Superdirective Closely-Spaced Arrays

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***Abstract*—In this paper we employ the concept of source currents to express the directivity of an array in generalized matrix form. It allows finding the excitation currents that maximize directivity for given array. First, we demonstrate this method on an array of two isotropic radiators and express the resulting superdirective currents in closed form for limiting separation going to zero. Then, using the proposed approach, a superdirective closely-spaced array of two loaded dipoles is designed using full-wave simulator CST Microwave Studio.**

***Index Terms*—antenna arrays, superdirectivity, minimum scattering antennas, optimal currents.**

I. Introduction

It is well known that an antenna array can show an increase in directivity (termed superdirectivity) [1]–[6] compared to a sole element. In case of end-fire directivity of isotropic elements, Uzkov [7] has shown that the directivity approaches N^2 for vanishing distance between them.

In this paper, we use the framework of self- and mutual intensities and powers of the antenna array to optimize its directivity. The proposed method is based on the work of Uzsoky and Solymar [8] and allows finding excitation currents that maximize directivity of arbitrary array. Particularly we show optimization of array of two closely-spaced capacitively loaded dipoles to produce maximum end-fire radiation. CST Microwave Studio [9] is used to design the array element and evaluate necessary quantities that are exported into MATLAB [10] for evaluating the optimal feeding currents.

II. Theory

A. Directivity of Antenna Array

In the case of antenna arrays, it is possible to write the directivity in a compact matrix form, where the vector of excitation currents $\mathbf{I} = [I_1 \cdots I_N]^{\mathrm{T}}$ is extracted [8], [11], [12]

$$D(\theta,\phi) = 4\pi \frac{\mathbf{I}^{\mathrm{H}}\mathbf{u}(\theta,\phi)\mathbf{I}}{\mathbf{I}^{\mathrm{H}}\mathbf{p}\mathbf{I}}. \tag{1}$$

In the above H stands for Hermitian transpose and $\mathbf{u}$ and $\mathbf{p}$ are matrices of the normalized radiation intensities and powers respectively. They contain the self and mutual interactions in the array structure with regard to total directivity D. Entries of the matrices are

$$u_{mn}(\theta,\phi) = \frac{15k^2}{4\pi} \int\limits_{V_m}\int\limits_{V_n} \Lambda(\mathbf{r}_m,\mathbf{r}_n)\mathrm{e}^{\mathrm{j}k(\Delta_m-\Delta_n)}\,\mathrm{d}\mathbf{r}_m\,\mathrm{d}\mathbf{r}_n, \tag{2}$$

where $k = 2\pi/\lambda$ is a wavenumber, integration is performed over the source currents,

$$\Delta_m - \Delta_n = \mathbf{r_0}\cdot(\mathbf{r}_m - \mathbf{r}_n), \tag{3}$$

$$\Lambda(\mathbf{r}_m,\mathbf{r}_n) = \mathbf{j}_m(\mathbf{r}_m)\cdot\mathbf{j}_n^*(\mathbf{r}_n) - \mathbf{r}_0\cdot\mathbf{j}_m(\mathbf{r}_m)\mathbf{r}_0\cdot\mathbf{j}_n^*(\mathbf{r}_n), \tag{4}$$

$\mathbf{j}_m$ and $\mathbf{j}_n$ are normalized current densities in cartesian coordinates and $\mathbf{r}_0$ is a unit vector in spherical coordinates. The normalized radiated power can be obtained by integrating the radiation intensity over the complete solid angle

$$p_{mn} = \int\limits_0^{2\pi}\int\limits_0^{\pi} u_{mn}(\theta,\phi)\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \tag{5}$$

or from impedance matrix of the given array.

Next, the eigenvalue equation is employed [13] to solve for pairs of excitation current vectors $\mathbf{I}$ and eigenvalues - directivities D

$$4\pi\mathbf{u}(\theta,\phi)\mathbf{I} = D\mathbf{p}\mathbf{I}. \tag{6}$$

It is shown in [8] that the largest eigenvalue, i. e. largest directivity, is associated with the superdirective optimal current

$$\mathbf{I}_{\mathrm{opt}} = \frac{1}{4\pi}\mathbf{p}^{-1}\mathbf{V}. \tag{7}$$

The elements of $\mathbf{V}$ are $V_m = f_m\mathrm{e}^{\mathrm{j}k\mathbf{r}_m\cdot\mathbf{r}_0}$, where f_m is a voltage radiation pattern of the m- th element with the others open-circuited. The reference point is considered to be the the origin of a cartesian system.

B. End-Fire Array of Two Isotropic Radiators

For some simple sources, (7) can be evaluated analytically. An illustrating example are two closely-spaced isotropic radiators. Their radiation pattern is therefore $f_1 = f_2 = 1$. Assume that they are spaced in the x axis by electrical distance $s = kd$ and obtain the superdirective currents for end-fire radiation.

The self radiated power can be set arbitrary, let $p_{11} = p_{22} = 1$. It can be shown that for two identical antennas, the mutual power is completely given by the self-radiated power of one element. It follows from (2) that

$$u_{mn}(\theta,\phi) = u_{11}(\theta,\phi)\mathrm{e}^{\mathrm{j}k(\Delta_m-\Delta_n)}, \tag{8}$$

which is then inserted into (5). Simple calculation yields $p_{12} = p_{21} = \frac{\sin s}{s}$.

Since the $\mathbf{p}$ matrix has a dimension of 2×2, its inverse can be easily written in closed form and we have (7)

$$\mathbf{I}_{\mathrm{opt}} = \frac{1}{4\pi}\frac{1}{p_{11}^2 - p_{12}^2}\begin{bmatrix} p_{11} & -p_{12} \\ -p_{12} & p_{11}\end{bmatrix}\begin{bmatrix}\mathrm{e}^{\mathrm{j}s/2} \\ \mathrm{e}^{-\mathrm{j}s/2}\end{bmatrix}. \tag{9}$$

For superdirective behavior, the elements have to be close ($< \lambda/4$). So, the difference between self and mutual power decreases and is even going to zero for zero spacing. Consequently, the condition number of the $\mathbf{p}$ matrix increases dramatically. Well known sensitivity of the superdirective array to feeding currents has therefore roots in the matrix inverse in (7).

Omitting the $1/4\pi$ factor in (7) and assuming that the separation s is very small, (9) may be evaluated to

$$\mathbf{I}_{\text{opt}} = \left[\tfrac{1}{2} + \mathrm{j}\tfrac{1}{3s} \quad \tfrac{1}{2} - \mathrm{j}\tfrac{1}{3s}\right] \tag{10}$$

Amplitudes of the current are the same and their relative phase is

$$\alpha_{\text{opt}} \approx \pi - \frac{1}{3}s. \tag{11}$$

Indeed, it can be checked that inserting (10) into (1) gives $D = 4$ for $s \to 0$ in accordance with the Uzkov's N^2 limit.

III. Array Feeding Synthesis for Maximum Directivity

Now consider more real example of a two-element array of capacitively loaded dipoles. The electrical size of the array is $ka = 0.8$ and separation $s = 0.063\lambda$, see Fig. 1.

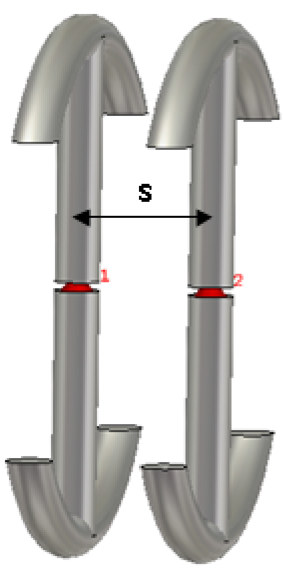

Fig. 1: Layout of the closely-spaced array.

The array is first simulated in CST and f_1 and f_2 are obtained by sequential excitation of current ports. This assures that the other antenna is open-circuited. The $\mathbf{p}$ matrix is gathered from the CST impedance matrix. The process is automated via VBA macro and MATLAB script. By solving (7), superdirective currents for given geometry are returned. In the presented case, the amplitude ratio is 1.081 and relative phase $\alpha_{\text{opt}} = 164.33°$.

It is interesting to note that even the array is symmetric, the amplitudes of optimal currents are different. The reason is that the open-circuit patterns f_1, f_2 in (7) are not identical in general. The only exception are Canonical minimum-scattering antennas [14], i.e. single-mode antennas (elementary dipoles/loops and of course isotropic radiators).

For comparison, the array was also fed by out-of-phase currents, $\alpha = 180°$. The results - farfield and Poynting streamlines [15] are shown in Fig. 2. It is seen that the power density has quite complicated vortex-like structure and is more closely bound to the antennas for the superdirective case.

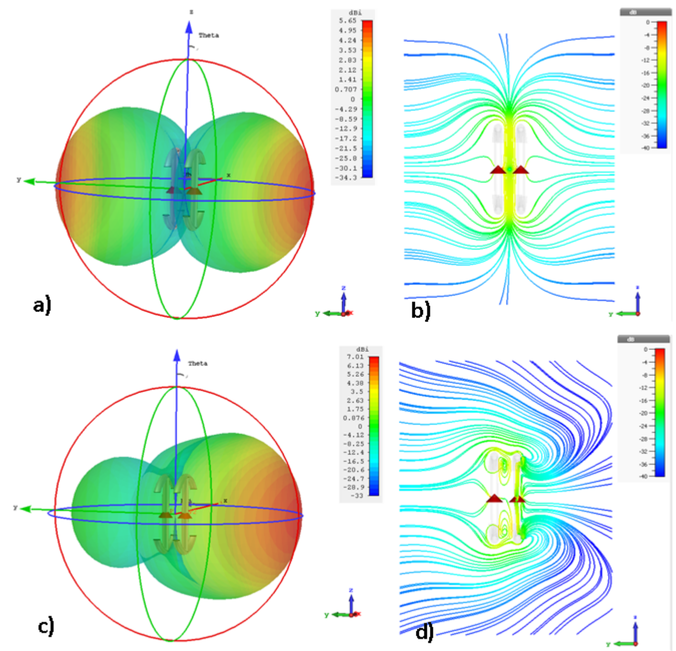

Fig. 2: Far field and Poynting streamlines for out-of-phase (top) and superdirective (bottom) excitation. The increase of directivity is 1.36 dB.

IV. Conclusion

A method for designing optimum currents with regard to the maximum directivity of an array has been shown. Those currents for close spacing produce superdirective radiation and therefore, due to the inverse of the power matrix, their settings are very sensitive. In further work, losses will be incorporated and more complicated arrays designed.

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