Q measurement of microstrip-coupled dielectric resonators

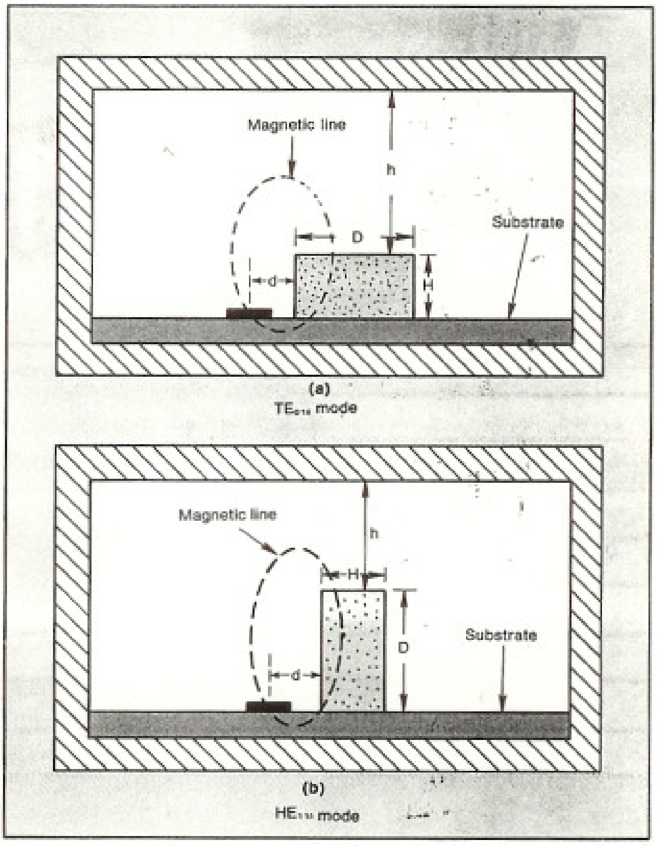
Two practical techniques allow accurate measurement of unloaded and loaded Q and the coefficient of coupling of dielectric resonators to microstrip lines.

A.P.S. Khanna, Senior Member, Technical Staff, LORAL Advanced Technology Laboratories, 5001-G Forbes Blvd., Washington Business Park, Lanham, MD 20706

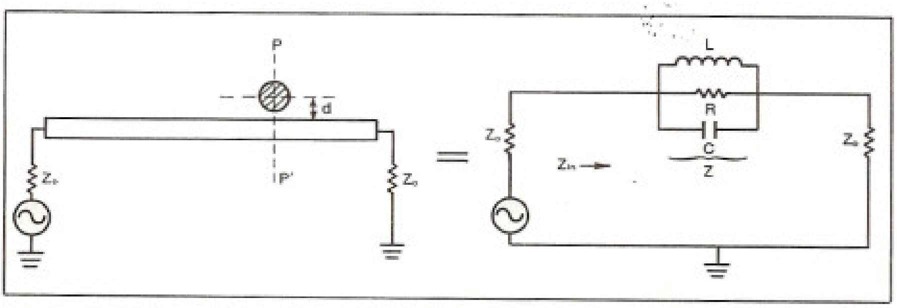
THE advent of temperature-stable dielectric materials has created much interest in the use of dielectric resonators in both active and passive microwave integrated circuits.[1] To date, the $TE_{01\delta}$ mode has been used primarily, but a method of coupling the $HE_{11\delta}$ mode to a shielded microstrip line has been reported.[2] To achieve the best results, an accurate characterization of the coupled dielectric resonator and microstrip line is essential.

Two practical methods have been devised to measure accurately the loaded and unloaded quality factor of both the $TE_{01\delta}$ mode and $HE_{11\delta}$ mode, as well as the coefficient of coupling of a dielectric resonator to a microstrip line. One method uses the complex S_{11} characteristic, while the other uses the S_{21} magnitude.

Figure 1 shows the coupling configurations for both modes. The coupled resonator and microstrip are enclosed in a metallic box that provides the necessary shielding. The distance between the dielectric resonator and the microstrip line, d, determines the coupling coefficient, β. The equivalent circuit in the resonant plane for both configurations is shown in Fig. 2.



1. Configurations for coupling the $TE_{01\delta}$ mode (a) and $HE_{11\delta}$ mode (b) to microstrip.



2. This equivalent circuit for a dielectric resonator *coupled to a microstrip line can be used for quality factor measurements.*

Using S_{11}

The normalized input impedance in the resonator plane of the equivalent circuit in Fig. 2, $z_{in} = Z_{in}/Z_o$, can be expressed as

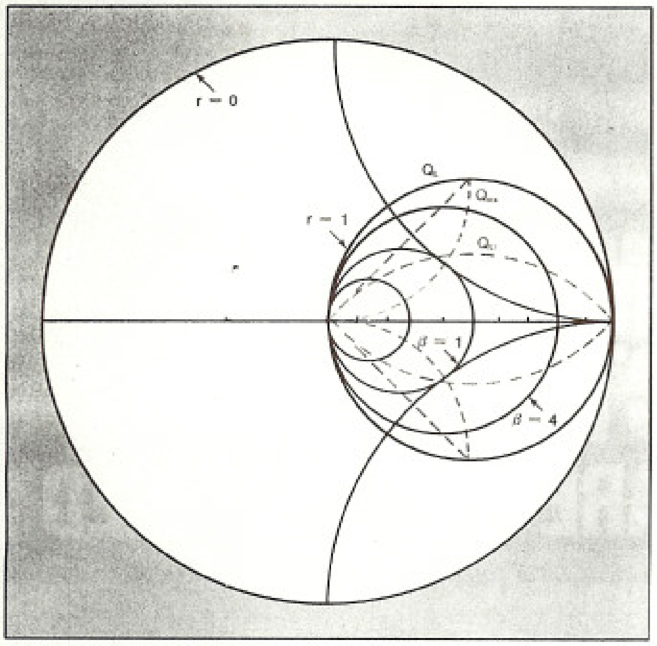
$$Z_{in} = 1 + \frac{2\beta}{1 + j2Q_u\,\delta} \qquad (1)$$

where

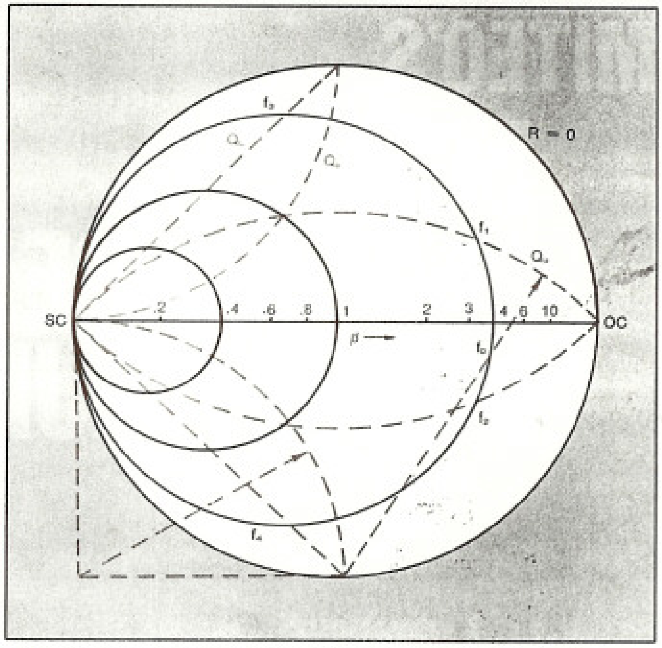
Q_u = unloaded quality factor and

(continued on p. 83)

 (continued from p. 81)



*3. **The loci of Q_u, Q_L, and Q_e** are shown plotted on the Smith chart reflection coefficient plane.*



*4. **To increase measurement accuracy,** the r = 1 plane can be extended to the r = 0 circle so it covers the entire polar display of the chart.*

δ = normalized frequency deviation, $(f - f_0)/f_0$.

The resonant frequency of the dielectric resonator is a function of its diameter, D, and height, H, and the distance, h, between the resonator and the upper metallic cover, in addition to the dielectric constant of the resonator and the substrate.[3] Under given shielding conditions—i.e., for a fixed h—the coupling coefficient between the dielectric resonator and the microstrip line is a function of d:

$$\beta = \frac{R}{2z_0} = \frac{S_{11_0}}{S_{21_0}} = \frac{S_{11_0}}{1-S_{11_0}} = \frac{1-S_{21_0}}{S_{21_0}} \qquad (2)$$

where S_{11_0} and S_{21_0} are the real reflection and transmission coefficients at the resonant frequency in the resonator plane, P-P′. The coupling coefficient also relates the various quality factors by

$$Q_u = Q_L\,(1 + \beta) = \beta Q_{ex} \qquad (3)$$

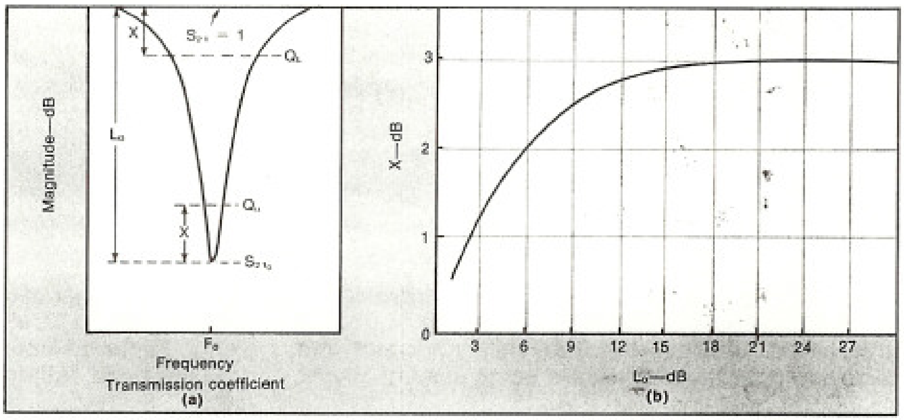
where

Q_L = loaded quality factor and

Q_{ex} = external quality factor.

In Eq. 1 the normalized frequency deviation for Q_u is $\delta_u = \pm 1/2Q_u$. Using Eqs. 1 through 3, the impedance locus for Q_u and Q_L in the S_{11} plane is given by[4]

$$S_{11_u} = \frac{S_{11_0}}{\sqrt{S^2\,11_0{}^{-2}\,S_{11_0}{}^{+2}}}\; e^{\pm j \tan^{-1}(1-S_{11_0})} \qquad (4)$$



*5. **The Q-factors can be determined** by finding their measuring point, x, from S_{21} and S_{21_0} as defined above.*

$$S_{11_L} = \frac{S_{11_0}}{2}\, e^{\pm j\pi/4} \qquad (5)$$

where

$S_{11} = (Z_i - Z_o)/(Z_i + Z_o)$.

The external quality factor can similarly be determined using Eq. 3.

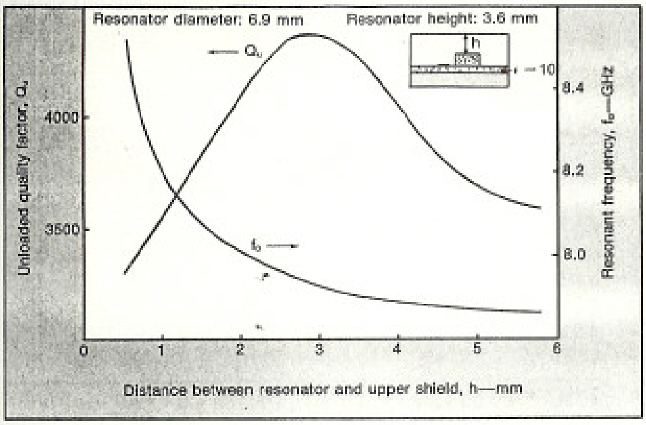
The loci of the three Q-factors for the S_{11} plane are shown in Fig. 3, which also depicts some S_{11} curves of the dielectric resonator coupled to microstrip for several different coupling coefficients. Note that the S_{11} plane of the coupled resonator and microstrip is restricted to the normalized $r = R/Z_o = 1$ circle of the Smith chart.

The accuracy of the Q_u measurement can be improved significantly by expanding the r = 1 circle to cover the entire polar display, using the Hewlett-Packard HP 8410C network analyzer to bring the r = 1 circle up to the r = 0 circle. This can be accomplished by the following steps:

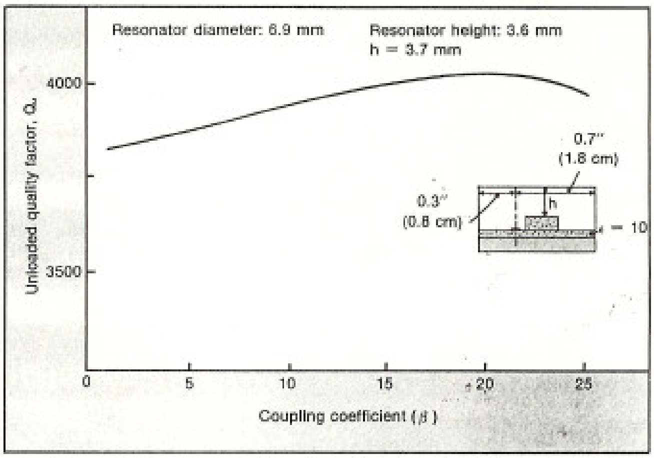
- Calibrate the network analyzer in the reflection mode, using a standard short circuit;
- Increase the test-channel gain control 6 dB and use the horizontal beam control to move the beam's zero point to the normal short circuit point on the polar display;
- Connect the test fixture containing the dielectric resonator coupled between the "unknown" port and the transmission return port of the HP 8743B S-parameter test set (use the reference plane extension control to bring the measurement reference

(concluded on p. 86)

 (continued from p. 83)



6. The unloaded quality factor *was found to be at a maximum where h was about 0.8 times the height of the resonator.*



7. With proper shielding, the coupling coefficient *had little effect on the unloaded quality factor.*

plane to the resonator plane).

The loci of the various Q-factors[5] can now be easily drawn on a Smith chart and used to measure the unloaded and loaded Qs for the desired coupling coefficient (Fig.4). Thus, for δ to equal 4,

$Q_u = f_0/(f_1 - f_2)$, and $Q_L = f_0/(f_3 - f_4)$.

Another advantage of the above procedure is that the coupling coefficient can now be read directly from the normalized impedance scale on the Smith chart's horizontal axis.

The unloaded and loaded Q-factors can also be found using the magnitude of the transmission coefficient, S_{21}, at the resonant frequency of the resonator. Since a dielectric resonator coupled to a microstrip line can be represented by a series impedance, the magnitudes of the transmission coefficients corresponding to the Q_u and Q_L measurement points, S_{21_u} and S_{21_L}, can be determined[6] from Eqs. 4 and 5 and the relationship $S_{21} = 1 - S_{11}$:

$$S_{21_u} = S_{21_0}\sqrt{\frac{2}{1 + S_{21_0}^2}} \qquad (6)$$

$$S_{21_L} = \sqrt{\frac{1 + S_{21_0}^2}{2}} \qquad (7)$$

Since S_{21_0} is related to the insertion loss, L_0, by L_0 (dB) $= -20\log S_{21_0}$, the Q_u and Q_L measuring point, X, defined in Fig. 5a can be determined as function of L_0:

$$X(\text{dB}) = 3 - 10\log(1 + 10^{-0.1L_0}) \qquad (8)$$

Fig. 5b illustrates the above relationship. Fig. 5 shows that for a high L_0 the Q_u and Q_L measuring points approach a 3-dB separation from S_{21_0} and $S_{21} = 1$, respectively.

The coupling coefficient can also be determined using L_0:

$$\beta = 10^{L_0/20} - 1 = \text{antilog}(L_0/20) - 1 \qquad (9)$$

The two methods of measuring unloaded Q were tested with good agreement between the results. The test setup used an alumina substrate 1 × 1 × 0.025 in. (2.54 × 2.54 × 0.0635 cm) and a dielectric resonator with an ϵ of 37 from Thomson-CSF. The $TE_{01\delta}$ mode of the resonator was coupled to a 50-ohm line etched on the substrate. Shielding was accomplished by enclosing the substrate in a metallic box with a cover whose height could be varied.

The frequency and the unloaded Q were measured as h was varied by moving the cover (Fig. 6). Note that the maximum Q_u was found at about h = 0.8H.

Another experiment measured the unloaded Q as a function of d, and therefore of β. If the shielding conditions were properly chosen, the effect of β on Q_u was insignificant. Figure 7 shows results for h = 3.7 mm. ••

References

1. J.K. Plourde and C.L. Ren, "Applications of Dielectric Resonators in Microwave Components," *IEEE Trans. on Microwave Theory and Techniques*, Vol. MTT-29, (August 1981), pp. 754-769.
2. A.P.S. Khanna, Y. Garault, and M. Guedira, "Dielectric Resonator HE_{11d}-mode Coupling to a Shielded Microstrip Line," *IEEE MTT-S Symposium Digest 1983*, pp. 527-529.
3. D. Kajfez, "Elementary Functions Procedure Simplifies Dielectric Resonators Design," *Microwave Systems News*, (June 1982), pp. 391-427.
4. A.P.S. Khanna and Y. Garault, "Determination of Loaded, Unloaded, and External Quality Factors of a Dielectric Resonator Coupled to a Microstrip Line," *IEEE Trans. on Microwave Theory and Techniques*, Vol. MTT-31, No. 3, pp. 261-264.
5. E.L. Ginzton, *Microwave Measurements*, (New York: McGraw-Hill, 1957), pp. 391-427.
6. R.E. Collins, *Foundations of Microwave Engineering*, (New York: McGraw-Hill, 1966), p. 178.