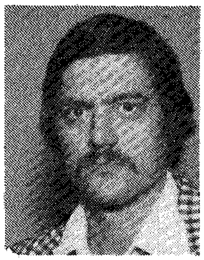
high-power microwave limiters, broad-band microwave detectors, switches, and mixers. He joined Varian Associates, Solid-State Microwave Division in October 1981. Currently, he is working on GaAs power FET amplifier design and automatic testing system development.

C. Gerald Gardner is a graduate of Mississippi State University and Vanderbilt University.

Following post-doctoral work at Oak Ridge National Laboratory and Lawrence Radiation Laboratory, he joined the Faculty of Texas Technological College. In 1964, he joined Southwest Research Institute where he worked in instrumentation research and development, especially for nondestructive evaluation. He was a Faculty Member of the Department of Electrical Engineering of the University of Houston from 1977 to 1981, working in the field of electromagnetic methods of nondestructive evaluation. He is presently employed by Welex, Inc., a Division of Haliburton, in Houston, TX.



Stuart A. Long (S'65–S'72–M'74–SM'80) received his public education in Snyder, TX and was granted the B.A. and M.E.E. degrees in electrical engineering from Rice University, Houston, TX, in 1967 and 1968, respectively, and the Ph.D. degree in applied physics from Harvard University, Cambridge, MA, in 1974.

He was employed as an Aerosystems Engineer in the Antenna Design Group of General Dynamics, Ft. Worth, TX, from 1968 to 1969. From 1970 to 1974 he was a Teaching Fellow and Research Assistant in Applied Mathematics and Applied Physics at Harvard University. He was also a Research Assistant at Los Alamos Scientific Laboratories, Los Alamos, NM, for the summers of 1970 and 1971 working on the design of linear accelerators. In 1974 he joined the faculty of the University of Houston, and is presently an Associate Professor and Chairman of the Department of Electrical Engineering. His current interests include printed-circuit antennas, electromagnetic methods of nondestructive evaluation, millimeter-wave guiding structures and radiators, and well-logging.

Dr. Long is a member of Phi Beta Kappa, Tau Beta Pi, and Sigma Xi. He was elected to the National Administrative Committee of the Antennas and Propagation Society in 1981, and is presently chairman of the Houston joint chapter of the AP, MTT, MAG, and ED societies of the IEEE.

Determination of Loaded, Unloaded, and External Quality Factors of a Dielectric Resonator Coupled to a Microstrip Line

APS KHANNA AND Y. GARAULT

Abstract—**In the case of a dielectric resonator coupled to a microstrip line, the relations for the determination of unloaded, loaded, and external quality factors, in terms of directly measurable reflection or transmission coefficient at the resonant frequency, have been derived and represented on the corresponding vectorial and scalar planes. Construction of a linear frequency scale and a graphical method to accurately determine the unloaded Q from the loaded Q measurement presented.**

I. Introduction

THE ADVENT OF temperature stable, high-Q and low-loss ceramic materials has created a considerable interest in the application of dielectric resonators in the microwave integrated circuits [1]. The dielectric resonator coupled to a microstrip line has been used for realizing a number of stable oscillators [2]–[4] and filters [5]. Complete characterization of a microstrip coupled dielectric resonator (Fig. 1) is necessary for the analysis and synthesis of these integrated circuits. Ginzton [6] introduced the plotting of loci of various quality factors and the construction of a linear frequency scale in the case of a single-port

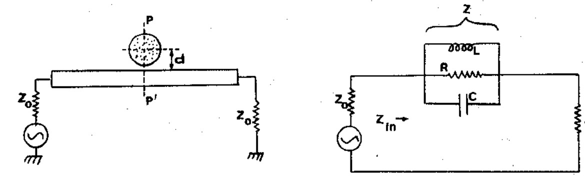


Fig. 1. Dielectric resonator coupled to a microstrip line and its equivalent circuit in symmetry plane PP'.

Manuscript received March 10, 1982; revised September 16, 1982.

A. Khanna, formerly in L.E.M. Limoges, is now with Giga Instrumentation Immeuble ATLAS, Avenue de la Baltique, Z. A. de Courtaboeuf, 91940 Les Ulis, France.

Y. Garault is with Laboratoire d'électronique des Microondes, E.R.A. au C.N.R.S. 535, U.E.R. des Sciences, 123 Rue Albert Thomas 87060, Limoges, France.

0018-9480/83/0300-0261$01.00

reflection mode resonant metallic cavity. Podcameni [7] recently suggested a relation for the determination of unloaded Q of the dielectric resonator, using an indirect method. In this paper, we present the necessary relations, in terms of directly measurable quantities, to draw loci of unloaded, loaded, and external quality factors, on the impedance as well as on the transmittance plane, in the case of a dielectric resonator coupled to a microstrip line (Fig. 1). The construction of a linear frequency scale is presented and a graphical method to accurately determine the unloaded Q from the loaded Q measurement is given.

II. Coupling Factor of a Dielectric Resonator Coupled to a Microstrip Line

Equivalent circuit in the symmetry plane PP' of a TE_{01p}-mode cylindrical resonator, magnetically coupled to a 50-Ω microstrip line, under necessary shielding conditions, is shown in Fig. 1 [5]. The coupling factor β is defined as the ratio of the resonator coupled resistance R at the resonant frequency to the resistance external to the resonator or

$$\beta = \frac{R}{R_{\text{ext}}} = \frac{R}{2Z_0} = \frac{S_{11_0}}{1-S_{11_0}} = \frac{1-S_{21_0}}{S_{21_0}} = \frac{S_{11_0}}{S_{21_0}} \tag{1}$$

where S_{11_0} and S_{21_0} are the real quantities representing the reflection and transmission coefficients respectively, at the resonant frequency, in the same symmetry plane PP' (Fig. 1).

The critical coupling ($\beta = 1$) occurs when the power dissipated in the resonator P_d is equal to the power dissipated in the external circuit, which is equally divided into the power reflected to the generator ($P_r = S_{11_0}^2$) and the power transmitted to the load ($P_t = S_{21_0}^2$), i.e., $P_t = P_r = P_d/2$. P_d, P_r, and P_t represent here the RF powers normalized with respect to the incident power.

In the shielded resonator configuration, from the conservation of energy, power dissipated in the resonator being given by

$$P_d = 1 - |S_{11_0}|^2 - |S_{21_0}|^2 \tag{2}$$

the critical coupling corresponds to $S_{11_0} = S_{21_0} = 0.5$.

The coupling factor β is a function of the distance between the dielectric resonator and the microstrip line under fixed shielding conditions. β also relates the various quality factors by the well-known relation

$$Q_u = Q_L(1+\beta) = \beta Q_{\text{ex}} \tag{3}$$

where Q_u, Q_L, and Q_{ex} represent unloaded, loaded, and external quality factor, respectively.

III. Loci of Unloaded, Loaded, and External Quality Factors

The aim is to draw the loci of the points, on the impedance (S_{11}) and transmittance (S_{21}) planes, for the frequency deviations corresponding to Q_u, Q_L, and Q_{ex}. In Fig. 1, Z_{in} in the plane PP' of the dielectric resonator is given by [5]

$$Z_{\text{in}} = Z_0 + \frac{R}{1+j2Q_u\delta} \tag{4}$$

where $\delta = (f-f_0)/f_0$ is the normalized frequency deviation.

Using (1) and (3) the normalized input impedance $z_{\text{in}} = Z_{\text{in}}/Z_0$ can be written as

$$\begin{aligned} z_{\text{in}} &= 1 + \frac{2\beta}{1+j2Q_u\delta} = 1 + \frac{2\beta}{1+j2Q_L(1+\beta)\delta} \\ &= 1 + \frac{2\beta}{1+j2Q_{\text{ex}}\beta\delta}. \end{aligned} \tag{5}$$

The normalized frequency deviations corresponding to various quality factors are given by

$$\delta_u = \pm\frac{1}{2Q_u}, \quad \delta_L = \pm\frac{1}{2Q_L}, \quad \text{and } \delta_{\text{ex}} = \pm\frac{1}{2Q_{\text{ex}}}. \tag{6}$$

The impedance locus of Q_u, for example, can be determined by using (6) in (5), and is given by

$$(z_{\text{in}})_u = 1 + \frac{2\beta}{1\pm j} \tag{7}$$

or using (1)

$$(z_{\text{in}})_u = \frac{1}{1-S_{11_0}} \pm j\frac{S_{11_0}}{1-S_{11_0}}. \tag{8}$$

The corresponding reflection coefficient S_{11_u} can be determined from

$$S_{11_u} = \frac{(z_{\text{in}})_u - 1}{(z_{\text{in}})_u + 1}$$

to be

$$S_{11_u} = \frac{S_{11_0}}{\sqrt{S_{11_0}^2 - 2S_{11_0} + 2}}\, e^{\pm j\tan^{-1}(1-S_{110})}. \tag{9}$$

The coupled dielectric resonator being a series impedance in the equivalent circuit, the reflection and transmission coefficients are related by [8]

$$S_{11} + S_{21} = 1. \tag{10}$$

The relation for the Q_u locus, in the transmittance (S_{21}) plane, can now be obtained, using the above relation.

In general, Table I represents the relations for the loci of various quality factors, for the reflection coefficient S_{11} and transmission coefficient S_{21} planes. Fig. 2(a) and (b) represent these loci in addition to some impedance (S_{11}) and transmittance (S_{21}) curves for various values of the coupling factor β between the resonator and the microstrip line. As an example, Q_u can be determined from

$$Q_u = f_0/(f_1 - f_2).$$

Care should be taken while making measurements using the scalar network measurement system giving only the modulus of reflection or transmission coefficients. It may be noted, for example, that in the reflection coefficient plane the Q_L points are always 3 dB below their value at

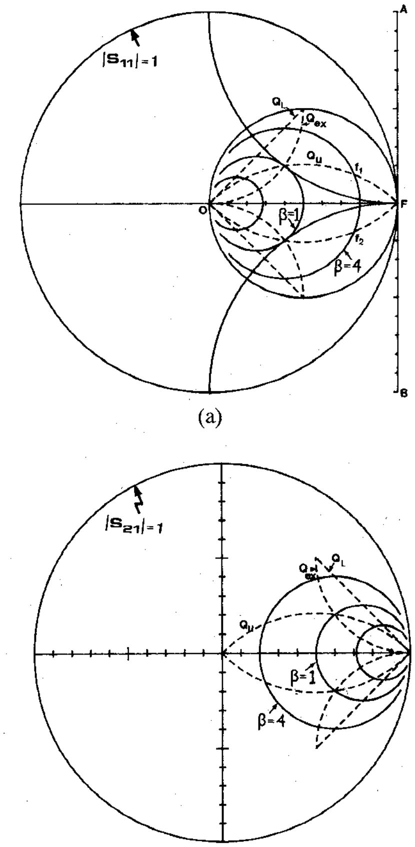


Fig. 2. Loci of Q_u, Q_L, and Q_{ex} on (a) reflection coefficient plane and on (b) transmission coefficient plane. Reflection and transmission coefficient curves for $\beta = 0.4$, 1, and 4.

TABLE I

	REFLECTION COEFFICIENT S_{11_i} $\lvert S_{11_i}\rvert$	$\angle S_{11_i}$	TRANSMISSION COEFFICIENT S_{21_i} $\lvert S_{21_i}\rvert$	$\angle S_{21_i}$
Q_u	$\dfrac{S_{11_0}}{\sqrt{S_{11_0}^2 - 2S_{11_0} + 2}}$	$\pm \tan^{-1}(1 - S_{11_0})$	$S_{21_0}\sqrt{\dfrac{2}{1 + S_{21_0}^2}}$	$\pm \tan^{-1}\left(\dfrac{1 - S_{21_0}}{1 + S_{21_0}}\right)$
Q_L	$\dfrac{S_{11_0}}{\sqrt{2}}$	$\pm \dfrac{\pi}{4}$	$\sqrt{\dfrac{1 + S_{21_0}^2}{2}}$	$\pm \tan^{-1}\left(\dfrac{1 - S_{21_0}}{1 + S_{21_0}}\right)$
Q_{ex}	$\dfrac{S_{11_0}}{\sqrt{1 + S_{11_0}^2}}$	$\pm \tan^{-1} S_{11_0}$	$\dfrac{\sqrt{2(S_{21_0}^4+1) - 6S_{21_0}(S_{21_0}^2+1) + 9S_{21_0}^2}}{S_{21_0}^2 - 2S_{21_0} + 2}$	$\pm \tan^{-1}\left(\dfrac{S_{21_0}^2 - 2S_{21_0} + 1}{S_{21_0}^2 - S_{21_0} + 1}\right)$

the resonance frequency, while in the transmission coefficient plane Q_L points are functions of the coupling factor (or S_{21_0}). Fig. 3(a) shows the Q_u and Q_L measurement points in the reflection and transmission coefficient magnitude planes as a function of coupling factor β and Fig. 3(b) represents various terms used in Fig. 3(a).

IV. Construction of a Linear Frequency Scale

For a given coupling factor β, a linear frequency scale is obtained along the straight line AB drawn perpendicular to the X-axis and passing through point F (Fig. 4).

The frequency deviation at any point R on the impedance locus can be shown to be proportional to the ordinate of the intersection H of OR with AB. The scale is established from the measurement of frequencies at any two points on the impedance locus.

Proof: OR being the reflection coefficient, from (5)

$$S_{11_R} = |S_{11_R}|e^{jS_{11R}} = \frac{z_{\text{in}} - 1}{z_{\text{in}} + 1}$$

$$= \frac{\beta(\beta+1) - j2\beta Q_u\delta}{(\beta+1)^2 + 4Q_u^2\delta^2} \tag{11}$$

$$\text{angle } ROF = \angle S_{11_R} = \tan^{-1}\left(\frac{2Q_u\delta}{\beta+1}\right). \tag{12}$$

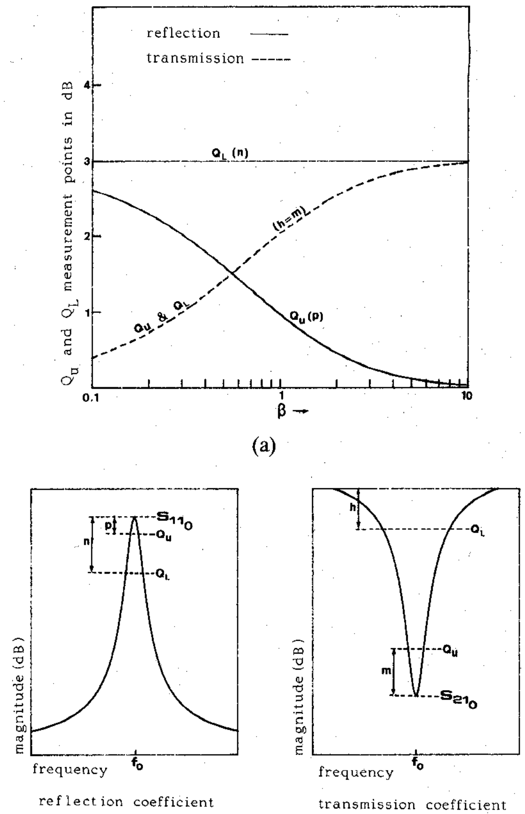


Fig. 3. (a) Q_u and Q_L measurement points in the reflection and transmission coefficient magnitude planes as a function of coupling factor β and (b) definition of various terms used.

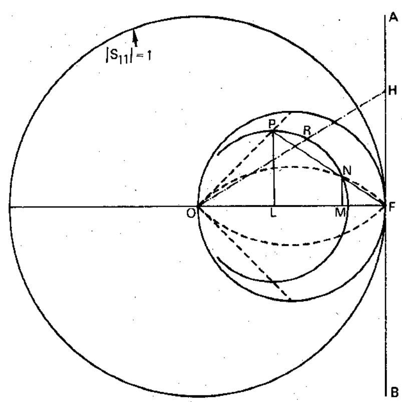


Fig. 4. Construction of a linear frequency scale and determination of unloaded Q from the loaded Q measurement.

The distance FH, being proportional to tan $\angle ROF$, is hence directly proportional to the frequency δ.

V. Graphical Method to Determine Unloaded Q From the Loaded Q Measurement

Unloaded quality factor of a shielded dielectric resonator being high, its manual direct measurement can cause considerable errors [7]. Q_u can, however, be accurately determined from the measurement of Q_L only, with the help of a simple graphical method using a Smith chart. This is possible because it can be shown that the Q_L point P (Fig. 4) when jointed to F, intersects the impedance locus at the point N which corresponds to Q_u. To prove this, it is sufficient to show that the angle PFO is the same as angle NFO (Fig. 4). Using the values given in Table I we have

In triangle OPF:

$$\text{angle } POF = 45^\circ$$
$$OP = S_{11_0}/\sqrt{2}.$$

Drawing PL perpendicular to OF

$$OL = PL = S_{11_0}/2$$
$$\text{and } LF = 1 - S_{11_0}/2.$$

$$\text{Hence, angle } PFO = \tan^{-1}\frac{PL}{LF} = \tan^{-1}\frac{S_{11_0}}{2 - S_{11_0}}.$$

In triangle ONF:

From Table I

$$\text{angle } NOF = \tan^{-1}(1 - S_{11_0}) = \theta_u$$
$$\text{and } ON = \frac{S_{11_0}}{\sqrt{S_{11_0}^2 - 2S_{11_0} + 2}}.$$

Drawing NM perpendicular to OF

$$NM = ON \sin\theta_u$$
$$OM = ON \cos\theta_u$$
$$\text{and } MF = 1 - ON\cos\theta_u$$
$$\text{angle } NFO = \tan^{-1}\frac{NM}{MF} = \tan^{-1}\frac{ON\sin\theta_u}{1 - ON\cos\theta_u}$$
$$= \tan^{-1}\frac{S_{11_0}}{2 - S_{11_0}}.$$

Hence, angle $\angle NFO$ is the same as angle $\angle PFO$.

Once the point N is determined, the corresponding frequency, and hence Q_u, can be evaluated accurately. The Q_u determined experimentally using this method did not show more than ± 1 percent variation for various values of coupling factor β.

References

[1] J. K. Plourde and C. L. Ren, "Applications of dielectric resonators in microwave components," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 754–769, Aug. 1981.

[2] Y. Abe *et al.*, "A highly stabilized low-noise GaAs FET integrated oscillator with a dielectric resonator in the *C*-band," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 156–162, Mar. 1978.

[3] A. Khanna *et al.*, "Efficient low-noise three-port *X*-band FET oscillator using two dielectric resonators," in *1982 IEEE MTT-S Int. Microwave Symp. Dig.*, pp. 277–279.

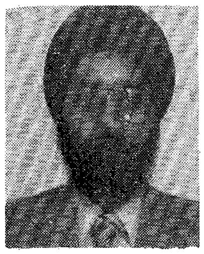
[4] A. Khanna *et al.*, "New very high *Q* microwave transistor oscillators using dielectric resonators," in *Proc. 11th Eur. Microwave Conf.*, (Amsterdam), Sept. 1981, pp. 554–561.

[5] P. Guillon *et al.*, "Microstrip bandstop filter using dielectric resonator," *Proc. Inst. Elec. Eng.*, part H, vol. 128, pp. 151–154, June 1981.

[6] E. L. Ginzton, *Microwave Measurements.* New York McGraw-Hill, 1957, pp. 391–427.

[7] A. Podcameni *et al.*, "Unloaded quality factor measurement for MIC dielectric resonator applications," *Electron. Lett.*, vol. 17, pp. 656–658, Sept. 1981.

[8] R. E. Collins, *Foundations of Microwave Engineering.* New York: McGraw-Hill, 1966, pp. 178.



Aps Khanna was born in Simla, India, in 1950. He received the B.Sc. (engineering) degree in electronics and electrical communications from Punjab Engineering college, Chandigarh, India, in 1971.

Between 1971 and 1973 he was with the Electronics and Radar Development Establishment, Bangalore, India, where he was involved in the development of radar subsystems. Between 1973 and 1977 he was with the RCDU Civil Aviation Department, Government of India, New Delhi, where he was involved in the planning and development of radar and radio navigation aids. From 1978 until September 1981 he was with Laboratoire d'électronique des Microondes, University of Limoges, France, where he completed his doctorate of engineering on "MIC transistor stable oscillators using dielectric resonators," and was Visiting Assistant Professor from October 1980 to September 1981. Between October 1981 and March 1982 he was post-doctoral fellow at the University of Limoges and Consultant to French National Telecommunication Research Center. Presently, he is with Giga Instrumentation, France, where he is involved in the design and development of various microwave integrated circuits for instrumentation applications.

Y. Garault graduated from the University of Paris.

He has been Professor of Electronics in the University of Limoges since 1965. He is Director of the Microwave Electronics Laboratory associated with the French National Scientific Research Center. His research work after the study of periodic structures for RF separators, was about MIC passive circuits and dielectric resonators and their application to microwave filters, oscillator stabilization, and dielectric material measurements. He is presently responsible for the coordination of French University research in microwaves.

Prof. Garault is President of the Center-West group of the French Societe des Electriciens et Electroniciens (S.E.E.).