Antenna Arrays

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Antenna Arrays

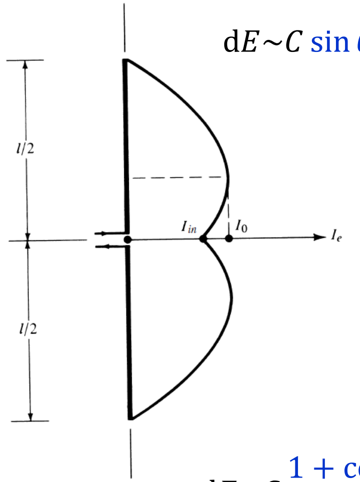
- increase the overall gain
- provide diversity reception, beamforming (MIMO)
- cancel out interference from a particular set of directions
- "steer" the array so that it is most sensitive in a particular direction
- determine the direction of arrival of the incoming signals

- Elementary arrays
- Superdirectivity

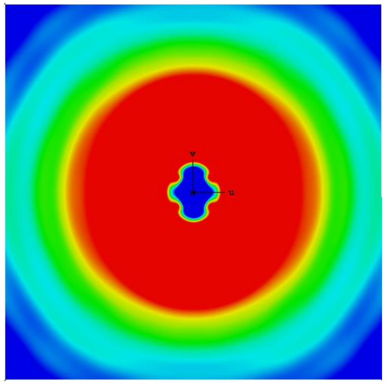
Motivation

- We have seen arrays already…

$$E_\theta(r,\theta,\phi) = \underbrace{\frac{jkZ_0}{4\pi}\frac{e^{-jkr}}{r}\sin\theta}_{\substack{\text{elementary dipole!} \\ \text{(element factor)}}} \underbrace{\int_{-l/2}^{l/2} I_z(z')e^{+jkz'\cos\theta}\,dz'}_{f(\theta)\text{ space (array) factor}}$$

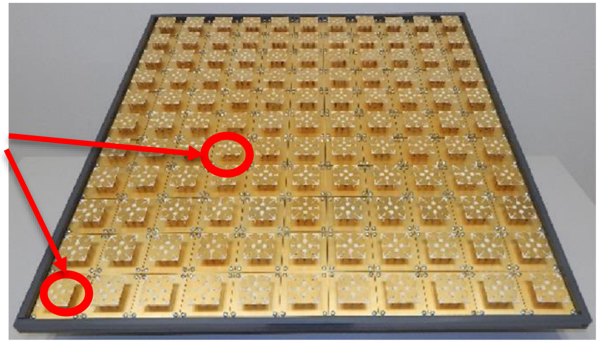


$$\boldsymbol{P}(\theta,\phi) = \iint_S \boldsymbol{E}_a(x',y')e^{jk(x'\sin\theta\cos\phi + y'\sin\theta\sin\phi)}\mathrm{d}x'\mathrm{d}y'$$



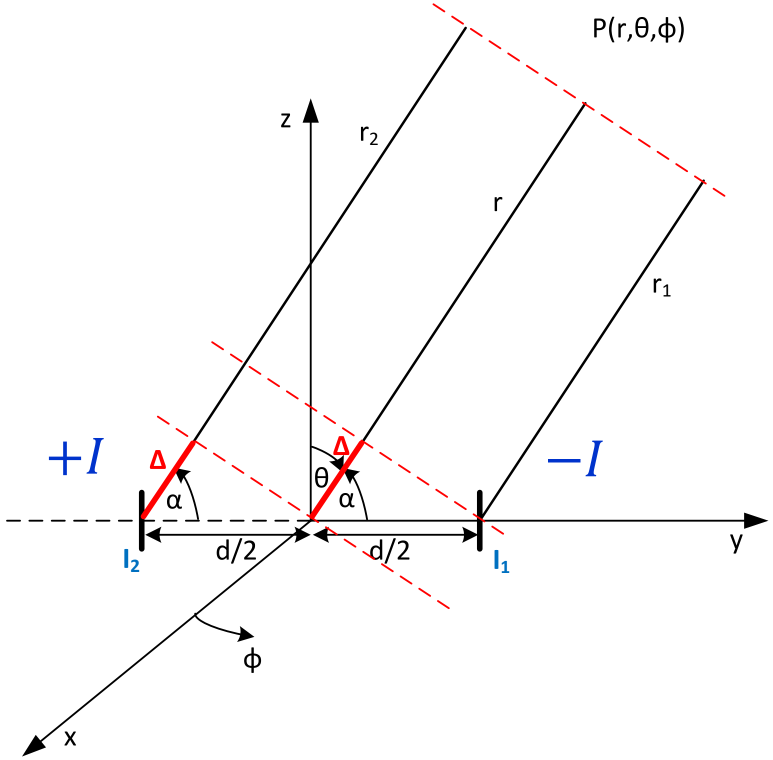
$$E_\theta(\theta,\phi,r) = \frac{j}{\lambda}\frac{e^{-jkr}}{r}\underbrace{\frac{1+\cos\theta}{2}}_{\substack{\text{elementary} \\ \text{Huygens surface} \\ \text{(element factor)}}}\underbrace{(P_x\cos\phi + P_y\sin\phi)}_{f(\theta,\phi)\text{ space (array) factor}}$$

Isolated pattern
Embedded pattern



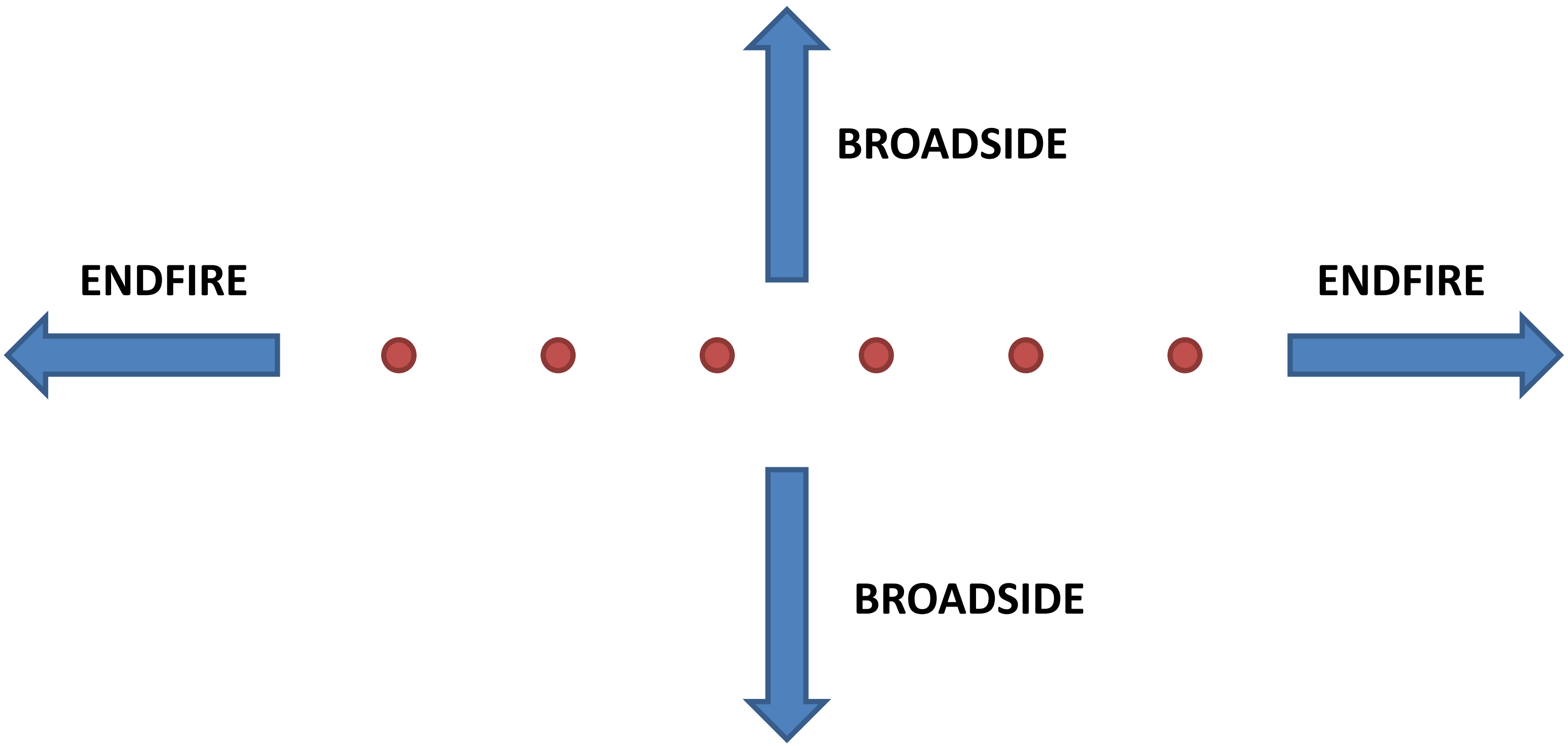
$$\boxed{f_{array} = \mathit{Element\ pattern} \cdot \mathit{Array\ factor}}$$

Motivation – simple array



$$E_\theta(r,\theta,\phi) = 60kIL\frac{e^{-jkr}}{r}\sin\theta\sin\left(\frac{1}{2}kd\sin\theta\sin\phi\right) = C\frac{e^{-jkr}}{r}\mathrm{ElementPattern}\cdot\mathrm{ArrayFactor}$$

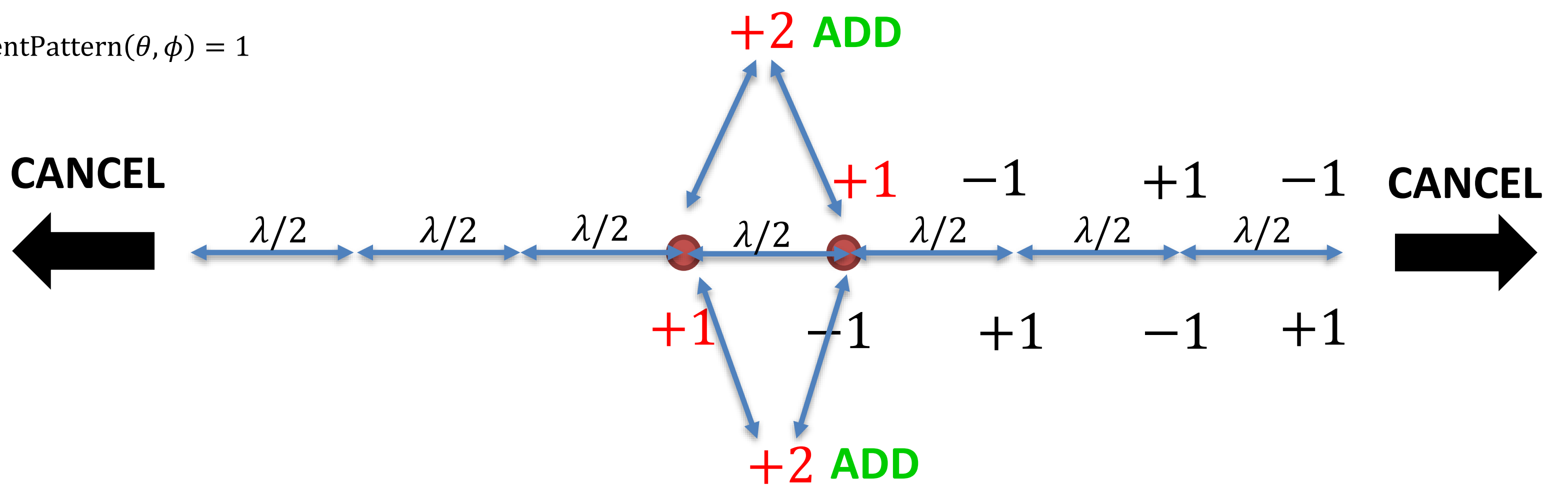
Broadside x Endfire



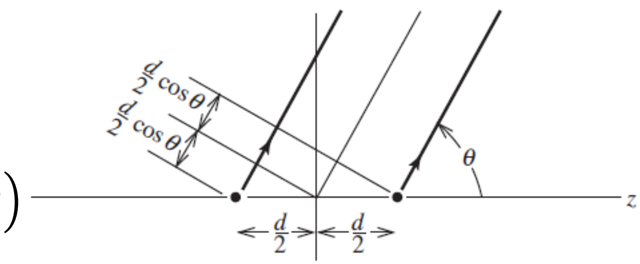
Inspection method (Stutzman)

Isotropic radiators λ/2 spaced – In-phase excitation

$\mathrm{ElementPattern}(\theta,\phi) = 1$



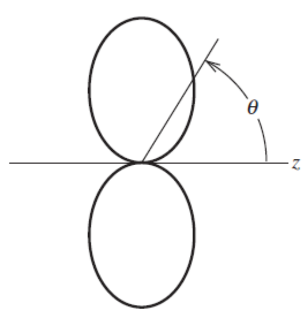
$$AF = 1e^{-jk\frac{d}{2}\cos\theta} + 1e^{jk\frac{d}{2}\cos\theta} = 2\cos\left(k\frac{d}{2}\cos\theta\right)$$

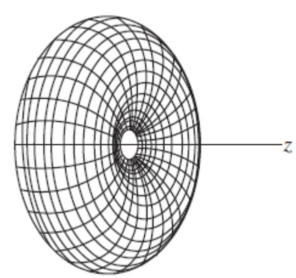


$d = \lambda/2$

$$AF = 2\cos\left(\frac{\pi}{2}\cos\theta\right) \xrightarrow{\text{norm. to 1}} f(\theta) = \cos\left(\frac{\pi}{2}\cos\theta\right)$$

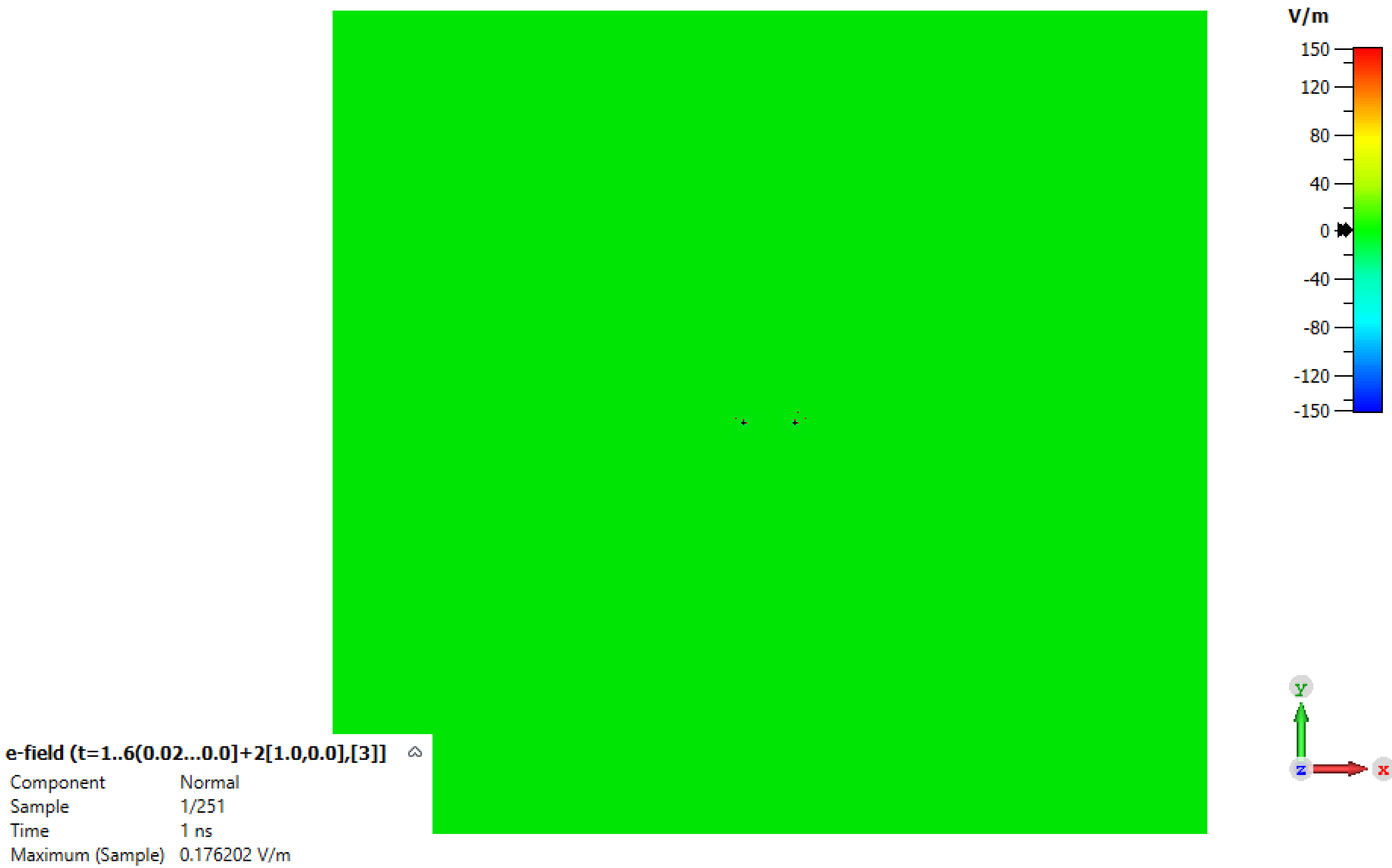
$k = 2\pi/\lambda$





Inspection method (Stutzman)

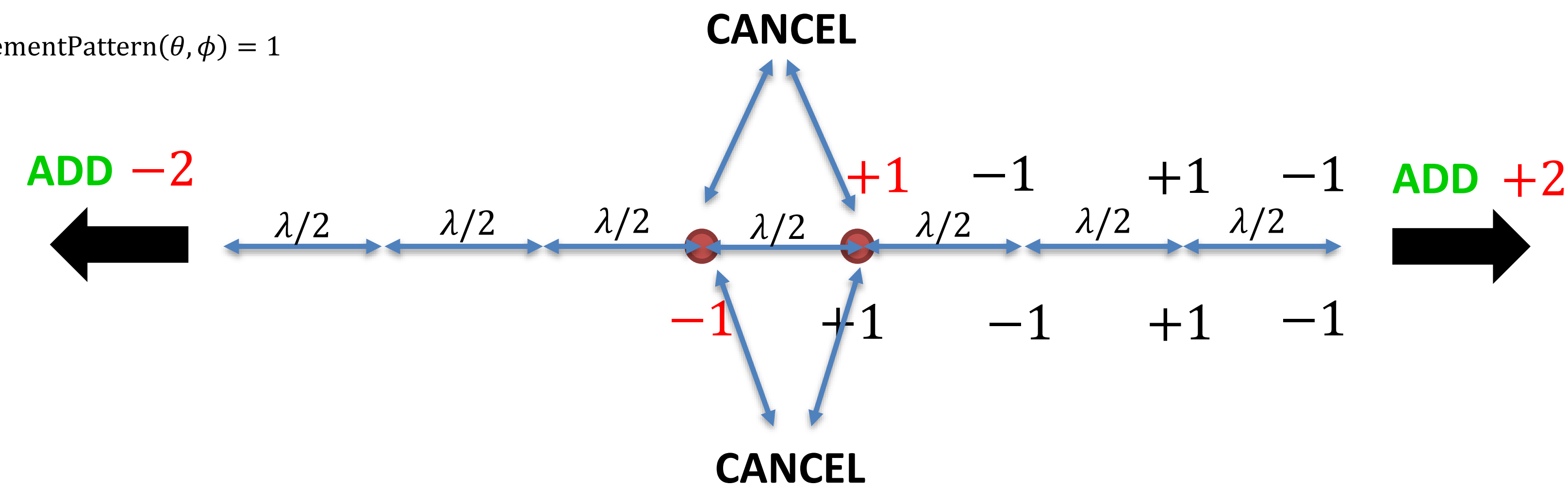
Isotropic radiators λ/2 spaced – In-phase excitation



Inspection method (Stutzman)

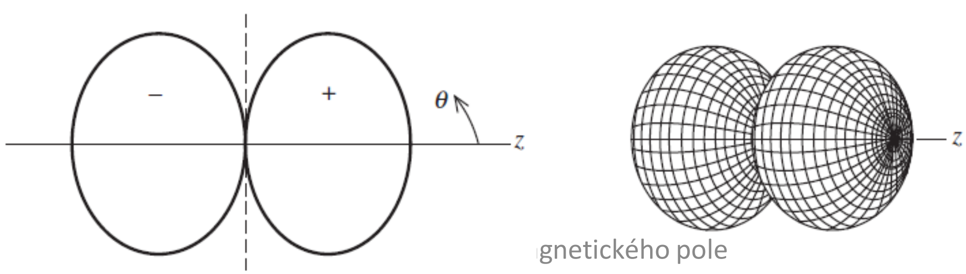
Isotropic radiators λ/2 spaced – Out-of-phase-phase excitation

$\text{ElementPattern}(\theta, \phi) = 1$



$$AF = -1e^{jk\frac{d}{2}\cos\theta} + 1e^{jk\frac{d}{2}\cos\theta} = 2j\sin\left(k\frac{d}{2}\cos\theta\right)$$

$$d = \lambda/2 \qquad AF = 2j\sin\left(\frac{\pi}{2}\cos\theta\right) \longrightarrow f(\theta) = \sin\left(\frac{\pi}{2}\cos\theta\right)$$



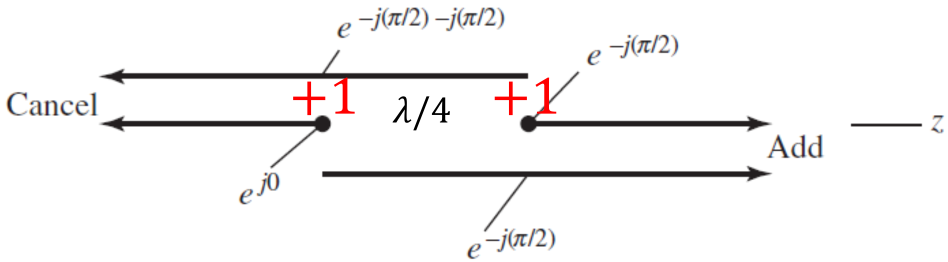
Inspection method (Stutzman)

Isotropic radiators λ/2 spaced – Out-of-phase excitation

Inspection method (Stutzman)

Isotropic radiators λ/4 spaced – phase 90 degrees

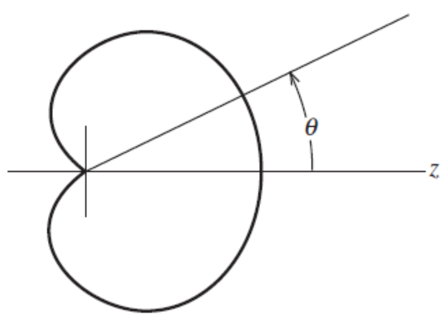
$\mathrm{ElementPattern}(\theta, \phi) = 1$

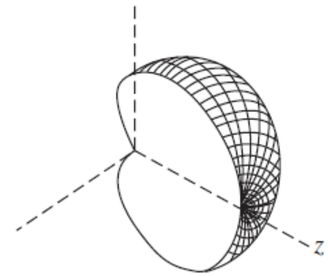


$$AF = 1e^{-jk\frac{d}{2}\cos\theta} + 1e^{-j\frac{\pi}{2}}e^{jk\frac{d}{2}\cos\theta} = e^{-j\frac{\pi}{4}}\,2\cos\left(k\frac{d}{2}\cos\theta - \frac{\pi}{4}\right)$$

$$d = \lambda/4 \longrightarrow f(\theta) = \cos\left(\frac{\pi}{2}\cos\theta - \frac{\pi}{4}\right)$$

Cardidoid





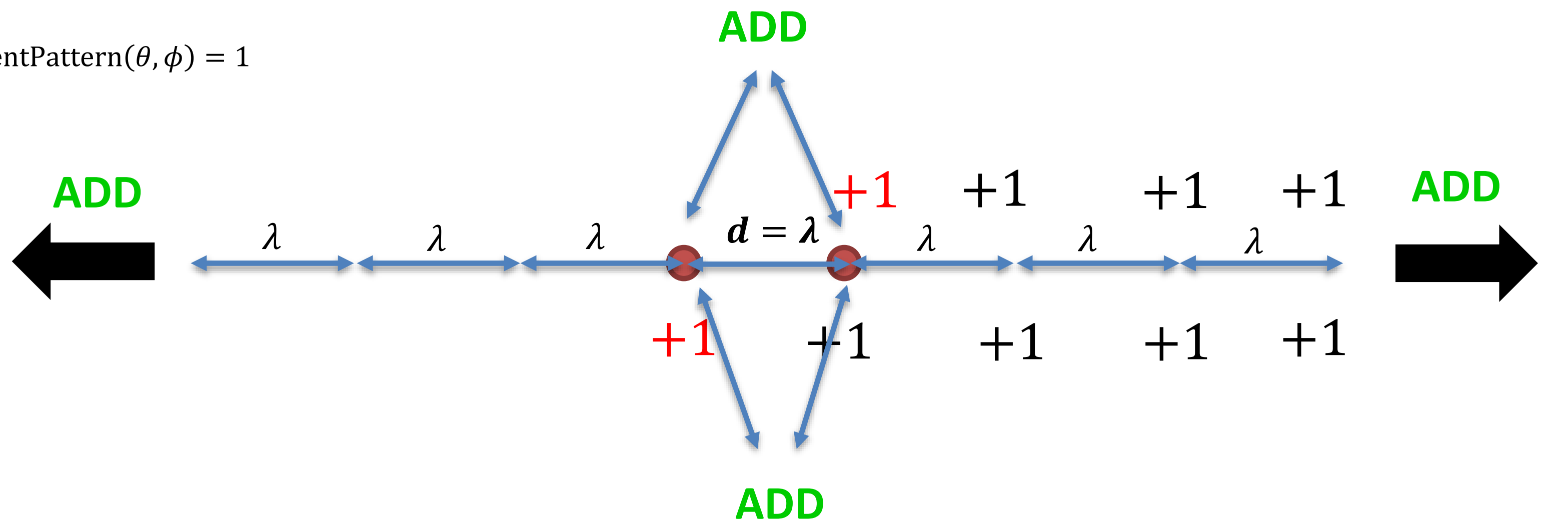
Inspection method (Stutzman)

Isotropic radiators λ/4 spaced – phase 90 degrees

Inspection method (Stutzman)

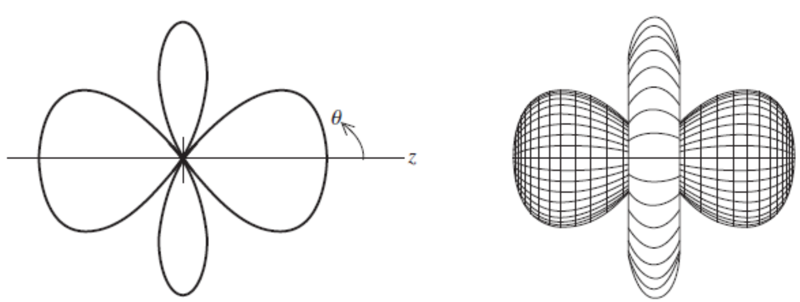
Isotropic radiators λ spaced – In-phase excitation $d > \lambda/2 \rightarrow$ more lobes (grating lobes)

$\text{ElementPattern}(\theta, \phi) = 1$



$$f(\theta) = \cos\left(k\frac{d}{2}\cos\theta\right) = \cos(\pi\cos\theta)$$

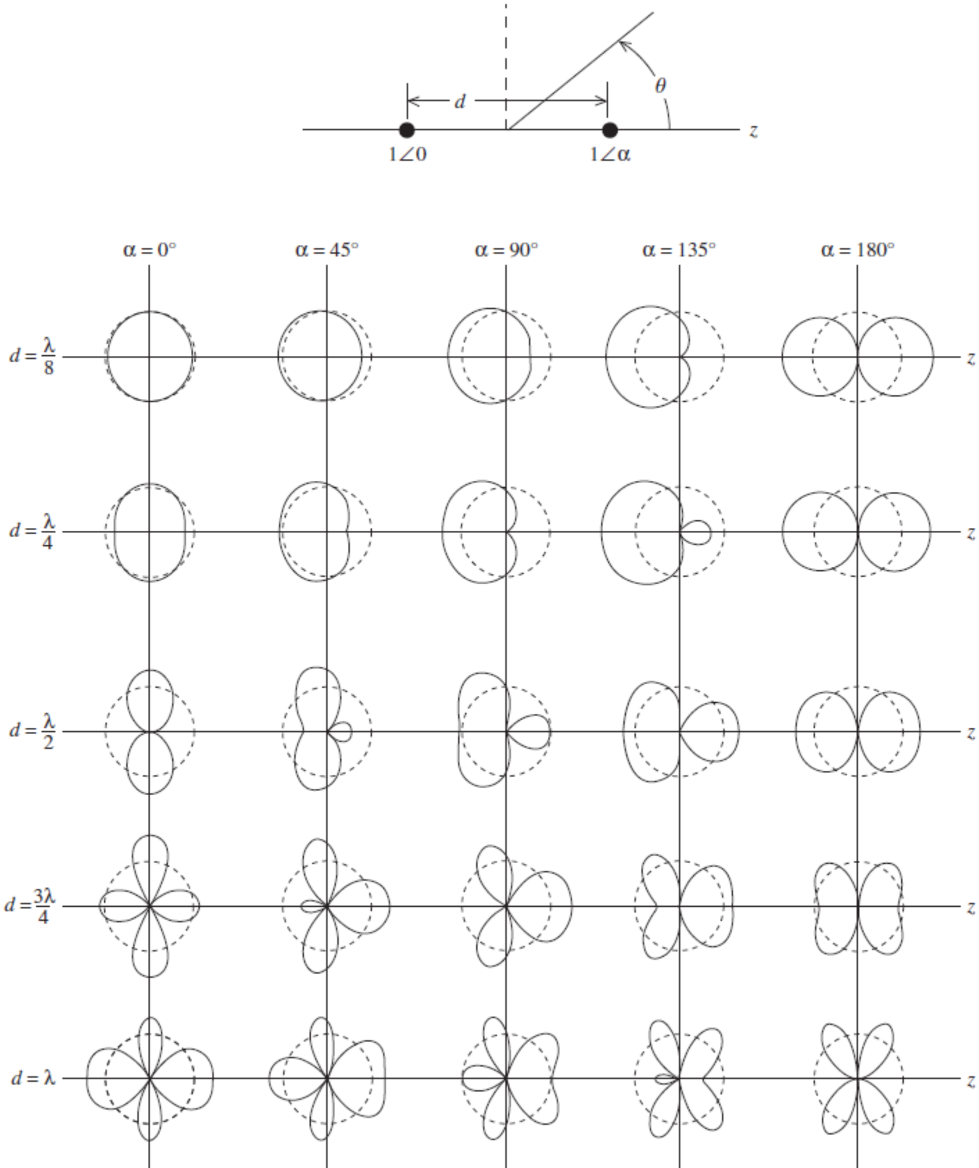
Nulls for $\theta = 60{,}120..$ degrees



Inspection method (Stutzman)

Isotropic radiators λ spaced – In-phase excitation

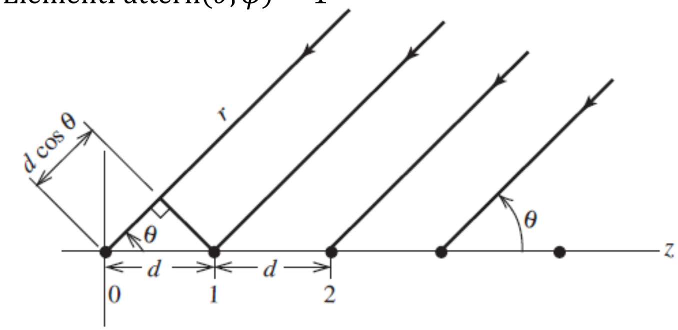
Two isotropic radiators



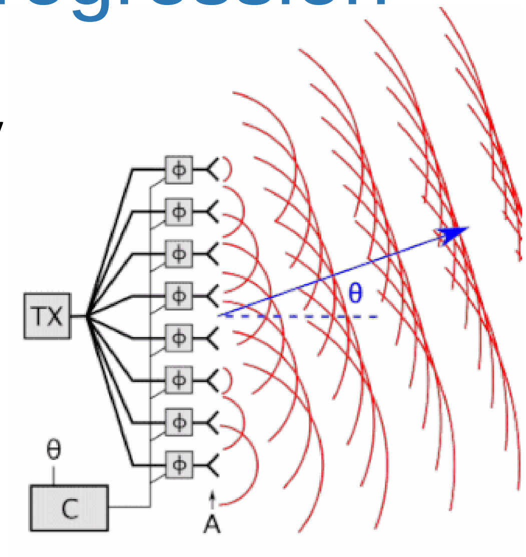
- $d > \lambda/2$ more lobes… (grating lobes)
- Array spacing usually $< \lambda/2$

Phased arrays – linear phase progression

$\mathrm{ElementPattern}(\theta, \phi) = 1$



equally spaced isotropic array



$$AF = I_0 + I_1 e^{jkd\cos\theta} + I_2 e^{jk2d\cos\theta} + \cdots = \sum_{n=0}^{N-1} I_n e^{jknd\cos\theta}$$

If current has linear phase progression

$$I_n = A_n e^{jn\alpha} \Rightarrow AF = \sum_{n=0}^{N-1} A_n e^{jn\overbrace{(kd\cos\theta+\alpha)}^{\psi}} = \sum_{n=0}^{N-1} A_n e^{jn\psi}$$

$\psi = kd\cos\theta + \alpha$

nonlinear

Uniform array (same amplitudes) $A_0 = A_1 = A_2 = \cdots$

amplitude

$$AF = A_0 \sum_{n=0}^{N-1} e^{jn\psi} = \underbrace{A_0\left(1 + e^{j\psi} + \cdots + e^{j(N-1)\psi}\right)} = A_0 \frac{1 - e^{jN\psi}}{1 - e^{j\psi}} = A_0 e^{j(N-1)\psi/2} \frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

AF maximum for $\psi = 0$ $\quad AF(\psi = 0) = A_0 N \Rightarrow f(\psi) = \dfrac{AF(\psi)}{AF(\psi = 0)} = \dfrac{\sin(N\psi/2)}{N\sin(\psi/2)}$

...periodic with 2π (ψ=0..2π) → visible region

Phased arrays – scanning

Scan angle

$$AF = \sum_{n=0}^{N-1} A_n e^{jn(kd\cos\theta+\alpha)} = \sum_{n=0}^{N-1} A_n e^{jn\psi}$$

Maximum of $AF(\psi)$ occurs for $\psi = 0$ $\Rightarrow$ $\psi = kd\cos\theta_0 + \alpha$

$$\alpha = -kd\cos\theta_0$$

Required phasing

Broadside array $\theta_0 = \pm 90°$, $\alpha = 0$

Endfire arrays more interesting in terms of directivity

Endfire array (ordinary) $\theta_0 = 0°$ or $180°$, $\alpha = \pm kd$ $\quad d < \frac{\lambda}{2}\left(1 - \frac{1}{2N}\right)$

2.94

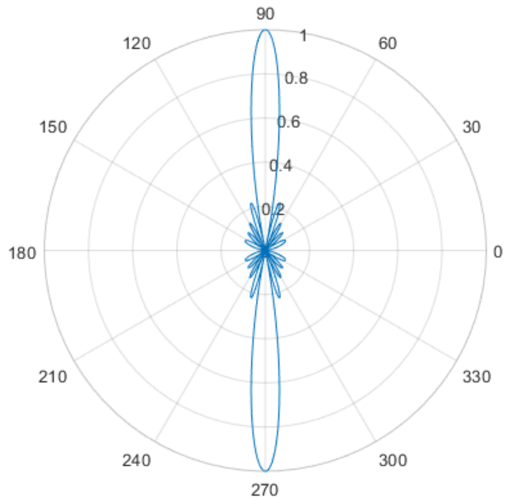
Hansen-Woodyard Endfire array (increased dir.) $\theta_0 = 0°$ or $180°$, $\alpha = \pm\left(kd + \frac{\pi}{N}\right)$ $\quad d < \frac{\lambda}{2}\left(1 - \frac{1}{N}\right)$

Superdirective Endfire array (increased dir.) $\theta_0 = 0°$ or $180°$, α optimized $\quad d \to 0$

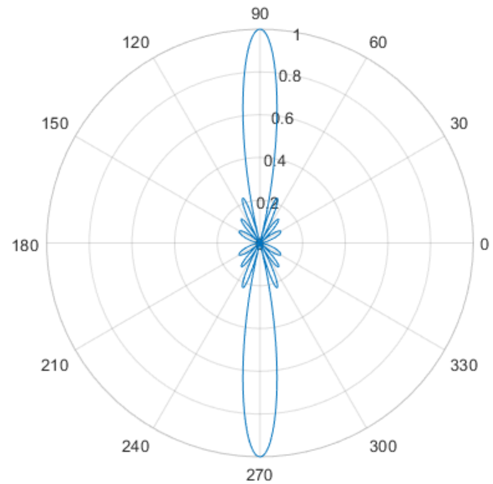
Uzkov limit for isotropic radiators: $D = N^2$ as $d \to 0$ requires unique phase for each separation

Phased arrays – N=10 isotropic elements

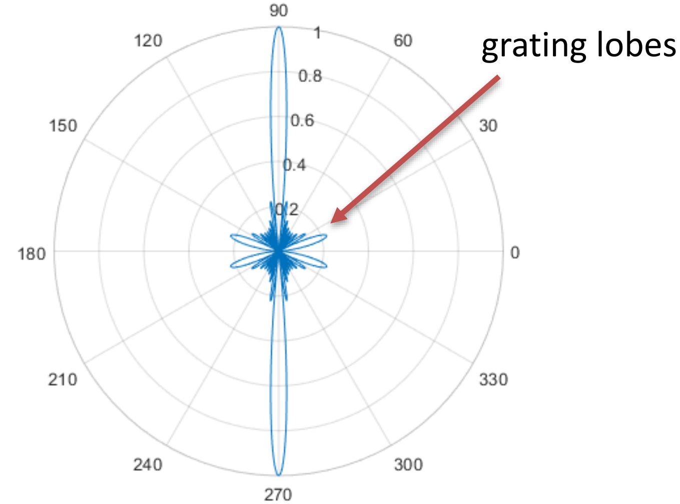
Broadside array $d = 0.5\,\lambda$



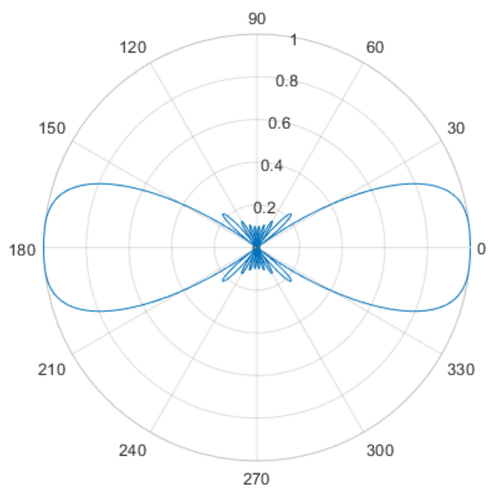
Broadside array $d = 0.4\,\lambda$



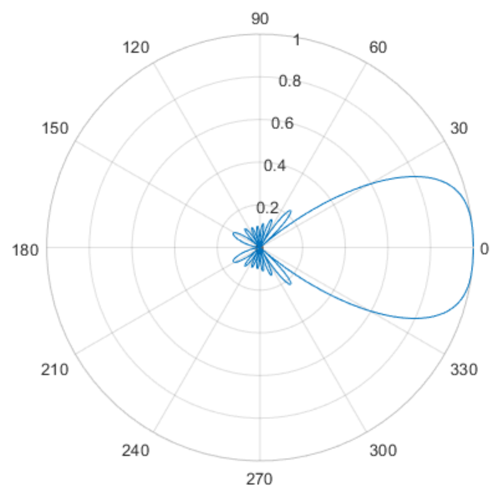
Broadside array $d = 0.9\,\lambda$



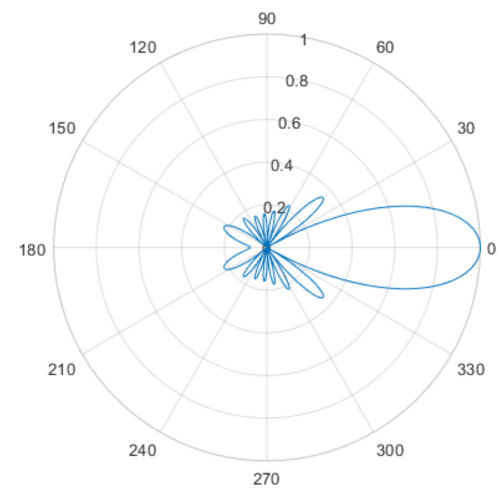
Endfire array (ordinary) $d = 0.5\,\lambda$



Endfire array (ordinary) $d = 0.4\,\lambda$

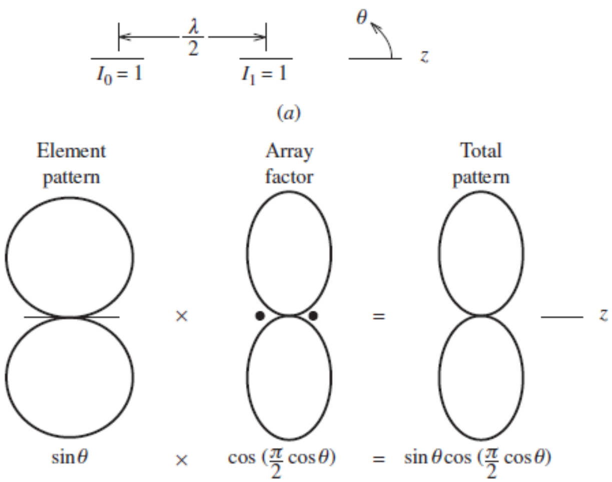


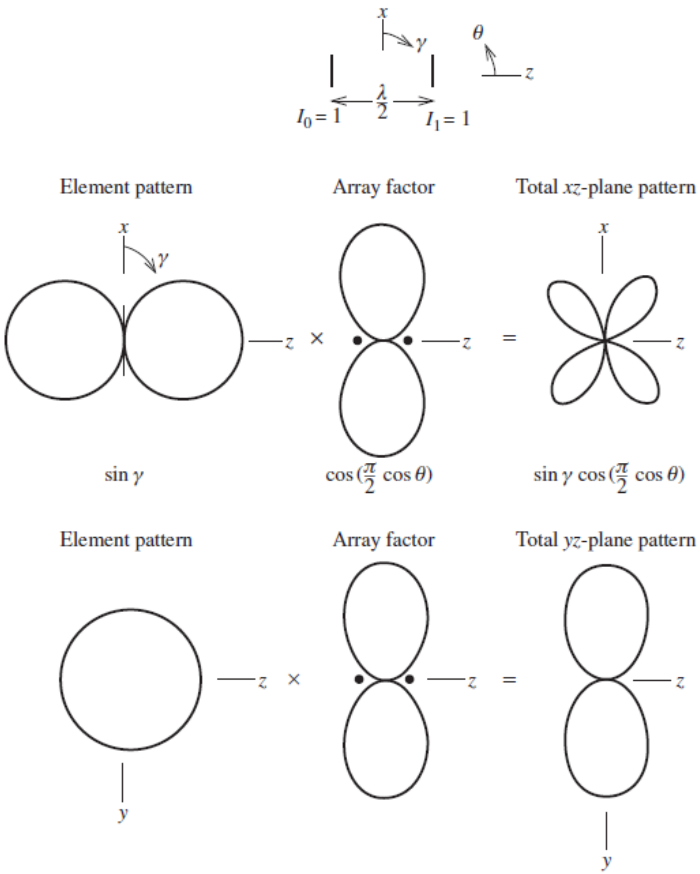
Endfire array (H-WD increased dir.) $d = 0.4\,\lambda$

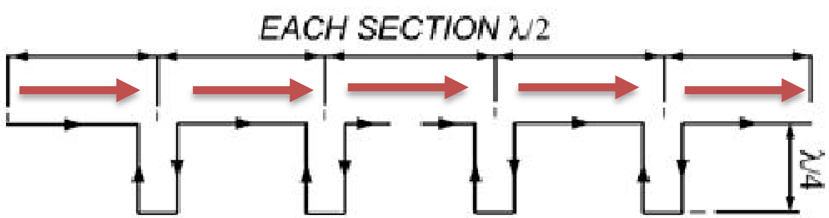


Non-ISOTROPIC: Array of two dipoles

Same geometry and feeding (+I, +I) → same AF but different dipole orientation

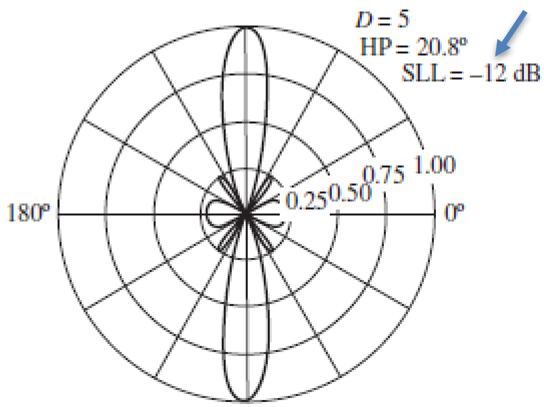






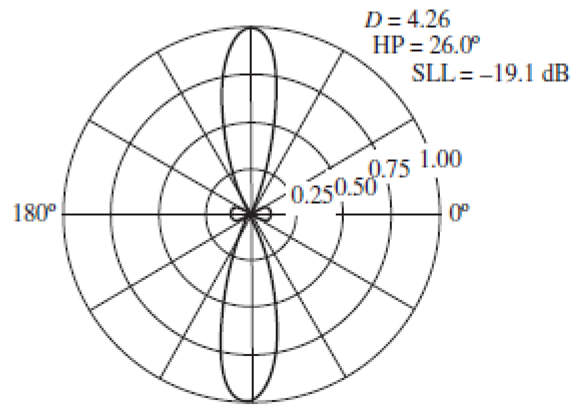
Franklin collinear array

Amplitude taper (non-uniform excitation)



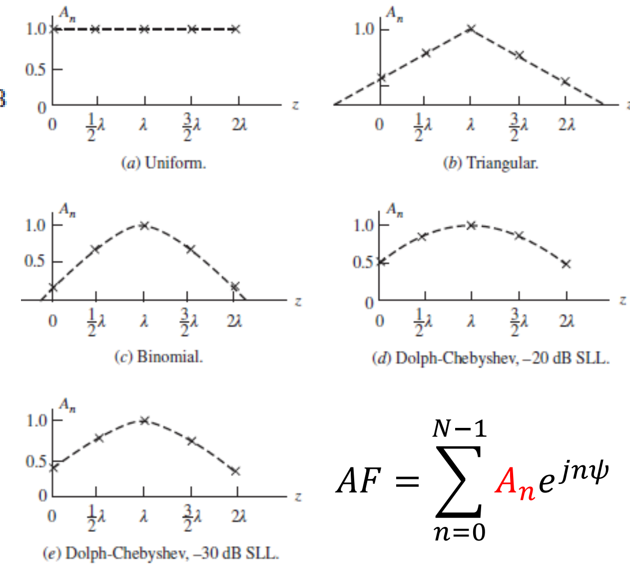
(a) Uniform.

1: 1: 1: 1: 1

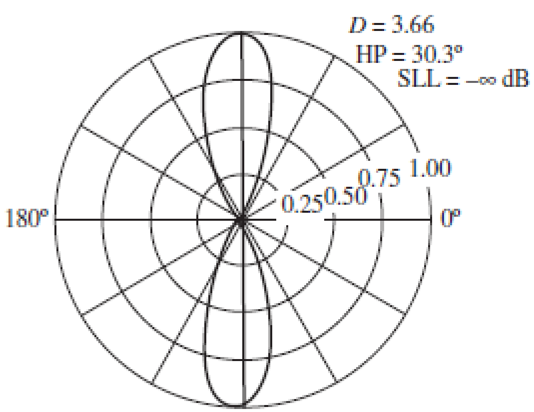


(b) Triangular.

1: 2: 3: 2: 1

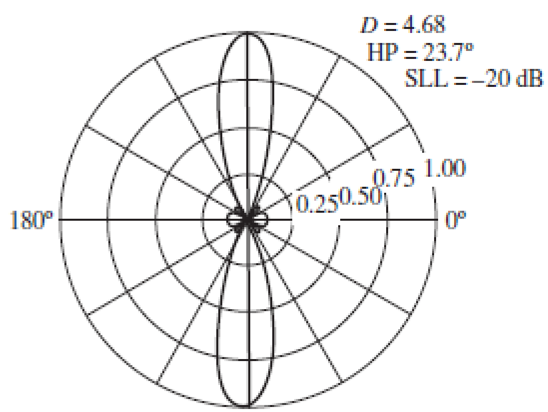


$$AF = \sum_{n=0}^{N-1} A_n e^{jn\psi}$$



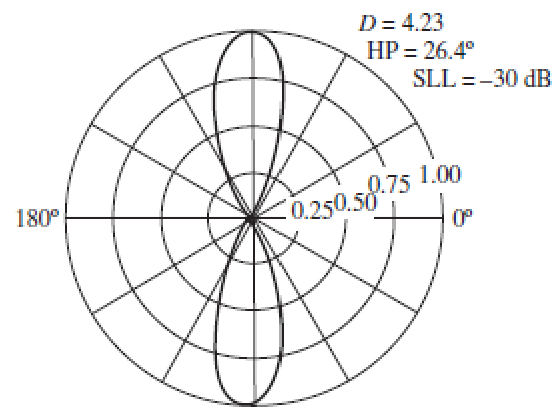
(c) Binomial.

1: 4: 6: 4: 1



(d) Dolph-Chebyshev, –20 dB SLL.

1: 1.61: 1.94: 1.61: 1



(e) Dolph-Chebyshev, –30 dB SLL.

1: 2.41: 3.14: 2.41: 1

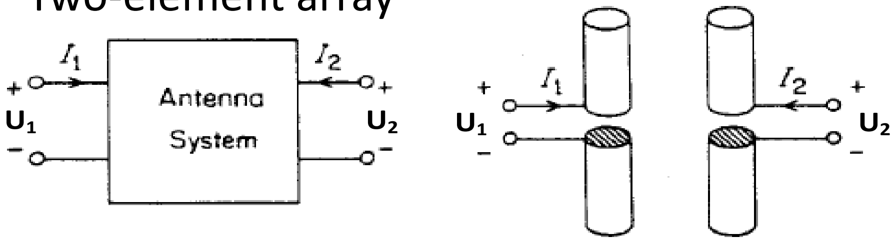
Pascal triangle…

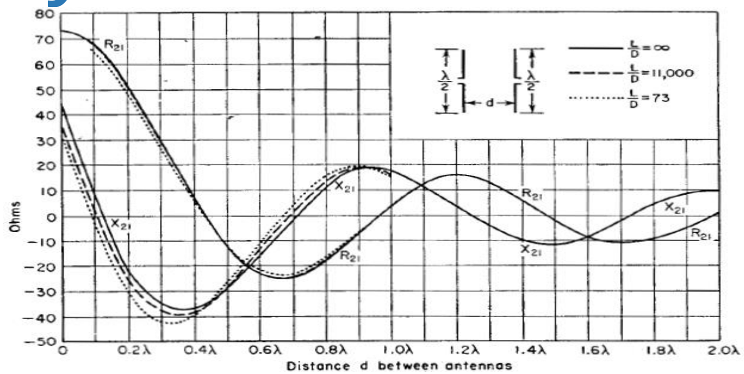
D-Ch: Polynomials with equal ripples -> Side lobe level (SLL) can be specified

More taper towards the edges -> More SLL reduction (as in continuous arrays)

Mutual coupling in arrays

Two-element array





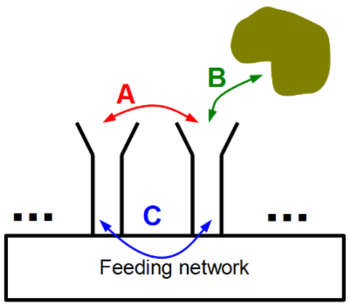
MATLAB imped

Self impedance

Mutual impedance

$$Z_{11} = \frac{U_1}{I_1}\bigg|_{I_2=0} \qquad Z_{22} = \frac{U_2}{I_2}\bigg|_{I_1=0} \qquad Z_{21} = \frac{U_2}{I_1}\bigg|_{I_2=0} \qquad Z_{12} = \frac{U_1}{I_2}\bigg|_{I_1=0} \qquad Z_{21} = Z_{12}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \qquad \begin{aligned} U_1 &= I_1 Z_{11} + I_2 Z_{12} \\ U_2 &= I_2 Z_{22} + I_1 Z_{21} \end{aligned}$$



$$I_1 = I_2 = I$$

$$U_1 = IZ_{11} + IZ_{12} = I(Z_{11} + Z_{12})$$
$$U_2 = IZ_{11} + IZ_{12} = I(Z_{11} + Z_{12})$$

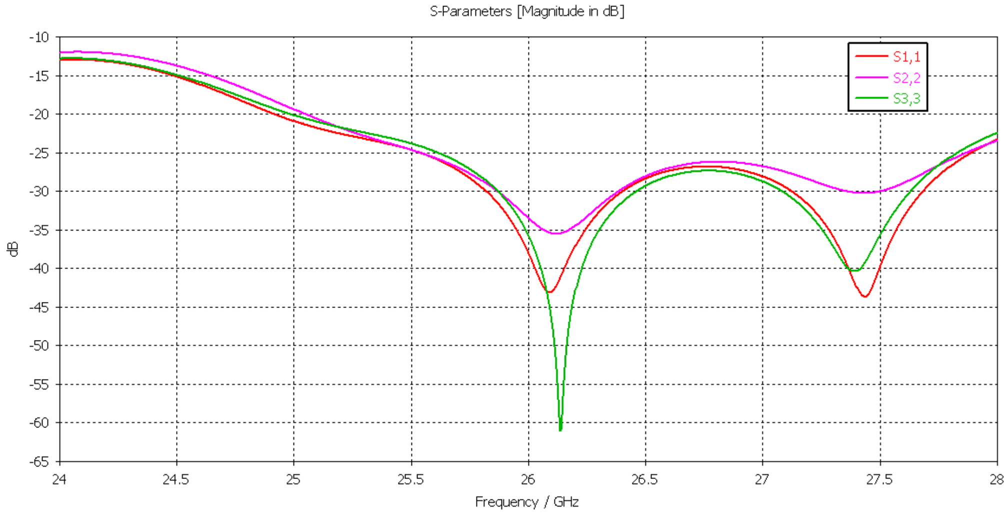
$$Z_{in1} = Z_{in2} = \frac{U_1}{I} = \frac{U_2}{I} = Z_{11} + Z_{12}$$

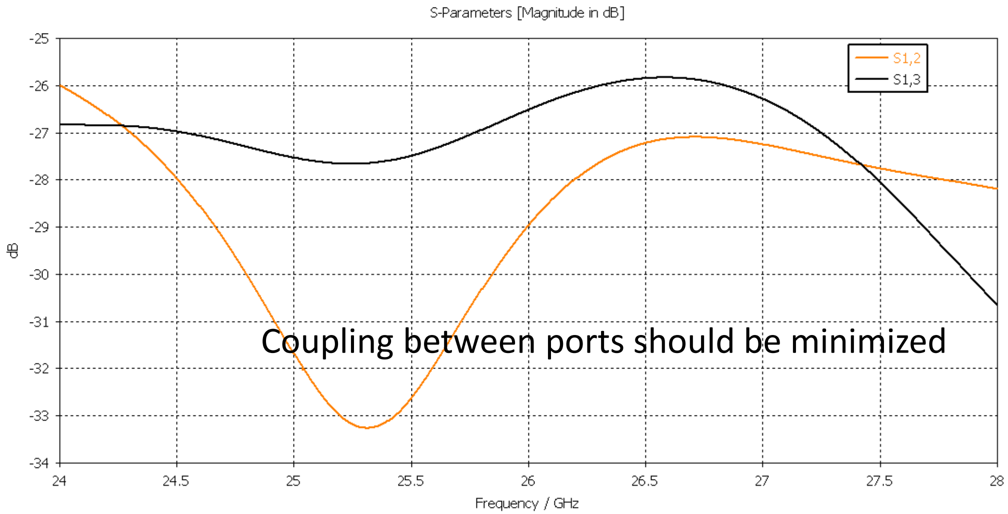
$$Z_{in,m} = \frac{V_m}{I_m} = Z_{m1}\frac{I_1}{I_m} + Z_{m2}\frac{I_2}{I_m} + \cdots + Z_{mN}\frac{I_N}{I_m} = \sum_{n=1}^{N} Z_{mn}\frac{I_n}{I_m}$$

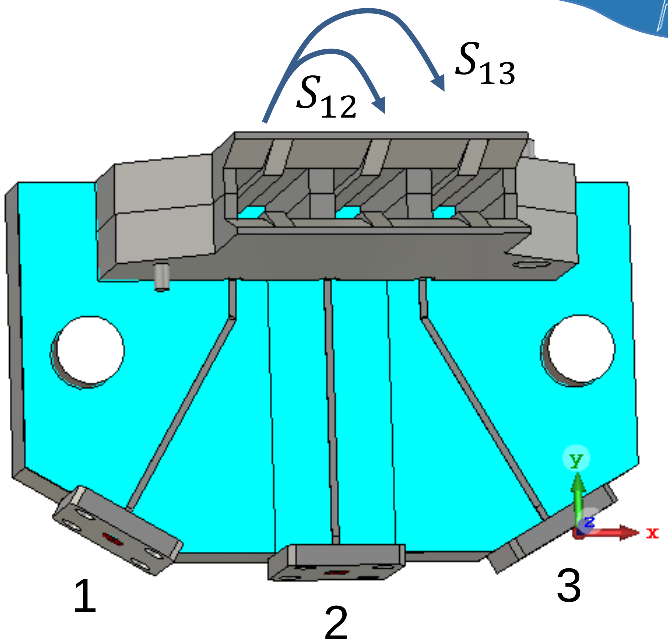
Active (driving, scan) impedance (S-parameters)

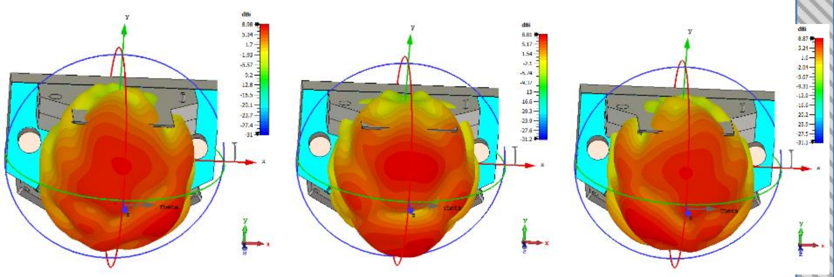
Mutual coupling in arrays

Embedded S parameters





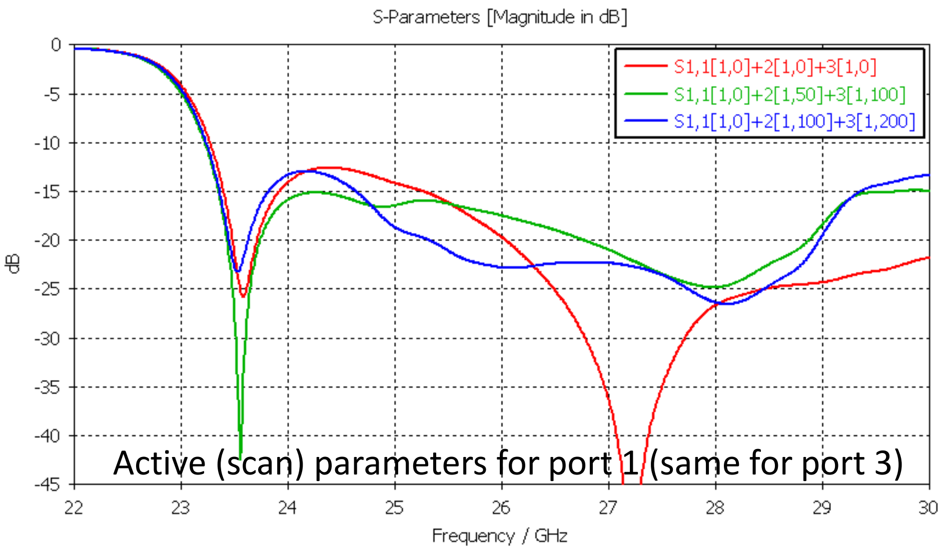




Embedded patterns – one port excited, others terminated by 50 Ω

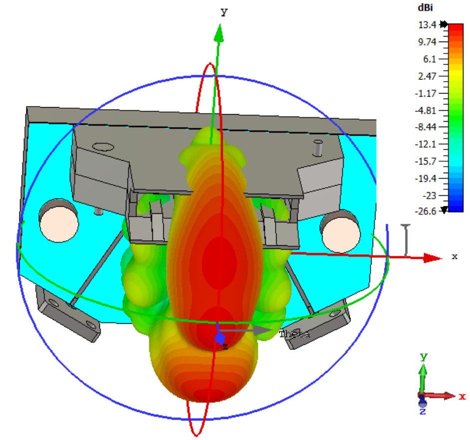
Arbitrary feeding = superposition of these states

Mutual coupling in arrays

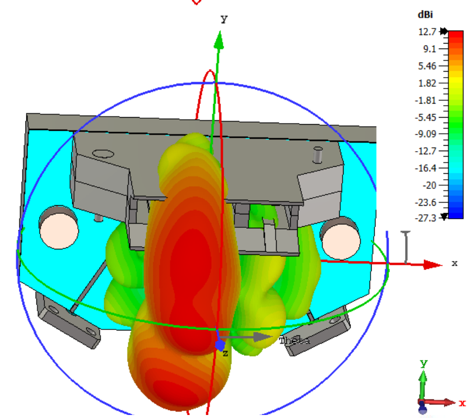


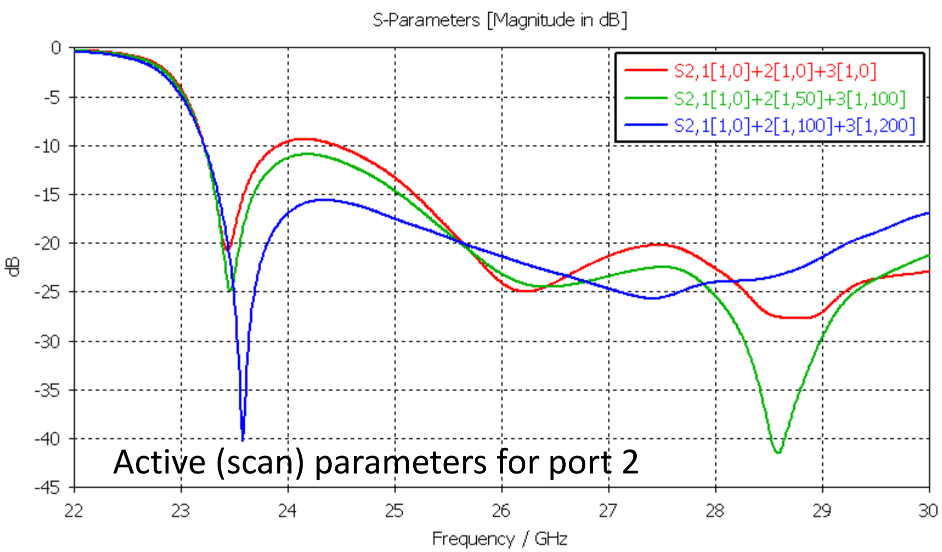
Active (scan) parameters for port 1 (same for port 3)

$0°, 0°, 0°$



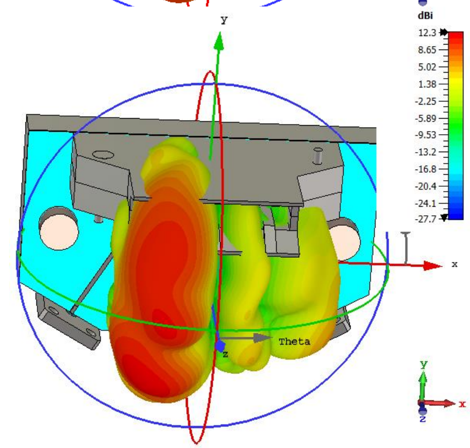
$0°, 50°, 100°$





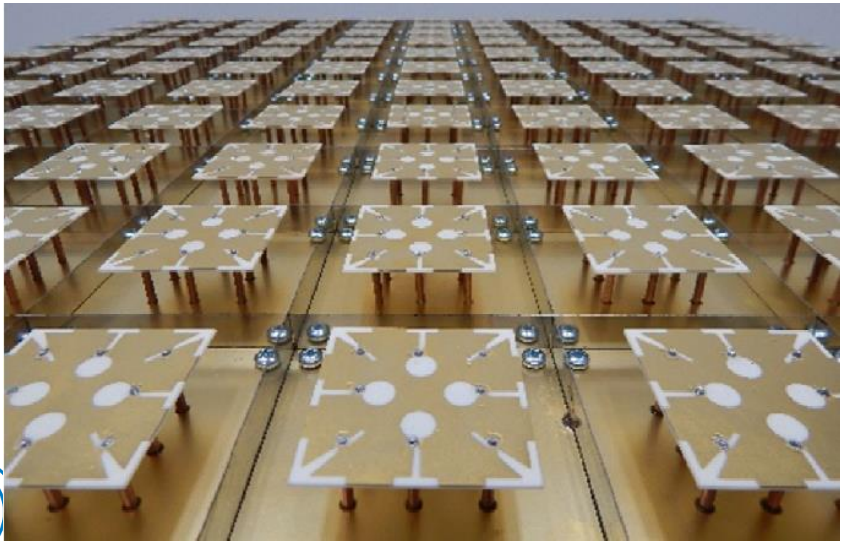
Active (scan) parameters for port 2

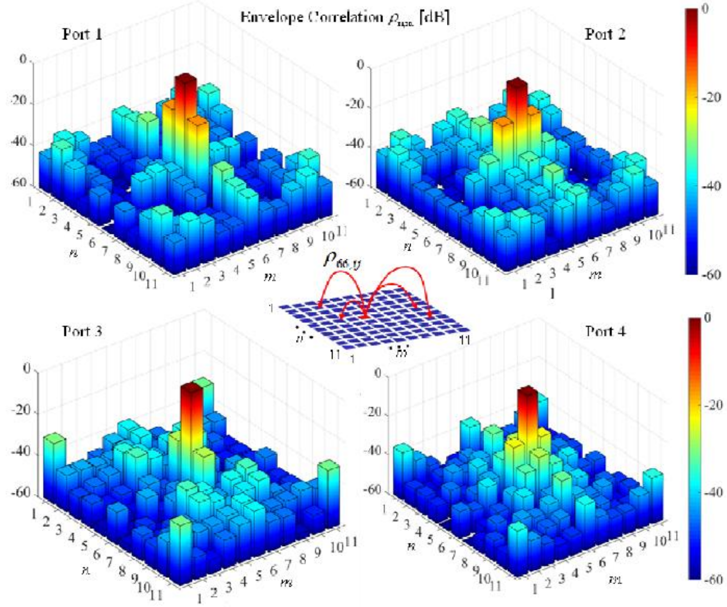
$0°, 100°, 200°$



Scan blindness – mismatch for some scan angle(s)

11x11 array, 484 ports





D. Manteuufel: Compact Multi Port Multi Element Antenna for Massive MIMO

Array directivity – one elementary dipole

- Various approaches, approximate equations, see e.g. Stutzman
- We use currents only to express the directivity (space integrals evaluated analytically)

$$D(\theta,\phi) = 4\pi \frac{[I]^H [u(\theta,\phi)][I]}{[I]^H [p][I]}$$

$$\boldsymbol{r}_0 \cdot (\boldsymbol{r} - \boldsymbol{r}') = (x-x')\sin\theta\cos\phi + (y-y')\sin\theta\sin\phi + (z-z')\cos\theta$$

$$u_{\mathrm{mn}}(\theta,\phi) = \frac{15k^2}{4\pi I_{\mathrm{m}} I_{\mathrm{n}}^*} \iint\limits_{\text{sources}} \Lambda(\theta,\phi)\, \mathrm{e}^{\mathrm{j}k(\Delta-\Delta')} \mathrm{d}\mathbf{r}\mathrm{d}\mathbf{r}'$$

$$p_{\mathrm{mn}} = \frac{1}{I_{\mathrm{m}} I_{\mathrm{n}}^*} \iint\limits_{\text{sphere}} u_{\mathrm{mn}}(\theta,\phi) \sin\theta \,\mathrm{d}\Omega$$

$= 1$ for isotropic radiator

$= \sin^2\theta$ for z-oriented elementary dipole

(polarization projection factors)

- Elementary dipole of length L and constant current (self directivity)

$$u_{11}(\theta,\phi) = \frac{15k^2}{4\pi} \sin^2\theta \int\limits_{-L/2}^{L/2} \int\limits_{-L/2}^{L/2} \mathrm{e}^{jk(\mathrm{z}-\mathrm{z}')\cos\theta}\, \mathrm{dz}\mathrm{dz}' \approx \frac{15k^2 L^2}{4\pi} \sin^2\theta$$

$$p_{11} = \frac{15k^2L^2}{2} \int\limits_0^{\pi} \sin^3\theta \,\mathrm{d}\theta = 10k^2L^2$$

$$D(\theta,\phi) = \frac{15}{10}\sin^2\theta$$

Array directivity – two elementary dipoles

- Two elementary dipoles – need to know mutual radiation intensities and powers
- Consider two elements separated by $s = kd$ in the x-axis
- Now in the directivity expression the matrices are 2x2 (self and mutual)

$$\frac{15k^2L^2}{4\pi}\sin^2\theta$$

$$u_{12}(\theta,\phi,s) = u_{11}(\theta,\phi)\mathrm{e}^{js\sin\theta\cos\phi}$$

interesting: everything about the array is known from pattern of sole element. Valid only for class of very special antennas called Minimum Scattering Antennas (MSA)

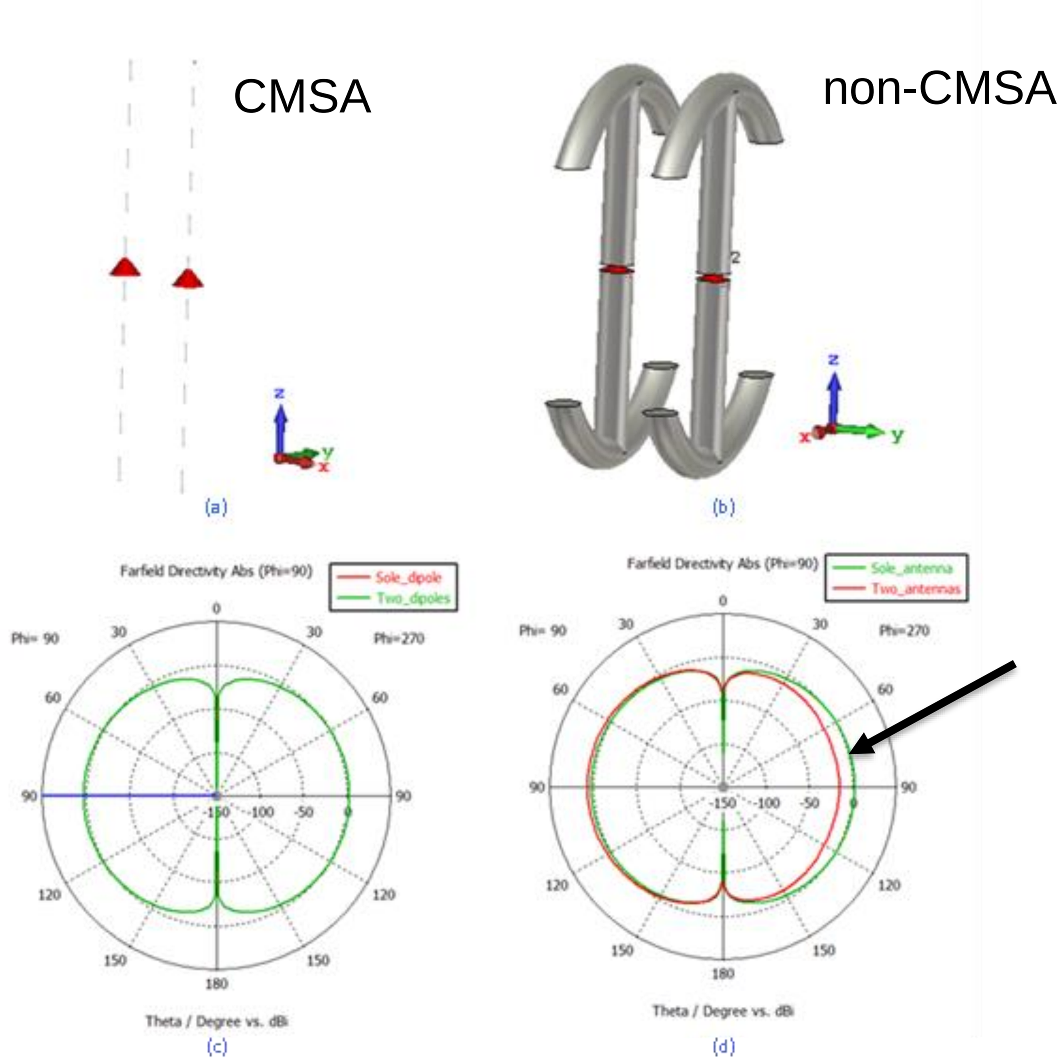
$$p_{12}(s) = \frac{15k^2L^2}{4\pi}\int_0^{2\pi}\int_0^{\pi}\sin^3\theta\,\mathrm{e}^{js\sin\theta\cos\phi}\,\mathrm{d}\theta\mathrm{d}\phi = 15k^2L^2\left(\frac{\sin s}{s}+\frac{\cos s}{s^2}-\frac{\sin s}{s^3}\right) = 15k^2L^2\left(j_0(s)-\frac{j_1(s)}{s}\right)$$

$$D(\theta,\phi,s) = 4\pi\frac{[I]^H[u(\theta,\phi,s)][I]}{[I]^H[p(s)][I]}$$

Enter arbitrary currents, spacing and angular direction…

- Special class of single-mode antennas (elementary dipole, isotropic radiator, elementary loop, half-wavelength dipole) that are basically “invisible” from the point of view of other antenna - the far field of a standalone antenna is identical with the far field of the same antenna embedded as an element of an array and influenced by its other open-circuited elements.

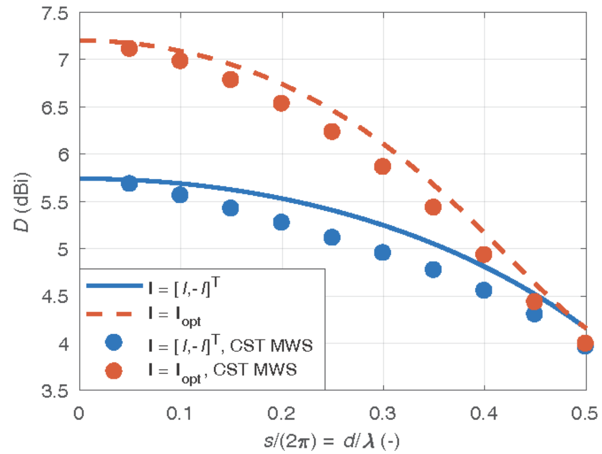
Canonical minimum scattering antennas

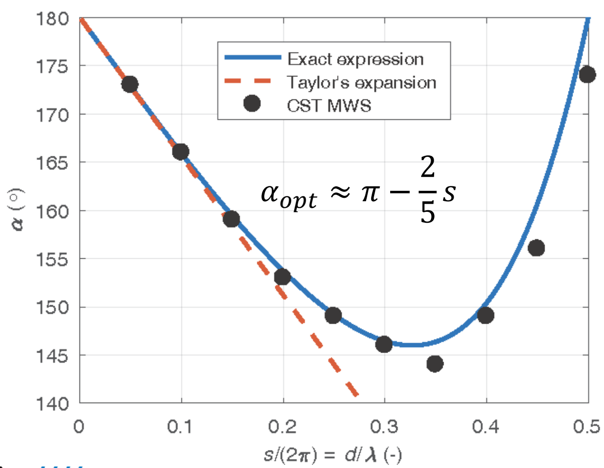


Directivity optimization - SuperDirectivity

$$D = 4\pi \frac{[I]^H[u][I]}{[I]^H[p][I]} \Rightarrow 4\pi \mathbf{uI}_{opt} = D\mathbf{pI}_{opt}$$

Eigenvalue equation eig(p,u) → $\mathbf{I}$, D

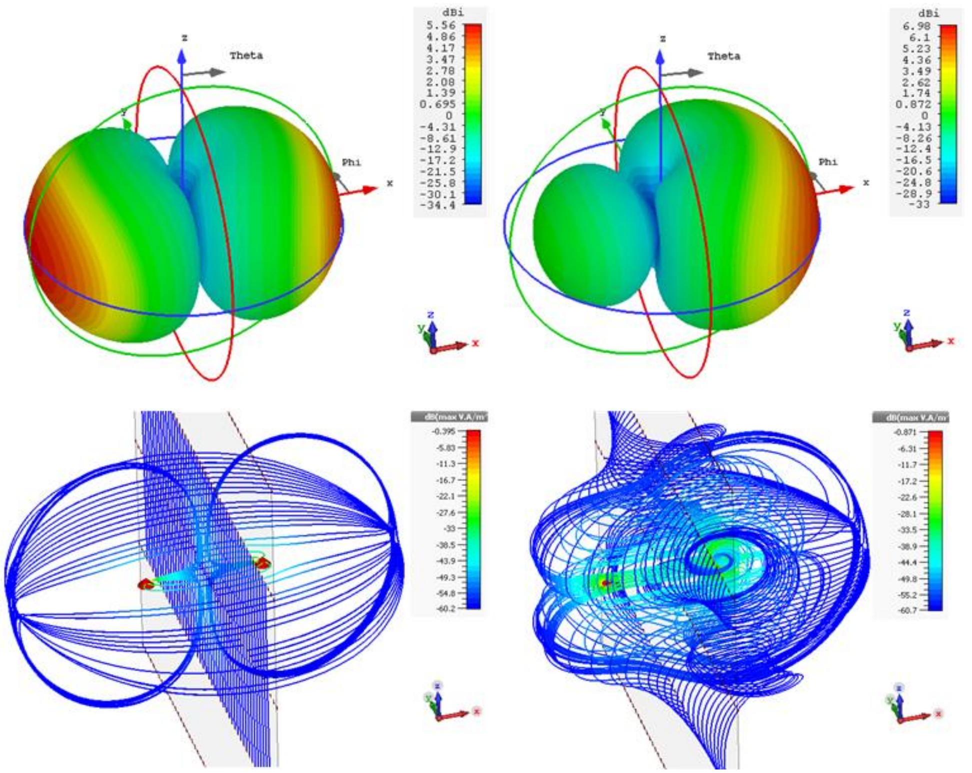




$$\alpha_{opt} \approx \pi - \frac{2}{5}s$$

Out-of-phase currents

superdirective currents



Optimizing the Directivity – elem. dipoles

Powerflow: Out-of phase currents

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Optimizing the Directivity – elem. dipoles

Powerflow: Superdirective currents

Directivity optimization - SuperDirectivity

$$D = 4\pi \frac{[I]^H[u][I]}{[I]^H[p][I]} \Rightarrow 4\pi\mathbf{u}\mathbf{I}_{opt} = D\mathbf{p}\mathbf{I}_{opt}$$

Eigenvalue equation eig(p,u)

$\mathbf{I}$

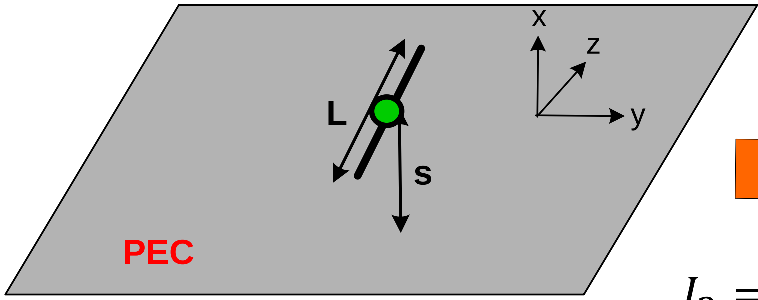
D

Highest eigenvalue (max. D) $\mathbf{I}_{opt} = \frac{1}{4\pi}\mathbf{p}^{-1}\mathbf{V}$

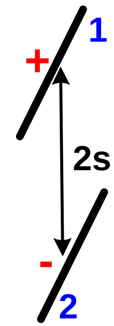
$$\mathbf{I}_{opt} = \frac{1}{4\pi}\frac{1}{p_{11}^2 - p_{12}^2}\begin{bmatrix} p_{11} & -p_{12} \\ -p_{12} & p_{11} \end{bmatrix}\begin{bmatrix} \mathrm{e}^{js/2} \\ \mathrm{e}^{-js/2} \end{bmatrix}$$

- For small spacing $p_{12} \to p_{11}$, currents have high amplitude (losses), condition number of the $\mathbf{p}$ matrix increases → high sensitivity of the array
- Directivity higher than that obtained with the same array elements uniformly excited
- Rapid phase variations across array
- Appears when elements of array are spaced <λ/2
- Sensitivity problems
- Efficiency!

Dipole above ground



$I_2 = -I_1$



$$U_1 = I_1 Z_{11} + I_2 Z_{12}$$

$$U_2 = I_2 Z_{22} + I_1 Z_{12}$$

$Z_{11} = Z_{22}$ Self impedance

$Z_{12} = Z_{21}$ Mutual impedance

Input driving impedance

For $s \to 0$ is $R_{11} - R_{12} \to 0$

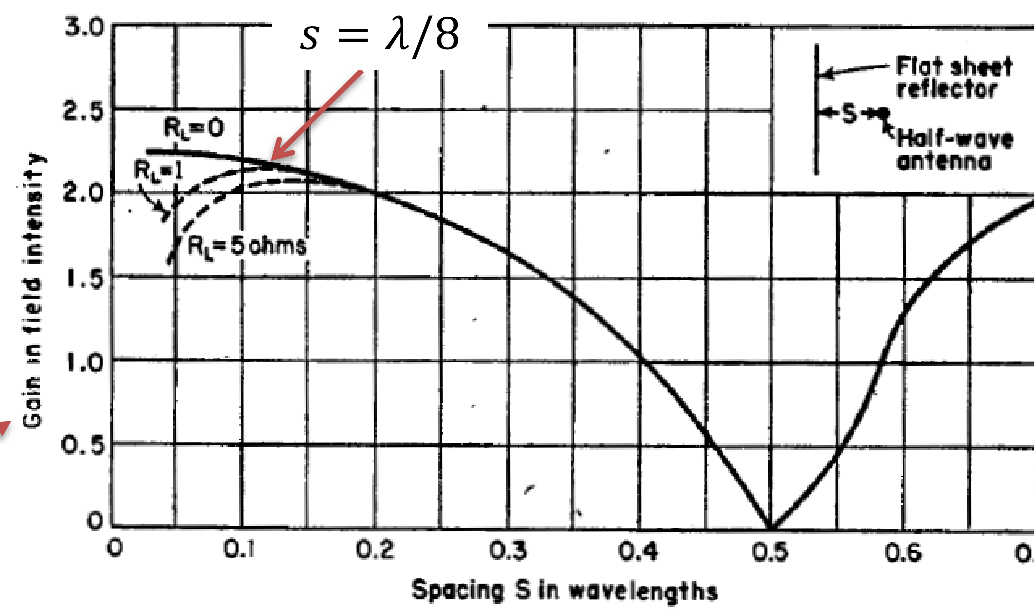
$$Z_{in} = \frac{U_1}{I_1} = \frac{U_2}{I_2} = Z_{11} - Z_{12}$$

Input current for constant power P $I_1 = \sqrt{\frac{P}{2(R_{11}+R_L-R_{12})}}$

$$s = \lambda/8$$

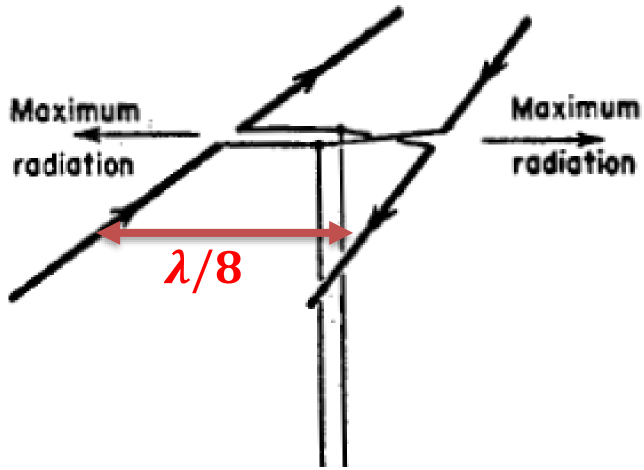
$$R_{in} = R_{11} - R_{12} = 8.95\ \Omega$$

$$G = 20 \log 2.13 + 2.15 = 8.7\ dBi$$



Above $\lambda/2$ dipole!

W8JK Kraus's array



N=2 Isotropic radiators – Uzkov limit

- **Out-of-phase currents (+I, -I) → end-fire radiation** $D(\theta,\phi) = 4\pi \frac{[I]^H[u][I]}{[I]^H[p][I]}$

Uzkov:

$$\mathrm{D}_{max} = N^2$$

$$u_{11} = \frac{p_{11}}{4\pi}$$

$$u_{12}(\theta,\phi,s) = u_{11}\mathrm{e}^{js\sin\theta\cos\phi}$$

$$p_{12}(s) = \frac{p_{11}}{4\pi}\int_0^{2\pi}\int_0^{\pi}\sin\theta\,\mathrm{e}^{js\sin\theta\cos\phi}\,\mathrm{d}\theta\mathrm{d}\phi = p_{11}\frac{\sin s}{s}$$

Out-of-phase currents

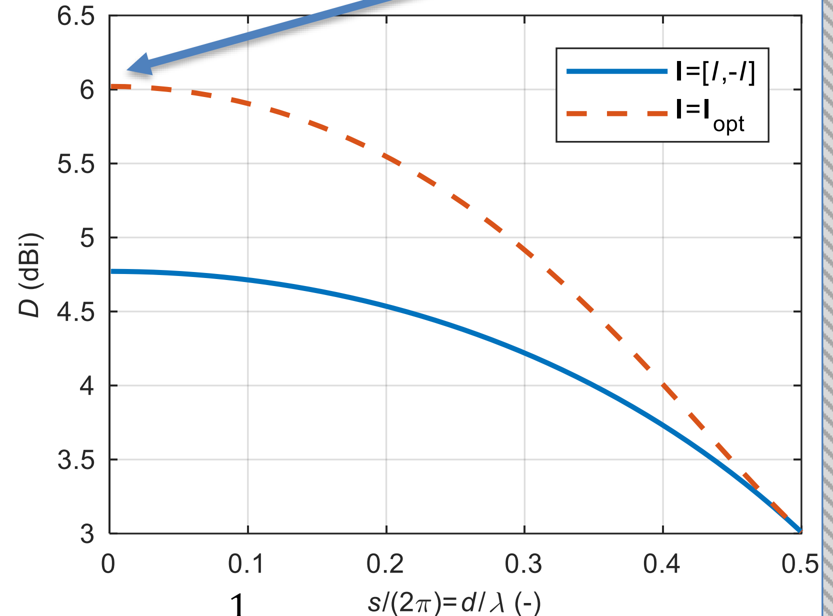
$$\boxed{D(s) = \frac{1-\cos s}{1-\frac{\sin s}{s}}}$$

$D = 3\ (4.77\ dBi),\ s \to 0$

Superdirective currents

$$\boxed{D(s) = \frac{\cos\alpha + \cos s}{\cos\alpha + \frac{\sin s}{s}}}$$

$D = 4\ (6.02\ dBi),\ s \to 0$



$$\alpha \approx \pi - \frac{1}{3}s$$