Chapter 4

Transmission Lines and Signal Integrity

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Outline

- Preview
- The Transmission-Line Equations
- The Per-Unit-Length Parameters
- The Time-Domain Solution
- High-Speed Digital Interconnects and Signal Integrity
- Sinusoidal Excitation of the Line and the Phasor Solution
- Lumped-Circuit Approximate Models

Preview

- Why Transmission Lines are so Important?
  - The transmission line will impose a time delay as the signal propagate from one end to the other. $T_D = \frac{\mathscr{L}}{v}$
  - The mismatch between impedance will cause reflections, which in turn will distort the original signal, and thus becomes the major aspect of degrading signal integrity.
- Signal Integrity
  - Signal integrity ensures that the waveforms at the input and the output of the line are identical or approximately so.

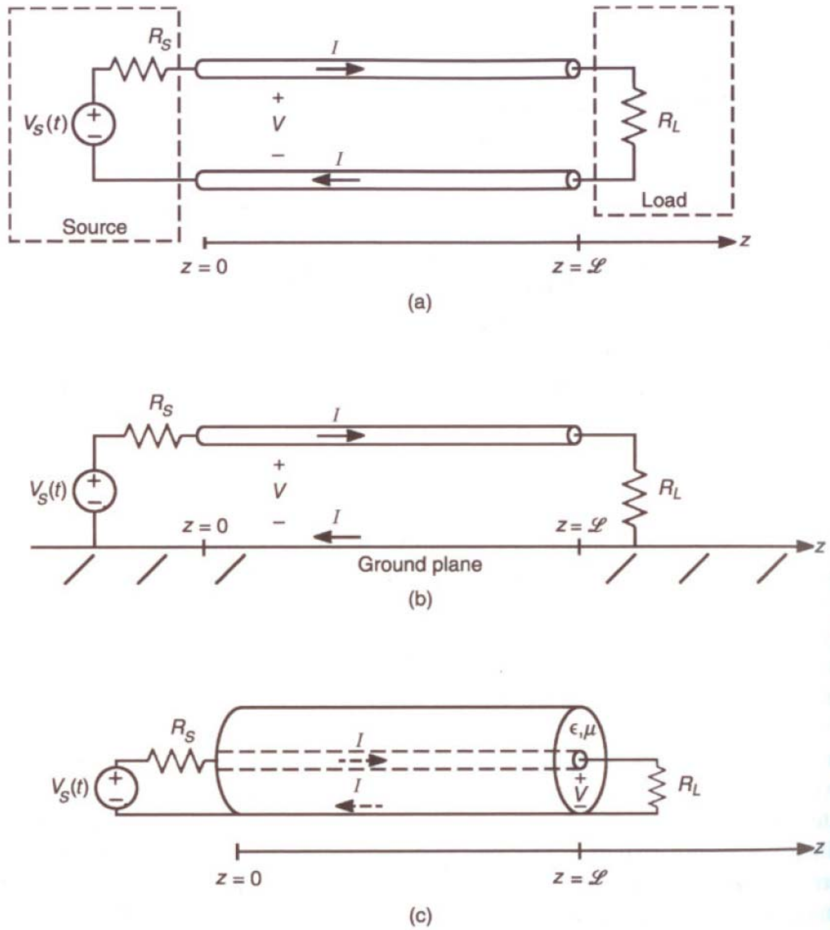
Preview

- Categories of Transmission Lines
  - Typical Wire-Type Transmission Lines

Two wires:
dispersive because of outer insulator

One wire above ground:
dispersive because of outer insulator

Coaxial cable:
not dispersive



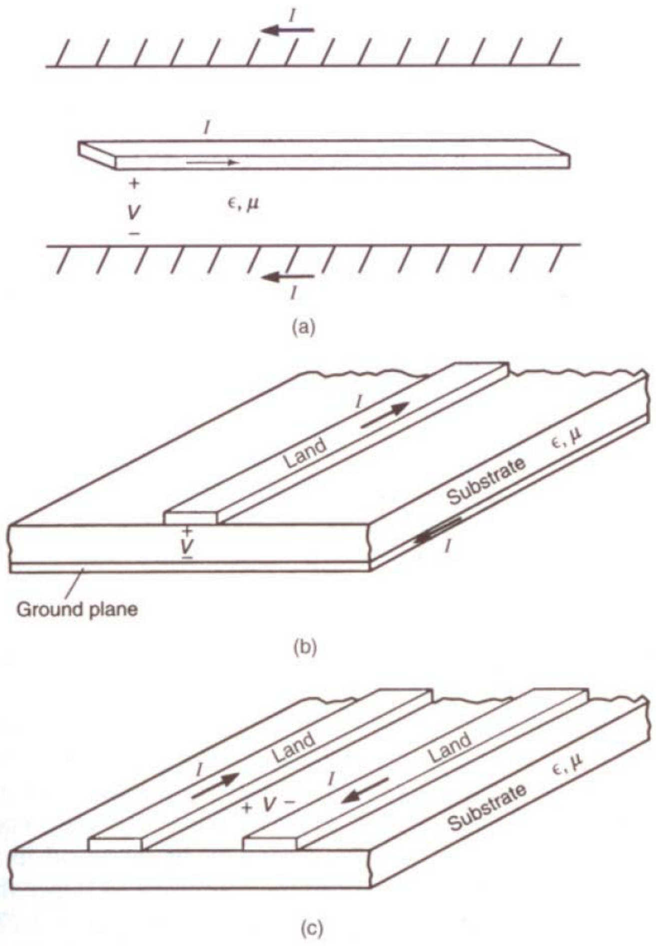
Preview

- Categories of Transmission Lines
  - Typical Printed Circuit Board (PCB)

Stripline (innerplane boards):
not dispersive

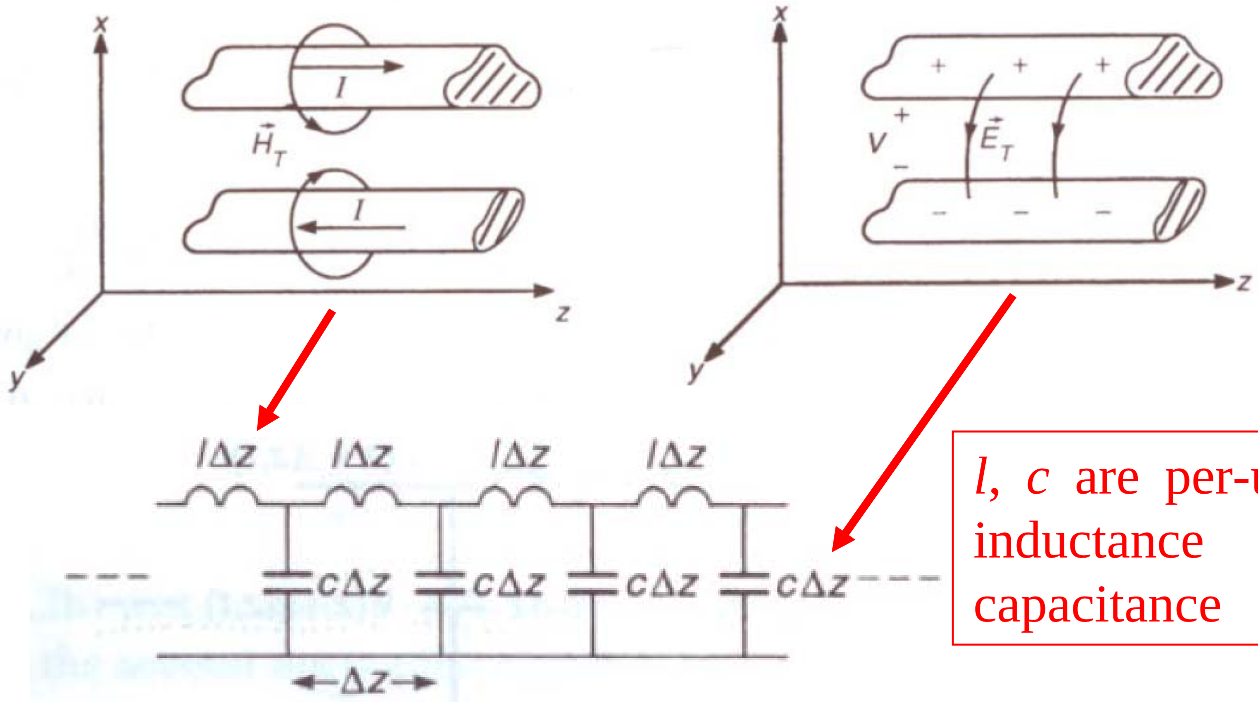
Microstrip
(outer lands of an innerplane board):
dispersive

Coplanar strip
(lands on a board without innerplanes):
dispersive



The Transmission-Line Equations

- Two-Conductor Transmission Line
  - Equivalent Circuit Model-Lossless
    - The line can be modeled as a *distributed parameter circuit* consisting of a sequence of inductors and capacitors.



l, *c* are per-unit inductance and capacitance

The Transmission-Line Equations

- Two-Conductor Transmission Line
  - Equivalent Circuit Model-Lossless
    - **Effect of *L* and *C*:** It takes a certain time to energize and deenergize these inductors and capacitors so that it will take a finite, nonzero time for the waves to transit the line. This will result in a time delay for a line of total length L of $T_D=L/v$.
    - **Effect of *R*:** At frequencies in the GHz range, the resistance of the conductors may become significant due to skin effect, thus could not be neglected.

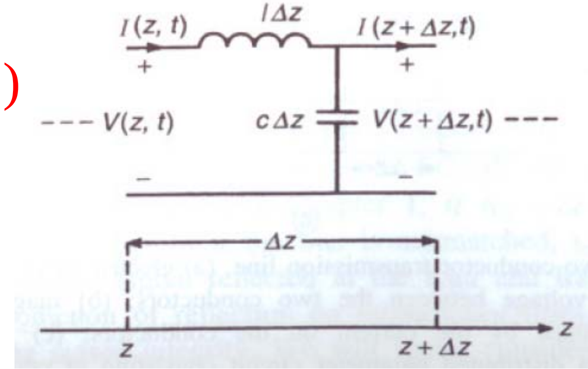
The Transmission-Line Equations

- Two-Conductor Transmission Line
  - Equivalent Circuit Model-Lossless
    - Consider a Δz section of the line model
    - Writing Kirchhoff's voltage law around the outside loop gives

$$V(z+\Delta z, t) - V(z, t) = -l\,\Delta z \frac{\partial I(z, t)}{\partial t}$$

$$\rightarrow \left.\frac{V(z+\Delta z, t) - V(z, t)}{\Delta z}\right|_{\lim \Delta z \to 0} = \frac{\partial V(z, t)}{\partial z}$$

$$\rightarrow \frac{\partial V(z, t)}{\partial z} = -l \frac{\partial I(z, t)}{\partial t} \quad (1)$$



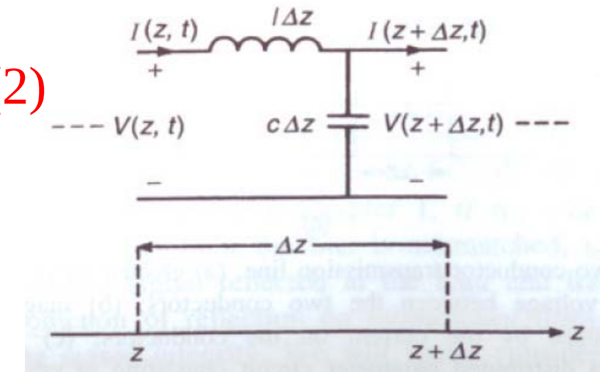
The Transmission-Line Equations

- Two-Conductor Transmission Line
  - Equivalent Circuit Model-Lossless
    - Writing Kirchhoff's current law at the upper node of the capacitor gives

$$I(z+\Delta z, t) - I(z, t) = -c\,\Delta z \frac{\partial V(z+\Delta z, t)}{\partial t}$$

$$\longrightarrow \left.\frac{I(z+\Delta z, t) - I(z, t)}{\Delta z}\right|_{\lim \Delta z \to 0} = \frac{\partial I(z, t)}{\partial z}$$

$$\longrightarrow \frac{\partial I(z, t)}{\partial z} = -c \frac{\partial V(z, t)}{\partial t} \quad (2)$$



(1) and (2) are called the ***coupled transmission-line equations*** in the 1st order form.

The Transmission-Line Equations

- Two-Conductor Transmission Line
  - Equivalent Circuit Model-Lossless
    - Differentiating (1) with respect to *z* to give

$$\frac{\partial V(z,t)}{\partial z} = -l\frac{\partial I(z,t)}{\partial t} \rightarrow \frac{\partial^2 V(z,t)}{\partial z^2} = -l\frac{\partial^2 I(z,t)}{\partial t\,\partial z}$$

    - Differentiating (2) with respect to *t* to give

$$\frac{\partial I(z,t)}{\partial z} = -c\frac{\partial V(z,t)}{\partial t} \rightarrow \frac{\partial^2 I(z,t)}{\partial z\,\partial t} = -c\frac{\partial^2 V(z,t)}{\partial t^2}$$

    - Combing the above two equations, we have

$$\frac{\partial^2 V(z,t)}{\partial z^2} = lc\frac{\partial^2 V(z,t)}{\partial t^2}$$

    - Similarly, we could obtain

$$\frac{\partial^2 I(z,t)}{\partial z^2} = lc\frac{\partial^2 I(z,t)}{\partial t^2}$$

    - These are called the ***uncoupled transmission line equations*** in 2nd order form.

The Per-Unit-Length Parameters

- Preview
  - Faraday's Law
    - Since TEM waves have no H_z, i.e. H_z=0, we have

$$\nabla \times E = -\frac{\partial B}{\partial t} \longrightarrow \oint_{C_{xy}} \vec{E}_T \bullet \vec{dl} = \underbrace{-\frac{d}{dt}\int_{S_{xy}} \mu \vec{H}_z \bullet \vec{ds}}_{=0}$$

    - which could be used to derive E_T.
  - Ampere's Law
    - Since TEM waves have no E_z, i.e. E_z=0, we have

$$\nabla \times H = J + \frac{\partial D}{\partial t} \longrightarrow \oint_{C_{xy}} \vec{H}_T \bullet \vec{dl} = \underbrace{I + \frac{d}{dt}\int_{S_{xy}} \epsilon \vec{E}_z \bullet \vec{ds}}_{=I}$$

    - which could be used to derive H_T.

$\boxed{\text{With } E_T \text{ and } H_T \text{ in hand, you can derive } L \text{ and } C.}$

The Per-Unit-Length Parameters

- Preview
  - Important Points
    - Due to the characteristics of TEM, we can compute the per-unit-length capacitance *c* and per-unit-length inductance *l* using dc field computations even though the fields will be varying with time.
    - If the medium surrounding the two conductors are homogeneous, then *c* and *l* are related as

$$lc = \mu\epsilon \longrightarrow \begin{cases} l = \dfrac{1}{cv^2} \\ c = \dfrac{1}{lv^2} \end{cases}$$

    - where $v = \dfrac{1}{\sqrt{\mu\epsilon}}$

$$= \frac{v_0 = 3 \times 10^8}{\sqrt{\epsilon_r}} \quad \text{m/s}$$

The Per-Unit-Length Parameters

- Preview
  - Effective Relative Permittivity
    - If the inhomogeneous surrounding medium were replaced by a homogeneous medium having an effective relative permittivity of ε_r', none of the properties of the line would be changed. The relation could be rewritten as

$$lc = \mu_0 \epsilon_0 \epsilon_r'$$

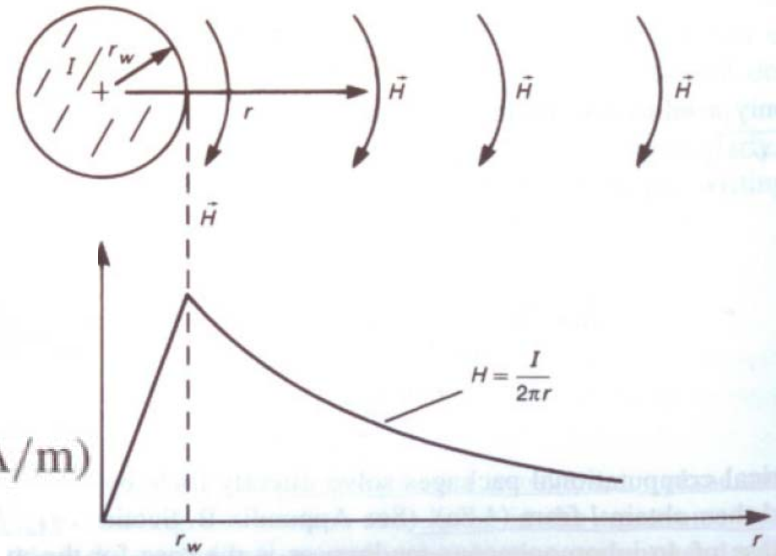
$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r'}} = \frac{v_0 = 3 \times 10^8}{\sqrt{\epsilon_r'}} \quad \frac{\text{m}}{\text{s}}$$

The Per-Unit-Length Parameters

- Wire-Type Structures
  - Magnetic Field about a Current-Carrying Wire
    - Assuming that the current is uniformly distributed across the wire cross section and applying Ampere's law, we have

$$\oint_C \vec{H}_T \cdot d\vec{l} = I_{\text{enclosed}}$$

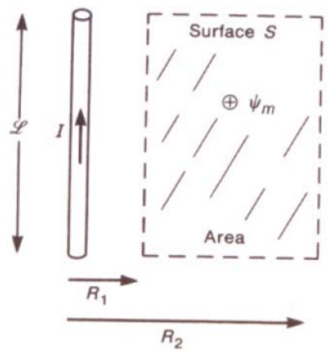
$$H_T = \frac{I}{\oint_C dl} = \frac{I}{2\pi r} \quad \text{(in A/m)}$$



The tangential magnetic field of the TEM wave is obtained from this equation.

The Per-Unit-Length Parameters

- Wire-Type Structures
  - Magnetic Flux through S
    - By applying Gauss' law, which states that the total magnetic flux leaving a closed surface is zero, we have

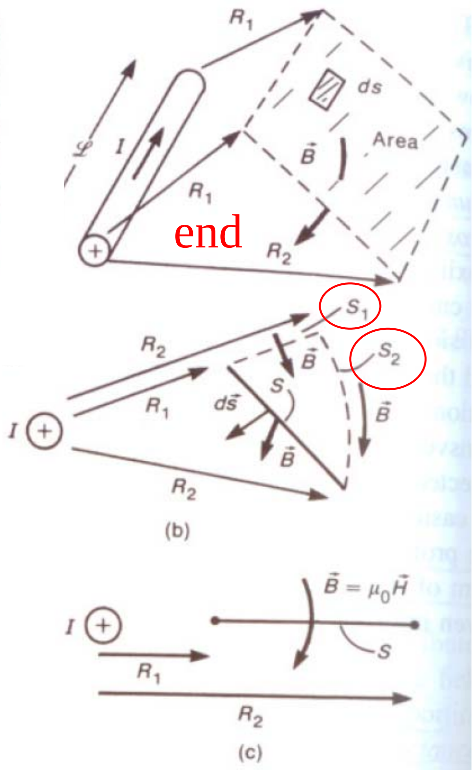


$$\psi_m = \int_S \vec{B}_T \cdot \vec{ds}$$

$$= \int_{S_1} \vec{B}_T \cdot \vec{ds} + \underbrace{\int_{S_2} \vec{B}_T \cdot \vec{ds}}_{0} + \underbrace{\int_{S_{\text{end}}} \vec{B}_T \cdot \vec{ds}}_{0}$$

$$= \int_{r=R_1}^{R_2} \frac{\mu_0 I}{2\pi r}\, dr$$

$$= \frac{\mu_0 I}{2\pi} \ln\left(\frac{R_2}{R_1}\right) \quad \text{(in Wb)} \quad R_2 > R_1$$



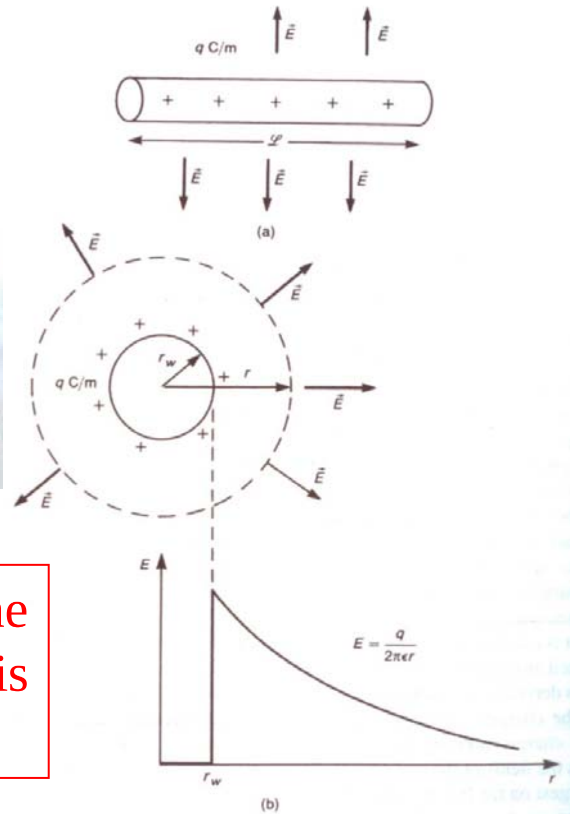
This could be used to solve for l through $\Psi = lI$

The Per-Unit-Length Parameters

- Wire-Type Structures
  - Electric Field about a Charged Wire
    - Assuming that the charge is uniformly distributed around the wire periphery and applying Gauss' law, we have

$$\oint_S \epsilon_0 \vec{E}_T \cdot \vec{ds} = Q_{\text{enclosed}}$$

$$E_T = \frac{q \times 1\,\text{m}}{\epsilon_0 \oint_S ds} = \frac{q}{2\pi\epsilon_0 r} \quad \text{(in V/m)}$$

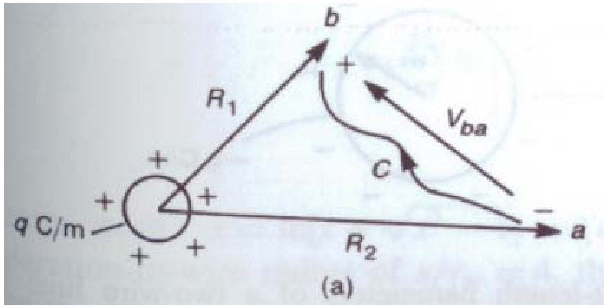


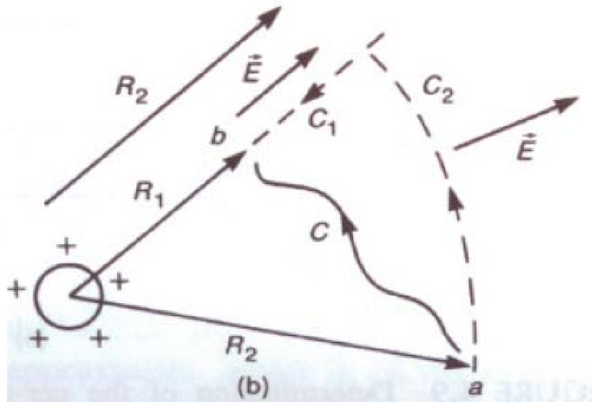
The tangential electric field of the TEM wave is obtained from this equation.

The Per-Unit-Length Parameters

- Wire-Type Structures
  - Voltage between Two Points
    - Changing the path to an appropriate one, the integral could be simplified as

$$
\begin{aligned}
V &= -\int_C \vec{E}_T \cdot \vec{dl} \\
&= -\int_{C_1} \vec{E}_T \cdot \vec{dl} - \underbrace{\int_{C_2} \vec{E}_T \cdot \vec{dl}}_{0} \\
&= -\int_{r=R_2}^{R_1} \frac{q}{2\pi\epsilon_0 r}\, dr \\
&= \frac{q}{2\pi\epsilon_0} \ln\left(\frac{R_2}{R_1}\right) \quad \text{(in V)} \qquad R_2 \geq R_1
\end{aligned}
$$





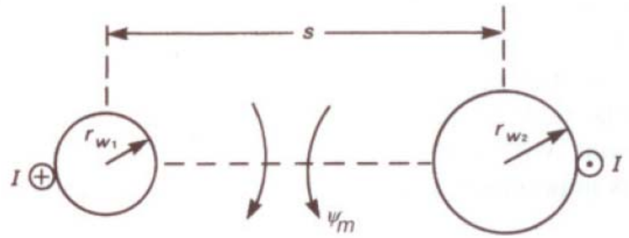
This could be used to solve for *c* through *q*=*cV*

The Per-Unit-Length Parameters

- Wire-Type Structures
  - Two-Wire Line
    - In order to prevent from the proximity effect, assume that the two wire are widely separated.
    - The total magnetic flux between the 2 wires is

$$\psi_m = \frac{\mu_0 I}{2\pi}\ln\left(\frac{s - r_{w2}}{r_{w1}}\right) + \frac{\mu_0 I}{2\pi}\ln\left(\frac{s - r_{w1}}{r_{w2}}\right)$$

$$= \frac{\mu_0 I}{2\pi}\ln\left[\frac{(s - r_{w2})(s - r_{w1})}{r_{w2} r_{w1}}\right]$$



$$\rightarrow \quad \psi_m = \frac{\mu_0 I}{2\pi}\ln\left(\frac{s^2}{r_{w2} r_{w1}}\right) \qquad s \gg r_{w1}, r_{w2}$$

The Per-Unit-Length Parameters

- Wire-Type Structures
  - Two-Wire Line
    - The approximate per-unit-length external inductance is

$$l = \frac{\psi_m}{I}$$

$$= \frac{\mu_0}{2\pi} \ln\left(\frac{s^2}{r_{w2} r_{w1}}\right) \quad \text{(in H/m)} \qquad s \gg r_{w1}, r_{w2}$$

$$l = \frac{\mu_0}{\pi} \ln\left(\frac{s}{r_w}\right), \quad r_{w1} = r_{w2} = r_w$$

$$= 0.4 \ln\left(\frac{s}{r_w}\right) \quad \text{(in } \mu\text{H/m)}$$

$$= 10.16 \ln\left(\frac{s}{r_w}\right) \quad \text{(in nH/in.)}$$

The Per-Unit-Length Parameters

- Wire-Type Structures
  - Two-Wire Line
    - The exact per-unit-length external inductance is

$$l = \frac{\mu_0}{\pi}\cosh^{-1}\left(\frac{s}{2r_w}\right) \quad \text{(in H/m)}$$
$$= \frac{\mu_0}{\pi}\ln\left[\frac{s}{2r_w} + \sqrt{\left(\frac{s}{2r_w}\right)^2 - 1}\right]$$

From the author's another book [1]

$$\cosh^{-1} x = \ln[x + \sqrt{x^2 - 1}]$$

- The exact per-unit-length capacitance is

$$c = \frac{\mu_0 \epsilon_0}{l}$$
$$= \frac{1}{v_0^2 l}$$
$$= \frac{\pi\epsilon_0}{\cosh^{-1}(s/2r_w)} \quad \text{(in F/m)}$$

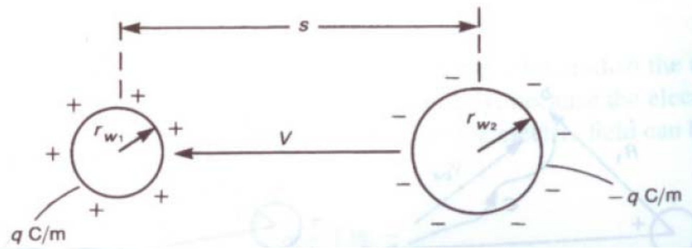
$$\cosh^{-1} x \cong \ln(2x) \quad \text{for} \quad x \gg 1$$

$$c \cong \frac{\pi\epsilon_0}{\ln(s/r_w)} \quad \text{(in F/m)} \qquad s \geq r_w$$

The Per-Unit-Length Parameters

- Wire-Type Structures
  - Two-Wire Line
    - In order to prevent from the proximity effect, assume that the two wire are widely separated.
    - The voltage between the two wires is given

$$
\begin{aligned}
V &= \frac{q}{2\pi\epsilon_0}\ln\left(\frac{s-r_{w2}}{r_{w1}}\right)+\frac{q}{2\pi\epsilon_0}\ln\left(\frac{s-r_{w1}}{r_{w2}}\right) \\
&= \frac{q}{2\pi\epsilon_0}\ln\left[\frac{(s-r_{w2})(s-r_{w1})}{r_{w2}r_{w1}}\right] \\
&\cong \frac{q}{2\pi\epsilon_0}\ln\left(\frac{s^2}{r_{w2}r_{w1}}\right)
\end{aligned}
$$



$$
\longrightarrow \quad V = \frac{q}{\pi\epsilon_0}\ln\left(\frac{s}{r_w}\right), \quad r_{w1} = r_{w2} = r_w
$$

The Per-Unit-Length Parameters

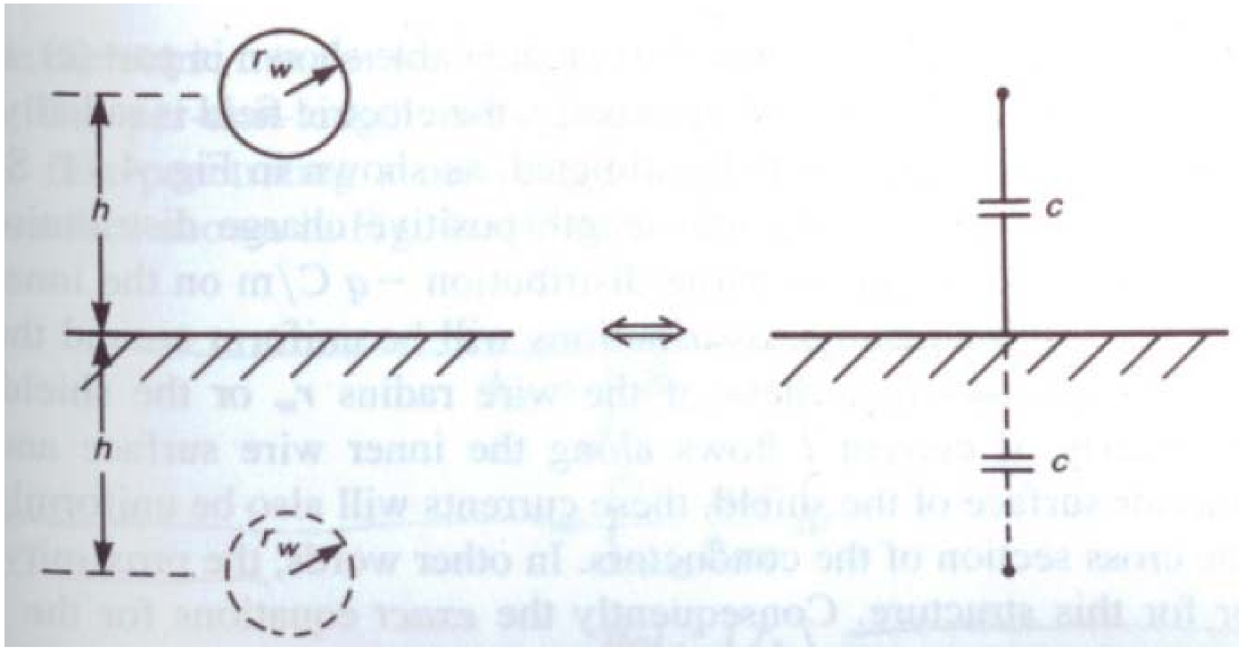
- Wire-Type Structures
  - Two-Wire Line
    - The approximate per-unit-length capacitance is then obtained as

$$
\begin{aligned}
c &= \frac{q}{V} \\
&= \frac{\pi \epsilon_0}{\ln(s/r_w)} \\
&= \frac{27.78}{\ln(s/r_w)} \quad \text{(in pF/m)} \\
&= \frac{0.706}{\ln(s/r_w)} \quad \text{(in pF/in.)}
\end{aligned}
$$

This is the same as that simplified from the formula for the exact per-unit-length capacitance.

The Per-Unit-Length Parameters

- Wire-Type Structures
  - A Wire above a Ground Plane
    - By using the method of image, we find that the capacitance between the wire and the ground plane is twice the capacitance of two wires separated a distance of $2h$.



The Per-Unit-Length Parameters

- Wire-Type Structures
  - A Wire above a Ground Plane
    - Thus, using the previous results, we obtain the exact

$$c = \frac{2\pi\epsilon_0}{\cosh^{-1}(h/r_w)} \quad \text{(in F/m)}$$

$$\longrightarrow c \cong \frac{2\pi\epsilon_0}{\ln(2h/r_w)} \quad \text{(in F/m)} \qquad h \gg r_w$$

    - The exact per-unit-length inductance can be obtained from

$$l = \frac{\mu_0 \epsilon_0}{c}$$

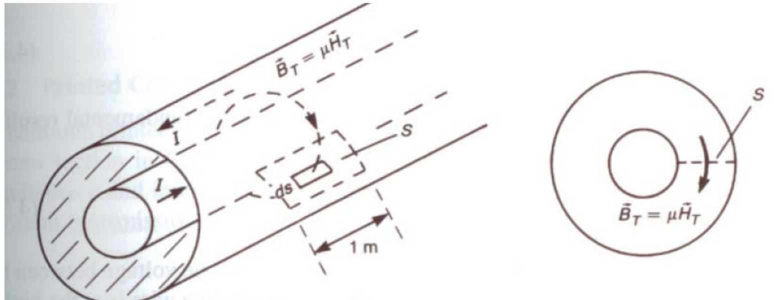
$$= \frac{\mu_0}{2\pi}\cosh^{-1}\left(\frac{h}{r_w}\right) \quad \text{(in H/m)}$$

Half of the inductance of two wires

$$\longrightarrow l \cong \frac{\mu_0}{2\pi}\ln\left(\frac{2h}{r_w}\right) \quad \text{(in H/m)} \qquad h \gg r_w$$

The Per-Unit-Length Parameters

- Wire-Type Structures
  - Coaxial Cable
    - Due to symmetry of this structure, there is no proximity effect.
    - Since
$$B_T = \mu_0 H_T = \frac{\mu_0 I}{2\pi r}$$



(b)

    - The total magnetic flux penetrating a unit-length surface is
$$\psi_m = \int_s \vec{B}_T \cdot \vec{ds} = \int_{r=r_w}^{r_s} \frac{\mu_0 I}{2\pi r}\, dr = \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_s}{r_w}\right)$$

The Per-Unit-Length Parameters

- Wire-Type Structures
  - Coaxial Cable
    - Therefore, the exact per-unit-length inductance is

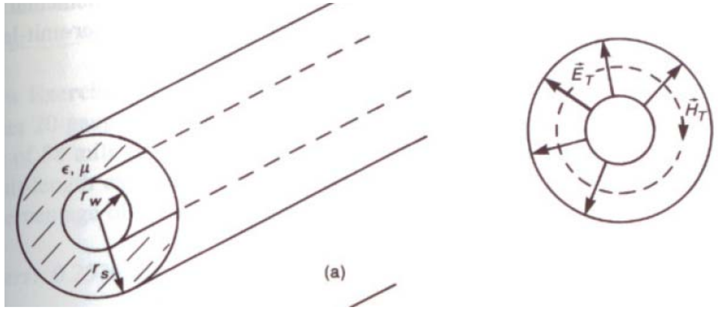
$$
\begin{aligned}
l &= \frac{\psi_m}{I} \\
&= \frac{\mu_0}{2\pi} \ln\left(\frac{r_s}{r_w}\right) \\
&= 0.2 \ln\left(\frac{r_s}{r_w}\right) \quad (\text{in } \mu\text{H/m}) \\
&= 5.08 \ln\left(\frac{r_s}{r_w}\right) \quad (\text{in nH/in.})
\end{aligned}
$$

    - Thus, the exact per-unit capacitance could be obtained through

$$c = \frac{1}{l v^2}$$

The Per-Unit-Length Parameters

- Wire-Type Structures
  - Coaxial Cable



Also, you could derive C using

- Since $E_T = \dfrac{q}{2\pi\epsilon r}$
- The voltage between the inner wire and the interior surface of the shield is

$$V = -\int_{r_s}^{r_w} \vec{E}_T \cdot \vec{dl}$$

$$= \frac{q}{2\pi\epsilon}\ln\left(\frac{r_s}{r_w}\right)$$

- The exact per-unit-length capacitance is obtained as

$$c = \frac{q}{V} \qquad = \frac{55.56\epsilon_r}{\ln(r_s/r_w)} \quad \text{(in pF/m)}$$

$$= \frac{2\pi\epsilon}{\ln(r_s/r_w} = \frac{1.4\epsilon_r}{\ln(r_s/r_w)} \quad \text{(in pF/in.)}$$

The Per-Unit-Length Parameters

- Printed Circuit Board (PCB) Structures
  - Commonly Used Parameters
    - In PCB configurations, we usually use the characteristic impedance

$$Z_C = \sqrt{\frac{l}{c}} \quad \Omega$$

    - and the velocity of propagation

$$v = \frac{1}{\sqrt{lc}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r'}} = \frac{v_0}{\sqrt{\epsilon_r'}} \quad \text{m/s}$$

$$\left\{ \begin{aligned} l &= \frac{Z_C}{v} \\ c &= \frac{1}{v Z_C} \end{aligned} \right.$$

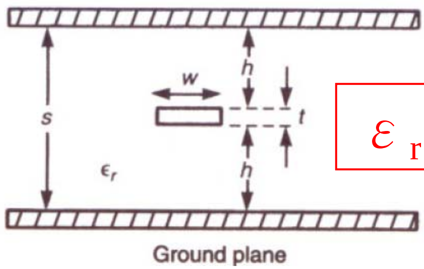
where ε_r' is an effective relative permittivity or effective dielectric constant

The Per-Unit-Length Parameters

- Printed Circuit Board (PCB) Structures
  - Commonly Used Parameters
    - Generally, exact per-unit-length parameters cannot be determined in formula form but can be obtained as approximate relations.
    - Some are obtained by conformal mapping and some are obtained by numerical methods.
  - Stripline
    - Assuming t=0, the characteristic impedance is

$$Z_C = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{1}{\left[\frac{w_e}{s} + 0.441\right]}$$

$$\frac{w_e}{s} = \begin{cases} \frac{w}{s} & \frac{w}{s} \geq 0.35 \\ \frac{w}{s} - \left(0.35 - \frac{w}{s}\right)^2 & \frac{w}{s} \leq 0.35 \end{cases}$$



$$\boxed{\varepsilon_r' = \varepsilon_r}$$

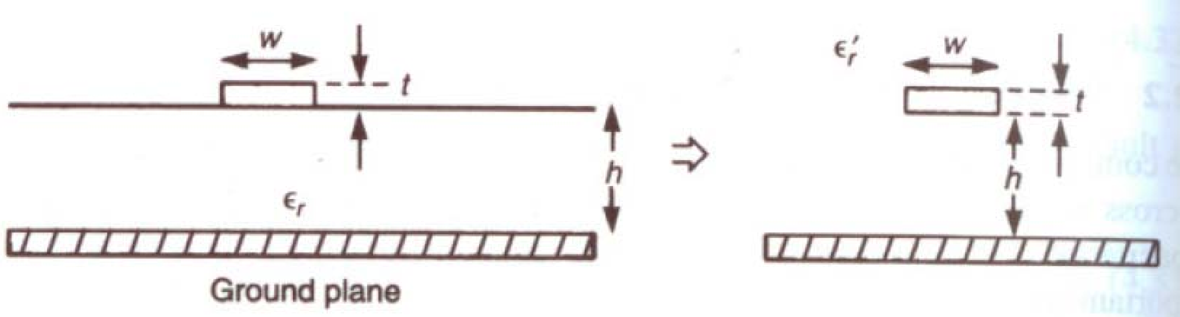
The Per-Unit-Length Parameters

- Printed Circuit Board (PCB) Structures
  - Microstrip Line
    - Assuming t=0, the characteristic impedance is

$$Z_C = \begin{cases} \dfrac{60}{\sqrt{\epsilon_r'}} \ln\left[\dfrac{8h}{w} + \dfrac{w}{4h}\right] & \dfrac{w}{h} \le 1 \\ \dfrac{120\pi}{\sqrt{\epsilon_r'}}\left[\dfrac{w}{h} + 1.393 + 0.667 \ln\left(\dfrac{w}{h} + 1.444\right)\right]^{-1} & \dfrac{w}{h} \ge 1 \end{cases}$$

    - where the effective relative permittivitv is

$$\epsilon_r' = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\frac{1}{\sqrt{1 + 10h/w}} \longrightarrow \epsilon_r' \cong \begin{cases} \epsilon_r & h \ll w \\ \dfrac{\epsilon_r + 1}{2} & h \gg w \end{cases}$$



The Per-Unit-Length Parameters

- Printed Circuit Board (PCB) Structures
  - Coplanar Strip
    - Assuming t=0, the characteristic impedance is

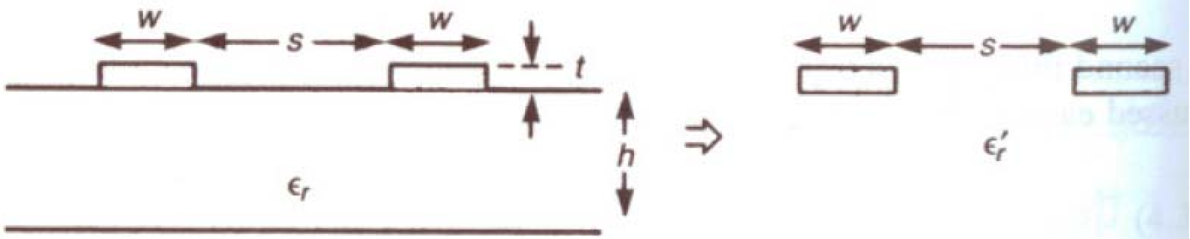
$$Z_C = \begin{cases} \dfrac{120}{\sqrt{\epsilon_r'}} \ln\left(2\dfrac{1+\sqrt{k}}{1-\sqrt{k}}\right) & \dfrac{1}{\sqrt{2}} \le k \le 1 \\ \dfrac{377\pi}{\sqrt{\epsilon_r'}\ln\left(2\dfrac{1+\sqrt{k'}}{1-\sqrt{k'}}\right)} & 0 \le k \le \dfrac{1}{\sqrt{2}} \end{cases}$$

$$k = \frac{s}{s+2w}$$

$$k' = \sqrt{1-k^2}$$

    - where the effective relative permittivity is

$$\epsilon_r' = \frac{\epsilon_r + 1}{2}\left\{\tanh\left[0.775\ln\left(\frac{h}{w}\right) + 1.75\right] + \frac{kw}{h}[0.04 - 0.7k + 0.01(1-0.1\epsilon_r)(0.25+k)]\right\}$$



The Per-Unit-Length Parameters

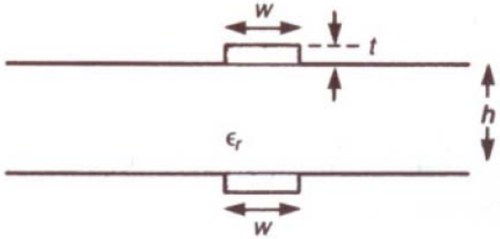
- Printed Circuit Board (PCB) Structures
  - Strips on Opposite Sides
    - Assuming t=0, the characteristic impedance is

$$Z_C = \frac{377}{\sqrt{\epsilon_r}\left\{\frac{w}{h} + 0.441 + \frac{\epsilon_r + 1}{2\pi\epsilon_r}\left[\ln\left(\frac{w}{h} + 0.94\right) + 1.451\right] + 0.082\frac{\epsilon_r - 1}{(\epsilon_r)^2}\right\}}$$

$$\text{for } \frac{w}{h} > 1$$

$$Z_C = \frac{377\sqrt{2}}{\pi\sqrt{\epsilon_r + 1}}\left[\ln\left(\frac{4h}{w}\right) + \frac{1}{8}\left(\frac{w}{h}\right)^2 - \frac{1}{2}\frac{\epsilon_r - 1}{\epsilon_r + 1}\left(0.452 + \frac{0.242}{\epsilon_r}\right)\right] \quad \text{for } \frac{w}{h} < 1$$

Pairs of wide lands placed on opposite sides of the board will give characteristic impedances lower than placing the lands on the same side of the board. Why?
Power plane need a low impedance. (v=Ldi/dt). Why?



The Time-Domain Solution

- Preview
  - Definition
    - The time-domain solution refers to the complete solution of those equations with no assumptions as to the time form of the line excitation.
    - The time-domain solution gives the total solution—transient plus steady state, which is often mistaken as transient solution only.
    - The time-domain solution is very important in solving the problems of signal integrity.

The Time-Domain Solution

- Graphical Solutions
  - Solution for Lossless Tx Line
    - The time-domain solution for the lossless Tx line is

$$V(z,t) = V^{+}\left(t - \frac{z}{v}\right) + V^{-}\left(t + \frac{z}{v}\right)$$

$$I(z,t) = \frac{1}{Z_C}V^{+}\left(t - \frac{z}{v}\right) - \frac{1}{Z_C}V^{-}\left(t + \frac{z}{v}\right)$$

    - where the characteristic impedance and velocity of propagation are

$$Z_C = \sqrt{\frac{l}{c}} = v\,l = \frac{1}{vc} \qquad v = \frac{1}{\sqrt{lc}} = \frac{1}{\sqrt{\mu\epsilon}}$$

V^+: forward-traveling wave
V^-: backward-traveling wave

The Time-Domain Solution

- Graphical Solutions
  - Solution for Lossless Tx Line
    - The current of each wave is related to the voltage of that wave by the characteristic impedance

$$I^+\left(t-\frac{z}{v}\right)=\frac{1}{Z_C}V^+\left(t-\frac{z}{v}\right)$$

$$I^-\left(t+\frac{z}{v}\right)=-\frac{1}{Z_C}V^-\left(t+\frac{z}{v}\right)$$

    - The forward- and backward- traveling waves are related at the load, *z=L*, by the ***load reflection coefficient*** as

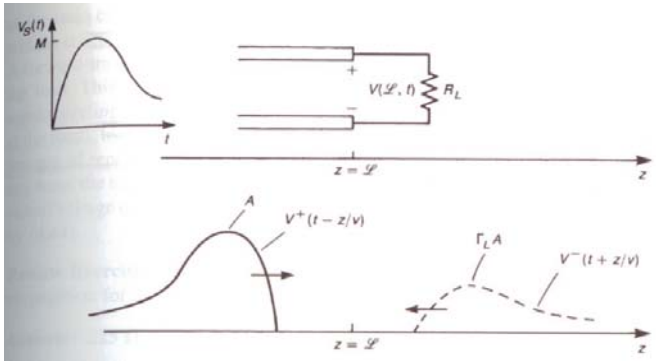
$$\Gamma_L=\frac{V^-(t+\mathscr{L}/v)}{V^+(t-\mathscr{L}/v)}=\frac{R_L-Z_C}{R_L+Z_C}$$

The Time-Domain Solution

- Graphical Solutions
  - Solution for Lossless Tx Line
    - Thus, we obtain

$$V^-\left(t+\frac{\mathscr{L}}{v}\right)=\Gamma_L V^+\left(t-\frac{\mathscr{L}}{v}\right)$$

$$I^-\left(t+\frac{\mathscr{L}}{v}\right)=-\Gamma_L I^+\left(t-\frac{\mathscr{L}}{v}\right)$$

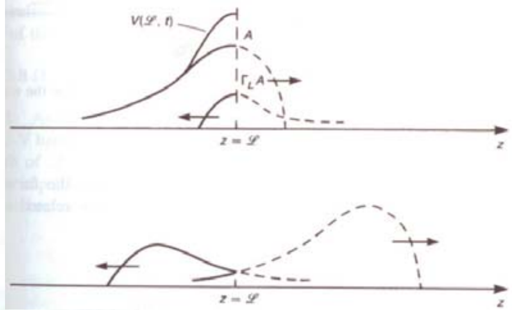


    - Note that the current reflection coefficient is the negative of the voltage reflection coefficient.
    - Since

$$V(0,t)=V^+\left(t-\frac{0}{v}\right)$$

$$I(0,t)=I^+\left(t-\frac{0}{v}\right)$$

$$=\frac{V^+(t-0/v)}{Z_C}\quad \text{for } 0\le t\le\frac{2\mathscr{L}}{v}$$

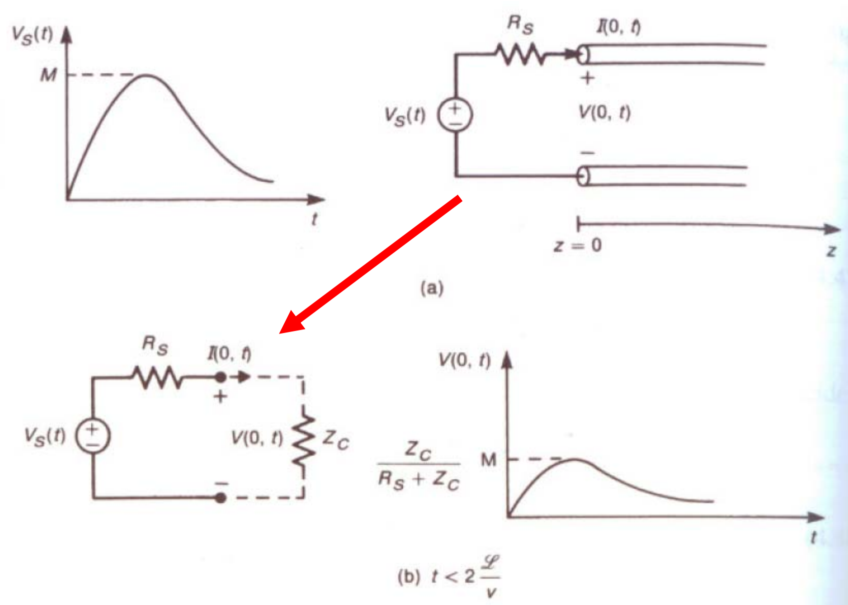


The Time-Domain Solution

- Graphical Solutions
  - Solution for Lossless Tx Line
    - Thus, the forward-traveling voltage and current waves that are initially launched are related to the source voltage by

$$V(0, t) = \frac{Z_C}{R_S + Z_C} V_S(t)$$

$$I(0, t) = \frac{V_S(t)}{R_S + Z_C}$$



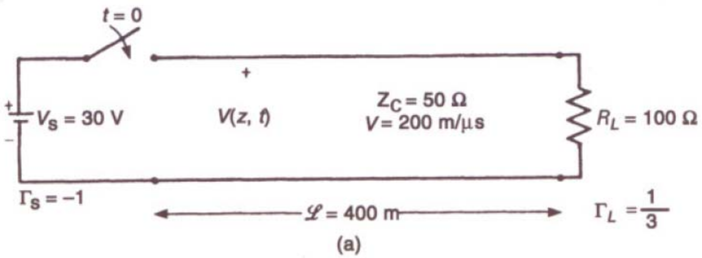
    - and

$$\Gamma_S = \frac{R_S - Z_C}{R_S + Z_C}$$

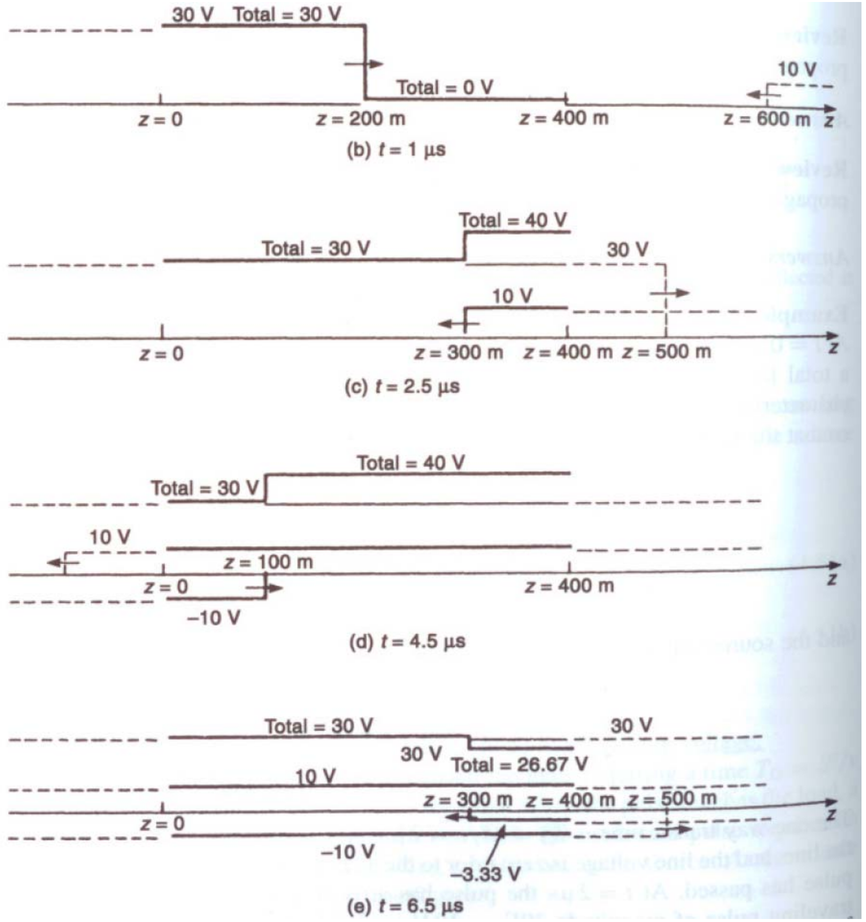
This process of repeated reflections continue as re-reflections at the source and load.

The Time-Domain Solution

- Graphical Solutions
  - Example 4.1 Voltage on a Line as a Function of *t*
    - $T_D = 2\,\mu s$

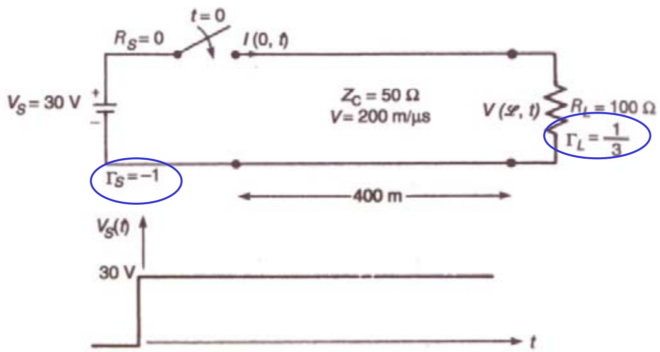


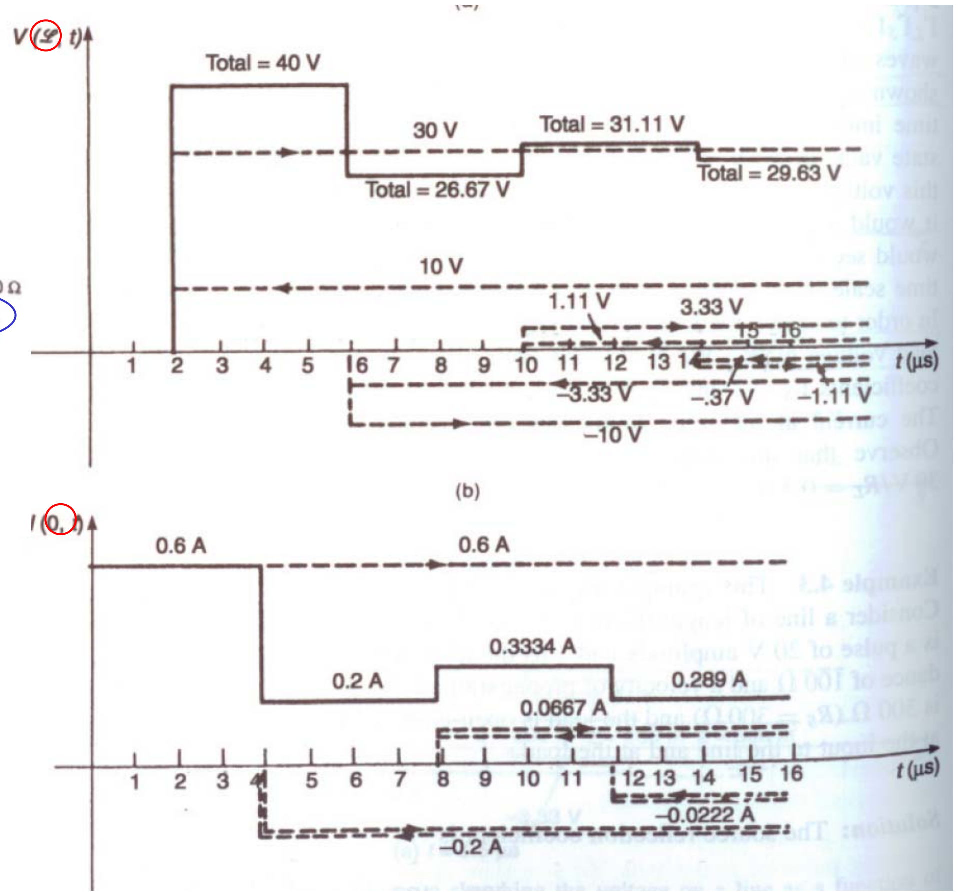
As time goes by, the load voltage will approach the steady state value of 30V



The Time-Domain Solution

- Graphical Solutions
  - Example 4.2 Voltage and Current on a Line as a Function of *t*
    - $T_D = 2\,\mu s$





For the current wave:
$\Gamma_S^I = 1,\ \Gamma_L^I = -1/3$

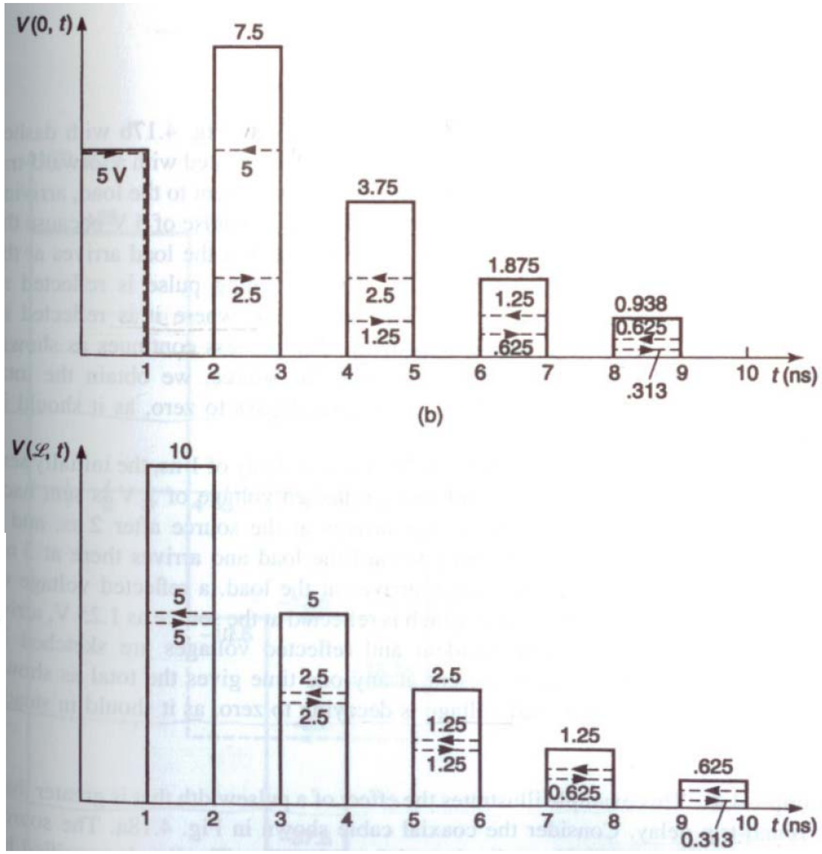
The current at $z=L$ could be obtained by $V(L,t)/R_L$

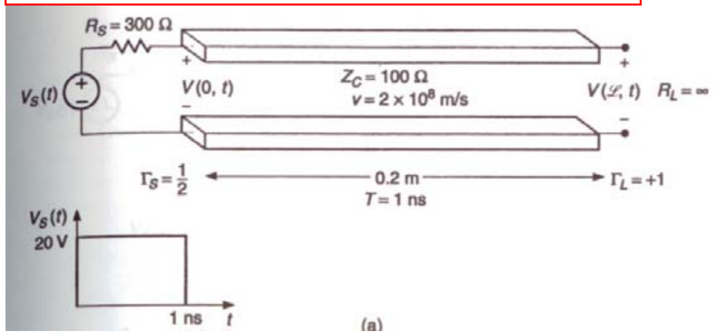
The Time-Domain Solution

- Graphical Solutions
  - Example 4.3 Short Pulsewidth
    - $T_D = 1\,\mu s$
    - The initially sent out V is

$$\frac{100}{300+100} \times 20 = 5\ \mathrm{V}$$

Both the total voltages of $V(0,t)$ and $V(L,t)$ decay to zero, as it should be in steady state

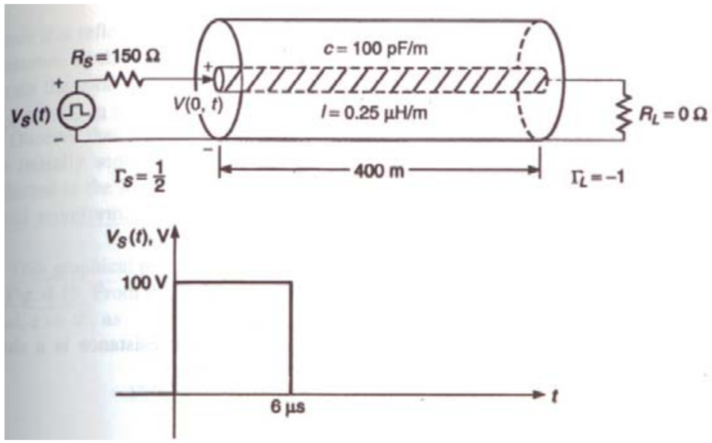


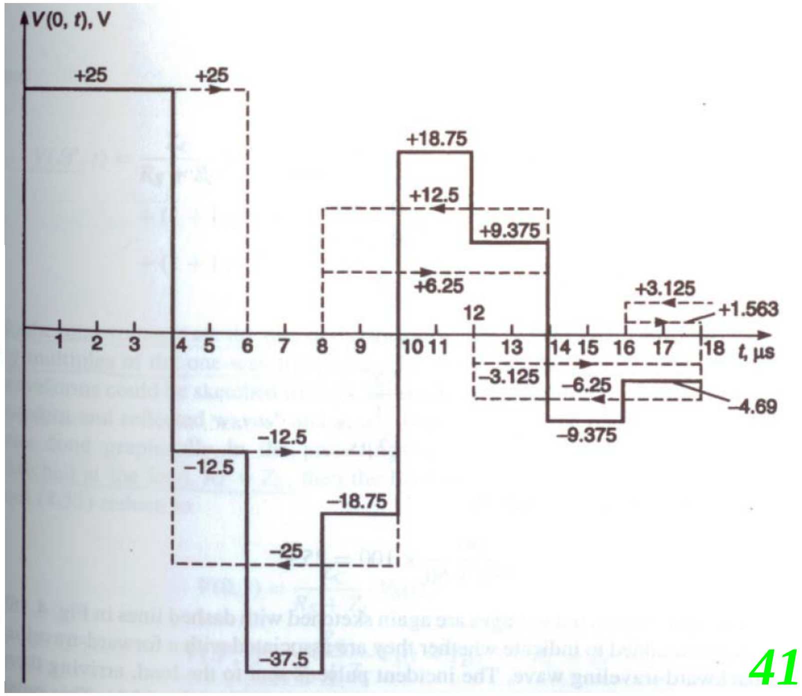


The Time-Domain Solution

- Graphical Solutions
  - Example 4.4 Long Pulsewidth
    - Z_C and v : $Z_C = \sqrt{\frac{l}{c}} = 50\,\Omega \quad v = \frac{1}{\sqrt{lc}} = 200\ \text{m}/\mu\text{s}$
    - The initially sent out V is
      $$\frac{50}{150+50} \times 100 = 25\ \text{V}$$
    - $T_D = L/v = 400/200 = 2\ \mu s$

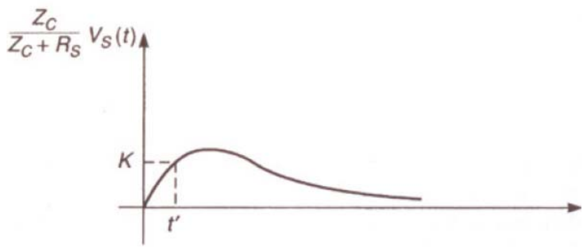
The overlap creates a rather interesting and complicated waveform. (ringing)

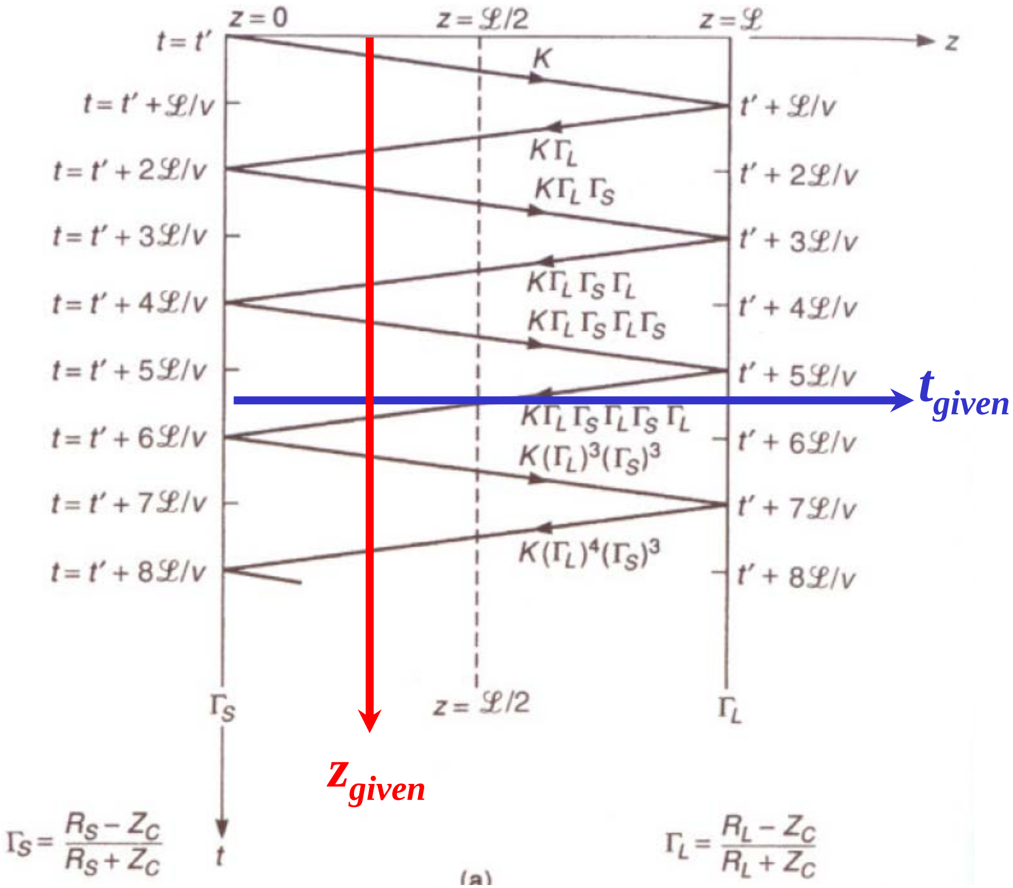




The Time-Domain Solution

- Graphical Solutions
  - Bounce or lattice Diagram





This diagram could easily be used to determine the $V(z, t_{given})$ or $V(z_{given}, t)$

Please see David K. Cheng, Field and Wave Electromagnetic, for details.

The Time-Domain Solution

- Graphical Solutions
  - Bounce or lattice Diagram
    - The expressions for the voltages at z=0 and z=L are

$$V(0,t) = \frac{Z_C}{R_S + Z_C}[V_S(t) + (1+\Gamma_S)\Gamma_L V_S(t-2T_D)$$
$$+ (1+\Gamma_S)(\Gamma_S\Gamma_L)\Gamma_L V_S(t-4T_D)$$
$$+ (1+\Gamma_S)(\Gamma_S\Gamma_L)^2\Gamma_L V_S(t-6T_D) + \cdots]$$

$$V(\mathscr{L},t) = \frac{Z_C}{R_S + Z_C}[(1+\Gamma_L)V_S(t-T_D) + (1+\Gamma_L)\Gamma_S\Gamma_L V_S(t-3T_D)$$
$$+ (1+\Gamma_L)(\Gamma_S\Gamma_L)^2 V_S(t-5T_D)$$
$$+ (1+\Gamma_L)(\Gamma_S\Gamma_L)^3 V_S(t-7T_D) + \cdots]$$

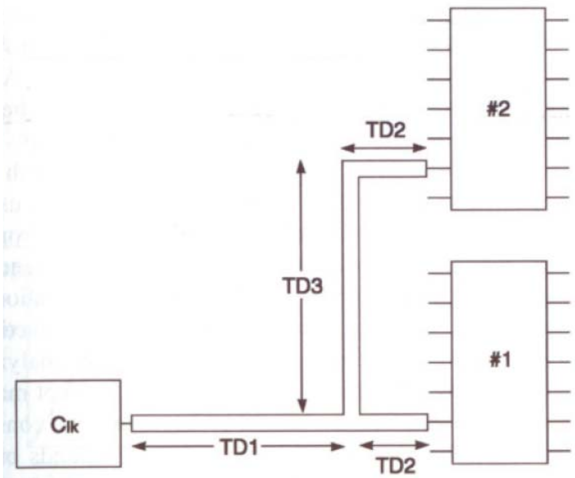
- which reduces to

$$V(0,t) = \frac{Z_C}{R_S + Z_C}V_S(t) \qquad R_L = Z_C$$
$$V(\mathscr{L},t) = \frac{Z_C}{R_S + Z_C}V_S(t-T_D) \qquad R_L = Z_C$$

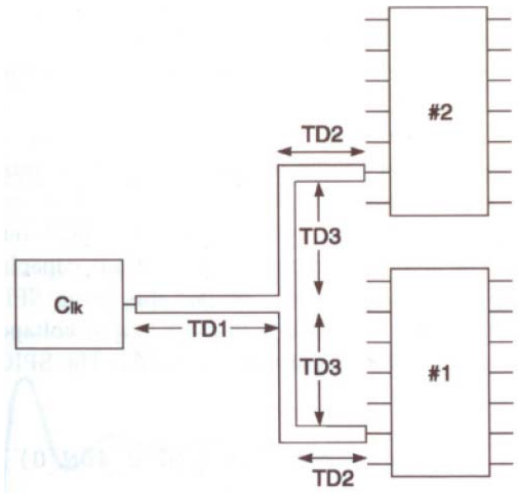
Only a time delay is imposed, the line "doesn't matter."

High-Speed Digital Interconnects and Signal Integrity

- Preview
  - Clock Skew
    - The time delay in traversing the lands is becoming a critical factor in the overall timing budget of the system with the time delays becoming on the order of the pulse rise/falltimes.



Time delays for clock 1 and 2 are not equal.



Time delays for clock 1 and 2 are equal.

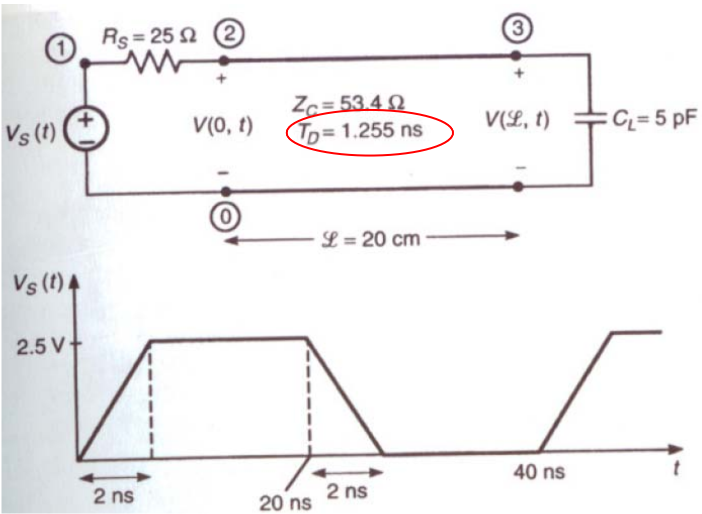
High-Speed Digital Interconnects and Signal Integrity

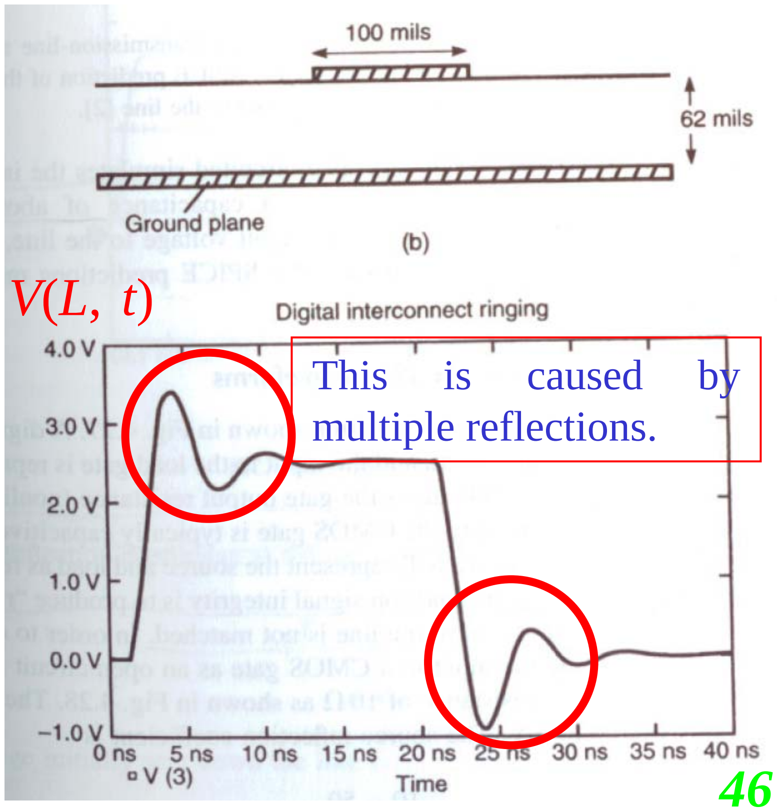
- Preview
  - Impedance Mismatch
    - A mismatched source and load will cause reflections, resulting in a distorted signal.
    - When the cross-sectional dimensions of transmission line change, the characteristic impedance of the line will change and hence we will have reflections at that discontinuity.
    - A signal passing from one layer to another along a via will have encountered a discontinuity and hence a change in characteristic impedance.

High-Speed Digital Interconnects and Signal Integrity

- Preview
  - A Typical Signal Integrity Problem
    - The mismatch of the line will cause a ringing effect.

Typical CMOS gates:
Output resistance: 10-30 Ω
Input capacitance: 5-15 pF

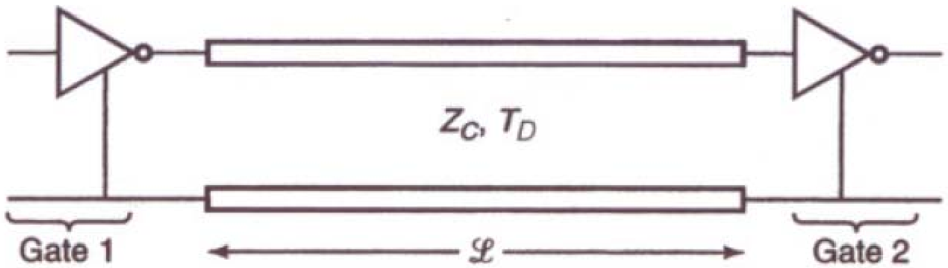




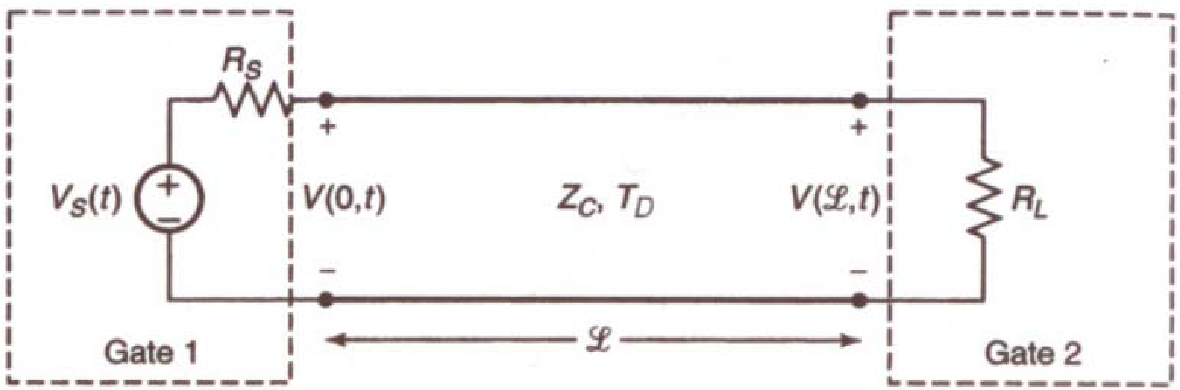
This is caused by multiple reflections.

High-Speed Digital Interconnects and Signal Integrity

- Effect of Resistive Termination
  - A Typical Digital Application of TX Line
    - A line connects two digital gates



(a)

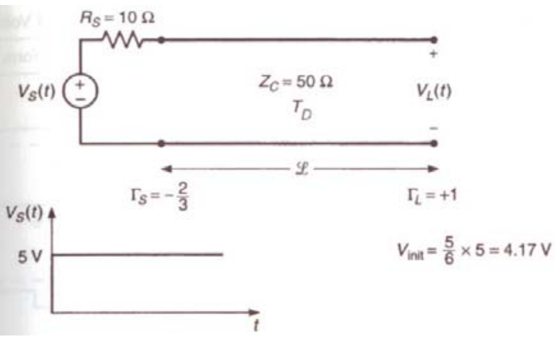


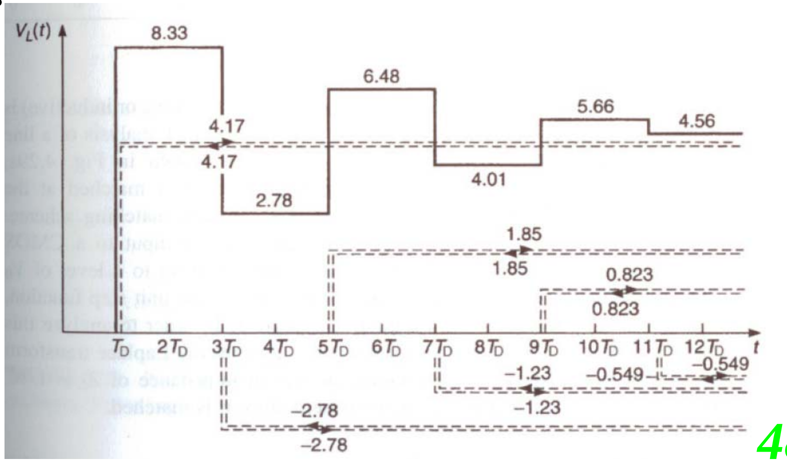
High-Speed Digital Interconnects and Signal Integrity

- Effect of Resistive Termination
  - A Typical Digital Application of TX Line
    - The load voltage could be written as

$$V(\mathscr{L},t) = \frac{Z_C}{R_S + Z_C}(1+\Gamma_L)[V_S(t-T_D) + \Gamma_S\Gamma_L V_S(t-3T_D) + (\Gamma_S\Gamma_L)^2 V_S(t-5T_D) + (\Gamma_S\Gamma_L)^3 V_S(t-7T_D) + \cdots]$$

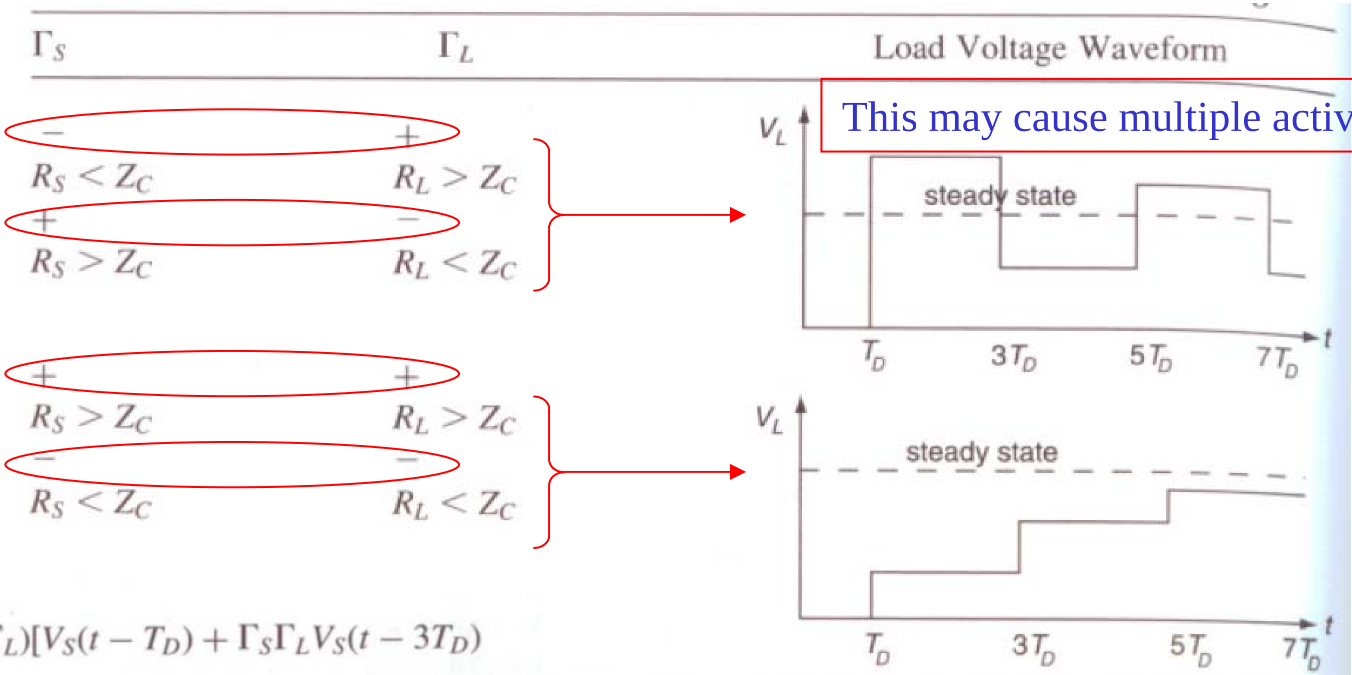
    - The mismatch of the line constitutes ringing and can cause the received voltage to lie outside the levels for a logic 0 and a logic 1.





High-Speed Digital Interconnects and Signal Integrity

- Effect of Resistive Termination
  - A Typical Digital Application of TX Line
    - Effects of the signs of the reflection coefficients on the load voltage.



This may cause multiple activations.

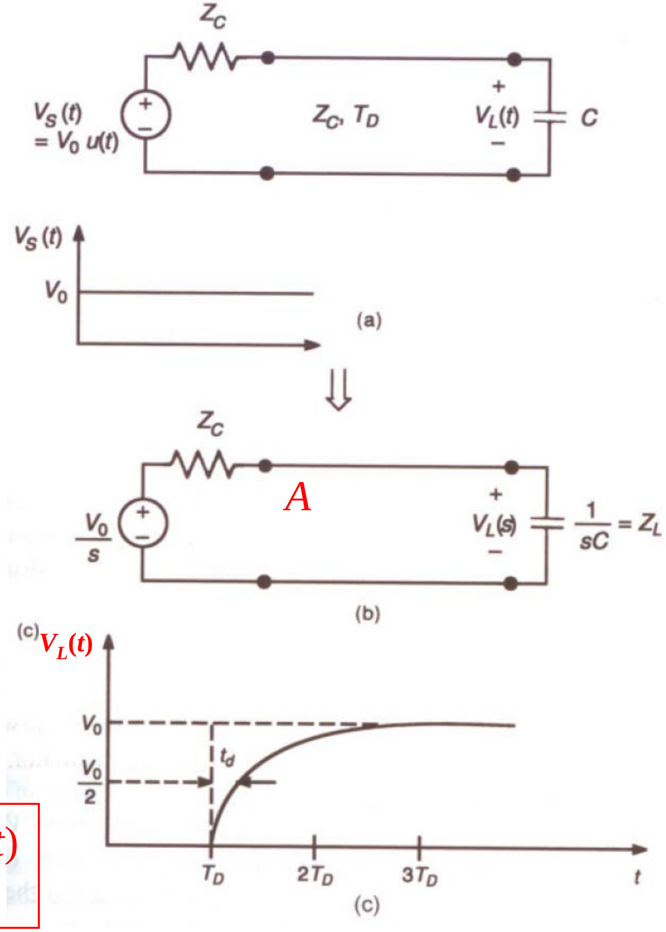
$$V(\mathscr{L},t) = \frac{Z_C}{R_S+Z_C}(1+\Gamma_L)[V_S(t-T_D)+\Gamma_S\Gamma_L V_S(t-3T_D) + (\Gamma_S\Gamma_L)^2 V_S(t-5T_D)+(\Gamma_S\Gamma_L)^3 V_S(t-7T_D)+\cdots]$$

This may postpone activation.

High-Speed Digital Interconnects and Signal Integrity

- Effect of Capacitive Termination
  - A Line Terminated in a Capacitive Load
    - Since $\Gamma_S = 0$

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{\frac{1}{sC} - Z_C}{\frac{1}{sC} + Z_C} = \frac{1 - sT_C}{1 + sT_C}$$



    - where the time constant

$$T_C = Z_C C$$

Known $V_L(t) = (V_0/2)u(t-T_D) + V_r(t)$ → $V_r(t)$
→ TDR: plot $V_A(t) = (V_0/2)u(t) + V_r(t)$

High-Speed Digital Interconnects and Signal Integrity

- Effect of Capacitive Termination
  - A Line Terminated in a Capacitive Load
    - Since the source is matched, we will only have a forward-traveling incident at the load and a reflected wave traveling back to the source.

$$V(\mathscr{L},t)=\frac{Z_C}{R_S+Z_C}(1+\Gamma_L)[V_S(t-T_D)+\cancel{\Gamma_S\Gamma_L V_S(t-3T_D)}$$

$$+\cancel{(\Gamma_S\Gamma_L)^2 V_S(t-5T_D)}+\cancel{(\Gamma_S\Gamma_L)^3 V_S(t-7T_D)}+\cdots]$$

$$\rightarrow \quad V_L(t)=(1+\Gamma_L)\frac{Z_C}{(R_S=Z_C)+Z_C}V_0u(t-T_D)$$

- Transforming this to Laplace domain, we have

$$V_L(s)=(1+\Gamma_L(s))\frac{1}{2}V_S(s)e^{-sT_D}$$

$$=\frac{\frac{1}{T_C}}{\left(s+\frac{1}{T_C}\right)s}V_0e^{-sT_D}=\left[\frac{1}{s}-\frac{1}{\left(s+\frac{1}{T_C}\right)}\right]V_0e^{-sT_D}$$

High-Speed Digital Interconnects and Signal Integrity

- Effect of Capacitive Termination
  - A Line Terminated in a Capacitive Load
    - Inverse transforming gives

$$V_L(t) = V_0 u(t - T_D) - e^{-(t-T_D)/T_C} V_0 u(t - T_D)$$

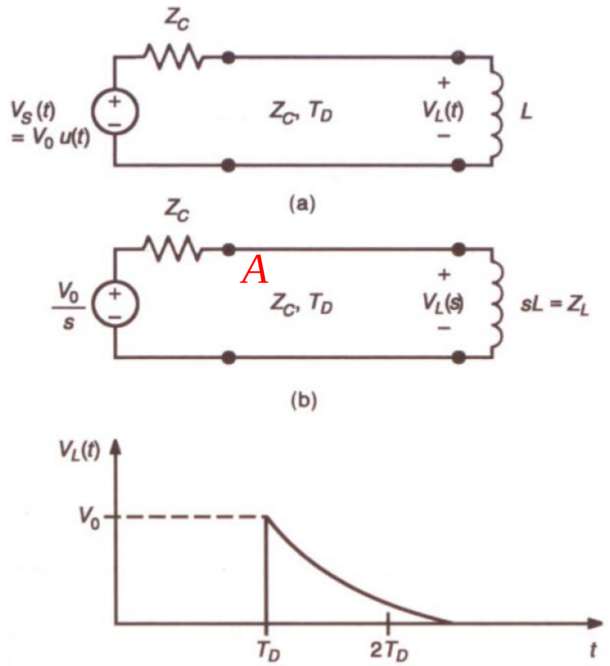
- Initially the capacitor looks like a short circuit gradually transitioning to an open circuit.
- The effect of the capacitor is to introduce a an additional time delay (measured at the 50% point) t_d, thereby giving the load voltage a risetime when the source had none.

$$\begin{aligned} t_d &= 0.693 T_C \\ &= 0.693 C Z_C \end{aligned}$$

High-Speed Digital Interconnects and Signal Integrity

- Effect of Inductive Termination
  - A Line Terminated in an Inductive Load
    - Since $\Gamma_S = 0$

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{sL - Z_C}{sL + Z_C} = \frac{sT_L - 1}{sT_L + 1}$$



    - where the time constant is

$$T_L = \frac{L}{Z_C}$$

    - Similarly, the transformed load voltage is

$$V_L(s) = (1 + \Gamma_L(s))\frac{1}{2}V_S(s)e^{-sT_D} = \frac{1}{s + \frac{1}{T_L}}V_0 e^{-sT_D}$$

$$V_L(t) = V_0 e^{-(t-T_D)/T_L}u(t - T_D)$$

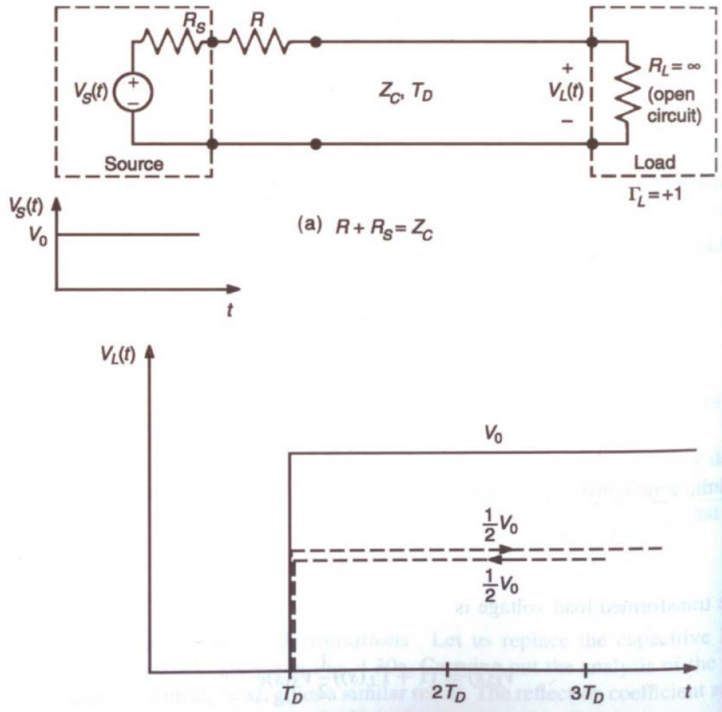
Known $V_L(t) = (V_0/2)u(t-T_D) + V_r(t) \rightarrow V_r(t)$
$\rightarrow$ TDR: plot $V_A(t) = (V_0/2)u(t) + V_r(t)$

Initially the inductor looks like an open circuit gradually becoming a short circuit.

High-Speed Digital Interconnects and Signal Integrity

- Matching Schemes for Signal Integrity
  - Series Match
    - The most common matching is the series match which is shown below.
    - A resistor R is added in series with the source such that

$$R_S + R = Z_C$$



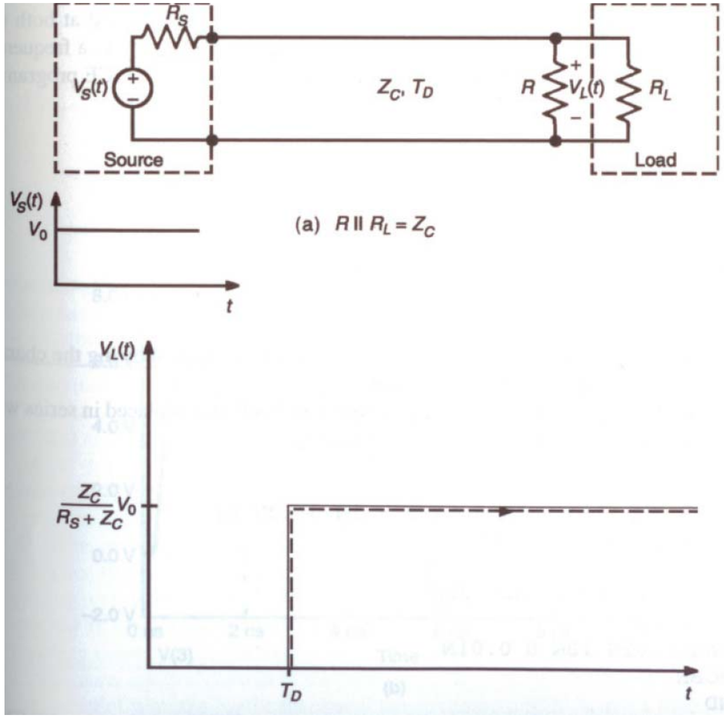
1. We see that the output has the same level as the input.
2. No current flows in the line and the resistor R and therefore the resistor dissipates no power.

High-Speed Digital Interconnects and Signal Integrity

- Matching Schemes for Signal Integrity
  - Parallel Match
    - A resistor R is added in parallel with the load such that

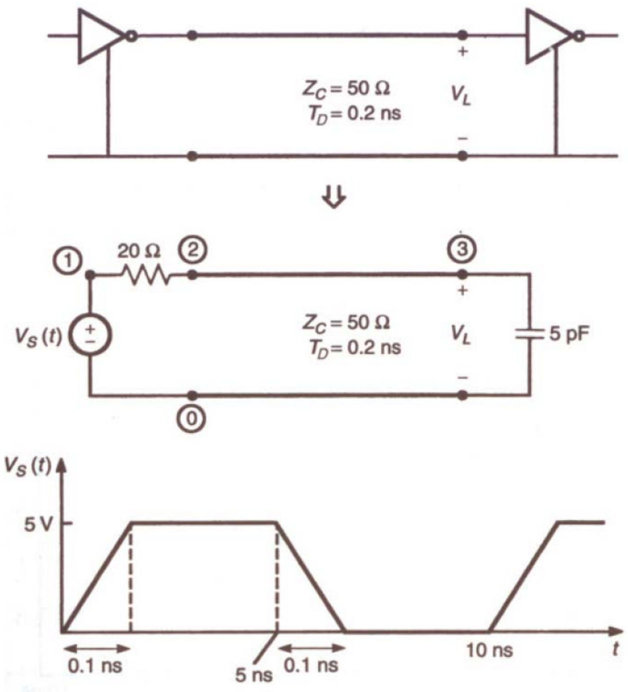
$$R \parallel R_L = \frac{RR_L}{R + R_L} = Z_C$$

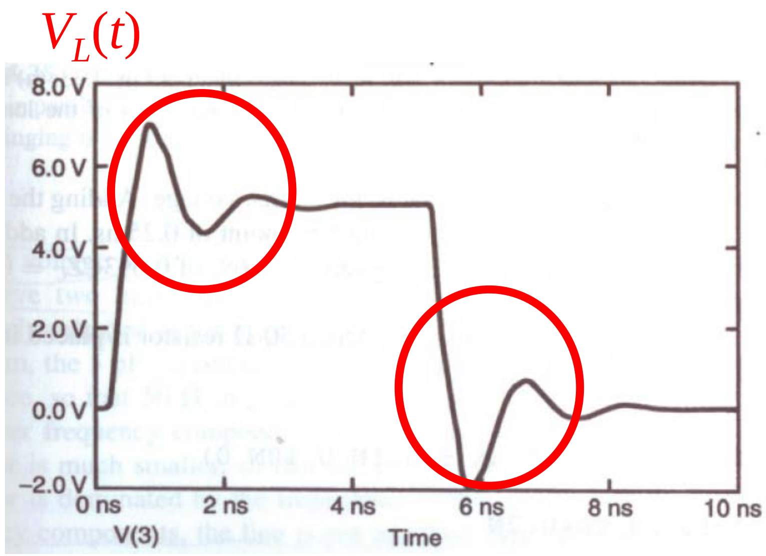
1. We see that the load voltage is always less than the source level V_0.
2. Even for an open-circuited load, the line will draw current when the source is in the high state. Hence the matching resistor R will consume power.



High-Speed Digital Interconnects and Signal Integrity

- Matching Schemes for Signal Integrity
  - Example for Series and Parallel Match
    - For the circuit without match, the load voltage shows the characteristic ringing caused by the mismatches.

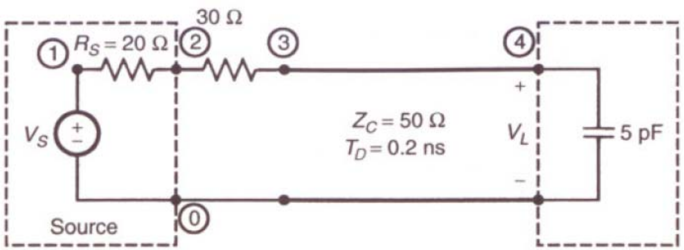




High-Speed Digital Interconnects and Signal Integrity

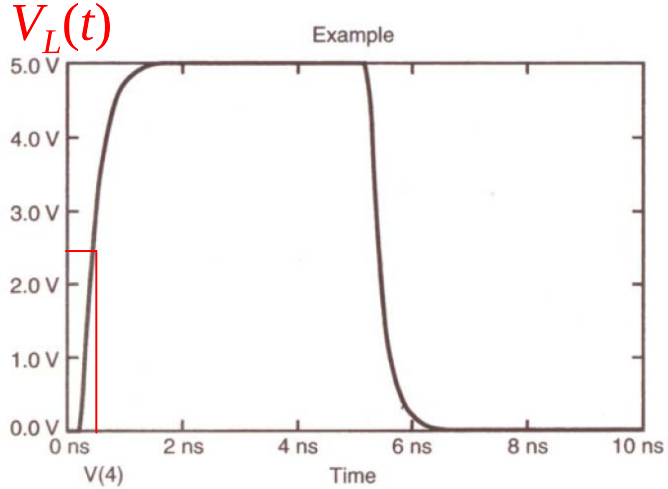
- Matching Schemes for Signal Integrity
  - Example for Series and Parallel Match
    - For the circuit with series match, the load voltage smoothly rises to the desired 5-V level.

At 50% point of the pulse there are 3 delays:
(1). 0.05ns: rise time of the source voltage (0.1/2)
(2). 0.2ns: one-way time delay (T_D)
(3). 0.1733ns ($0.693CZ_C$): capacitive load delay
The total delay is the sum of the above three values, which is 0.4233ns.



(a)

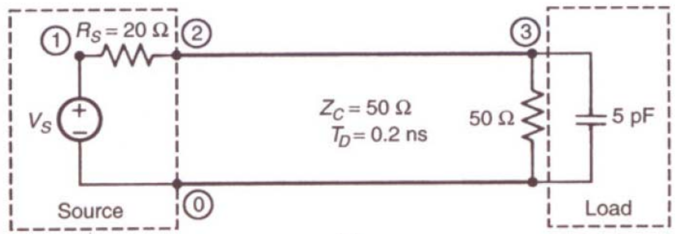
$V_L(t)$



High-Speed Digital Interconnects and Signal Integrity

- Matching Schemes for Signal Integrity
  - Example for Series and Parallel Match
    - For the circuit with parallel match, the load voltage still has ringing.

1. The line is not matched at the higher frequencies since the capacitor dominates and shows an short circuit at these higher frequencies.
2. The steady state level of the load voltage falls significantly below the desired 5-V level, and logic errors may occur.



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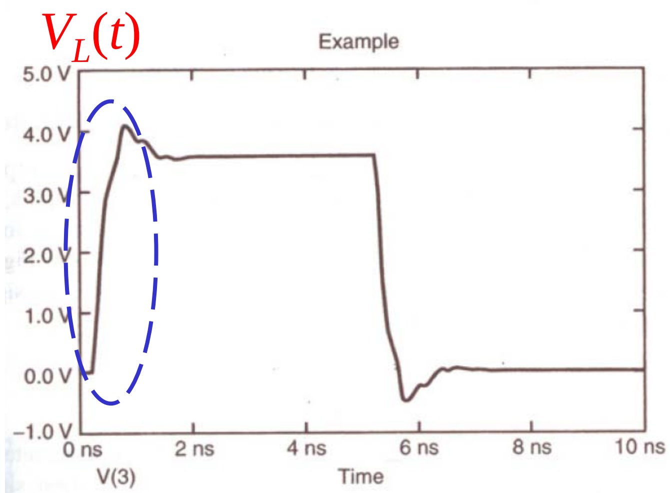


High-Speed Digital Interconnects and Signal Integrity

- When Does the Line Not Matter?
  - Criterion
    - When the line is electrically short at the highest significant frequency, the distributed parameter effects of the line is negligible.

$$\mathscr{L} < \frac{1}{10}\frac{v}{f_{\max}} \quad \lambda \quad \xrightarrow[\mathrm{BW} = \frac{1}{\tau_r}]{f_{\max} = 1/\tau_r} \quad T_D = \frac{\mathscr{L}}{v} < \frac{1}{10}\tau_r$$

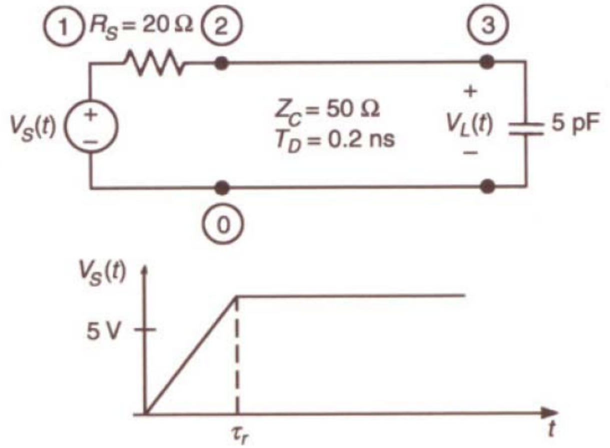
$$\downarrow$$

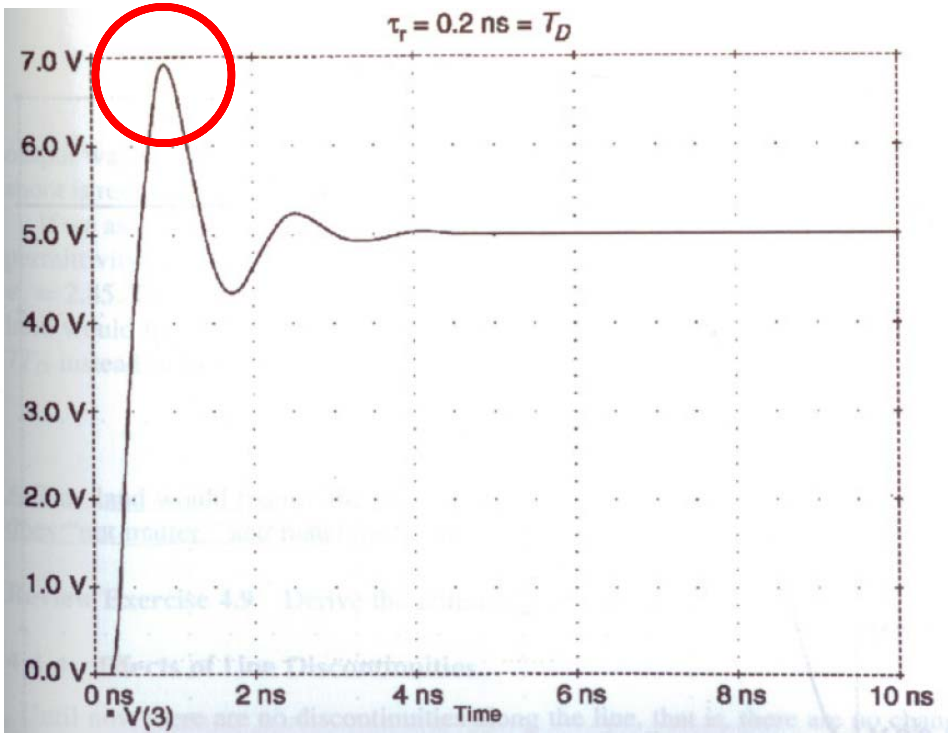
$$\boxed{\tau_r > 10T_D}$$

    - If the pulse risetime is greater than 10 times one-way time delays of the line, the line and any mismatches would not significantly degrade the output waveform.

High-Speed Digital Interconnects and Signal Integrity

- When Does the Line Not Matter?
  - Example of $\tau_r = kT_D$
    - When $\tau_r = T_D$, the overshoot is 7 V.



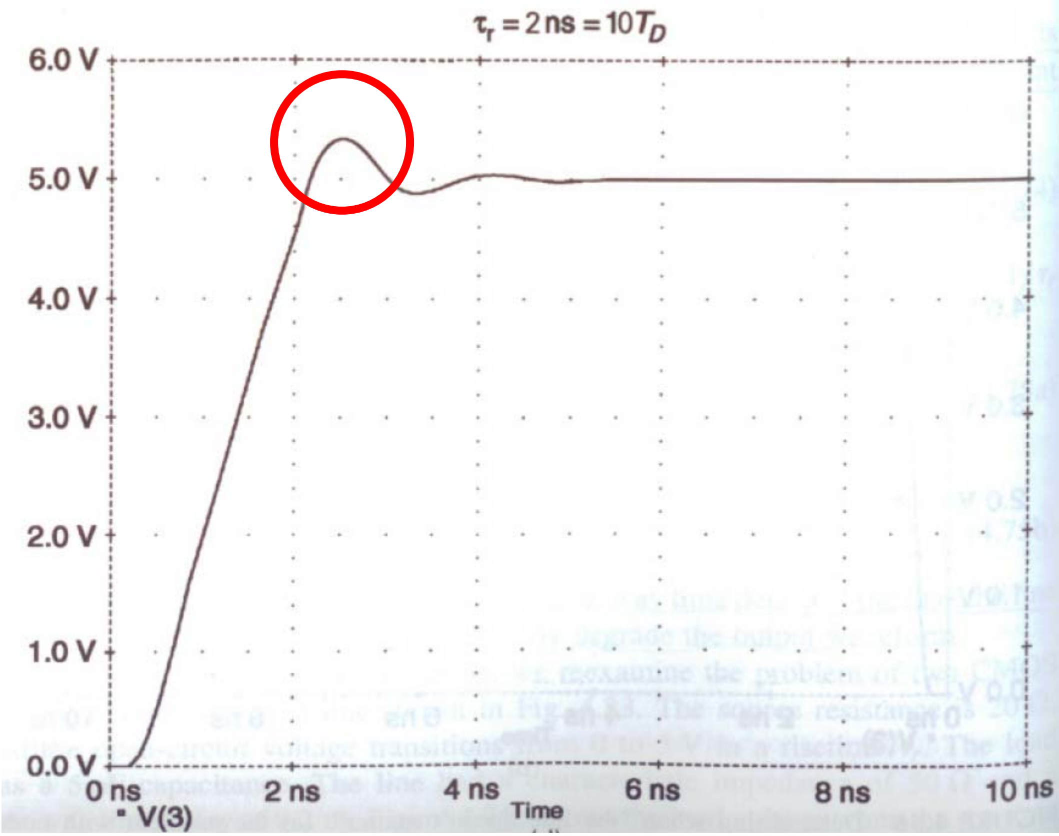


High-Speed Digital Interconnects and Signal Integrity

- When Does the Line Not Matter?
  - Example of $\tau_r = kT_D$
    - When $\tau_r = 5T_D$, the overshoot reduces to 6 V.

High-Speed Digital Interconnects and Signal Integrity

- When Does the Line Not Matter?
  - Example of $\tau_r = kT_D$
    - When $\tau_r = 10T_D$, the overshoot reduces to 5.3 V.

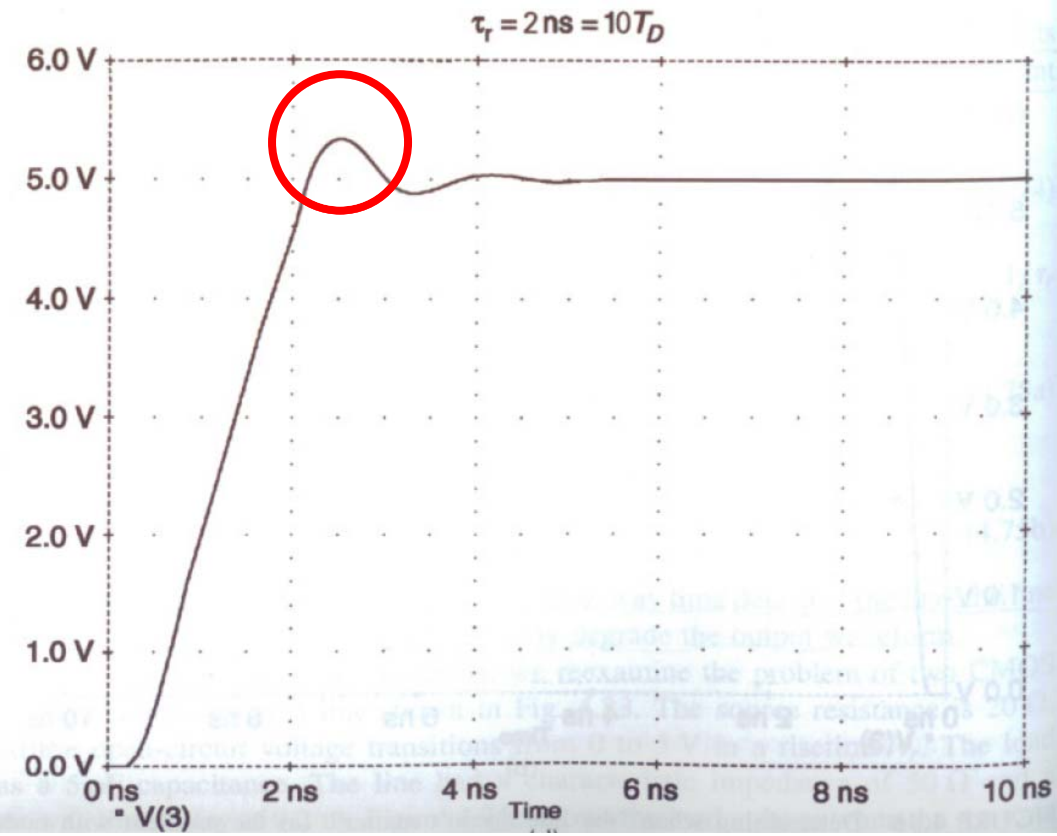


High-Speed Digital Interconnects and Signal Integrity

- When Does the Line Not Matter?
  - – Example of $\tau_r = kT_D$
    - When $\tau_r = 20T_D$, the overshoot reduces to 5.2 V.

$$f_{\max} = 1/\tau_r$$

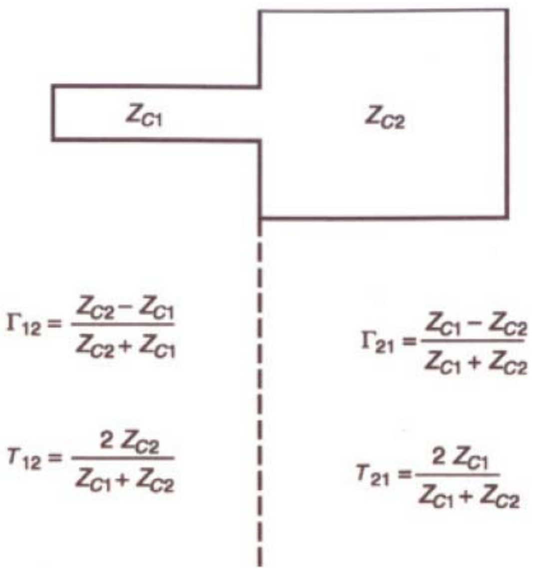
$\tau_r \uparrow \rightarrow f_{max} \downarrow$
(The high frequencies components are reduced.)

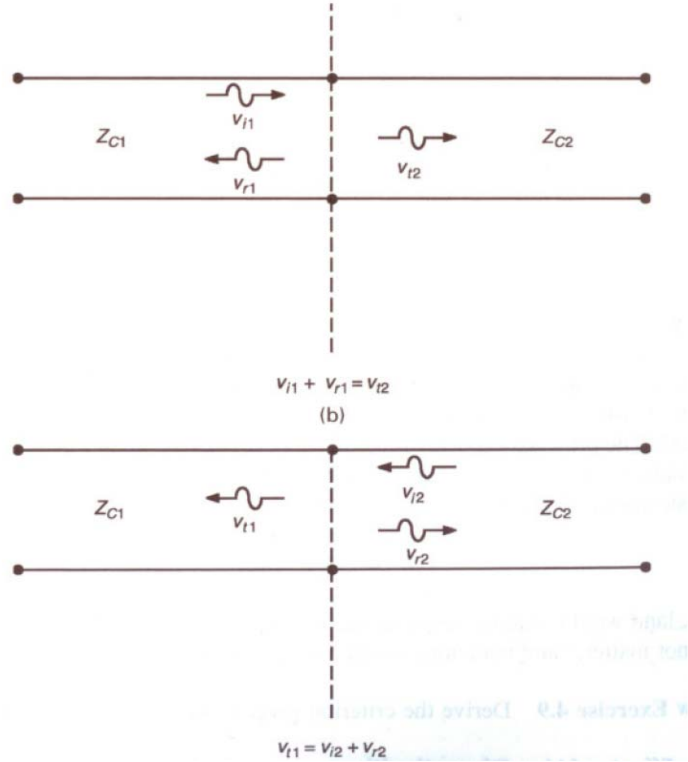


High-Speed Digital Interconnects and Signal Integrity

- Effects of Line Discontinuities
  - – Discontinuities Caused by Different Characteristic Impedances
    - The reflection incident from the left is

$$\Gamma_{12} = \frac{Z_{C2} - Z_{C1}}{Z_{C2} + Z_{C1}}$$





High-Speed Digital Interconnects and Signal Integrity

- Effects of Line Discontinuities
  - Discontinuities Caused by Different Characteristic Impedances
    - Since the total voltage on each side of the junction must be equal, we have

$$v_{i1} + v_{r1} = v_{t2} \longrightarrow 1 + \Gamma_{12} = T_{12} \longrightarrow T_{12} = \frac{2Z_{C2}}{Z_{C2} + Z_{C1}}$$

    - Similarly, the reflection incident from the right is

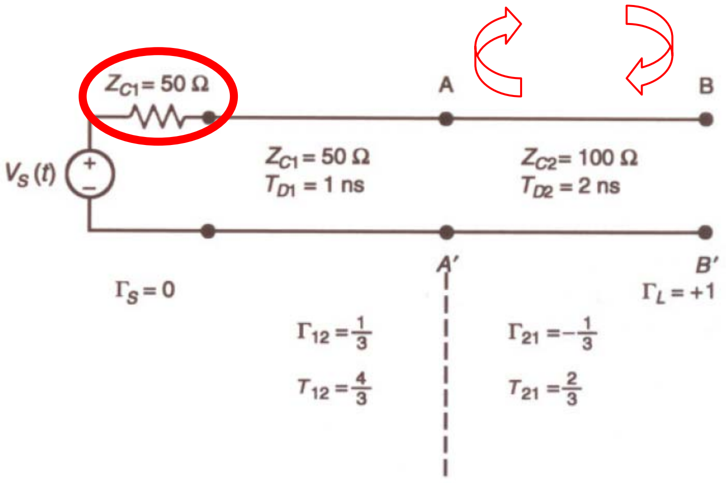
$$\Gamma_{21} = \frac{Z_{C1} - Z_{C2}}{Z_{C1} + Z_{C2}}$$

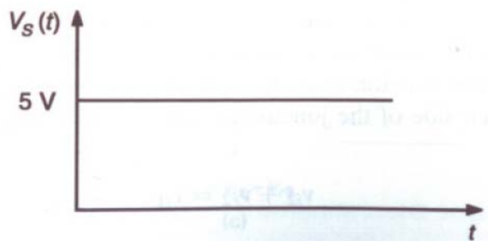
    - And since the total voltage on each side of the junction must be equal, we have

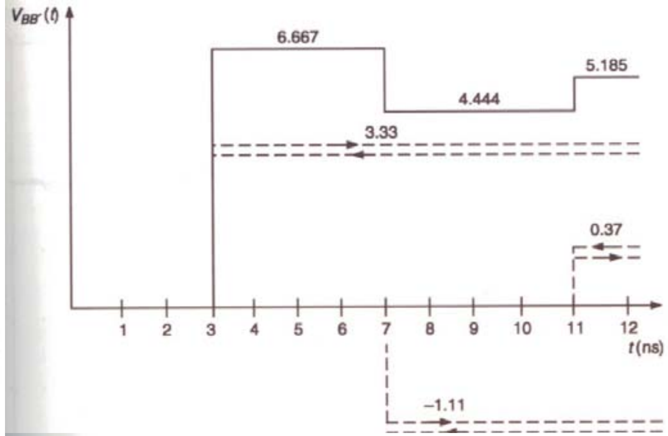
$$v_{i2} + v_{r2} = v_{t1} \longrightarrow 1 + \Gamma_{21} = T_{21} \longrightarrow T_{21} = \frac{2Z_{C1}}{Z_{C1} + Z_{C2}}$$

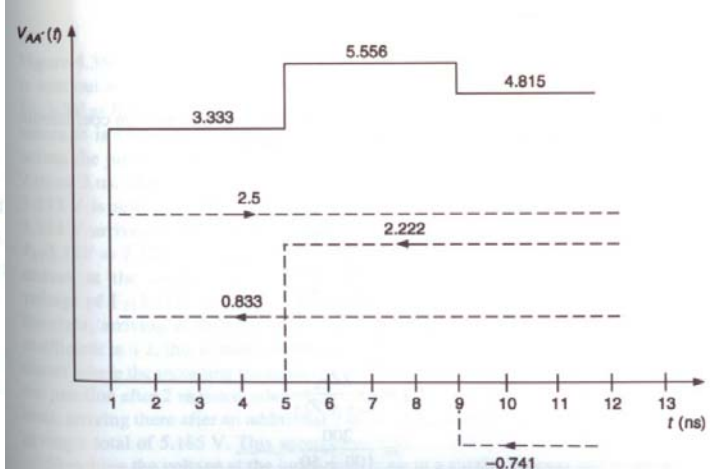
High-Speed Digital Interconnects and Signal Integrity

- Effects of Line Discontinuities
  - Example 4.8 A Typical Case with Series Match
    - We see that series match only at the source does not eliminate reflections.



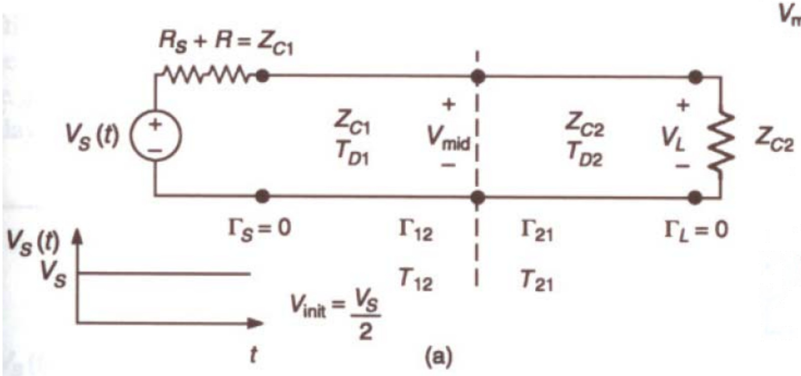


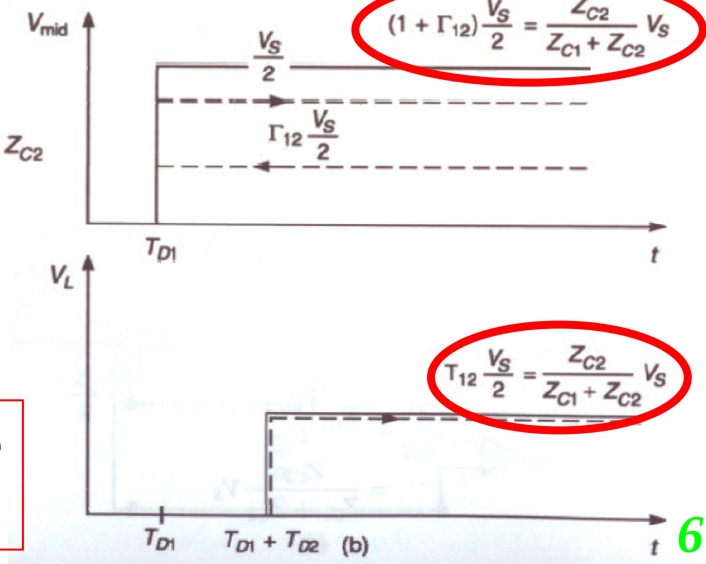




High-Speed Digital Interconnects and Signal Integrity

- Effects of Line Discontinuities
  - Example 4.9 Series and Parallel Match Both
    - In addition, with parallel match only at the load, the reflections are not eliminated also.
    - Hence, in order to eliminate reflections on lines having discontinuities, we must have series match at the source and parallel match at the load.

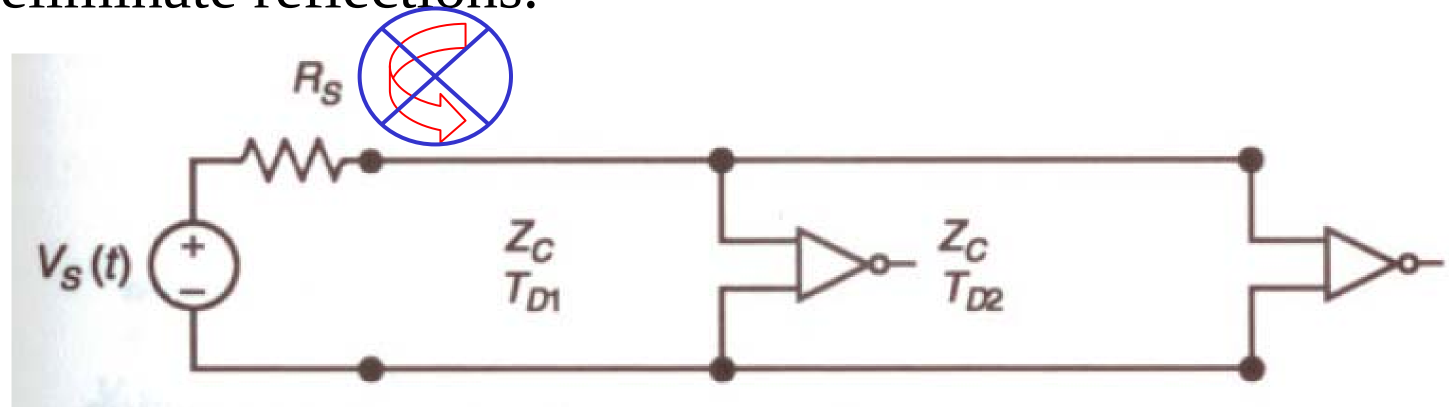




The values in the circle are the same steady-state values.

High-Speed Digital Interconnects and Signal Integrity

- Effects of Feedline Discontinuities
  - Series Multiple Feedlines
    - Clearly, a series match at the source, $R_S+R=Z_C$, will eliminate all reflections. (this would not be the case if the line characteristic impedances were not equal.)
    - We could also have parallel match at the load to eliminate reflections.

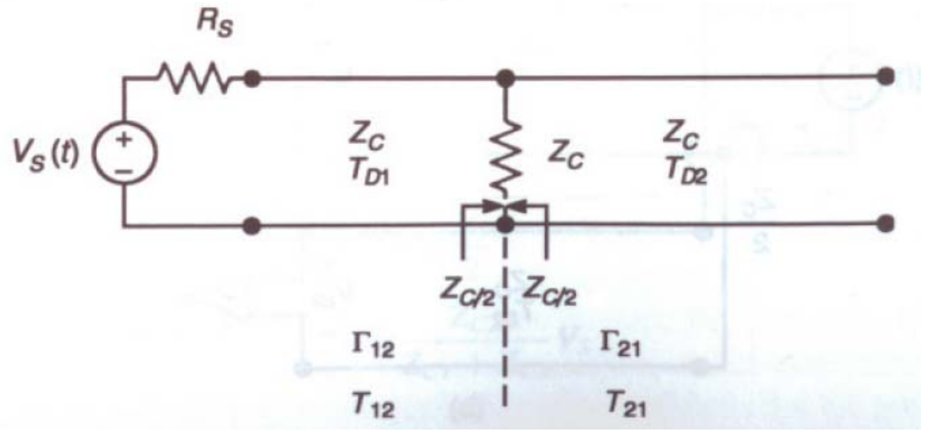


High-Speed Digital Interconnects and Signal Integrity

- Effects of Feedline Discontinuities
  - Series Multiple Feedlines
    - A parallel match at the midpoint would not eliminate the reflections.
    - Since

$$\Gamma_{12} = \frac{(Z_C/2) - Z_C}{(Z_C/2) + Z_C} = \Gamma_{21} = -\frac{1}{3} \qquad T_{12} = \frac{2(Z_C/2)}{(Z_C/2) + Z_C} = T_{21} = \frac{2}{3}$$

Hence there will be multiple reflections and transmissions at the junction.

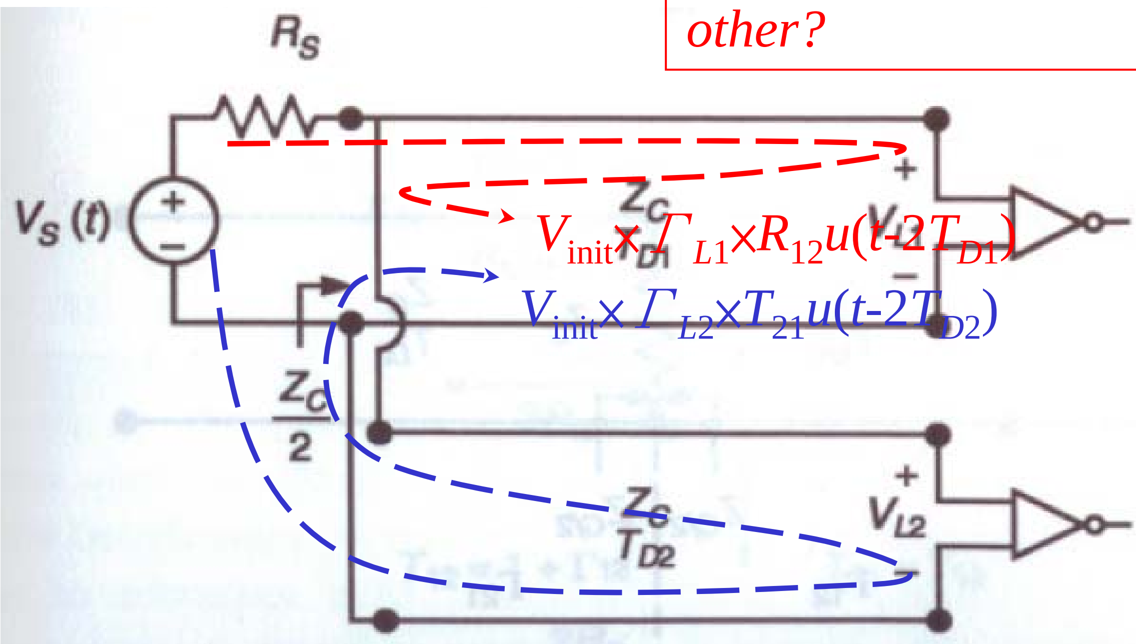


High-Speed Digital Interconnects and Signal Integrity

- Effects of Feedline Discontinuities
  - Parallel Multiple Feedlines
    - The initially sent out voltage for each parallel line is

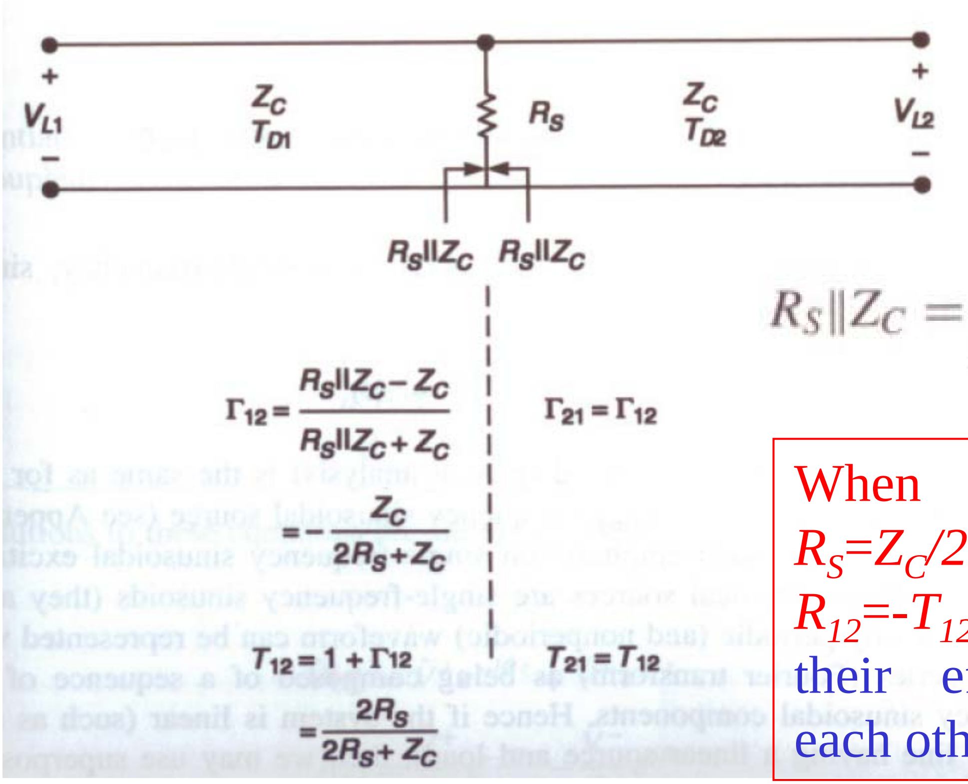
$$V_{\text{init}} = \frac{Z_C/2}{R_S + (Z_C/2)} V_S$$

How could these two signals cancel with each other?



High-Speed Digital Interconnects and Signal Integrity

- Effects of Feedline Discontinuities
  - Parallel Multiple Feedlines
    - If we see from the output of the two parallel lines, the reflection and transmission coefficients are



$$R_S \| Z_C = \frac{R_S Z_C}{R_S + Z_C}$$

When $T_{D1}=T_{D2}$ and $R_S=Z_C/2$ → we have $R_{12}=-T_{12}=-T_{21}$ Thus, their effects cancel each other.

High-Speed Digital Interconnects and Signal Integrity

- Effects of Feedline Discontinuities
  - Parallel Multiple Feedlines
    - First, suppose that we use a series match at the source, making $R_S=Z_C$, the reflections would not be eliminated.
    - Second, suppose series-match at the source but choose $R_S=Z_C/2$, the reflections would not be eliminated only if the two time delays are equal ($T_{D1}=T_{D2}$).
    - An alternative matching scheme would be to parallel match at the loads, placing resistors of value Z_C across the loads. Due to power loss, the parallel match is less desirable than the series match.

Sinusoidal Excitation of the Line and the Phasor Solution

- Phasor Form of Lossless TX Line
  - Sinusoidal Steady-State Analysis of TX Lines
    - From the results of time-domain, we have

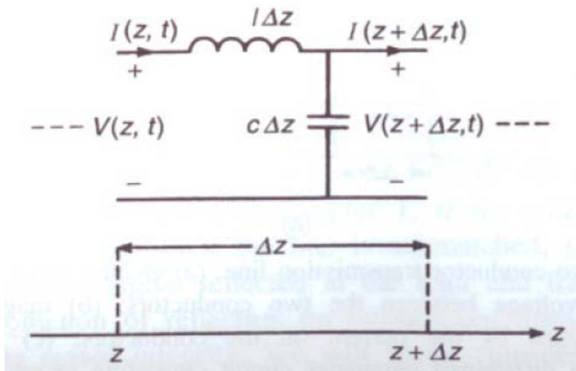
$$\frac{\partial V(z,t)}{\partial z} = -l\frac{\partial I(z,t)}{\partial t}$$
$$\frac{\partial I(z,t)}{\partial z} = -c\frac{\partial V(z,t)}{\partial t}$$

From time-domain to frequency-domain

$$\frac{d\hat{V}(z)}{dz} = -j\omega l\hat{I}(z)$$
$$\frac{d\hat{I}(z)}{dz} = -j\omega c\hat{V}(z)$$

$$\frac{d^2\hat{V}(z)}{dz^2} + \omega^2 lc\hat{V}(z) = 0$$
$$\frac{d^2\hat{I}(z)}{dz^2} + \omega^2 lc\hat{I}(z) = 0$$

$$\hat{V}(z) = \hat{V}^+ e^{-j\beta z} + \hat{V}^- e^{j\beta z}$$
$$\hat{I}(z) = \frac{\hat{V}^+}{Z_C}e^{-j\beta z} - \frac{\hat{V}^-}{Z_C}e^{j\beta z}$$



Sinusoidal Excitation of the Line and the Phasor Solution

- Phasor Form of Lossless TX Line
  - Sinusoidal Steady-State Analysis of TX Lines
    - Where the characteristic impedance and phase constant are

$$Z_C = \sqrt{\frac{l}{c}} \qquad v = \frac{1}{\sqrt{lc}}$$

$$\beta = \frac{\omega}{v} \qquad = \frac{1}{\sqrt{\mu\epsilon}}$$

    - The voltage reflection coefficient at a particular point z is defined as

$$\hat{\Gamma}_L = \frac{\hat{Z}_L - Z_C}{\hat{Z}_L + Z_C}$$

$$\hat{\Gamma}(z) = \frac{\hat{V}^- e^{j\beta z}}{\hat{V}^+ e^{-j\beta z}} = \frac{\hat{V}^-}{\hat{V}^+} e^{j2\beta z} \longrightarrow \hat{\Gamma}(z) = \hat{\Gamma}_L e^{j2\beta(z-\mathscr{L})}$$

Sinusoidal Excitation of the Line and the Phasor Solution

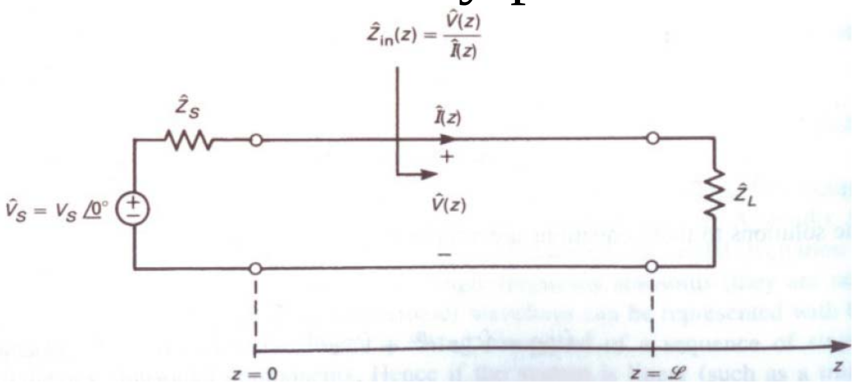
- Phasor Form of Lossless TX Line
  - Sinusoidal Steady-State Analysis of TX Lines
    - In terms of this reflection coefficient, the voltage and current could be rewritten as

$$\hat{V}(z) = \hat{V}^{+} e^{-j\beta z}[1 + \hat{\Gamma}(z)]$$

$$\hat{I}(z) = \frac{\hat{V}^{+}}{Z_C} e^{-j\beta z}[1 - \hat{\Gamma}(z)]$$

    - And the input impedance to the line at any point on the line is

$$\hat{Z}_{\text{in}}(z) = \frac{\hat{V}(z)}{\hat{I}(z)} = Z_C \frac{1 + \hat{\Gamma}(z)}{1 - \hat{\Gamma}(z)}$$



Sinusoidal Excitation of the Line and the Phasor Solution

- Phasor Form of Lossless TX Line
  - Sinusoidal Steady-State Analysis of TX Lines
    - Also, the voltage and current could be rewritten as

$$\hat{\Gamma}(0) = \hat{\Gamma}_{\text{in}} = \hat{\Gamma}_L e^{-j2\beta\mathscr{L}} \longrightarrow \hat{Z}_{\text{in}}(0) = Z_C \frac{1+\hat{\Gamma}(0)}{1-\hat{\Gamma}(0)} \longrightarrow \hat{V}(0) = \frac{\hat{Z}_{\text{in}}(0)}{\hat{Z}_{\text{in}}(0)+\hat{Z}_S}\hat{V}_S \longrightarrow \hat{V}^+ = \frac{\hat{V}(0)}{1+\hat{\Gamma}(0)}$$

$$\left.\begin{aligned} \hat{V}(z) &= \hat{V}^+ e^{-j\beta z}[1+\hat{\Gamma}(z)] \\ \hat{I}(z) &= \frac{\hat{V}^+}{Z_C} e^{-j\beta z}[1-\hat{\Gamma}(z)] \end{aligned}\right\} \quad \hat{V}^+ = \frac{\hat{V}(0)}{1+\hat{\Gamma}(0)}, \quad \hat{\Gamma}(z) = \hat{\Gamma}_L e^{j2\beta(z-\mathscr{L})} \longrightarrow \left\{\begin{aligned} \hat{V}(z) &= \frac{1+\hat{\Gamma}_L e^{-j2\beta\mathscr{L}} e^{j2\beta z}}{1-\hat{\Gamma}_S \Gamma_L e^{-j2\beta\mathscr{L}}} \frac{Z_C}{Z_C+\hat{Z}_S}\hat{V}_S e^{-j\beta z} \\ \hat{I}(z) &= \frac{1-\hat{\Gamma}_L e^{-j2\beta\mathscr{L}} e^{j2\beta z}}{1-\hat{\Gamma}_S \Gamma_L e^{-j2\beta\mathscr{L}}} \frac{1}{Z_C+\hat{Z}_S}\hat{V}_S e^{-j\beta z} \end{aligned}\right.$$

    - Thus, the input impedance could be written as

$$\begin{aligned} \hat{Z}_{\text{in}}(0) &= Z_C \frac{1+\hat{\Gamma}_L e^{-j2\beta\mathscr{L}}}{1-\hat{\Gamma}_L e^{-j2\beta\mathscr{L}}} \\ &= Z_C \frac{\hat{Z}_L + jZ_C \tan\beta\mathscr{L}}{Z_C + j\hat{Z}_L \tan\beta\mathscr{L}} \end{aligned}$$

Sinusoidal Excitation of the Line and the Phasor Solution

- Phasor Form of Lossless TX Line
  - Sinusoidal Steady-State Analysis of TX Lines
    - When the line is matched, the load reflection coefficient $\Gamma_L=0$, there exists forward-traveling wave on the line only.

$$\hat{V}(z) = \frac{Z_C}{Z_C + \hat{Z}_S}\hat{V}_S e^{-j\beta z}$$

$$\hat{Z}_L = Z_C \text{ (matched line)}$$

$$\hat{I}(z) = \frac{1}{Z_C + \hat{Z}_S}\hat{V}_S e^{-j\beta z}$$

    - The voltage standing-wave ratio (VSWR), which is a measure of reflection, is defined as

$$\text{VSWR} = \frac{|\hat{V}(z)|_{\max}}{|\hat{V}(z)|_{\min}} = \frac{1+|\hat{\Gamma}_L|}{1-|\hat{\Gamma}_L|}$$

$$1 \le \text{VSWR} \le \infty$$

Sinusoidal Excitation of the Line and the Phasor Solution

- Phasor Form of Lossless TX Line
  - Power Flow
    - The average power flow in the +z direction is given by

$$\hat{V}(z) = \hat{V}^{+} e^{-j\beta z}[1 + \hat{\Gamma}(z)]$$

$$\hat{I}(z) = \frac{\hat{V}^{+}}{Z_C} e^{-j\beta z}[1 - \hat{\Gamma}(z)]$$

$$P_{av}(z) = \frac{1}{2}\mathcal{R}e\{\hat{V}(z)\hat{I}^{*}(z)\} \longrightarrow$$

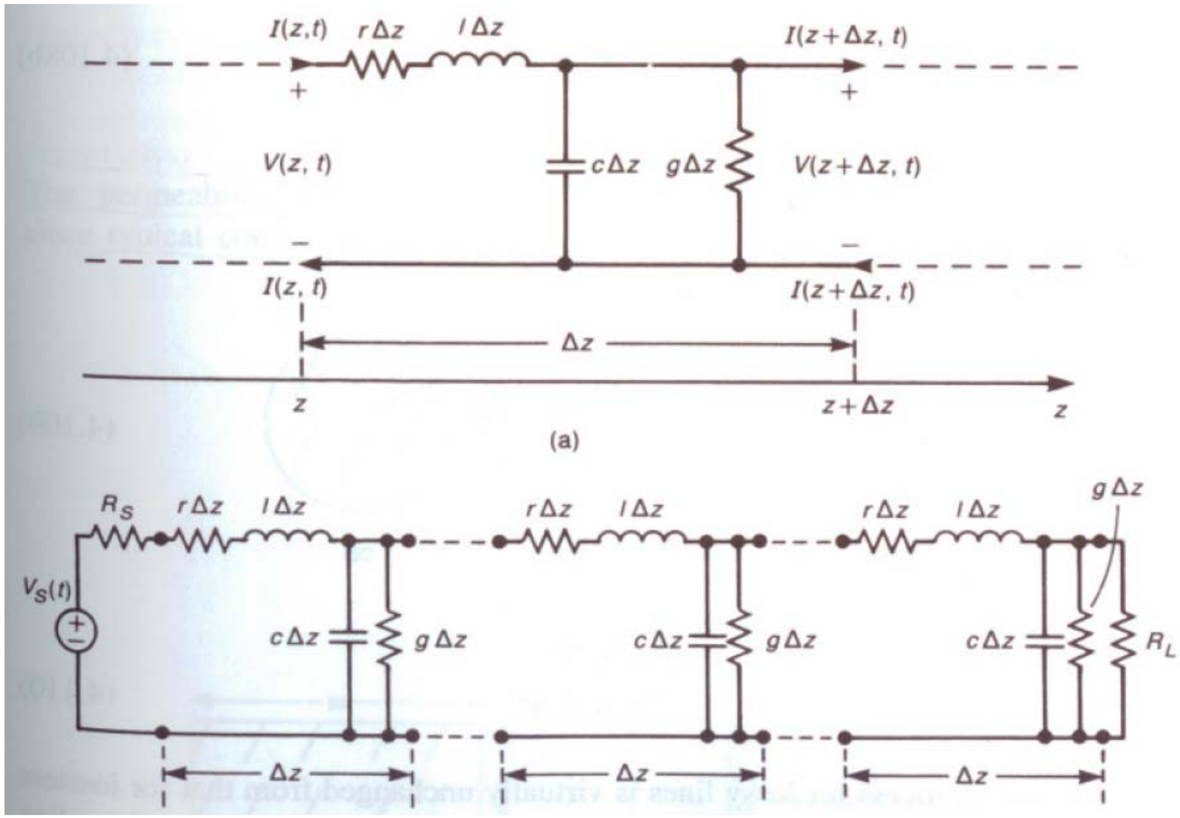
$$P_{av}(z) = \frac{1}{2}\frac{|\hat{V}^{+}|^2}{Z_C}(1 - |\hat{\Gamma}_L|^2)$$

    - This result could have been derived by adding the average powers of the forward- and backward-traveling waves.
    - From the above results, we also have

$$\frac{P_{\text{av, reflected}}}{P_{\text{av, incident}}} = |\hat{\Gamma}_L|^2$$

Sinusoidal Excitation of the Line and the Phasor Solution

- Phasor Form of Lossy TX Line
  - Derivation
    - The equivalent circuit of the lossy TX line is shown below



Sinusoidal Excitation of the Line and the Phasor Solution

- Phasor Form of Lossy TX Line
  - Derivation
    - For lossy Tx line, we have

$$\frac{d\hat{V}(z)}{dz} = -\hat{z}\hat{I}(z) \qquad \frac{d\hat{I}(z)}{dz} = -\hat{y}\hat{V}(z)$$

    - where the per-unit-length impedance and admittance are given by

$$\hat{z} = r(f) + j\omega l$$

$$\hat{y} = g + j\omega c$$

    - in which $r(f)$ is a function of frequency showing its dependence on skin effect.
    - The voltage and current could be decoupled as

$$\frac{d^2\hat{V}(z)}{dz^2} - \hat{z}\hat{y}\hat{V}(z) = 0 \qquad \hat{V}(z) = \hat{V}^+ e^{-\alpha z} e^{-j\beta z} + \hat{V}^- e^{\alpha z} e^{j\beta z}$$

$$\frac{d^2\hat{I}(z)}{dz^2} - \hat{y}\hat{z}\hat{I}(z) = 0 \quad \longrightarrow \quad \hat{I}(z) = \frac{\hat{V}^+}{\hat{Z}_C} e^{-\alpha z} e^{-j\beta z} - \frac{\hat{V}^-}{\hat{Z}_C} e^{\alpha z} e^{j\beta z}$$

Sinusoidal Excitation of the Line and the Phasor Solution

- Phasor Form of Lossy TX Line
  - Derivation
    - where the characteristic impedance and the propagation constant are

$$\hat{Z}_C = \sqrt{\frac{\hat{z}}{\hat{y}}} = \sqrt{\frac{r(f)+j\omega l}{g+j\omega c}}$$

$$\hat{\gamma} = \sqrt{\hat{z}\hat{y}} = \alpha + j\beta$$

    - We observe that the forward- and backward-traveling voltage and current waves are attenuated as they travel alone the line.

Sinusoidal Excitation of the Line and the Phasor Solution

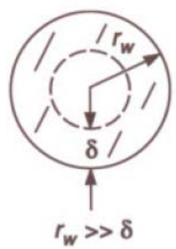
- Effect of Losses on Signal Integrity
  - Loss of Line Conductors — Circular Wire
    - At dc, the current will be uniformly distributed over the wire cross section, and the dc resistance per unit length is

$$r_{dc} = \frac{1}{\sigma \pi r_w^2} \quad \Omega/\mathrm{m}$$

    - As the frequency of the current is increased, the current crowds toward the outer edge of the wire, and the majority of the current is confined to an annulus equal to the skin depth

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}} \quad \mathrm{m}$$

Decreases with the square root of *f*.

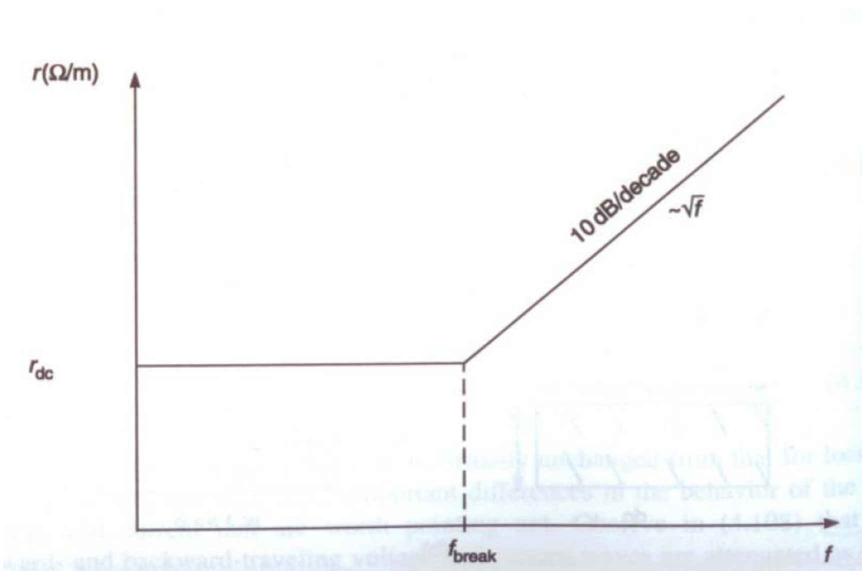


Sinusoidal Excitation of the Line and the Phasor Solution

- Effect of Losses on Signal Integrity
  - Loss of Line Conductors — Circular Wire
    - Hence for $r_w >> \delta$, the per-unit-length resistance is

$$r_{hf} = \frac{1}{\sigma\left(\pi r_w^2 - \pi(r_w - \delta)^2\right)} \cong \frac{1}{\sigma 2\pi r_w \delta} \quad \Omega/\mathrm{m}$$

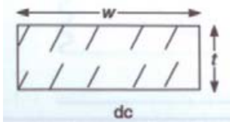
Increases with the square root of f or 10dB/decade



Sinusoidal Excitation of the Line and the Phasor Solution

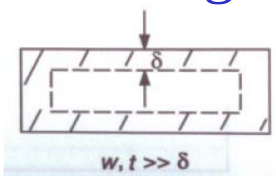
- Effect of Losses on Signal Integrity
  - Loss of Line Conductors — Rectangular Land
    - Similarly, for the rectangular cross-section PCB land, at dc the current is uniformly distributed over the land cross section and the dc resistance is

$$r_{dc} = \frac{1}{\sigma w t} \quad \Omega/\mathrm{m}$$



    - As the frequency is increased, the current becomes concentrated in a thickness equal to one skin depth and for $r_w >> \delta$, the per-unit-length resistance is

$$r_{hf} = \frac{1}{\sigma(2\delta t + 2\delta w)} = \frac{1}{2\sigma\delta(w+t)} \quad \Omega/\mathrm{m}$$



The turning point is obtained from $r_{dc}=r_{hf}$.

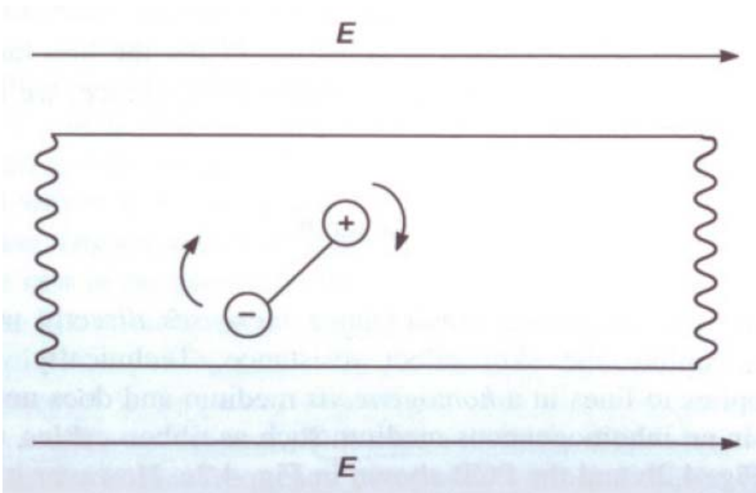
$$\delta = \frac{1}{2}\frac{wt}{(w+t)} \cong \frac{t}{2} \qquad w \gg t$$

    - Actually, the current peaks at the corners.

Sinusoidal Excitation of the Line and the Phasor Solution

- Effect of Losses on Signal Integrity
  - Loss of Dielectric
    - Dielectrics are characterized by having bound charge in the form of microscopic dipoles and have no appreciable free charge, unlike conductors.
    - The dielectric loss results from the inability of the dipoles to completely follow the time-varying rate of the electric field.

Thus, we have an intuition that the loss is proportional to the frequency *f*.



Sinusoidal Excitation of the Line and the Phasor Solution

- Effect of Losses on Signal Integrity
  - Loss of Dielectric
    - Dielectric loss could be described by a complex permittivity

$$\epsilon_r = \epsilon_r' - j\epsilon_r''$$

    - Since the capacitance is proportional to the complex permittivity, the per-unit-length capacitive reactance could be written as

$$j\omega c = j\omega\epsilon K \longrightarrow j\omega\epsilon K = j\omega\epsilon_0\epsilon_r K$$

$$= \underbrace{\omega\epsilon_0\epsilon_r'' K}_{g} + j\omega \underbrace{\epsilon_0\epsilon_r' K}_{c}$$

    - The real part of this is equivalent to a per-unit-length conductance, while the imaginary part is equivalent to a per-unit-length capacitive reactance.

Sinusoidal Excitation of the Line and the Phasor Solution

- Effect of Losses on Signal Integrity
  - Loss of Dielectric
    - The loss tangent is defined as

$$\tan\theta = \frac{\epsilon_r''}{\epsilon_r'} = \frac{g}{\omega c} \longrightarrow g = \omega c \tan\theta$$

    - Observe that the per-unit-length conductance increases directly with frequency, unlike the skin effect resistance.
    - From the propagation constant, we see that both α and β are frequency-dependent, which means that we might have two kinds of dispersions due to α and β.

Sinusoidal Excitation of the Line and the Phasor Solution

- Effect of Losses on Signal Integrity
  - Loss on Dispersions of α and β
    - Due to β: If each of these sinusoidal components travel down the line at different speeds, they will arrive at the load at different time, resulting in a distortion of the pulse.
    - Due to α: Since each sinusoidal component will be attenuated by a different amount, this also results in distortion.
    - We also know that losses attenuate the higher frequency components more than the lower frequency components. Hence the bandwidth of the pulse is reduced and the pulse rise/fall-time are increased.

Sinusoidal Excitation of the Line and the Phasor Solution

- Effect of Losses on Signal Integrity
  - Special Cases
    - If all the sinusoidal components traveled at the same velocity and are attenuated at the same rate, the received pulse will retain its shape and only the amplitude would be lowered.
    - Power loss in a line for a matched load is

$$e^{-2\alpha\mathscr{L}} \longrightarrow \text{Loss}_{\text{dB}} = 8.686\alpha\mathscr{L}$$

    - When will the loss be neglected? r should be compared with $j\omega l$ and g should be compared with $j\omega c$.

Sinusoidal Excitation of the Line and the Phasor Solution

- Effect of Losses on Signal Integrity
  - Low Loss Case
    - When the below criterion is satisfied, the transmission line is said to be low loss.

$$r \ll \omega l \qquad g \ll \omega c$$

    - The propagation could be simplified as

$$\begin{aligned}
\hat{\gamma} &= \alpha + j\beta \\
&= \sqrt{(r + j\omega l)(g + j\omega c)} \\
&= \sqrt{(j\omega l)(j\omega c)\left(1 + \frac{r}{j\omega l}\right)\left(1 + \frac{g}{j\omega c}\right)} \\
&= j\omega\sqrt{lc}\sqrt{\left(1 - \frac{rg}{\omega^2 lc}\right) - j\left(\frac{r}{\omega l} + \frac{g}{\omega c}\right)} \\
&\cong j\omega\sqrt{lc}\sqrt{1 - j\left(\frac{r}{\omega l} + \frac{g}{\omega c}\right)} \qquad \left\{\begin{matrix} r \ll \omega l \\ g \ll \omega c \end{matrix}\right\}
\end{aligned}$$

$$\longrightarrow \hat{\gamma} \cong j\omega\sqrt{lc}\left[1 - j\left(\frac{r}{2\omega l} + \frac{g}{2\omega c}\right)\right]$$

Sinusoidal Excitation of the Line and the Phasor Solution

- Effect of Losses on Signal Integrity
  - Low Loss Case
    - where the attenuation and phase constants are

$$\alpha \cong \frac{1}{2}\left(\frac{r}{Z_C} + gZ_C\right)$$

Constant at low frequency, r_{dc}

$$\beta \cong \omega\sqrt{lc}$$

Linear function of frequency

    - In a similar fashion, the characteristic impedance could be simplified as

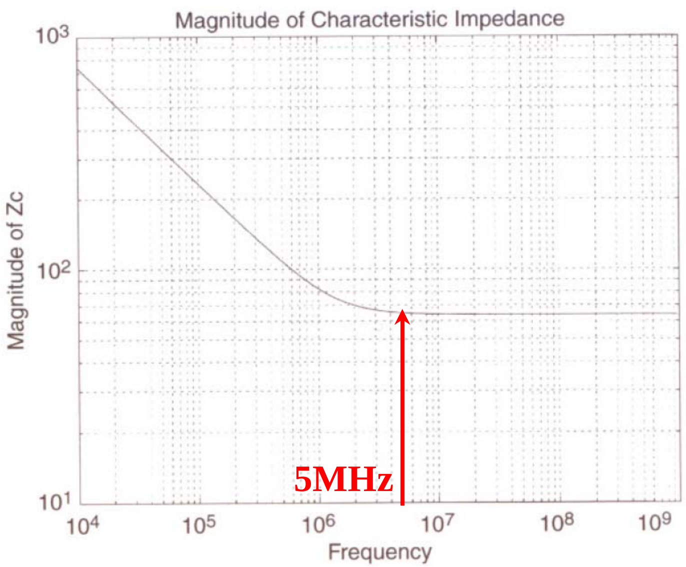
$$Z_C = \sqrt{\frac{r + j\omega l}{g + j\omega c}}$$

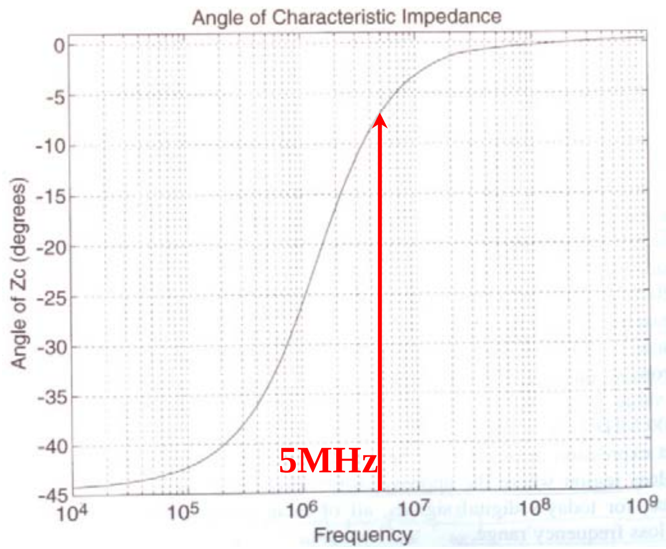
$$\cong \sqrt{\frac{l}{c}} \qquad \left\{\begin{matrix} r \ll \omega l \\ g \ll \omega c \end{matrix}\right\}$$

Independent of frequency

Sinusoidal Excitation of the Line and the Phasor Solution

- Effect of Losses on Signal Integrity
  - Example for Typical Board Dimensions
    - For typical board dimensions, the low loss region is above ~5MHz. $r \ll \omega l \qquad g \ll \omega c$
    - Magnitude and phase of the characteristic impedance



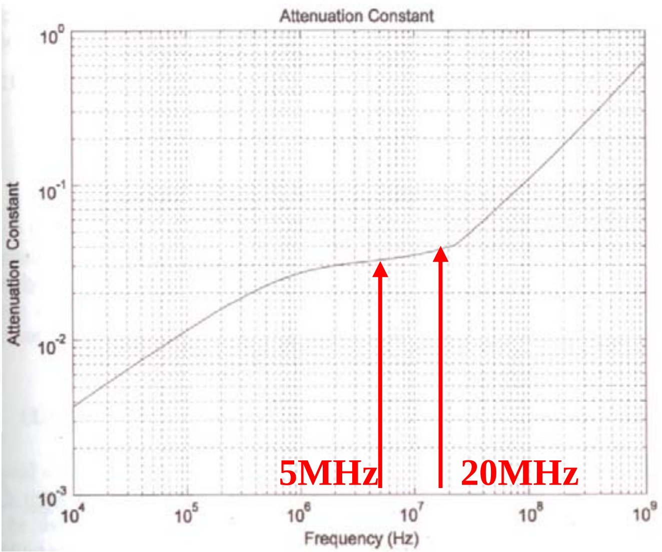


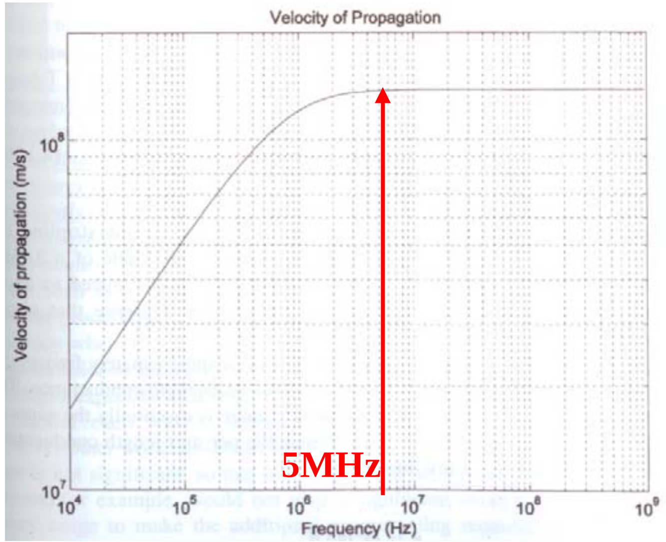
Sinusoidal Excitation of the Line and the Phasor Solution

- Effect of Losses on Signal Integrity
  - Example for Typical Board Dimensions
    - Attenuation constant and velocity of propagation

$$\alpha \cong \frac{1}{2}\left(\frac{r}{Z_C} + gZ_C\right)$$

$$Z_C = \sqrt{\frac{r + j\omega l}{g + j\omega c}} \cong \sqrt{\frac{l}{c}} \qquad \left\{ \begin{array}{l} r \ll \omega l \\ g \ll \omega c \end{array} \right\}$$



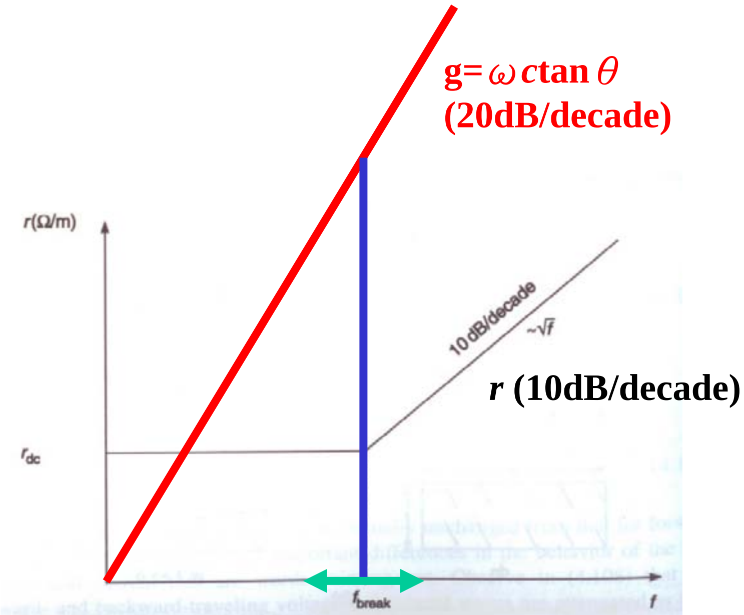


From 5MHz to 20MHz, we don't have α and β dispersions
Above 20MHz, we don't have β dispersion, but we have α dispersion.

Sinusoidal Excitation of the Line and the Phasor Solution

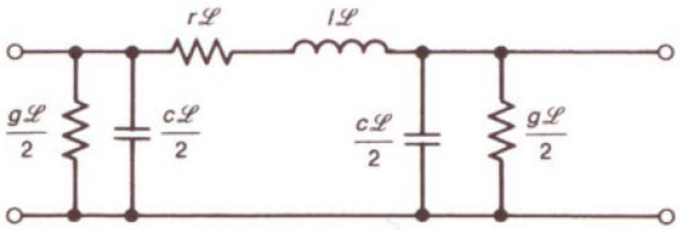
- Effect of Losses on Signal Integrity
  - Example for Typical Board Dimensions
    - We know that g is always smaller than $j\omega c$ by a factor of $\tan\theta$, which is 0.02 for FR-4. $g = \omega c \tan\theta$
    - 5MHz~20MHz corresponds to the frequency region around the blue line.

Please compare this figure with the figure of α v.s. f.
Also, please note the turning points f_{0max} in these two figures.

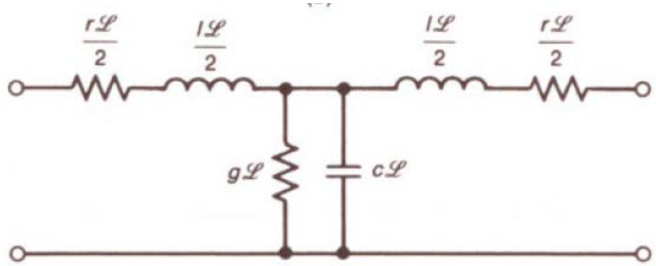


Lumped-Circuit Approximate Models

- TX Line Equivalent Circuit Model
  - Criterion for Usage of Lumped-Circuit Models
    - In order to use the lumped-circuit models to model the transmission line, the line must be electrically short at the highest frequency of interest of the input source waveform.
    - The typical lumped-circuit model are lumped-π or lumped-T circuits.



lumped-π

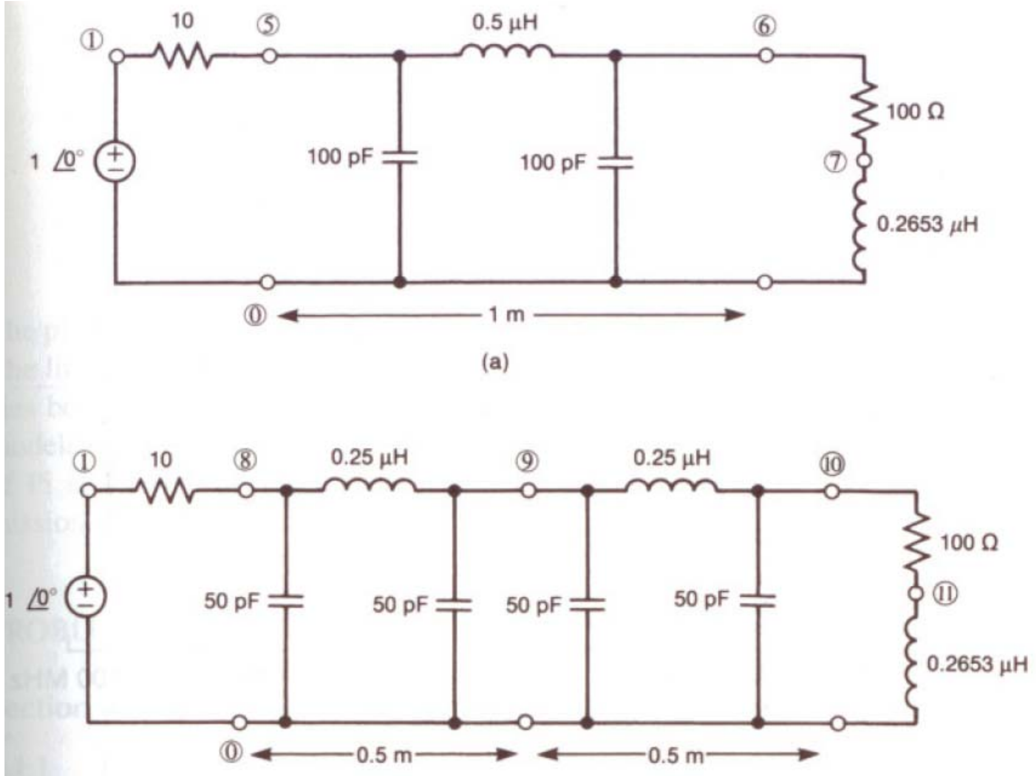


lumped-T

Difficult to be used in time-domain, since losses are not easily handled.

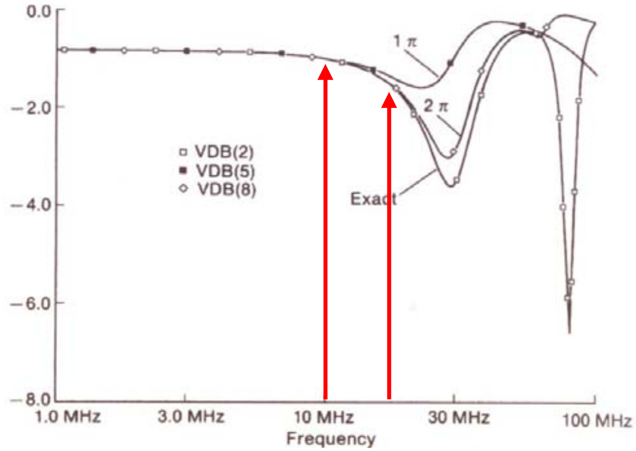
Lumped-Circuit Approximate Models

- TX Line Equivalent Circuit Model
  - Example 4.12 How Many Subsections Needed?
    - Consider the distributed tx-line model and lumped-π model with one or two sections.

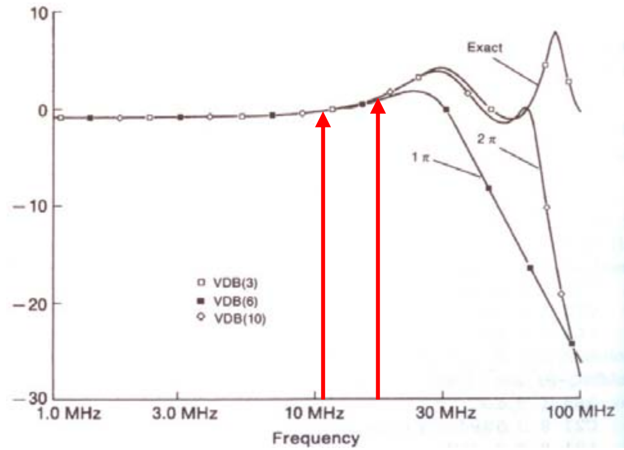


Lumped-Circuit Approximate Models

- TX Line Equivalent Circuit Model
  - Example 4.12 How Many Subsections Needed?
    - The lumped-π model with one section is valid up to 10MHz where the line is one-tenth of a wavelength.
    - The lumped-π model with two sections is valid up to 20MHz where the line is one-tenth of a wavelength.



Input voltage



Load voltage