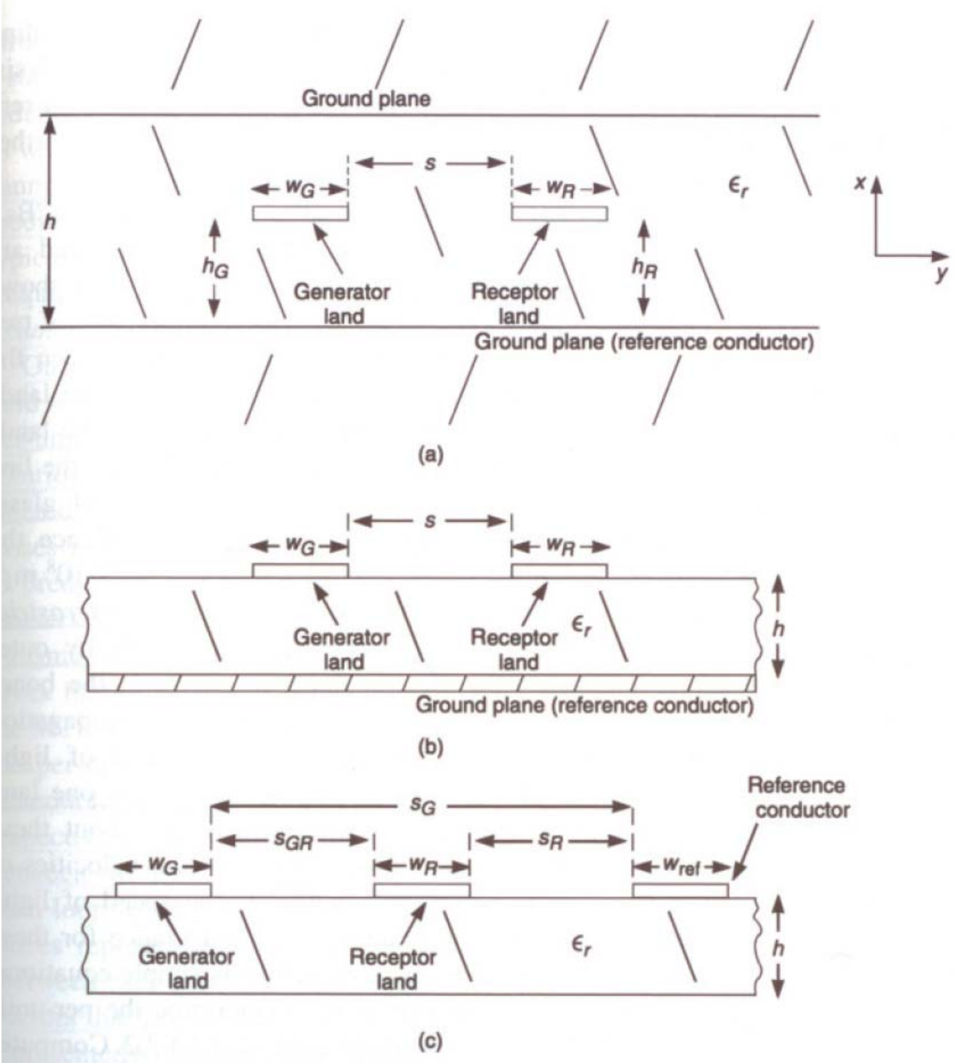
Three-Conductor Transmission Lines and Crosstalk

- PCB Land Cross Sections

Two striplines: homogeneous

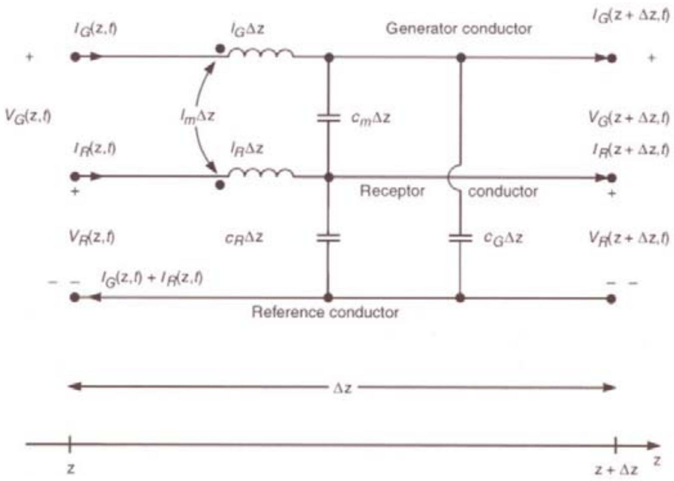
Two microstrip lines: inhomogeneous

Three microstrip lines: inhomogeneous



The Transmission-Line Equations for Lossless Lines

- Equivalent Circuit
  - Quasi-TEM Mode Assumption
    - Using KVL, we have



$$\frac{\partial V_G(z,t)}{\partial z} = -l_G \frac{\partial I_G(z,t)}{\partial t} - l_m \frac{\partial I_R(z,t)}{\partial t}$$

$$\frac{\partial V_R(z,t)}{\partial z} = -l_m \frac{\partial I_G(z,t)}{\partial t} - l_R \frac{\partial I_R(z,t)}{\partial t}$$

- Using KCL, we have

$$\frac{\partial I_G(z,t)}{\partial z} = -(c_G + c_m) \frac{\partial V_G(z,t)}{\partial t} + c_m \frac{\partial V_R(z,t)}{\partial t}$$

$$\frac{\partial I_R(z,t)}{\partial z} = c_m \frac{\partial V_G(z,t)}{\partial t} - (c_R + c_m) \frac{\partial V_R(z,t)}{\partial t}$$

The Transmission-Line Equations for Lossless Lines

- Equivalent Circuit
  - Quasi-TEM Mode Assumption
    - Combing in matrix form, we have

$$\frac{\partial}{\partial z}\mathbf{V}(z,t) = -\mathbf{L}\frac{\partial}{\partial t}\mathbf{I}(z,t)$$

$$\frac{\partial}{\partial z}\mathbf{I}(z,t) = -\mathbf{C}\frac{\partial}{\partial t}\mathbf{V}(z,t)$$

$$\mathbf{V}(z,t) = \begin{bmatrix} V_G(z,t) \\ V_R(z,t) \end{bmatrix}$$

$$\mathbf{I}(z,t) = \begin{bmatrix} I_G(z,t) \\ I_R(z,t) \end{bmatrix}$$

    - and the per-unit-length parameter matrices are

$$\mathbf{L} = \begin{bmatrix} l_G & l_m \\ l_m & l_R \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} (c_G + c_m) & -c_m \\ -c_m & (c_R + c_m) \end{bmatrix}$$

$$= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

The Transmission-Line Equations for Lossless Lines

- Equivalent Circuit
  - Quasi-TEM Mode Assumption
    - Transferring into phasor form, we have

$$\frac{d}{dz}\hat{\mathbf{V}}(z) = -j\omega\mathbf{L}\hat{\mathbf{I}}(z)$$

$$\frac{d}{dz}\hat{\mathbf{I}}(z) = -j\omega\mathbf{C}\hat{\mathbf{V}}(z)$$

    - where

$$\mathbf{V}(z,t) = \mathscr{R}e\{\hat{\mathbf{V}}(z)e^{j\omega t}\}$$

$$\mathbf{I}(z,t) = \mathscr{R}e\{\hat{\mathbf{I}}(z)e^{j\omega t}\}$$

    - and *L* and *C* matrix are unchanged.

The Per-Unit-Length Parameters

- Homogeneous versus Inhomogeneous Media
  - Homogeneous Media
    - If the surrounding medium is homogeneous, we have the relationship

$$\mathbf{LC} = \mathbf{CL} = \mu\epsilon\mathbf{1}_2 \qquad \mathbf{1}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

    - Thus, the capacitance matrix is obtained from

$$\mathbf{C} = \mu\epsilon\mathbf{L}^{-1} = \frac{1}{v^2}\mathbf{L}^{-1}$$

$$\longrightarrow \begin{bmatrix} c_G + c_m & -c_m \\ -c_m & c_R + c_m \end{bmatrix} = \frac{1}{v^2(l_G l_R - l_m^2)}\begin{bmatrix} l_R & -l_m \\ -l_m & l_G \end{bmatrix}$$

$$\longrightarrow c_m = \frac{l_m}{v^2(l_G l_R - l_m^2)}$$

$$c_G + c_m = \frac{l_R}{v^2(l_G l_R - l_m^2)}$$

$$c_R + c_m = \frac{l_G}{v^2(l_G l_R - l_m^2)}$$

The Per-Unit-Length Parameters

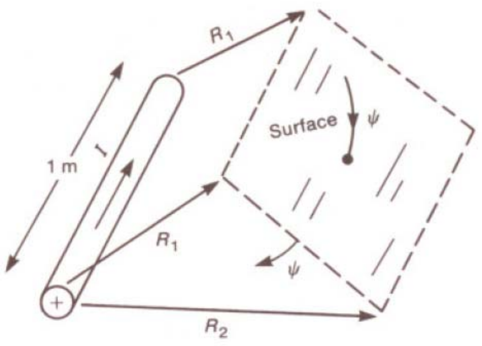
- Homogeneous versus Inhomogeneous Media
  - Inhomogeneous Media
    - Since the per-unit-length inductance matrix L is not affected by the dielectric inhomogeneity, it could be determined from the per-unit-length capacitance matrix with the dielectric removed $\boldsymbol{C_0}$, which is $\boldsymbol{L}=\mu_0 \varepsilon_0 \boldsymbol{C_0}^{-1}$
    - And the real capacitance matrix $\boldsymbol{C}$ is obtained with the dielectric exists.

Hence, we only need to develop one program for calculating both the capacitance C and C_0.

The Per-Unit-Length Parameters

- Wide-Separation Approximations for Wires
  - Three Wires
    - Assuming that the wires are separated sufficiently such that the charge and current distributions around the peripheries of the wires are essentially uniform.
    - The flux with direction shown is

$$\psi = \frac{\mu_0 I}{2\pi} \ln\left(\frac{R_2}{R_1}\right)$$



    - Since

$$\boldsymbol{\psi} = \mathbf{L}\mathbf{I} \longrightarrow \begin{bmatrix} \psi_G \\ \psi_R \end{bmatrix} = \begin{bmatrix} l_G & l_m \\ l_m & l_R \end{bmatrix} \begin{bmatrix} I_G \\ I_R \end{bmatrix}$$

$$\longrightarrow \begin{aligned} \psi_G &= l_G I_G + l_m I_R \\ \psi_R &= l_m I_G + l_R I_R \end{aligned}$$

The Per-Unit-Length Parameters

- Wide-Separation Approximations for Wires
  - Three Wires
    - The self and mutual inductance are obtained from

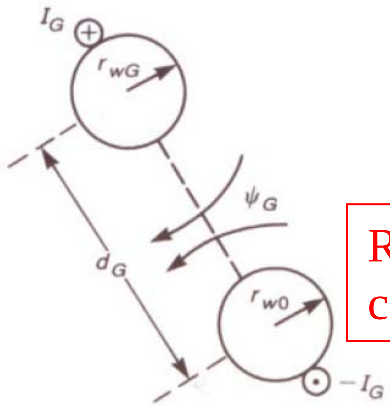
$$l_G = \left.\frac{\psi_G}{I_G}\right|_{I_R=0} \qquad l_m = \left.\frac{\psi_G}{I_R}\right|_{I_G=0}$$

$$l_m = \left.\frac{\psi_R}{I_G}\right|_{I_R=0} \qquad l_R = \left.\frac{\psi_R}{I_R}\right|_{I_G=0}$$

    - For the case of three wires, the self inductances are

$$l_G = \left.\frac{\psi_G}{I_G}\right|_{I_R=0} \rightarrow l_G = \frac{\mu_0}{2\pi}\ln\left(\frac{d_G}{r_{wG}}\right) + \frac{\mu_0}{2\pi}\ln\left(\frac{d_G}{r_{w0}}\right) = \frac{\mu_0}{2\pi}\ln\left(\frac{d_G^2}{r_{wG}r_{w0}}\right)$$

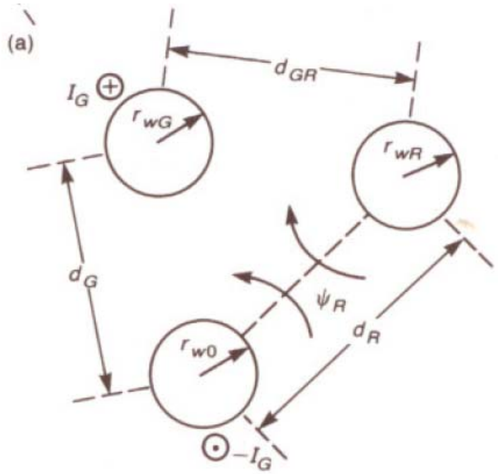
$$l_R = \left.\frac{\psi_R}{I_R}\right|_{I_G=0} \rightarrow l_R = \frac{\mu_0}{2\pi}\ln\left(\frac{d_R^2}{r_{wR}r_{w0}}\right)$$



Reference conductor

The Per-Unit-Length Parameters

- Wide-Separation Approximations for Wires
  - Three Wires
    - The mutual inductance is

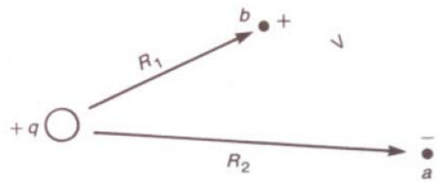


$$l_m = \frac{\psi_R}{I_G}\bigg|_{I_R=0} \longrightarrow l_m = \frac{\mu_0}{2\pi}\ln\left(\frac{d_G}{d_{GR}}\right) + \frac{\mu_0}{2\pi}\ln\left(\frac{d_R}{r_{w0}}\right)$$

$$= \frac{\mu_0}{2\pi}\ln\left(\frac{d_G d_R}{d_{GR} r_{w0}}\right)$$

    - The voltage between two points is

$$V = \frac{q}{2\pi\epsilon_0}\ln\left(\frac{R_2}{R_1}\right)$$



    - Since

$$\mathbf{q} = \mathbf{CV} \longrightarrow \begin{bmatrix} q_G \\ q_R \end{bmatrix} = \begin{bmatrix} c_G + c_m & -c_m \\ -c_m & c_R + c_m \end{bmatrix}\begin{bmatrix} V_G \\ V_R \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} V_G \\ V_R \end{bmatrix} = \begin{bmatrix} p_G & p_m \\ p_m & p_R \end{bmatrix}\begin{bmatrix} q_G \\ q_R \end{bmatrix} \qquad \mathbf{V} = \mathbf{Pq} = \mathbf{C}^{-1}\mathbf{q}$$

The Per-Unit-Length Parameters

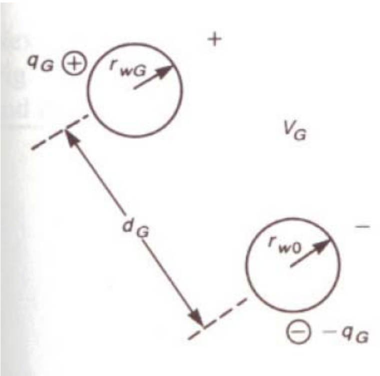
- Wide-Separation Approximations for Wires
  - Three Wires

$$V_G = p_G q_G + p_m q_R$$
$$V_R = p_m q_G + p_R q_R$$

- The entries of the inverse of the capacitance matrix are

$$p_G = \left.\frac{V_G}{q_G}\right|_{q_R=0} \qquad p_m = \left.\frac{V_G}{q_R}\right|_{q_G=0}$$

$$p_m = \left.\frac{V_R}{q_G}\right|_{q_R=0} \qquad p_R = \left.\frac{V_R}{q_R}\right|_{q_G=0}$$



- For P_G and P_R, we have

$$p_G = \left.\frac{V_G}{q_G}\right|_{q_R=0}$$

$$p_G = \frac{1}{2\pi\epsilon_0}\ln\left(\frac{d_G}{r_{wG}}\right) + \frac{1}{2\pi\epsilon_0}\ln\left(\frac{d_G}{r_{w0}}\right) = \frac{1}{2\pi\epsilon_0}\ln\left(\frac{d_G^2}{r_{wG}r_{w0}}\right)$$

$$p_R = \frac{1}{2\pi\epsilon_0}\ln\left(\frac{d_R}{r_{wR}}\right) + \frac{1}{2\pi\epsilon_0}\ln\left(\frac{d_R}{r_{w0}}\right) = \frac{1}{2\pi\epsilon_0}\ln\left(\frac{d_R^2}{r_{wR}r_{w0}}\right)$$

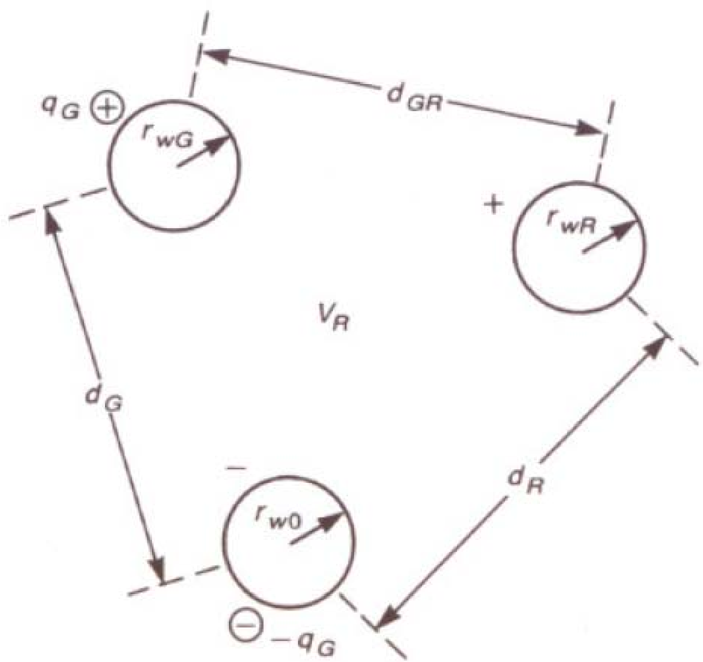
The Per-Unit-Length Parameters

- Wide-Separation Approximations for Wires
  - Three Wires
    - For P_m, we have

$$p_m = \left.\frac{V_R}{q_G}\right|_{q_R=0} \longrightarrow p_m = \frac{1}{2\pi\epsilon_0}\ln\left(\frac{d_G}{d_{GR}}\right) + \frac{1}{2\pi\epsilon_0}\ln\left(\frac{d_R}{rw_0}\right)$$

$$= \frac{1}{2\pi\epsilon_0}\ln\left(\frac{d_G d_R}{d_{GR} r_{w0}}\right)$$

The capacitance matrix ***C*** is obtained by taking the inverse of the matrix ***P***. In addition, we see that ***L***$=\mu_0\varepsilon_0$***P***. This means we only need to calculate the matrix ***P*** for obtaining both matrices ***L*** and ***C***.



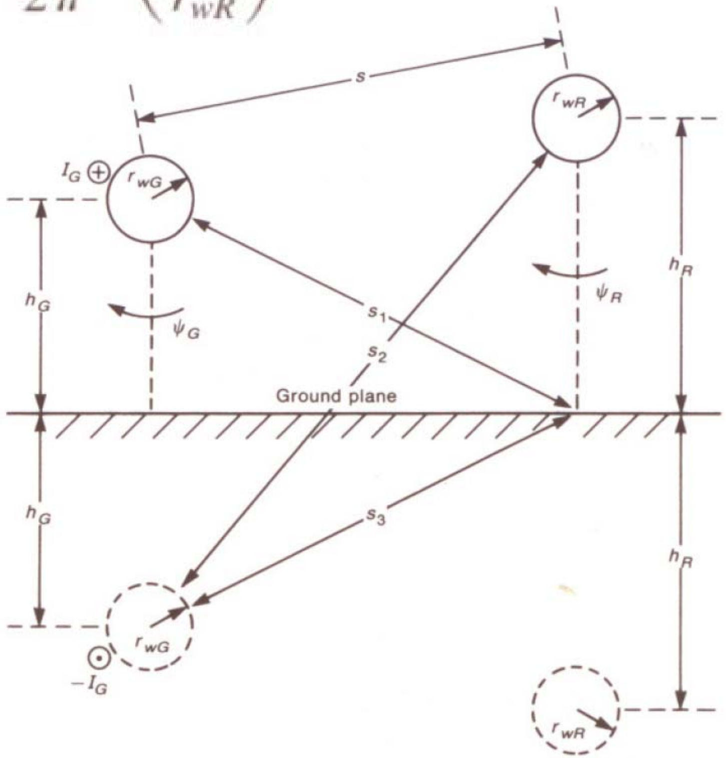
The Per-Unit-Length Parameters

- Wide-Separation Approximations for Wires
  - Two Wires above PEC
    - The self inductances are $l_R = \frac{\mu_0}{2\pi}\ln\left(\frac{2\,h_R}{r_{wR}}\right)$

$$l_G = \frac{\psi_G}{I_G}\bigg|_{I_R=0}$$

$$= \frac{\mu_0}{2\pi}\ln\left(\frac{h_G}{r_{wG}}\right) + \frac{\mu_0}{2\pi}\ln\left(\frac{2\,h_G}{h_G}\right)$$

$$= \frac{\mu_0}{2\pi}\ln\left(\frac{2\,h_G}{r_{wG}}\right)$$



    - and the mutual inductance is

$$l_m = \frac{\psi_R}{I_G}\bigg|_{I_R=0}$$

$$= \frac{\mu_0}{2\pi}\ln\left(\frac{s_1}{s}\right) + \frac{\mu_0}{2\pi}\ln\left(\frac{s_2}{s_3}\right)$$

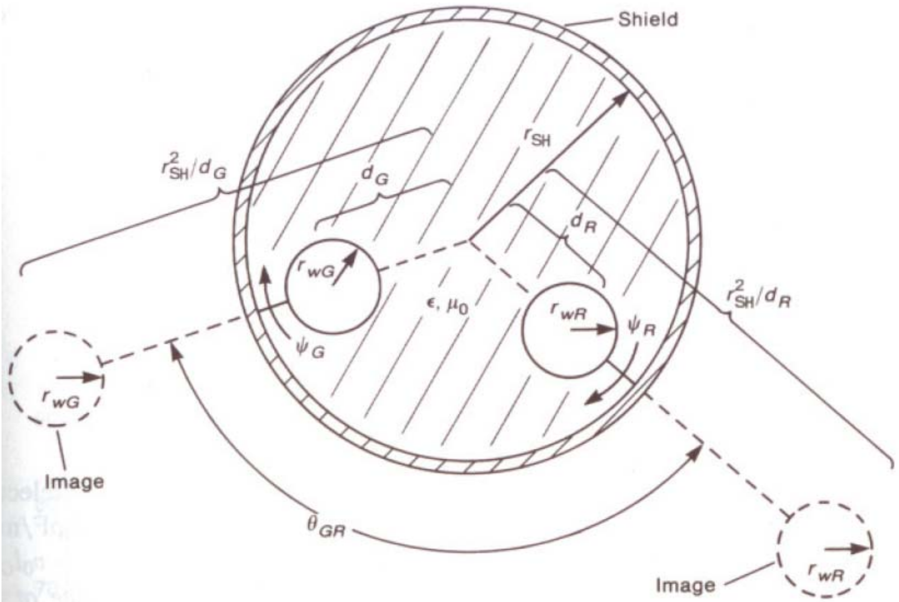
$$= \frac{\mu_0}{2\pi}\ln\left(\frac{s_2}{s}\right) \xrightarrow{s_2 = \sqrt{s^2 + 4h_G h_R}} l_m = \frac{\mu_0}{4\pi}\ln\left(1 + 4\frac{h_G h_R}{s^2}\right)$$

The Per-Unit-Length Parameters

- Wide-Separation Approximations for Wires
  - Two Wires in a Shield
    - The self and mutual inductances are

$$l_G = \frac{\mu_0}{2\pi} \ln\left(\frac{r_{SH}^2 - d_G^2}{r_{SH} r_{wG}}\right)$$

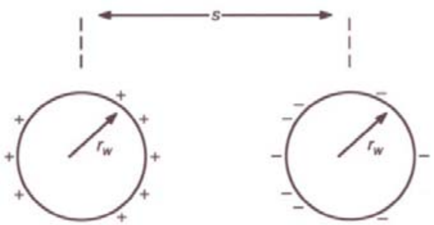
$$l_R = \frac{\mu_0}{2\pi} \ln\left(\frac{r_{SH}^2 - d_R^2}{r_{SH} r_{wR}}\right)$$



$$l_m = \frac{\mu_0}{2\pi} \ln\left[\frac{d_R}{r_{SH}} \sqrt{\frac{(d_G d_R)^2 + r_{SH}^4 - 2 d_G d_R r_{SH}^2 \cos\theta_{GR}}{(d_G d_R)^2 + d_R^4 - 2 d_G d_R^3 \cos\theta_{GR}}}\right]$$

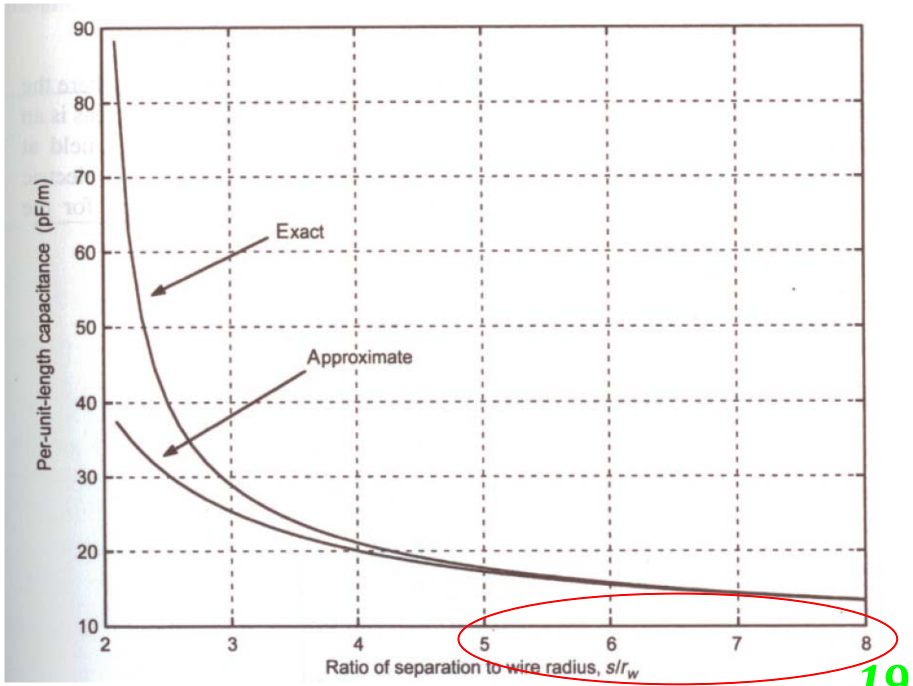
The Per-Unit-Length Parameters

- Numerical Methods for Other Structures
  - Two Wires in Homogeneous Medium
    - When the wire separation to wire radius is on the order of 5:1 or greater, the exact solution and approximate solution are nearly equal.



$$c_{\text{exact}} = \frac{\pi\epsilon_0}{\ln\left[\dfrac{s}{2r_w} + \sqrt{\left(\dfrac{s}{2r_w}\right)^2 - 1}\right]}$$

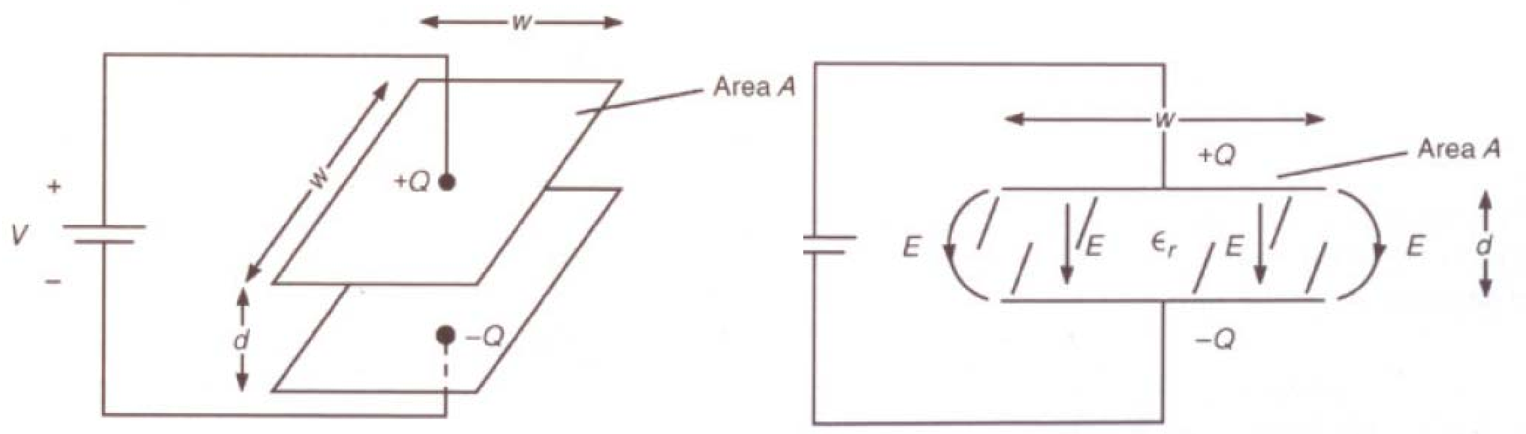
$$c_{\text{approximation}} = \frac{\pi\epsilon_0}{\ln\left[\dfrac{s}{r_w}\right]} \qquad s \gg r_w$$



The Per-Unit-Length Parameters

- Numerical Methods for Other Structures
  - Parallel-Plates in Homogeneous Medium
    - When *w>>d*, the fringing effect could be neglected. Thus, the capacitance could be approximated as

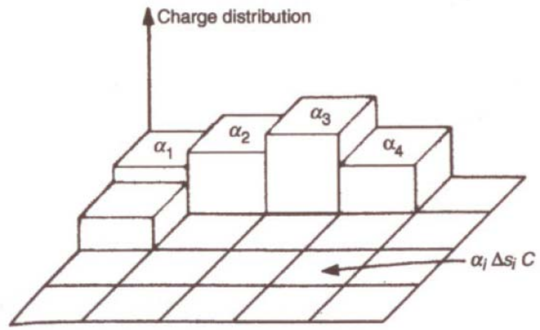
$$C = \epsilon \frac{A}{d}$$



The Per-Unit-Length Parameters

- Numerical Methods for Other Structures
  - Parallel-Plates in Homogeneous Medium
    - Assuming the two plates are divided into 2N subsections and each subsection having an constant with an unknown level α_i.
    - The total charge on each plate is

$$Q \cong \sum_{i=1}^{N} \alpha_i \, \Delta s_i \quad \text{C (coulombs)}$$



- The potential on the plate contributing from all the subsection charges is

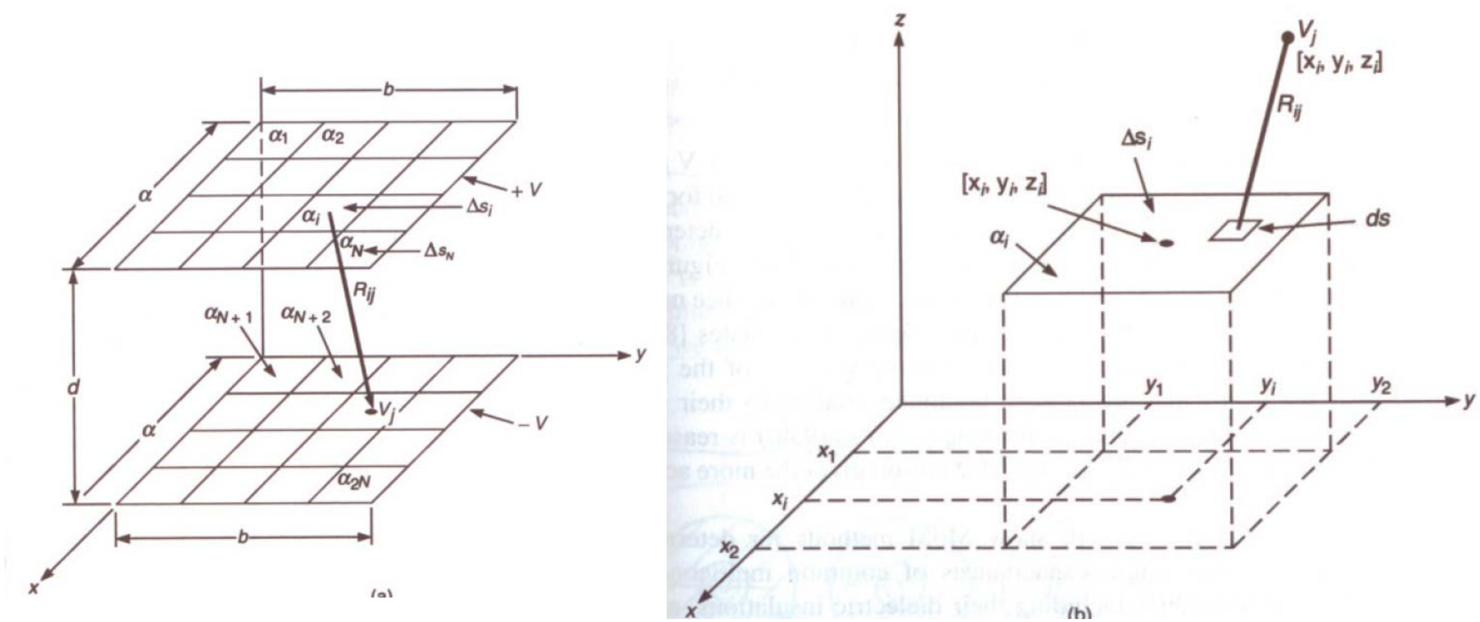
$$V_j = \underbrace{K_{j1}\alpha_1 + \cdots + K_{jN}\alpha_N}_{\text{top-plate contributions}} + \underbrace{K_{jN+1}\alpha_{N+1} + \cdots + K_{j2N}\alpha_{2N}}_{\text{bottom-plate contributions}}$$

The Per-Unit-Length Parameters

- Numerical Methods for Other Structures
  - Parallel-Plates in Homogeneous Medium
    - where

$$K_{ji} = \frac{V_j}{\alpha_i} \qquad K_{ji} = \frac{\Delta s_i}{4\pi \epsilon R_{ij}}$$

$$K_{ii} = \frac{\Delta w}{\pi \epsilon_0} \ln(1+\sqrt{2}) = 0.8814 \frac{\Delta w}{\pi \epsilon_0}$$

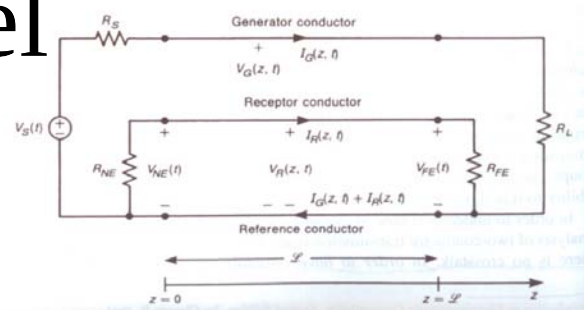


The Per-Unit-Length Parameters

- Numerical Methods for Other Structures
  - Parallel-Plates in Homogeneous Medium
    - Writing V_j in matrix form and solving for α_i, the capacitance could be obtained from $C = \frac{Q}{2V}$

$$
\begin{bmatrix}
K_{11} & K_{12} & \cdots & K_{1(2N)} \\
K_{21} & K_{22} & \cdots & K_{2(2N)} \\
\vdots & \vdots & \cdots & \\
K_{N1} & K_{N2} & \cdots & K_{N(2N)} \\
\vdots & \vdots & \cdots & \vdots \\
\vdots & \vdots & \cdots & \vdots \\
K_{(2N)1} & K_{(2N)2} & \cdots & K_{(2N)(2N)}
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \\ \alpha_{N+1} \\ \vdots \\ \alpha_{2N}
\end{bmatrix}
=
\begin{bmatrix}
+V \\ +V \\ \vdots \\ +V \\ -V \\ \vdots \\ -V
\end{bmatrix}
=
\begin{bmatrix}
+1 \\ +1 \\ \vdots \\ +1 \\ -1 \\ \vdots \\ -1
\end{bmatrix}
$$

The Inductive-Capacitive Coupling Approximate Model



- Derivation of Model
  - Weak Coupling
    - By assuming weak coupling, we mean that the voltages and currents induced in the generator circuit due to the currents and voltages that were induced in the receptor circuit may be ignored.
    - The MTL equations could be simplified as

$$\frac{\partial V_G(z,t)}{\partial z} = -l_G \frac{\partial I_G(z,t)}{\partial t} \cancel{- l_m \frac{\partial I_R(z,t)}{\partial t}}$$

$$\frac{\partial V_R(z,t)}{\partial z} = -l_m \frac{\partial I_G(z,t)}{\partial t} - l_R \frac{\partial I_R(z,t)}{\partial t}$$

$$\frac{\partial I_G(z,t)}{\partial z} = -(c_G + c_m) \frac{\partial V_G(z,t)}{\partial t} \cancel{+ c_m \frac{\partial V_R(z,t)}{\partial t}}$$

$$\frac{\partial I_R(z,t)}{\partial z} = c_m \frac{\partial V_G(z,t)}{\partial t} - (c_R + c_m) \frac{\partial V_R(z,t)}{\partial t}$$

For the generator circuit, the coupling from the receptor circuit is ignored.

The Inductive-Capacitive Coupling Approximate Model

- Derivation of Model
  - Equivalent Circuit for Receptor
    - Rearranging these equations, we have the approximate equations for the generator circuit as

$$\frac{\partial V_G(z,t)}{\partial z} + l_G \frac{\partial I_G(z,t)}{\partial t} = 0$$

$$\frac{\partial I_G(z,t)}{\partial z} + (c_G + c_m) \frac{\partial V_G(z,t)}{\partial t} = 0$$

V_G and I_G could solved by using the two-conductor transmission line solution.

    - And the approximate equations for the receptor circuit are

$$\frac{\partial V_R(z,t)}{\partial z} + l_R \frac{\partial I_R(z,t)}{\partial t} = -l_m \frac{\partial I_G(z,t)}{\partial t}$$

$$\frac{\partial I_R(z,t)}{\partial z} + (c_R + c_m) \frac{\partial V_R(z,t)}{\partial t} = c_m \frac{\partial V_G(z,t)}{\partial t}$$

V_G and I_G are substituted into the following equations to solved for V_R and I_R.

The Inductive-Capacitive Coupling Approximate Model

- Derivation of Model
  - Equivalent Circuit for Receptor
    - For magnetic field or inductive coupling, using the Faraday's law, we have the per-unit-length magnetic flux penetrating the receptor circuit as $\psi_R = l_R I_R + l_m I_G$
    - The per-unit-length voltages due to the time rate of change of the magnetic flux are

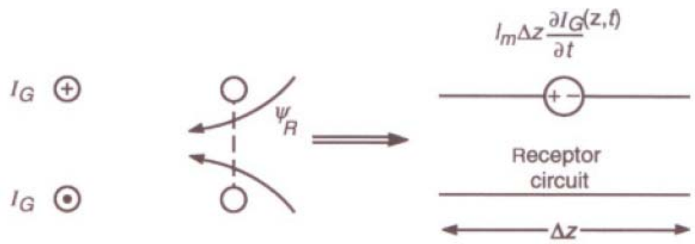
$$V_{S1} = l_R \frac{\partial I_R}{\partial t}$$

Produced by the self-inductance of the receptor circuit and the receptor current I_R

$$V_{S2} = l_m \frac{\partial I_G}{\partial t}$$

Produced by the mutual inductance between the two circuits and the generator current I_G

- Neglecting V_{S1}, we have



The Inductive-Capacitive Coupling Approximate Model

- Derivation of Model
  - Equivalent Circuit for Receptor
    - For electric field or capacitive coupling, The per-unit-length charges induced on the receptor circuit are
$$\begin{aligned} q_R &= (c_R + c_m)V_R - c_m V_G \\ &= c_R V_R - c_m(V_G - V_R) \end{aligned}$$
    - The per-unit-length current sources are the time rate-of-change of charge, which are

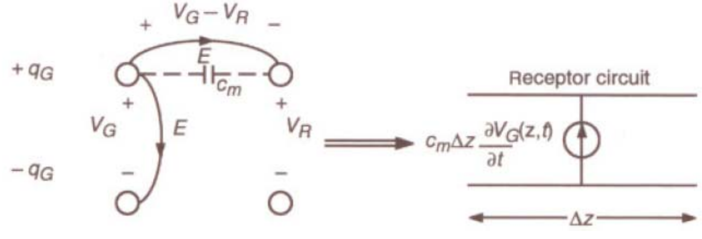
$$I_{S1} = (c_R + c_m)\frac{\partial V_R}{\partial t}$$

Produced by the self and mutual capacitance of the receptor circuit and V_R

$$I_{S2} = -c_m \frac{\partial V_G}{\partial t}$$

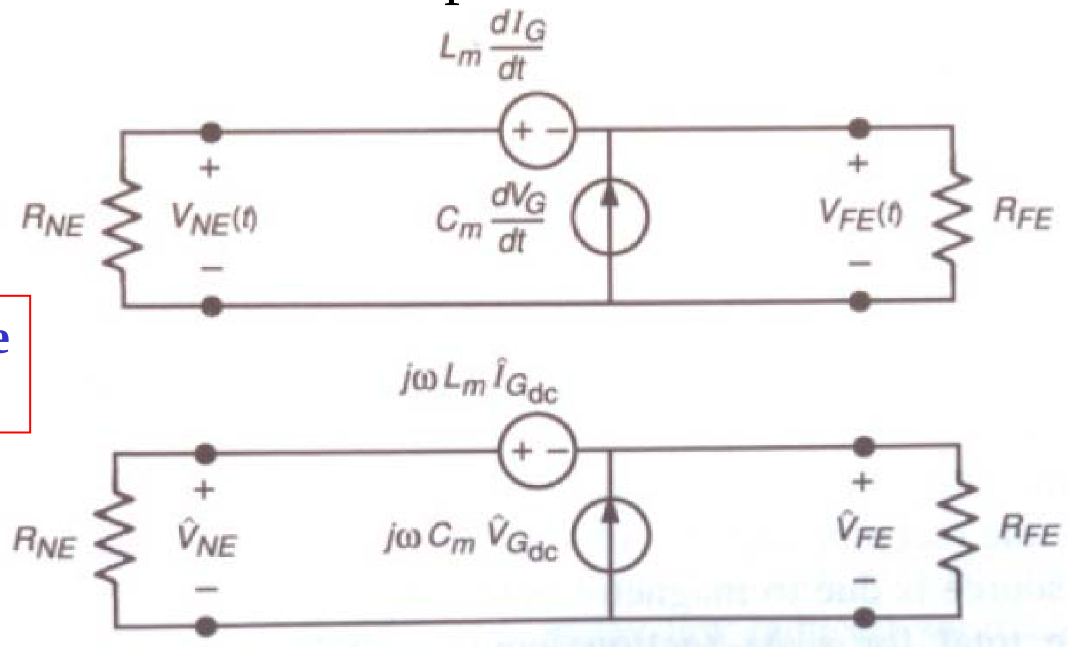
Produced by the mutual capacitance between the two circuits and the generator voltage V_G

    - Neglecting I_{S1}, we have



The Inductive-Capacitive Coupling Approximate Model

- Derivation of Model
  - Equivalent Circuit for Receptor
    - Assuming that the line is electrically short at the frequency of the driving source in the generator circuit, equivalent circuit for the receptor circuit could be further simplified as



Time Domain

Frequency Domain

Transmission line effects are simplified.

The Inductive-Capacitive Coupling Approximate Model

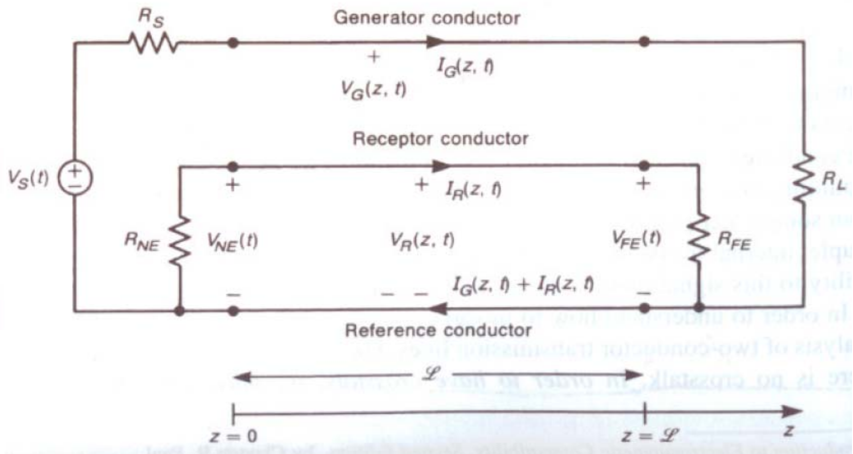
- Derivation of Model
  - Equivalent Circuit for Receptor
    - This is referred to as the ***inductive-capacitive coupling model***.
    - There are two key assumptions in this mode: (1) we assume weak coupling between the generator and receptor circuit, that is, the coupling is a one-way effect from the generator circuit to the receptor circuit. (2) the line is assumed to be electrically short at the frequency of the driving source in the generator circuit, V_S, that is, $L \ll \lambda = v/f$.

The Inductive-Capacitive Coupling Approximate Model

- Frequency-Domain Inductive-Capacitive Coupling Model
  - Near-End and Far-End Voltages
    - For the generator circuit, since we assume that line is electrically short at the frequency of the driving source, i.e., $L \ll \lambda$, the voltage and current do not vary appreciably in magnitude along the generator line. Thus, the voltage and currents could be viewed as dc

$$\hat{I}_{G_{dc}} \cong \frac{1}{R_S + R_L}\hat{V}_S$$

$$\hat{V}_{G_{dc}} \cong \frac{R_L}{R_S + R_L}\hat{V}_S$$

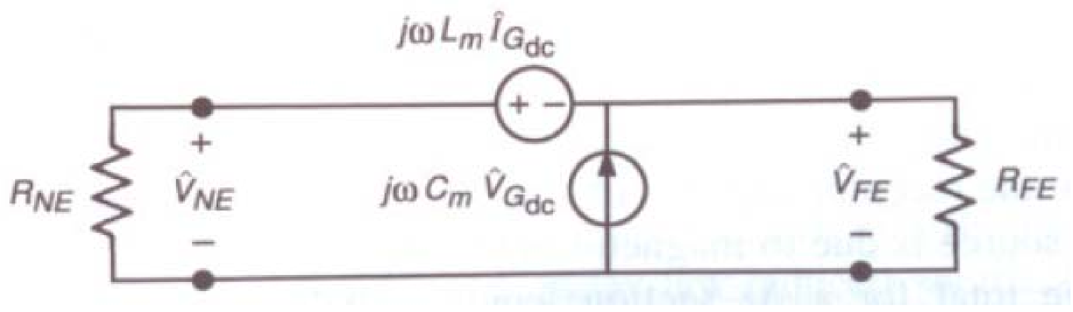


The Inductive-Capacitive Coupling Approximate Model

- Frequency-Domain Inductive-Capacitive Coupling Model
  - Near-End and Far-End Voltages
    - For the receptor circuit, the near-end and far-end phasor crosstalk voltages using superposition are

$$\hat{V}_{NE} = \underbrace{\frac{R_{NE}}{R_{NE}+R_{FE}} j\omega L_m \hat{I}_{G_{dc}}}_{\text{inductive coupling}} + \underbrace{\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}} j\omega C_m \hat{V}_{G_{dc}}}_{\text{capacitive coupling}}$$

$$\hat{V}_{FE} = \underbrace{-\frac{R_{FE}}{R_{NE}+R_{FE}} j\omega L_m \hat{I}_{G_{dc}}}_{\text{inductive coupling}} + \underbrace{\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}} j\omega C_m \hat{V}_{G_{dc}}}_{\text{capacitive coupling}}$$



The Inductive-Capacitive Coupling Approximate Model

- Frequency-Domain Inductive-Capacitive Coupling Model
  - Near-End and Far-End Voltages
    - which could be rearranged as

$$\hat{V}_{NE} = \underbrace{\frac{R_{NE}}{R_{NE}+R_{FE}} j\omega L_m \frac{1}{R_S+R_L}\hat{V}_S}_{\text{inductive coupling}} + \underbrace{\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}} j\omega C_m \frac{R_L}{R_S+R_L}\hat{V}_S}_{\text{capacitive coupling}}$$

$$\hat{V}_{FE} = \underbrace{-\frac{R_{FE}}{R_{NE}+R_{FE}} j\omega L_m \frac{1}{R_S+R_L}\hat{V}_S}_{\text{inductive coupling}} + \underbrace{\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}} j\omega C_m \frac{R_L}{R_S+R_L}\hat{V}_S}_{\text{capacitive coupling}}$$

    - These could be rewritten as ***transfer functions*** as

$$\frac{\hat{V}_{NE}}{\hat{V}_S} = j\omega\left(\frac{R_{NE}}{R_{NE}+R_{FE}}\frac{L_m}{R_S+R_L} + \frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}}\frac{R_L C_m}{R_S+R_L}\right)$$

$$\frac{\hat{V}_{FE}}{\hat{V}_S} = j\omega\left(-\frac{R_{FE}}{R_{NE}+R_{FE}}\frac{L_m}{R_S+R_L} + \frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}}\frac{R_L C_m}{R_S+R_L}\right)$$

The Inductive-Capacitive Coupling Approximate Model

- Frequency-Domain Inductive-Capacitive Coupling Model
  - Near-End and Far-End Voltages
    - which could be written as

$$\frac{\hat{V}_{NE}}{\hat{V}_S} = j\omega(M_{NE}^{\mathrm{IND}} + M_{NE}^{\mathrm{CAP}}) \qquad M_{NE}^{\mathrm{IND}} = \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{L_m}{R_S + R_L}$$

$$M_{NE}^{\mathrm{CAP}} = \frac{R_{NE}R_{FE}}{R_{NE} + R_{FE}} \frac{R_L C_m}{R_S + R_L}$$

$$\frac{\hat{V}_{FE}}{\hat{V}_S} = j\omega(M_{FE}^{\mathrm{IND}} + M_{FE}^{\mathrm{CAP}})$$

$$M_{FE}^{\mathrm{IND}} = -\frac{R_{FE}}{R_{NE} + R_{FE}} \frac{L_m}{R_S + R_L}$$

$$M_{FE}^{\mathrm{CAP}} = M_{NE}^{\mathrm{CAP}} = \frac{R_{NE}R_{FE}}{R_{NE} + R_{FE}} \frac{R_L C_m}{R_S + R_L}$$

The Inductive-Capacitive Coupling Approximate Model

- Frequency-Domain Inductive-Capacitive Coupling Model
  - Near-End and Far-End Voltages
    - For near-end crosstalk voltage, the inductive coupling dominates the capacitive coupling when

$$M_{NE}^{\mathrm{IND}} > M_{NE}^{\mathrm{CAP}}, \text{ if } L_m/C_m > R_{FE}R_L$$

homogeneous $\longrightarrow$

$$\frac{R_{FE}R_L}{(L_m/C_m)} = \frac{R_{FE}R_L}{Z_{CG}Z_{CR}} < 1$$

    - For far-end crosstalk voltage, the inductive coupling dominates the capacitive coupling when

$$M_{FE}^{\mathrm{IND}} > M_{FE}^{\mathrm{CAP}}, \text{ if } L_m/C_m > R_{NE}R_L$$

homogeneous $\longrightarrow$

$$\frac{R_{NE}R_L}{(L_m/C_m)} = \frac{R_{NE}R_L}{Z_{CG}Z_{CR}} < 1$$

$$Z_{CG} = \sqrt{\frac{l_G}{c_G + c_m}}$$

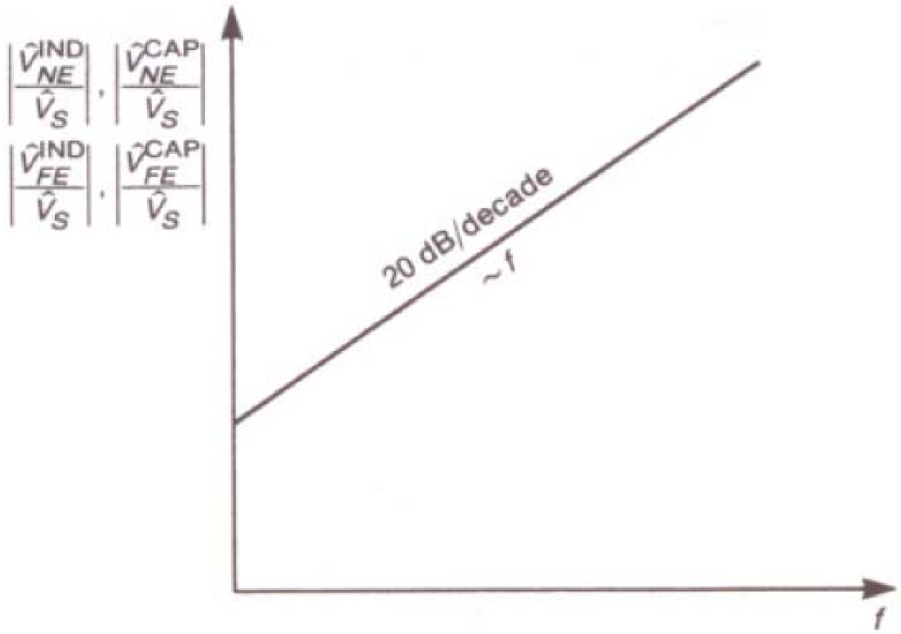
$$Z_{CR} = \sqrt{\frac{l_R}{c_R + c_m}}$$

The Inductive-Capacitive Coupling Approximate Model

- Frequency-Domain Inductive-Capacitive Coupling Model
  - Near-End and Far-End Voltages
    - From the above two inequalities, we see that inductive coupling dominates capacitive coupling for termination impedances that are ***low impedances*** (with respect the circuit characteristic impedance). This makes sense since low impedance result in large current. → inductive coupling.
    - Similarly, we see that capacitive coupling dominates inductive coupling for termination impedances that are ***high impedances***. This makes sense since large impedance result in large voltage. → capacitive coupling.

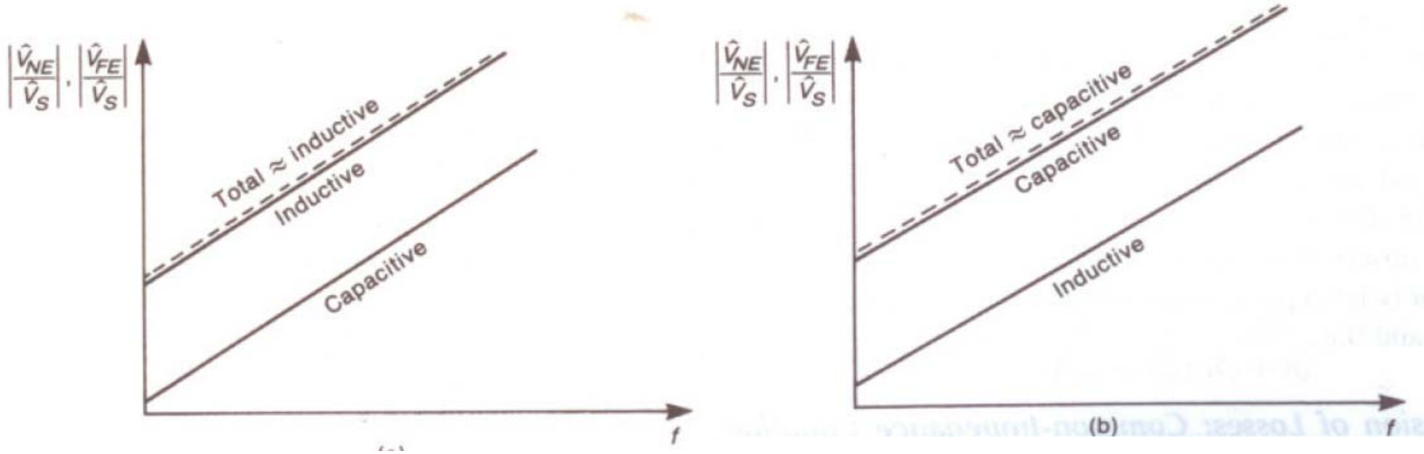
The Inductive-Capacitive Coupling Approximate Model

- Frequency-Domain Inductive-Capacitive Coupling Model
  - Bode Plot of Transfer Functions
    - The transfer function increase linearly with frequency at a rate of 20dB/decade.



The Inductive-Capacitive Coupling Approximate Model

- Frequency-Domain Inductive-Capacitive Coupling Model
  - Bode Plot of Transfer Functions
    - Depending on the load impedances, this frequency response may be due to totally to one component.



Low-impedance loads

High-impedance loads

The Inductive-Capacitive Coupling Approximate Model

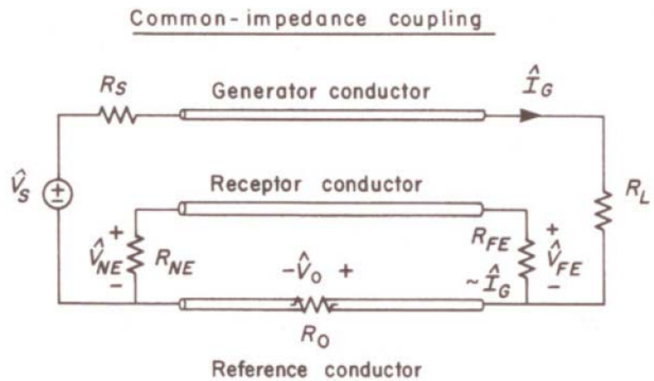
- Inclusion of Losses: Common-Impedance Coupling
  - Effect of Imperfect Conductors
    - For low GHz range, the dielectric loss is small, thus, could be neglected. However, imperfect conductors can produce significant crosstalk at the lower frequencies. This is referred to as ***common-impedance coupling***.
    - At low frequencies, for an electrically short line, we may lump the per-unit-length resistance of the reference conductor, r_0, as a single resistance $R_0 = r_0 L$.

The Inductive-Capacitive Coupling Approximate Model

- Inclusion of Losses: Common-Impedance Coupling
  - Effect of Imperfect Conductors
    - The voltage drop across the reference conductor is given by

$$\hat{V}_0 = R_0 \hat{I}_G$$

$$= \frac{R_0}{R_S + R_L} \hat{V}_S$$



- The voltage appears directly in the receptor circuit as

$$\frac{\hat{V}_{NE}^{CI}}{\hat{V}_S} = M_{NE}^{CI} \qquad M_{NE}^{CI} = \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{R_0}{R_S + R_L}$$

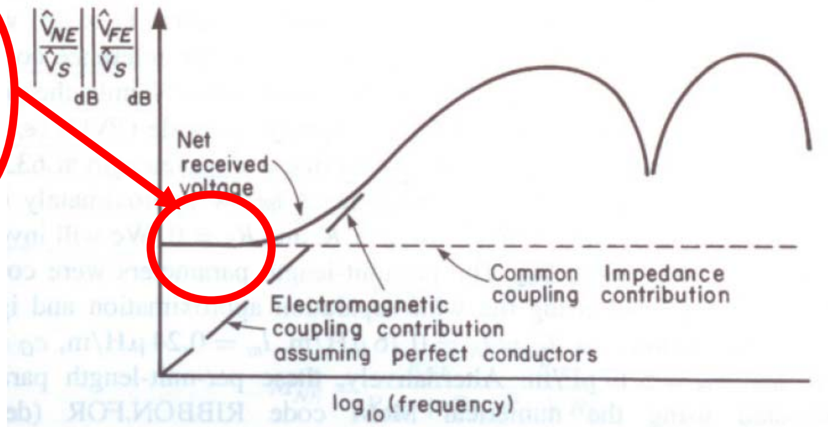
$$\frac{\hat{V}_{FE}^{CI}}{\hat{V}_S} = M_{FE}^{CI} \qquad M_{FE}^{CI} = -\frac{R_{FE}}{R_{NE} + R_{FE}} \frac{R_0}{R_S + R_L}$$

The Inductive-Capacitive Coupling Approximate Model

- Inclusion of Losses: Common-Impedance Coupling
  - Effect of Imperfect Conductors
    - The total coupling is approximately the sum of the inductive, capacitive, and common-impedance coupling:

$$\frac{\hat{V}_{NE}}{\hat{V}_S} = j\omega\left(M_{NE}^{\mathrm{IND}} + M_{NE}^{\mathrm{CAP}}\right) + M_{NE}^{CI}$$

$$\frac{\hat{V}_{FE}}{\hat{V}_S} = j\omega\left(M_{FE}^{\mathrm{IND}} + M_{FE}^{\mathrm{CAP}}\right) + M_{FE}^{CI}$$



The Inductive-Capacitive Coupling Approximate Model

- Time-Domain Inductive-Capacitive Coupling Model
  - Near-End and Far-End Voltages
    - Since the phasor crosstalk relations are

$$\hat{V}_{NE}(j\omega) = j\omega M_{NE}\hat{V}_S(j\omega)$$

$$\hat{V}_{FE}(j\omega) = j\omega M_{FE}\hat{V}_S(j\omega)$$

- where the crosstalk coefficients are

$$M_{NE} = \underbrace{\frac{R_{NE}}{R_{NE}+R_{FE}}\frac{L_m}{R_S+R_L}}_{M_{NE}^{\mathrm{IND}}} + \underbrace{\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}}\frac{R_L C_m}{R_S+R_L}}_{M_{NE}^{\mathrm{CAP}}}$$

$$M_{FE} = \underbrace{-\frac{R_{FE}}{R_{NE}+R_{FE}}\frac{L_m}{R_S+R_L}}_{M_{FE}^{\mathrm{IND}}} + \underbrace{\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}}\frac{R_L C_m}{R_S+R_L}}_{M_{FE}^{\mathrm{CAP}}}$$

The Inductive-Capacitive Coupling Approximate Model

- Time-Domain Inductive-Capacitive Coupling Model
  - Near-End and Far-End Voltages
    - Transforming from frequency-domain to time-domain, we have

$$\hat{V}_{NE}(j\omega) = j\omega M_{NE}\hat{V}_S(j\omega) \qquad \hat{V}_{FE}(j\omega) = j\omega M_{FE}\hat{V}_S(j\omega)$$

$$j\omega \rightarrow \frac{d}{dt}$$

$$V_{NE}(t) = M_{NE}\frac{dV_S(t)}{dt} \qquad V_{FE}(t) = M_{FE}\frac{dV_S(t)}{dt}$$

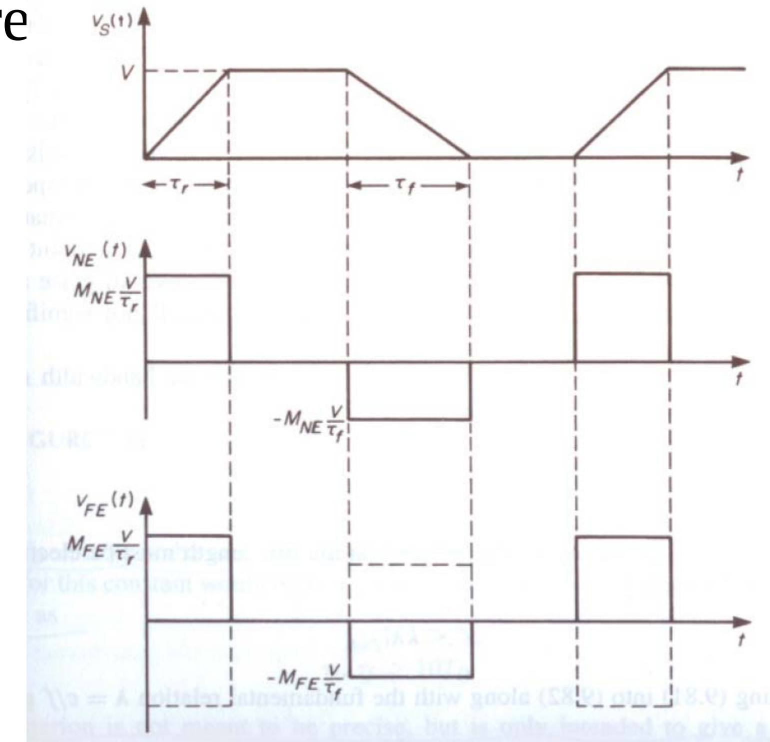
    - The frequency components of the input signal for which the line is electrically short are processed by the line to give an output that is the derivative of the source voltage multiplied by the crosstalk coefficients M_{NE} and M_{FE}.

The Inductive-Capacitive Coupling Approximate Model

- Time-Domain Inductive-Capacitive Coupling Model
  - – Near-End and Far-End Voltages
    - For a periodic, trapezoidal pulse train source voltage, the crosstalk voltages are

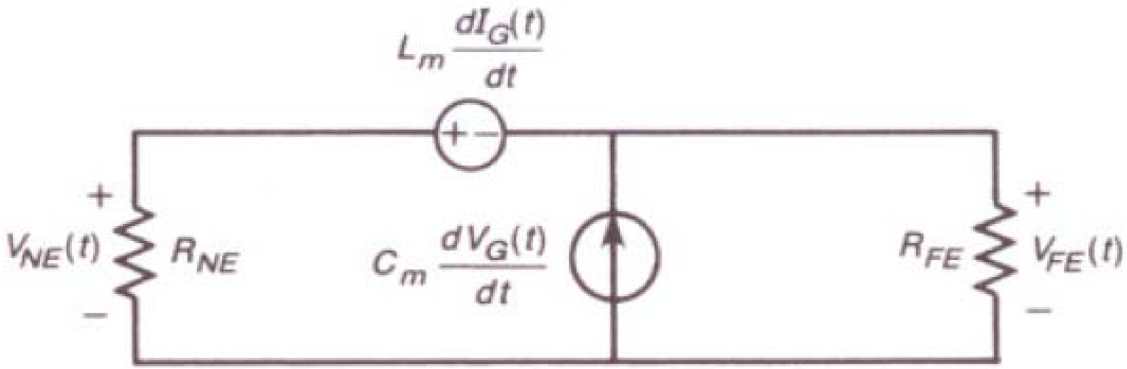
$$V_{NE}(t) = M_{NE}\frac{dV_S(t)}{dt}$$

$$V_{FE}(t) = M_{FE}\frac{dV_S(t)}{dt}$$



The Inductive-Capacitive Coupling Approximate Model

- Time-Domain Inductive-Capacitive Coupling Model
  - Near-End and Far-End Voltages
    - A simple time-domain equivalent circuit for the receptor circuit is shown below

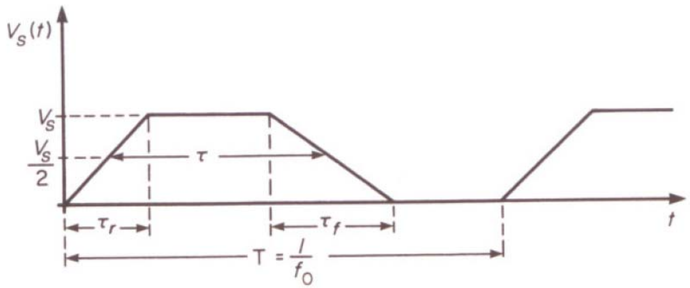


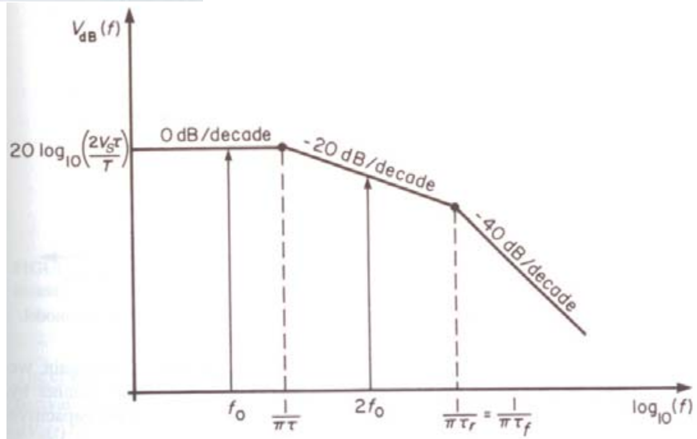
    - Remember the primary restrictions that this model is valid only if the line be electrically short and weakly coupled.

The Inductive-Capacitive Coupling Approximate Model

- Time-Domain Inductive-Capacitive Coupling Model
  - Important Restriction on τ_r and τ_f
    - For a periodic, trapezoidal pulse train source voltage, the bandwidth of this signal is $f_u = \frac{1}{\tau_r}$
    - Since the line length must be electrically short at this frequency

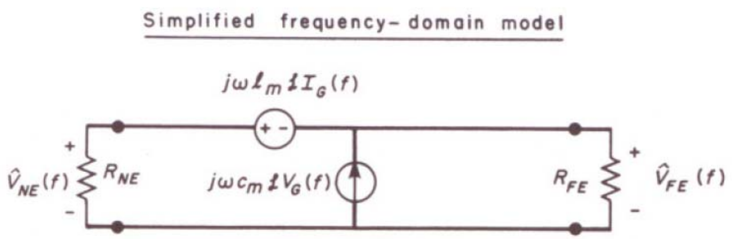
$$\mathscr{L} < k\lambda|_{f=f_u} \xrightarrow[T_D = \mathscr{L}/v]{\lambda = v/f} \tau_r > \frac{1}{k} T_D \xrightarrow{k=1/10} \tau_r, \tau_f > 10T_D$$

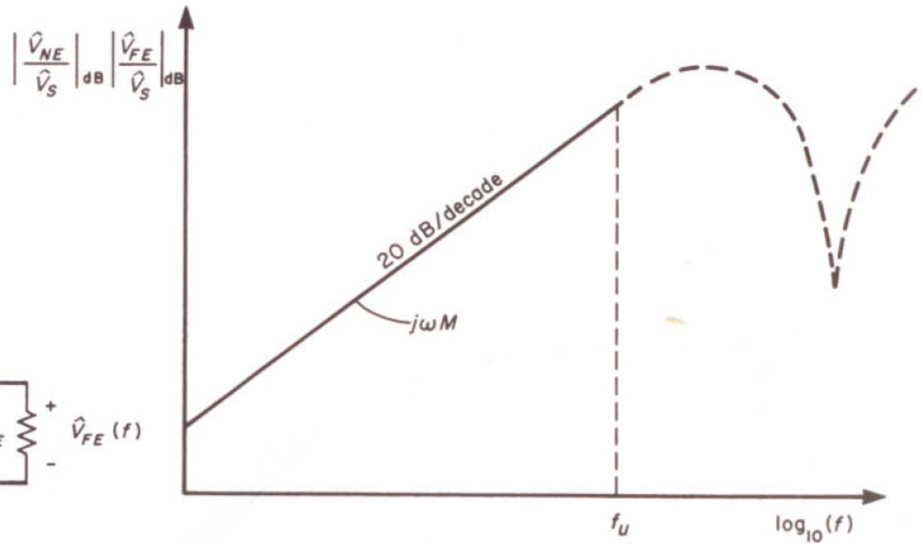




The Inductive-Capacitive Coupling Approximate Model

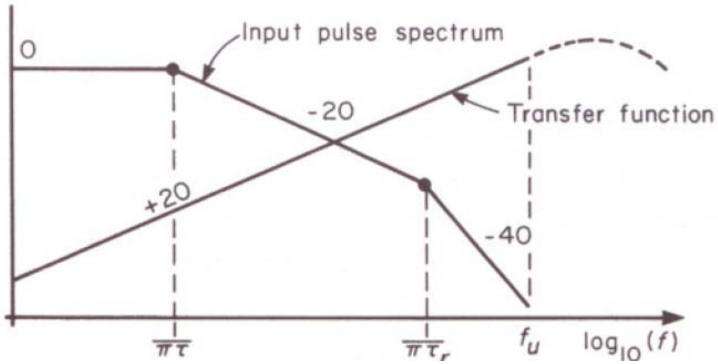
- Time-Domain Inductive-Capacitive Coupling Model
  - – Important Restriction on τ_r and τ_f
    - Since the frequency response of the simple inductive-capacitive coupling model is



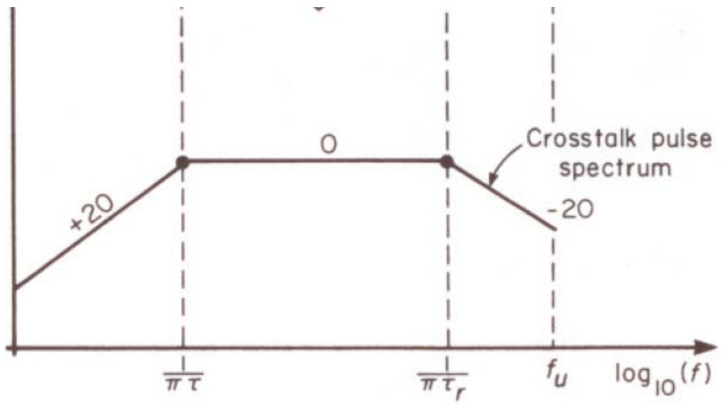


The Inductive-Capacitive Coupling Approximate Model

- Time-Domain Inductive-Capacitive Coupling Model
  - Important Restriction on τ_r and τ_f
    - The output crosstalk spectrum is the sum of the trapezoidal pulse train and the transfer function



The spectrum of the source signal above f_u is small, thus, is ignored.



The Inductive-Capacitive Coupling Approximate Model

- Inclusion of Losses: Common-Impedance Coupling
  - Near-End And Far-End Voltages
    - The total coupling contributed from the inductive, capacitive and common-impedance couplings, which is

$$V_{NE}(t) = (M_{NE}^{\text{IND}} + M_{NE}^{\text{CAP}})\frac{dV_S(t)}{dt} + M_{NE}^{CI} V_S(t)$$

$$V_{FE}(t) = (M_{FE}^{\text{IND}} + M_{FE}^{\text{CAP}})\frac{dV_S(t)}{dt} + M_{FE}^{CI} V_S(t)$$

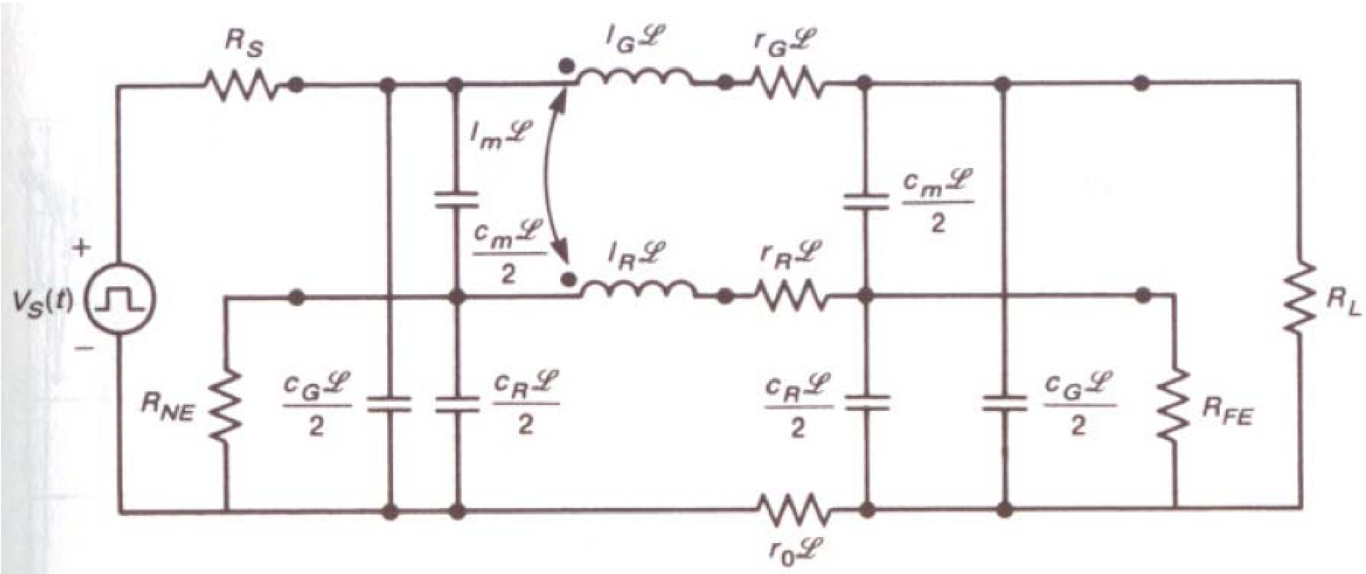
    - where the common-impedance coupling are

$$M_{NE}^{CI} = \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{R_0}{R_S + R_L}$$

$$M_{FE}^{CI} = -\frac{R_{FE}}{R_{NE} + R_{FE}} \frac{R_0}{R_S + R_L}$$

Lumped-Circuit Approximate Models

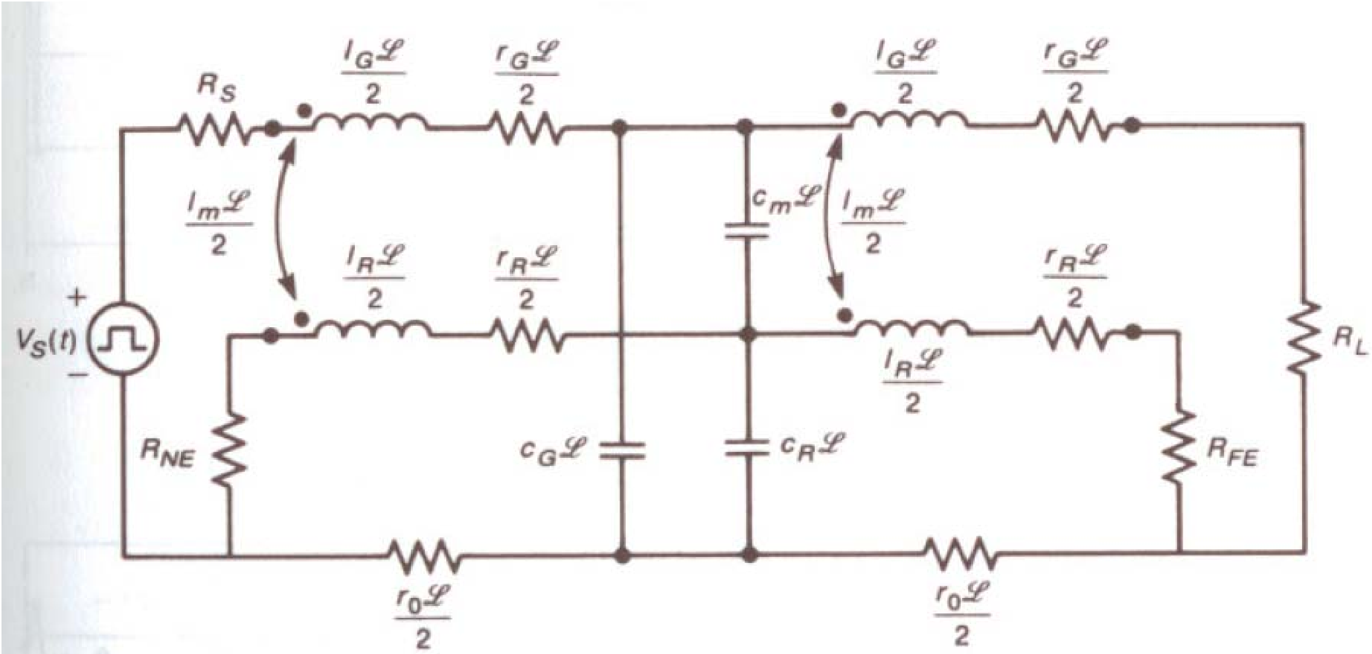
- Lumped-π Model
  - Equivalent Circuit for a Three-Conductor Line
    - In this model, we have included the dc resistances of the conductors.



More accurate for large load impedances

Lumped-Circuit Approximate Models

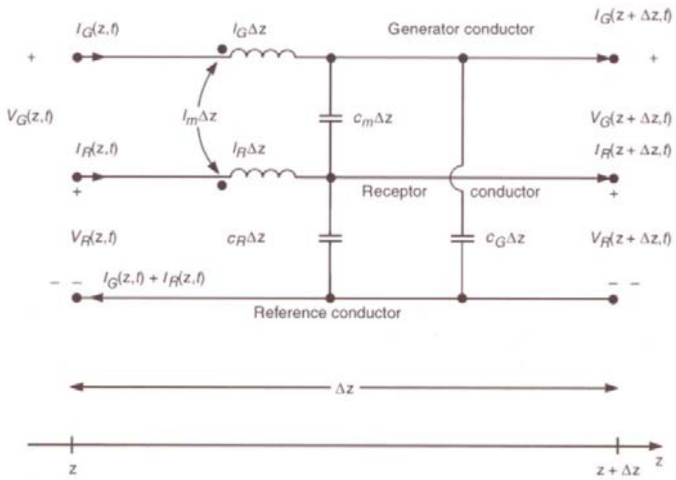
- Lumped-T Model
  - Equivalent Circuit for a Three-Conductor Line
    - In this model, we have included the dc resistances of the conductors.



More accurate for small load impedances

An Exact SPICE Model for Lossless, Coupled Lines

- Lossless Lines
  - Coupled MTL Equations
    - The multiconductor transmission line equations, in matrix form, are



$$\frac{\partial}{\partial z}\mathbf{V}(z,t) = -\mathbf{L}\frac{\partial}{\partial t}\mathbf{I}(z,t)$$

$$\frac{\partial}{\partial z}\mathbf{I}(z,t) = -\mathbf{C}\frac{\partial}{\partial t}\mathbf{V}(z,t)$$

- where

$$\mathbf{V}(z,t) = \begin{bmatrix} V_G(z,t) \\ V_R(z,t) \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} l_G & l_m \\ l_m & l_R \end{bmatrix}$$

$$\mathbf{I}(z,t) = \begin{bmatrix} I_G(z,t) \\ I_R(z,t) \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} c_G + c_m & -c_m \\ -c_m & c_R + c_m \end{bmatrix}$$

An Exact SPICE Model for Lossless, Coupled Lines

- Lossless Lines
  - Decoupled MTL Equations
    - If **V**(z,t) and **I**(z,t) could be written as

$$\mathbf{V}(z,t) = \mathbf{T}_V \mathbf{V}_m(z,t)$$

$$\mathbf{I}(z,t) = \mathbf{T}_I \mathbf{I}_m(z,t)$$

    - Where $\mathbf{V}_m$ and $\mathbf{I}_m$ are vectors of mode voltages and currents, respectively.

$$\mathbf{V}_m(z,t) = \begin{bmatrix} V_{mG}(z,t) \\ V_{mR}(z,t) \end{bmatrix} \qquad \mathbf{T}_V = \begin{bmatrix} T_{VGG} & T_{VGR} \\ T_{VRG} & T_{VRR} \end{bmatrix}$$

$$\mathbf{I}_m(z,t) = \begin{bmatrix} I_{mG}(z,t) \\ I_{mR}(z,t) \end{bmatrix} \qquad \mathbf{T}_I = \begin{bmatrix} T_{IGG} & T_{IGR} \\ T_{IRG} & T_{IRR} \end{bmatrix}$$

An Exact SPICE Model for Lossless, Coupled Lines

- Lossless Lines
  - Decoupled MTL Equations
    - The coupled MTL equations could be rewritten as

$$\frac{\partial}{\partial z}\mathbf{V}(z,t) = -\mathbf{L}\frac{\partial}{\partial t}\mathbf{I}(z,t)$$

$$\frac{\partial}{\partial z}\mathbf{I}(z,t) = -\mathbf{C}\frac{\partial}{\partial t}\mathbf{V}(z,t)$$

$$\mathbf{V}(z,t) = \mathbf{T}_V\mathbf{V}_m(z,t)$$

$$\mathbf{I}(z,t) = \mathbf{T}_I\mathbf{I}_m(z,t)$$

$$\frac{\partial}{\partial z}\mathbf{V}_m(z,t) = -\mathbf{T}_V^{-1}\mathbf{L}\mathbf{T}_I\frac{\partial}{\partial t}\mathbf{I}_m(z,t)$$

$$\frac{\partial}{\partial z}\mathbf{I}_m(z,t) = -\mathbf{T}_I^{-1}\mathbf{C}\mathbf{T}_V\frac{\partial}{\partial t}\mathbf{V}_m(z,t)$$

    - Suppose that we can find these transformation matrices such that they simultaneously diagonalize the per-unit-length inductance and capacitance matrices as

An Exact SPICE Model for Lossless, Coupled Lines

- Lossless Lines
  - Decoupled MTL Equations

$$\mathbf{T}_V^{-1}\mathbf{L}\mathbf{T}_I = \mathbf{l}_m = \begin{bmatrix} l_{mG} & 0 \\ 0 & l_{mR} \end{bmatrix}$$

$$\mathbf{T}_I^{-1}\mathbf{C}\mathbf{T}_V = \mathbf{c}_m = \begin{bmatrix} c_{mG} & 0 \\ 0 & c_{mR} \end{bmatrix}$$

- The transmission-line equations for the mode voltages and currents are uncoupled as

$$\frac{\partial}{\partial z} V_{mG}(z,t) = -l_{mG}\frac{\partial}{\partial t} I_{mG}(z,t) \qquad \frac{\partial}{\partial z} V_{mR}(z,t) = -l_{mR}\frac{\partial}{\partial t} I_{mR}(z,t)$$

$$\frac{\partial}{\partial z} I_{mG}(z,t) = -c_{mG}\frac{\partial}{\partial t} V_{mG}(z,t) \qquad \frac{\partial}{\partial z} I_{mR}(z,t) = -c_{mR}\frac{\partial}{\partial t} V_{mR}(z,t)$$

- where the characteristic impedances and velocities of propagation are

$$Z_{CmG} = \sqrt{l_{mG}/c_{mG}} \qquad v_{mG} = \frac{1}{\sqrt{l_{mG}c_{mG}}}$$

$$Z_{CmR} = \sqrt{l_{mR}/c_{mR}} \qquad v_{mR} = \frac{1}{\sqrt{l_{mR}c_{mR}}}$$

An Exact SPICE Model for Lossless, Coupled Lines

- Lossless Lines

Do you know how to implement the coupled line equations in spice? Through the decoupled model

  - Spice Model for Uncoupled MTL
    - The voltages V_G and V_R could be represented as the mode-voltage-controlled voltage sources

$$V_G = T_{VGG} V_{mG} + T_{VGR} V_{mR}$$

$$V_R = T_{VRG} V_{mG} + T_{VRR} V_{mR}$$

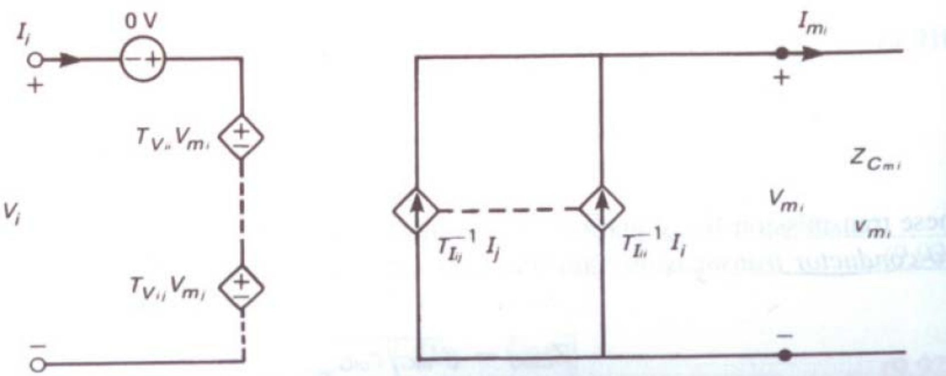
    - For the currents I_G and I_R, we have

$$\mathbf{I}_m(z,t) = \mathbf{T}_I^{-1}\,\mathbf{I}(z,t)$$

$$I_{mG} = T_{IGG}^{-1} I_G + T_{IGR}^{-1} I_R$$

$$I_{mR} = T_{IRG}^{-1} I_G + T_{IRR}^{-1} I_R$$

$$\mathbf{T}_I^{-1} = \begin{bmatrix} T_{IGG}^{-1} & T_{IGR}^{-1} \\ T_{IRG}^{-1} & T_{IRR}^{-1} \end{bmatrix}$$

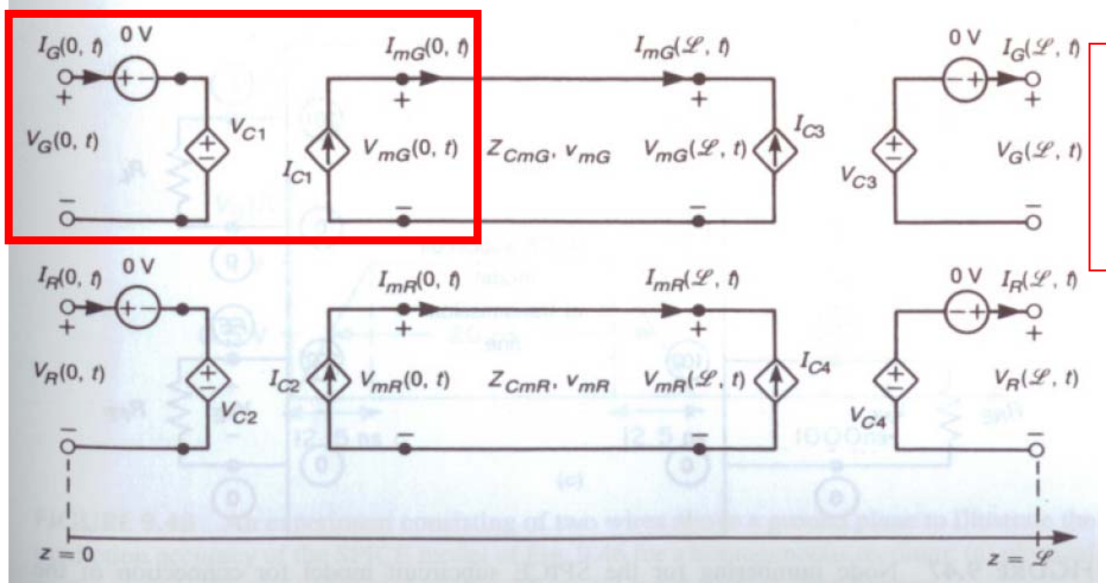


An Exact SPICE Model for Lossless, Coupled Lines

- Lossless Lines
  - Spice Model for Uncoupled MTL
    - For the overall circuit, we have

$$\boxed{V_{C1} = T_{VGG} V_{mG}(0,t) + T_{VGR} V_{mR}(0,t)}$$
$$V_{C2} = T_{VRG} V_{mG}(0,t) + T_{VRR} V_{mR}(0,t)$$
$$V_{C3} = T_{VGG} V_{mG}(\mathscr{L},t) + T_{VGR} V_{mR}(\mathscr{L},t)$$
$$V_{C4} = T_{VRG} V_{mG}(\mathscr{L},t) + T_{VRR} V_{mR}(\mathscr{L},t)$$

$$\boxed{I_{C1} = T_{IGG}^{-1} I_G(0,t) + T_{IGR}^{-1} I_R(0,t)}$$
$$I_{C2} = T_{IRG}^{-1} I_G(0,t) + T_{IRR}^{-1} I_R(0,t)$$
$$I_{C3} = T_{IGG}^{-1} I_G(\mathscr{L},t) + T_{IGR}^{-1} I_R(\mathscr{L},t)$$
$$I_{C4} = T_{IRG}^{-1} I_G(\mathscr{L},t) + T_{IRR}^{-1} I_R(\mathscr{L},t)$$



Use iteration in t to solve for the problem.

Shielded Wires

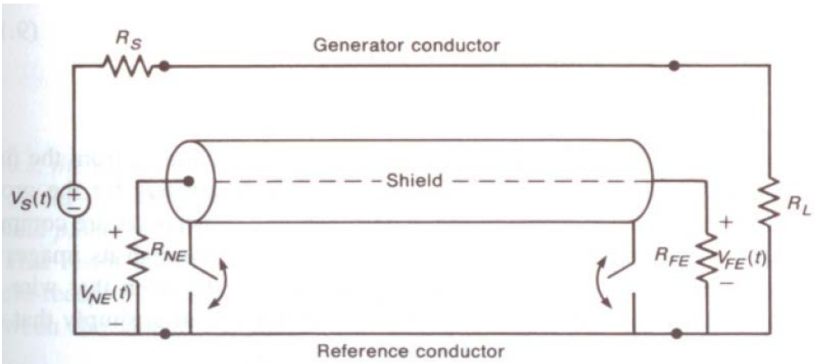
- Per-Unit-Length Parameters
  - Resistances
    - For braided-wire shields, the per-unit-length of resistance of the shield is

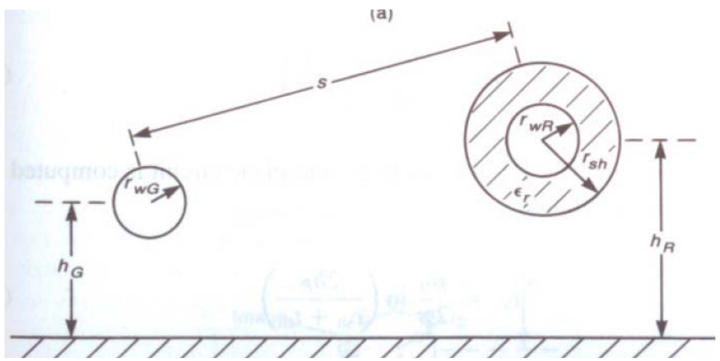
$$r_S = \frac{r_b}{BW \cos \theta_w}$$

    - For solid shields, the per-unit-length resistance of the shield is

$$r_S = \frac{1}{\sigma 2 \pi r_{\mathrm{sh}} t_{\mathrm{sh}}}$$

    - These results were derived previously.





Shielded Wires

- Per-Unit-Length Parameters
  - Self-Inductances
    - The self-inductance of the generator circuit is

$$l_G = \frac{\mu_0}{2\pi} \ln\left(\frac{2h_G}{r_{wG}}\right)$$

    - The self-inductance of the shield ground plane circuit is

$$l_S = \frac{\mu_0}{2\pi} \ln\left(\frac{2h_R}{r_{sh} + t_{sh}}\right)$$

    - The self-inductance of the receptor circuit is

$$l_R = \frac{\mu_0}{2\pi} \ln\left(\frac{2h_R}{r_{wR}}\right)$$

    - Remember all these values are computed with respect to ground.

Shielded Wires

- Per-Unit-Length Parameters
  - Mutual-Inductances
    - The mutual inductance between the generator wire and the shield l_{GS} and that between the generator wire and the receptor wire l_{GR} are

$$l_{GS} = \frac{\mu_0}{4\pi} \ln\left(1 + 4\frac{h_G h_R}{s^2}\right)$$

$$= l_{GR}$$

    - The mutual inductance between the shield and receptor wire circuit is

$$l_{RS} = \frac{\mu_0}{2\pi} \ln\left(\frac{2h_R}{r_{sh} + t_{sh}}\right)$$

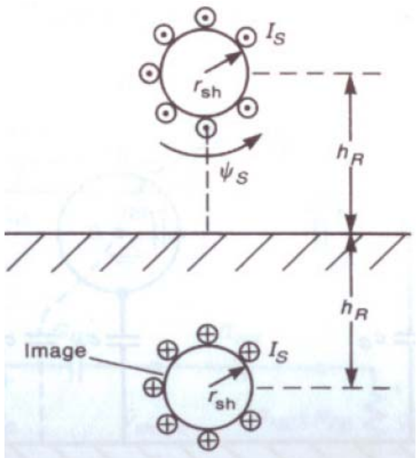
$$= l_S$$

The mutual inductance between the receptor wire-ground plane circuit and the shield-ground plane circuit l_{RS} is identical to the self–inductance of the shield-ground plane circuit l_S.

Shielded Wires

- Per-Unit-Length Parameters
  - Mutual-Inductances
    - The mutual inductance between the shield and receptor circuits can be obtained (1) by placing a current on the shield and determining the magnetic flux through the receptor circuit or (2) by placing a current on the receptor wire and determining the magnetic flux through the shield circuit:

$$l_{RS} = \left.\frac{\psi_S}{I_R}\right|_{I_S=0} = \left.\frac{\psi_R}{I_S}\right|_{I_R=0}$$



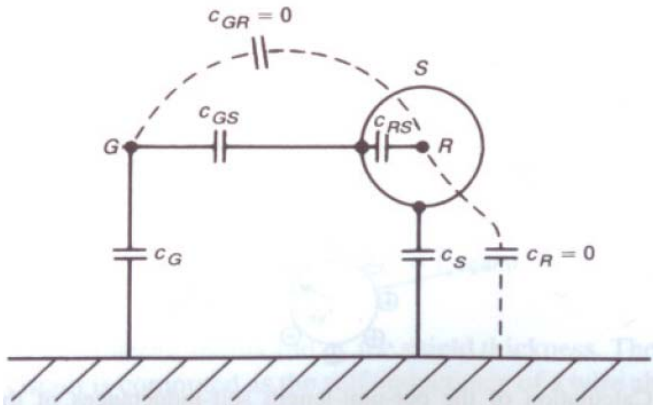
Shielded Wires

- Per-Unit-Length Parameters
  - Capacitances
    - The capacitance between the shield and receptor wire is the same as for a coaxial cable, which is

$$c_{RS} = \frac{2\pi\epsilon_0\epsilon_r}{\ln(r_{sh}/r_{wR})}$$

    - The other capacitances can be found using the reciprocal relationship for conductors in a homogeneous medium, which are

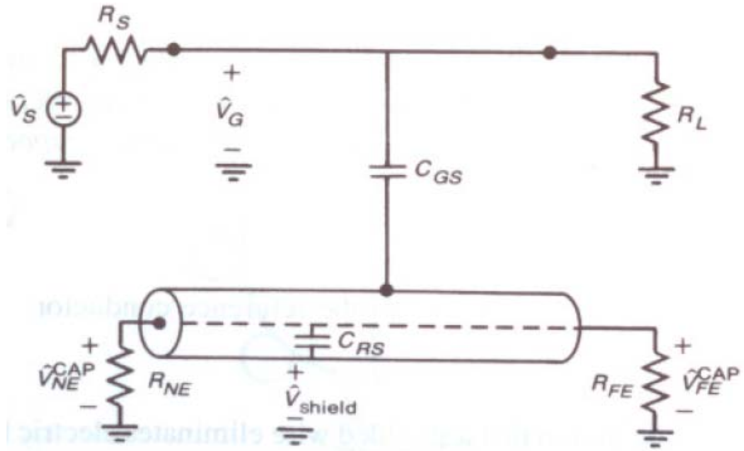
$$\begin{bmatrix} c_G + c_{GS} & -c_{GS} \\ -c_{GS} & c_S + c_{GS} \end{bmatrix} = \mu_0\epsilon_0 \begin{bmatrix} l_G & l_{GS} \\ l_{GS} & l_S \end{bmatrix}^{-1}$$



Shielded Wires

- Inductive and Capacitive Coupling
  - Capacitive Coupling
    - For weakly coupled lines that are electrically short, the capacitive coupling is (by voltage division)

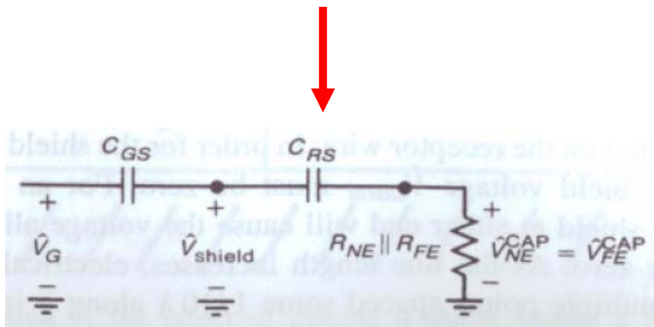
$$\hat{V}_{NE}^{CAP} = \hat{V}_{FE}^{CAP} = \frac{j\omega R C_{RS} \parallel C_{GS}}{1 + j\omega R C_{RS} \parallel C_{GS}} \hat{V}_G$$



- where

$$R = \frac{R_{NE} R_{FE}}{R_{NE} + R_{FE}}$$

$$C_{RS} \parallel C_{GS} = \frac{C_{RS} C_{GS}}{C_{RS} + C_{GS}}$$



Shielded Wires

- Inductive and Capacitive Coupling
  - Capacitive Coupling
    - For sufficiently small frequency this reduces to

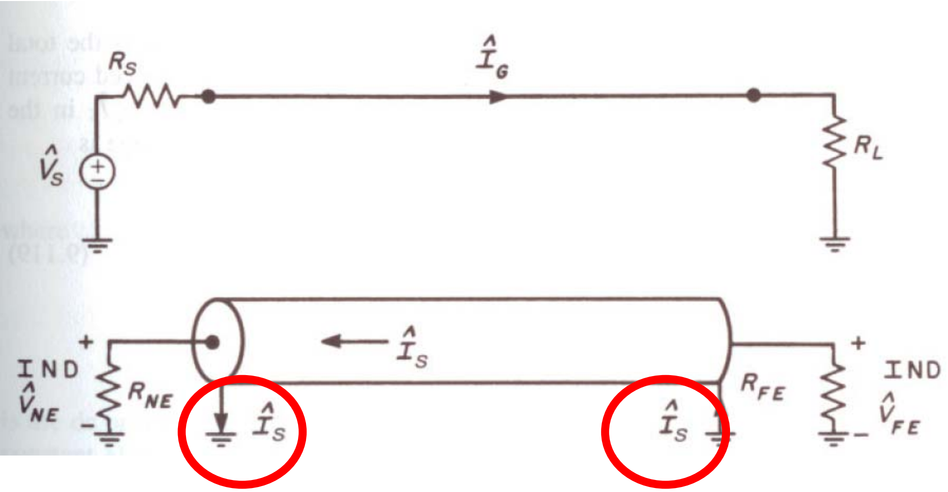
$$\hat{V}_{NE}^{CAP} = \hat{V}_{FE}^{CAP}$$
$$\cong j\omega \frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}} \frac{C_{RS}C_{GS}}{C_{RS}+C_{GS}} \hat{V}_{G_{dc}} \qquad \hat{V}_{G_{dc}} = \frac{R_L}{R_S+R_L}\hat{V}_S$$

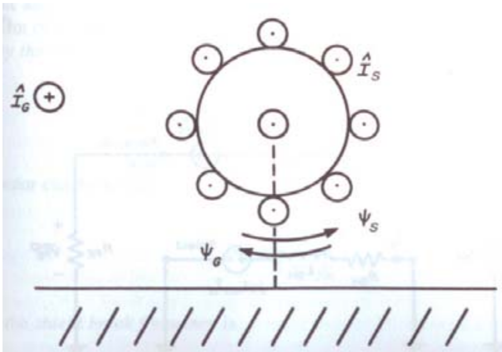
    - Also, for typical shielded wires $C_{RS} >> C_{GS}$, so that $C_{RS} \| C_{GS} \doteqdot C_{GS} \doteqdot C_{GR}$. Thus, the capacitive coupling is basically unchanged from the unshielded case.
    - If the shield is connected to the reference conductor at either end, the shield voltage is reduced to zero and the capacitive coupling contribution is removed.

$$\hat{V}_{NE}^{CAP} = \hat{V}_{FE}^{CAP}$$
$$= 0 \quad \text{(shield connected to the reference conductor at either end)}$$

Shielded Wires

- Inductive and Capacitive Coupling
  - Inductive Coupling
    - A shield must be grounded at both ends in order to eliminate inductive coupling.





Shielded Wires

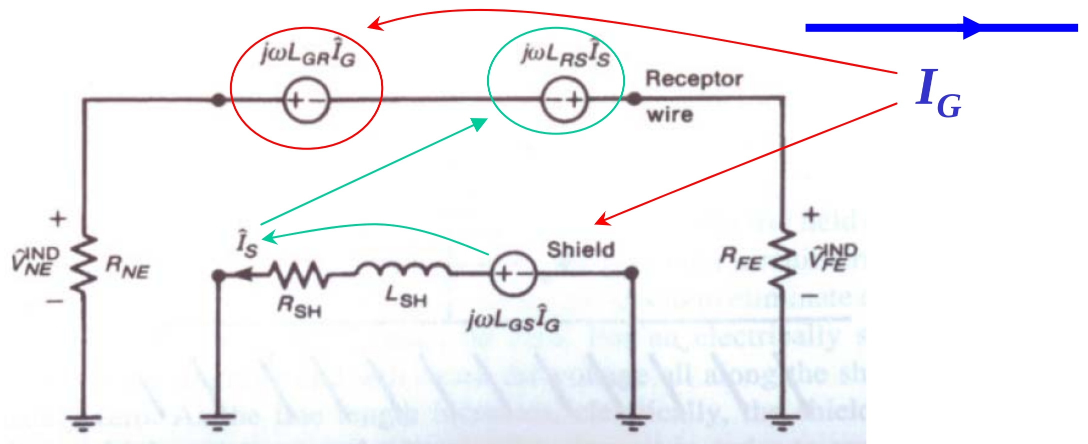
- Inductive and Capacitive Coupling
  - Inductive Coupling
    - Since the current flowing back along the shield is

$$\hat{I}_S = \frac{j\omega L_{GS}}{R_{SH} + j\omega L_{SH}} \hat{I}_G$$

    - The near-end induced crosstalk voltage is

$$\hat{V}_{NE}^{IND} = \frac{R_{NE}}{R_{FE} + R_{NE}} j\omega\left(L_{GR}\hat{I}_G - L_{RS}\hat{I}_S\right)$$

$$\hat{V}_{NE}^{IND} = \frac{R_{NE}}{R_{FE} + R_{NE}} \frac{j\omega R_{SH} L_{GR} + \omega^2 (L_{GS}L_{RS} - L_{GR}L_{SH})}{R_{SH} + j\omega L_{SH}} \hat{I}_G$$



Shielded Wires

- Inductive and Capacitive Coupling
  - Inductive Coupling
    - Since

$$L_{GR} = L_{GS}$$
$$L_{RS} = L_{SH}$$

    - we obtain

$$\hat{V}_{NE}^{\mathrm{IND}} = \underbrace{\frac{R_{NE}}{R_{FE} + R_{NE}} j\omega L_{GR} \hat{I}_G}_{\substack{\text{crosstalk} \\ \text{with shield} \\ \text{removed}}} \underbrace{\frac{R_{\mathrm{SH}}}{R_{\mathrm{SH}} + j\omega L_{\mathrm{SH}}}}_{\substack{\text{effect of} \\ \text{shield}}}$$

$$\hat{I}_G = \frac{1}{R_S + R_L} \hat{V}_S$$

$$\hat{V}_{FE}^{\mathrm{IND}} = -\underbrace{\frac{R_{FE}}{R_{FE} + R_{NE}} j\omega L_{GR} \hat{I}_G}_{\substack{\text{crosstalk} \\ \text{with shield} \\ \text{removed}}} \underbrace{\frac{R_{\mathrm{SH}}}{R_{\mathrm{SH}} + j\omega L_{\mathrm{SH}}}}_{\substack{\text{effect of} \\ \text{shield}}}$$

Shielded Wires

- Inductive and Capacitive Coupling
  - Inductive Coupling
    - These equations are the results for the same case with shield removed but multiplied by the shielding factor

$$\mathrm{SF} = \frac{R_{\mathrm{SH}}}{R_{\mathrm{SH}} + j\omega L_{\mathrm{SH}}} \longrightarrow \mathrm{SF} = \frac{1}{1 + j\frac{f}{f_{\mathrm{SH}}}}$$

    - where the shield break frequency is

$$f_{\mathrm{SH}} = \frac{R_{\mathrm{SH}}}{2\pi L_{\mathrm{SH}}} \longrightarrow \mathrm{SF} \cong \begin{cases} 1 & \text{for } f < f_{\mathrm{SH}} \\ \dfrac{R_{\mathrm{SH}}}{j\omega L_{\mathrm{SH}}} & \text{for } f > f_{\mathrm{SH}} \end{cases}$$

Shielded Wires

- Inductive and Capacitive Coupling
  - Inductive Coupling
    - Case 1: The shield is not grounded at both ends:

$$\hat{V}_{NE}^{\mathrm{IND}} = j\omega \left\{ \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{L_{GR}}{R_S + R_L} \right\} \hat{V}_S$$

$$\hat{V}_{FE}^{\mathrm{IND}} = j\omega \left\{ -\frac{R_{FE}}{R_{NE} + R_{FE}} \frac{L_{GR}}{R_S + R_L} \right\} \hat{V}_S$$

    - Case 2: The shield is grounded at both ends and $f < f_{SH}$:

$$\hat{V}_{NE}^{\mathrm{IND}} = j\omega \left\{ \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{L_{GR}}{R_S + R_L} \right\} \hat{V}_S$$

$$\hat{V}_{FE}^{\mathrm{IND}} = j\omega \left\{ -\frac{R_{FE}}{R_{NE} + R_{FE}} \frac{L_{GR}}{R_S + R_L} \right\} \hat{V}_S$$

    - Case 3: The shield is grounded at both ends and $f > f_{SH}$:

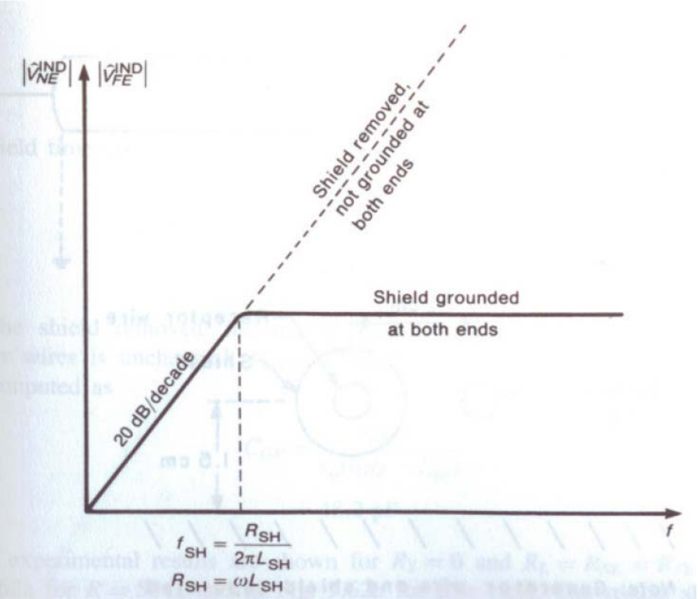
$$\hat{V}_{NE}^{\mathrm{IND}} = \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{L_{GR}}{R_S + R_L} \frac{R_{\mathrm{SH}}}{L_{\mathrm{SH}}} \hat{V}_S$$

$$\hat{V}_{FE}^{\mathrm{IND}} = -\frac{R_{FE}}{R_{NE} + R_{FE}} \frac{L_{GR}}{R_S + R_L} \frac{R_{\mathrm{SH}}}{L_{\mathrm{SH}}} \hat{V}_S$$

Shielded Wires

- Inductive and Capacitive Coupling
  - Inductive Coupling

• When $f<f_{SH}$, the generator current finds the lowest-impedance return path through the ground plane.
• When $f>f_{SH}$, the generator current finds the lowest-impedance return path to be back along the shield instead of through the ground plane.

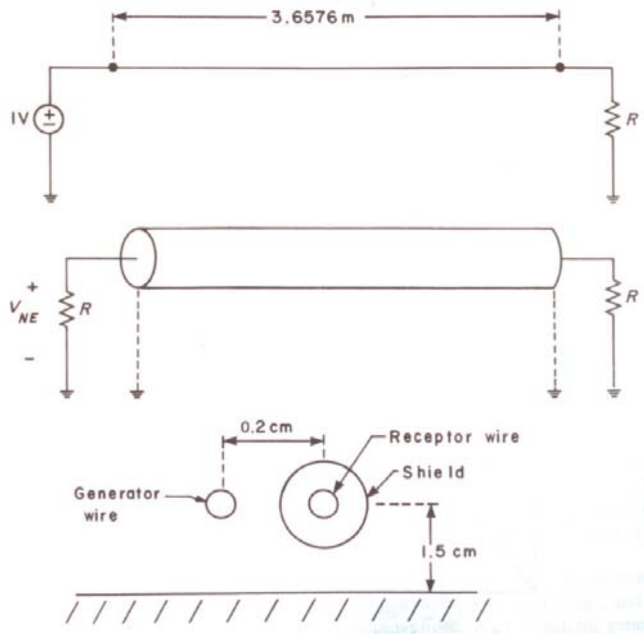


- The total crosstalk function is the sum of inductive and capacitive coupling contributions:

$$\frac{\hat{V}_{NE}}{\hat{V}_S} = \frac{\hat{V}_{NE}^{IND}}{\hat{V}_S} + \frac{\hat{V}_{NE}^{CAP}}{\hat{V}_S} \qquad \frac{\hat{V}_{FE}}{\hat{V}_S} = \frac{\hat{V}_{FE}^{IND}}{\hat{V}_S} + \frac{\hat{V}_{FE}^{CAP}}{\hat{V}_S}$$

Shielded Wires

- Effect of Shield Grounding
  - Capacitive Coupling



- The crosstalk transfer ratio is

$$\frac{\hat{V}_{NE}}{\hat{V}_S} = j2\pi f\left[M_{NE}^{\text{IND}} + M_{NE}^{\text{CAP}}\right]$$

$$\frac{\hat{V}_{FE}}{\hat{V}_S} = j2\pi f\left[M_{FE}^{\text{IND}} + M_{FE}^{\text{CAP}}\right]$$

- For R_S=0, R_L=R_{NE}=R_{FE}=R, if the shield is not grounded at either end, the capacitive coupling is essentially the same as with the shield removed

$$C_{RS} \| C_{GS} \cong C_{GS} \cong C_{GR}$$

$$\begin{aligned} M_{NE}^{\text{CAP}} &= M_{FE}^{\text{CAP}} \\ &= \frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}}\frac{C_{RS}C_{GS}}{C_{RS}+C_{GS}}\frac{R_L}{(R_S=0)+R_L} \\ &= \frac{R}{2}\frac{C_{RS}C_{GS}}{C_{RS}+C_{GS}} \\ &\cong \frac{R}{2}C_{GR} \qquad \text{OO} \end{aligned}$$

right open and left open

Shielded Wires

- Effect of Shield Grounding
  - Inductive Coupling
    - If the shield is grounded at at least one end, the capacitive coupling is eliminated.

$$M_{NE}^{\mathrm{CAP}} = M_{FE}^{\mathrm{CAP}}$$
$$= 0 \qquad \mathrm{OS, SO, SS}$$

    - If the shield is not grounded at both ends, the inductive coupling is essentially the same as with the shield removed.

$$M_{NE}^{\mathrm{IND}} = \frac{R_{NE}}{R_{NE}+R_{FE}} \frac{L_{GR}}{(R_S=0)+R_L}$$
$$= \frac{L_{GR}}{2R} \qquad \mathrm{OO, OS, SO}$$
$$M_{FE}^{\mathrm{IND}} = -\frac{R_{FE}}{R_{NE}+R_{FE}} \frac{L_{GR}}{(R_S=0)+R_L}$$
$$= -\frac{L_{GR}}{2R} \qquad \mathrm{OO, OS, SO}$$

Shielded Wires

- Effect of Shield Grounding
  - Inductive Coupling
    - If the shield is grounded at both ends, the a shield factor

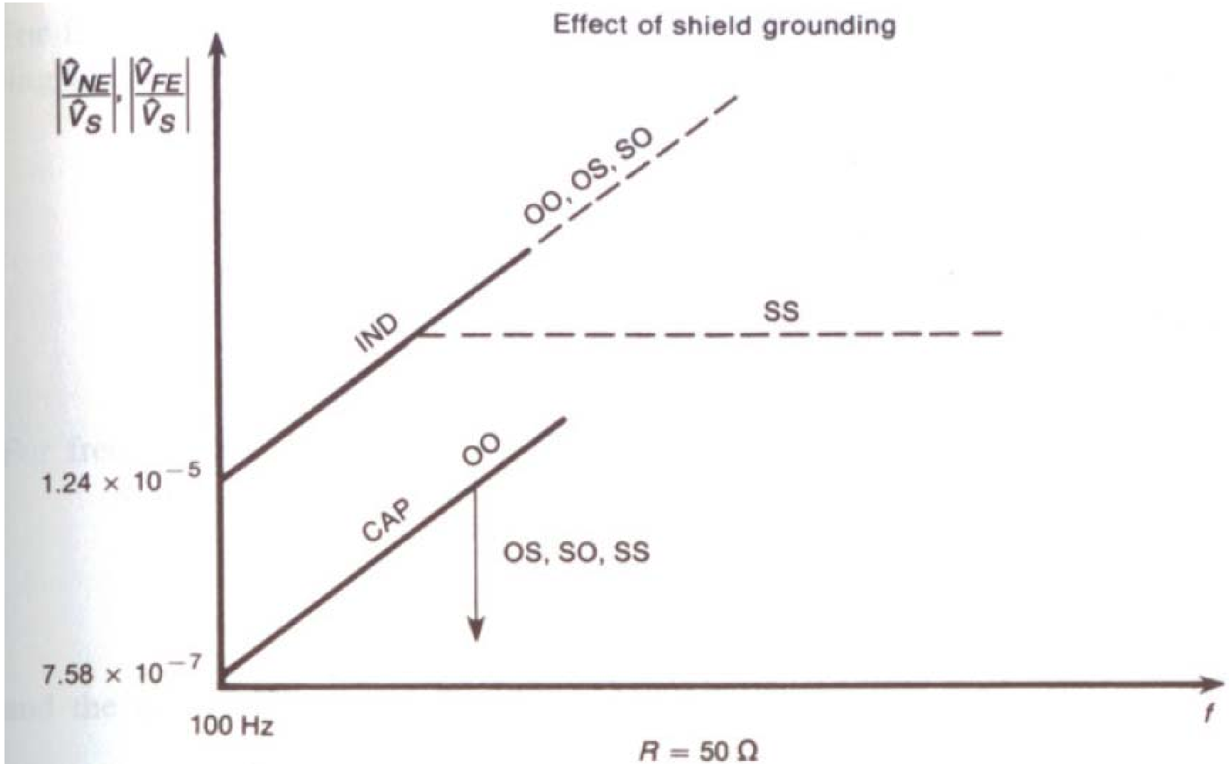
$$\mathrm{SF} = \frac{1}{1 + j\dfrac{f}{f_{\mathrm{SH}}}}$$

    - is multiplied by the inductive coupling.

$$\begin{aligned} M_{NE}^{\mathrm{IND}} &= \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{L_{GR}}{(R_S = 0) + R_L} \mathrm{SF} \\ &= \frac{L_{GR}}{2R} \mathrm{SF} \qquad \mathrm{SS} \\ M_{FE}^{\mathrm{IND}} &= -\frac{R_{FE}}{R_{NE} + R_{FE}} \frac{L_{GR}}{(R_S = 0) + R_L} \mathrm{SF} \\ &= \frac{-L_{GR}}{2R} \mathrm{SF} \qquad \mathrm{SS} \end{aligned}$$

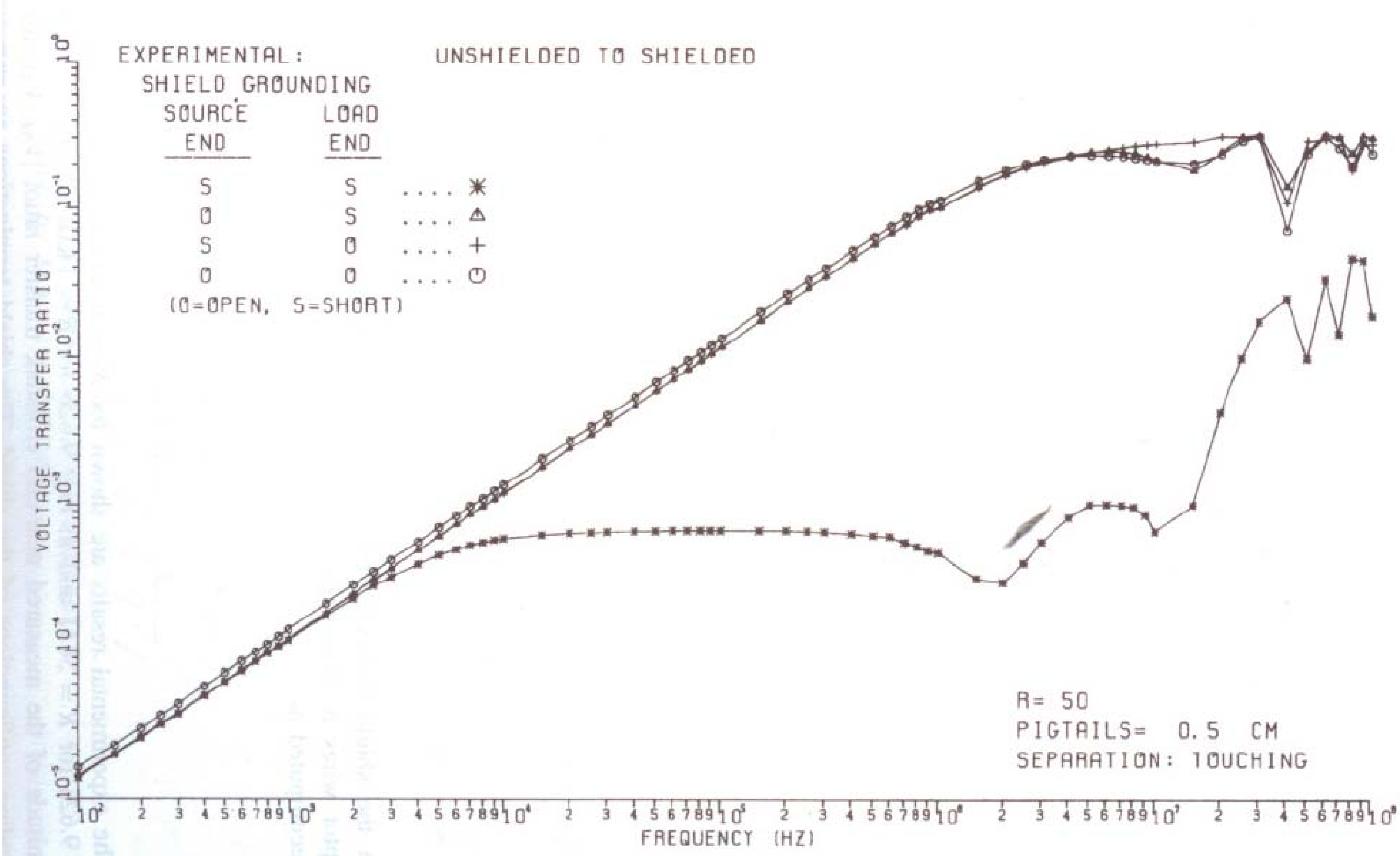
Shielded Wires

- Effect of Shield Grounding
  - Low-Impedance Load R=50 Ω
    - Initially, inductive coupling will dominate capacitive coupling for the shield removed.



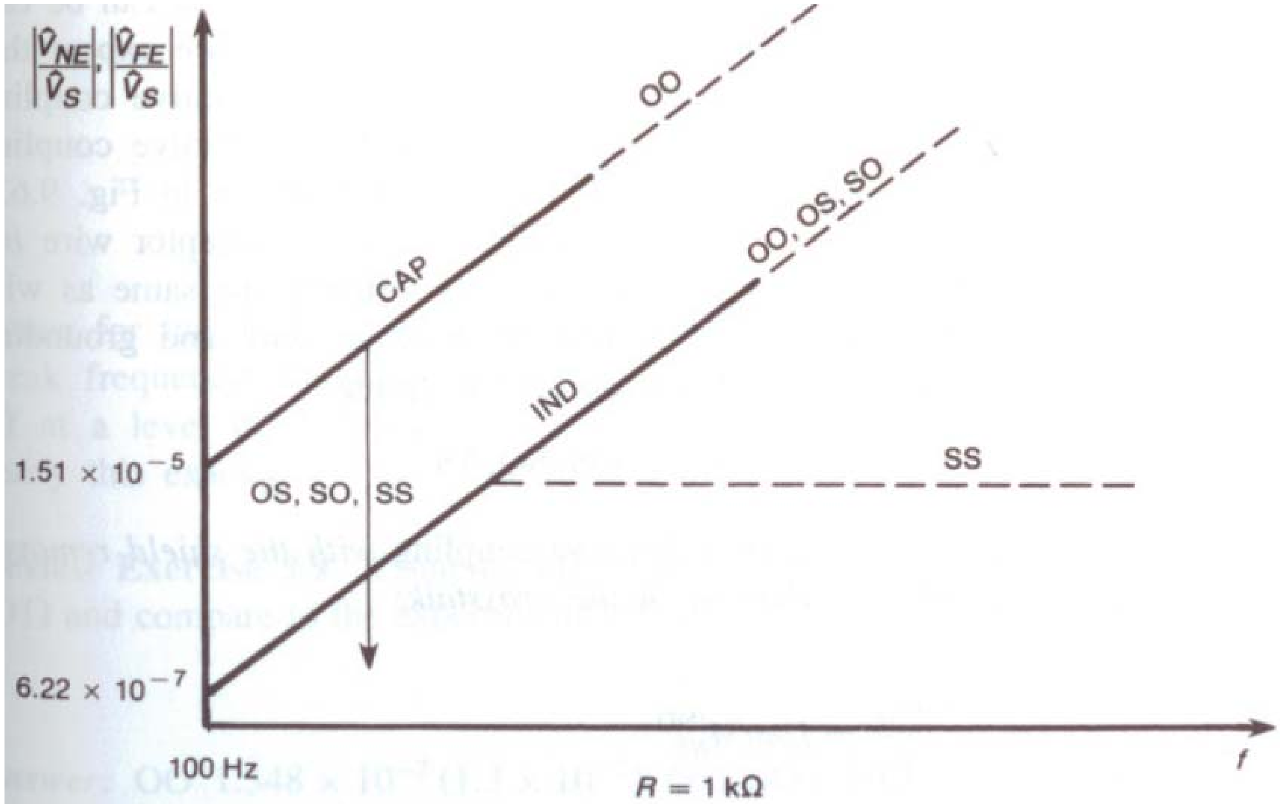
Shielded Wires

- Effect of Shield Grounding
  - Low-Impedance Load R=50 Ω



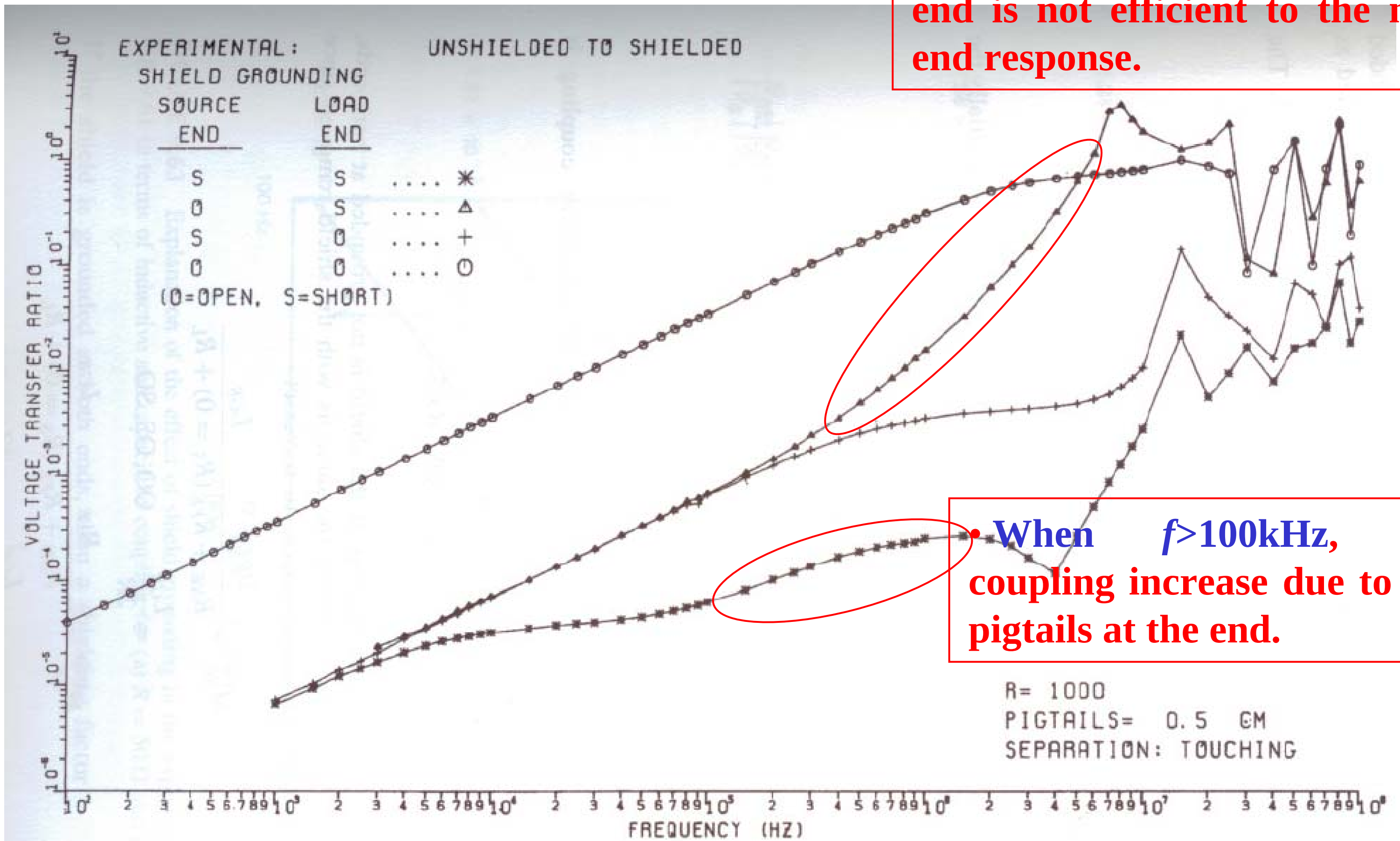
Shielded Wires

- Effect of Shield Grounding
  - High-Impedance Load R=1kΩ
    - Initially, capacitive coupling will dominate inductive coupling for the shield removed.



Shielded Wires

- Effect of Shield Grounding
  - High-Impedance Load R=1kΩ

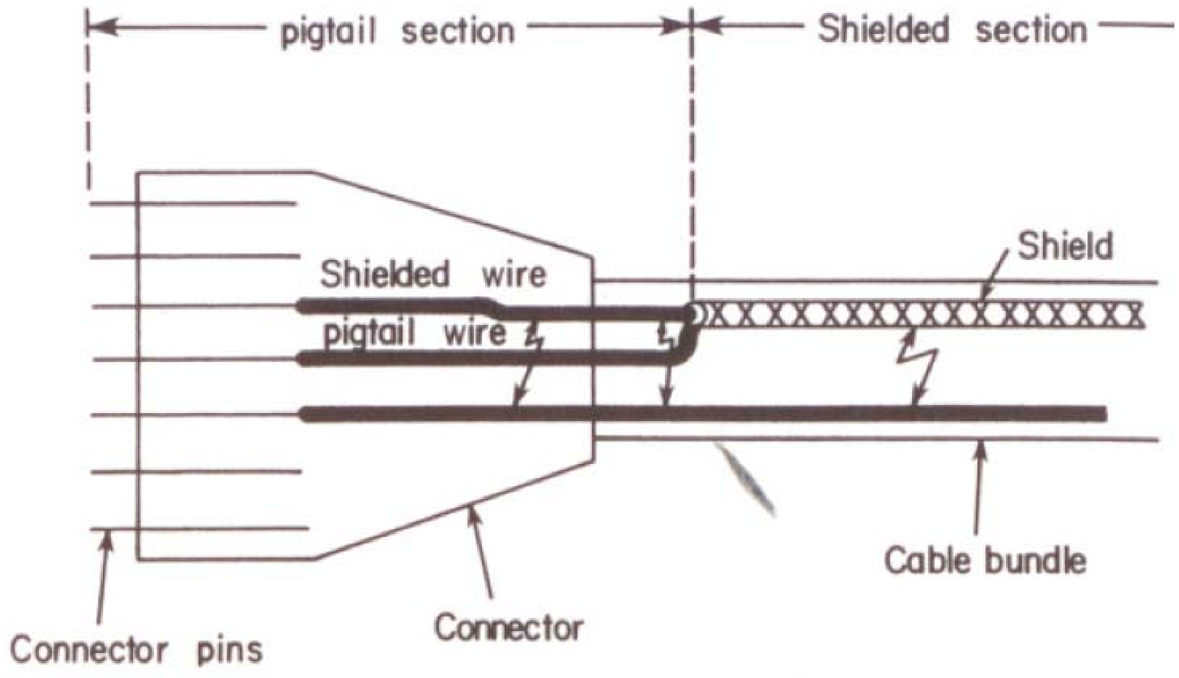


• When *f*>200kHz, the coupling increase since shorting at the far end is not efficient to the near end response.

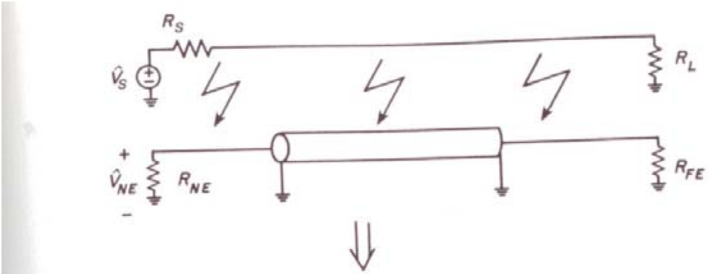
• When *f*>100kHz, the coupling increase due to the pigtails at the end.

Shielded Wires

- Effect of Pigtails
  - Definition
    - The term "pigtail" is commonly used to refer to the break in a shield required to terminate it to a "grounding point."

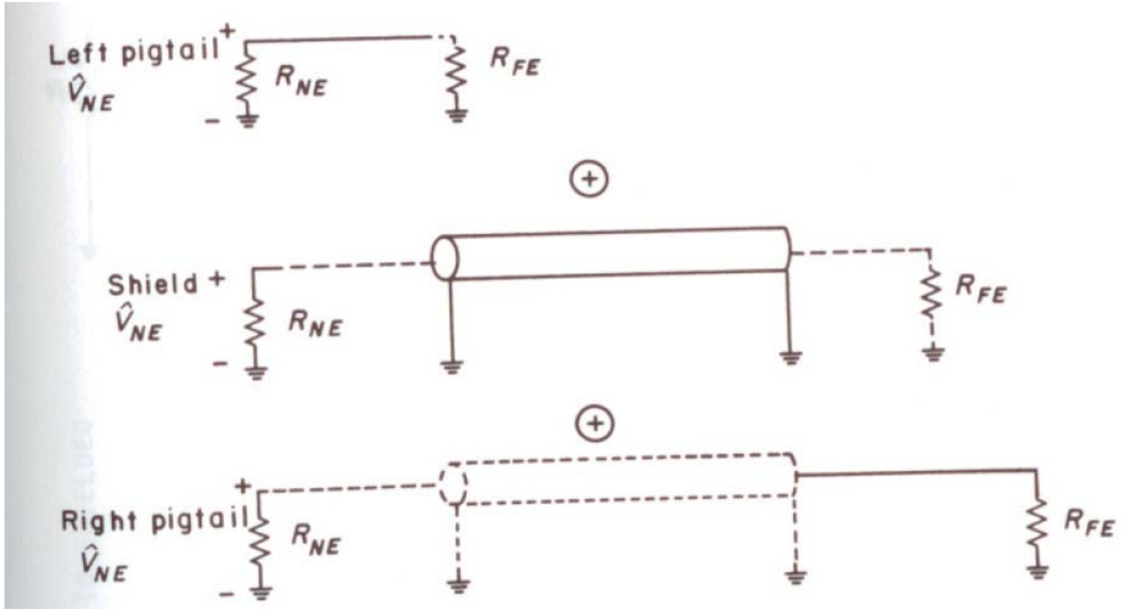


Shielded Wires



- Effect of Pigtails
  - Definition
    - For electrically short line, the total cross coupling could be obtained using superposition, which is

$$\hat{V}_{NE} = \hat{V}_{NE}^{\text{left pigtail}} + \hat{V}_{NE}^{\text{shielded section}} + \hat{V}_{NE}^{\text{right pigtail}}$$

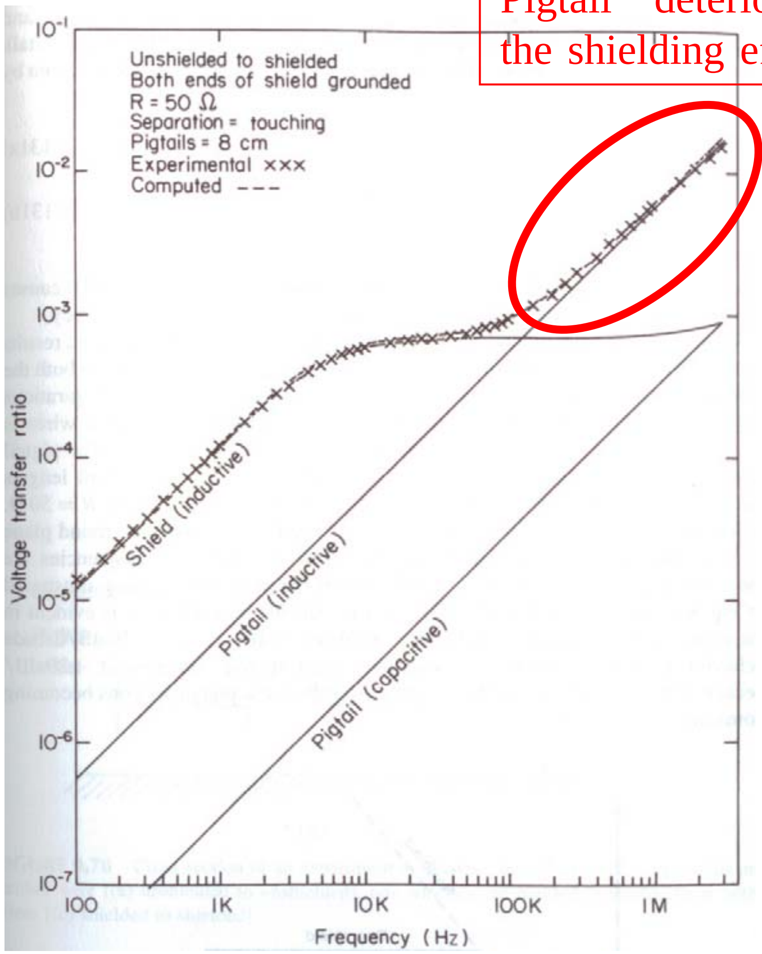


Shielded Wires

- Effect of Pigtails
  - Low-Impedance Load R=50 Ω

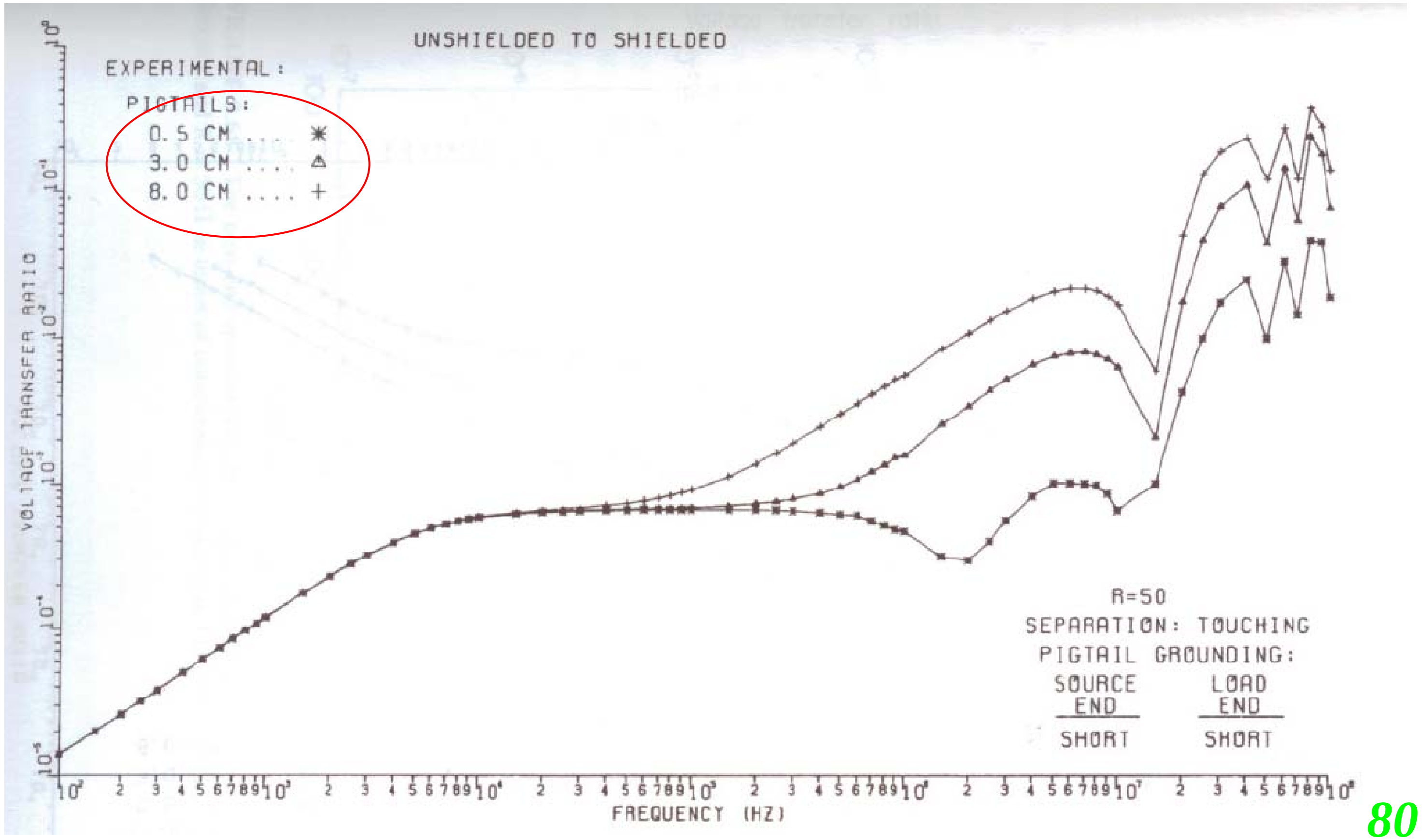
• Initially, without shield, the inductive coupling will dominate capacitive coupling.
• With either end of the shield grounded, the capacitive coupling of the shielded receptor wire is removed.
• With both ends of the shield grounded, the inductive coupling of the shielded wire includes a shielding factor.

Pigtail deteriorate the shielding effect.



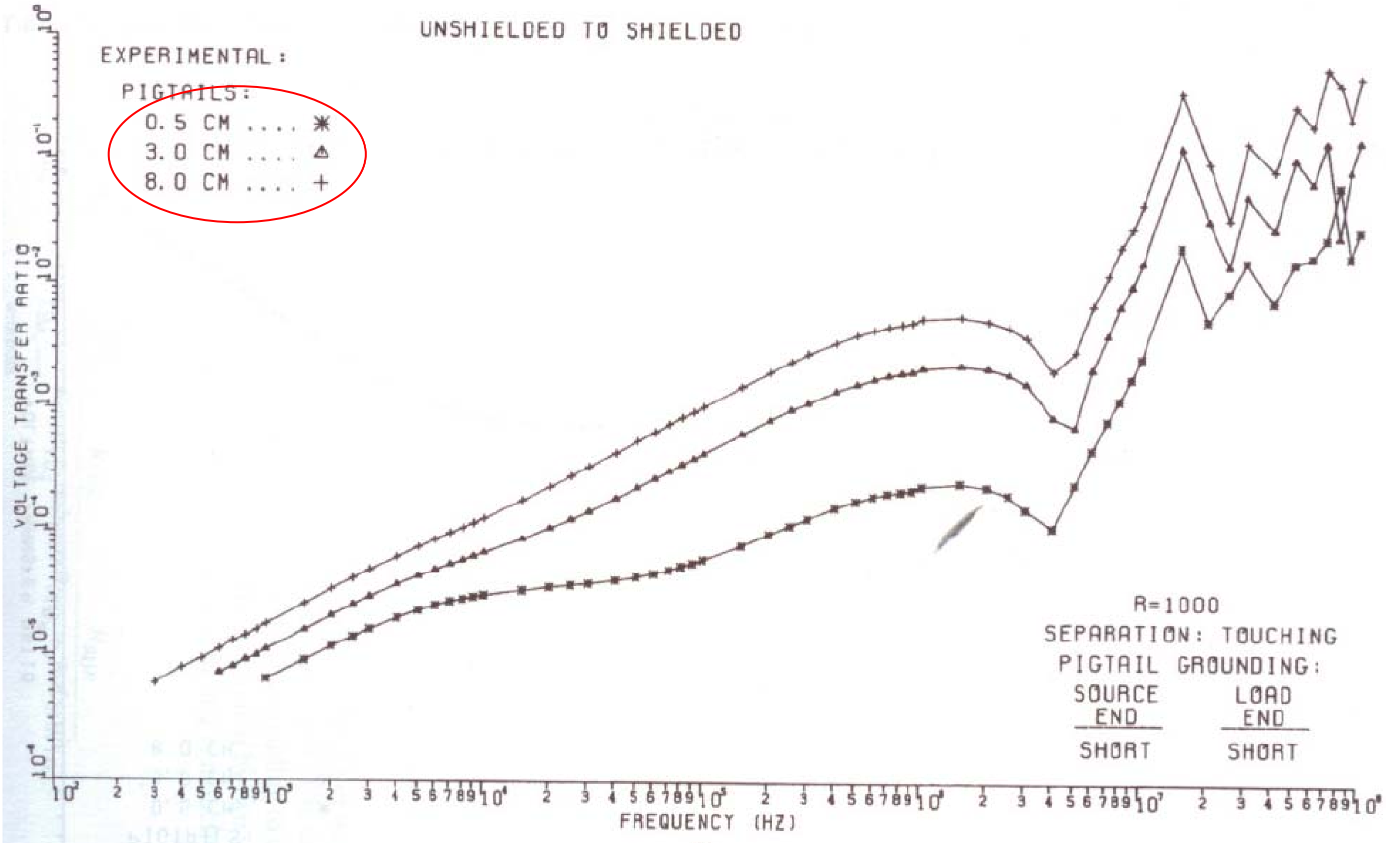
Shielded Wires

- Effect of Pigtails
  - Low-Impedance Load R=50 Ω
    - Different lengths of pigtails



Shielded Wires

- Effect of Pigtails
  - High-Impedance Load R=1kΩ
    - Different lengths of pigtails



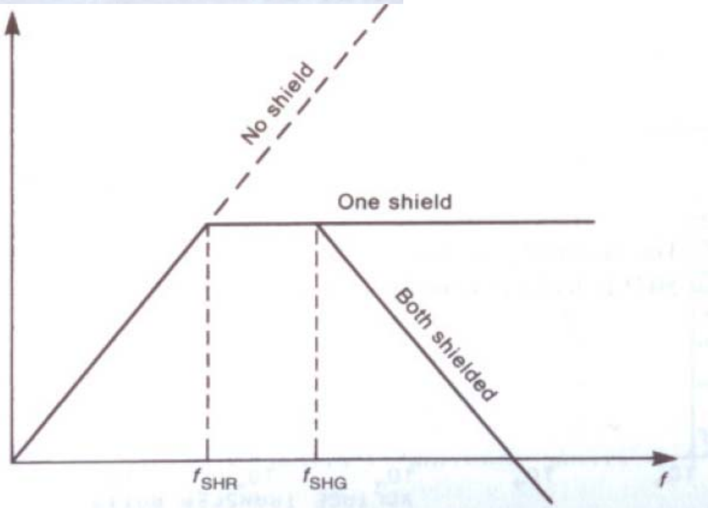
Shielded Wires

- Effect of Multiple Shields
  - Generator and Receptor Both Shielded
    - Assuming that both ends of each shield for the generator and receptor are grounded, the total coupling is inductive and is

$$\frac{\hat{V}_{NE,FE}}{\hat{V}_S} = \underbrace{\frac{\hat{V}^{\mathrm{IND}}_{NE,FE}}{\hat{V}_S}}_{\substack{\text{with both}\\ \text{shields}\\ \text{removed}}} \frac{R_{\mathrm{SHG}}}{R_{\mathrm{SHG}} + j\omega L_{\mathrm{SHG}}} \frac{R_{\mathrm{SHR}}}{R_{\mathrm{SHR}} + j\omega L_{\mathrm{SHR}}}$$

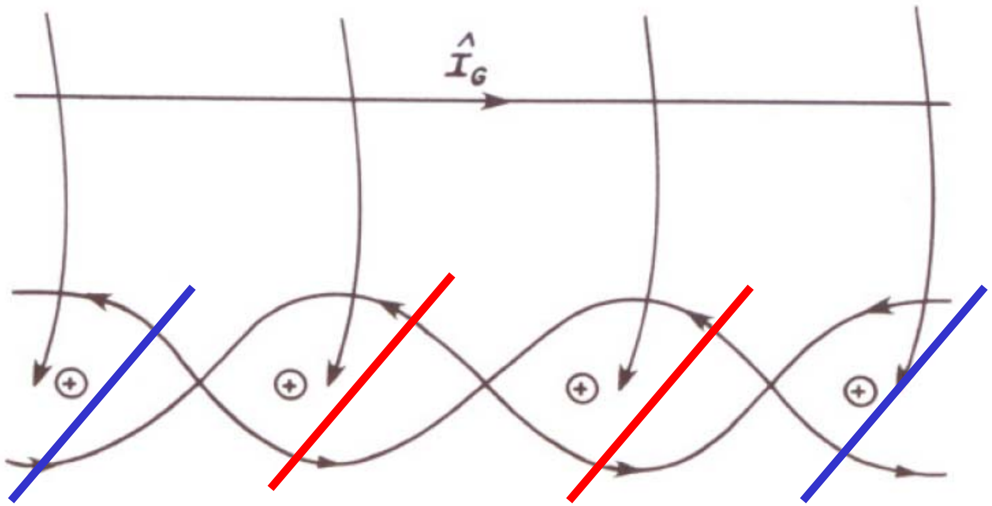
$$f_{\mathrm{SHG}} = \frac{R_{\mathrm{SHG}}}{2\pi L_{\mathrm{SHG}}}$$

$$f_{\mathrm{SHR}} = \frac{R_{\mathrm{SHR}}}{2\pi L_{\mathrm{SHR}}}$$



Twisted Wires

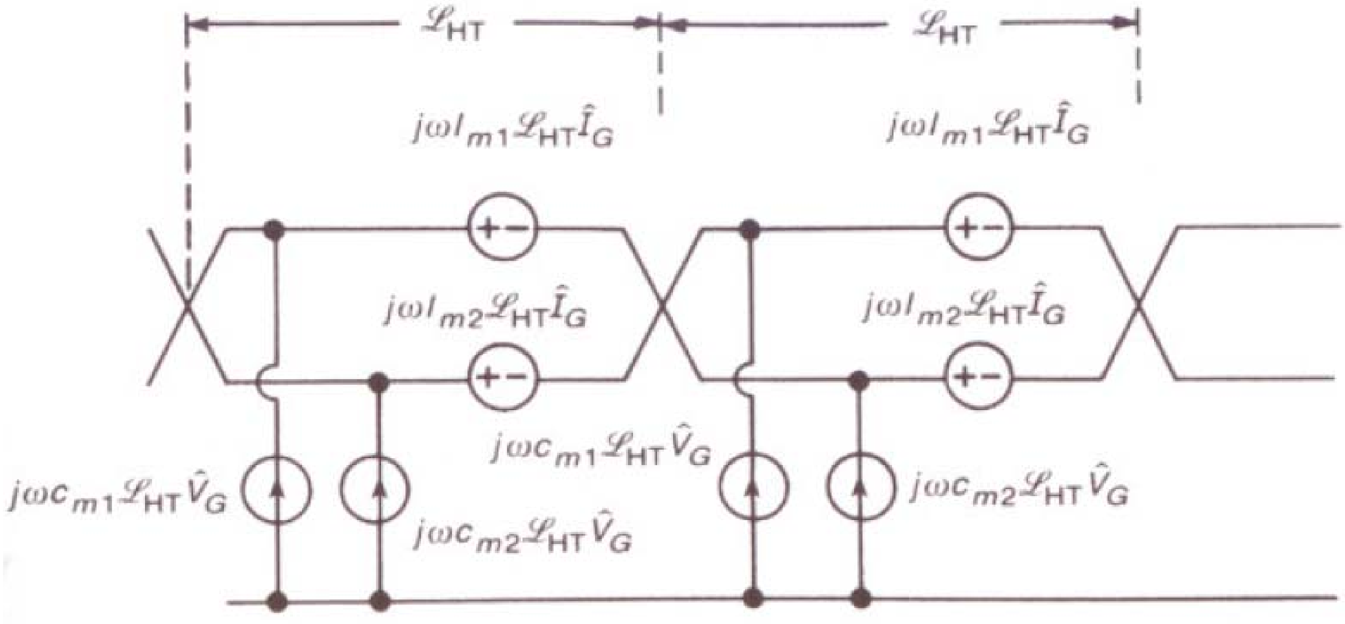
- Principle
  - Odd and Even Number of Half-Twists
    - Replacing a receptor wire with a twisted pair and using one wire of the pair as the return for the receptor circuit inherently reduces inductive or magnetic field coupling because of the twist.



The inductive coupling is zero for even number of half-twists or symmetry structure.

Twisted Wires

- Principle
  - Simple Inductive-Capacitive Coupling Model
    - We assume once again that the line is electrically short in using the following lumped model.

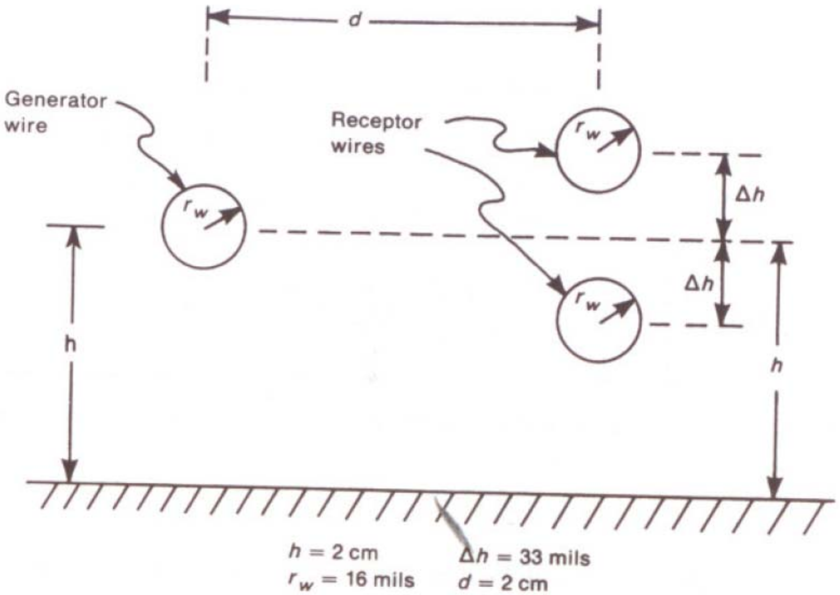


Twisted Wires

- Per-Unit-Length Parameters
  - Inductances
    - The mutual inductances are obtained by treating each wire of the twisted pair with ground plane as a circuit.

$$l_{m1} = \frac{\mu_0}{4\pi} \ln\left[1 + \frac{4h(h+\Delta h)}{d^2 + \Delta h^2}\right]$$

$$l_{m2} = \frac{\mu_0}{4\pi} \ln\left[1 + \frac{4h(h+\Delta h)}{d^2 + \Delta h^2}\right]$$



- The self-inductances are

$$l_G = \frac{\mu_0}{2\pi} \ln\left(\frac{2h}{r_w}\right)$$

$$l_{R1} = \frac{\mu_0}{2\pi} \ln\left[\frac{2(h+\Delta h)}{r_w}\right]$$

$$l_{R2} = \frac{\mu_0}{2\pi} \ln\left[\frac{2(h-\Delta h)}{r_w}\right]$$

Twisted Wires

- Per-Unit-Length Parameters
  - Inductances
    - The remaining mutual inductance is computed by placing a current on one wire of the twisted pair (and returning through the ground), and computing the flux through the circuit formed by the other wire and the ground plane, which is

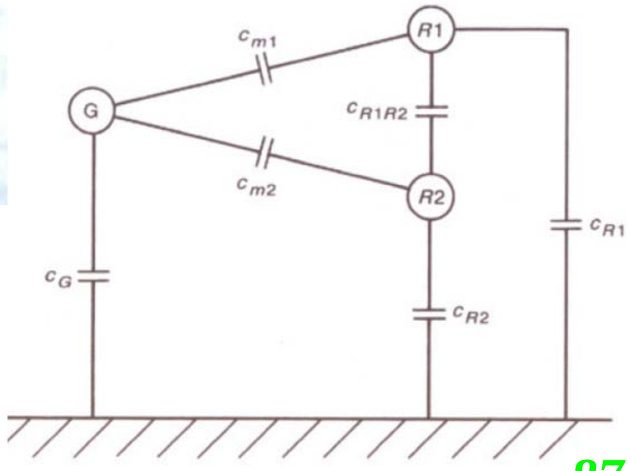
$$l_{R1R2} = \frac{\mu_0}{2\pi}\ln\left(\frac{h+\Delta h}{2\Delta h}\right) + \frac{\mu_0}{2\pi}\ln\left[\frac{(h+\Delta h)+(h-\Delta h)}{h+\Delta h}\right]$$
$$= \frac{\mu_0}{2\pi}\ln\left(\frac{h}{\Delta h}\right)$$

Twisted Wires

- Per-Unit-Length Parameters
  - Capacitances
    - The mutual capacitances are computed by ignoring the dielectric insulations as

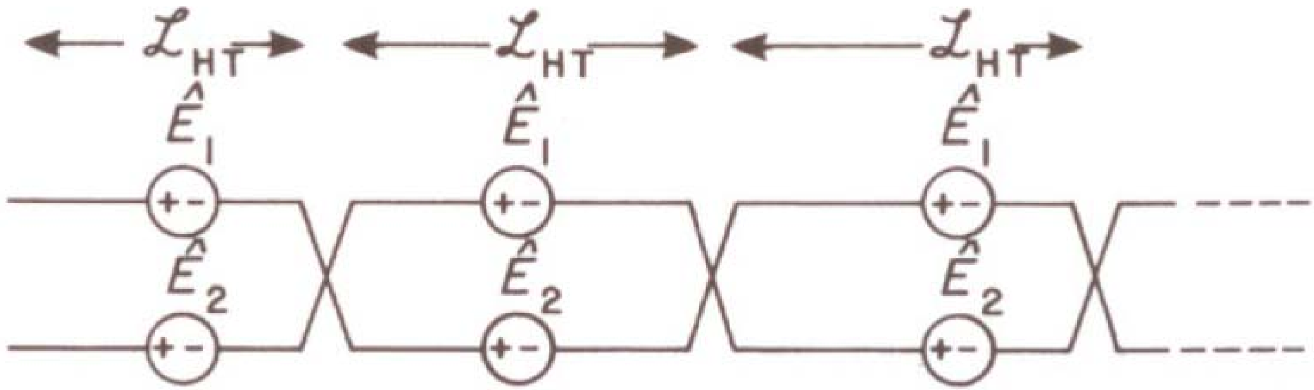
$$\begin{bmatrix} c_G + c_{m1} + c_{m2} & -c_{m1} & -c_{m2} \\ -c_{m1} & c_{R1R2} + c_{m1} + c_{R1} & -c_{R1R2} \\ -c_{m2} & -c_{R1R2} & c_{R1R2} + c_{m2} + c_{R2} \end{bmatrix}$$

$$= \mu_0 \epsilon_0 \begin{bmatrix} l_G & l_{m1} & l_{m2} \\ l_{m1} & l_{R1} & l_{R1R2} \\ l_{m2} & l_{R1R2} & l_{R2} \end{bmatrix}^{-1}$$



Twisted Wires

- Inductive and Capacitive Coupling
  - Inductive Coupling Model
    - The induced voltage sources E_1 and E_2 are due to the mutual inductance between the generator and the twisted pair receptor and are essentially induced emfs resulting from Faraday's law.

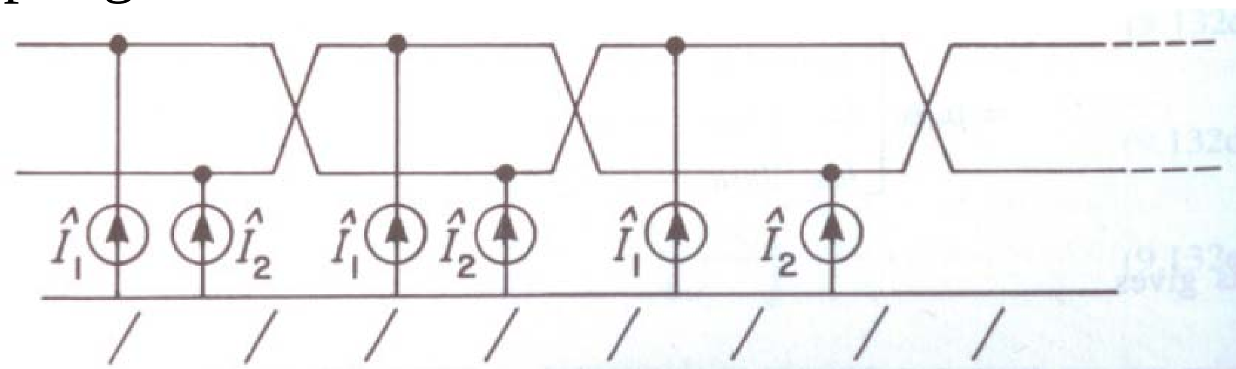


$$\hat{E}_1 = j\omega l_{m1} \mathscr{L}_{HT} \hat{I}_G$$

$$\hat{E}_2 = j\omega l_{m2} \mathscr{L}_{HT} \hat{I}_G$$

Twisted Wires

- Inductive and Capacitive Coupling
  - Capacitive Coupling Model
    - The induced current sources I_1 and I_2 are due to the mutual capacitance between the generator and twisted-pair receptor, and their contribution to crosstalk voltages are referred to as capacitive coupling.

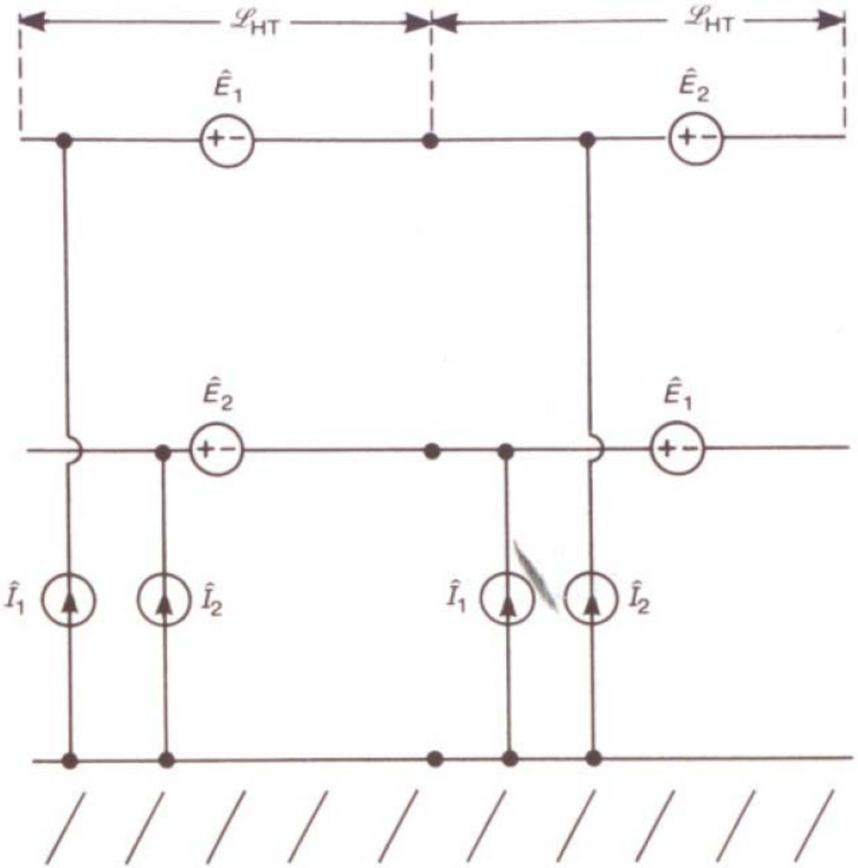


$$\hat{I}_1 = j\omega c_{m1} \mathscr{L}_{HT} \hat{V}_G$$

$$\hat{I}_2 = j\omega c_{m2} \mathscr{L}_{HT} \hat{V}_G$$

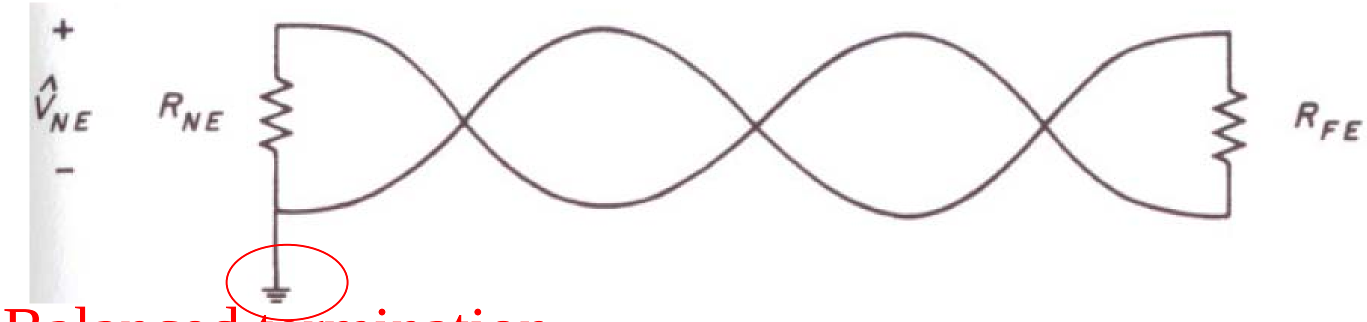
Twisted Wires

- Inductive and Capacitive Coupling
  - Untwisting Coupling Model
    - The untwisting model is shown below

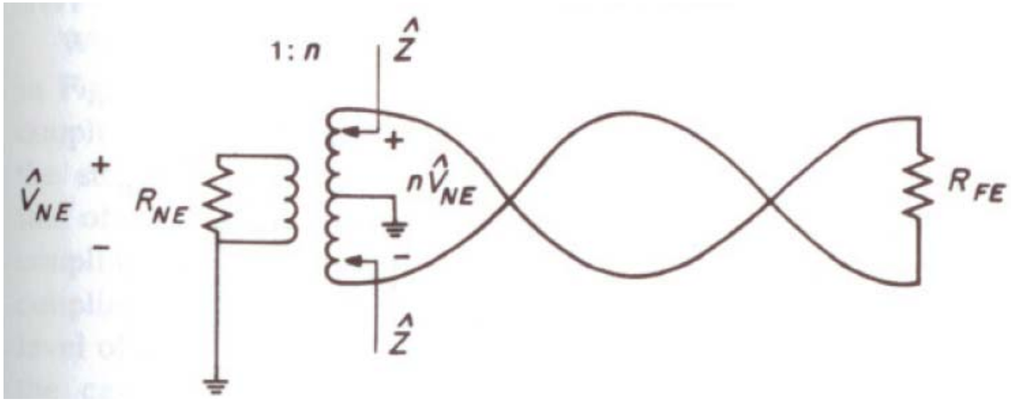


Twisted Wires

- Inductive and Capacitive Coupling
  - Unbalanced and Balanced Terminations
    - Unbalanced termination: grounded at only one end to avoid ground loops.



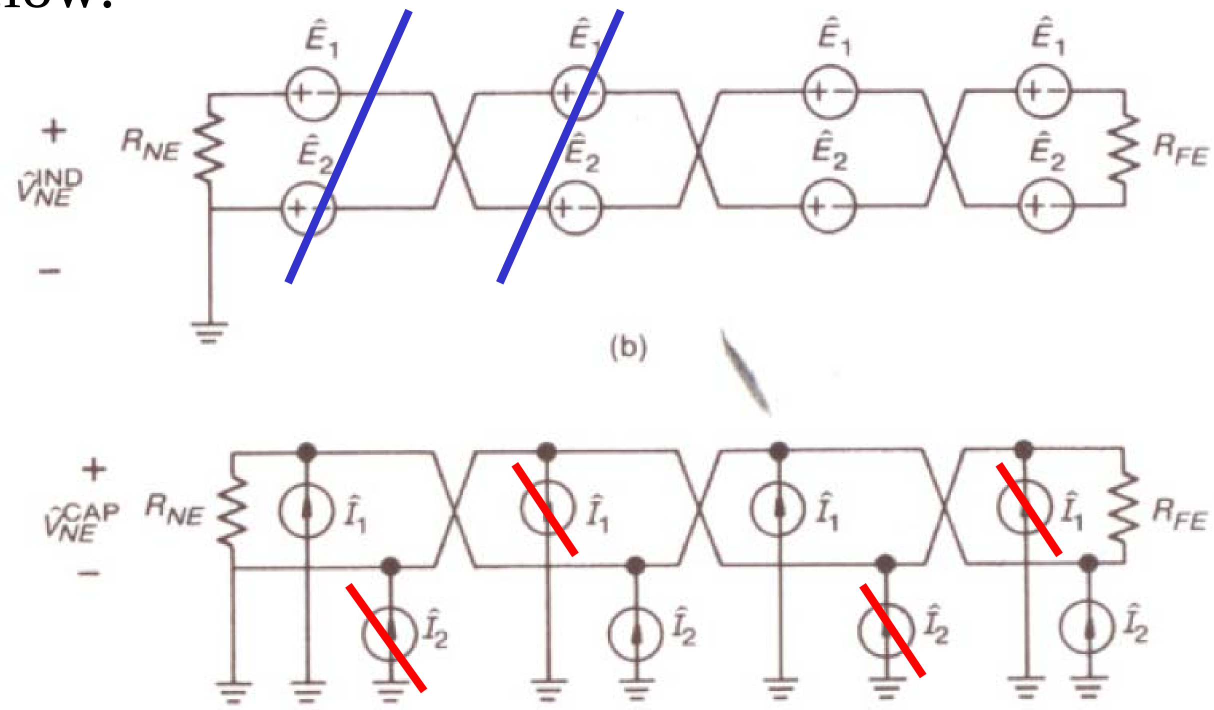
    - Balanced termination



For this case, the induced currents on the two wires cancel due to symmetry, thus, the capacitive coupling reduces to zero.

Twisted Wires

- Inductive and Capacitive Coupling
  - Unbalanced Termination Model
    - Assuming an odd number of half-twists, the inductive and capacitive coupling models are shown below.



Twisted Wires

- Inductive and Capacitive Coupling
  - Unbalanced Termination Model
    - The near-end and far-end crosstalk voltage transfer ratios are

$$\frac{\hat{V}_{NE}}{\hat{V}_S} = \frac{R_{NE}}{R_{NE}+R_{FE}} j\omega(l_{m1}-l_{m2})\mathscr{L}_{HT}\frac{1}{R_S+R_L} + \frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}} j\omega(c_m)\mathscr{L}\frac{R_L}{R_S+R_L}$$

$$\frac{\hat{V}_{FE}}{\hat{V}_S} = -\frac{R_{FE}}{R_{NE}+R_{FE}} j\omega(l_{m1}-l_{m2})\mathscr{L}_{HT}\frac{1}{R_S+R_L} + \frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}} j\omega(c_m)\mathscr{L}\frac{R_L}{R_S+R_L}$$

$$\mathscr{L} = N\mathscr{L}_{HT}$$

$$\hat{I}_G = \frac{1}{R_S+R_L}\hat{V}_S$$

$$\hat{V}_G = \frac{R_L}{R_S+R_L}\hat{V}_S$$

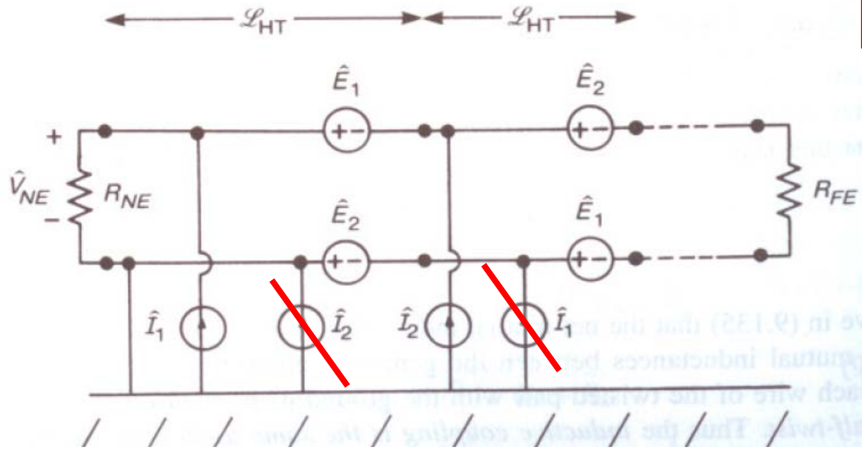
Essentially, the capacitive coupling of the twisted pair is the same as that of the untwisted pair.

Twisted Wires

- Inductive and Capacitive Coupling
  - Unbalanced Termination Model
    - Therefore, the near-end or far-end crosstalk becomes

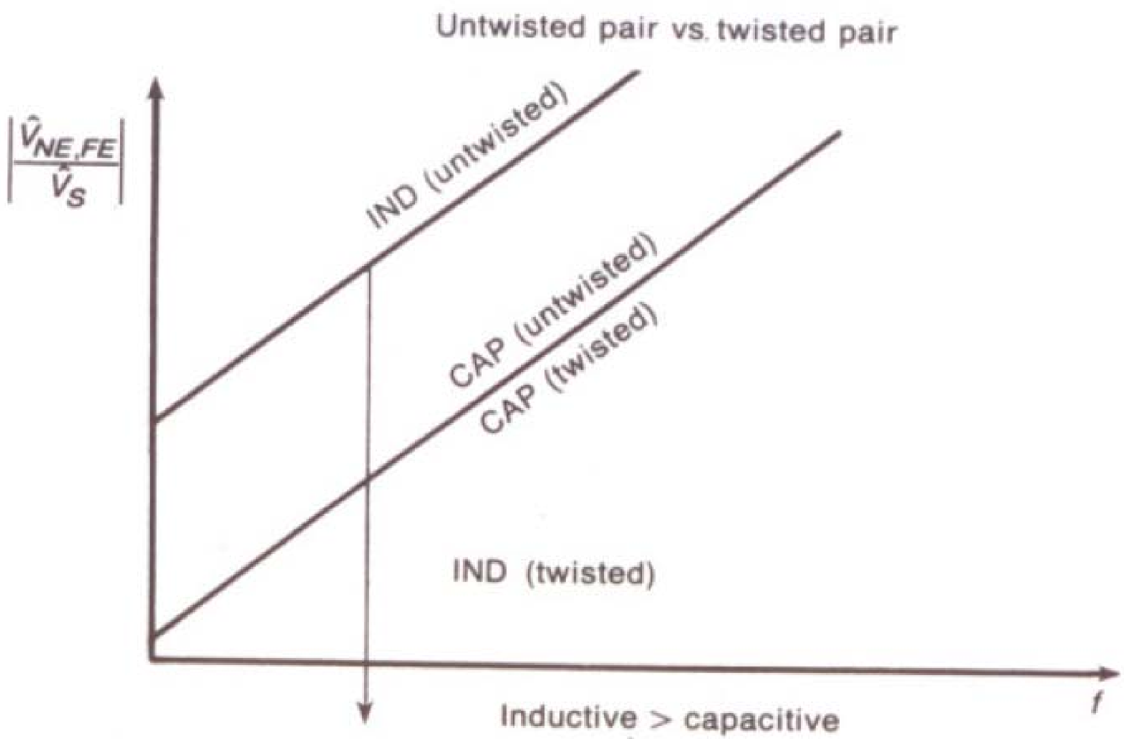
$$\frac{\hat{V}_{NE,FE}}{\hat{V}_S} = \underbrace{\left.\frac{\hat{V}^{\mathrm{IND}}_{NE,FE}}{\hat{V}_S}\right|^{SWP}_{\mathscr{L}_{SWP}=\mathscr{L}_{HT}}}_{\substack{\text{inductive coupling}\\ \text{for SWP whose}\\ \text{length is one}\\ \text{half-twist}}} + \underbrace{\left.\frac{\hat{V}^{\mathrm{CAP}}_{NE,FE}}{\hat{V}_S}\right|^{SWP}_{\mathscr{L}_{SWP}=\mathscr{L}}}_{\substack{\text{capacitive coupling}\\ \text{for SWP whose length}\\ \text{is the total line length}}}$$

SWP: Straight Wire Pair



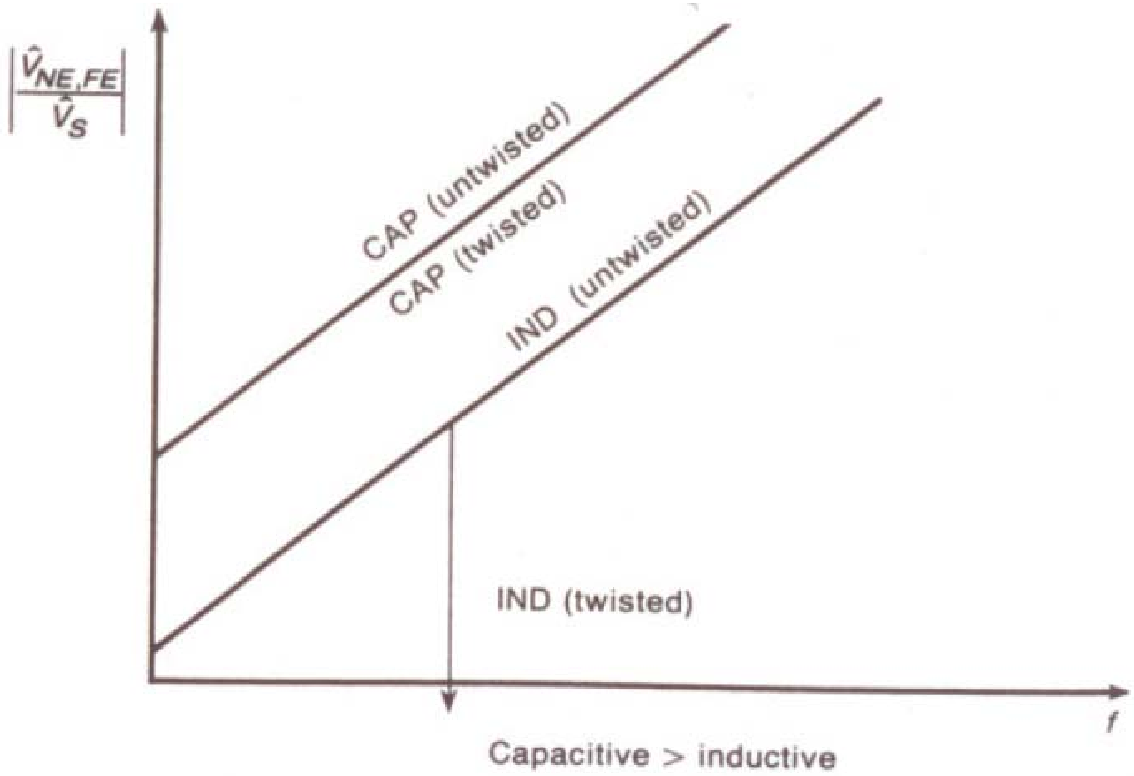
Twisted Wires

- Inductive and Capacitive Coupling
  - Unbalanced Termination Model
    - For low-impedance load, the crosstalk transfer function is



Twisted Wires

- Inductive and Capacitive Coupling
  - Unbalanced Termination Model
    - For high-impedance load, the crosstalk transfer function is



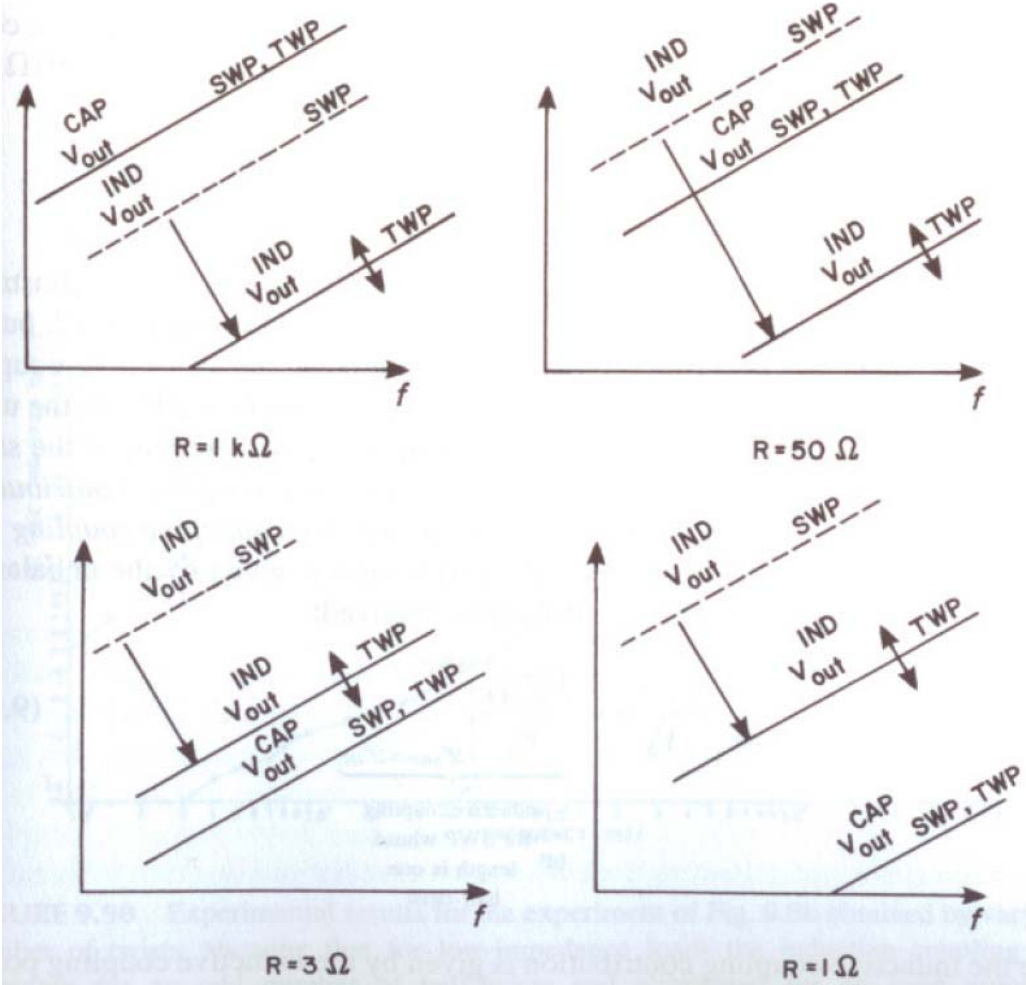
Twisted Wires

- Effects of Twist
  - Responses for SWP and TWP

For high load impedances, capacitive coupling dominates, thus, line twist number have no influence on the reduction of coupling.

The capacitive coupling is the same for SWP and TWP.

The inductive coupling for TWP is only $1/N$ that for SWP where N is the number of half-twists.



Twisted Wires

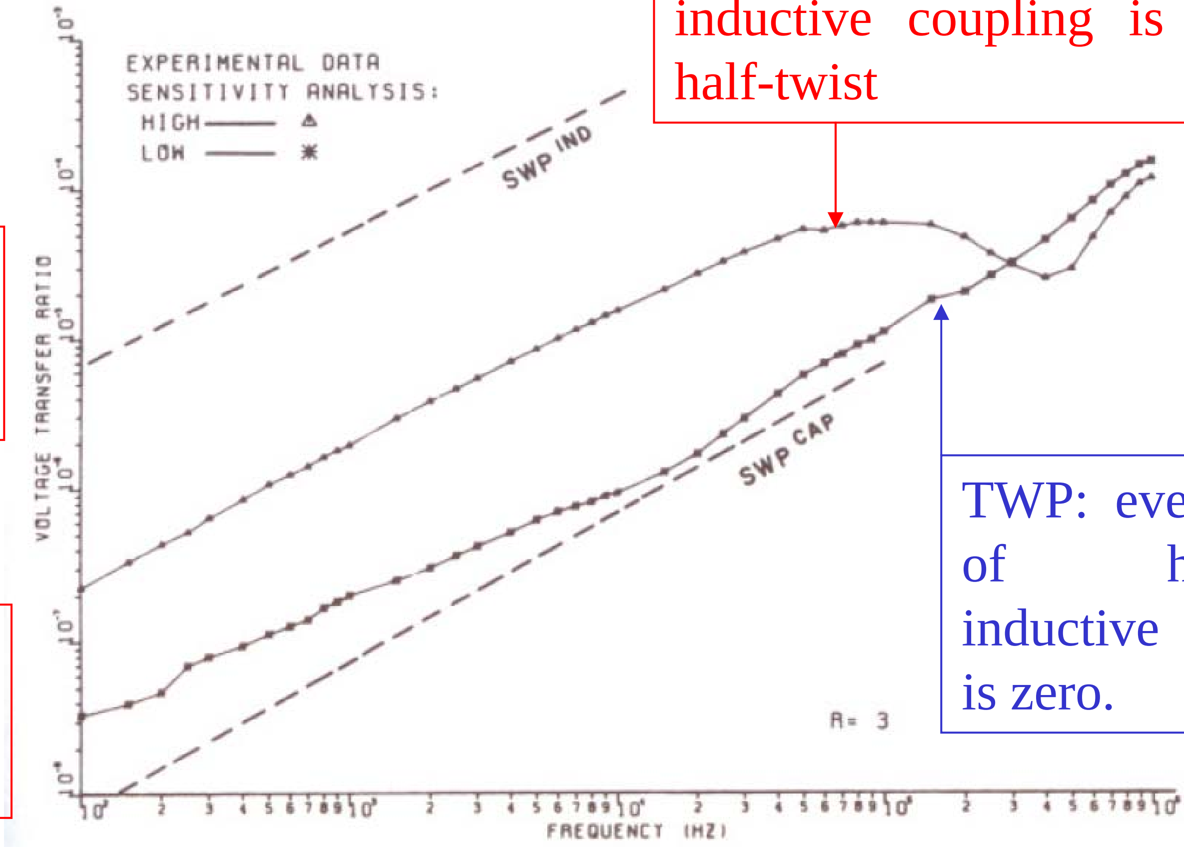
- Effects of Twist
  - Sensitivity to Line Twist Number
    - Load impedance R=3 Ω

For high load impedances, capacitive coupling dominates, thus, line twist number have no influence on the reduction of coupling.

TWP: odd number of half-twists, inductive coupling is only one half-twist

SWP: inductive coupling is one line length

SWP: capacitive coupling is one line length



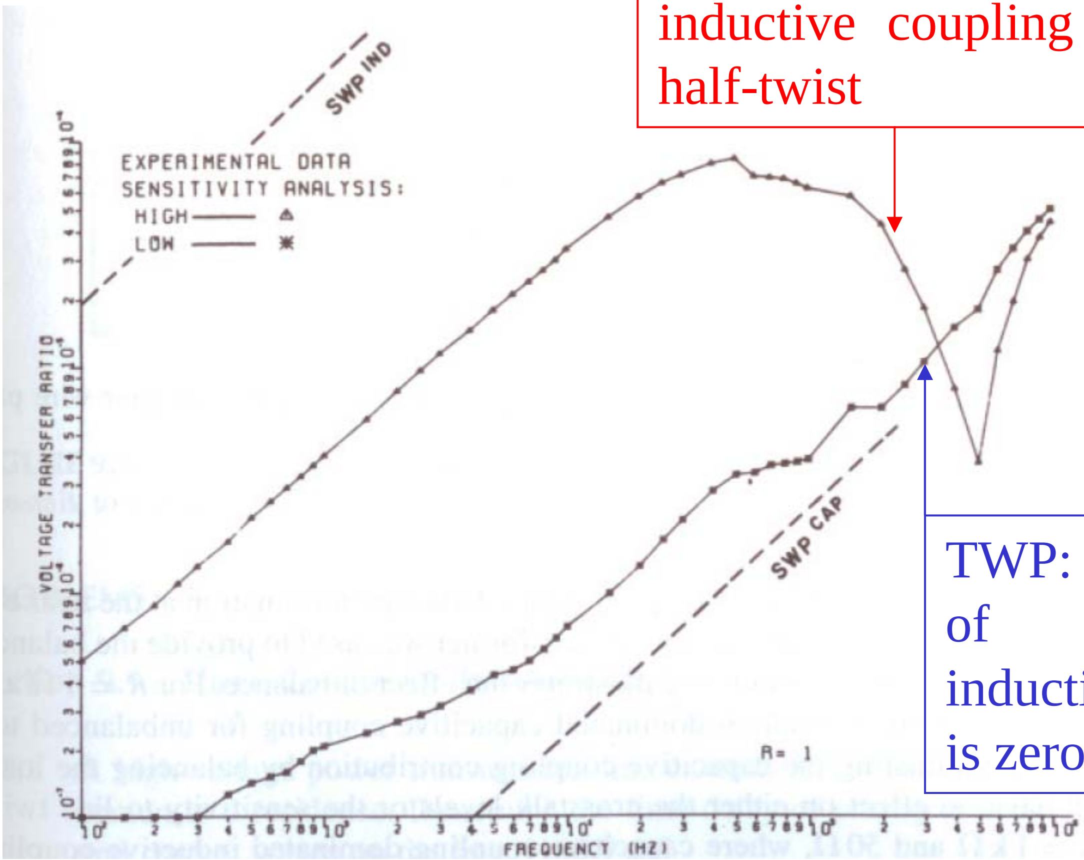
TWP: even number of half-twists, inductive coupling is zero.

Twisted Wires

- Effects of Twist
  - Sensitivity to Line Twist Number
    - Load impedance R=1 Ω

TWP: odd number of half-twists, inductive coupling is only one half-twist

SWP: inductive coupling is one line length



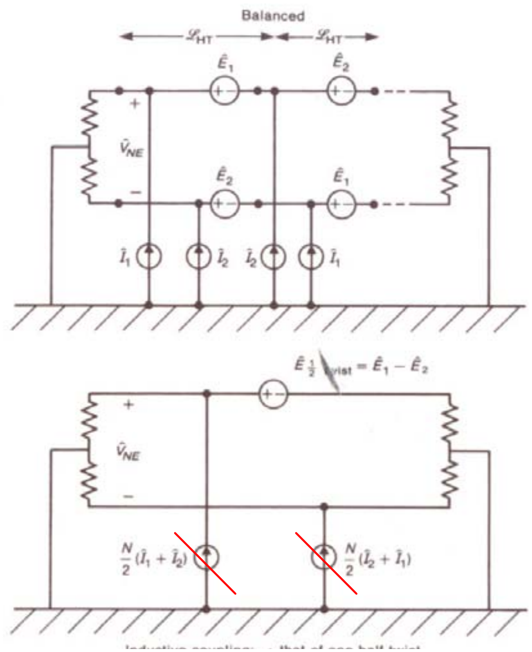
SWP: capacitive coupling is one line length

TWP: even number of half-twists, inductive coupling is zero.

Twisted Wires

- Effects of Balancing
  - Balanced Termination Coupling Model
    - Because of the balanced loads, the capacitive coupling contributions cancel out. The resulting crosstalk voltage transfer ratio is given by the unbalanced case with the capacitive coupling contribution removed.

$$\frac{\hat{V}_{NE,FE}}{\hat{V}_S} = \underbrace{\left.\frac{\hat{V}^{\mathrm{IND}}_{NE,FE}}{\hat{V}_S}\right|^{SWP}_{\mathscr{L}_{SWP}=\mathscr{L}_{HT}}}_{\substack{\text{inductive coupling}\\ \text{for SWP whose}\\ \text{length is one}\\ \text{half-twist}}}$$



Twisted Wires

- Effects of Balancing
  - Effects of Balanced versus Unbalanced Terminations

For low impedances R=3 or 1 Ω, the inductive coupling dominates capacitive coupling.

For high impedances R=50 or 1000 Ω, the capacitive coupling dominates inductive coupling.