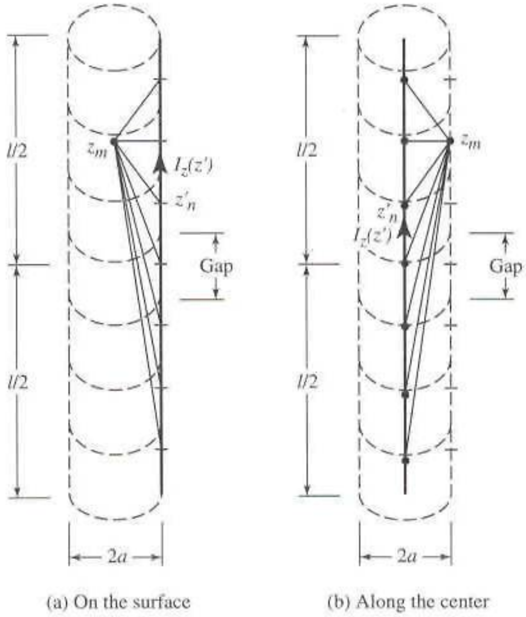

Figure 8.6 Dipole segmentation and its equivalent current.

where $I_z(z')$ is assumed to be an equivalent filament line-source current located a radial distance $\rho = a$ from the z axis, as shown in Figure 8.6(a). Thus (8-16) reduces to

$$A_z = \frac{\mu}{4\pi}\int_{-l/2}^{+l/2}\left[\frac{1}{2\pi a}\int_0^{2\pi} I_z(z')\frac{e^{-jkR}}{R}a\,d\phi'\right]dz' \tag{8-18}$$

$$\begin{aligned} R &= \sqrt{(x-x')^2+(y-y')^2+(z-z')^2} \\ &= \sqrt{(\rho^2+a^2-2\rho a\cos(\phi-\phi')+(z-z')^2} \end{aligned} \tag{8-18a}$$

where ρ is the radial distance to the observation point and a is the radius.

Because of the symmetry of the scatterer, the observations are not a function of ϕ. For simplicity, let us then choose $\phi = 0$. For observations on the surface $\rho = a$ of the scatterer (8-18) and (8-18a) reduce to

$$\begin{aligned} A_z(\rho = a) &= \mu\int_{-l/2}^{+l/2} I_z(z')\left(\frac{1}{2\pi}\int_0^{2\pi}\frac{e^{-jkR}}{4\pi R}d\phi'\right)dz' \\ &= \mu\int_{-l/2}^{+l/2} I_z(z')G(z,z')\,dz' \end{aligned} \tag{8-19}$$

$$G(z,z') = \frac{1}{2\pi}\int_0^{2\pi}\frac{e^{-jkR}}{4\pi R}d\phi' \tag{8-19a}$$

$$R(\rho = a) = \sqrt{4a^2\sin^2\left(\frac{\phi'}{2}\right)+(z-z')^2} \tag{8-19b}$$

Thus for observations at the surface $\rho = a$ of the scatterer, the z component of the scattered electric field can be expressed as

$$E_z^s(\rho = a) = -j\frac{1}{\omega\epsilon}\left(k^2 + \frac{d^2}{dz^2}\right)\int_{-l/2}^{+l/2} I_z(z')G(z, z')\,dz' \tag{8-20}$$

which by using (8-13a) reduces to

$$-j\frac{1}{\omega\epsilon}\left(\frac{d^2}{dz^2} + k^2\right)\int_{-l/2}^{+l/2} I_z(z')G(z, z')\,dz' = -E_z^i(\rho = a) \tag{8-21}$$

or

$$\left(\frac{d^2}{dz^2} + k^2\right)\int_{-l/2}^{+l/2} I_z(z')G(z, z')\,dz' = -j\omega\epsilon E_z^i(\rho = a) \tag{8-21a}$$

Interchanging integration with differentiation, we can rewrite (8-21a) as

$$\int_{-l/2}^{+l/2} I_z(z')\left[\left(\frac{\partial^2}{\partial z^2} + k^2\right)G(z, z')\right]dz' = -j\omega\epsilon E_z^i(\rho = a) \tag{8-22}$$

where $G(z, z')$ is given by (8-19a).

Equation (8-22) is referred to as *Pocklington's integral equation* [1], and it can be used to determine the equivalent filamentary line-source current of the wire, and thus current density on the wire, by knowing the incident field on the surface of the wire.

If we assume that the wire is very thin ($a \ll \lambda$) such that (8-19a) reduces to

$$G(z, z') = G(R) = \frac{e^{-jkR}}{4\pi R} \tag{8-23}$$

(8-22) can also be expressed in a more convenient form as [22]

$$\int_{-l/2}^{+l/2} I_z(z')\frac{e^{-jkR}}{4\pi R^5}[(1 + jkR)(2R^2 - 3a^2) + (kaR)^2]\,dz' = -j\omega\epsilon E_z^i(\rho = a) \tag{8-24}$$

where for observations along the center of the wire ($\rho = 0$)

$$R = \sqrt{a^2 + (z - z')^2} \tag{8-24a}$$

In (8-22) or (8-24), $I_z(z')$ represents the equivalent filamentary line-source current located on the surface of the wire, as shown in Figure 8.5(b), and it is obtained by knowing the incident electric field on the surface of the wire. By point-matching techniques, this is solved by matching the boundary conditions at discrete points on the surface of the wire. Often it is easier to choose the matching points to be at the interior of the wire, especially along the axis as shown in Figure 8.6(a), where $I_z(z')$ is located on the surface of the wire. By reciprocity, the configuration of Figure 8.6(a) is analogous to that of Figure 8.6(b) where the equivalent filamentary line-source current is assumed to be located along the center axis of the wire and the matching points are selected on the surface of the wire. Either of the two configurations can be used to determine the equivalent filamentary line-source current $I_z(z')$; the choice is left to the individual.