Chapter 7

Antennas

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Outline

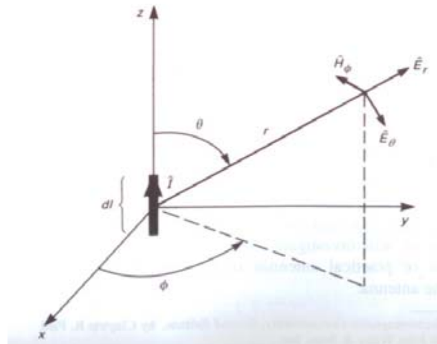
- Preview
- Elemental Dipole Antennas
- The Half-Wave Dipole and Quarter-Wave Monopole Antennas
- Antenna Arrays
- Characterization of Antennas
- The Friis Transmission Equation
- Effects of Reflections
- Broadband Measurement Antennas

Preview

- Intentional Antennas
  - AM, FM, and radar antennas generate electromagnetic fields that couple to electronic devices and result in susceptibility problems.
- Unintentional Antennas
  - These types of antennas are responsible for producing the radiated emissions that are measured by the measurement antennas and may result in the product being out of compliance. (For example: PCB Lands)

Elemental Dipole Antennas

- The Electric (Hertzian) Dipole
  - Electromagnetic Fields
    - The components of the magnetic field intensity vector are



$$\hat{H}_r = 0$$

$$\hat{H}_\theta = 0$$

$$\hat{H}_\phi = \frac{\hat{I}\,dl}{4\pi}\beta_0^2 \sin\theta \left(j\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} \right) e^{-j\beta_0 r}$$

- Similarly, the component of the electric field intensity vector are

$$\hat{E}_r = 2\frac{\hat{I}\,dl}{4\pi}\eta_0 \beta_0^2 \cos\theta \left(\frac{1}{\beta_0^2 r^2} - j\frac{1}{\beta_0^3 r^3} \right) e^{-j\beta_0 r}$$

$$\hat{E}_\theta = \frac{\hat{I}\,dl}{4\pi}\eta_0 \beta_0^2 \sin\theta \left(j\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} - j\frac{1}{\beta_0^3 r^3} \right) e^{-j\beta_0 r}$$

$$\hat{E}_\phi = 0$$

Elemental Dipole Antennas

- The Electric (Hertzian) Dipole
  - Electromagnetic Fields
    - The boundary between the near field and the far field is $\boxed{1/(\beta_0 r)^2 = 1/\beta_0 r}$ or $r = \lambda_0/2\pi \cong \frac{1}{6}\lambda_0$
    - Typically, a more realistic choice for the boundary is $3\lambda_0$ for wire-type antennas or $2D^2/\lambda_0$ for surface-type antennas such as parabolic or horns.
    - The far-field vectors are

$$\vec{\hat{E}}_{\text{far field}} = j\eta_0\beta_0 \frac{\hat{I}\,dl}{4\pi}\sin\theta\,\frac{e^{-j\beta_0 r}}{r}\,\vec{a}_\theta = j\frac{f\mu_0}{2}\,\hat{I}\,dl\,\sin\theta\left\{\frac{e^{-j[2\pi(r/\lambda_0)]}}{r}\right\}$$

$$\vec{\hat{H}}_{\text{far field}} = j\beta_0 \frac{\hat{I}\,dl}{4\pi}\sin\theta\,\frac{e^{-j\beta_0 r}}{r}\,\vec{a}_\phi = j\frac{f\mu_0}{2\eta_0}\,\hat{I}\,dl\,\sin\theta\left\{\frac{e^{-j[2\pi(r/\lambda_0)]}}{r}\right\}$$

    - where

$$\beta_0 = 2\pi/\lambda_0 = \omega\sqrt{\mu_0\epsilon_0} \text{ and } \eta_0 = \sqrt{\mu_0/\epsilon_0}$$

Elemental Dipole Antennas

- The Electric (Hertzian) Dipole
  - Electromagnetic Fields
    - Transforming into time-domains, we have

$$
\begin{aligned}
\vec{E}_{\text{far field}} &= \mathcal{Re}\{\hat{\vec{E}}_{\text{far field}} e^{j\omega t}\} \\
&= \frac{E_m}{r}\cos\left[\omega\left(t - \frac{r}{v_0}\right) + 90^\circ\right]\vec{a}_\theta \\
&= -\frac{E_m}{r}\sin\left[\omega\left(t - \frac{r}{v_0}\right)\right]\vec{a}_\theta
\end{aligned}
$$

$$
\begin{aligned}
E_m &= \frac{\eta_0 \beta_0 \hat{I}\, dl}{4\pi}\sin\theta \\
&= \frac{f\mu_0}{2}\hat{I}\, dl\, \sin\theta
\end{aligned}
$$

$$
\begin{aligned}
\vec{H}_{\text{far field}} &= \mathcal{Re}\{\hat{\vec{H}}_{\text{far field}} e^{j\omega t}\} \\
&= \frac{E_m}{\eta_0 r}\cos\left[\omega\left(t - \frac{r}{v_0}\right) + 90^\circ\right]\vec{a}_\phi \\
&= -\frac{E_m}{\eta_0 r}\sin\left[\omega\left(t - \frac{r}{v_0}\right)\right]\vec{a}_\phi
\end{aligned}
$$

Elemental Dipole Antennas

- The Electric (Hertzian) Dipole
  - Properties of the Far Fields
    - The far fields are spherical waves which locally resemble uniform plane waves.
    - The fields are proportional to $1/r$, I, dl, and $\sin\theta$.
    - $|E_{far\ field}|/|H_{far\ field}| = \eta_0$.
    - $E_{far\ field}$ and $H_{far\ field}$ are locally orthogonal.
    - $E_{far\ field} \times H_{far\ field}$ → in the direction of propagation.
    - A phase term $e^{-j\beta_0 r}$ translates to a time delay in the time domain of $\sin[\omega(t-r/v_0)]$
    - The inverse-distance rule holds only if both D_1 and D_2 are in the far field of the radiating element. $(E_{D1}/E_{D2}=D_2/D_1)$

Elemental Dipole Antennas

- The Electric (Hertzian) Dipole
  - Radiation Power
    - The average power density vector is

$$\vec{S}_{av} = \tfrac{1}{2}\mathscr{Re}\left\{\hat{\vec{E}} \times \hat{\vec{H}}^*\right\}$$
$$= \tfrac{1}{2}\mathscr{Re}\{\hat{E}_\theta \hat{H}^*_\phi \vec{a}_r - \hat{E}_r \hat{H}^*_\phi \vec{a}_\theta\}$$
$$= 15\pi\left(\frac{dl}{\lambda_0}\right)^2 |\hat{I}|^2 \frac{\sin^2\theta}{r^2}\vec{a}_r \quad (\text{in W/m}^2)$$

    - We see that this could have been obtained solely from the far-field expressions.
    - The total average radiation power is

$$P_{rad} = \oint_s \vec{S}_{av}\, \vec{ds}$$
$$= 80\pi^2\left(\frac{dl}{\lambda_0}\right)^2 \frac{|\hat{I}|^2}{2} \quad (\text{in W})$$

Elemental Dipole Antennas

- The Electric (Hertzian) Dipole
  - Radiation Power
    - The radiation resistance is defined as

$$R_{\text{rad}} = \frac{P_{\text{rad}}}{|\hat{I}_{\text{rms}}|^2}$$

$$= 80\pi^2 \left(\frac{dl}{\lambda_0}\right)^2 \quad (\text{in } \Omega) \qquad \hat{I}/\sqrt{2} = \hat{I}_{\text{rms}}$$

    - The radiation resistance represents a fictitious resistance that dissipates the same amount of power as that radiated by the Hertzian dipole when both carry the same value of current.
    - The Hertzian dipole is a very ineffective radiator since the effective resistance is small.

The far fields of a Hertzian dipole are virtually identical to the far fields of most other practical antennas.

Elemental Dipole Antennas

- The Magnetic Dipole (Loop)
  - Electromagnetic Fields
    - This loop constitutes a magnetic dipole moment

$$\hat{m} = \hat{I}\pi b^2 \quad (\text{in A m}^2)$$

    - The electric (magnetic) fields are dual to the magnetic (electric) fields of a Hertzian dipole, which are

$$\hat{E}_r = 0$$

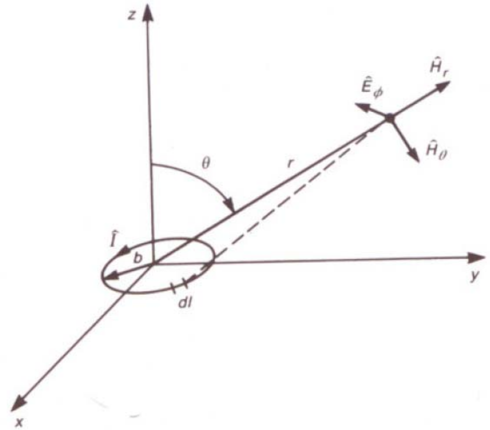
$$\hat{E}_\theta = 0$$

$$\hat{E}_\phi = -j\frac{\omega\mu_0\hat{m}\beta_0^2}{4\pi}\sin\theta\left(j\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2}\right)e^{-j\beta_0 r}$$

$$\hat{H}_r = j2\frac{\omega\mu_0\hat{m}\beta_0^2}{4\pi\eta_0}\cos\theta\left(\frac{1}{\beta_0^2 r^2} - j\frac{1}{\beta_0^3 r^3}\right)e^{-j\beta_0 r}$$

$$\hat{H}_\theta = j\frac{\omega\mu_0\hat{m}\beta_0^2}{4\pi\eta_0}\sin\theta\left(j\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} - j\frac{1}{\beta_0^3 r^3}\right)e^{-j\beta_0 r}$$

$$\hat{H}_\phi = 0$$



Elemental Dipole Antennas

- The Magnetic Dipole (Loop)
  - Electromagnetic Fields
    - The far fields are characterized by the $1/r$-dependent terms, which are

$$\vec{\hat{E}}_{\text{far field}} = \frac{\omega\mu_0 \hat{m} \beta_0}{4\pi} \sin\theta \, \frac{e^{-j\beta_0 r}}{r} \, \vec{a}_\phi$$

$$= \frac{\pi^2 f^2 \mu_0 \hat{I} b^2}{v_0} \sin\theta \, \frac{e^{-j[2\pi(r/\lambda_0)]}}{r}$$

$$\vec{\hat{H}}_{\text{far field}} = -\frac{\omega\mu_0 \hat{m} \beta_0}{4\pi\,\eta_0} \sin\theta \, \frac{e^{-j\beta_0 r}}{r} \, \vec{a}_\theta$$

$$= \frac{\pi^2 f^2 \mu_0 \hat{I} b^2}{\eta_0 \, v_0} \sin\theta \, \frac{e^{-j[2\pi(r/\lambda_0)]}}{r}$$

    - These far fields have the same characteristics as those of a Hertzian dipole.

Elemental Dipole Antennas

- The Magnetic Dipole (Loop)
  - Electromagnetic Fields
    - The radiation resistance of the magnetic dipole is

$$R_{rad} = \frac{P_{av}}{|\hat{I}_{RMS}|^2}$$

$$= 31{,}170 \left(\frac{A}{\lambda_0^2}\right)^2$$

    - where $A = \pi b^2$ is the area of the loop.
    - Like the Hertzian dipole, the magnetic dipole is not an efficient radiator.

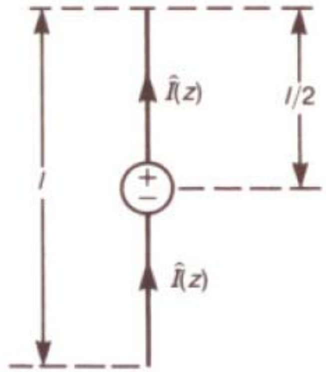
Commonly Used Circuit

    - From example 7.2, a 1×1cm^2 current loop on a PCB carrying a 100mA current and operating at 50MHz will cause a radiated emission that will fail to comply with the FCC Class B regulatory limit.

The Half-Wave Dipole and Quarter-Wave Monopole Antennas

- Long-Dipole Antenna
  - Far Fields
    - Assume the currents on the dipole are

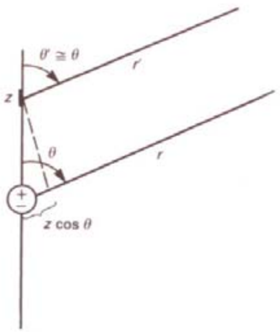
$$\hat{I}(z) = \begin{cases} \hat{I}_m \sin[\beta_0(\frac{1}{2}l - z)] & \text{for } 0 < z < \frac{1}{2}l \\ \hat{I}_m \sin[\beta_0(\frac{1}{2}l + z)] & \text{for } -\frac{1}{2}l < z < 0 \end{cases}$$

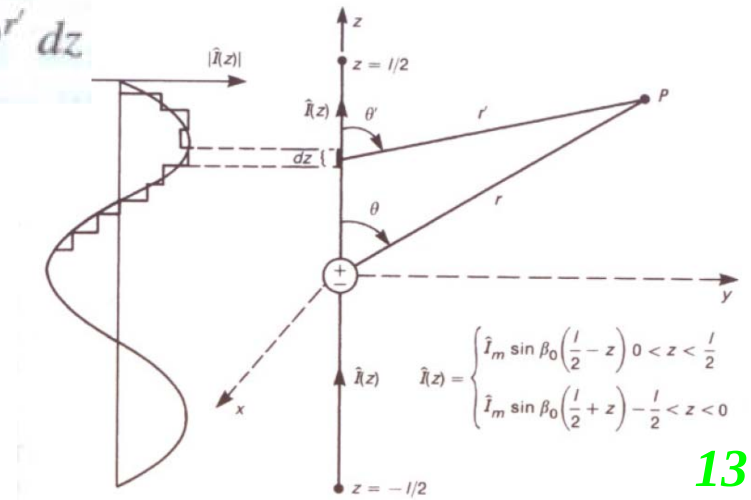


    - The far field due to a elemental segment *dz* (from the far field of a Hertzian dipole) is Please see p. 5

$$d\hat{E}_\theta = j\eta_0\beta_0 \frac{\hat{I}(z)\sin\theta'}{4\pi r'} e^{-j\beta_0 r'}\, dz$$

    - Since $r' \cong r - z\cos\theta$





The Half-Wave Dipole and Quarter-Wave Monopole Antennas

- Long-Dipole Antenna
  - Far Fields
    - Thus, the far field due to a elemental segment *dz* becomes

$$d\hat{E}_\theta = j\eta_0\beta_0 \frac{\hat{I}(z)\sin\theta}{4\pi r} e^{-j\beta_0(r-z\cos\theta)}\,dz$$

    - The total far electric field is the sum of these contributions, which is

$$\hat{E}_\theta = \int_{z=-l/2}^{z=l/2} j\eta_0\beta_0 \frac{\hat{I}(z)\sin\theta}{4\pi r} e^{-j\beta_0 r} e^{j\beta_0 z\cos\theta}\,dz$$

$$\hat{I}(z) = \begin{cases} \hat{I}_m \sin[\beta_0(\frac{1}{2}l - z)] & \text{for } 0 < z < \frac{1}{2}l \\ \hat{I}_m \sin[\beta_0(\frac{1}{2}l + z)] & \text{for } -\frac{1}{2}l < z < 0 \end{cases}$$

$$\longrightarrow \hat{E}_\theta = j\frac{\eta_0\hat{I}_m e^{-j\beta_0 r}}{2\pi r}F(\theta) = j\frac{60\hat{I}_m e^{-j\beta_0 r}}{r}F(\theta)$$

The Half-Wave Dipole and Quarter-Wave Monopole Antennas

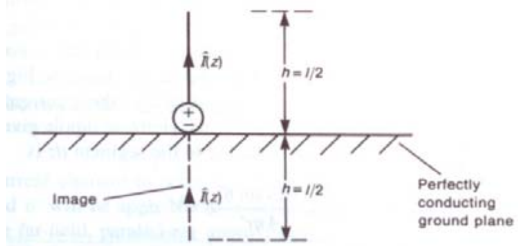
- Long-Dipole Antenna
  - Far Fields
    - where the θ -variation term is denoted by

$$F(\theta) = \frac{\cos[\beta_0(\frac{1}{2}l)\cos\theta] - \cos\beta_0(\frac{1}{2}l)}{\sin\theta} = \frac{\cos[(\pi l/\lambda_0)\cos\theta] - \cos(\pi l/\lambda_0)}{\sin\theta}$$

    - The magnetic field in the far-field region is obtained from

$$\hat{H}_\phi = \frac{\hat{E}_\theta}{\eta_0}$$

    - The monopole antenna can be calculated via a dipole using the image theory.



The Half-Wave Dipole and Quarter-Wave Monopole Antennas

- Half-Wave Dipole Antenna $l = \lambda_0/2$
  - Electric Field Pattern
    - For the half-wave dipole, the θ-variation term becomes

$$F(\theta) = \frac{\cos(\frac{1}{2}\pi\cos\theta)}{\sin\theta} \quad \text{(half-wave dipole, } l = \tfrac{1}{2}\lambda_0\text{)}$$

    - The maximum electric field is at the broadside

$$|\hat{E}|_{\max} = 60\frac{|\hat{I}_m|}{r} \quad (\theta = 90^\circ)$$

    - and the current at the input terminals is

$$z = 0: \ \hat{I}(0) = \hat{I}_m \sin[\beta_0 l/2] = \hat{I}_m \sin[\pi/2] = \hat{I}_m$$

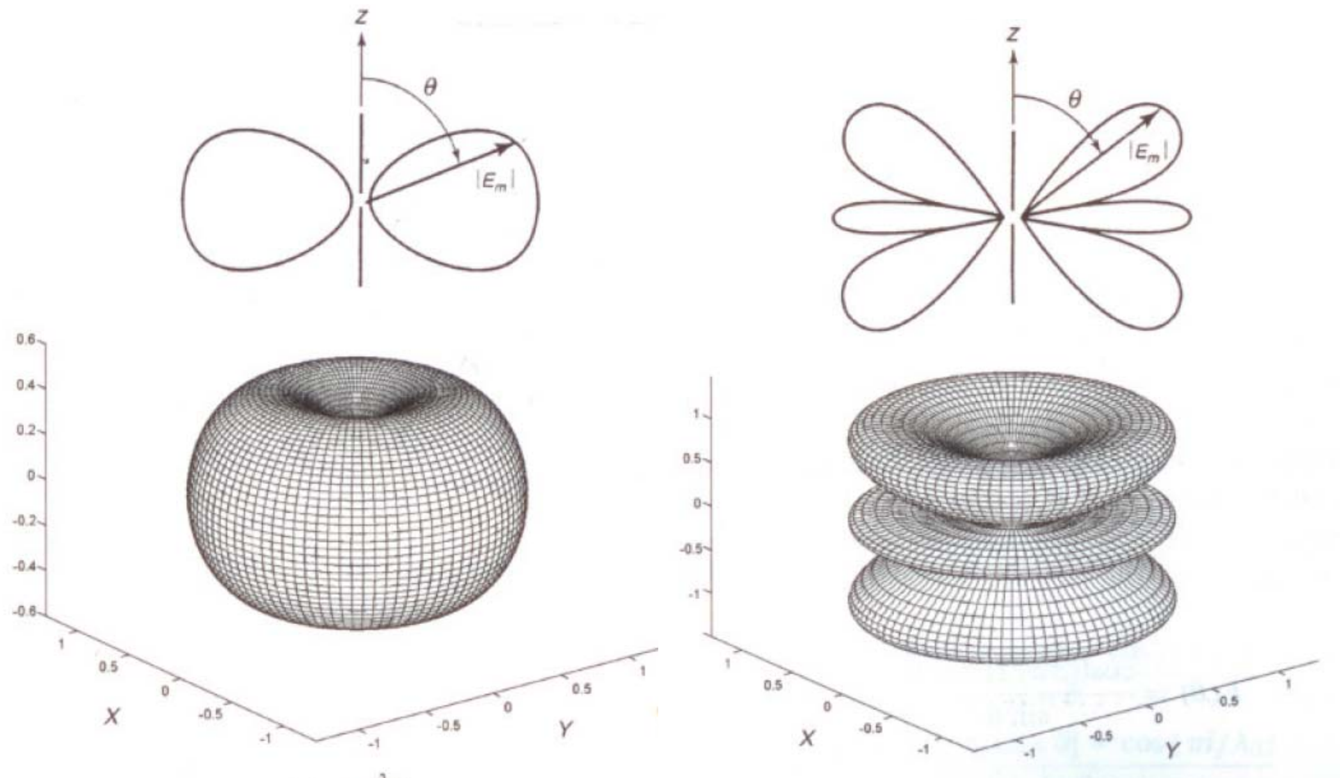
Peak value of the input signal

    - For $l = 3\lambda_0/2$, the θ-variation term becomes

$$F(\theta) = \frac{\cos[(3\pi/2)\cos\theta]}{\sin\theta} \quad l = \frac{3}{2}\lambda_0$$

The Half-Wave Dipole and Quarter-Wave Monopole Antennas

- Half-Wave Dipole Antenna $l = \lambda_0/2$
  - Electric Field Pattern
    - The field patterns are plotted below for



(a) $l = \frac{\lambda_0}{2}$ (b) $l = \frac{3}{2}\lambda_0$

The Half-Wave Dipole and Quarter-Wave Monopole Antennas

- Half-Wave Dipole Antenna $l = \lambda_0/2$
  - Radiation Power
    - The average power density is

$$\begin{aligned}\vec{S}_{\text{av}} &= \tfrac{1}{2}\,\mathcal{R}e\left\{\hat{\vec{E}} \times \hat{\vec{H}}^*\right\} \\ &= \tfrac{1}{2}\,\mathcal{R}e\left\{\hat{E}_\theta \hat{H}_\phi^*\right\}\vec{a}_r \\ &= \frac{1}{2}\frac{|\hat{E}_\theta|^2}{\eta_0}\,\vec{a}_r \\ &= \underbrace{\left(\frac{\eta_0}{8\pi^2}\right)}_{4.77}\frac{|\hat{I}_m|^2}{r^2}F^2(\theta)\,\vec{a}_r\end{aligned}$$

    - The total radiated power is obtained by integrating over a sphere of radius r as

$$P_{\text{av}} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \vec{S}_{\text{av}} \cdot \underbrace{r^2 \sin\theta\, d\theta\, d\phi\, \vec{a}_r}_{\vec{ds}} = 73\,\frac{|\hat{I}_m|^2}{2}$$

The Half-Wave Dipole and Quarter-Wave Monopole Antennas

- Half-Wave Dipole Antenna $l = \lambda_0/2$
  - Radiation Power
    - which could be rewritten as

$$P_{\text{rad}} = 73\frac{|\hat{I}_m|^2}{2}$$

$$= 73|\hat{I}_{\text{in, rms}}|^2 \quad \text{(in W) (half-wave dipole)}$$

    - This suggests that we define a radiation resistance of the half-wave dipole as

$$R_{\text{rad}} = 73\,\Omega \quad \text{(half-wave dipole)}$$

    - Since the monopole radiates only half the power of the dipole, the radiation resistance for the monopole is half that of the corresponding dipole, which is

$$R_{\text{rad}} = 36.5\,\Omega \quad \text{(quarter-wave monopole)}$$

The Half-Wave Dipole and Quarter-Wave Monopole Antennas

- Half-Wave Dipole Antenna $l = \lambda_0/2$
  - Input Impedance
    - Consider the total input impedance seen at the terminals of the dipole or monopole antenna, we have

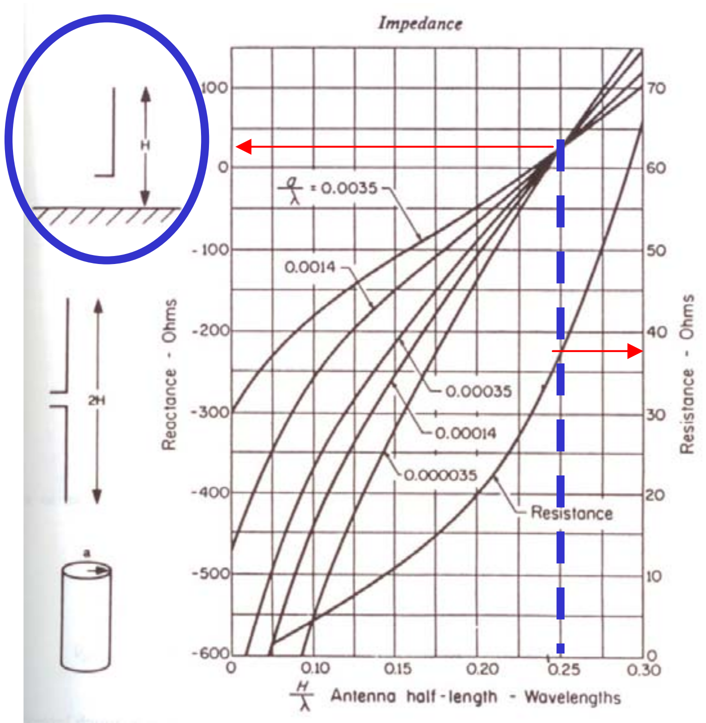
$$\hat{Z}_{\text{in}} = R_{\text{in}} + jX_{\text{in}} \longrightarrow \hat{Z}_{\text{in}} = R_{\text{loss}} + R_{\text{rad}} + jX_{\text{in}}$$

    - which consist of the sum of the radiation resistance and the resistance of the imperfect wires used to construct the dipole.
    - We know X_{in} should be made zero or canceled to result in maximum radiation power.

The Half-Wave Dipole and Quarter-Wave Monopole Antennas

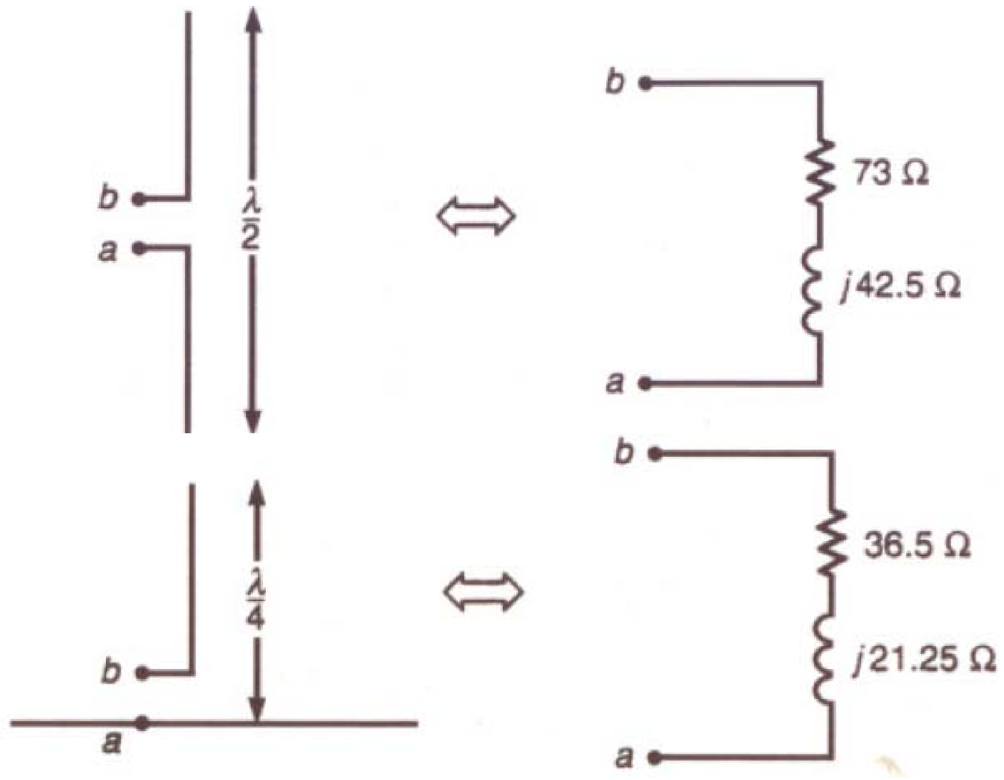
- Half-Wave Dipole Antenna $l = \lambda_0/2$
  - Input Impedance
    - The input reactance for a half-wave dipole (quarter-wave monopole) is X_{in}=42.5Ω (X_{in}=21.25Ω). This reduces to zero when the dipole is made slightly shorter.

The monopoles that are shorter than one-quarter wavelength appear at their input as a small resistance in series with a capacitance, thus loading coils or inductors are inserted in series to cancel this capacitive reactance and increase the radiated power.



The Half-Wave Dipole and Quarter-Wave Monopole Antennas

- Half-Wave Dipole Antenna $l = \lambda_0/2$
  - Input Impedance
    - From the figure, the input impedances to a half-wave dipole and a quarter-wave monopole.

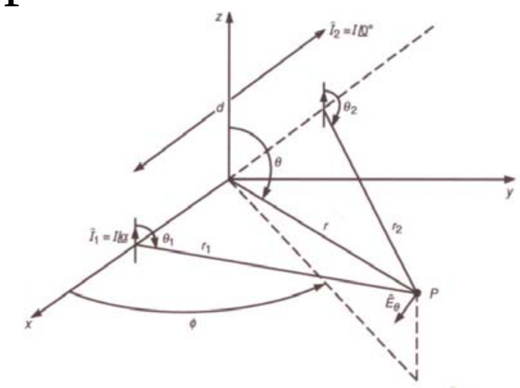


Antenna Arrays

- Array Pattern
  - Two Elements
    - Assuming the two elements are separated by a distance *d* and excited with a phase difference α in the currents, the far fields at point *P* due to each antenna are of the form

$$\hat{E}_{\theta 1} = \frac{\hat{M} I \angle \alpha}{r_1} e^{-j\beta_0 r_1}$$

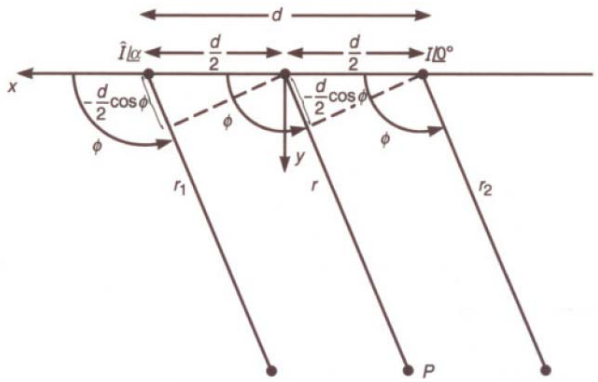
$$\hat{E}_{\theta 2} = \frac{\hat{M} I \angle 0}{r_2} e^{-j\beta_0 r_2}$$



    - where *M* depends on the type of antennas used.
    - For Hertzian dipoles and long dipoles, *M* are

$$\hat{M} = j\eta_0\beta_0(dl/4\pi)\sin\theta \qquad \hat{M} = j60F(\theta)$$

Antenna Arrays



- Array Pattern
  - Two Elements
    - The total field at point P is the sum of the fields of the two antennas, which is

$$\hat{E}_\theta = \hat{E}_{\theta 1} + \hat{E}_{\theta 2}$$

$$= \hat{M}I\left(\frac{e^{-j\beta_0 r_1}}{r_1}e^{j\alpha} + \frac{e^{-j\beta_0 r_2}}{r_2}\right)$$

$$= \hat{M}Ie^{j\alpha/2}\left(\frac{e^{-j\beta_0 r_1}e^{j\alpha/2}}{r_1} + \frac{e^{-j\beta_0 r_2}e^{-j\alpha/2}}{r_2}\right) \longrightarrow \begin{aligned} r_1 &\cong r - \frac{d}{2}\cos\phi \\ r_2 &\cong r + \frac{d}{2}\cos\phi \end{aligned}$$

$$\hat{E}_\theta = \hat{M}I\frac{e^{-j\beta_0 r}}{r}\underbrace{\left(e^{j[\beta_0(d/2)\cos\phi + (\alpha/2)]} + e^{-j[\beta_0(d/2)\cos\phi + (\alpha/2)]}\right)}_{2\cos[\beta_0(d/2)\cos\phi + (\alpha/2)]} e^{j(\alpha/2)}$$

$$= 2\hat{M}I\frac{e^{-j\beta_0 r}}{r}e^{j(\alpha/2)}\cos\left(\pi\frac{d}{\lambda_0}\cos\phi + \frac{\alpha}{2}\right)$$

Antenna Arrays

- Array Pattern
  - Two Elements
    - At a fix distance r, the electric field depends on angle φ as

$$|\hat{E}_\theta| \propto \cos\left(\frac{\pi d}{\lambda_0}\cos\phi + \frac{\alpha}{2}\right)$$

    - Hence the array factor is

$$F(\phi) = \cos\left(\frac{\pi d}{\lambda_0}\cos\phi + \frac{\alpha}{2}\right)$$

    - The location of the nulls in the pattern can be found by solving

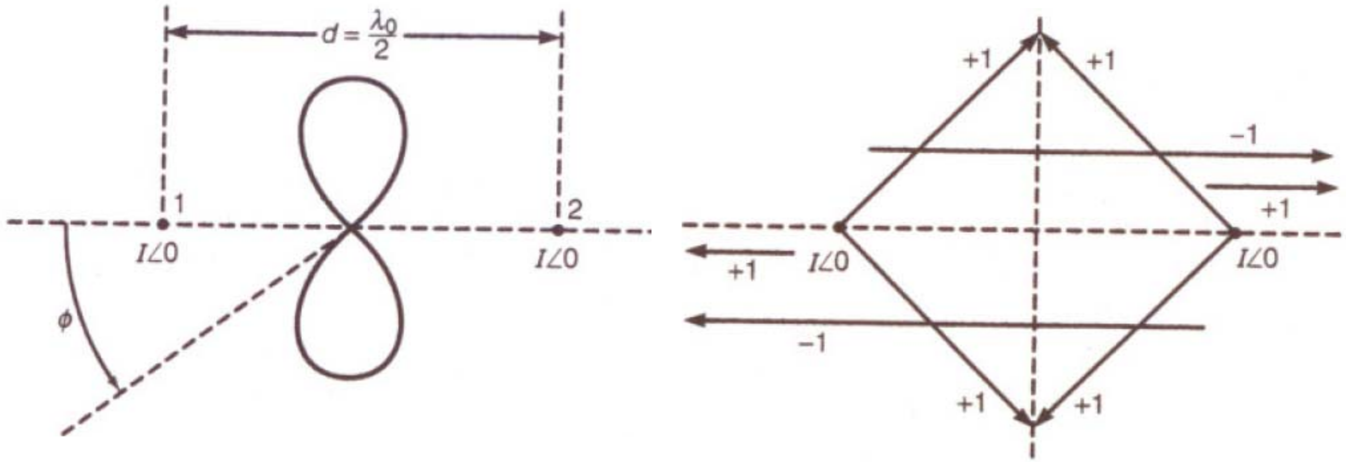
$$F(\phi) = \cos\left(\frac{\pi d}{\lambda_0}\cos\phi + \frac{\alpha}{2}\right) = 0 \longrightarrow \frac{\pi d}{\lambda_0}\cos\phi + \frac{\alpha}{2} = \pm\frac{\pi}{2}$$

Antenna Arrays

- Array Pattern
  - Two Elements: $d = \lambda_0/2,\ \alpha = 0^0$
    - The array factor becomes

$$F(\phi) = \cos\left(\frac{\pi}{2}\cos\phi\right)$$

    - The array pattern is as shown below



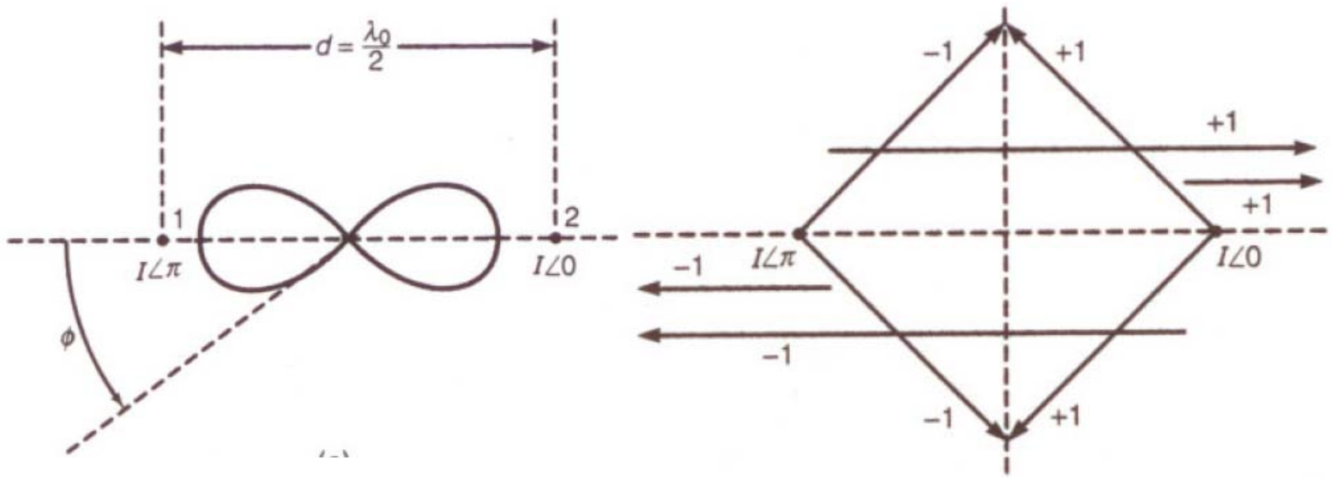
The phase difference between the two antennas is $\beta d = \pi$

Antenna Arrays

- Array Pattern
  - Two Elements: $d = \lambda_0/2$, $\alpha = \pi$
    - The array factor becomes

$$F(\phi) = \cos\left(\frac{\pi}{2}\cos\phi + \frac{\pi}{2}\right)$$

    - The array pattern is as shown below



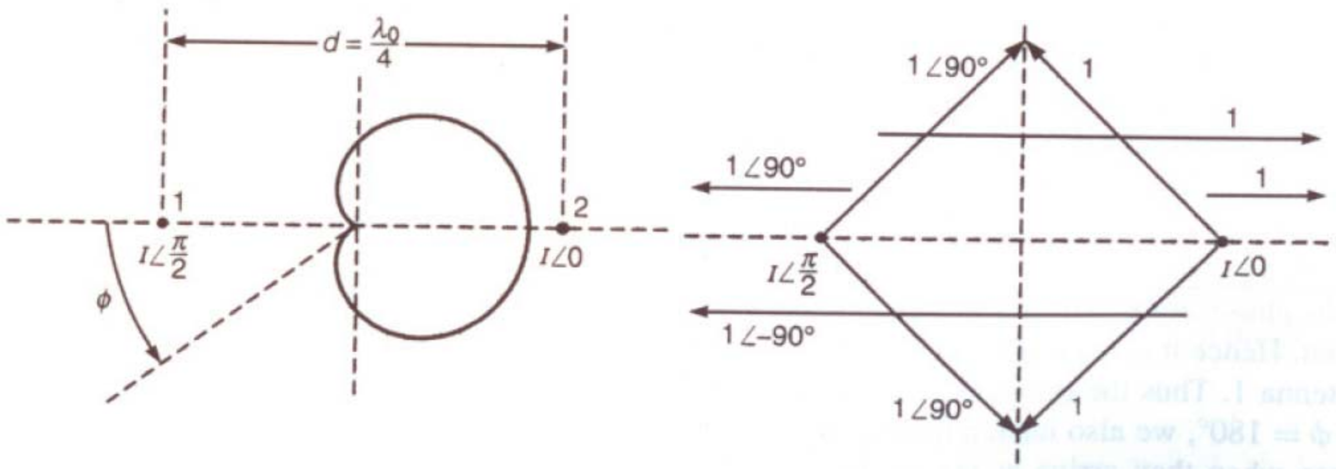
The phase difference between the two antennas is $\beta d = \pi$

Antenna Arrays

- Array Pattern
  - Two Elements: $d = \lambda_0/4$, $\alpha = \pi/2$
    - The array factor becomes

$$F(\phi) = \cos\left(\frac{\pi}{4}\cos\phi + \frac{\pi}{4}\right)$$

    - The array pattern is as shown below



The phase difference between the two antennas is $\beta d = \pi/2$

Characterization of Antennas

- Directivity and Gain
  - Directivity
    - The directivity of an antenna, $D(\theta, \varphi)$, is a measure of the concentration of the radiated power in a particular θ, φ direction at a fixed distance r away from the antenna.
    - Since the far-field, radiated average power densities are

$$\vec{S}_{av} = \frac{|\hat{\vec{E}}_{\text{far field}}|^2}{2\eta_0}\vec{a}_r \quad (\text{in W/m}^2)$$

$$= \frac{E_0^2}{2\eta_0 r^2}\vec{a}_r$$

    - The radiation intensity is defined as

$$U(\theta, \phi) = r^2 S_{av}$$

Independent of r

Characterization of Antennas

- Directivity and Gain
  - Directivity
    - The total average power radiated is

$$P_{\text{rad}} = \oint \vec{S}_{\text{av}} \cdot \vec{ds}$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} S_{\text{av}} \underbrace{r^2 \sin\theta \, d\theta \, d\phi}_{ds}$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} U(\theta, \phi) \sin\theta \, d\phi \, d\theta$$

$$= \oint_S U(\theta, \phi) \, d\Omega$$

The units of U are watts per steradian (W/sr)

The total radiated power is therefore the integral of the radiation intensity over a solid angle of 4π sr.

Characterization of Antennas

- Directivity and Gain
  - Directivity
    - The average radiation intensity is defined as

$$U_{\mathrm{av}} = \frac{P_{\mathrm{rad}}}{4\pi}$$

    - The directivity of an antenna in a particular direction is the ratio of the radiation intensity in that direction to the average radiation intensity, which is

$$D(\theta, \phi) = \frac{U(\theta, \phi)}{U_{\mathrm{av}}} = \frac{4\pi U(\theta, \phi)}{P_{\mathrm{rad}}}$$

Simply a function of the shape of the antenna pattern

    - The directivity is defined as the maximum of $D(\theta, \varphi)$, which is

$$D_{\max} = \frac{U_{\max}}{U_{\mathrm{av}}}$$

Characterization of Antennas

- Directivity and Gain
  - Gain
    - The gain $G(\theta, \varphi)$ takes into account the losses of the antenna, which is referenced to the total power P_{app} applied to the antenna.

$$G(\theta, \phi) = eD(\theta, \phi) \longrightarrow G(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_{\mathrm{app}}}$$

    - where e is the efficiency of the antenna.

$$e = \frac{P_{\mathrm{rad}}}{P_{\mathrm{app}}}$$

    - For most antennas, the efficiency is nearly 100%, and thus the gain and directivity are nearly equal.

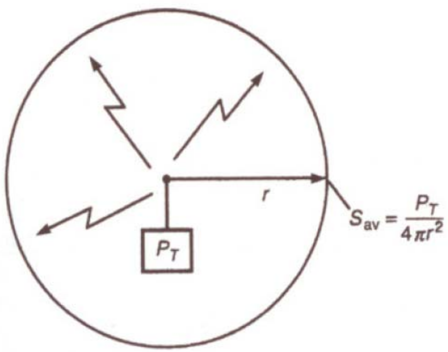
Characterization of Antennas

- Directivity and Gain
  - Isotropic Point Source
    - An isotropic point source is a fictitious lossless antenna that radiates power equally in all directions.
    - The average power density is defined as

$$\vec{S}_{av} = \frac{P_T}{4\pi d^2}\vec{a}_r$$

    - Since the waves resemble (locally) uniform plane waves, the power density is

$$\vec{S}_{av} = \frac{|\vec{\hat{E}}|^2}{2\eta_0} \quad (\text{in W/m}^2)$$



    - Equating the above two equations gives

$$|\vec{\hat{E}}| = \frac{\sqrt{60 P_T}}{d}\vec{a}_\theta \quad (\text{in V/m})$$

    - The gain becomes

$$G_0(\theta, \phi) = \frac{4\pi U_0(\theta, \phi)}{P_T} = 1$$

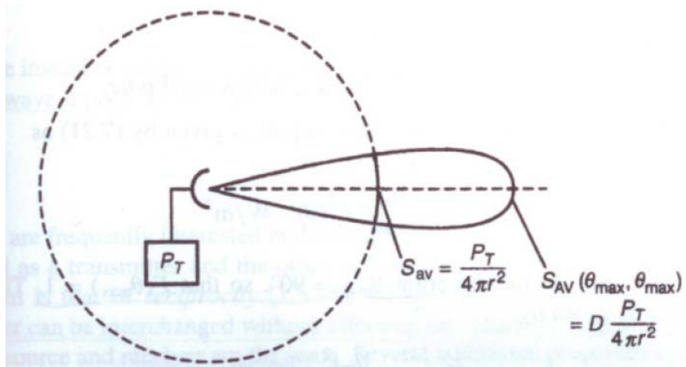
Characterization of Antennas

- Directivity and Gain
  - Directivity—Another Definition
    - The directivity is the ratio of the power density of the antenna in the direction of the main beam to the power density of an isotropic point source that is transmitting the same total power P_T in that direction, both measured at the same distance r

$$D = \frac{S_{av}(\theta_{max}, \phi_{max})}{P_T / 4\pi r^2}$$

$$S_{av} = G \frac{P_{app}}{4\pi r^2}$$

$$= D \frac{P_T}{4\pi r^2}$$



Characterization of Antennas

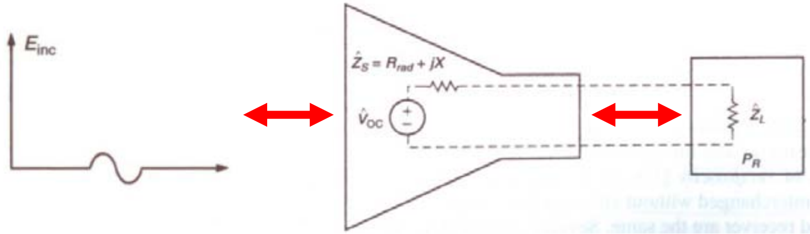
- Directivity and Gain
  - Difference between Half-Wave and Hertzian Dipoles
    - The half-wave dipole is only slightly better than the Hertzian dipole in its ability to focus the radiated power ($G_{half\text{-}wave}$=1.64, $G_{Hertzian}$=1.5). → Directivity is only concerned with the antenna pattern.
    - The radiation resistance of the half-wave dipole is considerably larger than that of the Hertzian dipole, and hence power can be transmitted using a much smaller input current. → Radiation power is proportional to $I^2_{rms}R_{rad}$.

Characterization of Antennas

- Reciprocity
  - Input Impedance
    - The impedance seen looking into an antenna terminals when it is used for transmission is the same as the Thevenin source impedance seen looking back into its terminals when it is used for reception.
  - Antenna Pattern
    - The transmission pattern of the antenna is the same as its reception pattern.

Characterization of Antennas

- Effective Aperture
  - Definition

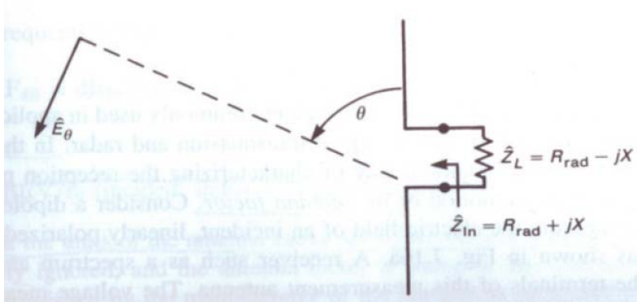


    - The effective aperture of an antenna is related to the ability of the antenna to extract energy from a passing wave.
    - Thus, the effective aperture A_e is defined as the ratio of the power received (in its load impedance) P_R to the power density of the incident wave S_{av}

$$A_e = \frac{P_R}{S_{av}} \quad (\text{in m}^2)$$

    - The maximum effective aperture A_{em} is the ratio when the load impedance is the conjugate of the antenna impedance. In addition to the load match, the polarization of the antenna must be matched.

Characterization of Antennas



- Effective Aperture
  - For a Hertzian Dipole
    - Since the polarization is matched, the open-circuit voltage produced at the terminals of the antenna is

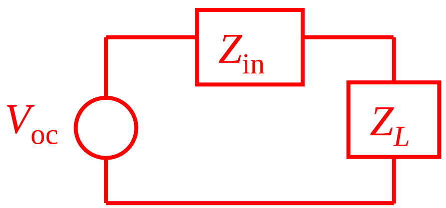
$$|\hat{V}_{OC}| = |\hat{E}_\theta|\, dl$$

    - The power density in the incident wave is

$$S_{\text{av}} = \frac{1}{2}\frac{|\hat{E}_\theta|^2}{\eta_0}$$

    - Since the load is matched, the power received is

$$P_R = \frac{|\hat{V}_{OC}|^2}{8\,R_{\text{rad}}}$$



$$= \frac{|\hat{E}_\theta|^2\, dl^2}{8\,R_{\text{rad}}} \longrightarrow P_R = \frac{|\hat{E}_\theta|^2 \lambda_0^2}{640\pi^2}$$

$$R_{\text{rad}} = \frac{P_{\text{rad}}}{|\hat{I}_{\text{rms}}|^2}$$

$$= 80\pi^2\left(\frac{dl}{\lambda_0}\right)^2 \quad (\text{in } \Omega)$$

Characterization of Antennas

- Effective Aperture
  - For a Hertzian Dipole
    - Thus, the maximum effective aperture is

$$A_{em} = \frac{P_R}{S_{av}} = 1.5\frac{\lambda_0^2}{4\pi} = \frac{\lambda_0^2}{4\pi}G$$

    - Observe that the maximum effective aperture of an antenna is not necessarily related to its "physical aperture".
    - The effective aperture of an antenna used for reception is related to the gain in the direction of the incoming wave of that antenna when it is used for transmission as

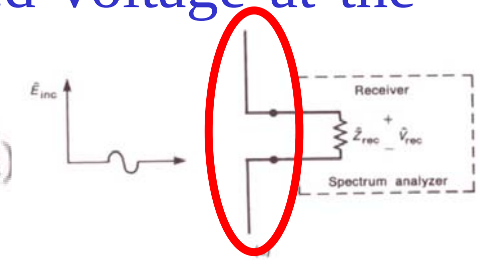
$$\boxed{G(\theta, \phi) = \frac{4\pi}{\lambda_0^2}A_e(\theta, \phi)}$$

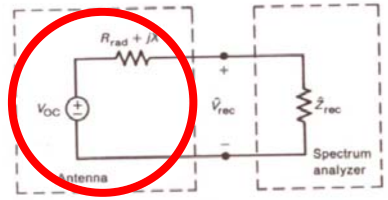
Characterization of Antennas

- Antenna Factor
  - Definition
    - The antenna factor is defined as the ratio of the incident electric field at the surface of the measurement antenna to the received voltage at the antenna terminals as

$$\mathrm{AF} = \frac{\mathrm{V/m\ in\ incident\ wave}}{\mathrm{V\ received}} \quad (\mathrm{in}\ 1/\mathrm{m})$$

$$= \frac{|\hat{E}_{\mathrm{inc}}|}{|\hat{V}_{\mathrm{rec}}|}$$





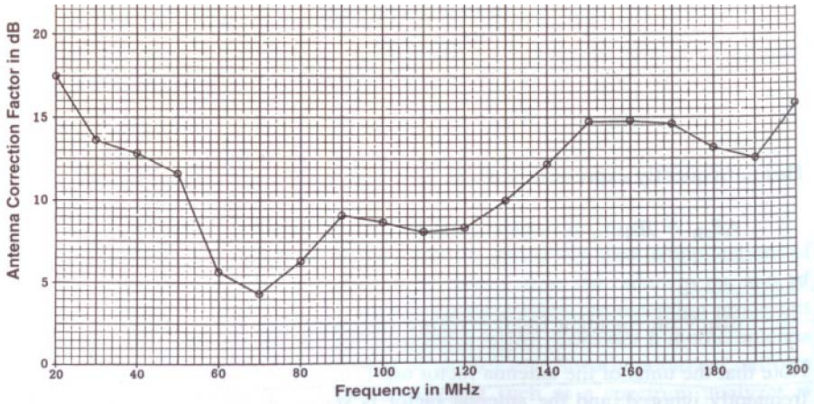
- Expressing in dB, we have

$$\mathrm{AF_{dB}} = \mathrm{dB\mu V/m\ (incident\ field)} - \mathrm{dB\mu V\ (received\ voltage)}$$

$$\leftrightarrow \mathrm{dB\mu V/m\ (incident\ field)} = \mathrm{dB\mu V\ (received\ voltage)} + \mathrm{AF_{dB}}$$

Characterization of Antennas

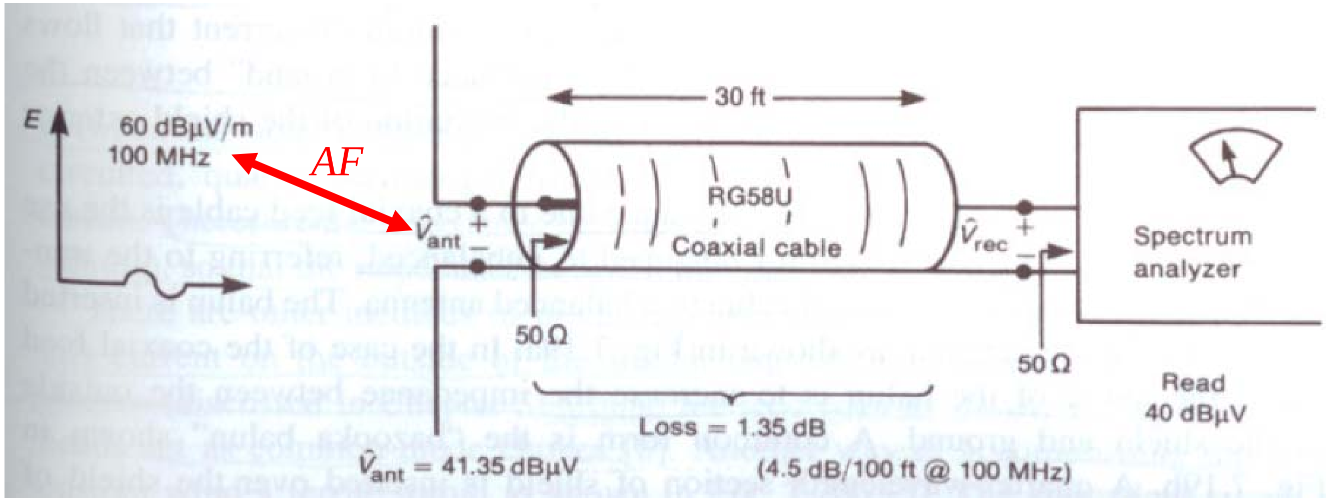
- Antenna Factor
  - Implicit Assumptions while Measuring
    - The incident field is polarized for maximum response of the antenna.
    - The input impedance of the receiver is used not only to make the measurement but also to calibrate the antenna. → This means that the antenna factor given by the manufacturer, which are based on the matched load, should be calibrated to the actual receiver load.



Characterization of Antennas

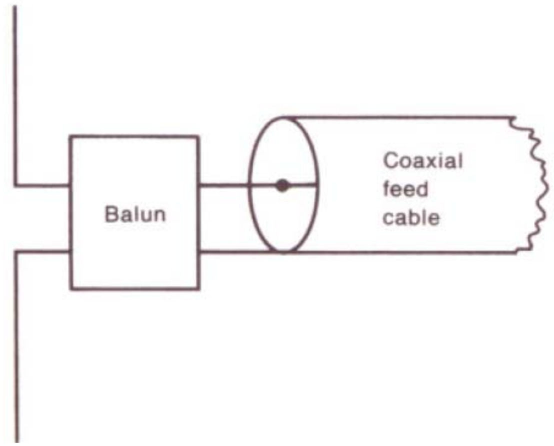
- Antenna Factor
  - With Cable Loss
    - The connection cable loss must be added since antenna factor is with respect to the base of the antenna and does not include any connection cable loss.

$$E\,(\mathrm{dB\mu V/m}) = \mathrm{AF}\,(\mathrm{dB}) + V_{SA}\,(\mathrm{dB\mu V}) + \text{cable loss}\,(\mathrm{dB})$$



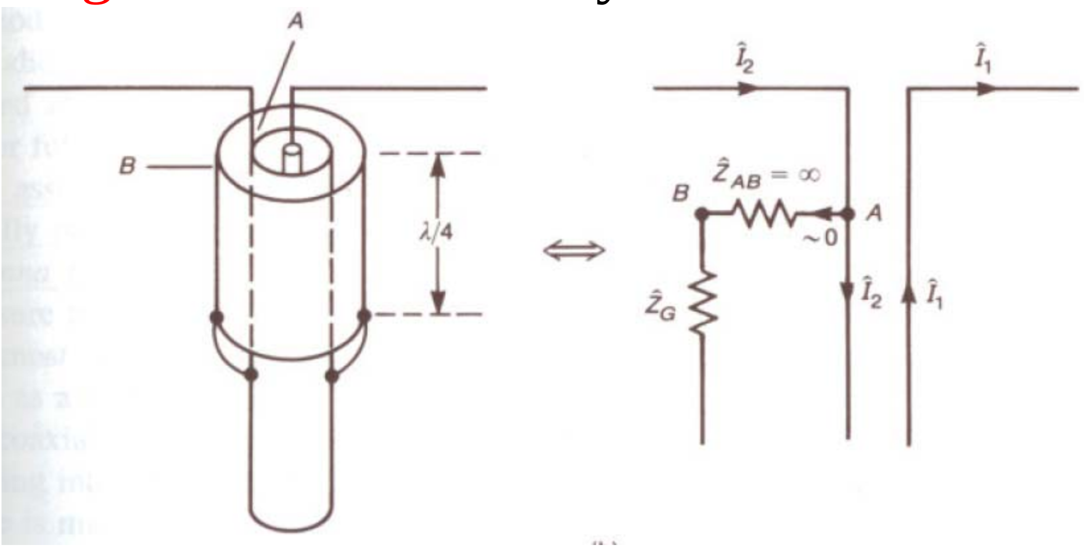
Characterization of Antennas

- Effects of Balancing and Baluns
  - Feeding with Coaxial Cable
    - If an antenna is fed with a coaxial cable, some current may flow on the outside of the shield, which in turn affects the antenna pattern.
    - The common way of preventing unbalance due to a coaxial feed cable is the use of a balun, which is inserted between the balanced antenna and unbalanced feedline.



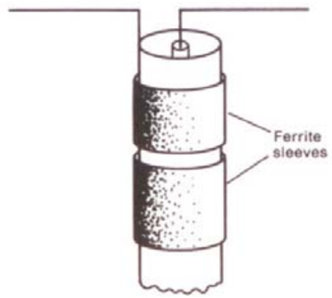
Characterization of Antennas

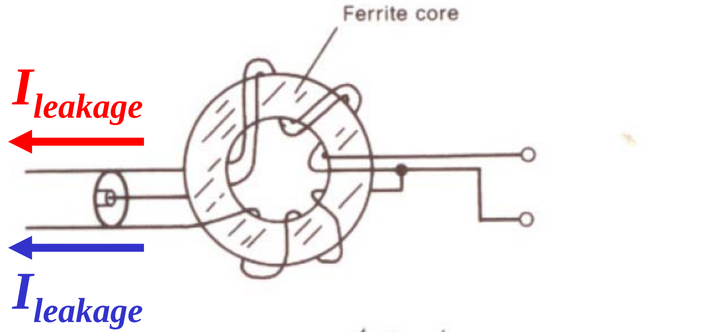
- Effects of Balancing and Baluns
  - Bazooka Balun – Narrow Band
    - A quarter-wavelength, short-circuited transmission line is formed between the outer coax and the inner coax.
    - Therefore the impedance between points *A* and *B* is very large and there is nearly no current flowing out.

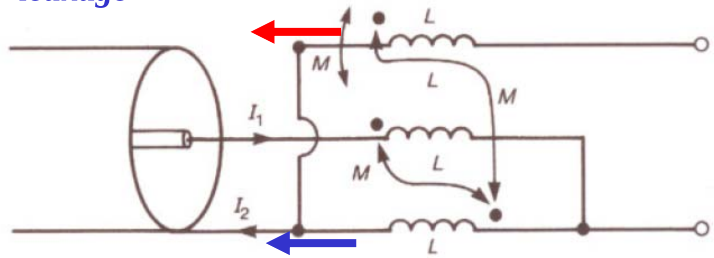


Characterization of Antennas

- Effects of Balancing and Baluns
  - Ferrite Sleeves and Ferrite Toroid– Wide Band
    - The current flowing on the outside of the shield sees a large impedance because of the ferrite sleeves.



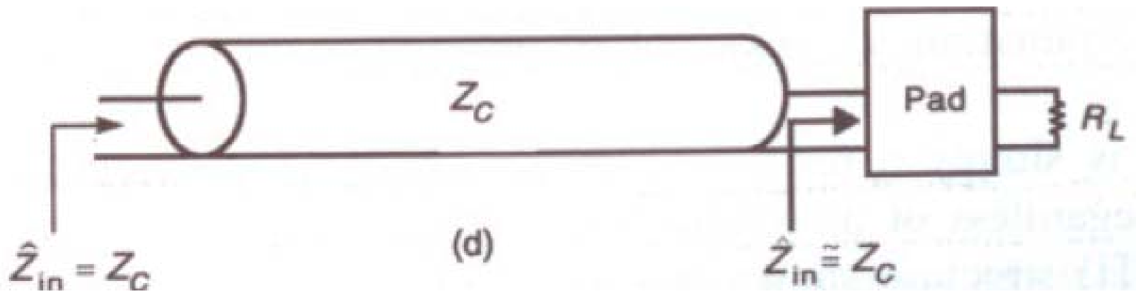




The leakage currents are common-mode currents, which sees an large impedance of $j\omega(L+M)$

Characterization of Antennas

- Impedance Matching and the Use of Pads
  - Function of Pad
    - A pad is simply a resistive network whose input impedance remains fairly constant regardless of its termination impedance.
    - Being resistive circuits, these pads provide matching over wide frequency ranges but they also give an insertion loss.

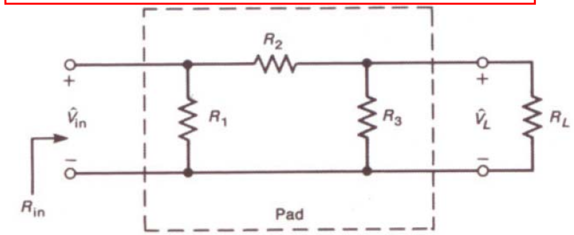


Characterization of Antennas

- Impedance Matching and the Use of Pads
  - Tradeoffs
    - Larger Bandwidth ←→ Larger Loss ←→ Smaller variation in VSWR
  - Insertion Loss
    - The insertion loss is obtained as

$$\mathrm{IL} = 20\log_{10}\left[\left(\frac{R_3 \| R_L}{R_2 + R_3 \| R_L}\right)^{-1}\right]$$

The three resistors could be replaced with three inductors.



- And since the impedance is matched, we have $Z_C = R_1 \| (R_2 + R_3 \| R_L)$
- From the above two equations, we can solve for

$$R_1 = R_3 = \frac{R_L(1+X)}{(R_L/Z_C)X - 1} \qquad R_2 = (R_3 \| R_L)(X-1) \qquad X = 10^{\mathrm{IL}/20}$$

The Friis Transmission Equation

- Relationship between P_R and P_T
  - Derivation
    - The power density at the receiving antenna is the power density of an isotropic point source multiplied by the gain of the transmitting antenna in the direction of transmission:

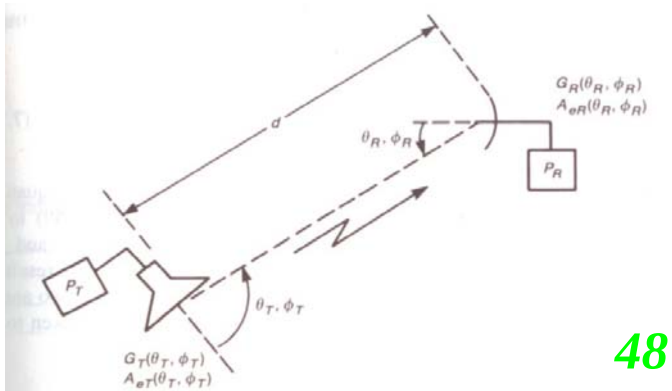
$$S_{av} = \frac{P_T}{4\pi d^2} G_T(\theta_T, \phi_T)$$

    - The received power is the product of this power density and the effective aperture of the receiving in the direction of transmission:

$$P_R = S_{av} A_{eR}(\theta_R, \phi_R)$$

    - Combing these equations gives

$$\frac{P_R}{P_T} = \frac{G_T(\theta_T, \phi_T) A_{eR}(\theta_R, \phi_R)}{4\pi d^2}$$



The Friis Transmission Equation

- Relationship between P_R and P_T
  - Derivation
    - Since $G(\theta, \phi) = \frac{4\pi}{\lambda_0^2} A_e(\theta, \phi)$
    - The Friis transmission equation is obtained as

$$\frac{P_R}{P_T} = G_T(\theta_T, \phi_T) G_R(\theta_R, \phi_R) \left(\frac{\lambda_0}{4\pi d}\right)^2$$

    - In dB form, we have

$$10 \log_{10}\left(\frac{P_R}{P_T}\right) = G_{T,\mathrm{dB}} + G_{R,\mathrm{dB}} - 20 \log_{10} f - 20 \log_{10} d + 147.56$$

    - Since the transmitted wave is that of a uniform plane wave (locally), the power density could also be written as

$$S_{av} = \frac{1}{2} \frac{|\hat{E}|^2}{\eta_0} \xrightarrow{S_{av} = \frac{P_T}{4\pi d^2} G_T(\theta_T, \phi_T)} |\hat{E}| = \frac{\sqrt{60 P_T G_T(\theta_T, \phi_T)}}{d}$$

The Friis Transmission Equation

- Relationship between P_R and P_T
  - Inherent Assumptions
    - The receiving antenna must be matched to its load impedance and the polarization of the incoming wave.
    - Also, the two antennas must be in the far field of each other. The far-field criterion is usually taken to be the larger of

$$d_{\text{far field}} > \frac{2D^2}{\lambda_0} \quad \text{(surface antennas)}$$

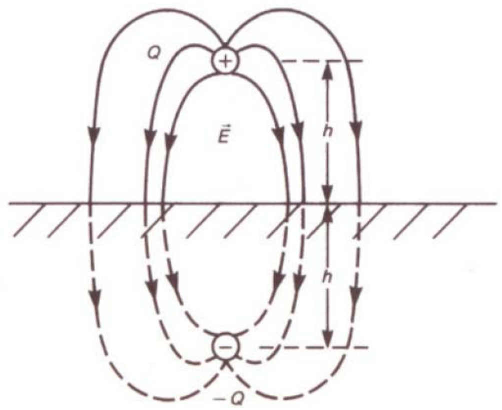
This criterion ensures that the phase difference on the aperture is less than $\lambda_0/16$.

$$d_{\text{far field}} > 3\lambda_0 \quad \text{(wire antennas)}$$

This criterion ensures that the wave impedance of the incoming wave is approximately that of free space.

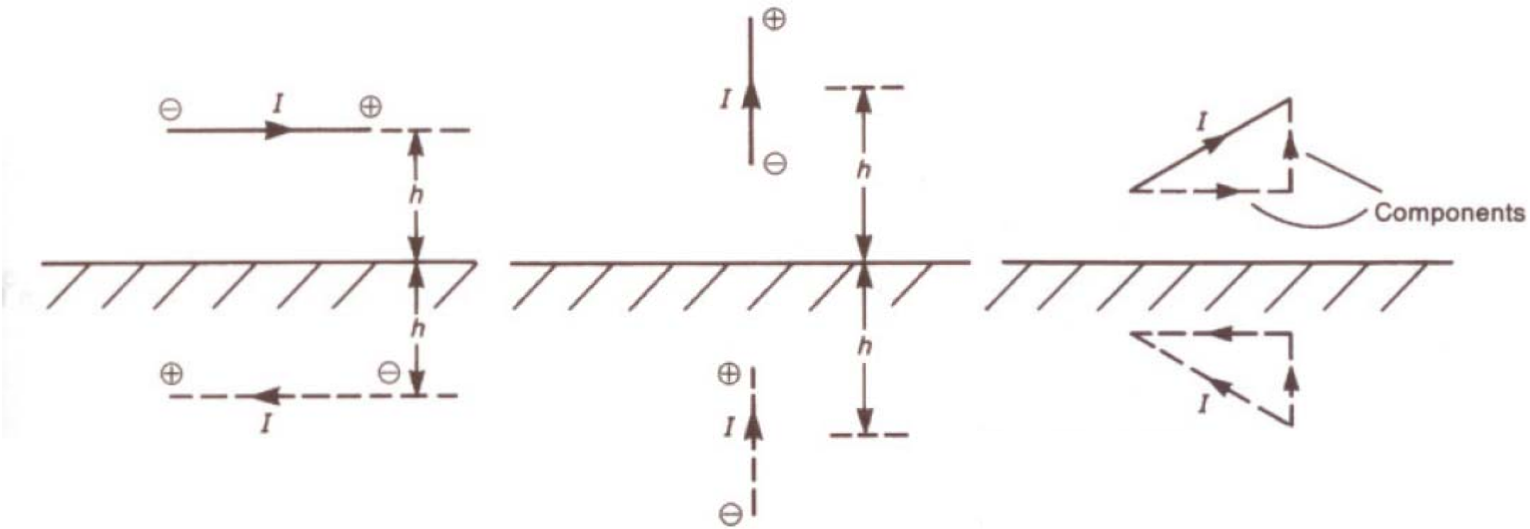
Effects of Reflections

- The Method of Images
  - Charges and Currents above Infinite PEC
    - The image of a static charge is shown below.
    - This image must be such that the electric field distribution in the space above the previous position of the ground plane remains unchanged.
    - The boundary condition that the electric field tangent to the ground plane be zero must be satisfied.



Effects of Reflections

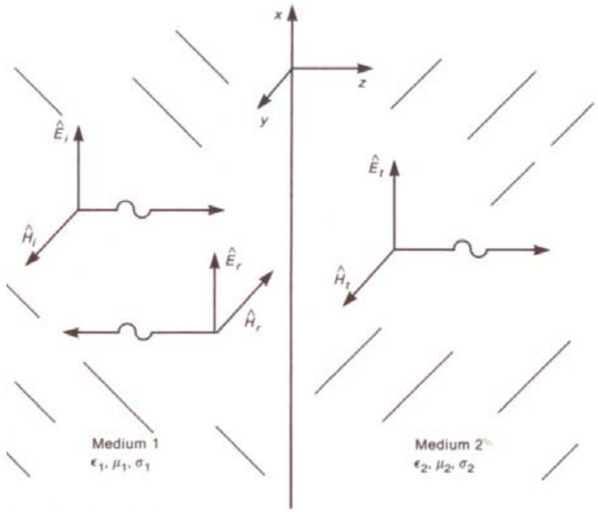
- The Method of Images
  - Charges and Currents above Infinite PEC
    - The image of currents are shown below.
    - The image of the current element is generated by analogy to static charge distributions.



What about the case for PMC which does not exist in reality?

Effects of Reflections

- Normal Incidence of Uniform Plane Waves on Plane, Material Boundaries
  - Fields Relationships
    - The incidence, reflection and transmission waves are shown below.
    - For the incidence wave



$$\vec{\hat{E}}_i = \hat{E}_i e^{-\hat{\gamma}_1 z} \vec{a}_x = \hat{E}_i e^{-\alpha_1 z} e^{-j\beta_1 z} \vec{a}_x$$

$$\vec{\hat{H}}_i = \frac{\hat{E}_i}{\hat{\eta}_1} e^{-\hat{\gamma}_1 z} \vec{a}_y = \frac{\hat{E}_i}{\eta_1} e^{-\alpha_1 z} e^{-j\beta_1 z} e^{-j\theta_{\eta 1}} \vec{a}_y$$

- where

$$\hat{\gamma}_1 = \sqrt{j\omega\mu_1(\sigma_1 + j\omega\epsilon_1)} = \alpha_1 + j\beta_1$$

$$\hat{\eta}_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\epsilon_1}} = \eta_1 \angle \theta_{\eta 1}$$

Effects of Reflections

- Normal Incidence of Uniform Plane Waves on Plane, Material Boundaries
  - Fields Relationships
    - For the reflected wave

$$\vec{\hat{E}}_r = \hat{E}_r e^{\hat{\gamma}_1 z} \vec{a}_x = \hat{E}_r e^{\alpha_1 z} e^{j\beta_1 z} \vec{a}_x$$

$$\vec{\hat{H}}_r = -\frac{\hat{E}_r}{\hat{\eta}_1} e^{\hat{\gamma}_1 z} \vec{a}_y = -\frac{\hat{E}_r}{\eta_1} e^{\alpha_1 z} e^{j\beta_1 z} e^{-j\theta_{\eta 1}} \vec{a}_y$$

    - For the transmitted wave

$$\vec{\hat{E}}_t = \hat{E}_t e^{-\hat{\gamma}_2 z} \vec{a}_x = \hat{E}_t e^{-\alpha_2 z} e^{-j\beta_2 z} \vec{a}_x$$

$$\vec{\hat{H}}_t = \frac{\hat{E}_t}{\hat{\eta}_2} e^{-\hat{\gamma}_2 z} \vec{a}_y = \frac{\hat{E}_t}{\eta_2} e^{-\alpha_2 z} e^{-j\beta_2 z} e^{-j\theta_{\eta 2}} \vec{a}_y$$

    - where

$$\hat{\gamma}_2 = \sqrt{j\omega\mu_2(\sigma_2 + j\omega\epsilon_2)} = \alpha_2 + j\beta_2 \qquad \hat{\eta}_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2}} = \eta_2 \angle \theta_{\eta 2}$$

Effects of Reflections

- Normal Incidence of Uniform Plane Waves on Plane, Material Boundaries
  - Fields Relationships
    - Matching the boundary conditions at z=0, we have

$$\left.\begin{aligned}\vec{\hat{E}}_i + \vec{\hat{E}}_r = \vec{\hat{E}}_t \quad \text{at } z = 0 \\ \vec{\hat{H}}_i + \vec{\hat{H}}_r = \vec{\hat{H}}_t \quad \text{at } z = 0\end{aligned}\right\} \rightarrow \begin{aligned}\hat{\Gamma} = \frac{\hat{E}_r}{\hat{E}_i} = \frac{\hat{\eta}_2 - \hat{\eta}_1}{\hat{\eta}_2 + \hat{\eta}_1} = \Gamma \angle \theta_\Gamma \\ \hat{T} = \frac{\hat{E}_t}{\hat{E}_i} = \frac{2\hat{\eta}_2}{\hat{\eta}_2 + \hat{\eta}_1} = T \angle \theta_T\end{aligned}$$

    - Thus, the fields are obtained as

$$\vec{\hat{E}}_i = E_m e^{-\hat{\gamma}_1 z} \vec{a}_x \qquad \vec{\hat{H}}_r = -\frac{\hat{\Gamma} E_m}{\hat{\eta}_1} e^{\hat{\gamma}_1 z} \vec{a}_y$$

$$\vec{\hat{H}}_i = \frac{E_m}{\hat{\eta}_1} e^{-\hat{\gamma}_1 z} \vec{a}_y \qquad \vec{\hat{E}}_t = \hat{T} E_m e^{-\hat{\gamma}_2 z} \vec{a}_x$$

$$\vec{\hat{E}}_r = \hat{\Gamma} E_m e^{\hat{\gamma}_1 z} \vec{a}_x \qquad \vec{\hat{H}}_t = \frac{\hat{T} E_m}{\hat{\eta}_2} e^{-\hat{\gamma}_2 z} \vec{a}_y$$

Effects of Reflections

- Normal Incidence of Uniform Plane Waves on Plane, Material Boundaries
  - Fields Relationships
    - In time-domain forms, the fields are

$$\vec{E}_i = E_m e^{-\alpha_1 z} \cos(\omega t - \beta_1 z)\vec{a}_x$$

$$\vec{H}_i = \frac{E_m}{\eta_1} e^{-\alpha_1 z} \cos(\omega t - \beta_1 z - \theta_{\eta 1})\vec{a}_y$$

$$\vec{E}_r = \Gamma E_m e^{\alpha_1 z} \cos(\omega t + \beta_1 z + \theta_\Gamma)\vec{a}_x$$

$$\vec{H}_r = -\frac{\Gamma E_m}{\eta_1} e^{\alpha_1 z} \cos(\omega t + \beta_1 z + \theta_\Gamma - \theta_{\eta 1})\vec{a}_y$$

$$\vec{E}_t = T E_m e^{-\alpha_2 z} \cos(\omega t - \beta_2 z + \theta_T)\vec{a}_x$$

$$\vec{H}_t = \frac{T E_m}{\eta_2} e^{-\alpha_2 z} \cos(\omega t - \beta_2 z + \theta_T - \theta_{\eta 2})\vec{a}_y$$

Effects of Reflections

- Normal Incidence of Uniform Plane Waves on Plane, Material Boundaries
  - Power Transmission
    - The average power density vector is

$$\vec{S}_{\mathrm{av},t} = \frac{1}{2}\mathscr{R}e\left\{\hat{\vec{E}}_t \times \hat{\vec{H}}_t^*\right\}$$

$$= \frac{1}{2}\mathscr{R}e\left\{\hat{T}E_m e^{-\hat{\gamma}_2 z}\frac{\hat{T}^* E_m e^{-\hat{\gamma}_2^* z}}{\hat{\eta}_2^*}\right\}\vec{a}_z$$

$$= \frac{1}{2}\frac{E_m^2 T^2}{\eta_2}e^{-2\alpha_2 z}\cos\theta_{\eta 2}\,\vec{a}_z$$

where we denote $|\hat{T}| = T$ and $|\hat{\eta}_2| = \eta_2$

Effects of Reflections

- Normal Incidence of Uniform Plane Waves on PEC
  - σ_1=0 and $\sigma_2=\infty \rightarrow \eta_2=0 \rightarrow \Gamma=-1$
    - Thus, the total fields in region 1 becomes

$$\vec{\hat{E}}_1 = \vec{\hat{E}}_i + \vec{\hat{E}}_r = E_m(e^{-j\beta_1 z} - e^{j\beta_1 z})\vec{a}_x = -2jE_m \sin(\beta_1 z)\vec{a}_x$$

$$\vec{\hat{H}}_i = \vec{\hat{H}}_i + \vec{\hat{H}}_r = \frac{E_m}{\eta_1}(e^{-j\beta_1 z} + e^{j\beta_1 z})\vec{a}_y = \frac{2E_m}{\eta_1}\cos(\beta_1 z)\vec{a}_y$$

    - The time-domain expressions become

$$\vec{E}_1 = \mathcal{R}e\left\{\vec{\hat{E}}_1 e^{j\omega t}\right\} = 2E_m \sin(\beta_1 z)\sin(\omega t)\vec{a}_x$$

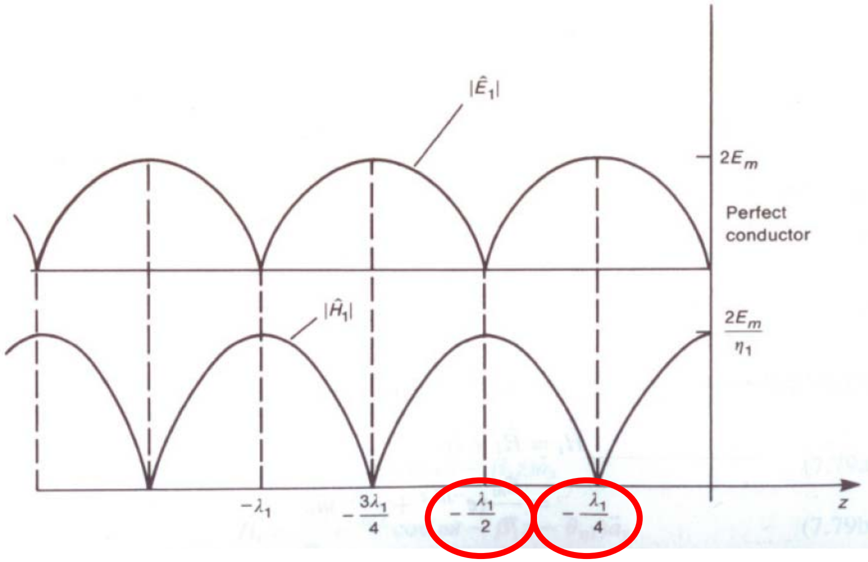
$$\vec{\hat{H}}_1 = \mathcal{R}e\left\{\vec{\hat{H}}_1 e^{j\omega t}\right\} = \frac{2E_m}{\eta_1}\cos(\beta_1 z)\cos(\omega t)\vec{a}_y$$

Effects of Reflections

- Normal Incidence of Uniform Plane Waves on PEC
  - σ_1=0 and σ_2=∞ → η_2=0 → Γ=-1
    - These total fields represent standing waves and the magnitudes of the fields are

$$|\hat{E}_1| = 2E_m|\sin(\beta_1 z)| = 2E_m\left|\sin\left(\frac{2\pi z}{\lambda_1}\right)\right|$$

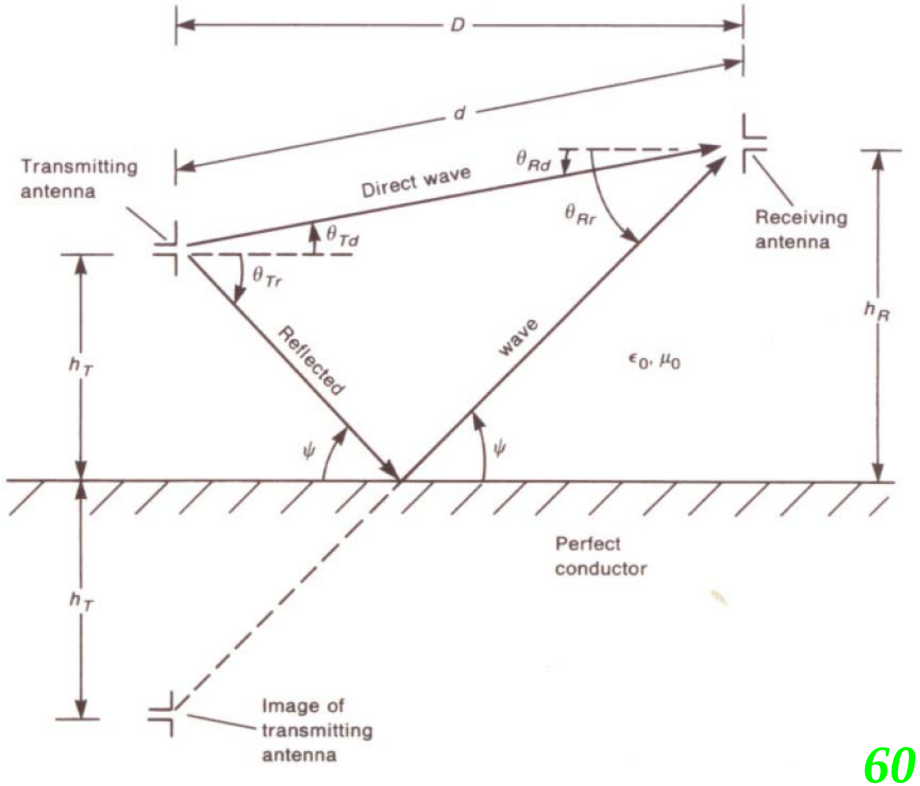
$$|\hat{H}_1| = 2\frac{E_m}{\eta_1}|\cos(\beta_1 z)| = 2\frac{E_m}{\eta_1}\left|\cos\left(\frac{2\pi z}{\lambda_1}\right)\right|$$



Effects of Reflections

- Multipath Effects
  - Communication between Two Antennas above a PEC
    - The equivalent problem is shown below.

The received signal at the measurement antenna consists of a direct wave and a reflected wave.



- For the direct path

$$d = \sqrt{D^2 + (h_R - h_T)^2}$$

- For the reflected path

$$d_r = \sqrt{D^2 + (h_R + h_T)^2}$$

Effects of Reflections

- Multipath Effects
  - Communication between Two Antennas above a PEC
    - Assuming the two antennas are in the far fields of each other, the received voltage due to the direct wave is

$$\hat{V}_d = \hat{V}_0 E_T(\theta_{Td}, \phi_{Td}) E_R(\theta_{Rd}, \phi_{Rd}) \frac{e^{-j\beta_0 d}}{d}$$

    - and the received voltage due to the reflected wave is

$$\hat{V}_r = \hat{V}_0 E_T(\theta_{Tr}, \phi_{Tr}) E_R(\theta_{Rr}, \phi_{Rr}) \hat{\Gamma} \frac{e^{-j\beta_0 d_r}}{d_r}$$

    - where E_T and E_R concerns with the pattern of the two antennas.

Effects of Reflections

- Multipath Effects
  - Communication between Two Antennas above a PEC
    - The total received voltage is the sum of the above two voltages

$$
\begin{aligned}
\hat{V} &= \hat{V}_d + \hat{V}_r \\
&= \hat{V}_0 E_T(\theta_{Td}, \phi_{Td}) E_R(\theta_{Rd}, \phi_{Rd}) \frac{e^{-j\beta_0 d}}{d} \\
&\quad + \hat{V}_0 E_T(\theta_{Tr}, \phi_{Tr}) E_R(\theta_{Rr}, \phi_{Rr}) \hat{\Gamma} \frac{e^{-j\beta_0 d_r}}{d_r} \\
&= \hat{V}_0 E_T(\theta_{Td}, \phi_{Td}) E_R(\theta_{Rd}, \phi_{Rd}) \frac{e^{-j\beta_0 d}}{d} \hat{F}
\end{aligned}
$$

The ground reflection modifies the free-space direct wave propagation by the multiplicative factor *F*.

- where

$$
\hat{F} = 1 + \frac{E_T(\theta_{Tr}, \phi_{Tr}) E_R(\theta_{Rr}, \phi_{Rr})}{E_T(\theta_{Td}, \phi_{Td}) E_R(\theta_{Rd}, \phi_{Rd})} \hat{\Gamma} \frac{d}{d_r} e^{-j\beta_0 (d_r - d)}
$$

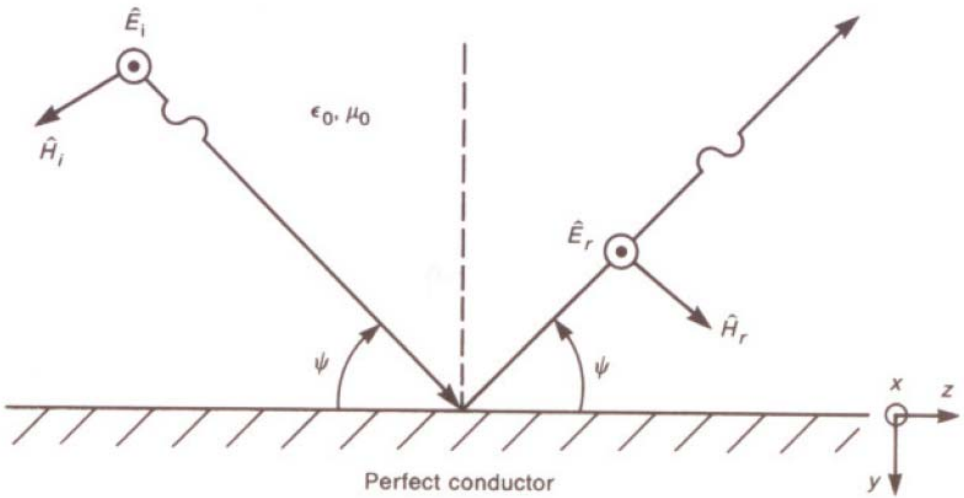
Effects of Reflections

- Multipath Effects
  - Communication between Two Antennas above a PEC
    - Consequently, the Friis transmission equation can be modifies to account for the ground reflection by multiplying it by the square of the magnitude of *F* (since the Friis transmission equation involves power).
    - Considering the reflection coefficient, two cases arise : parallel (vertical) polarization and perpendicular (horizontal) polarization.

Effects of Reflections

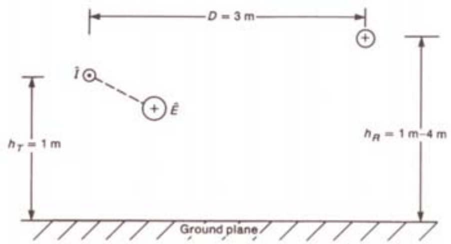
- Multipath Effects
  - Perpendicular (Horizontal) Polarization
    - The reflection coefficient at the ground plane becomes

$$\hat{\Gamma}_H = \frac{\hat{E}_r}{\hat{E}_i} = -1$$



- Since E_T and E_R are omnidirectional in this plane, the reflection factor becomes

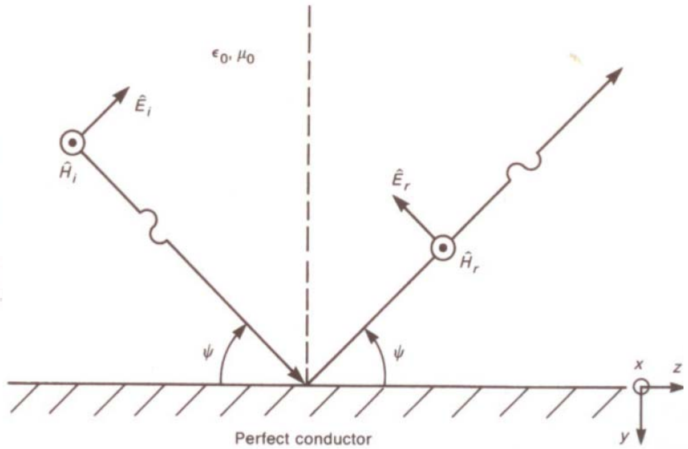
$$\hat{F}_H = 1 - \frac{d}{d_r} e^{-j(2\pi/\lambda_0)(d_r - d)}$$

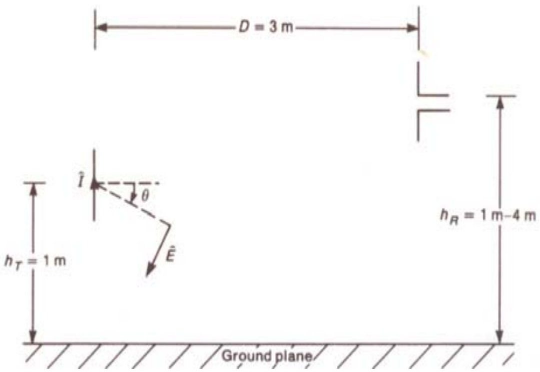


Effects of Reflections

- Multipath Effects
  - Parallel (Vertical) Polarization
    - The reflection coefficient at the ground plane becomes

$$\hat{\Gamma}_V = \frac{\hat{E}_r}{\hat{E}_i}$$
$$= +1$$





- Since E_T and E_R are functions of cosine in this plane the reflection factor becomes

$$\hat{F}_V = 1 + \frac{\cos\theta_{Tr}\cos\theta_{Rr}}{\cos\theta_{Td}\cos\theta_{Rd}}\hat{\Gamma}_V\frac{d}{d_r}e^{-j(2\pi/\lambda_0)(d_r-d)}$$

$$\hat{F}_V = 1 + \left(\frac{d}{d_r}\right)^3 e^{-j(2\pi/\lambda_0)(d_r-d)}$$

$$\cos\theta_{Tr} = \frac{D}{d_r}$$
$$\cos\theta_{Rr} = \frac{D}{d_r}$$
$$\cos\theta_{Td} = \frac{D}{d}$$
$$\cos\theta_{Rd} = \frac{D}{d}$$

Effects of Reflections

- Broadband Measurement Antennas
  - Tuned, Half-Wave Dipoles
    - FCC prefers this kind of antenna which needs to be tuned for each frequency but not practical for vertical measurement at low frequencies since it’s too long.
  - Definition of Broadband Antennas
    - The input impedance is fairly constant over the frequency of interest.
    - The pattern is fairly constant over the frequency of interest.

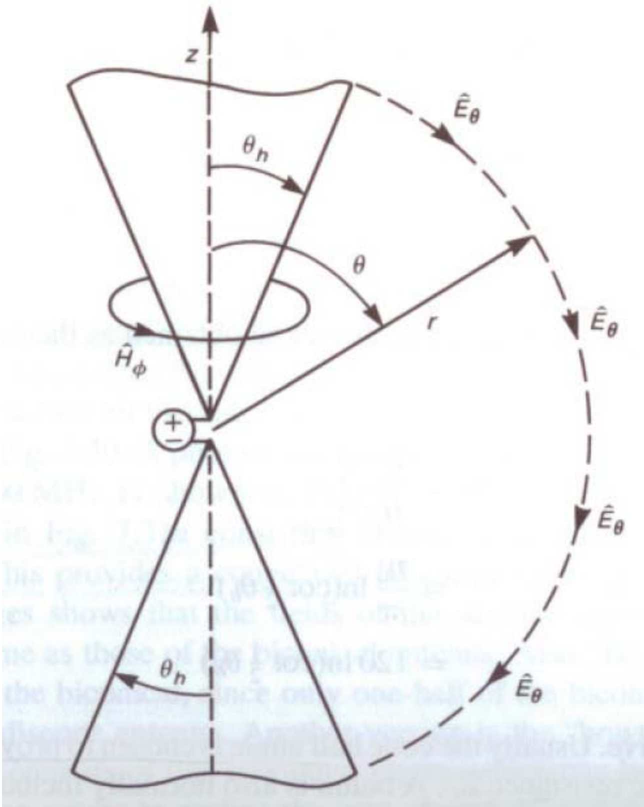
Effects of Reflections

- Broadband Measurement Antennas
  - The Biconical Antenna
    - Due to symmetry, the fields of the biconical antenna are

$$\hat{H}_\phi = \frac{H_0}{\sin\theta}\frac{e^{-j\beta_0 r}}{r}$$

$$\hat{E}_\theta = \frac{\beta_0}{\omega\epsilon_0}\frac{H_0}{\sin\theta}\frac{e^{-j\beta_0 r}}{r}$$

$$= \eta_0 \hat{H}_\phi$$



Since the radiated fields are TEM, we may uniquely define a voltage between two points on the cones.

Effects of Reflections

- Broadband Measurement Antennas
  - The Biconical Antenna
    - The voltage produced between two points on the two cones that are a distance *r* from the feedpoint is

$$\hat{V}(r) = -\int_{\theta=\pi-\theta_h}^{\theta_h} \hat{\vec{E}} \cdot \vec{dl}$$

$$= 2\eta_0 H_0 e^{-j\beta_0 r} \ln(\cot \tfrac{1}{2}\theta_h)$$

    - The current on the surface of the cones is

$$\hat{I}(r) = \int_{\phi=0}^{2\pi} \hat{H}_\phi r \sin\theta \, d\phi$$

$$= 2\pi H_0 e^{-j\beta_0 r}$$

    - Thus, the input impedance at the feed terminals is

$$\hat{Z}_{\text{in}} = \left. \frac{\hat{V}(r)}{\hat{I}(r)} \right|_{r=0} = \frac{\eta_0}{\pi} \ln(\cot \tfrac{1}{2}\theta_h)$$

$$= 120 \ln(\cot \tfrac{1}{2}\theta_h)$$

Effects of Reflections

- Broadband Measurement Antennas
  - The Biconical Antenna
    - If the cones are lossless, we can derive that $R_{\text{rad}} = \hat{Z}_{\text{in}}$
    - The total radiated average power is

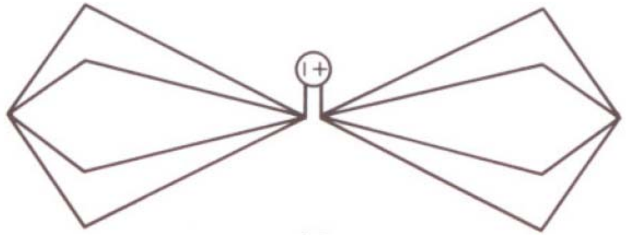
$$
\begin{aligned}
P_{\text{rad}} &= \oint_S \vec{S}_{\text{av}} \cdot d\vec{s} \\
&= \int_{\phi=0}^{2\pi} \int_{\theta=\theta_h}^{\pi-\theta_h} \frac{|\hat{E}_\theta|^2}{2\eta_0} r^2 \sin\theta \, d\theta \, d\phi \\
&= \pi\eta_0 H_0^2 \int_{\theta=0}^{\theta_h} \frac{d\theta}{\sin\theta} \\
&= 2\pi\eta_0 H_0^2 \ln(\cot \tfrac{1}{2}\theta_h)
\end{aligned}
$$

    - The radiation resistance is defined by

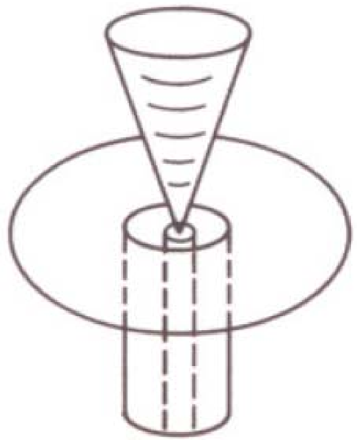
$$P_{\text{rad}} = \tfrac{1}{2} |\hat{I}(0)|^2 R_{\text{rad}} \longrightarrow R_{\text{rad}} = \hat{Z}_{\text{in}}$$

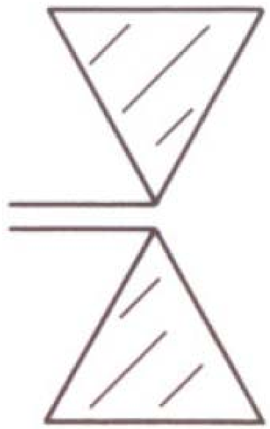
Effects of Reflections

- Broadband Measurement Antennas
  - The Biconical Antenna – Other Deformation
    - Truncated cones: standing wave on the antenna → complex input impedance
    - Wired cones surfaces



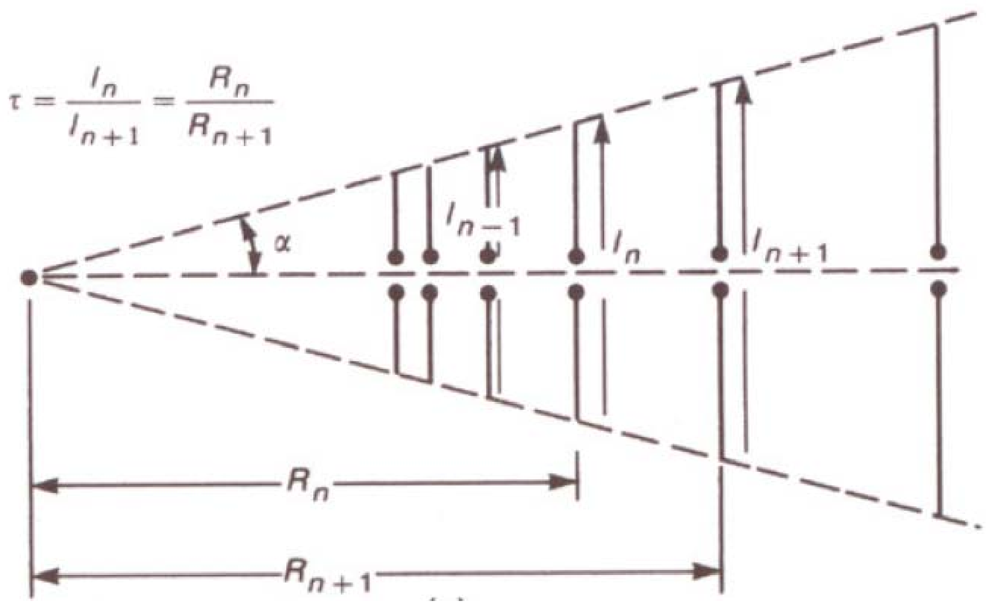
    - One cone above a PEC: balanced feed with coaxial cable
    - Bowtie antenna





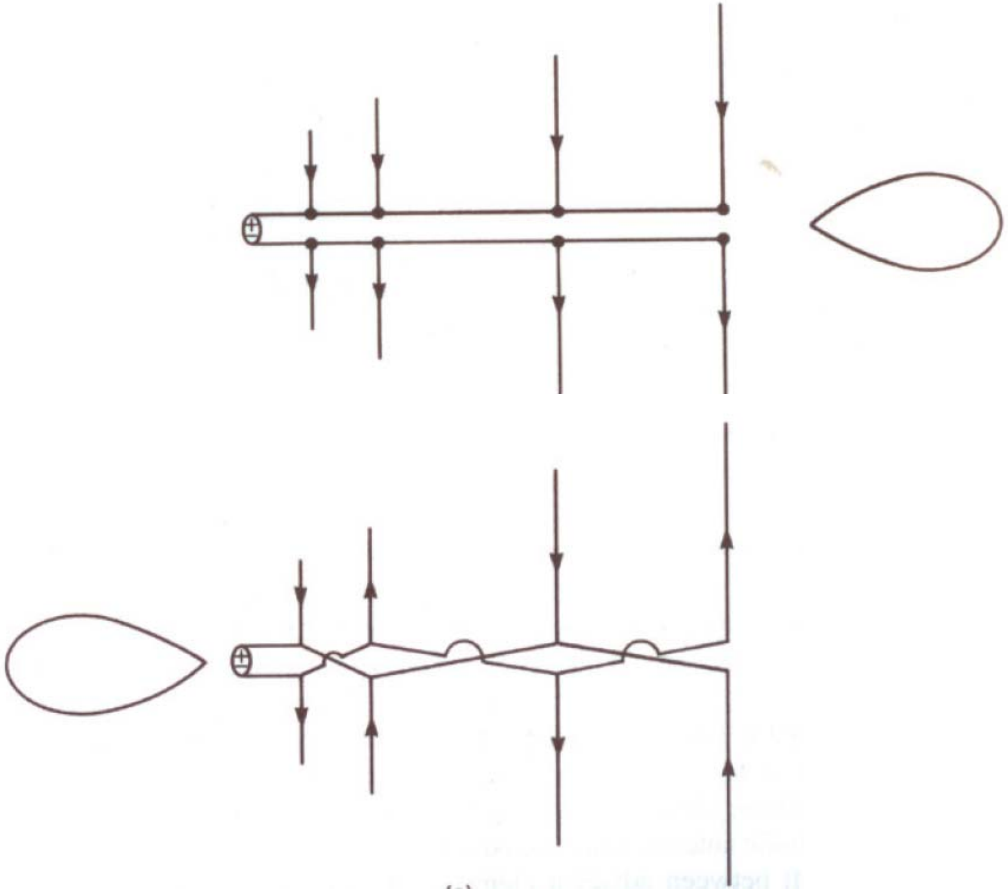
Effects of Reflections

- Broadband Measurement Antennas
  - The Log-Periodic Antenna
    - The input impedance and radiation properties repeating periodically as the logarithm of frequency.
    - The element distances and lengths must satisfy



Effects of Reflections

- Broadband Measurement Antennas
  - The Log-Periodic Antenna – Excitation Ways

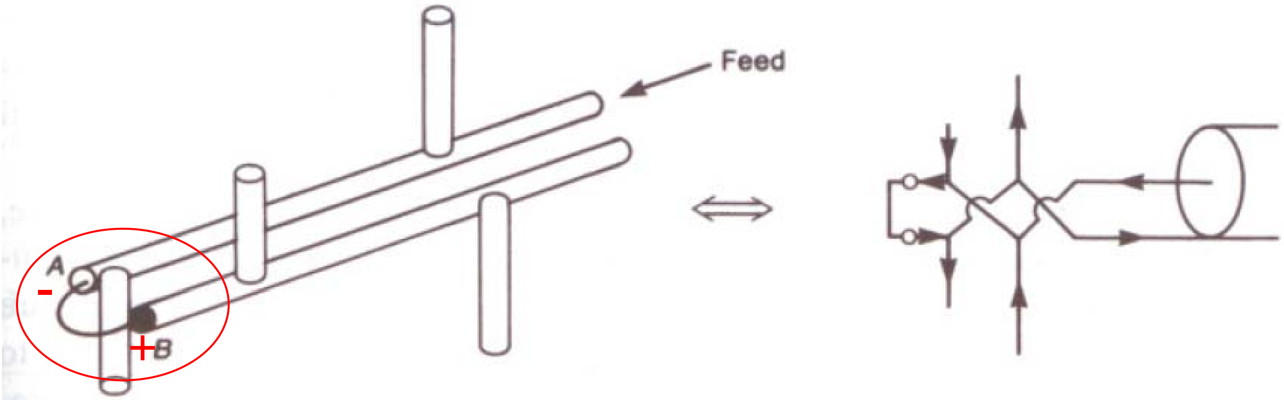


Nonpreferred excitation method

Preferred excitation method

Effects of Reflections

- Broadband Measurement Antennas
  - The Log-Periodic Antenna – Excitation Ways



Preferred excitation method

- The cutoff frequencies of the log-periodic dipole array (its bandwidth) can be approximately computed by determining the frequency of the shortest elements and longest elements in one-half wavelength.