We prove that (i) and (ii) are equivalent by recalling that γ is a geodesic for ∇, if

$$\frac{d^2\gamma^k}{dt^2} + \Gamma_{ij}^k \frac{d\gamma^i}{dt} \cdot \frac{d\gamma^j}{dt} = 0$$

Correct this on p 32 of your text, if I have overlooked it.

holds. And γ is a geodesic for $\tilde{\nabla}$, if, analogously

$$\frac{d^2\gamma^k}{dt^2} + \tilde{\Gamma}_{ij}^k \frac{d\gamma^i}{dt} \cdot \frac{d\gamma^j}{dt} = 0$$

Above, Γ_{ij}^k and $\tilde{\Gamma}_{ij}^k$ are the Christoffel symbols belonging to ∇ and $\tilde{\nabla}$, respectively.

By the existence and uniqueness theorem for ordinary differential equations, for each $p \in U$ and each $v_p \in T_pM$ there is a geodesic such that $\gamma(0) = p$ and $\left.\frac{d\gamma}{dt}\right|_0 = v_p$. Thus, $\Gamma(v,v) = \tilde{\Gamma}(v,v)$ for all v.

→ You can cite, e.g., the classic book Philip Hartman, Ordinary differential equations, John Wiley & Sons, New York, 1964.