Microwave Circuits and Sub-Systems

A2M17MOS

Narrow-Band Amplifier Design – Part B

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Absolute Stability Conditions

- **Absolute stability** conditions:

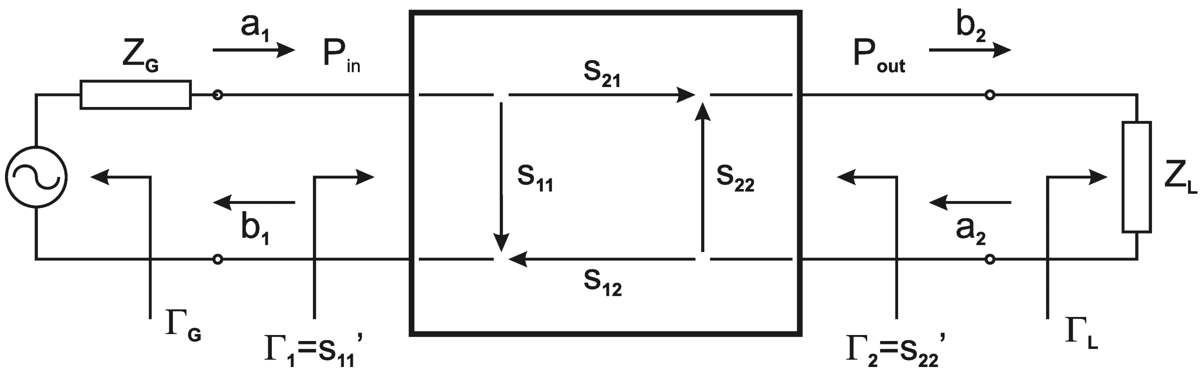
$$|\Gamma_1| \le 1 \qquad \text{for all passive loads} \quad |\Gamma_L| \le 1$$

$$|\Gamma_2| \le 1 \qquad \text{for all passive sources} \quad |\Gamma_G| \le 1$$

- Stability factor $\quad k = \dfrac{1 - |s_{11}|^2 - |s_{22}|^2 + |D|^2}{2|s_{12}s_{21}|} \qquad D = s_{11}s_{22} - s_{12}s_{21}$

- Absolutely stable transistor $k \ge 1$ → **ideal amplifier** can be designed:
  - o Input reflections $|\Gamma_{in}| \to 0$ output reflections $|\Gamma_{out}| \to 0$
  - o Gain $G_t = G_{a\max}$

- **Resulting amplifier** → ideally matched, highest possible gain (at the given frequency and transistor biasing point)



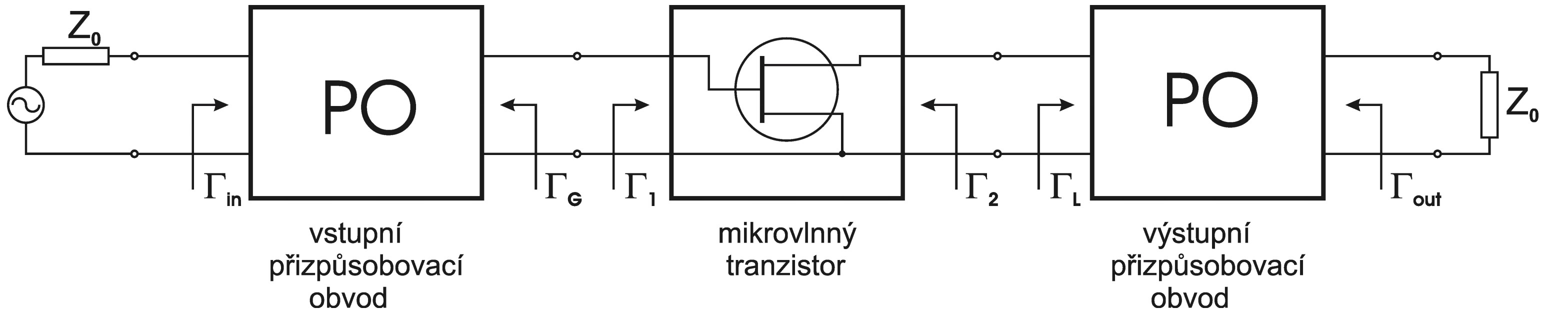
Amplifier with Absolutely Stable Transistor

- Basic impedance matching conditions:

$$\Gamma_{Gopt}^{\ *} = \Gamma_1 = s_{11} + \frac{s_{12}s_{21}\Gamma_{Lopt}}{1 - s_{22}\Gamma_{Lopt}}$$

$$\Gamma_{Lopt}^{\ *} = \Gamma_2 = s_{22} + \frac{s_{12}s_{21}\Gamma_{Gopt}}{1 - s_{11}\Gamma_{Gopt}}$$

- Set of two equations with two unknowns → Γ_{Gopt} , Γ_{Lopt}



Amplifier with Absolutely Stable Transistor

- Solution:

$$\Gamma_{Gopt} = C_1^* \left[B_1 \pm \left(B_1^2 - 4|C_1|^2 \right)^{\frac{1}{2}} \right] \frac{1}{2|C_1|^2}$$

$$\Gamma_{Lopt} = C_2^* \left[B_2 \pm \left(B_2^2 - 4|C_2|^2 \right)^{\frac{1}{2}} \right] \frac{1}{2|C_2|^2}$$

- Auxiliary variables:

$$B_1 = 1 + |s_{11}|^2 - |s_{22}|^2 - |D|^2$$

$$B_2 = 1 - |s_{11}|^2 + |s_{22}|^2 - |D|^2$$

$$C_1 = s_{11} - D.s_{22}^*$$

$$C_2 = s_{22} - D.s_{11}^*$$

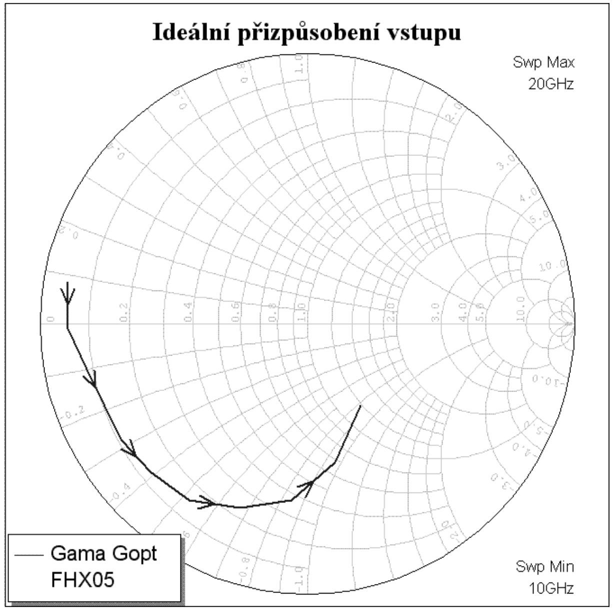
$$D = s_{11}s_{22} - s_{12}s_{21}$$

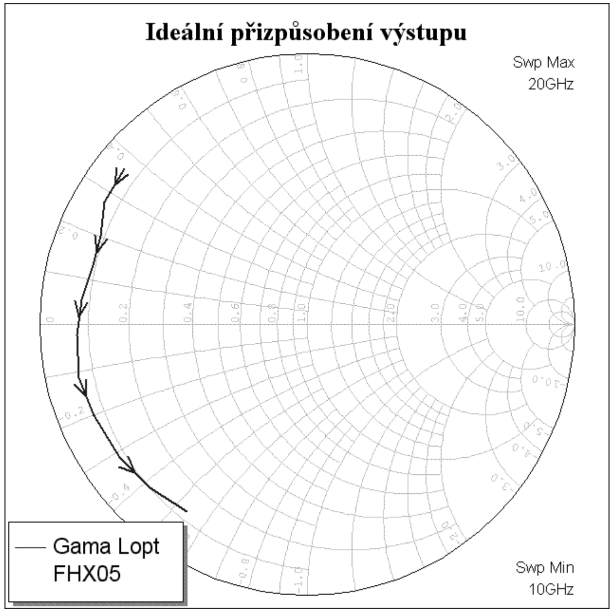
- Signs: + if $B_j < 0$

  \- if $B_j > 0$

Wideband Impedance Matching

- **AWR-MO:** $\Gamma_{Gopt} = GM1 \qquad \Gamma_{Lopt} = GM2$



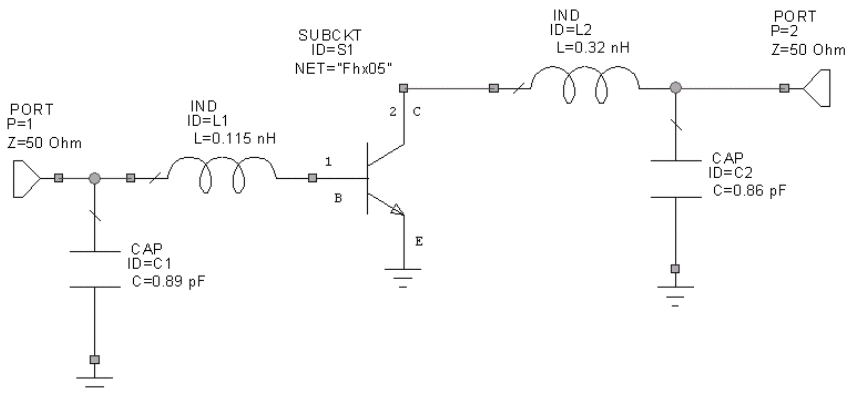


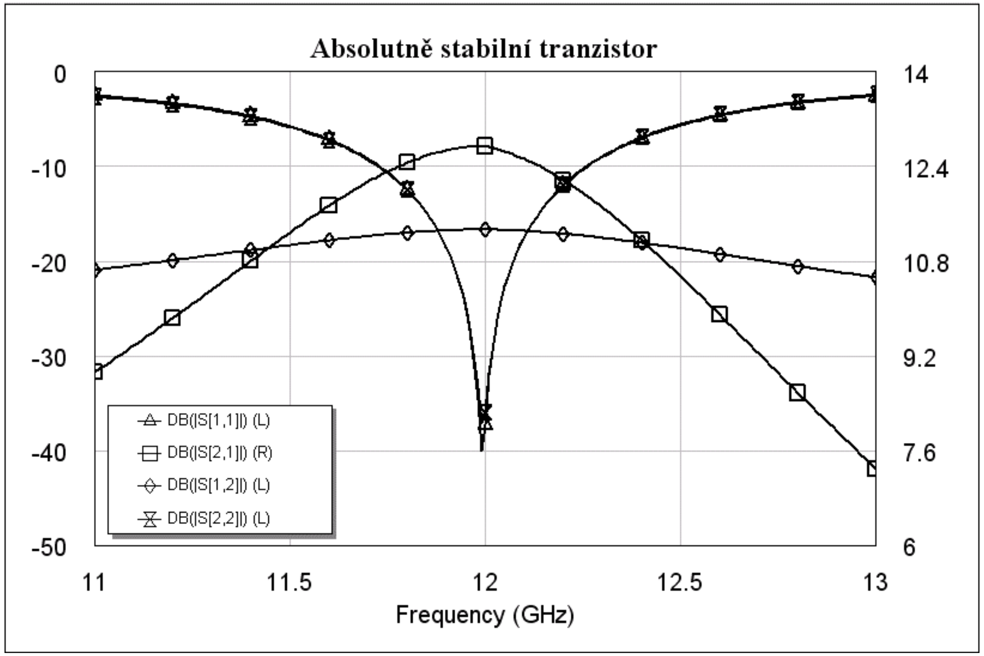
- Frequency plots of Γ_{Gopt} and Γ_{Lopt} run **counter-clockwise**
- Frequency plot of ANY matching circuit run clockwise
- Ideal impedance matching can be reached only at 1 frequency, or at several discrete and distant frequencies
- It is NOT possible to design wideband amplifiers with ideal impedance matching

Design Steps – Example 1

- With the Γ_{Gopt} and Γ_{Lopt} values known (GM1, GM2 provided by AWR-MO) it is possible to synthesize the whole **absolutely stable amplifier**:
  - o Input MC → transforms Z_0 to Γ_{Gopt}
  - o Output MC → transforms Z_0 to Γ_{Lopt}
  - o Input reflection coeff. $\Gamma_{in} \to 0$
  - o Output reflection coeff. $\Gamma_{out} \to 0$
  - o Gain is equal to G_{amax}

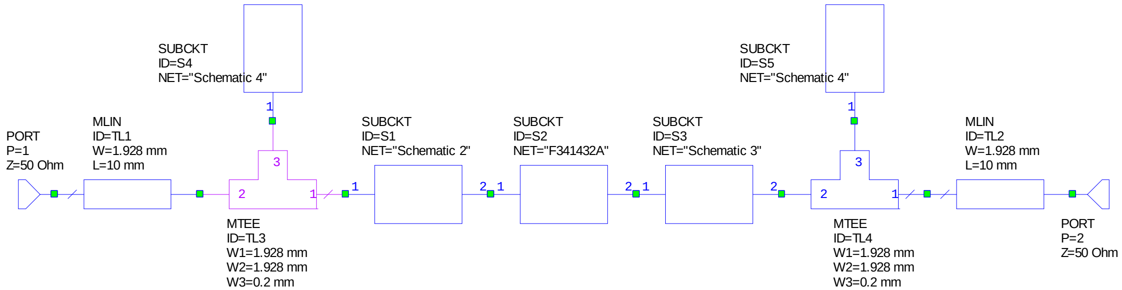
- **Example: FHX05, 12 GHz**

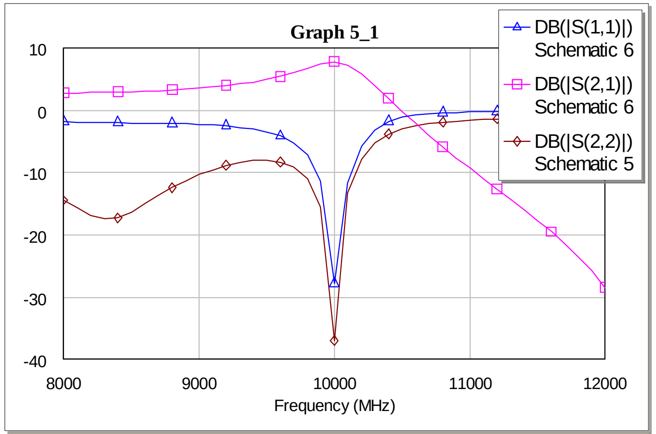


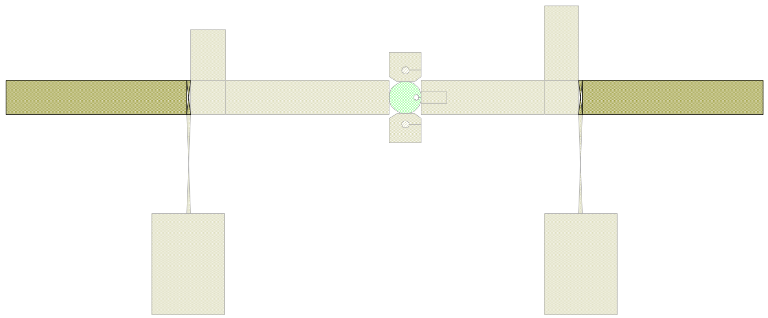


Example 2

- Task No.2 – amplifier with absolutely stable transistor
  - ATF-34143 (file F341432A.S2P), FET, 2V/20mA
  - f=10GHz
  - k>1
  - GM1= 0,821 / -36°
  - GM2=0,4975 / -85°

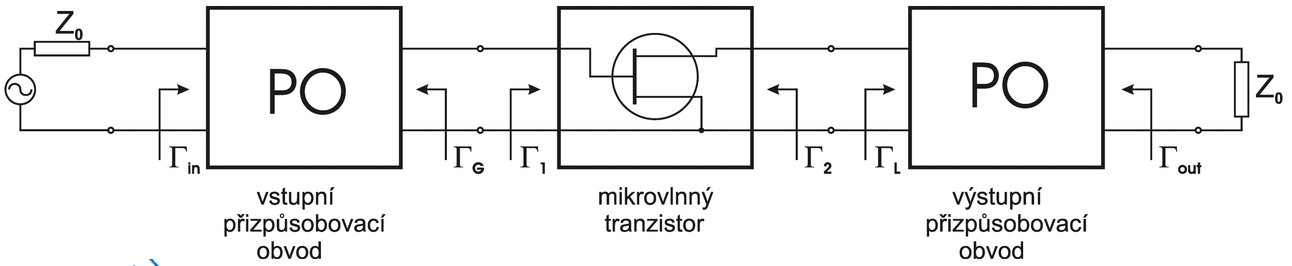


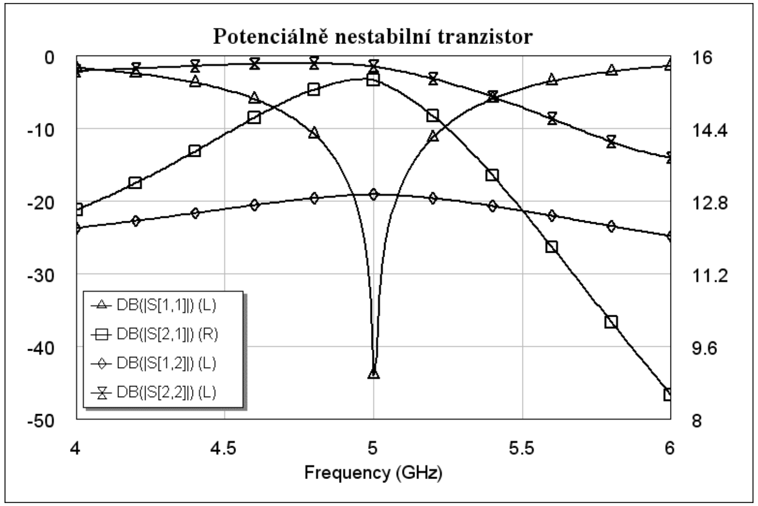




Amplifier with Potentially Unstable Transistor

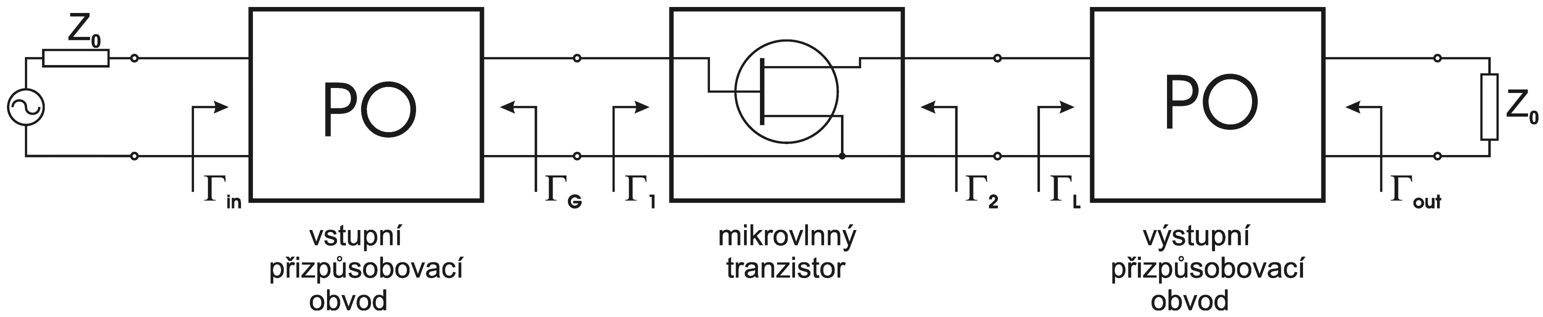
- **More complicated design:**
  - Definite Γ_G and Γ_L values can cause the amplifier to oscillate
  - The $G_{a\max}$ parameter is not defined, G_{ms} must be used
  - Stability must be treated
- **Worse attainable parameters:**
  - Cannot be simultaneously matched both at input and output
  - If matched at input $\Gamma_{in} \to 0$ there appears reflection at output $|\Gamma_{out}| > 0$
  - If matched at output $\Gamma_{out} \to 0$ there appears reflection at input $|\Gamma_{in}| > 0$
  - There can be reflections both at input $|\Gamma_{in}| > 0$ and output $|\Gamma_{out}| > 0$
  - Gain must be lower than G_{ms} (by 1 – 2 dB)
  - Amplifier is only **CONDITIONALLY stable**





Amplifier with Potentially Unstable Transistor

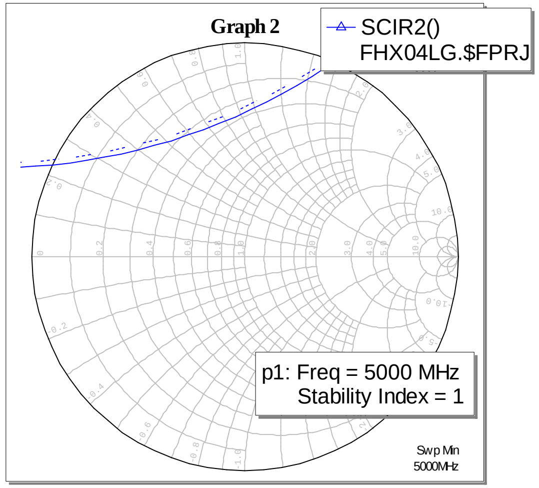
- Design with **ideal impedance matching at the input** $\Gamma_{in} \to 0$:
- Resulting parameters:
  - Ideal impedance matching at input $\Gamma_{in} \to 0$
  - General reflection at output $|\Gamma_{out}| \leq 1$ (example: dBs22≈-3dB)
  - Operational gain is by 1-2 dB lower than G_{ms}
- Design steps:
  - Ensuring of stability
  - Impedance matching



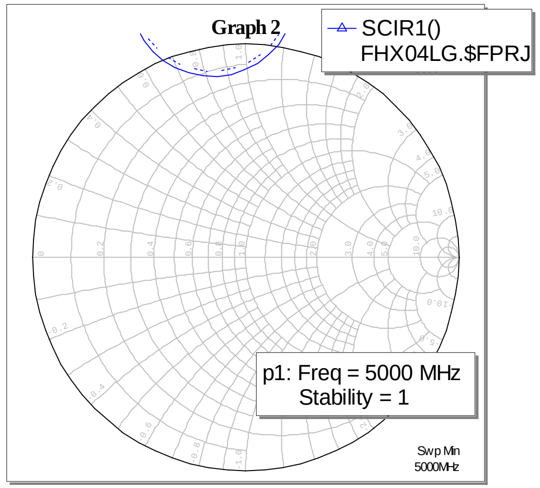
Stability Circles

- **Stability** → will be treated in the Γ_L plane (due to the required imp. matching at input)
- Stability circles can be used
- Equation of the output stability circle in the Γ_L plane:

$$|\Gamma_1| = 1 \qquad \Gamma_1 = s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1 - s_{22}\Gamma_L} \qquad 1 = \left| s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1 - s_{22}\Gamma_L} \right|$$

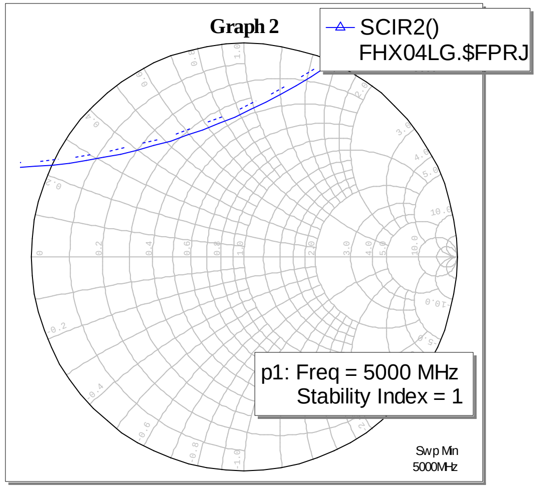


- Solution → circle $|\Gamma_1| = 1$ in the Γ_L plane
- Centers $$C_L = \frac{s_{22}^* - s_{11}D^*}{|D|^2 - |s_{22}|^2}$$
- Radiuses $$r_L = \frac{|s_{12}s_{21}|}{|D|^2 - |s_{22}|^2}$$
- Stable/unstable region → can be differentiated according to s_{11} of the given transistor
- **AWR-MO:** **SCIR1** (in the Γ_G plane)
  SCIR2 (in the Γ_L plane)



Conditional Stability

- Ensuring **conditional amplifier stability** → the selected Γ_{Lz} must lie in the STABLE region
- **AWR-MO** → the **SCIR2** plot, the UNSTABLE region is marked by the dashed circle
- The Γ_{Lz} chosen from the stable region – lead to $|\Gamma_1| < 1$
- The Γ_{Lz} chosen from the unstable region – lead to $|\Gamma_1| > 1$

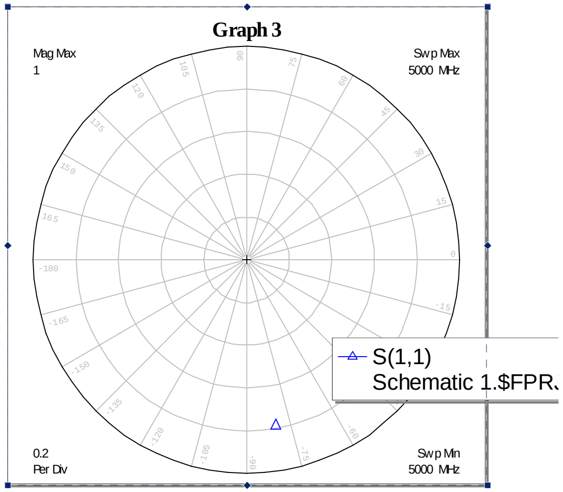


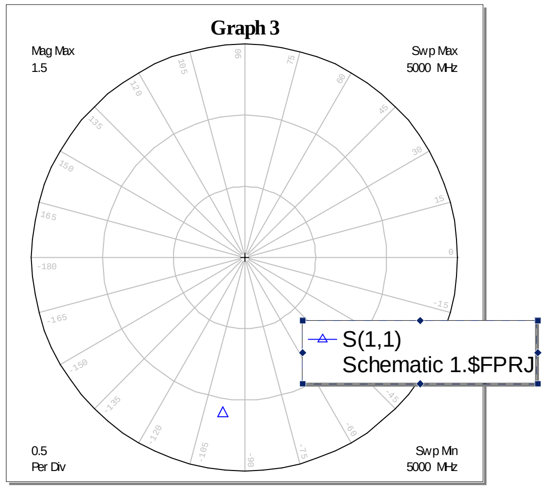
- **Example:** FHX04LG @ 5 GHz
- Stable region:

$$\Gamma_{Lz} = 0{,}9 \,/\, -90^0 \text{ lead to } \Gamma_1 = 0{,}79 \,/\, -80^0$$

- Unstable region:

$$\Gamma_{Lz} = 0{,}9 \,/\, 90^0 \text{ lead to } \Gamma_1 = 1{,}1 \,/\, -98^0$$





- **AWR-MO** calculations → use the **LTUNER** model

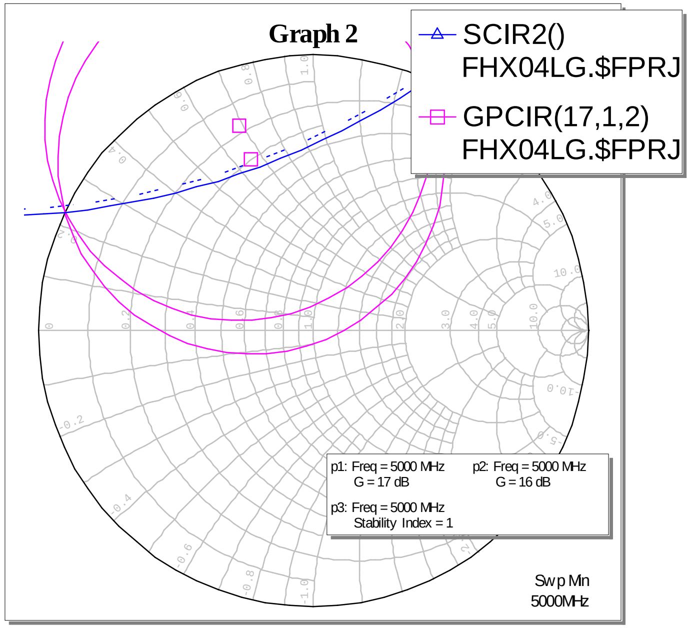
Constant G_p Circles

- Operational gain → must be by 1-2 dB lower than G_{ms}
- Can be ensured by using constant power gain G_p circles
- Power gain G_p → depends on s-parameters and Γ_L (+ input impedance matching)
- **Power gain circles** → solution $G_p = const.$ in the Γ_L plane

$$G_P = const = \frac{\left(1-|\Gamma_L|^2\right)|s_{21}|^2}{|1-s_{22}\Gamma_L|^2\left(1-|\Gamma_1|^2\right)} \qquad \Gamma_1 = s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1-s_{22}\Gamma_L}$$

- Solution:
  - o Centers $$C_P = \frac{g\left(s_{22}^* - D^* s_{11}\right)}{1+g\left(|s_{22}|^2 - |D|^2\right)}$$
  - o Radiuses $$r_P = \frac{\left(1-2k|s_{12}s_{21}|g + |s_{12}s_{21}|^2 g^2\right)^{\frac{1}{2}}}{\left|1+g\left(|s_{22}|^2 - |D|^2\right)\right|}$$

$$g = \frac{G_P}{|s_{21}|^2} \qquad D = s_{11}s_{22} - s_{12}s_{21}$$



- **AWR-MO**: **GPCIR**

Optimum Transistor Loading - Γ_{Lz}

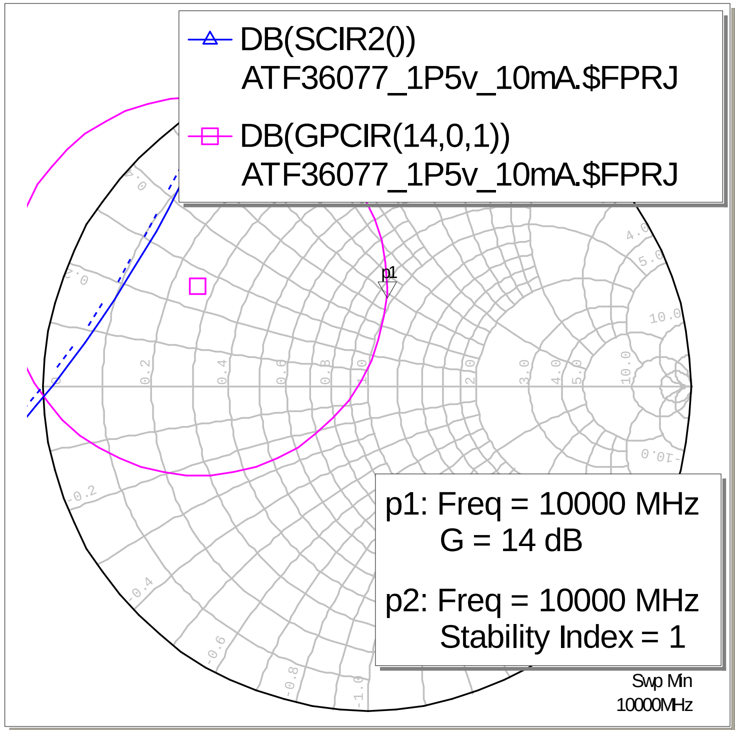
- The selected Γ_{Lz} load:
  - Lies in the stable region
  - Lies on the required constant power gain circle
  - Close to the SD centre
- Often – the $\Gamma_{Lz} = 0$ value can be chosen → amplifier has no output matching circuit

- Input impedance matching → **must be ideal** $\Gamma_G = \Gamma_1^*$

- Calculation of Γ_1 :

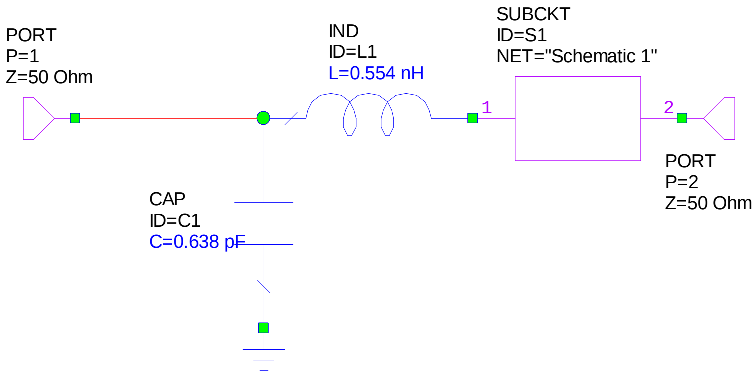
- Formula $$\Gamma_1 = s_{11} + \frac{s_{12}s_{21}\Gamma_{Lz}}{1 - s_{22}\Gamma_{Lz}}$$

- **AWR-MO:**
  - transistor
  - **LTUNER** at output simulates Γ_{Lz}
  - S11= Γ_1

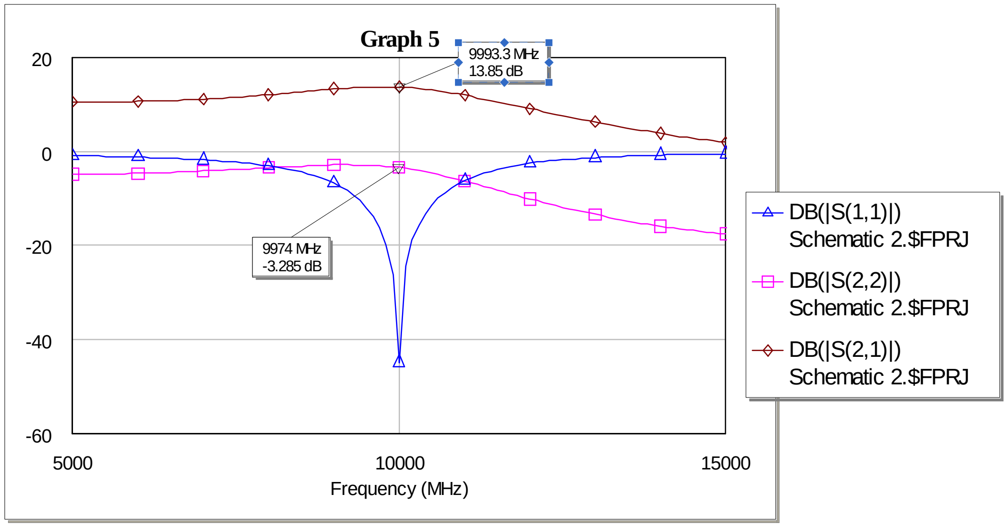


Example – Task No. 3

- With the Γ_{Lz} and $\Gamma_G = \Gamma_1^*$ values known → input and output MCs can be synthesized
- **Example:** ATF36077 @ 10GHz
  - FET 1,5V / 10mA
  - k=0,74
  - MSG=16,38dB
  - Chosen G_p=14dB, $\Gamma_{Lz} = 0$



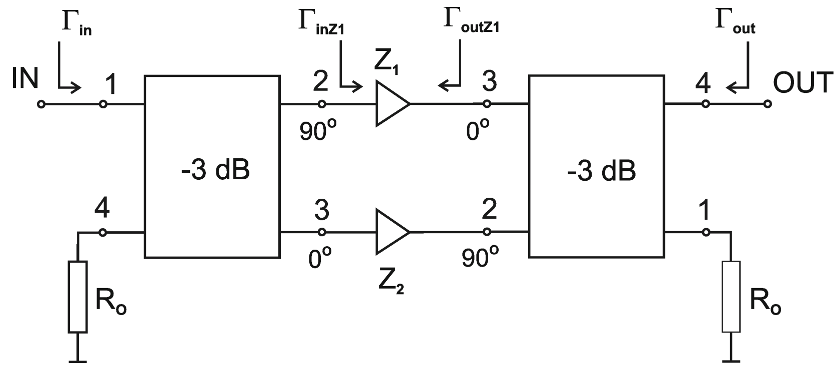
- Lumped input matching circuit
- Obtained parameters @10GHz:
  - Gain $G_t = G_p = 13{,}85\ dB$
  - Input matching $RL_{in} = -dBs11 = 42dB$
  - Output matching $RL_{out} = -dBs22 = 3{,}2dB$
  - **Conditionally stable**



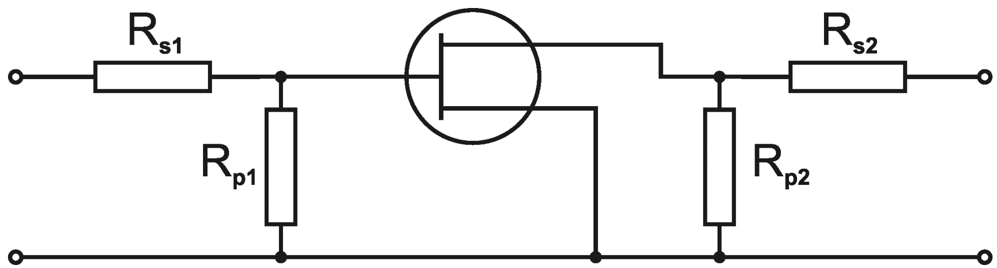
Problems

- **Amplifiers based on potentially un-stable transistors show several significant disadvantages:**
- High reflection at output, commonly $|\Gamma_{out}| \approx 0{,}8 \div 0{,}9$
- Conditional stability
  - Under definite circumstances → the amplifier can **start to oscillate**:
    - When disconnecting the input 50Ω source with active transistor biasing
    - When disconnecting the output 50Ω load with active transistor biasing
- Reason:
  - Γ_L is realized from the $|\Gamma| = 0$ load by the output MC
  - If $|\Gamma| = 1$ → the real Γ_L can fall into the unstable region
  - The amplifier can start to oscillate
  - The same is valid for Γ_G
  - Only hardly acceptable in the technical praxis

Stabilization

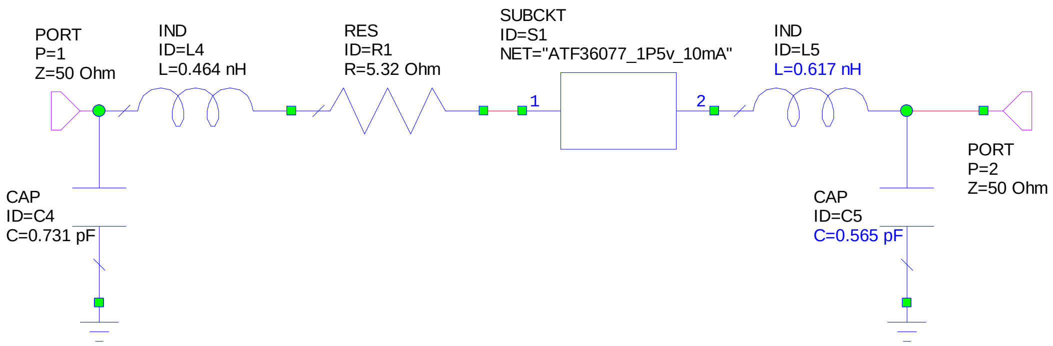


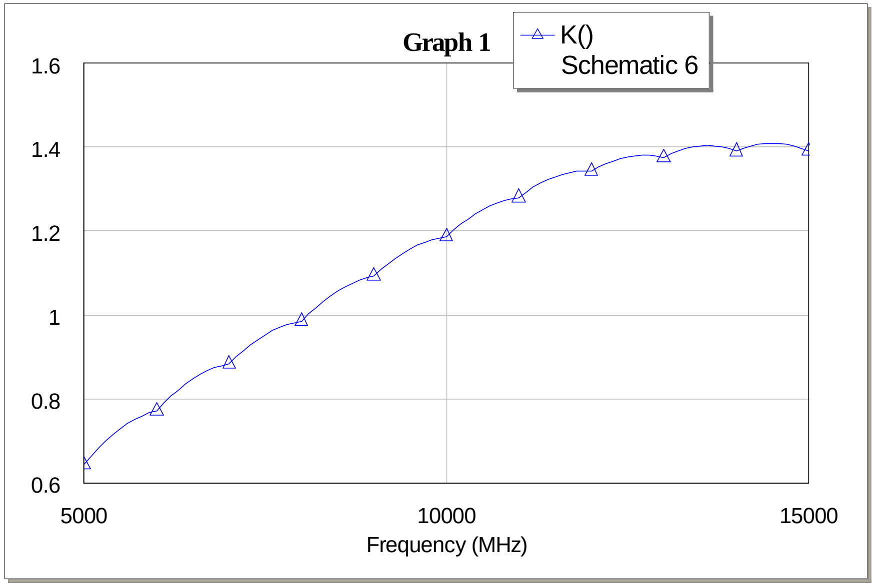
- Possible solutions:
  - Balanced amplifier structure
  - Stabilization of the transistor used
- Transistor stabilization – connection of 1 or 2 **stabilizing resistors**
  - Low-noise amplifiers → R_{p2}, R_{s2}
  - Power amplifiers → R_{s1}, R_{p1}
  - Increase the stability coefficient from $k < 1$ to $k \cong 1{,}1$
- Combination transistor + stabilizing resistor = **new absolutely stable transistor**
- Design amplifier with absolutely stable transistor
- Resulting parameters:
  - $|\Gamma_{in}| \to 0$
  - $|\Gamma_{out}| \to 0$
  - $G_t \approx G_{ms} - 2dB$
  - Professional solution

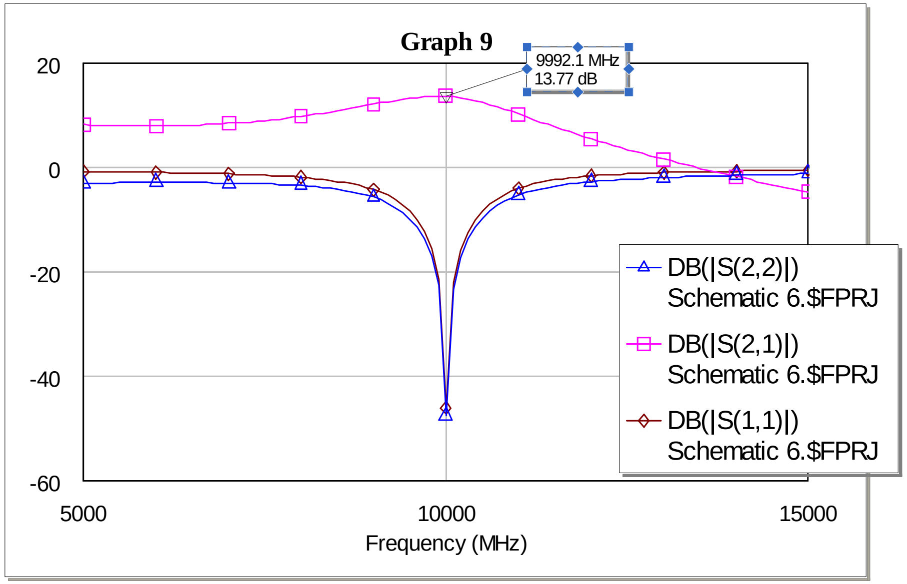


Stabilization - Example

- **Example:** ATF36077 @10GHz
  - Stability coeff. $k = 0{,}74$
  - Stabilized using R_{s1}=5,3Ω
  - Resulting parameters:
    - Stability $k = 1{,}2$
    - Gain $G_t = 13{,}77\ dB$
    - Input matching $\left|\Gamma_{in}\right|_{dB} > 42$
    - Output matching $\left|\Gamma_{out}\right|_{dB} > 45$







Low-Noise Amplifiers

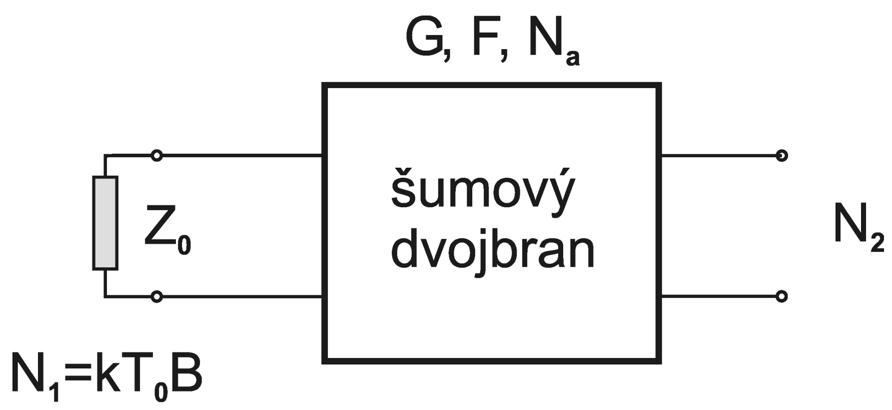
- **LNAs**
- Important at inputs of circuits or sub-systems processing very low-level signals
- Usually, at inputs of communication, satellite or radar receivers
- Input signal levels are lower than -100dBm often
- The receiver must ensure sufficient S/N ratio → in order to:
    - Ensure the required communication quality e.g. BER
    - Ensure the required radar detection range

- Basic noise parameters:
    - **Noise figure**
    - **Equivalent noise temperature**
    - **Design noise parameters**
- The above stated parameters are important for the LNA design

S/N Definition of F

- Noise figure → the S/N definition: $$F = \frac{\dfrac{S_1}{N_1}}{\dfrac{S_2}{N_2}}$$
- S_1 power of signal at the 2-port input
- N_1 power of noise at the 2-port input $N_1 = k_B T_0 B$
- S_2 power of signal at the 2-port output
- N_2 power of noise at the 2-port output

- Important: **The definition is valid only if:** $N_1 = k_B T_0 B$
  - N_1 Available noise power generated by the ideal Z_0 impedance termination (black-body)
  - k_B Boltzman's constant $k_B = 1{,}38.10^{-23}\ J/K$
  - T_0 Definition temperature $T_0 = 290K$
  - B Frequency bandwidth



S/N Definition of F

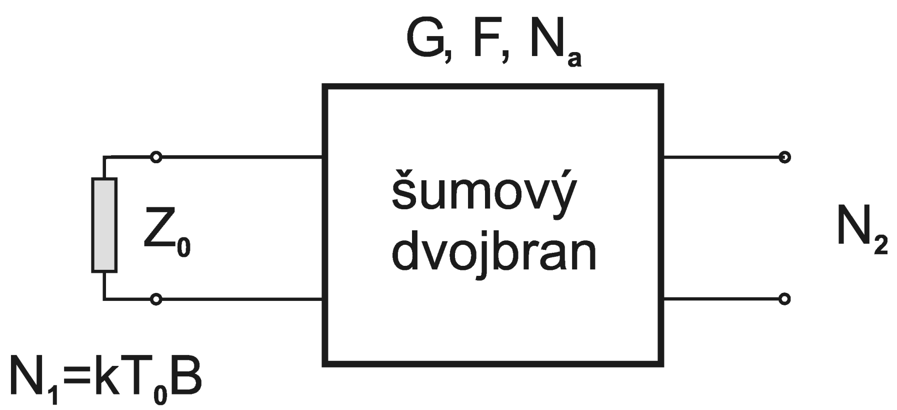
- Noise figure - the S/N definition:
  - Important for the communication, radar,… system design
  - S/N is one of the parameters decisive for BER
  - Cannot be used easily in cascades → N_1 is no more $k_B T_0 B$
  - More complex formulas must be used for analysis of the cascaded noise 2-ports

- All powers = **available powers from the generator** P_{AL} (into the conjugate load)
- If all components are well matched → insertion gain can be used

- Amplifier design → S_1 is not known usually
- That is why the **power F definition** can be more suitable

- F often stated in dB:

$$F_{dB} = 10\log(F)$$



Power Definition of F

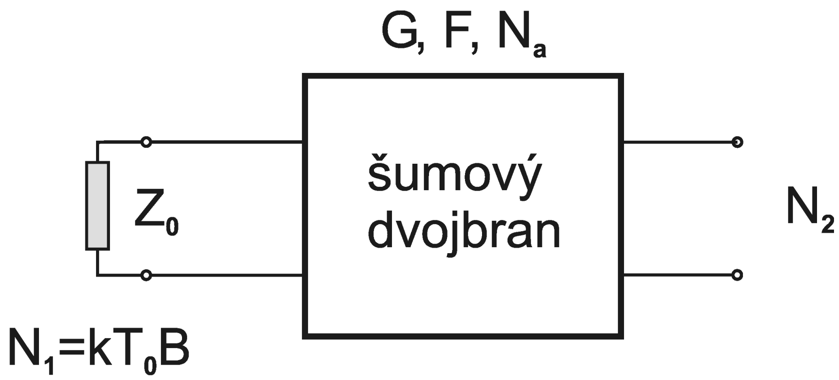
- **Power F definition:**

$$F = \frac{N_2}{GN_1} = \frac{GN_1 + N_a}{GN_1} = \frac{kT_0BG + N_a}{kT_0BG} = 1 + \frac{N_a}{kT_0BG}$$

- Total output noise power divided by the amplified noise $N_1 = k_B T_0 B$ from the input Z_0 impedance termination
- N_2 consists of the amplified N_1 noise and the N_a noise added by the 2-port itself (referred to output)
- Can be derived from the S/N definition using the (available) gain $G = S_2 / S_1$

$$F = \frac{S_1 N_2}{S_2 N_1} = \frac{N_2}{GN_1}$$

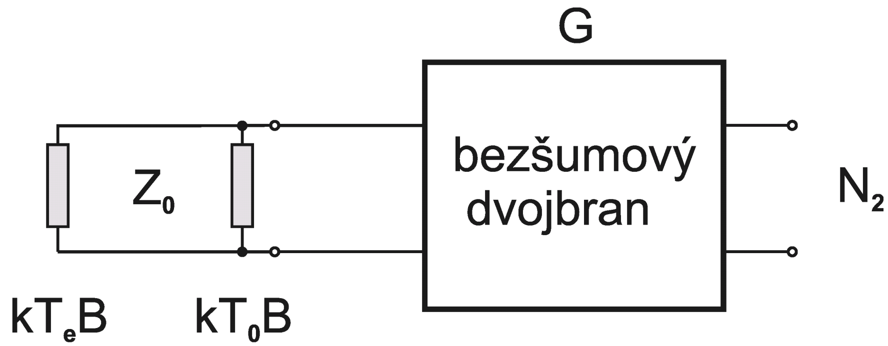
- **Noise added** ref. to input $N_a = (F-1)k_B T_0 B$
- **Noise added** ref. to output $N_a = (F-1)k_B T_0 BG$
- Suitable for calculations of cascades

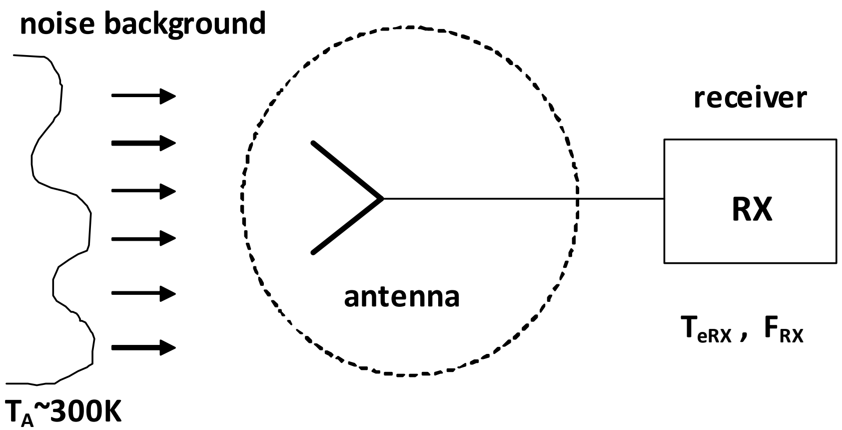


Equivalent Noise Temperature - T_e

- All noise sources are removed from the 2-port and replaced by an additional virtual impedance termination kept at equivalent temperature T_e
- Corresponding output noise power: $N_2 = k_B T_0 BG + k_B T_e BG$
- From the F definition: $N_2 = k_B T_0 BG + N_a = k_B T_0 BG + (F-1) k_B T_0 BG$
- Comparison: $k_B T_e BG = (F-1) k_B T_0 BG$
- **Definition:** $T_e = (F-1) T_0$
- **Applications:**
  - Low F values (radio-astronomy)
    - F=1,1 (0,41dB) T_e=29K
    - F=1,15 (0,61dB) T_e=43,5K
  - Noise description of receiver input circuits
  - Enable to take into account the noise background „seen“ by the antenna

$$T_{sys} = T_A + T_{eRX} \qquad F_{sys} = T_{sys} / T_0 + 1$$



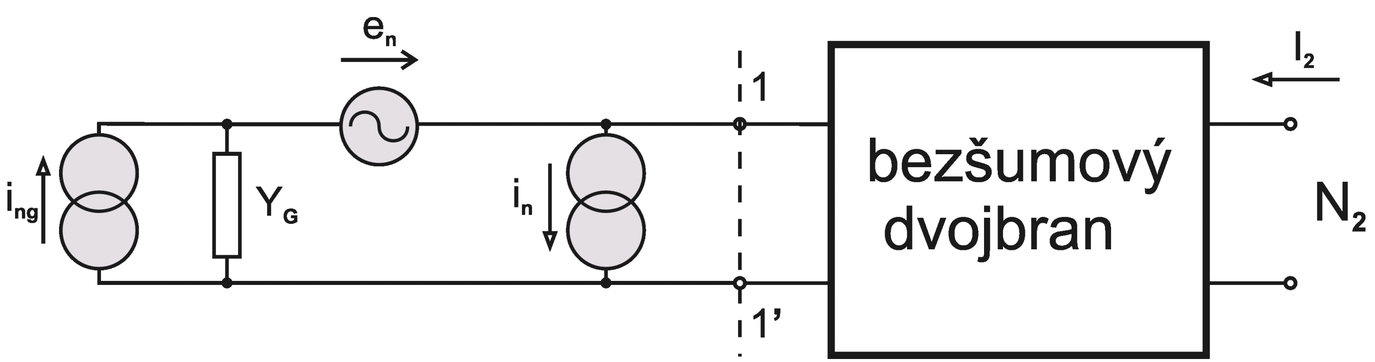


Design Noise Parameters

- Design noise parameters → necessary for LNA design
- 1 complex number + 2 scalar numbers → together **4 scalar numbers + frequency**
- Can be derived from the noisy 2-port equivalent circuit
- All 2-port inner noise sources are replaced with 2 noise sources in the input circuit
- The solution can be derived in the 1-1' plane → the rest of circuit is noise-less

- Noise figure $F = \frac{P_{na1}}{P_{ng}}$

- P_{na1} total available noise power in the 1-1' plane
- P_{ng} noise power of the Y_G (general admitance) generator at $T_0 = 290\,K$



Noise Equivalent Circuit

- The P_{na1} noise power can be calculated from the i_{nc} total **noise current in the short-circuited 1-1' plane**
- Using superposition and the Thevenin rules:

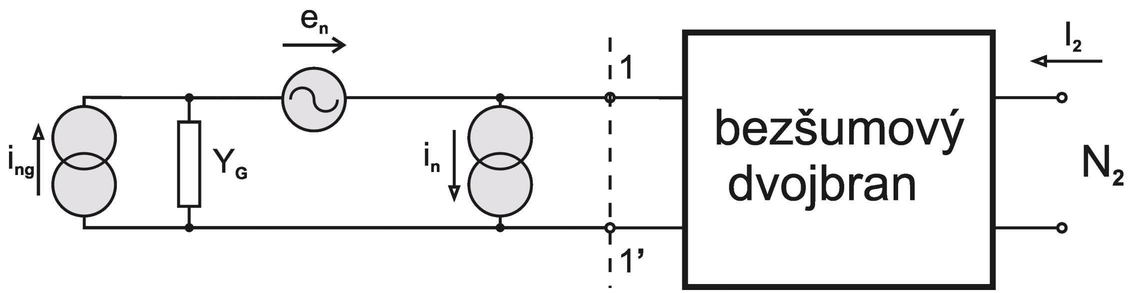
$$i_{nc1} = i_{ng} \qquad i_{nc2} = -e_n Y_G \qquad i_{nc3} = -i_n \qquad i_{nc} = i_{ng} - (i_n + e_n Y_G)$$

- Noise power → corresponds to the mean-square value

$$E\left[|i_{nc}|^2\right] = E\left[\left(i_{ng} - (i_n + e_n Y_G)\right)\left(i_{ng} - (i_n + e_n Y_G)\right)^*\right] = E\left[|i_{ng}|^2\right] - 2\,\mathrm{Re}\,E\left[i_{ng}(i_n + e_n Y_G)^*\right] + E\left[|i_n + e_n Y_G|^2\right]$$

- Uncorrelated variables: $E[x.y] = E[x]E[y]$

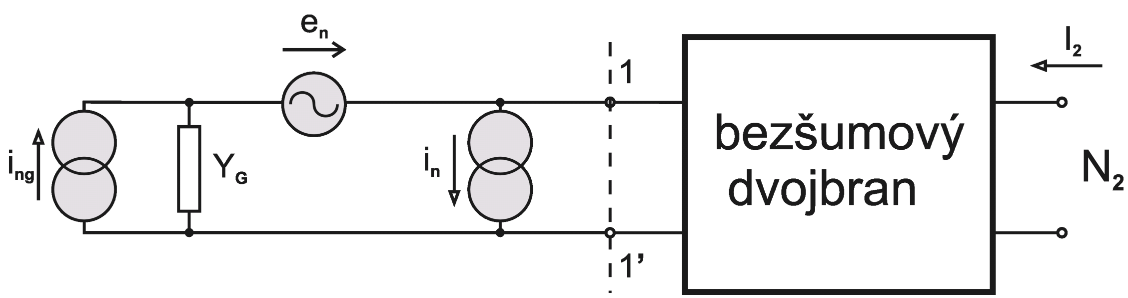
- White noise $E[i_{ng}] = 0$



Correlated Noise Sources

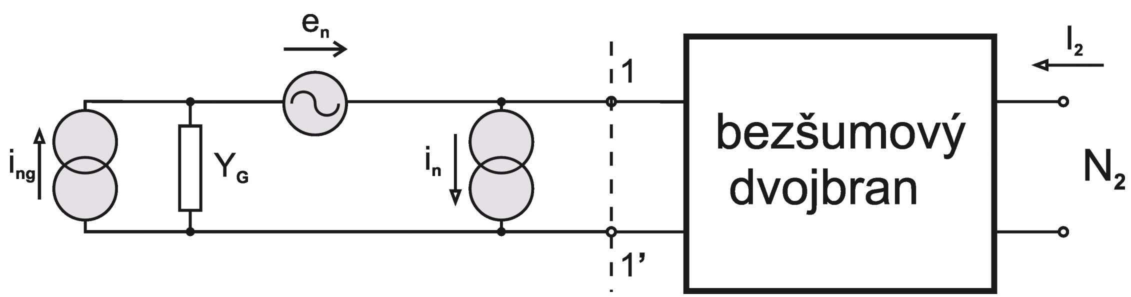
- From that $$E\left[\left|i_{nc}\right|^2\right] = E\left[\left|i_{ng}\right|^2\right] + E\left[\left|i_n + e_n Y_G\right|^2\right]$$
- Inner noise sources i_n and e_n can be **partially correlated**
- The correlation can be described using: $i_n = i_{nn} + Y_{cor} e_n$
- Y_{cor} = **correlation admitance** $Y_{cor} = G_{cor} + jB_{cor}$
- i_{nn} = 100% **uncorrelated component**
- $Y_{cor} e_n$ = 100% **correlated component**
- From that: $$i_{nc} = i_{ng} - \left(i_{nn} + Y_{cor} e_n + Y_G e_n\right) = i_{ng} - i_{nn} - e_n\left(Y_{cor} + Y_G\right)$$
- All 3 components are non-correlated → the mean-square value can be expressed as:

$$E\left[\left|i_{nc}\right|^2\right] = E\left[\left|i_{ng}\right|^2\right] + E\left[\left|i_{nn}\right|^2\right] + \left|Y_{cor} + Y_G\right|^2 E\left[\left|e_n\right|^2\right]$$



Noise Parameter Set: G_n, R_n, G_{cor}, B_{cor}

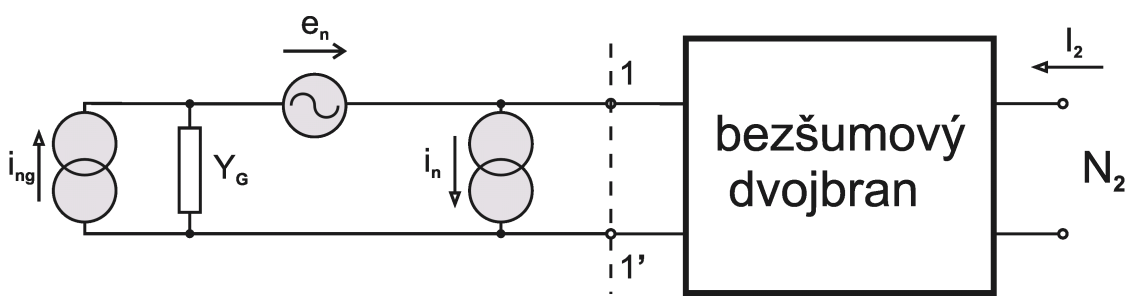
- Noise power P_{na1} : $P_{na1} = E\left[\left|i_{nc}\right|^2\right] = E\left[\left|i_{ng}\right|^2\right] + E\left[\left|i_{nn}\right|^2\right] + \left|Y_{cor} + Y_G\right|^2 E\left[\left|e_n\right|^2\right]$
- Noise power P_{ng} : $P_{ng} = E\left[\left|i_{ng}\right|^2\right]$
- Noise figure: $F = \dfrac{E\left[\left|i_{nc}\right|^2\right]}{E\left[\left|i_{ng}\right|^2\right]} = 1 + \dfrac{E\left[\left|i_{nn}\right|^2\right]}{E\left[\left|i_{ng}\right|^2\right]} + \dfrac{E\left[\left|e_n\right|^2\right]}{E\left[\left|i_{ng}\right|^2\right]}\left|Y_{cor} + Y_G\right|^2$
- Equivalent expressions: $E\left[\left|e_n\right|^2\right] = 4kT_0BR_n$ $\quad E\left[\left|i_{nn}\right|^2\right] = 4kT_0BG_n$ $\quad E\left[\left|i_{ng}\right|^2\right] = 4kT_0BG_G$
- From that: $F = 1 + \dfrac{G_n}{G_G} + \dfrac{R_n}{G_G}\left|Y_{cor} + Y_G\right|^2$
- **4 noise parameter set:** $G_n \quad R_n \quad Y_{cor} = G_{cor} + jB_{cor}$



Noise Parameter Set: G_n, R_n, G_{cor}, B_{cor}

- **Noise figure:** $$F = 1 + \frac{G_n}{G_G} + \frac{R_n}{G_G}\left|Y_{cor} + Y_G\right|^2$$
- Set of 4 design noise parameters: G_n R_n $Y_{cor} = G_{cor} + jB_{cor}$
- Describe dependence of F on $Y_G = G_G + jB_G$
- This set of noise parameters → often used in **calculation cores of noise analysis programs**
- But Y_{cor} is **difficult to be measured**
- That is why another sets of design noise parameters are also used

- The most frequently used set can be derived from the above presented noise parameters by finding minimum value of the noise figure F as a function of $Y_G = G_G + jB_G$



Noise Parameter Set: F_{min}, R_n, G_{opt}, B_{opt}

- Noise figure: $$F = 1 + \frac{G_n}{G_G} + \frac{R_n}{G_G}\left|Y_{cor} + Y_G\right|^2$$
- Minimum of F: $$\frac{\partial F}{\partial B_G} = \frac{R_n}{G_G}2\left(B_G + B_{cor}\right) = 0$$

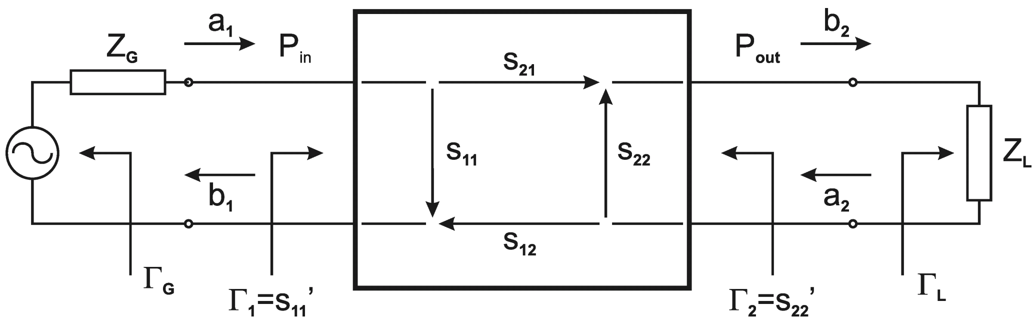
$$\frac{\partial F}{\partial G_G} = -\frac{G_n}{G_G^{\ 2}} + \left\{-\frac{R_n}{G_G^{\ 2}}\left[\left(G_G + G_{cor}\right)^2 + \left(B_G + B_{cor}\right)^2\right] + 2\frac{R_n}{G_G}\left(G_G + G_{cor}\right)\right\} = 0$$

- From that → the **optimum generator conductance** G_{Gopt} and the **optimum generator susceptance** B_{Gopt} providing the **lowest achievable** $F = F_{\min}$ value can be derived:

$$B_{Gopt} = -B_{cor}$$

$$G_{Gopt} = \left(\frac{G_n}{R_n} + G_{cor}\right)^{\frac{1}{2}}$$

$$F_{\min} = 1 + 2R_n\left(G_{Gopt} + G_{cor}\right)$$

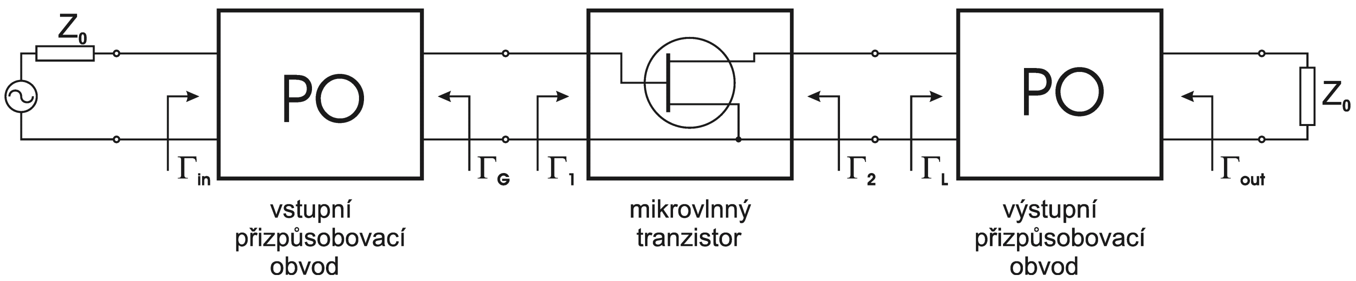


Noise Parameter Set: F_{min}, R_n, $|\Gamma_{Gopt}|$, φ_{Gopt}

- **Noise figure:** $$F = F_{\min} + \frac{R_n}{G_G}\left[\left(G_G - G_{Gopt}\right)^2 + \left(B_G - B_{Gopt}\right)^2\right] = F_{\min} + \frac{R_n}{G_G}\left|Y_G - Y_{Gopt}\right|^2$$
- If gate or base of the transistor used „sees“ $Y_{Gopt} = G_{Gopt} + jB_{Gopt}$ the **optimum generator admitance** → its noise figure $F = F_{\min}$
- If not, the resulting noise figure increases
- The R_n **noise resistance** → defines how fast F increases
- Low R_n values are advantageous
- For design of RF & microwave amplifiers → noise parameters based on Γ_G and Γ_{Gopt} are beneficial

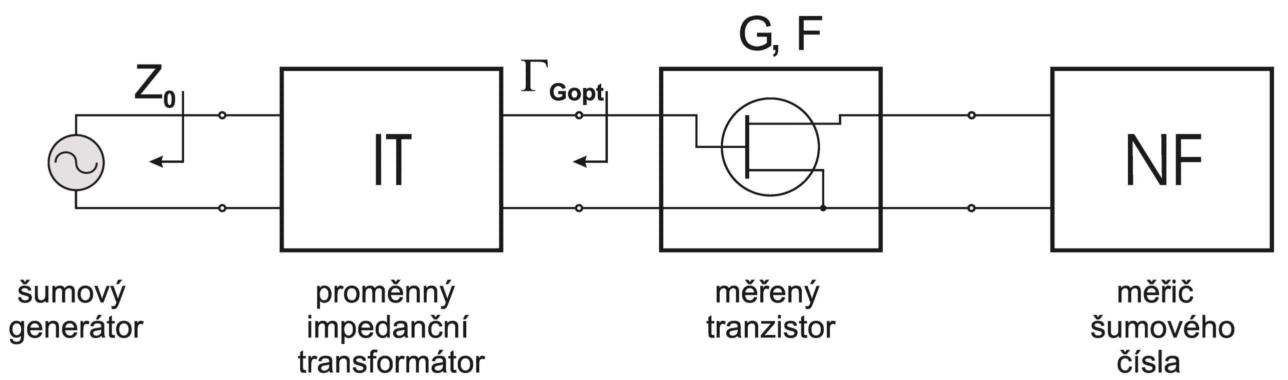
$$F = F_{\min} + 4\frac{R_n}{Z_0}\frac{\left|\Gamma_G - \Gamma_{Gopt}\right|^2}{\left|1 + \Gamma_{Gopt}\right|^2\left(1 - \left|\Gamma_G\right|^2\right)}$$

- Corresponding design noise parameters: R_n $F_{\min}$ $\Gamma_{Gopt} = \left|\Gamma_{Gopt}\right| e^{j\varphi_{Gopt}}$



Noise Parameter Set: F_{min}, R_n, $|\Gamma_{Gopt}|$, φ_{Gopt}

- Measurement:
  - Using the **variable impedance transformer** (IT) and the **noise figure measurement device** (NF)
  - Closer description in A0M17MMS
  - Using the variable IT → the minimum noise figure $F = F_{\min}$ of the DUT is found
  - The corresponding $\Gamma_G = \Gamma_{Gopt}$ is measured using the VNA
  - The R_n value can be calculated from the additional $\Gamma_G = 0$ measurement
  - In practice:
    - Measurement of F for more Γ_G values
    - Solution of set of equations
  - Measurement of design noise parameters → **demanding**
- For the LNA design → noise parameters appended to the s-parameter file
- Signal and noise analysis run in parallel



Example: AWR-MO transistor data file

```
!fhx04lg.s2p
!8/88
!FHX04/05/06LG
!@2V-10mA
!.1GHZ 20GHZ 22
# GHZ S MA R 50
! S-parameter data
  1.0   .990  -19.3   4.232  162.1   .016   75.1   .576   -14.3
  2.0   .965  -37.5   4.115  144.1   .030   64.8   .563   -28.1
  3.0   .928  -55.2   3.923  127.4   .042   53.3   .546   -41.2
  ..........................................................................
 16.0  .557   151.8   2.151  -43.2   .066  -22.2   .642   177.8
 17.0  .522   140.9   2.142  -56.9   .067  -29.4   .673   169.5
 18.0  .480   128.4   2.136  -71.2   .068  -39.2   .694   159.7
! Noise data 4/90
  2   0.33  0.99   29.0  .43
  4   0.35  0.97   53.0  .30
  6   0.45  0.93   77.0  .20
  ..............................
 14   0.88  0.63  178.0  .03
 16   1.05  0.53 -156.0  .05
 18   1.30  0.42 -129.0  .09
```

Transistor Fujitsu

FHX04LG, FHX05LG, FX06LG

2V/10mA

f [GHz] s11 s21 s12 s22

Design noise parameters

f [GHz] F_{mindB} $|\Gamma_{opt}|$ $\arg(\Gamma_{opt})$ R_n/Z_0

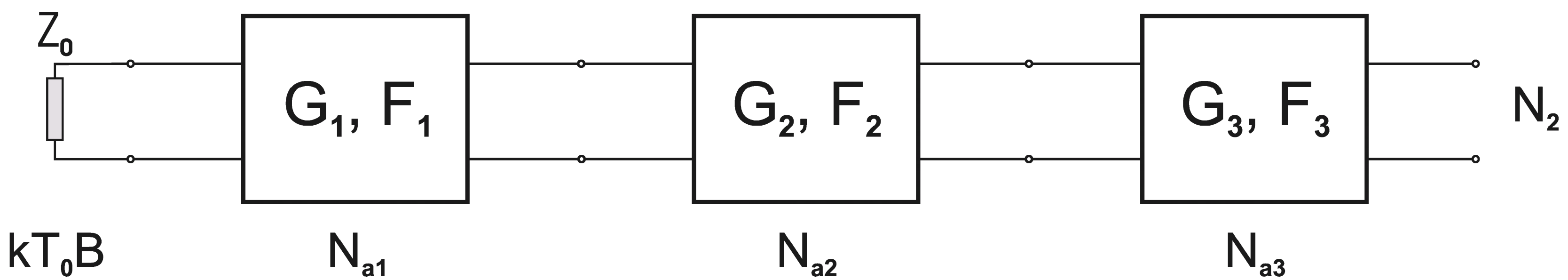
Cascade of Noisy 2-Ports

- Each circuit or system contain more noisy elements always
- In case of their cascade connection → the Frii's formula can be used
- The cascade noise figure → can be derived using the **N_a added noise powers**:

$$N_{a1} = (F_1 - 1)kT_0B \qquad N_{a2} = (F_2 - 1)kT_0B \qquad N_{a3} = (F_3 - 1)kT_0B$$

$$N_2 = kT_0BG_1G_2G_3 + N_{a1}G_1G_2G_3 + N_{a2}G_2G_3 + N_{a3}G_3 =$$
$$= kT_0B\left[G_1G_2G_3 + (F_1 - 1)\,G_1G_2G_3 + (F_2 - 1)\,G_2G_3 + (F_3 - 1)\,G_3\right]$$

- **The Frii's formula:** $F = \dfrac{N_2}{kT_0BG_1G_2G_3} = F_1 + \dfrac{F_2 - 1}{G_1} + \dfrac{F_3 - 1}{G_1G_2}$



Noise Figure of Passive 2-Ports

- Noise of any **passive lossy matched 2-port** (e.g. attenuator):
  - Behaves as the Z_0 impedance termination $N_2 = k_B T_0 B$
  - The signal is amplified by: $G = 1/L$
- **Noise figure:** $$F = \frac{N_2}{GN_1} = \frac{kT_0B}{\frac{kT_0B}{L}} = L$$
- The same applies to all lossy 2-ports with very good impedance matching: **Attenuators, cables, inter-connecting lines, RF filters (in the pass-band), RF switches, …**
- But also to multi-ports with only 2 ports considered: **Splitters, directional couplers, …**
- **Consequences** of the Frii's formula:
  - LNAs must show high associate gain
  - Cascade connection of the attenuator L and amplifier F_2: $F = L + (F_2 - 1)L = L + LF_2 - L = LF_2$
  - Any attenuation L at input increases the overall noise figure: $F_{dB} = L_{dB} + F_{2dB}$
  - This also concerns input matching circuits → their loss directly increase F

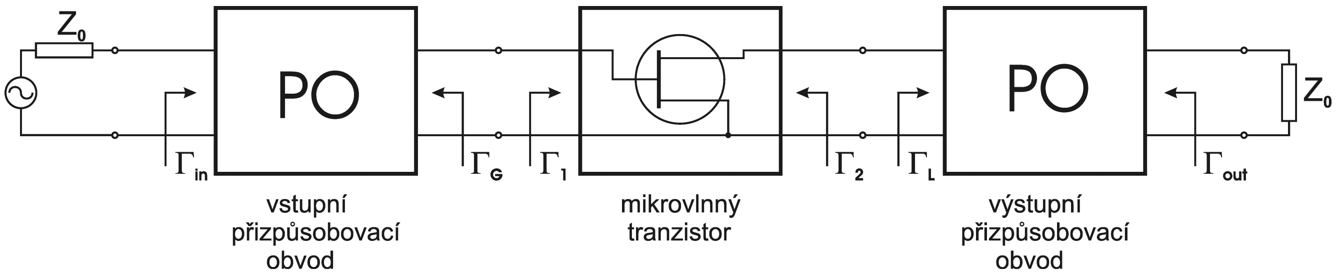
Recommended LNA Design Steps

1. Choice of the suitable transistor

- Specialized low-noise transistors (HEMT) → acceptable $F_{\min}$ and G_{as} , low R_n = advantage
- Proper biasing → F_{min} depends upon I_{CE} or I_{DS}
- Design noise parameters: R_n $F_{\min}$ $\Gamma_{Gopt} = |\Gamma_{Gopt}| e^{j\varphi_{Gopt}}$ → from the catalog or by measurement

2. Noise matching

- Performed in the Γ_G plane
- Optimum noise matching $\Gamma_G = \Gamma_{Gopt}$
- Absolutely stable transistor → output impedance matching $\Gamma_L = \Gamma_2^*$
- Potentially unstable transistor
  - Both $\Gamma_G = \Gamma_{Gopt}$ and $\Gamma_L = \Gamma_2^*$ must lie in the stable regions
  - Or stabilization of the transistor by using R_{p2} or R_{s2}
- Since $\Gamma_{Goptnoise} \neq \Gamma_{Goptpower}$, the LNAs show high input reflection



LNA design

3. Possible compromise → noise matching versus input reflection:

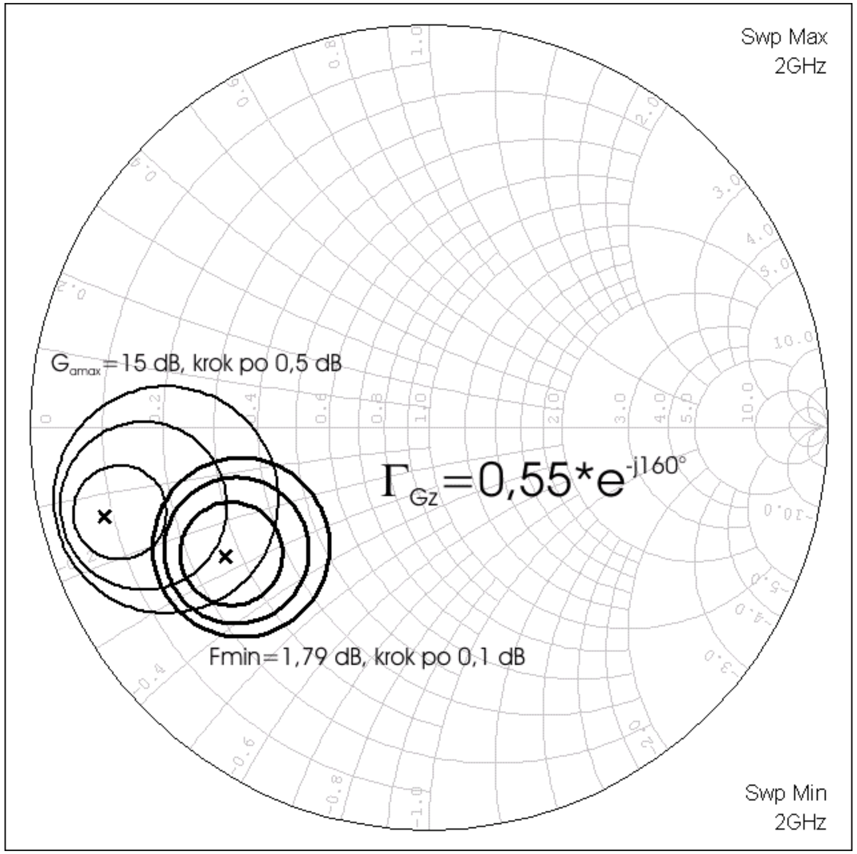
- **Constant noise figure** F_i **circles** can be plotted into the Γ_G plane

Centers $C_F = \dfrac{\Gamma_{Gopt}}{1+N_i}$

Diameters $r_F = \dfrac{1}{1+N_i}\left[N_i^2 + N_i\left(1-\left|\Gamma_{Gopt}\right|^2\right)\right]^{\frac{1}{2}}$

$$N_i = \frac{(F_i - F_{\min})Z_0}{4R_n}\left|1+\Gamma_{Gopt}\right|^2$$

- **AWR-MO:** NFCIR



LNA design

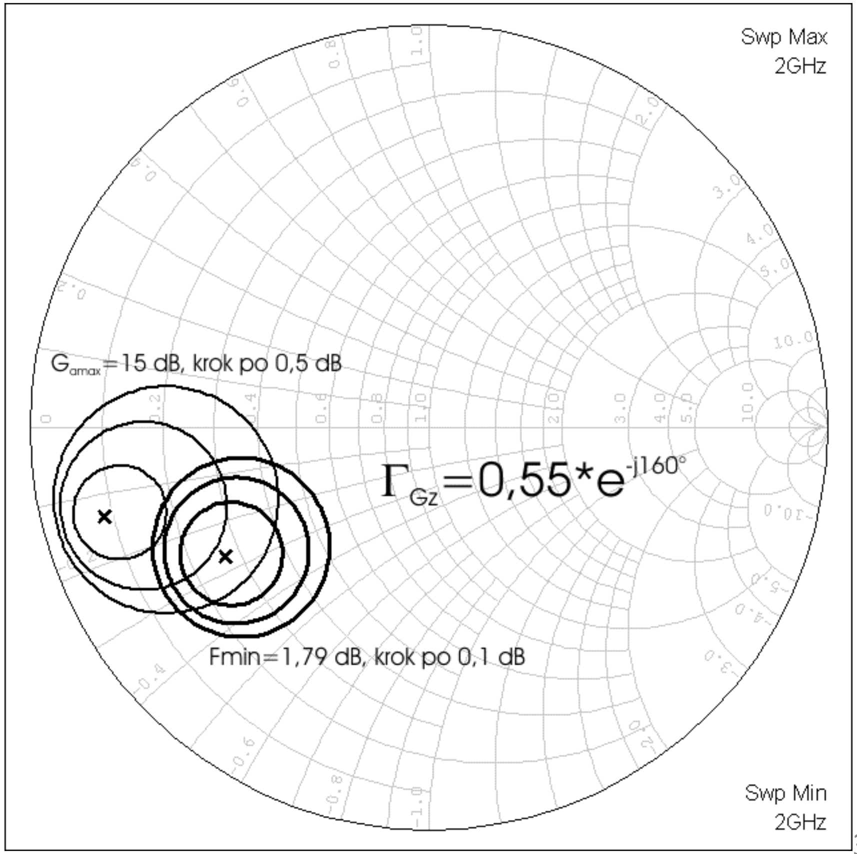
- **Constant available gain circles** can also be plotted into the Γ_G plane:

Centers
$$C_a = \frac{g_a\left(s_{11}^* - D^* s_{22}\right)}{1+g_a\left(|s_{11}|^2 - |D|^2\right)}$$

Diameters
$$r_a = \frac{\left(1-2k|s_{12}s_{21}|g_a + |s_{12}s_{21}|^2 g_a^2\right)^{\frac{1}{2}}}{\left|1+g_a\left(|s_{11}|^2-|D|^2\right)\right|}$$

Norm. gains
$$g_a = \frac{G_a}{|s_{21}|^2}$$

- **AWR-MO:** GACIR
- The compromise $\Gamma_G = \Gamma_{Gz}$ can be found
- Higher F but lower Γ_{in}
- Not much recommended
- Lower Γ_{in} can be obtained by R_{s2}



LNA design

4. Output impedance matching is beneficial

$$\Gamma_L = \Gamma_2^* \qquad \Gamma_2 = s_{22} + \frac{s_{12} s_{21} \Gamma_{Gz}}{1 - s_{11}\Gamma_{Gz}}$$

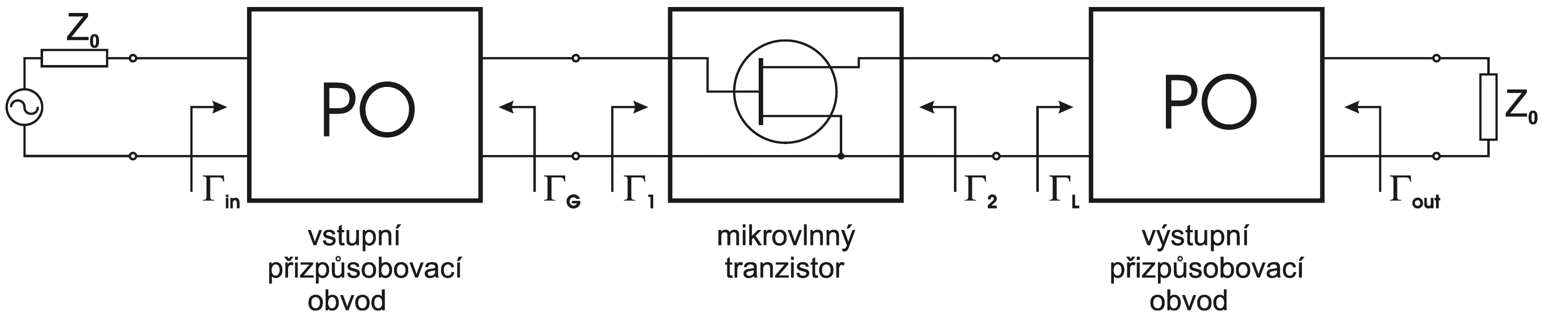
AWR-MO: LTUNER

With Γ_{Gz} and Γ_L known, the input and output matching circuits can be synthesized

- Input MC transforms Z_0 to Γ_{Gz} or Γ_{Gopt}
- Output MC transforms Z_0 to Γ_L

5. Resulting amplifier parameters:

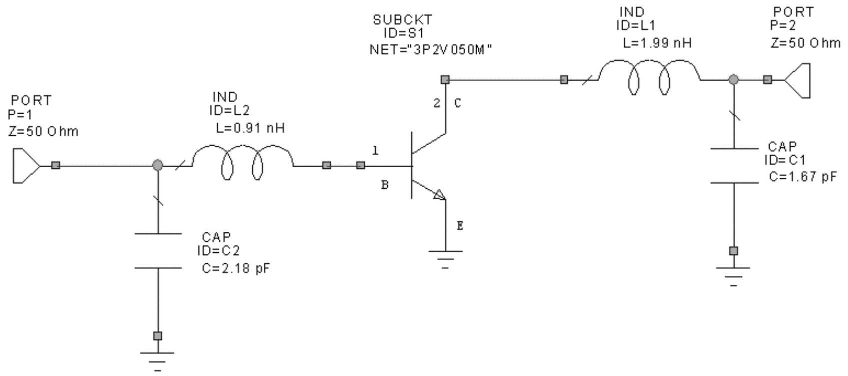
- Since $\Gamma_{Goptnoise} \neq \Gamma_{Goptpower}$ $\rightarrow$ significant input reflections appear $|\Gamma_{in}| \approx 0{,}6 \div 0{,}9$
- Output can be ideally matched $|\Gamma_{out}| \rightarrow 0$
- Gain is equal to associate gain G_{as}

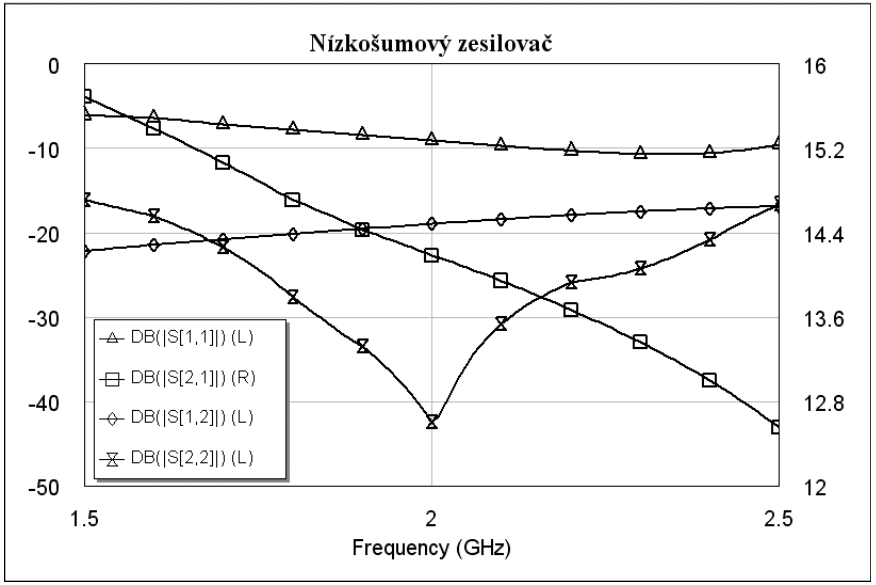


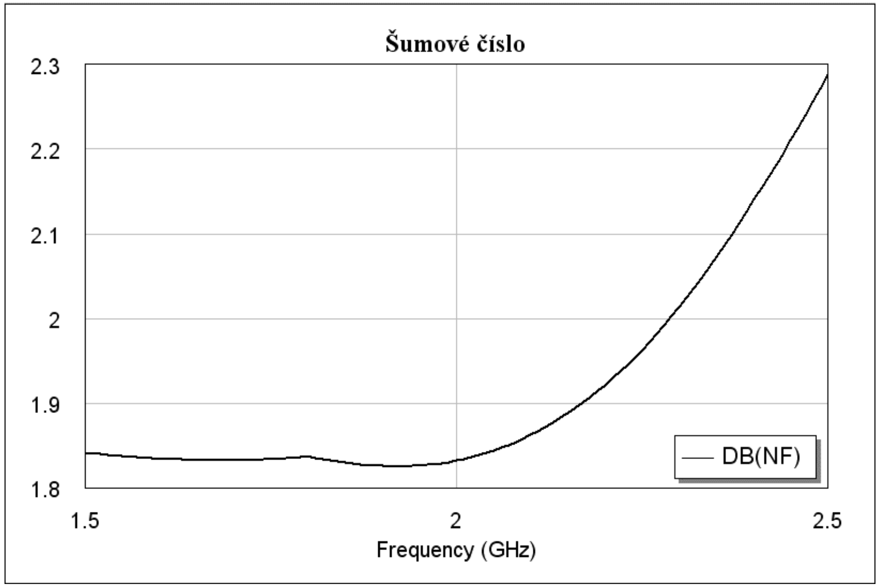
LNA Example 1

6. Example:

- BJT BFP450
- Frequency 2 GHz
- Chosen generator $\Gamma_G = \Gamma_{Gz} = 0{,}55\, e^{-j160^o}$







LNA Example 2

6. Example: ATF-34143, FET 2V / 20mA

- f=8GHz , k=1,1 , F_{min}=0,95dB
- GMN=0,47 / -86°
- Minimum F design
- Optimum output matching
- Resulting parameters:
  - F=1,05dB
  - Input matching
  - Output matching