10. week
19.4. 2011

MoM
Wire dipol antenna

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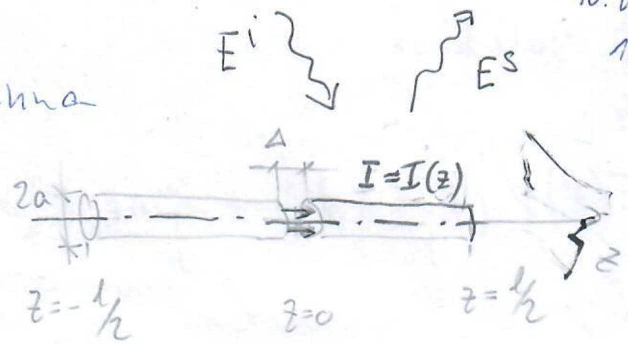
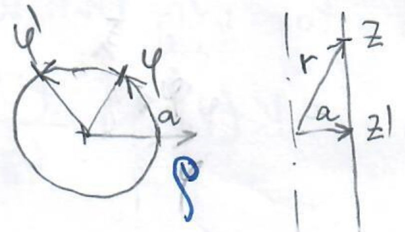
boundary condition on the surface



$E_z^s = -E_z^i$ for $-\frac{l}{2} \le z \le \frac{l}{2}, \rho = a,$

$E_t = 0$ (PEC) $\quad 0 \le \varphi \le 2\pi$

Fig. 1



for $a \ll l$ $\; J = J_z(z)$

IE may be obtained from boundary condition.

$$r = \sqrt{2a^2 - 2a^2\cos(\varphi - \varphi') + (z-z')^2} = \Big|_{\varphi=0} \sqrt{4a^2 \sin^2\left(\frac{\varphi'}{2}\right) + (z-z')^2} \cong \sqrt{a^2 + (z-z')^2}$$

El. field produced by current

$$j\omega\varepsilon_0 \bar{E} = \nabla\nabla \bar{A} + k^2 \bar{A} \qquad \text{p. 470, 15}$$

$$I(z') = \int_0^{2\pi} J_z(z')\, a\, d\phi = 2\pi a J_z(z')$$

where vect. potencial A is related to J_z

$$\Delta A_z + k^2 A_z = -\mu J_z \quad \Rightarrow \quad A_z = \int_{-l/2}^{l/2} I(z') G(z,z')\, dz' = \int_{-l/2}^{l/2} I(z') \frac{e^{-jkr}}{4\pi r}\, dz'$$

here

$$E_z = \frac{1}{j\omega\varepsilon_0}\left[\frac{\partial^2 A_z}{\partial z^2} + k^2 A_z\right] \qquad G(z,z') = \frac{e^{-jkr}}{4\pi r} \qquad r \cong \sqrt{a^2 + (z-z')^2}$$

p. 457, 471

substitute A_z to E_z and put equal to $-E_z^i$, we obtain

current supposed to be on the axis of the dipole !

$$\frac{1}{j\omega\varepsilon_0} \int_{-l/2}^{l/2} \left[\frac{\partial^2 G(z,z')}{\partial z^2} + k^2 G(z,z')\right] I(z')\, dz' = -E_z^i \quad \text{at } \rho = a$$

Pocklington integro-differential eq.

excitation - here delta gap voltage V_i

$$E_z^i = \begin{cases} V_i/\Delta & \text{over feed gap } \Delta \\ 0 & \text{elsewhere} \end{cases}$$

p. 473

Math solution

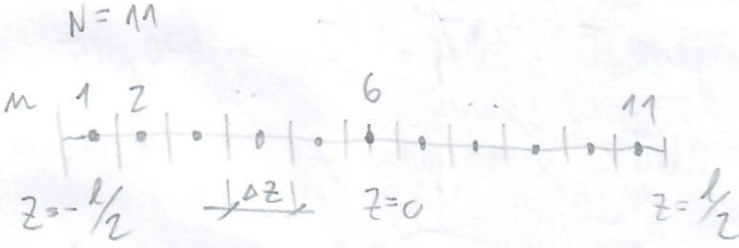
$$\int_{-l/2}^{l/2} I(z')\,K(z,z')\,dz' = -j\omega\varepsilon_0 E_z^i \quad \text{at } \rho = a \quad , \; K(z,z') - \text{kernel}$$

for very thin wires

$$K(z,z') = \frac{e^{-jkr}}{4\pi r^5}\left[(1+jkr)(2r^2-3a^2)+(kar)^2\right]$$

using pulse basis function and point matching at midpoints of segments z_m (see fig. 2) IE reduces to

$$\sum_{n=1}^{N} \alpha_n \int_{\Delta z_n} K(z_m, z')\,dz' = -j\omega\varepsilon_0 E_z^i(z_m) \qquad m = 1, 2, .. N$$



in simplified form

$$\sum_{n=1}^{N} \alpha_n I_{mn} = -E_z^i(z_m) \qquad I_{mn} = \frac{1}{j\omega\varepsilon_0}\int_{\Delta z_n} K(z_m, z')\,dz' =$$

or

$$\alpha_1 I_{m1} + \alpha_2 I_{m2} + .. + \alpha_n I_{mn} = -E_z^i(z_m)$$

$$= \frac{1}{j\omega\varepsilon_0}\frac{1}{4\pi}\int_{z_n-\Delta z/2}^{z_n+\Delta z/2} \frac{e^{-jkr}}{4\pi r^5}\left[(1+jkr)(2r^2-3a^2)+(kar)^2\right]dz'$$

$$r = \sqrt{a^2 + (z_m - z')^2}$$

or

$$[I][\alpha]$$

$$[\cancel{\alpha}][\cancel{I}] = [-E_z^i]$$

$$[\alpha] = [I]^{-1}[-E_z^i]$$

input impedance $Z_{in} = \dfrac{V_i}{\alpha_6}$ - center of the wire $\quad \alpha_{\frac{N-1}{2}+1}$

here