Radar Signal Detection

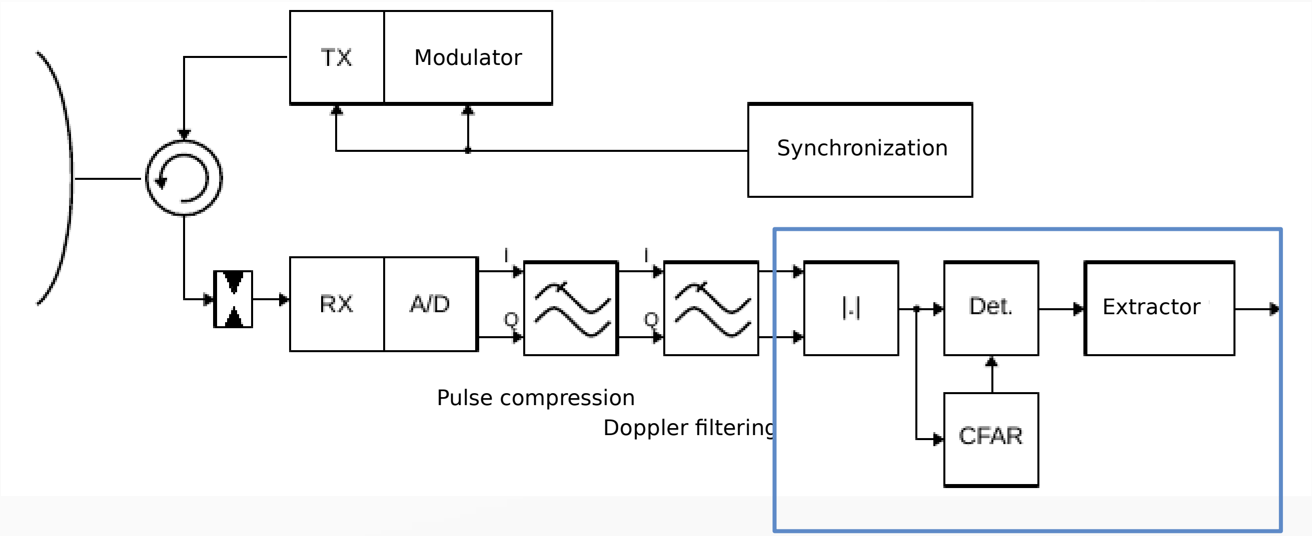
Radar systems (Radarové systémy)

Radar signal processing

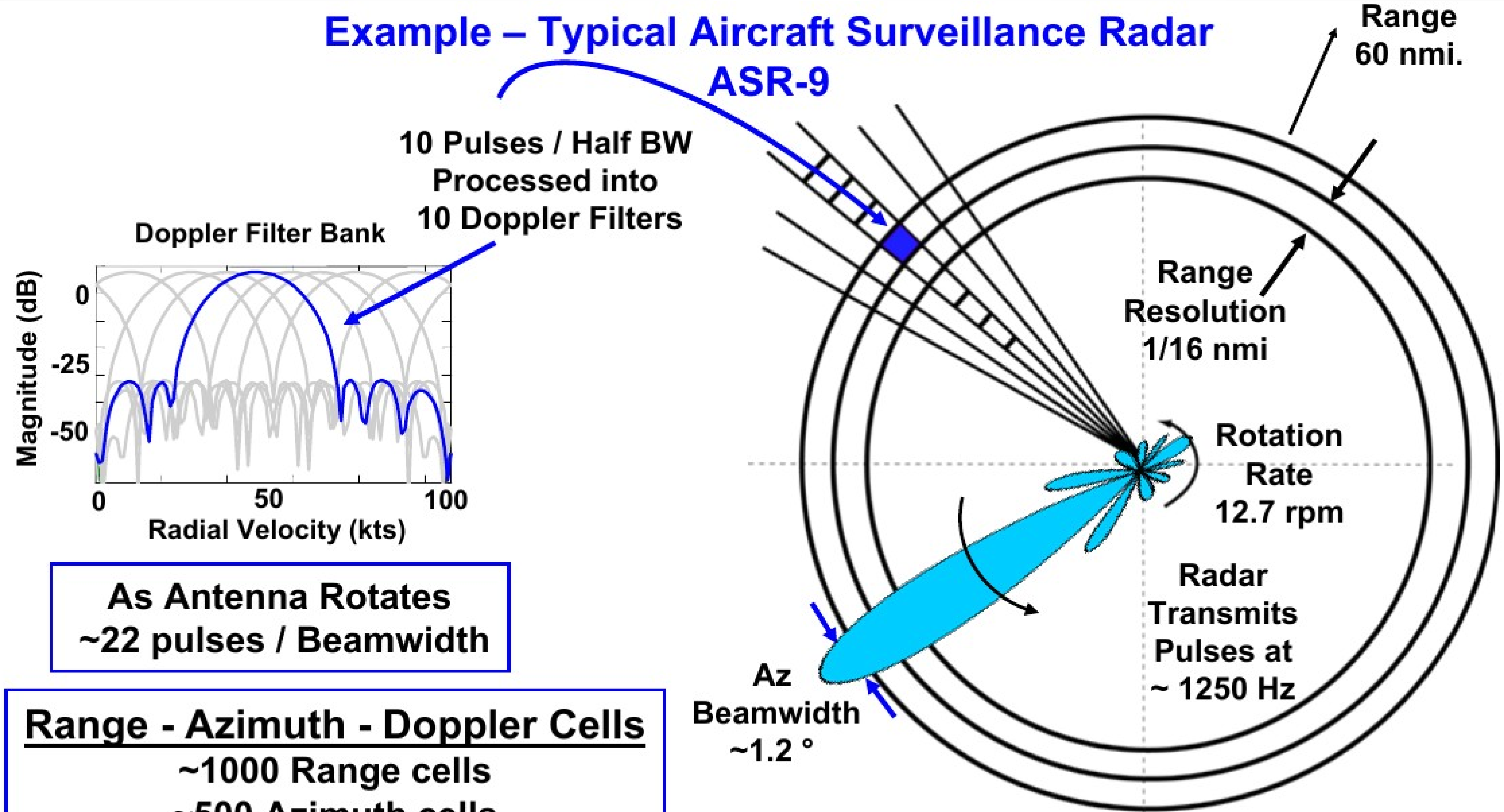
- Primary processing
  - Processing of signal within one antenna revolution – one PRI or several adjacent PRIs
    - Pulse compression
    - Doppler filtration
    - Target detection
- Secondary processing
  - Processsing of several revolutions
    - Target tracking
    - Clutter mapping for adaptive processing
- Tertial processing
  - Data fusion – signal processing from several radar sites

Radar signal processing

- Primary processing
  - Signal detection
  - Extraction



Range-Azimuth-Doppler Cells to Be Thresholded

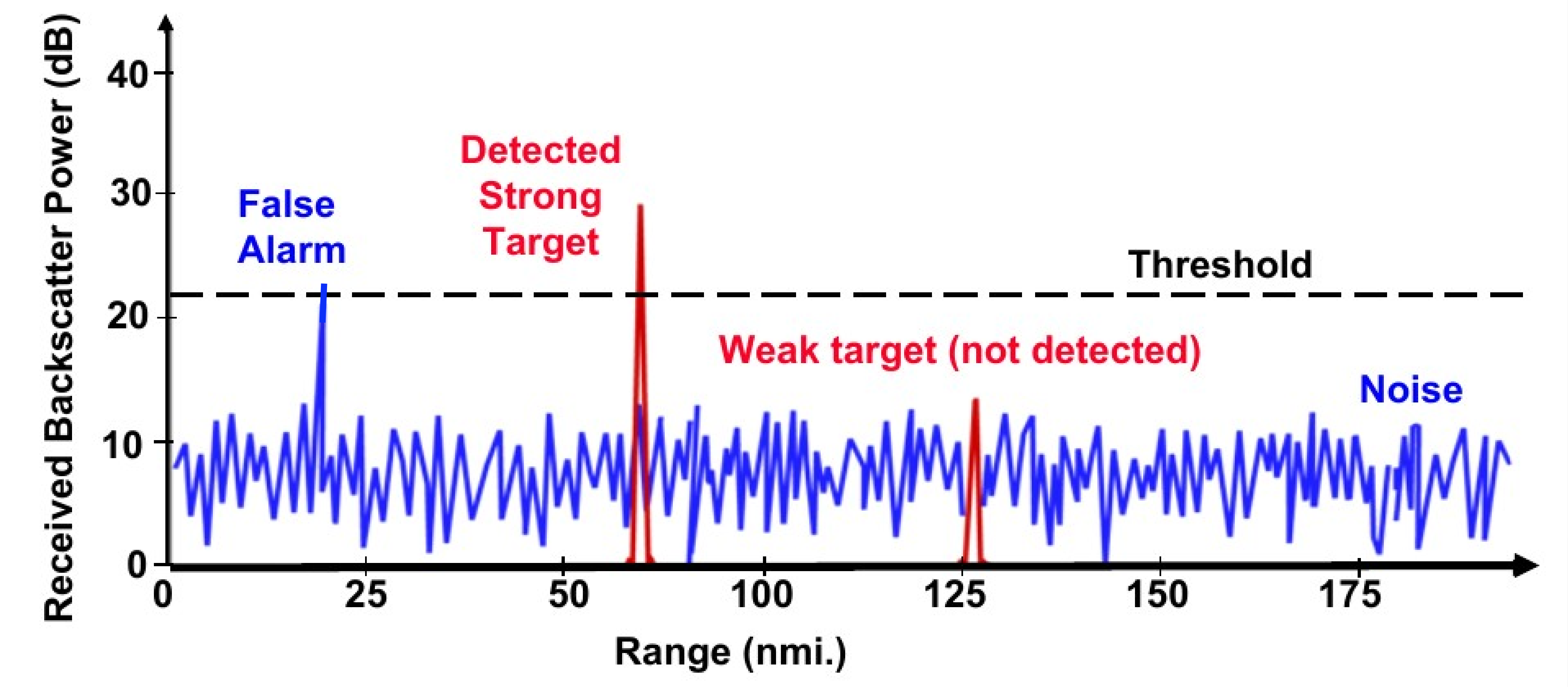


As Antenna Rotates
~22 pulses / Beamwidth

Range - Azimuth - Doppler Cells
~1000 Range cells
~500 Azimuth cells
~8-10 Doppler cells
5,000,000 Range-Az-Doppler Cells
to be threshold every 4.7 sec.

Is There a Target Present
in Each Cell?

Target Detection in Noise



- Received background noise and target echo fluctuate randomly → Both are random variables
- To decide if a target is present, at a given range, we need to set a threshold (constant or variable)
- Detection performance (**Probability of Detection, P_D**) depends of the strength of the target relative to that of the noise and the threshold setting (***Signal-To Noise Ratio*** and ***Probability of False Alarm, P_{fa}***)

Signal Detection

- Decision about traget presence for each evaluated cell from measurement x
- Two hypotheses H_0, H_1
- Neyman-Paerson theorem/detector
- – Maximize P_D for chosen P_{fa}
- Based on information about characteristics of noise, clutter, signal

Statistics Theory	**Radar Theory**
Tested statistics $U(x)$ and threshold γ	Detector
Zero hypothesis $\boldsymbol{H_0}$	Target absent, Noise only (x=n), PDF: p(x|H_0)
Alternative hypothesis $\boldsymbol{H_1}$	Target present, Signal+noise (x=s+n), PDF: p(x|H_1)
1st case error (decision $\boldsymbol{H_1}$ for real $\boldsymbol{H_0}$)	False alarm, ($\boldsymbol{P_{fa}}$)
2nd case error (decision $\boldsymbol{H_0}$ for real $\boldsymbol{H_1}$)	Missed detection, (**1**-$\boldsymbol{P_D}$)

Optimum Threshold Test

		Decision	
		H_0	H_1
Reality	H_0	Do not report	False alarm
	H_1	Missed detection	Detection

Probability of Detection:

P_D Probability we choose H_1 for reality H_1

Probability of False alarm:

P_{fa} Probability we choose H_1 for reality H_0

Neyman-Pearson criterion: Maximize $\boldsymbol{P_D}$ for $\boldsymbol{P_{fa}}$ not greater than specified value

Likelihood function (ratio)

$$\boxed{L(x) = \frac{p(x|H_1)}{p(x|H_0)} : \quad \begin{array}{l} \geq T \rightarrow H_1 \\ < T \rightarrow H_0 \end{array}}$$

Threshold

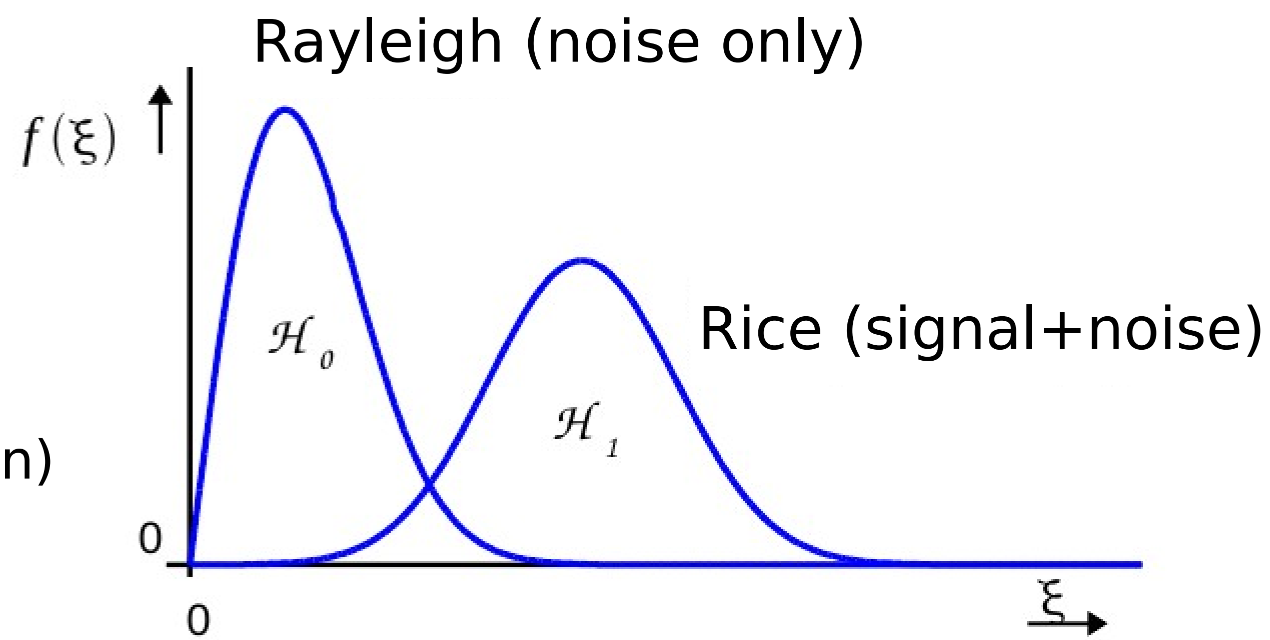
Statistic description

- Rayleigh distribution - noise probability density function (e.g. voltages)
- Rice distribution - signal+noise probability density function
- AWGN envelope and impacts of environment, non-fluctuating target

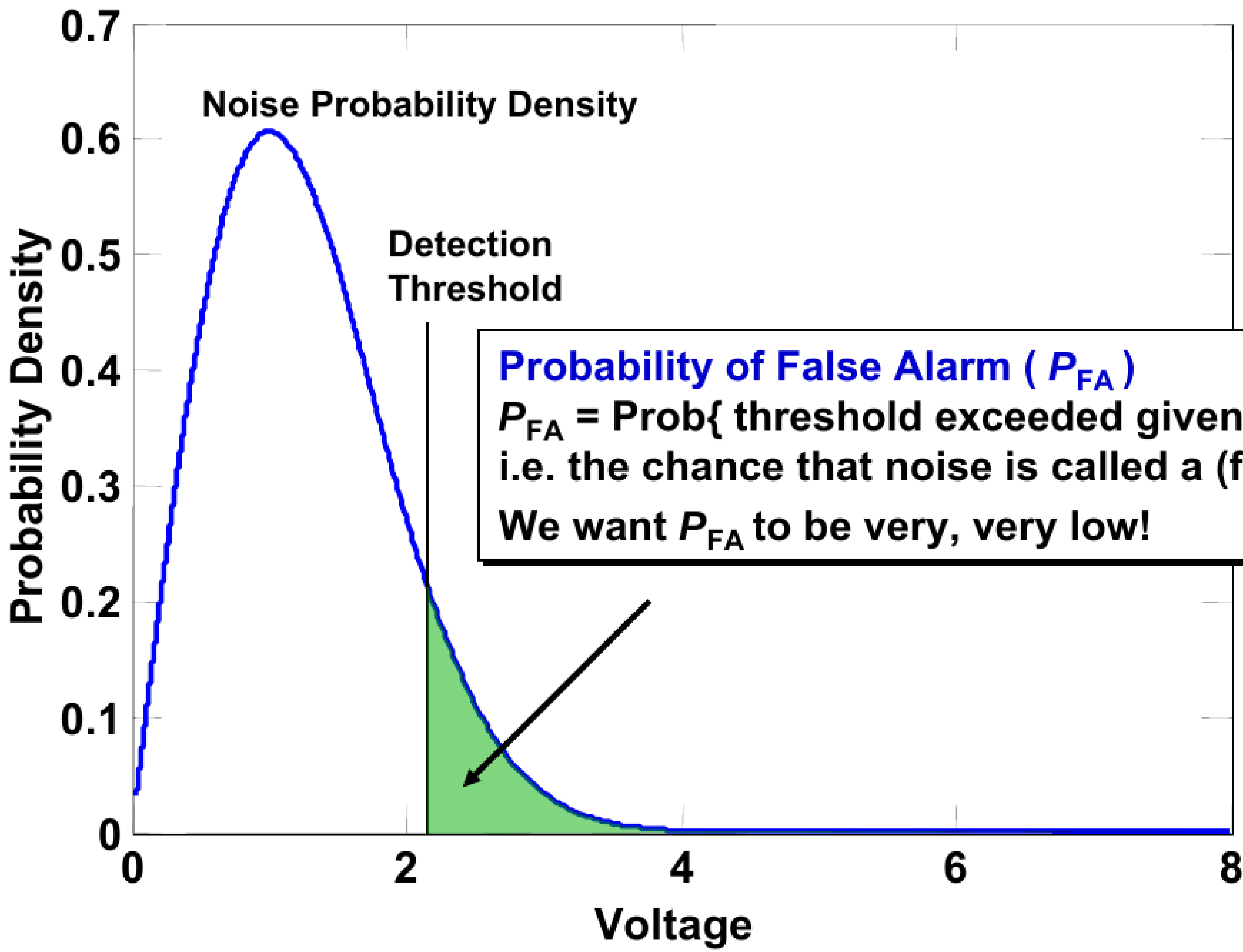
$$f(\xi|\mathcal{H}_0)=\frac{\xi}{\sigma^2}e^{-\frac{\xi^2}{2\sigma^2}}$$

$$f(\xi|\mathcal{H}_1)=\frac{\xi}{\sigma^2}e^{-\xi^2-\frac{a^2}{2\sigma^2}}I_0\left(a\frac{\xi}{\sigma^2}\right)$$

I_0 (modified Bessel function)



Detection statistics



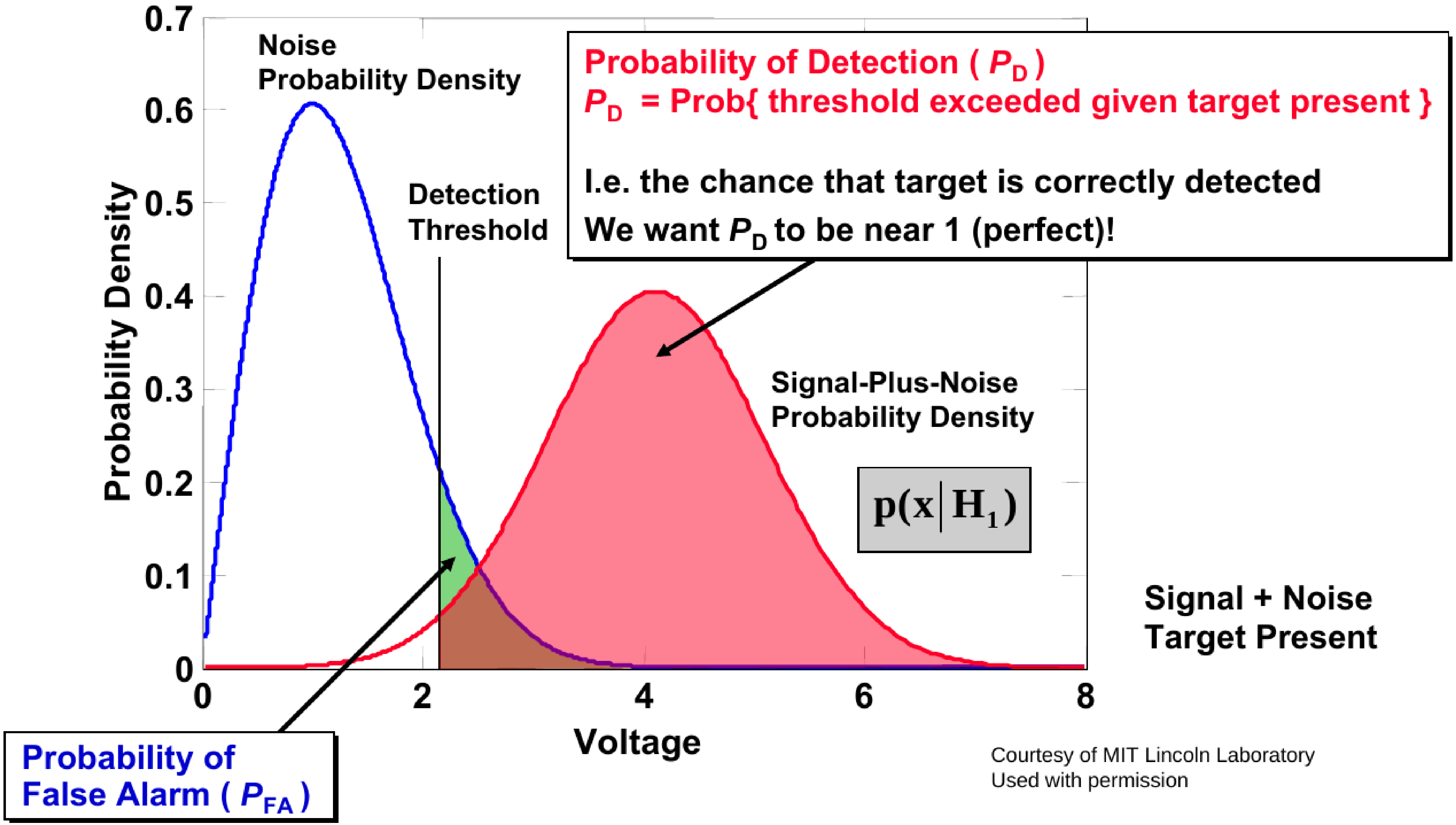
Noise Only
Target Absent

Probability of False Alarm (P_{FA})

P_{FA} = Prob{ threshold exceeded given target absent }
i.e. the chance that noise is called a (false) target

We want P_{FA} to be very, very low!

Detection statistics



Courtesy of MIT Lincoln Laboratory
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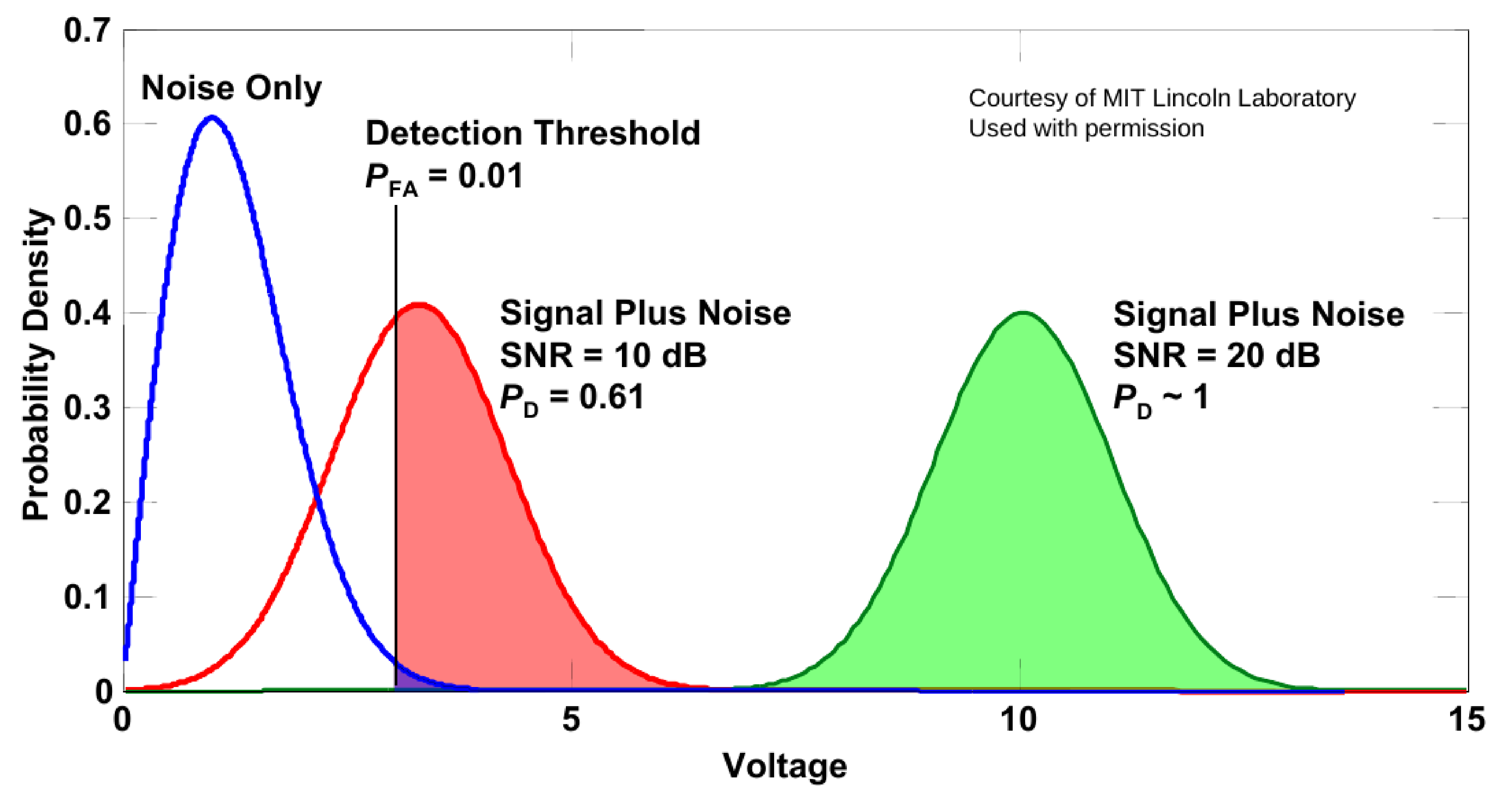
Impact of SNR



P_D increases with SNR for given P_{fa} (threshold)

Non-fluctuating target detection

$$L(x)=\frac{P(x|\mathcal{H}_1)}{P(x|\mathcal{H}_0)} \overset{\mathcal{H}_1}{>} T$$

$$P_{fa}=Pr\{L(x)>T;\mathcal{H}_0\}=\int_T^{\infty} p(\xi|\mathcal{H}_0)\,d\xi$$

$$P_d=Pr\{L(x)>T;\mathcal{H}_1\}=\int_T^{\infty} p(\xi|\mathcal{H}_1)\,d\xi$$

$$P_{fa}=\int_T^{\infty}\frac{\xi}{\sigma^2}e^{\frac{-\xi^2}{2\sigma^2}}d\xi=e^{\frac{-T^2}{2\sigma^2}}$$

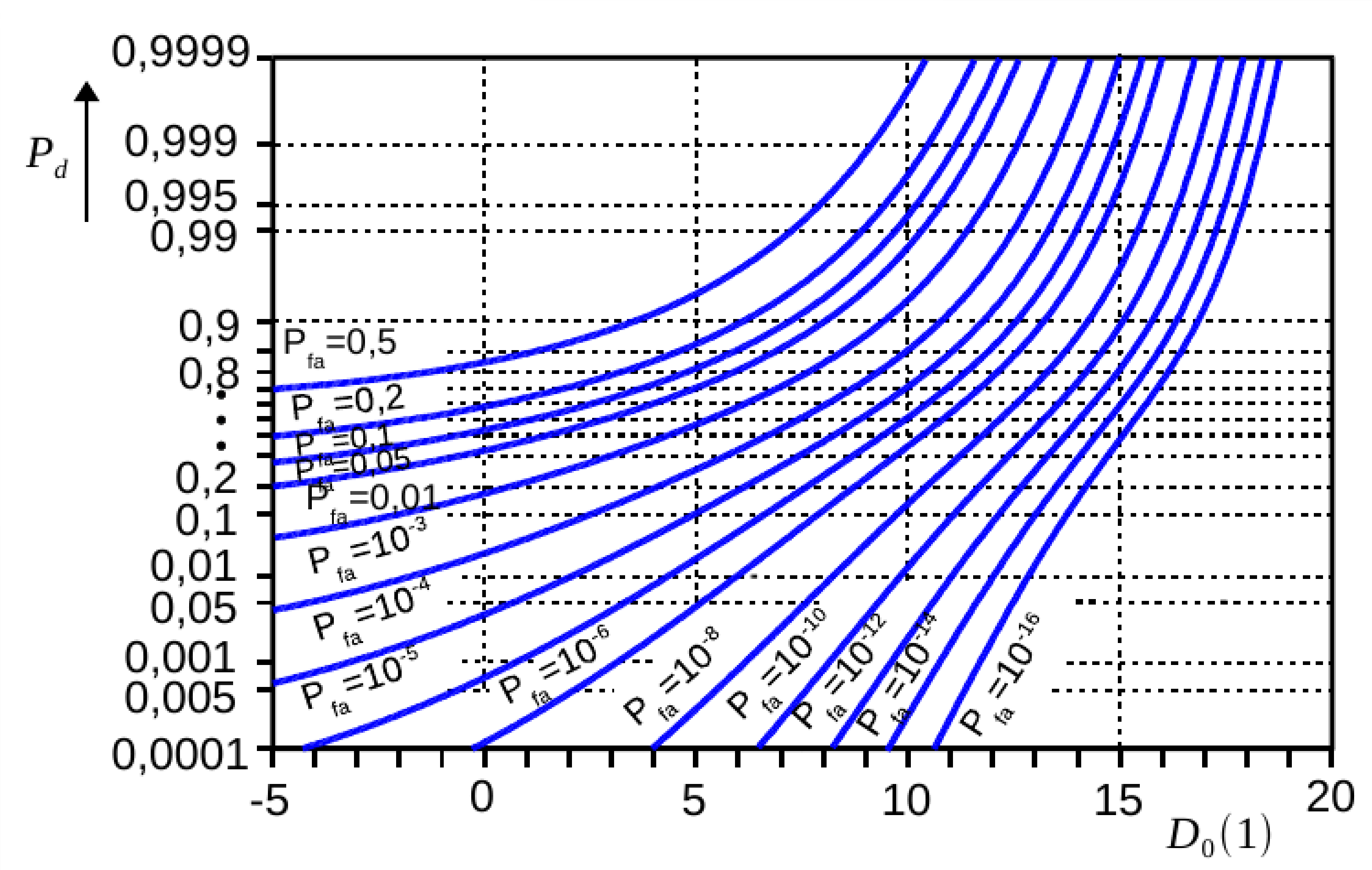
$$T=\sigma\sqrt{-2\ln(P_{fa})}$$

$$S/N=\frac{s^2}{\sigma^2}$$

$$P_d=\int_T^{\infty}\frac{\xi}{\sigma^2}e^{\frac{-\xi^2+s^2}{2\sigma^2}}I_0\left(\frac{\xi s}{\sigma^2}\right)d\xi$$

Detection quality

- System parameters for required detection parameters
- Detection coeficient ($D_0(1)$ for pro SW0 – non-fluctuating target,single pulse detection)
  - Necessary SNR to reach P_d for given P_{fa}
  - Usual conditions
    - Non-fluctuating
    - $P_d = 0.9$; $P_{fa} = 10^{-6}$
    - Fluctuating
    - $P_d = 0.8$; $P_{fa} = 10^{-6}$



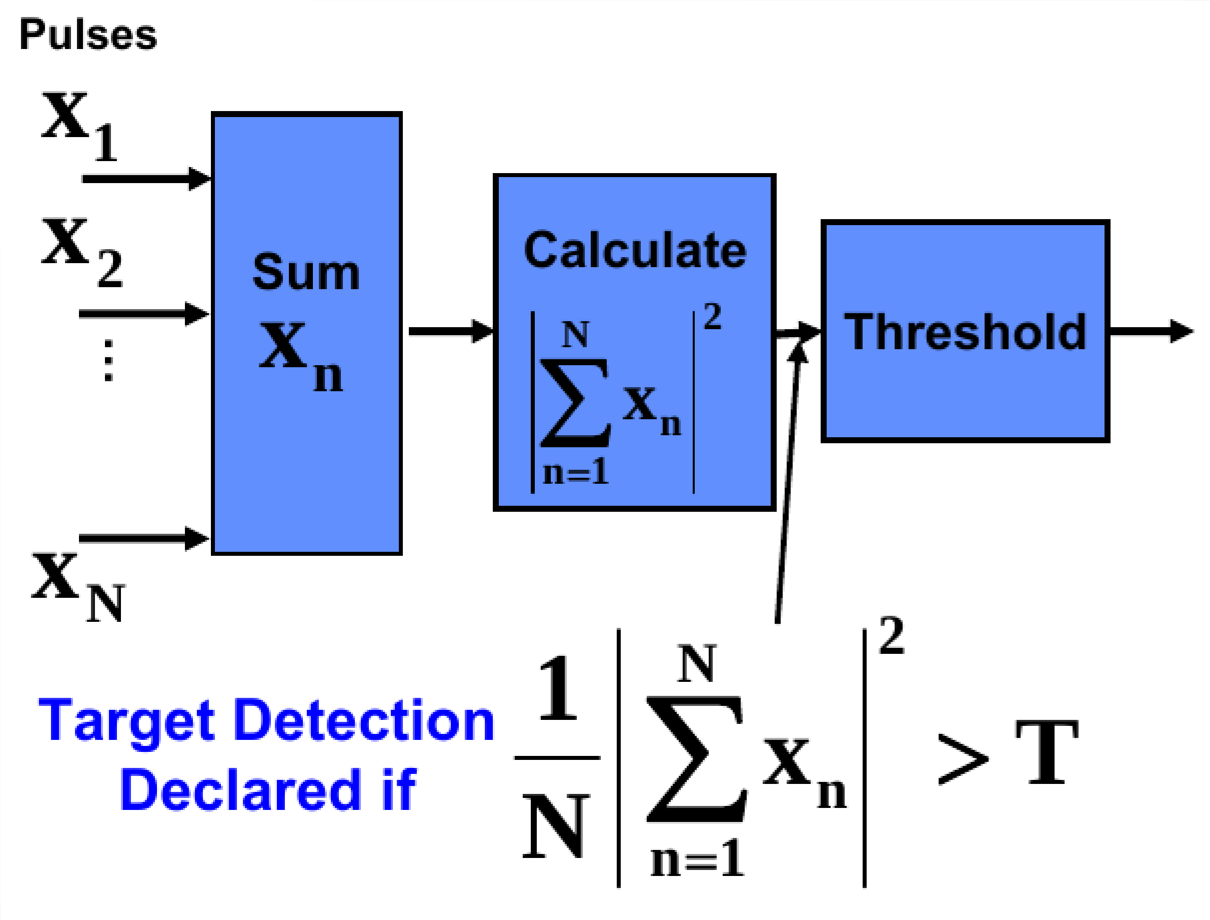
e.g. P_d=0.9, P_{fa} = 10^{-6}, D_0: SNR=13.2 dB

Improvement by integration

- Use multiple pulses from same target
- Impact of statistical dependence of samples
- Motivation
  - Increase Pd
  - Decrease Pfa
  - Decrease of necessary SNR
- Detection
  - Single pulse
  - Multiple pulses
    - Coherent integration
    - Non-coherent (envelope,video) integration
    - Binary Integration (M of N)
    - Cumulative detection (1 of N)

Coherent integration

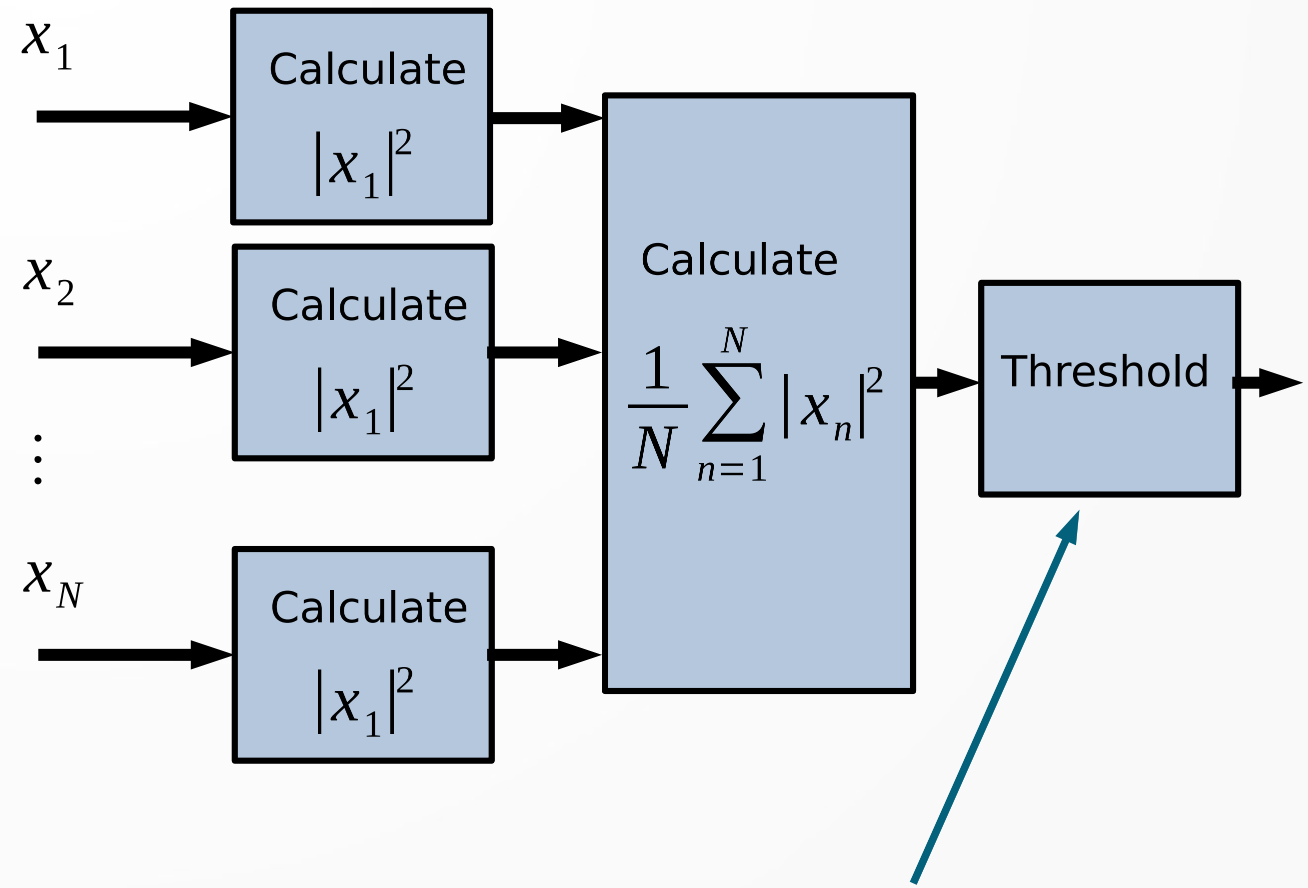
- Doppler compensation prior to integration (MTD)
- Integration (sum) od N pulses – samples of complex envelope
- Amplitude increase N-times
- Noise power (uncorrelated) increase by N-times
- SNR increase N-times
- Integration gain $G_i = N$



$$D_0(n) = \frac{D_0(1)}{G_i} = \frac{D_0(1)}{n}$$

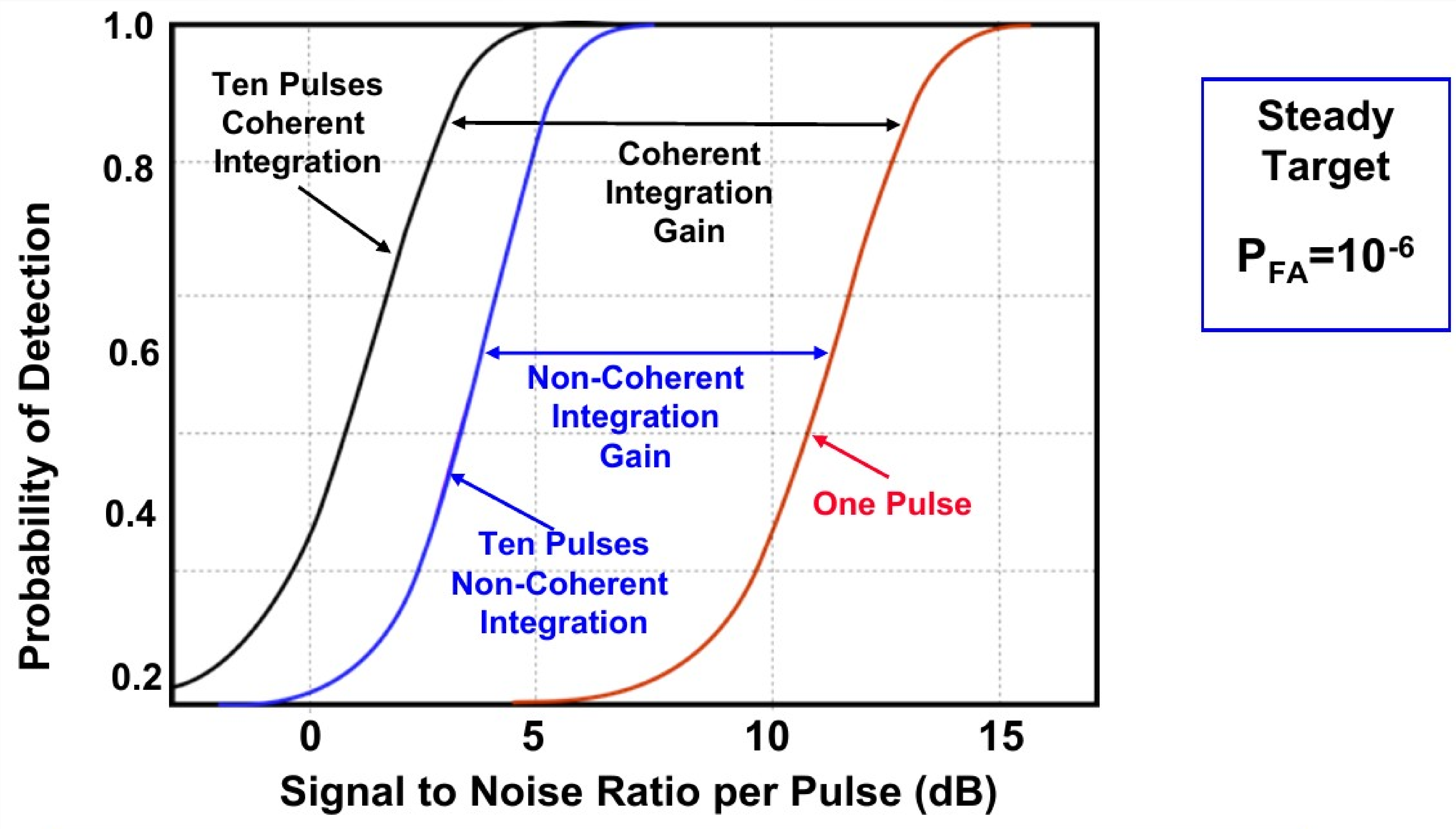
Non-coherent Integration of envelope (video signal)

- Sum of amplitudes (powers)
- Ignores phase - simplification
- Envelope detector
- Higher SNR compared to coherent case
- Integration loses w.r.t. ideal (coherent) integrator



Target detection declared for $\frac{1}{N}\sum_{n=1}^{N}|x_n|^2 > T$

Coherent vs. Non-coherent Integration



Integration of binary output (M of N)

- Simplification of Detector realization
- Integration behind detector, count threshold binary outputs
- Secondary threshold for binary counts (M of N filter)
- First Detector can be set for higher P_{fa}

$$P_d = \sum_{k=m}^{n} \frac{n!}{k!(n-k)!} P_{d1}^k (1-P_{d1})^{n-k}$$

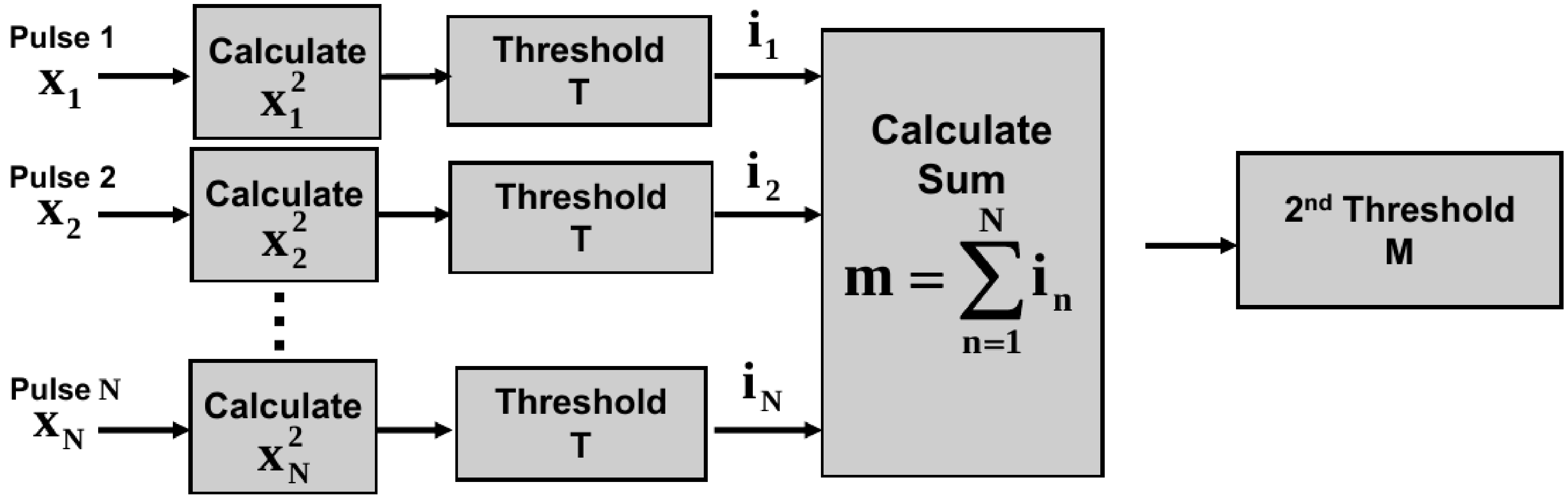
$$P_{fa} = \sum_{k=m}^{n} \frac{n!}{k!(n-k)!} P_{fa1}^k (1-P_{fa1})^{n-k}$$

$$P_{fa} \approx \frac{n!}{m!(n-m)!} P_{fa1}^m ; P_{fa1} \ll 1$$

Can be found optimal M for given N

$$m_{opt} \approx 1{,}5\sqrt{n}$$

M of N integration



Individual pulse detectors:

$$\begin{aligned} |x_n|^2 &\geq T, \quad i_n = 1 \\ |x_n|^2 &< T, \quad i_n = 0 \end{aligned}$$

2nd thresholding:

$m \geq M$, target present
$m < M$, target absent

Target present if at least *M* detections in *N* pulses

Binary Integration

At Least M of N Detections $\quad P_{M/N} = \sum_{k=M}^{N} \frac{N!}{k!(N-k)!} p^k (1-p)^{N-k}$

Cumulative Detection

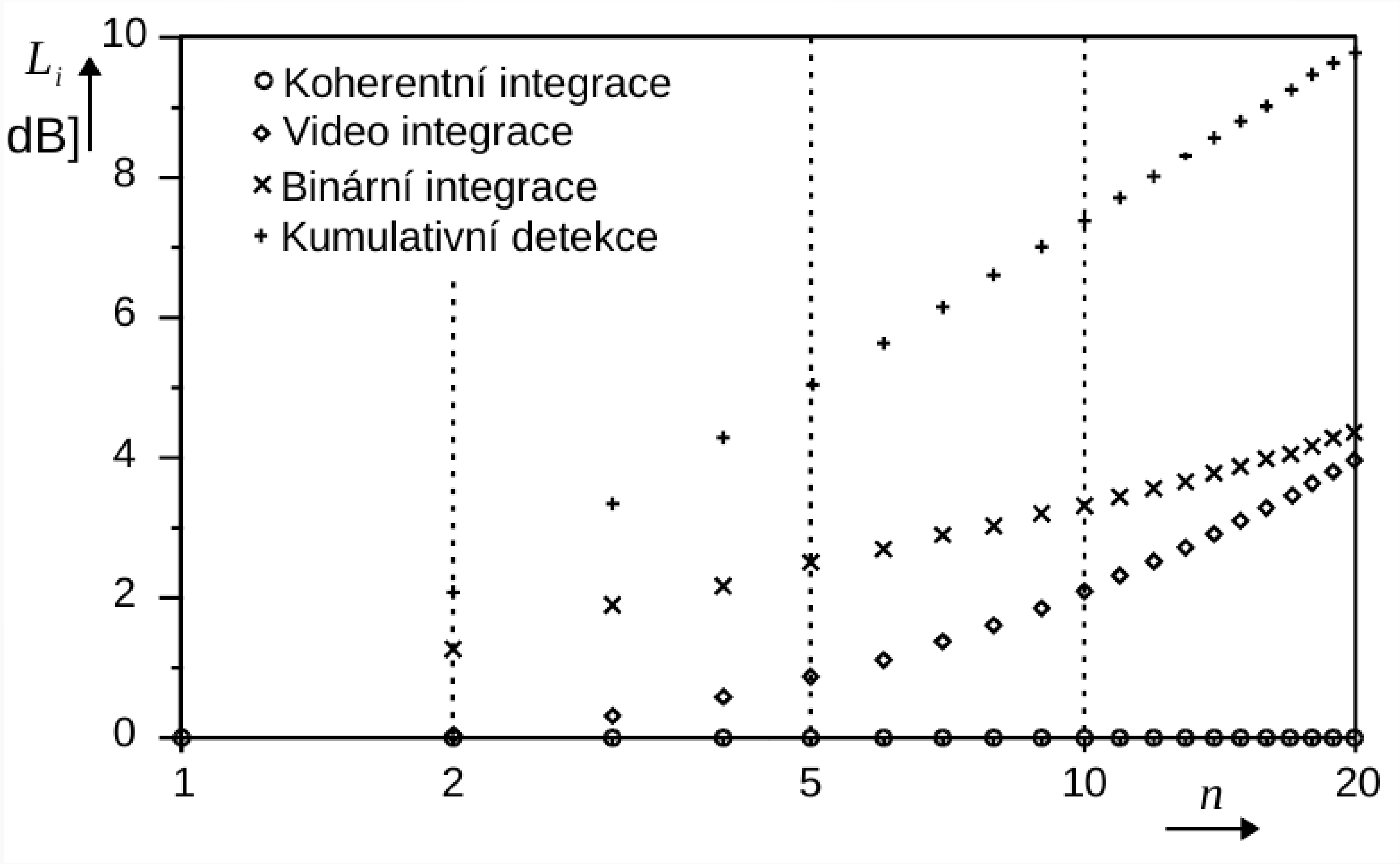
At Least 1 of N $\quad P_C = 1 - (1-p)^N$

1 of N Detection – cummulative detection

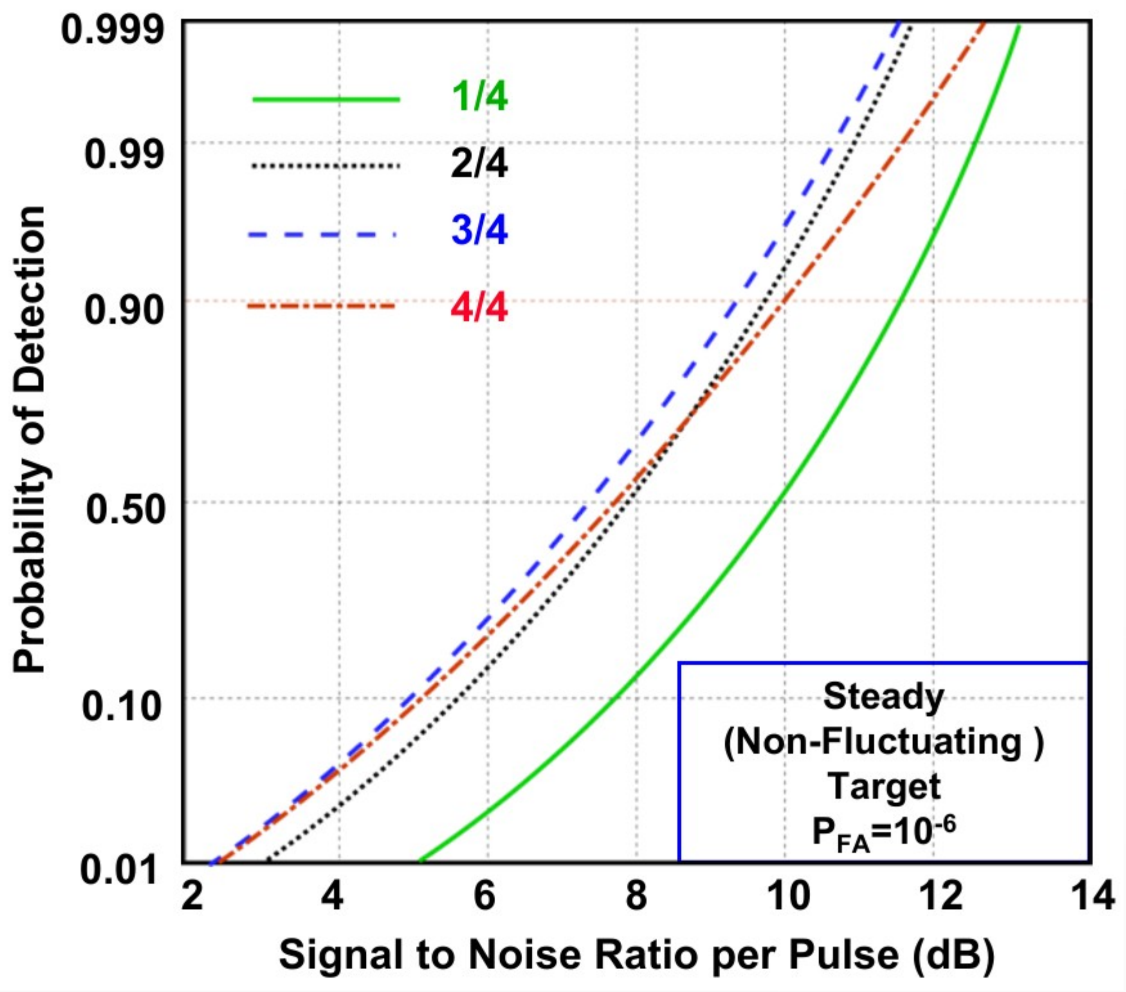
- N independent decisions for N pulses
- Detection positive if at least one of N decisions positive
- Higher requirements to P_{fa} setup in comparison to coherent and non-coherent integrations
- Little improvement of P_d compared to single pulse → low integration gain

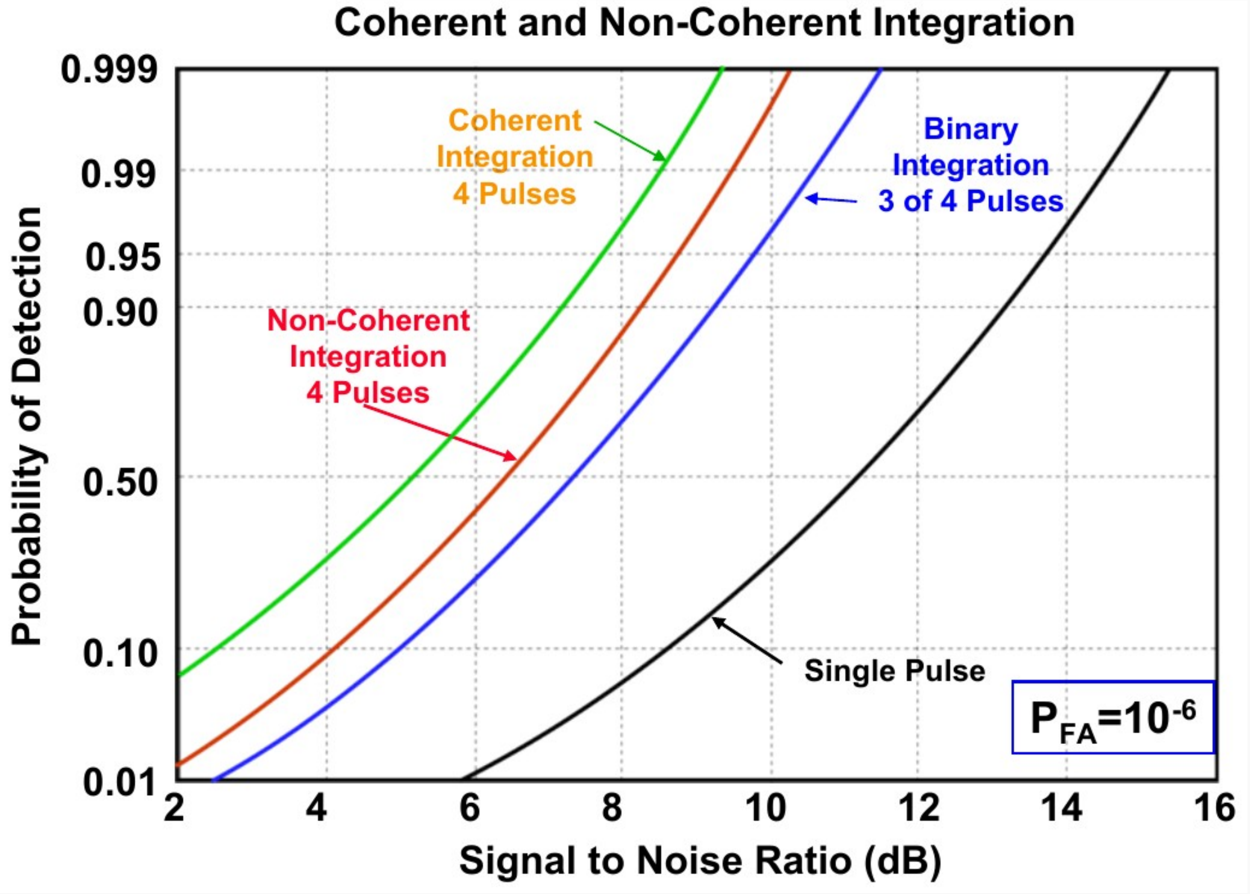
$$P_D = 1 - (1 - P_{d1})^N$$

Loss by integration



Detection comparison

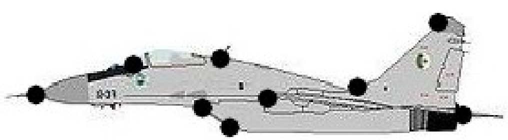
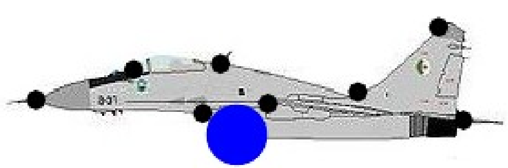




Fluctuation of target (Fluctuation of RCS)

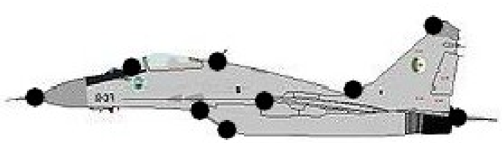
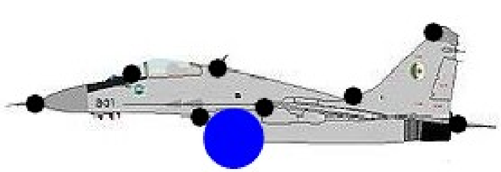
- Swerling models
- SW0 non-fluctuating (steady) target
- SW1 slow fluctuating target (scan to scan)
  - Rayleigh distribution of amplitudes, Exponential distribution of powers (RCS)
  - Mono-frequency radar observations example: Target consists from many comparable scatterers, no dominance
- SW2 fast fluctuating target (pulse to pulse)
  - SW1 target observed by agile radar
- SW3 slow fluctuating target (scan to scan)
  - Power distribution (RCS) $\chi 2$ with 4 degrees of freedom (DOF)
  - Mono-frequency radar observations example: Target consists from one dominant scatterer and many secondary scatterers
- SW4 fast fluctuating target (pulse to pulse)
  - SW3 target observed by agile radar

Swerling Target Models

Nature of Scattering	RCS Model	Fluctuation Rate	
		Slow Fluctuation “Scan-to-Scan”	Fast Fluctuation “Pulse-to-Pulse”
Similar amplitudes 	Exponential (Chi-Squared DOF=2) $p(\sigma)=\frac{1}{\bar{\sigma}}\exp\left(-\frac{\sigma}{\bar{\sigma}}\right)$	Swerling I	Swerling II
One scatterer much Larger than others 	(Chi-Squared DOF=4) $p(\sigma)=\frac{4\sigma}{\bar{\sigma}^2}\exp\left(-\frac{2\sigma}{\bar{\sigma}}\right)$	Swerling III	Swerling IV

$\bar{\sigma} =$ Average RCS (m^2)

Swerling Target Models

Nature of Scattering	Amplitude Model	Fluctuation Rate: Slow Fluctuation "Scan-to-Scan"	Fluctuation Rate: Fast Fluctuation "Pulse-to-Pulse"
Similar amplitudes 	Rayleigh $p(a)=\frac{2a}{\overline{\sigma}}\exp\left(-\frac{a^2}{\overline{\sigma}}\right)$	Swerling I	Swerling II
One scatterer much Larger than others 	Central Rayleigh, DOF=4 $p(a)=\frac{8a^3}{\overline{\sigma}^2}\exp\left(-\frac{2a^2}{\overline{\sigma}}\right)$	Swerling III	Swerling IV

$\overline{\sigma}=$ Average RCS (m^2)

Impact of fluctuations to Probability of detection

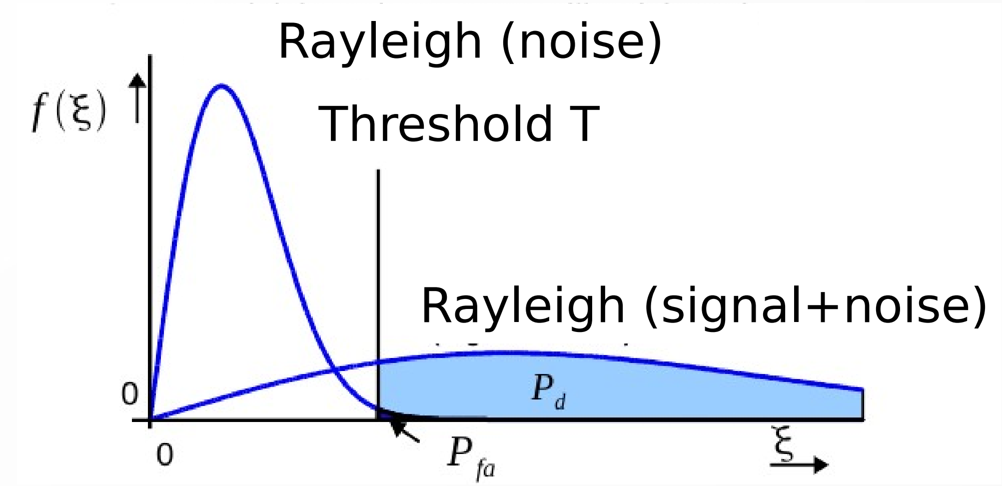
- PSD of fluctuating target echo – Rayleigh (non-fluctuating - Rice)

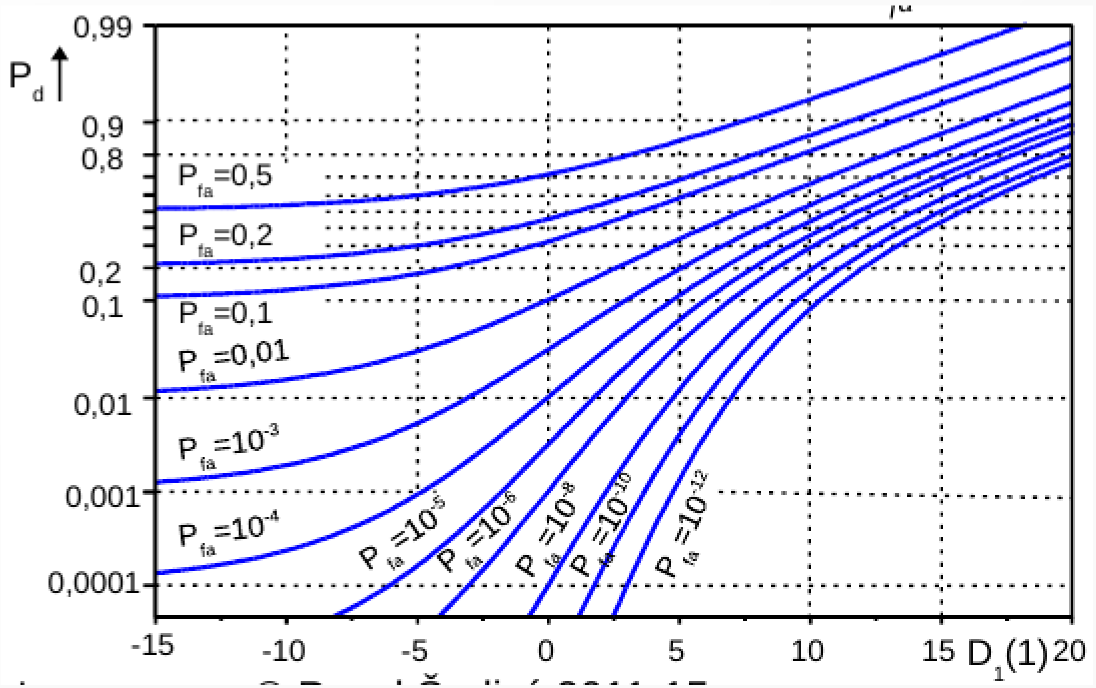
$$f_s(\xi|\mathcal{H}_1)=\frac{\xi}{\sigma^2+\alpha^2}e^{\frac{-\xi^2}{2(\sigma^2+\alpha^2)}}$$

$$P_d=e^{\frac{-T^2}{2(S+N)}}=e^{\frac{\ln(P_{fa})}{1+S/N}}$$

- Detection cefficient

$$D_1=\frac{\ln P_{fa}}{P_d}-1$$



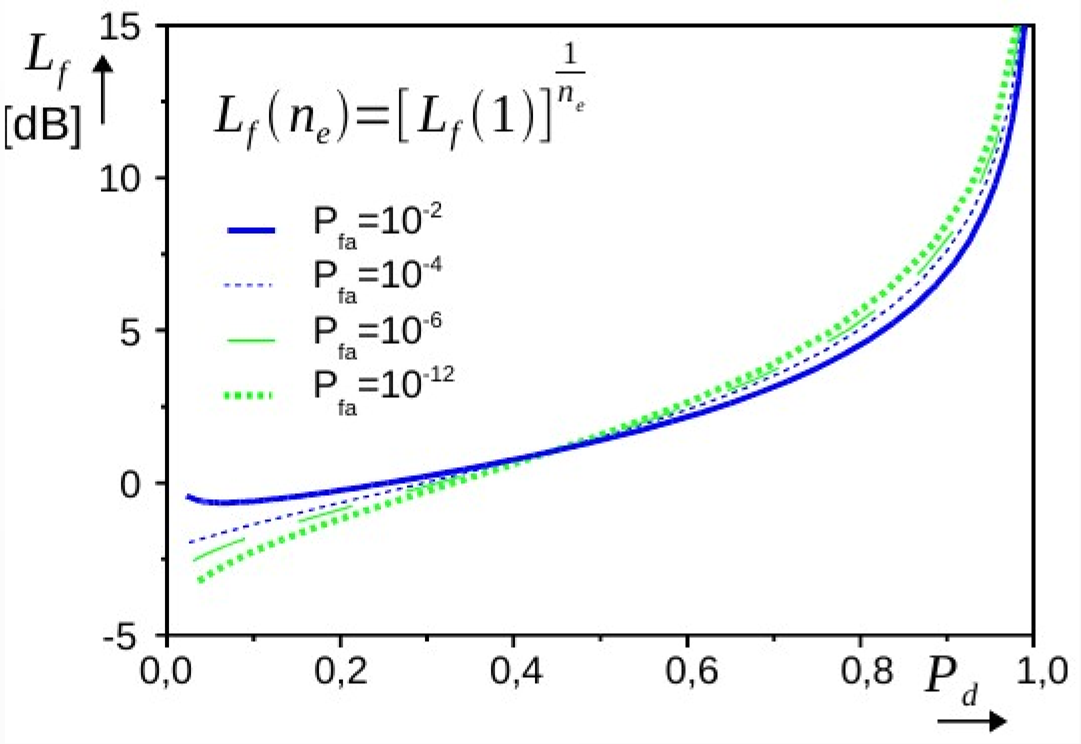


Fluctuation loss

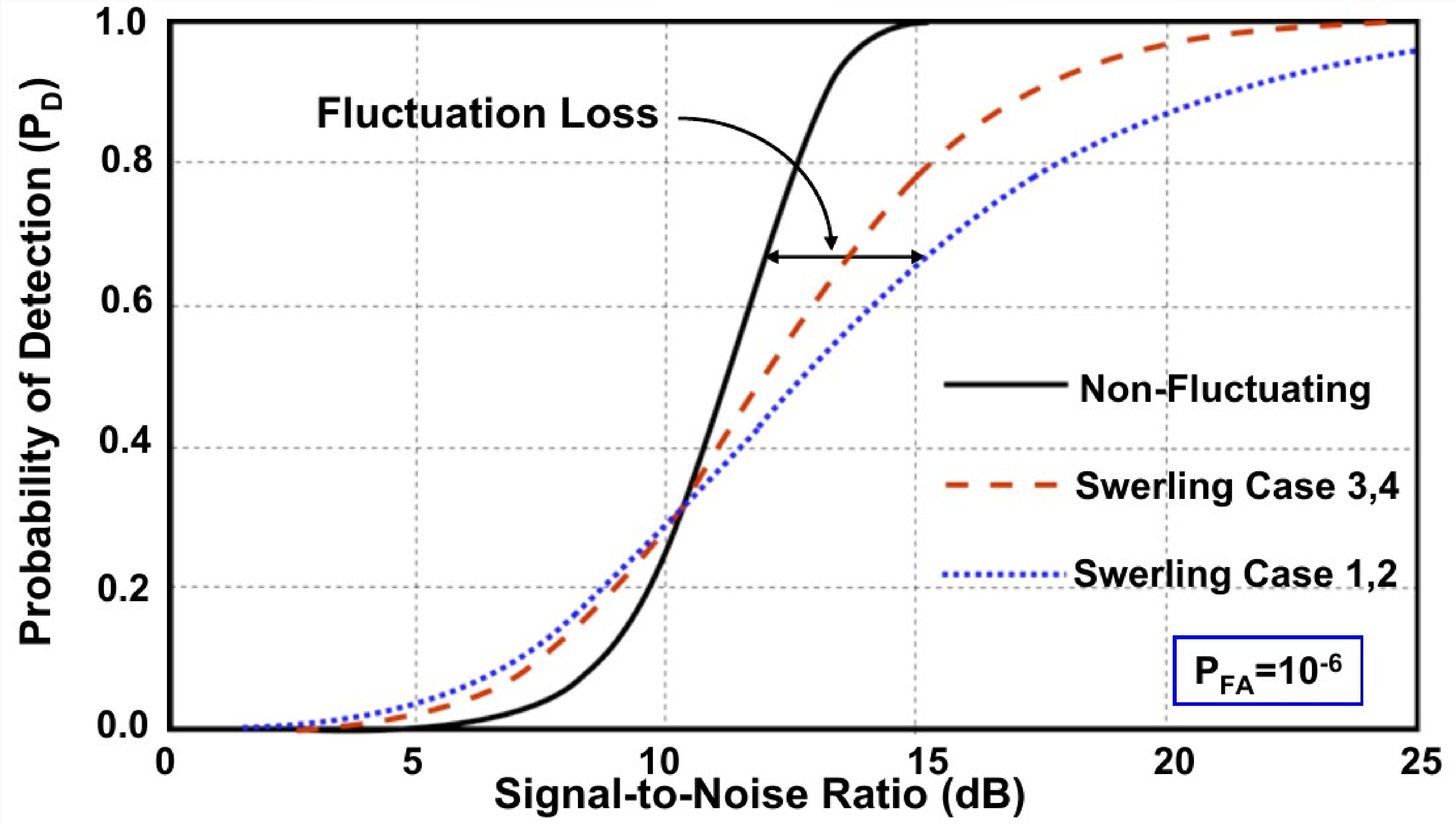
- SNR difference necessary to achieve same P_d for SW0 and (e.g.) SW1

$$L_f = \frac{D_1(1)}{D_0(1)}$$

- Integration of echoes from fluctuating targets
  - Dependent on mutual correlation of samples
- Diversity gain
  - time – observation time is longer than correlation length of samples
  - frequency (detection on more frequencies)

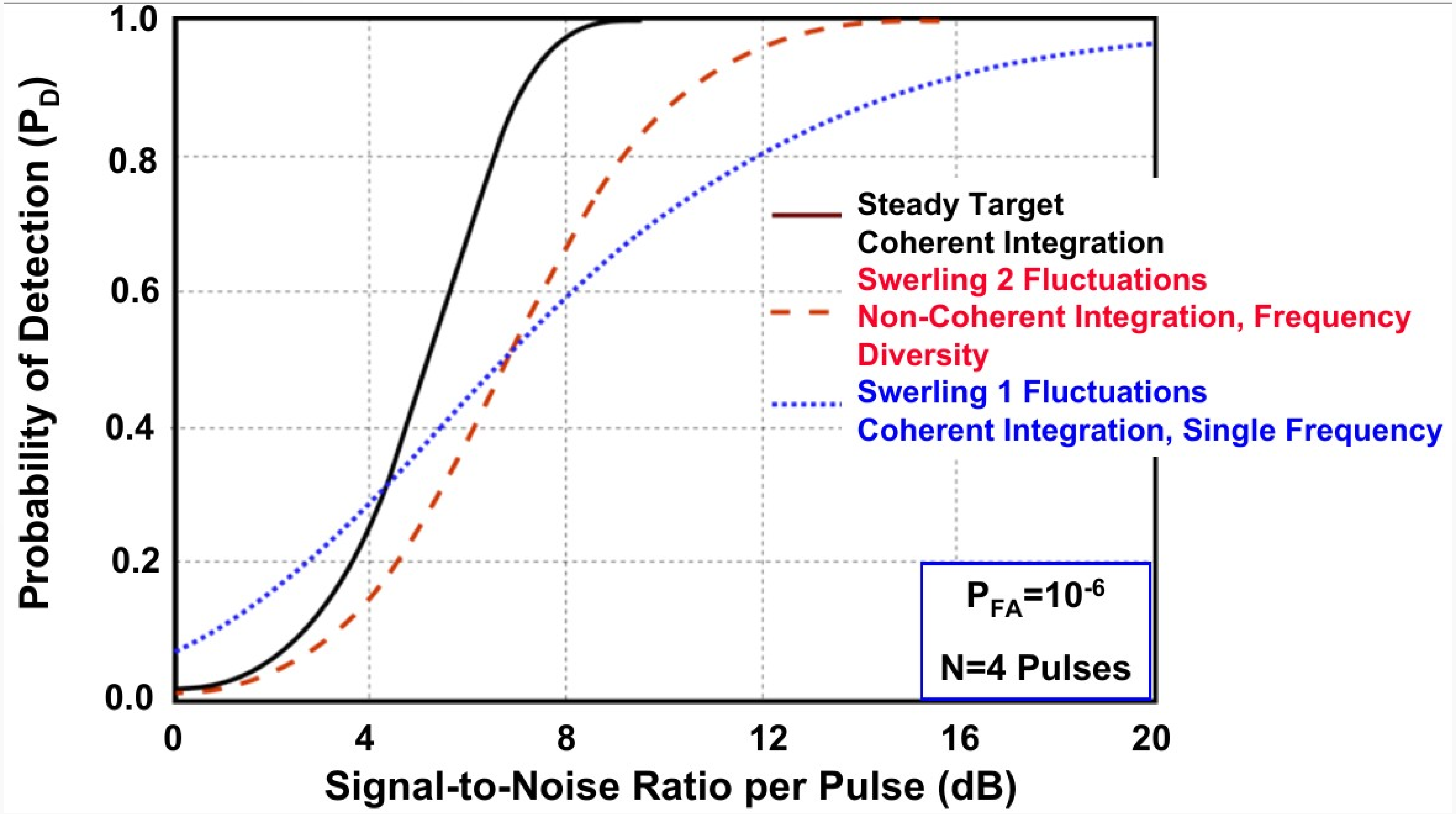


Fluctuation Loss



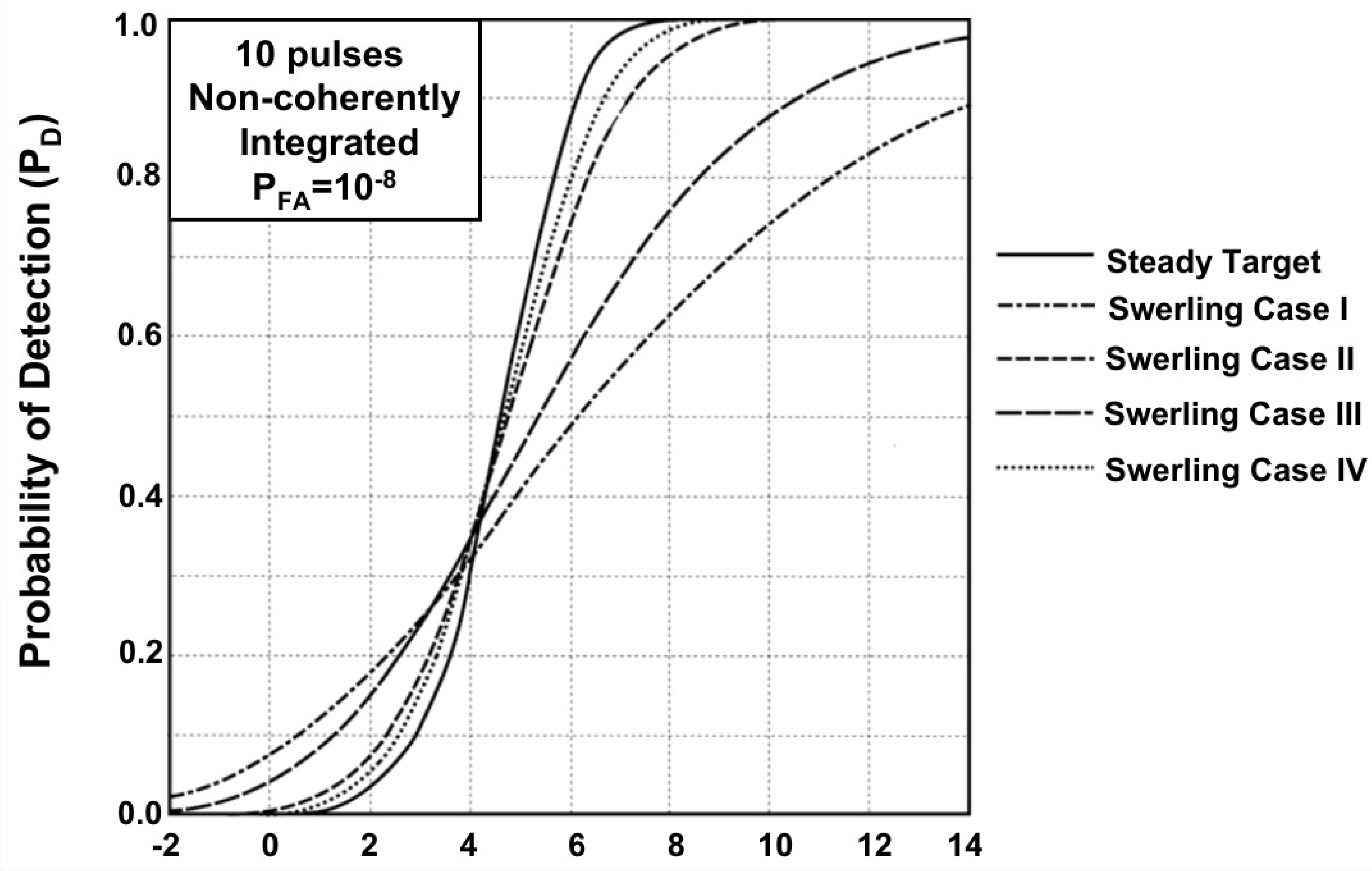
The fluctuation loss depends on the target fluctuations, probability of detection, and probability of false alarm.

Fluctuation Loss – Integration impact



For some targets (fast fluctuating), the non-coherent integration is better

Non-coherent integration for fluctuating targets



SNR [dB]

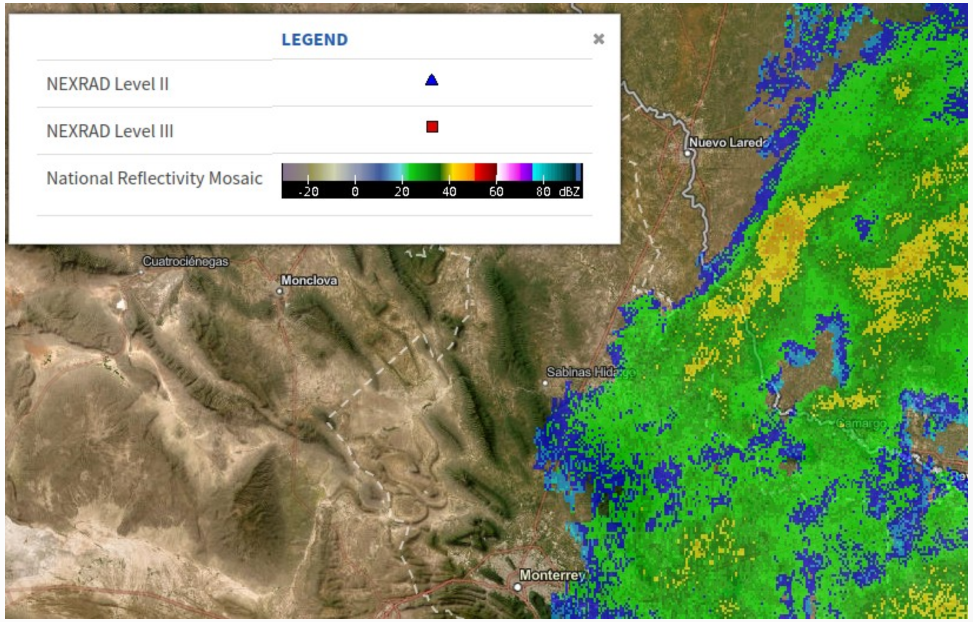
Other Fluctuation Models

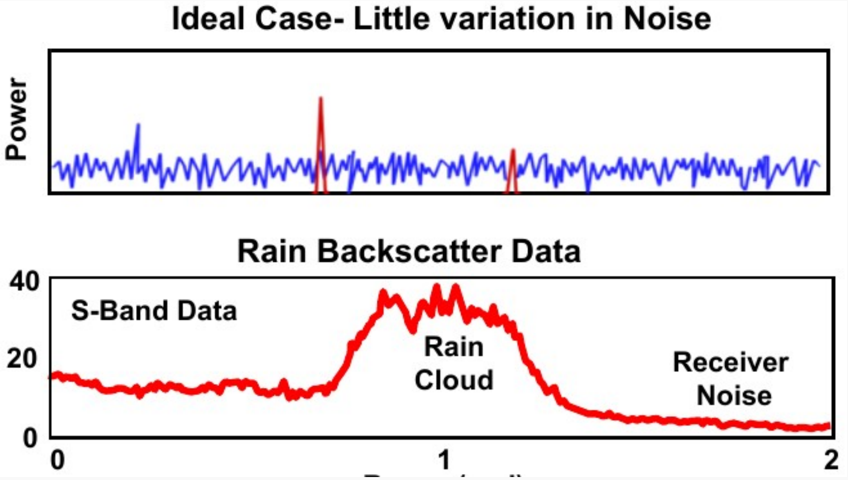
- Detection Statistics Calculations
  - Steady and Swerling 1,2,3,4 Targets in Gaussian Noise
  - Chi- Square Targets in Gaussian Noise
  - Log Normal Targets in Gaussian Noise
  - Steady Targets in Log Normal Noise
  - Log Normal Targets in Log Normal Noise
  - Weibel Targets in Gaussian Noise
- Chi Square, Log Normal and Weibel Distributions have long tails
  - One more parameter to specify distribution
  - Mean to median ratio for log normal distribution
- When used
  - Ground clutter Weibel
  - Sea Clutter - Log Normal
  - HF noise - Log Normal
  - Birds - Log Normal

Adaptive detection - CFAR

Constant False Alarm Rate, CFAR

- Adaptive estimation of the threshold to keep consistent (constant) P_{fa}
- Problems
  - Clutter borders
  - Clutter residuals
  - More targets
  - Interference, jamming
  - High sensitivity to model accuracy (statistics)



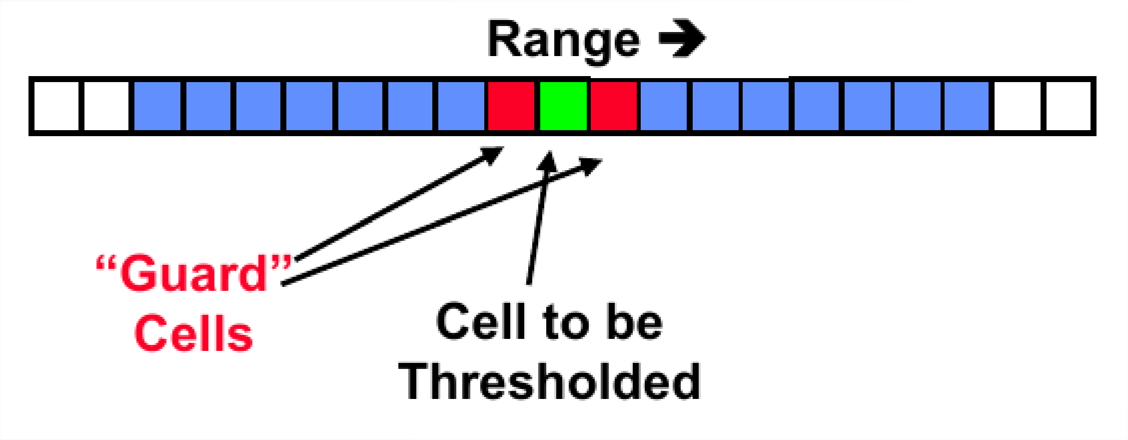


Adaptive detection - CFAR

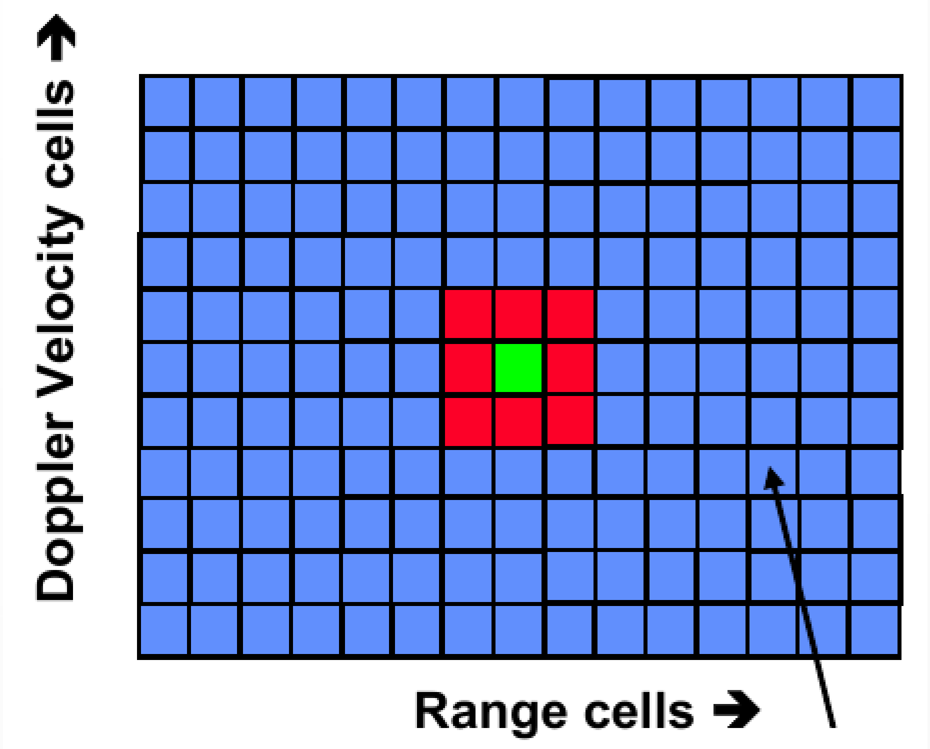
- Realization
  - Parametric – e.g. CA (Cell Averaging) CFAR for Rayleigh
  - Non-parametric – ad hoc structures – logCACFAR, time averaging, median-based detector
- Rayleigh model suitable for
  - Amplitude based AWGN detection
  - Water reflections (vertical polarization)
  - Terrain reflections

CFAR Window

CFAR Window – Range cells



CFAR Window – Range and Doppler cells

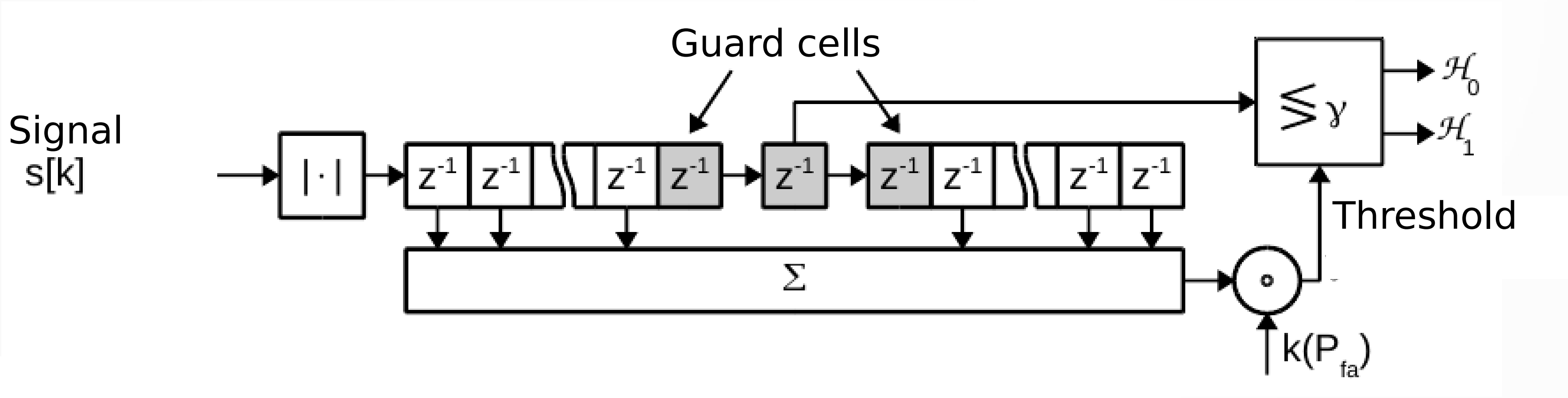


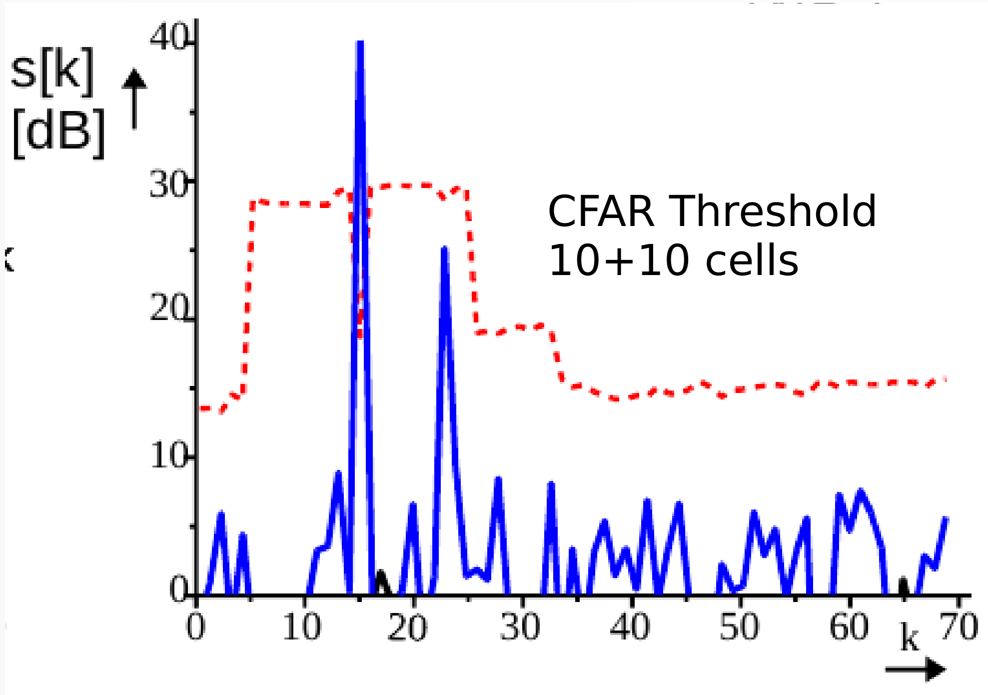
Blue cells for estimation of background and threshold

Estimate background mean from Range data only or Range/Doppler space data

Set threshold as background mean multiplied by a constant (to adjust P_{fa})

Adaptive detection – CA CFAR



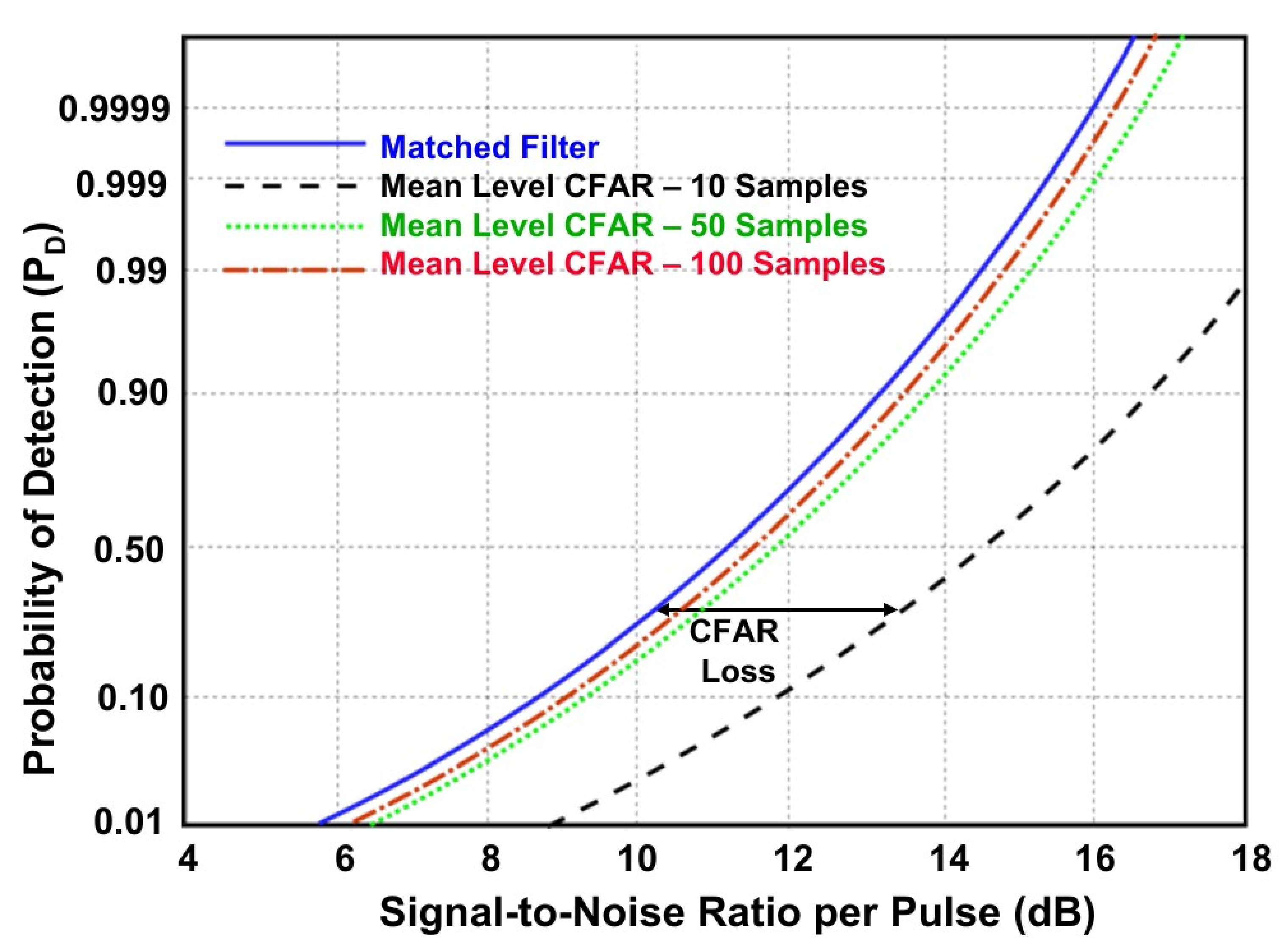


$$E[\xi]=\sigma\frac{\sqrt{\pi}}{2}$$

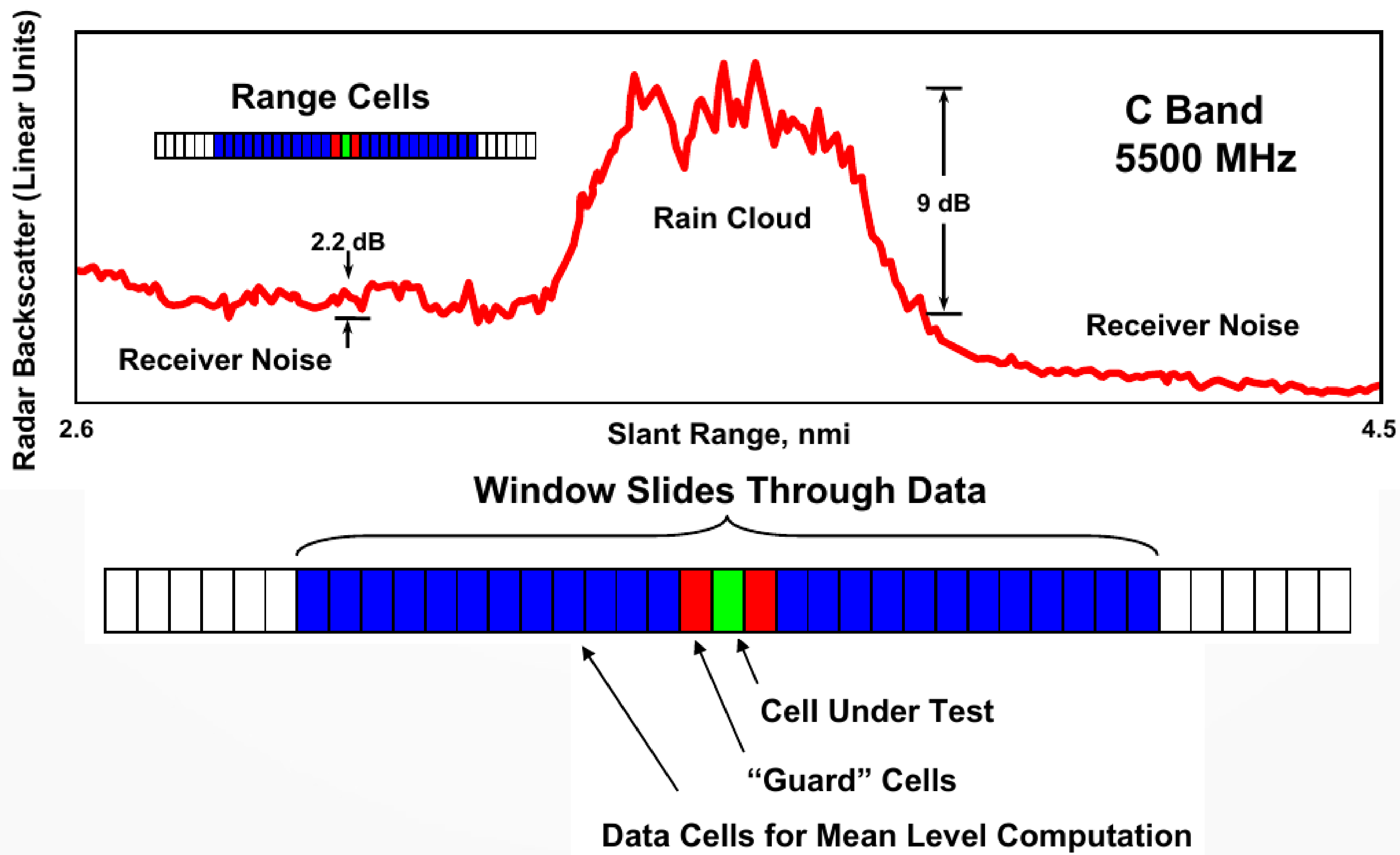
$$P_{fa}=\int_T^{\infty}\frac{\xi}{\sigma^2}e^{\frac{-\xi^2}{2\sigma^2}}d\xi=e^{\frac{-T^2}{2\sigma^2}}$$

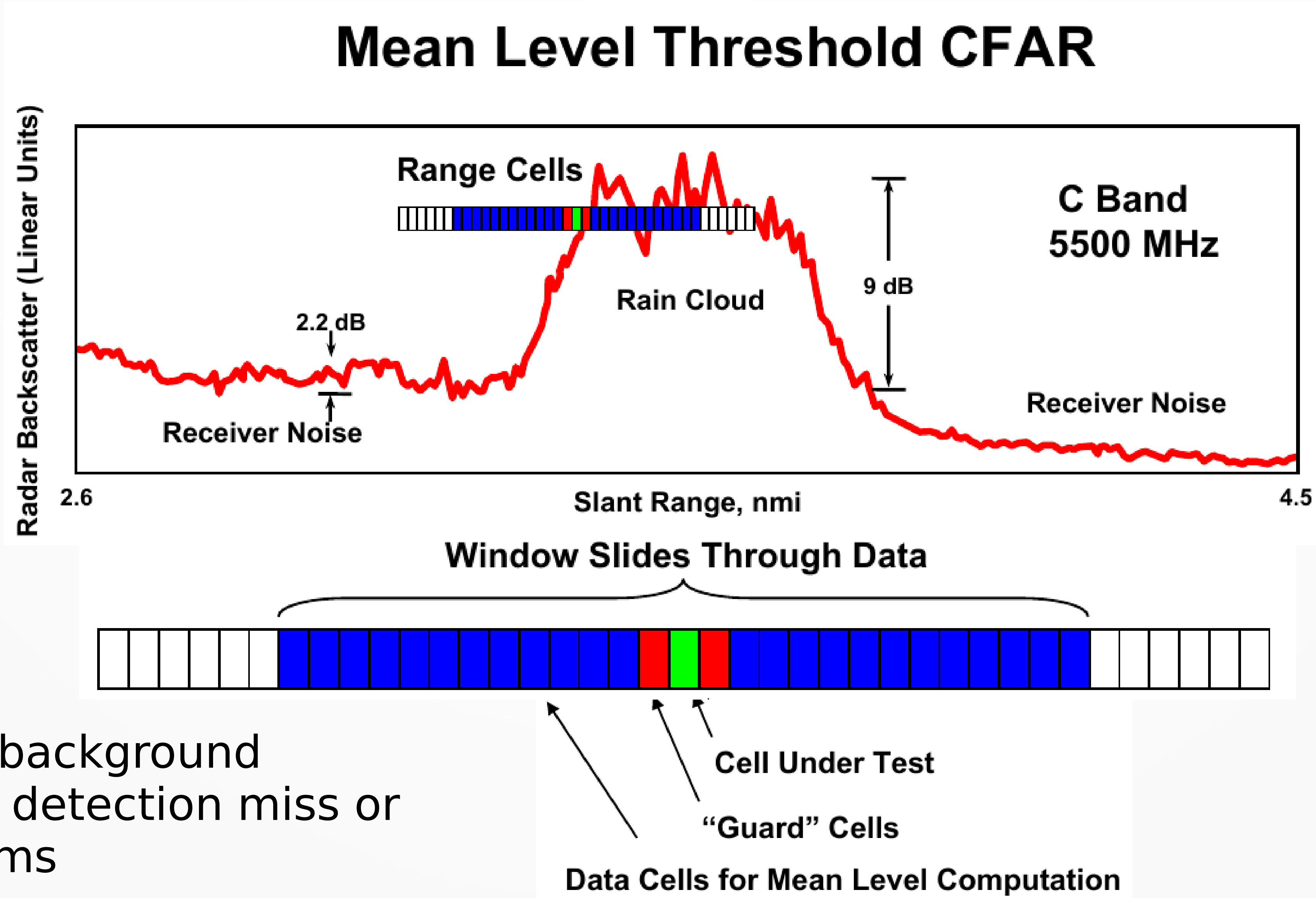
$$T=\sigma\sqrt{-2\ln(P_{fa})}=E[\xi]\sqrt{\frac{-4\ln(P_{fa})}{\pi}}$$

CA CFAR – impact of number of samples (cells)



Mean Level Threshold CFAR

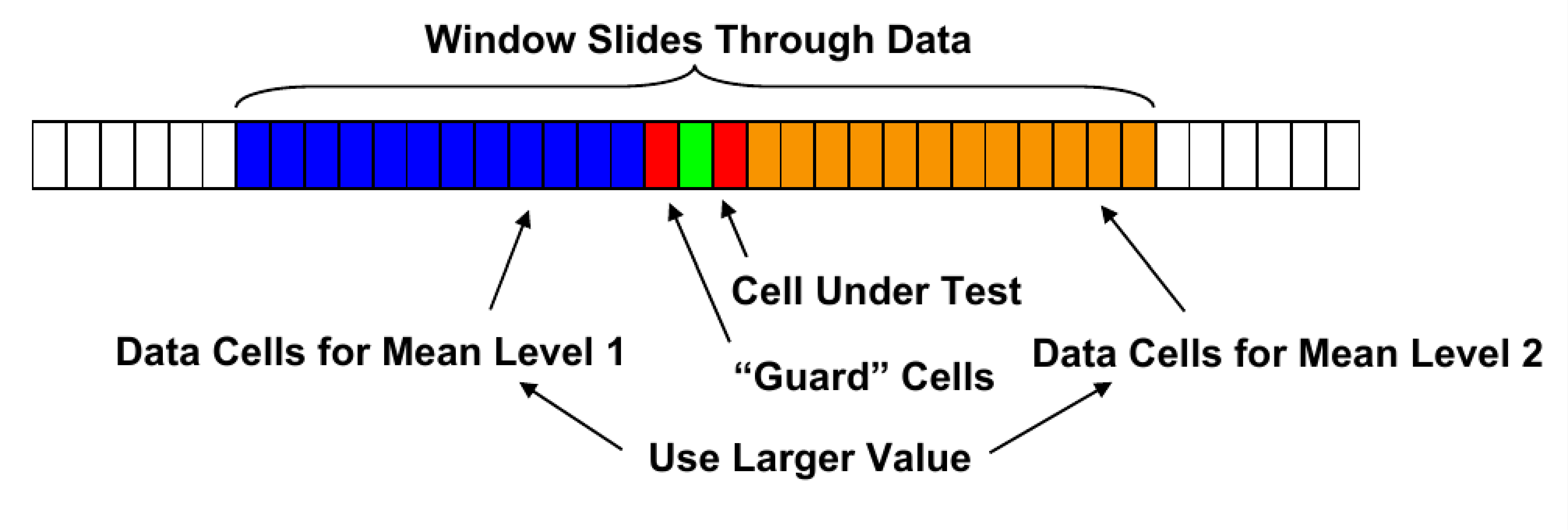




Mean for background increases detection miss or false alarms

Greatest of Means CFAR

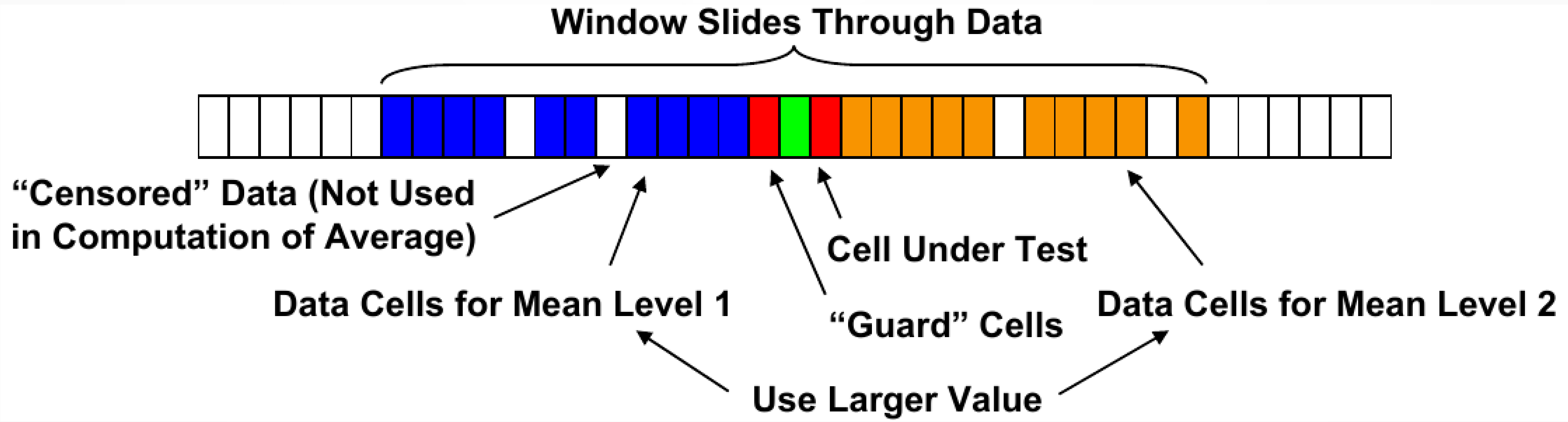
Compare mean value of N/2 cells before and after test cell and use larger value to determine threshold



- Helps reduce false alarms near sharp clutter or interference boundaries
- Nearby targets still raise threshold and suppress detection

Censored Greatest of Means CFAR

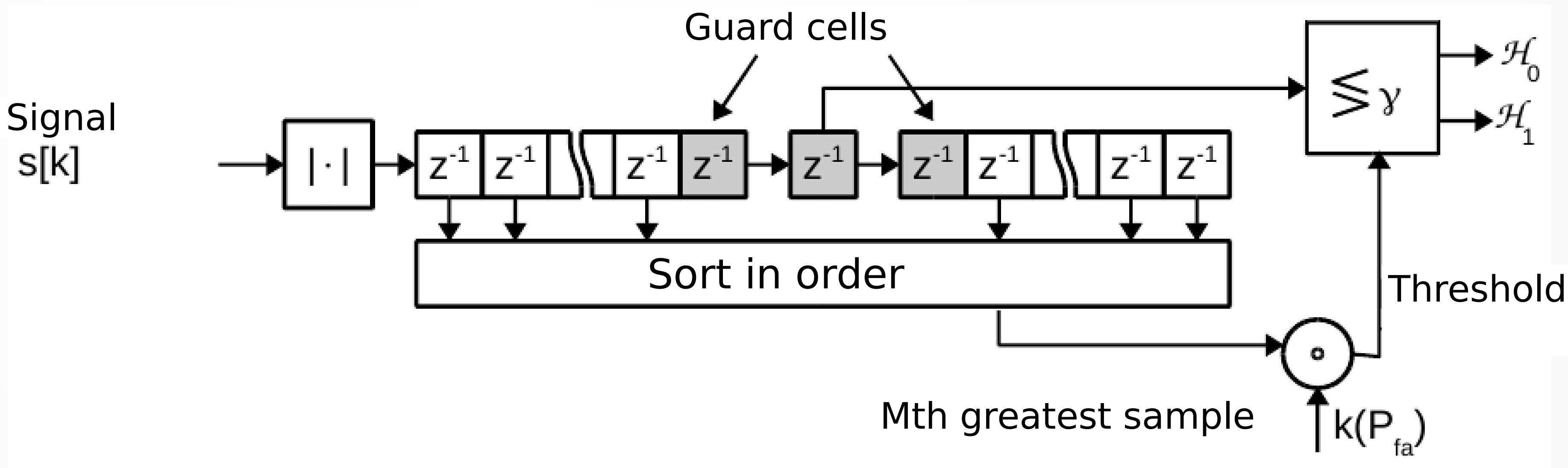
Compute and use noise estimates as in Greatest-of, but remove the largest M samples before computing each average

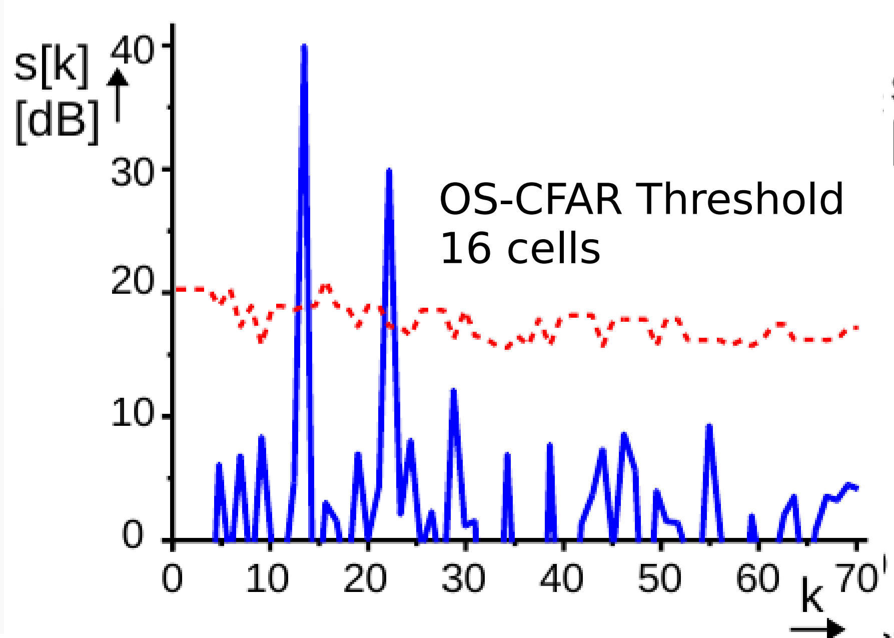


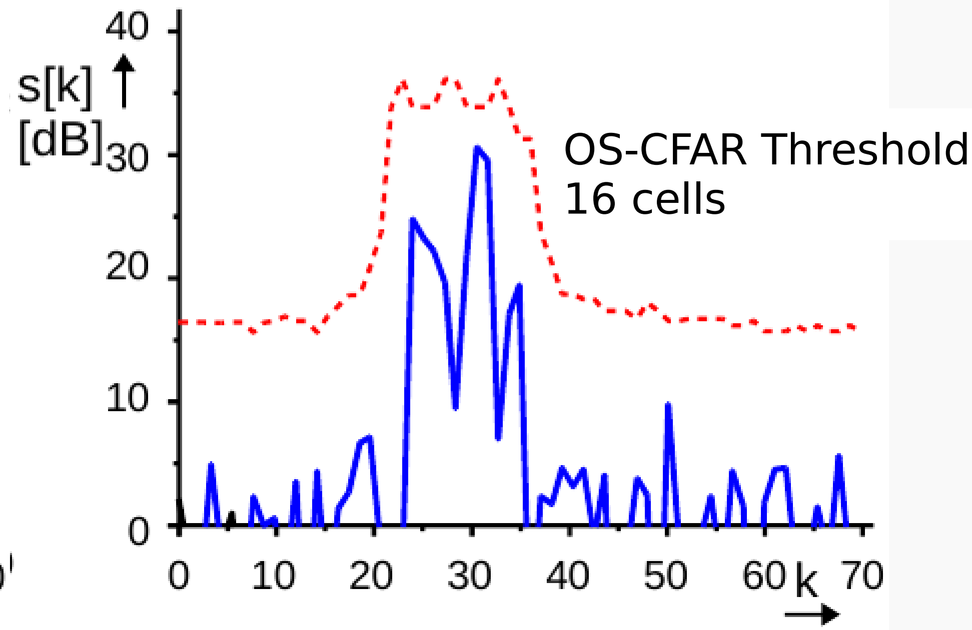
- Up to M nearby targets can be in each window without affecting threshold – dependent on target statistics
- Ordering the samples from each window is more computationally expensive than plain averaging

Adaptive detection – OS CFAR

- Ordered statistics (OS)
- Better in case of multiple targets or at clutter border
- Lower gain







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Thank You!