

## **Chapter 8**

# *Radiated Emissions and Susceptibility*

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# Outline

- Simple Emission Models for Wires and PCB Lands
  - Differential-Mode versus Common-Mode Currents
  - Differential-Mode Current Emission Model
  - Common-Mode Current Emission Model
  - Current Probes
  - Experimental Results
- Simple Susceptibility Models for Wires and PCB Lands
  - Experimental Results
  - Shield Cables and Surface Transfer Impedance

# Preview

- Assumptions

- Form simplicity, we will assume that the measurement antenna is in the far field of the emission (product), although this is not necessarily the case over the entire frequency range of the regulatory limit.
- In the far field, the inverse-distance rules applies, which is used to translate an emission measured at one distance to another distance.

# Simple Emission Models for Wires and PCB Lands

- Differential-Mode versus Common-Mode Currents

- Decomposition of Currents

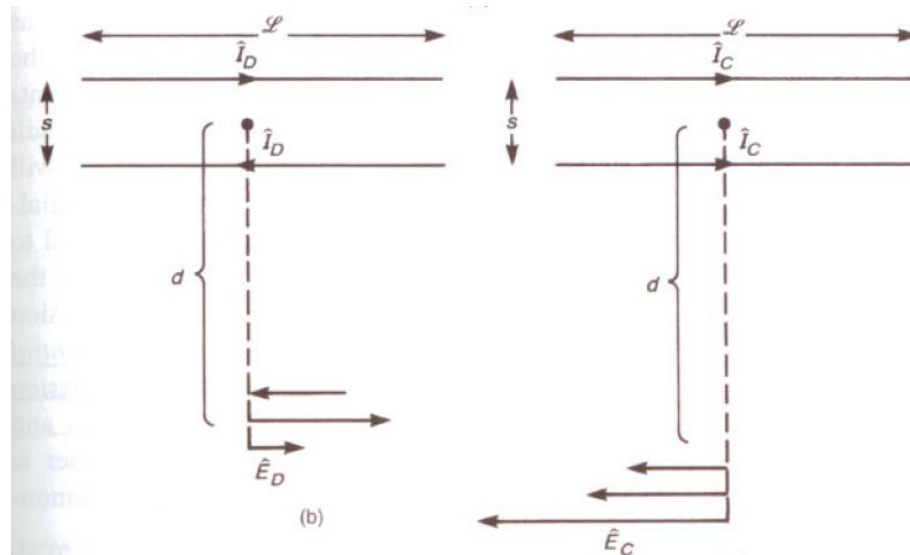
- The currents could be decomposed into **differential-mode and common-mode components** by writing

$$\left. \begin{aligned} \hat{I}_1 &= \hat{I}_C + \hat{I}_D \\ \hat{I}_2 &= \hat{I}_C - \hat{I}_D \end{aligned} \right\} \rightarrow \begin{aligned} \hat{I}_D &= \frac{\hat{I}_1 - \hat{I}_2}{2} \\ \hat{I}_C &= \frac{\hat{I}_1 + \hat{I}_2}{2} \end{aligned}$$



# Simple Emission Models for Wires and PCB Lands

- Differential-Mode versus Common-Mode Currents
  - Radiation Emissions of Different Currents
    - Common-mode currents are not inconsequential in typical products, and, moreover, they often produce larger radiated emissions than do the differential-mode currents.



# Simple Emission Models for Wires and PCB Lands

- Differential-Mode versus Common-Mode Currents

- Far Fields of Wire Currents

- The total radiated electric field will be the sum of each radiated electric field, which is

$$\hat{E}_\theta = \hat{E}_{\theta,1} + \hat{E}_{\theta,2}$$

- where the far fields of each antenna are of the form

$$\hat{E}_{\theta,i} = \hat{M} \hat{l}_i \frac{e^{-j\beta_0 r_i}}{r_i} F(\theta)$$

- and  $\left. \begin{aligned} \hat{M} &= j \frac{\eta_0 \beta_0}{4\pi} \mathcal{L} = j 2\pi \times 10^{-7} f \mathcal{L} \\ F(\theta) &= \sin \theta \end{aligned} \right\} \text{ (Hertzian dipoles)}$

$$\left. \begin{aligned} \hat{M} &= j \frac{\eta_0}{2\pi} = j 60 \\ F(\theta) &= \frac{\cos(\frac{1}{2} \pi \cos \theta)}{\sin \theta} \end{aligned} \right\} \text{ (half-wave dipoles, } \mathcal{L} = \frac{1}{2} \lambda_0)$$

# Simple Emission Models for Wires and PCB Lands

- Differential-Mode versus Common-Mode Currents

- Far Fields of Wire Currents

- Since

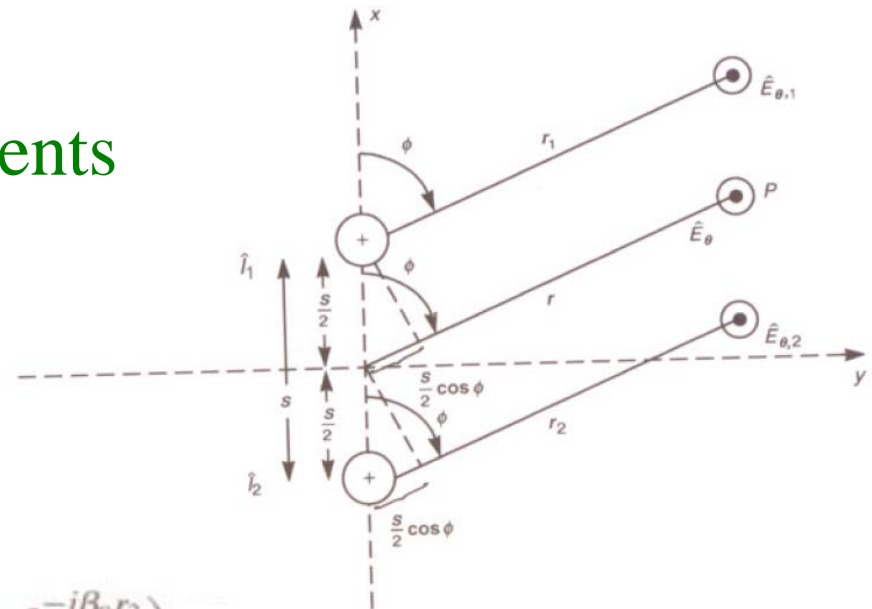
$$r_1 = r - \frac{s}{2} \cos \phi$$

$$r_2 = r + \frac{s}{2} \cos \phi$$

- The total field becomes

$$\hat{E}_\theta = \hat{M} \left( \hat{I}_1 \frac{e^{-j\beta_0 r_1}}{r_1} + \hat{I}_2 \frac{e^{-j\beta_0 r_2}}{r_2} \right) F(\theta)$$

$$\longrightarrow \hat{E}_\theta = \hat{M} \frac{e^{-j\beta_0 r}}{r} (\hat{I}_1 e^{+j\beta_0 s/2 \cos \phi} + \hat{I}_2 e^{-j\beta_0 s/2 \cos \phi}) F(\theta) \quad (a)$$



# Simple Emission Models for Wires and PCB Lands

- Differential-Mode Current Emission Model

- Far Fields

- In order to simplify the resulting model, a **Hertzian dipole** is used.
- For  $I_1 = -I_2$ , **the maximum** will occur in the plane of the wires and on a line perpendicular to the wires.

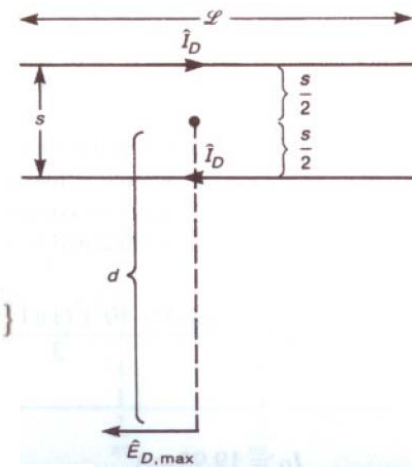
- Since  $\hat{I}_1 = \hat{I}_D$   
 $\hat{I}_2 = -\hat{I}_D$

- The radiated field becomes

(a)  $\xrightarrow{\varphi=0}$  
$$\hat{E}_{D,\max} = j2\pi \times 10^{-7} \frac{f\hat{I}_D\mathcal{L}}{d} e^{-j\beta_0 d} \{e^{j\beta_0 s/2} - e^{-j\beta_0 s/2}\}$$

$$= -4\pi \times 10^{-7} \frac{f\hat{I}_D\mathcal{L}}{d} e^{-j\beta_0 d} \sin(\tfrac{1}{2}\beta_0 s)$$

$\longrightarrow |\hat{E}_{D,\max}| = 1.316 \times 10^{-14} \frac{|\hat{I}_D| f^2 \mathcal{L} s}{d}$





# Simple Emission Models for Wires and PCB Lands

- Differential-Mode Current Emission Model

- Example 8.1

- Considering a ribbon cable of  $s=50\text{mil}$  and  $L=1\text{m}$ , operating at  $f=30\text{MHz}$ , the current  $I_D$  required to satisfy the FCC Class B limit ( $100\text{ }\mu\text{V/m}$  at  $30\text{MHz}$ ) is

$$100\text{ }\mu\text{V/m} = 1.316 \times 10^{-14} \frac{|\hat{I}_D|(3 \times 10^7)^2(1)(1.27 \times 10^{-3})}{3}$$

→  $I_D = 19.95\text{ mA}$

which is large

- Generally, the formula for the maximum emission given is sufficient for estimation purposes.

# Simple Emission Models for Wires and PCB Lands

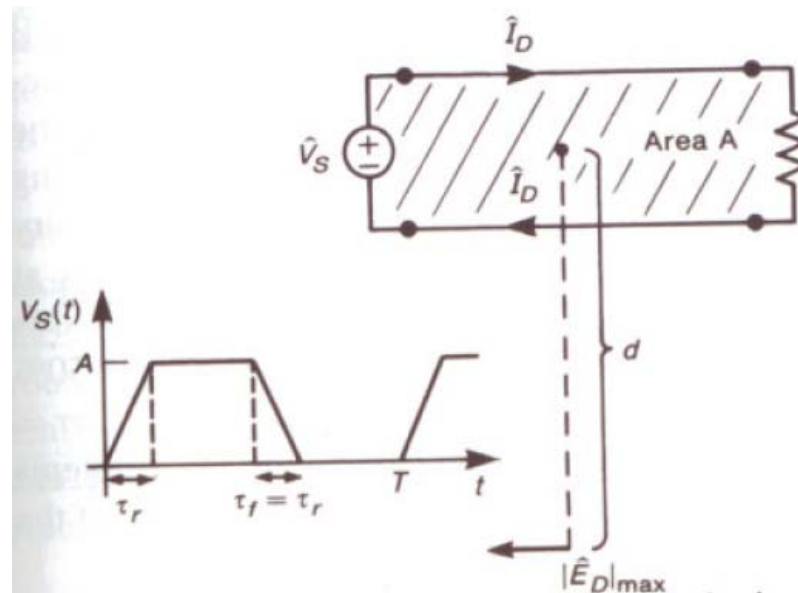
- Differential-Mode Current Emission Model

- Example 8.1

- Consider a current  $I$  of a **trapezoidal waveform**
- **The transfer function** relating the maximum received electric field to the current is

$$\left| \frac{\hat{E}_{D, \max}}{\hat{I}_D} \right| = K f^2 A$$

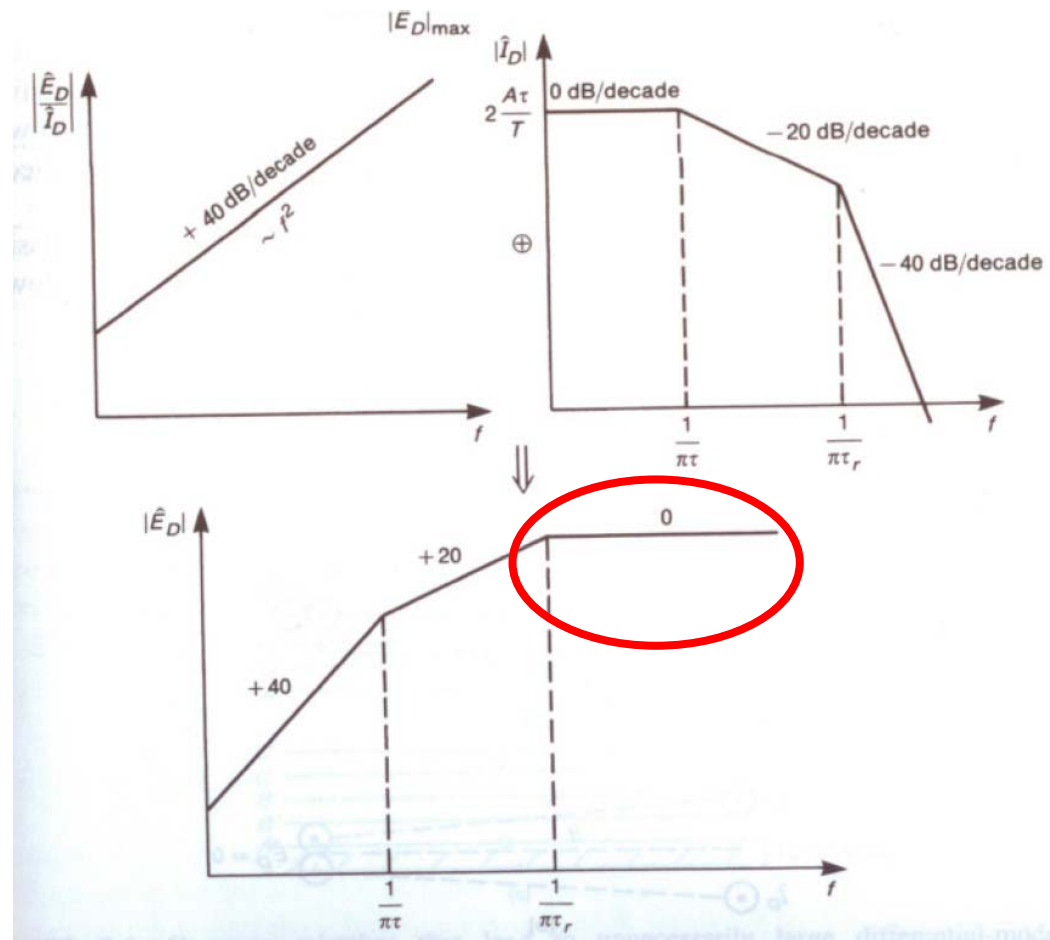
The frequency response is shown in the next page



# Simple Emission Models for Wires and PCB Lands

- Differential-Mode Current Emission Model
  - Example 8.1

If the regulatory limit of the red circle is satisfied, the all other portions are satisfied.



# Simple Emission Models for Wires and PCB Lands

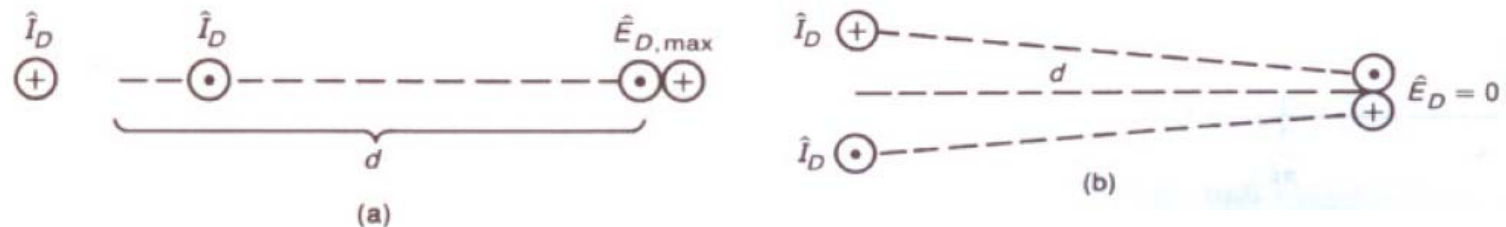
- Differential-Mode Current Emission Model

- How to Reduce the Emission Level

- The radiated emission is proportional to (1) the square of the frequency, (2) the loop area  $A=Ls$ , (3) the current level  $I_D$ .

$$|\hat{E}_{D,\max}| = 1.316 \times 10^{-14} \frac{|\hat{I}_D| f^2 \mathcal{L} s}{d}$$

- Thus, ways to reduce the radiated emission are (1) increase the values of  $\tau_r$  and/or  $\tau$ , (2) reduce the loop area, and (3) reduce the current level.

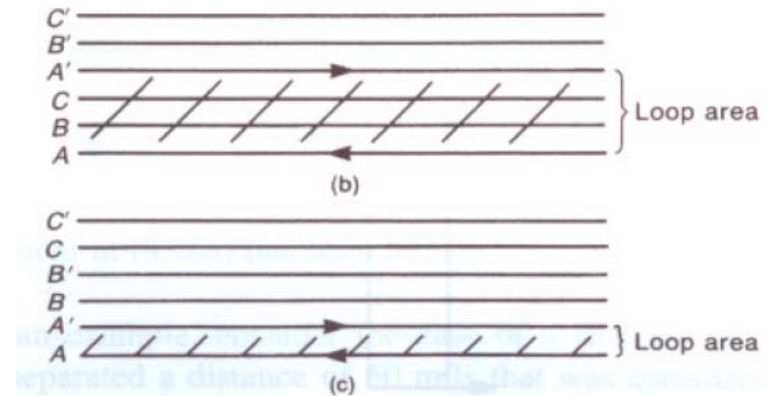
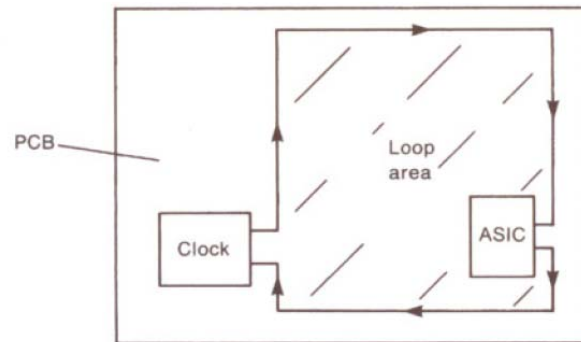


# Simple Emission Models for Wires and PCB Lands

- Differential-Mode Current Emission Model

- For Example

- Place the clock **as close as possible** to the ASIC to lessen the loop area.
    - Place the differential pair **as close as possible** to lessen the loop area.



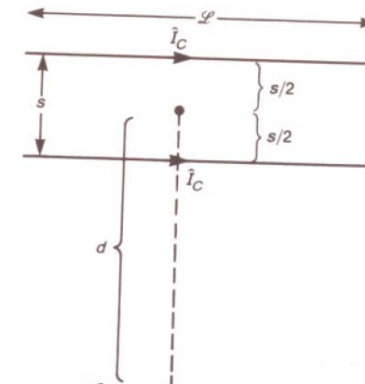
# Simple Emission Models for Wires and PCB Lands

- Common-Mode Current Emission Model

- Far Fields

- Since  $\hat{I}_1 = \hat{I}_C$   
 $\hat{I}_2 = \hat{I}_C$

- The far field as the figure shown is



$$\begin{aligned}
 \text{(a) } \xrightarrow{\varphi=0} \hat{E}_{C, \max} &= j2\pi \times 10^{-7} \frac{f \hat{I}_C \mathcal{L}}{d} e^{-j\beta_0 d} \{e^{j\beta_0 s/2} + e^{-j\beta_0 s/2}\} \\
 &= j4\pi \times 10^{-7} \frac{f \hat{I}_C \mathcal{L}}{d} e^{-j\beta_0 d} \cos\left(\frac{1}{2} \beta_0 s\right) \\
 &\xrightarrow{\quad} |\hat{E}_{C, \max}| = 1.257 \times 10^{-6} \frac{|\hat{I}_C| f \mathcal{L}}{d} \quad \text{(b)}
 \end{aligned}$$

- Although this is not the maximum field point, **this approximately is.**
- Since  **$s$  is electrical small**, the field is omnidirectional in  $\varphi$ .

# Simple Emission Models for Wires and PCB Lands

- Common-Mode Current Emission Model

- Far Fields

- For common-mode currents and electrically small wire separations, the “**pattern**” is virtually **omnidirectional around the wires**.
    - Also, we may replace **the two wires**, each carrying current  **$I_C$** , **with one wire** carrying current  **$2I_C$**  without substantially changing the radiated fields at any point around it.
    - If the total current is  $I_{probe}=2I_C$ , we have

(b)  $\longrightarrow |\hat{E}_{C,max}| = 6.283 \times 10^{-7} \frac{|\hat{I}_{probe}| f \mathcal{L}}{d}$

# Simple Emission Models for Wires and PCB Lands

- Common-Mode Current Emission Model

- Example 8.2

- Considering a ribbon cable of  $s=50\text{mil}$  and  $L=1\text{m}$ , operating at  $f=30\text{MHz}$ , the current  $I_C$  required to satisfy the FCC Class B limit ( $100\mu\text{V/m}$  at  $30\text{MHz}$ ) is

$$100\mu\text{V/m} = 1.257 \times 10^{-6} \frac{|\hat{I}_C|(3 \times 10^7)(1)}{3}$$

$$\longrightarrow I_C = 7.96\mu\text{A} \quad \boxed{\text{which is small}}$$

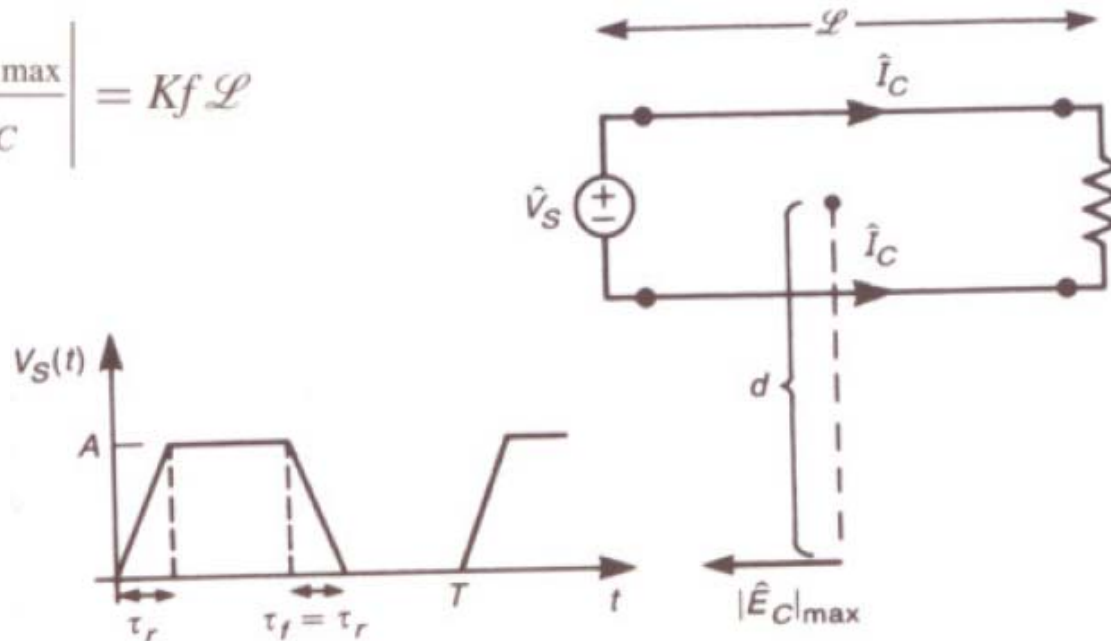
- Generally, the formula for the maximum emission given is sufficient for estimation purposes.



# Simple Emission Models for Wires and PCB Lands

- Common-Mode Current Emission Model
  - Example 8.2
    - Consider a current  $I$  of a **trapezoidal waveform**
    - **The transfer function** relating the maximum received electric field to the current is

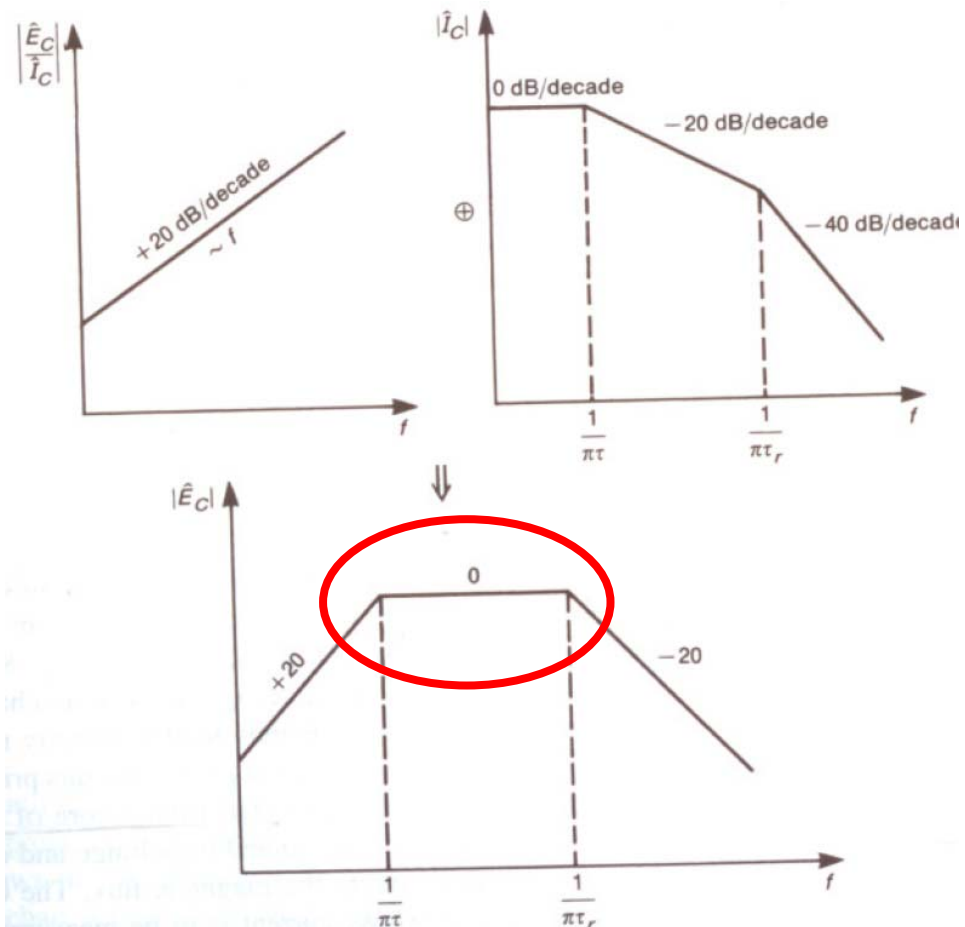
$$\left| \frac{\hat{E}_{C,\max}}{I_C} \right| = Kf \mathcal{L}$$



# Simple Emission Models for Wires and PCB Lands

- Common-Mode Current Emission Model
  - Example 8.2

If the regulatory limit of the red circle is satisfied, the all other portions are satisfied.



# Simple Emission Models for Wires and PCB Lands

- Common-Mode Current Emission Model

- How to Reduce the Emission Level

- The radiated emission is proportional to (1) the frequency, (2) the line length  $L$ , and (3) the current level  $I_C$ .

$$|\hat{E}_{C,\max}| = 1.257 \times 10^{-6} \frac{|\hat{I}_C| f \mathcal{L}}{d}$$

- Thus, ways to reduce the radiated emission are (1) increase the values of  $\tau_r$  and/or  $\tau$ , (2) reduce the line length, and (3) reduce the current level.

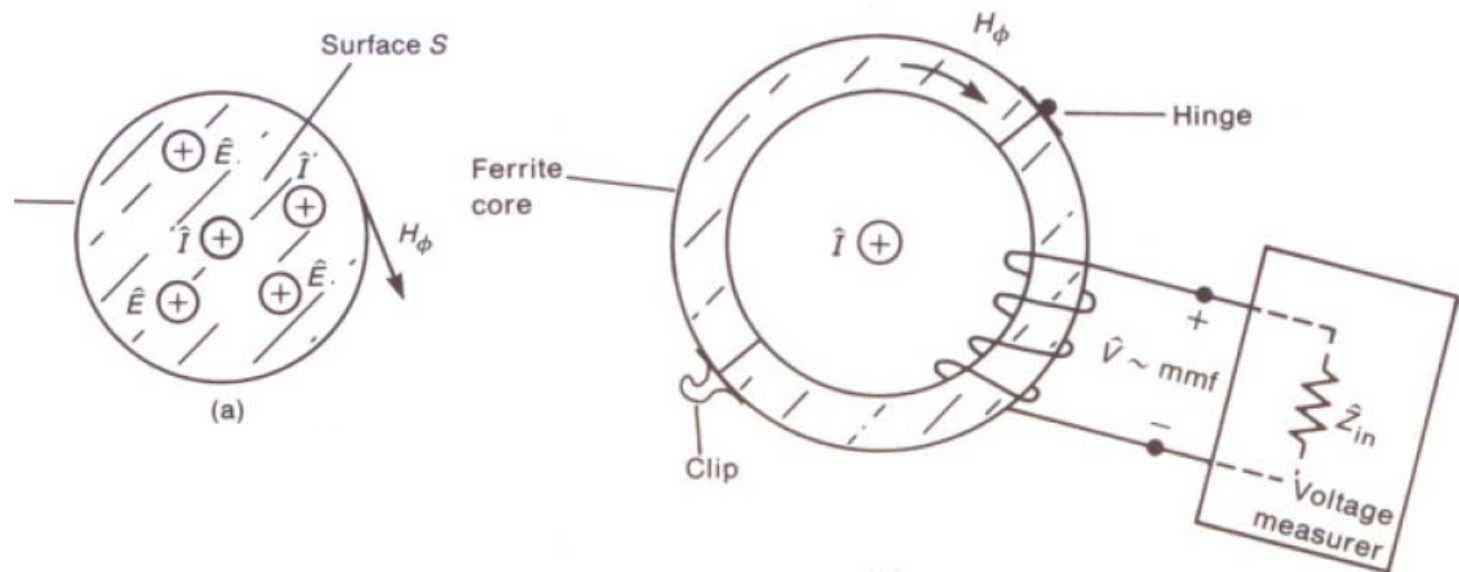
# Simple Emission Models for Wires and PCB Lands

- Current Probes

- Principle

- Current probes make use of Ampere's law

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s} + \frac{d}{dt} \epsilon \int_S \vec{E} \cdot d\vec{s}$$



# Simple Emission Models for Wires and PCB Lands

- Current Probes

- Principle

- Simply pass a current of **known magnitude and frequency** through the probe and measure the **resulting voltage** produced at the terminals, we can obtain the transfer impedance as

$$\hat{Z}_T = \frac{\hat{V}}{\hat{I}} \longrightarrow |\hat{Z}_T|_{\text{dB}\Omega} = |\hat{V}|_{\text{dB}\mu\text{V}} - |\hat{I}|_{\text{dB}\mu\text{A}}$$

- The calibration curve of the current probe is valid only when the probe is **terminated in the same impedance** as was used in the course of its calibration (usually  $50\Omega$ )

# Simple Emission Models for Wires and PCB Lands

- Current Probes

- Principle

- The current probe **will not measure differential-mode current** unless it is placed around each individual wire.
- The radiated emission due to the total current  $I_{C, net}$  could be obtained by **dividing the  $I_{C, net}$  in two equal currents  $I_C = I_{C, net}/2$**  and using the radiated field equation of the **common-mode currents**, which is

$$|\hat{E}_C|_{\max} = 6.28 \times 10^{-7} \frac{|\hat{I}_{C, net}| f \mathcal{L}}{d}$$

$$\xrightarrow{\hat{Z}_T = \frac{\hat{V}}{\hat{I}}} |\hat{E}_C|_{\max} = 6.28 \times 10^{-7} \frac{|\hat{V}_{SA}| f \mathcal{L}}{|\hat{Z}_T| d}$$

# Simple Emission Models for Wires and PCB Lands

- Current Probes

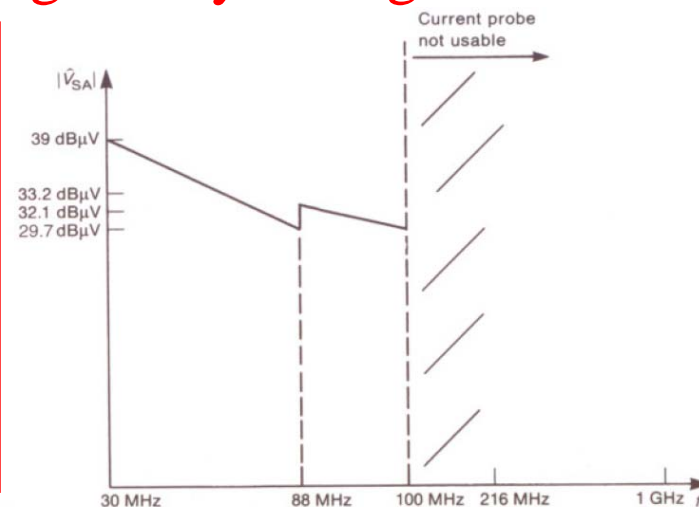
- Principle

- In dB form, we obtain

$$|\hat{V}_{SA}|_{\text{dB}\mu\text{V}} = |\hat{E}|_{\text{limit}, \text{dB}\mu\text{V/m}} + |\hat{Z}_T|_{\text{dB}\Omega} + 20 \log_{10} d \\ - 20 \log_{10} f_{\text{MHz}} - 20 \log_{10} \mathcal{L} + 4.041$$

- This could be used to transform the **regulatory electric field limits** into **regulatory voltage limits**.

By simply measuring the voltage or current, we could determine if the product meets the regulatory limits with using the complex and expensive semianechoic chamber.



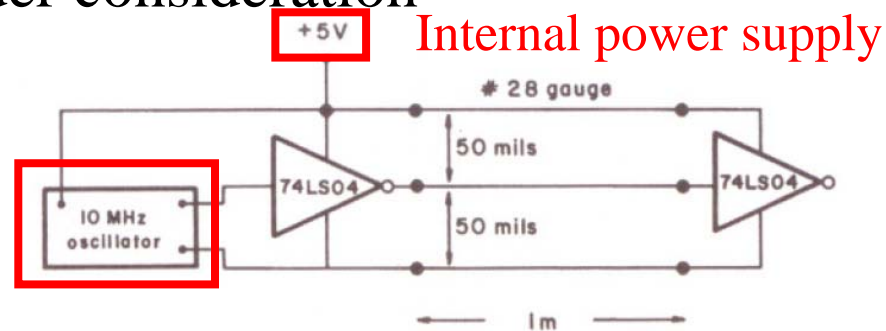
# Experimental Results

- Cable Emissions

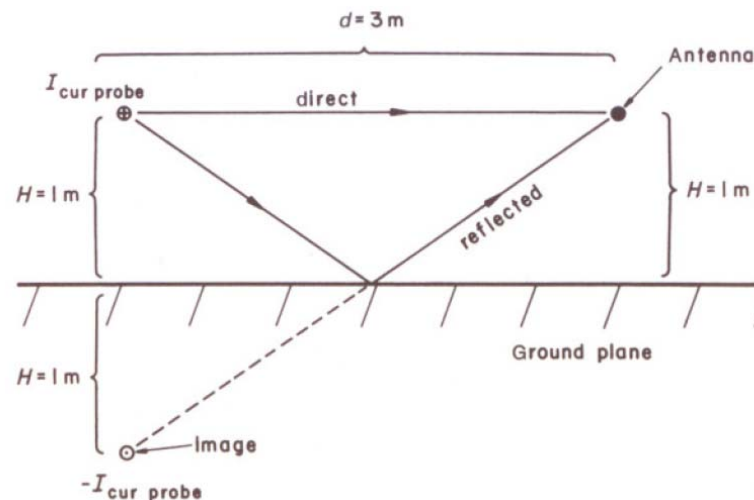
- Circuit and Measurement Setup

- Circuit under consideration

DIP-  
packaged  
oscillator



- Measurement site





# Experimental Results

- Cable Emissions

- Prediction and Measurement Results

- The **predicted emission** is described as follows

$$|\hat{E}_C|_{\max} = 6.28 \times 10^{-7} \frac{|\hat{I}_{\text{probe}}| f \mathcal{L}}{d} \hat{F}_{\text{GP}}$$

- Since **the measured probe current** is

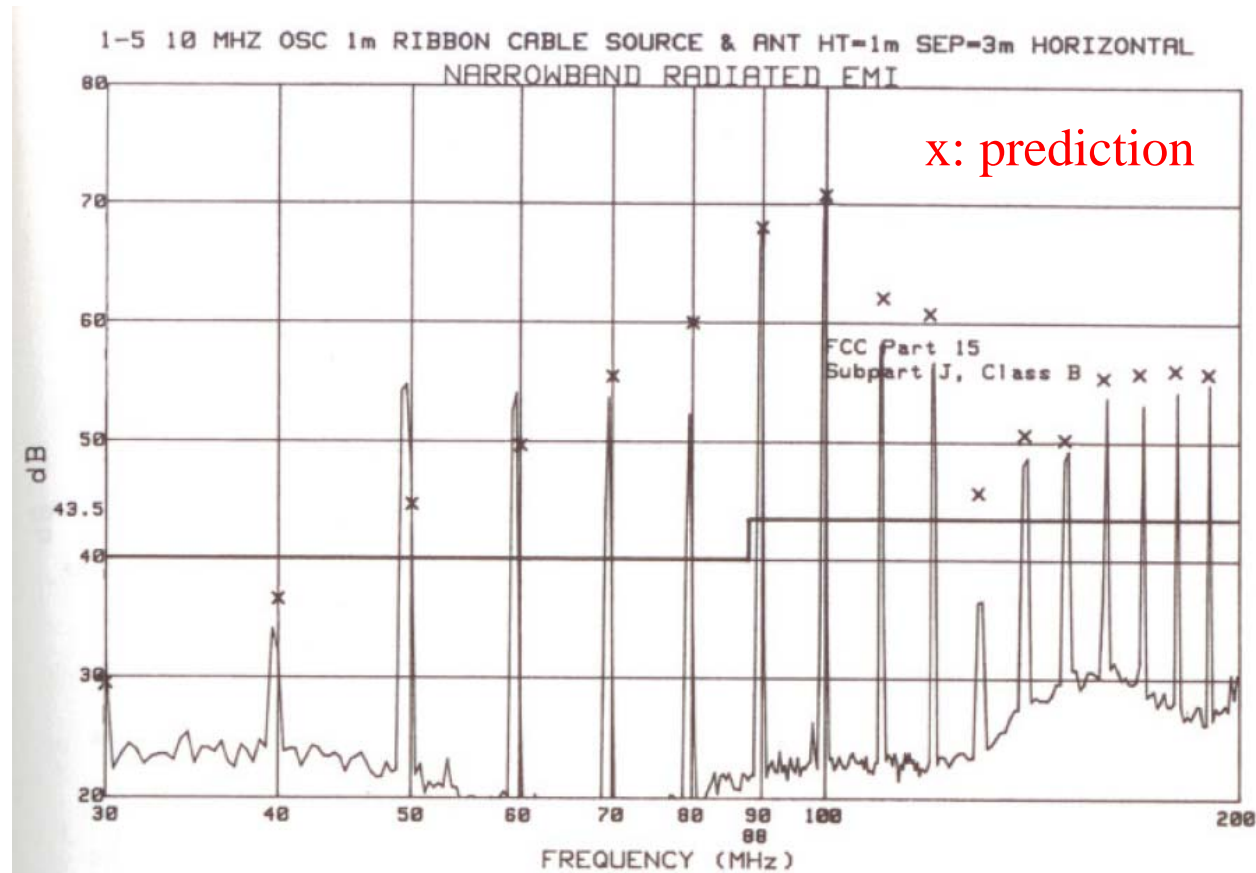
$$\begin{aligned} |\hat{I}_{\text{probe}}|_{\text{dB}\mu\text{A}} &= |\hat{V}_{\text{SA}}|_{\text{dB}\mu\text{V}} + \text{cable loss}_{\text{dB}} - |\hat{Z}_T|_{\text{dB}\Omega} \\ &= |\hat{V}_{\text{SA}}|_{\text{dB}\mu\text{V}} + \text{cable loss}_{\text{dB}} - 15 \end{aligned}$$

- **Combing the above two equations** and translating into dB form, we have

$$\begin{aligned} |\hat{E}_C|_{\text{dB}\mu\text{V/m}} &= |\hat{V}_{\text{SA}}|_{\text{dB}\mu\text{V}} + \text{cable loss}_{\text{dB}} - |\hat{Z}_T|_{\text{dB}\Omega} + 20 \log_{10} f_{\text{MHz}} \\ &\quad + |\hat{F}_{\text{GP}}|_{\text{dB}} - 13.58 \end{aligned}$$

# Experimental Results

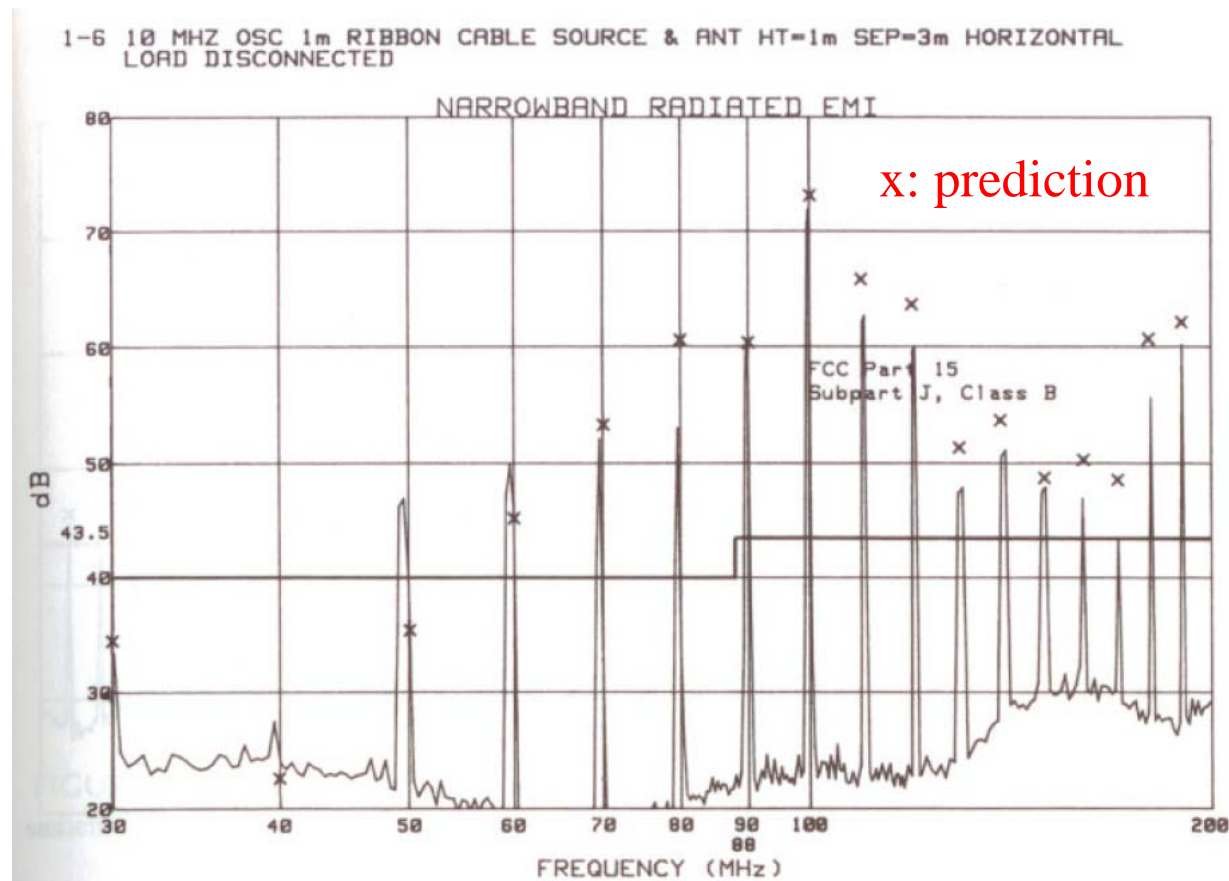
- Cable Emissions
  - Prediction and Measurement Results – with Load



# Experimental Results

- Cable Emissions
  - Prediction and Measurement Results – without Load

Only common-mode current is excited.

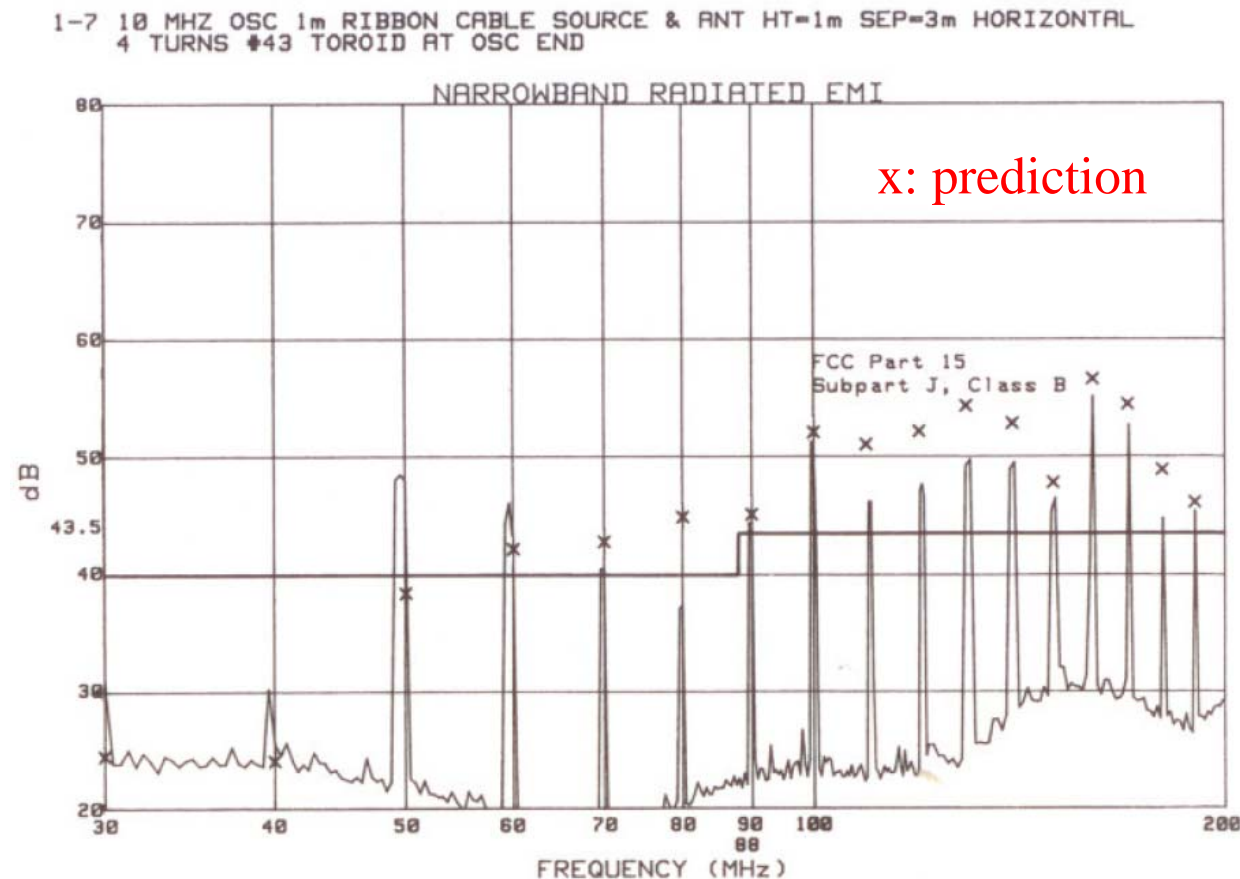


# Experimental Results

- Cable Emissions

- Prediction and Measurement Results – with Load and Ferrite Toroid

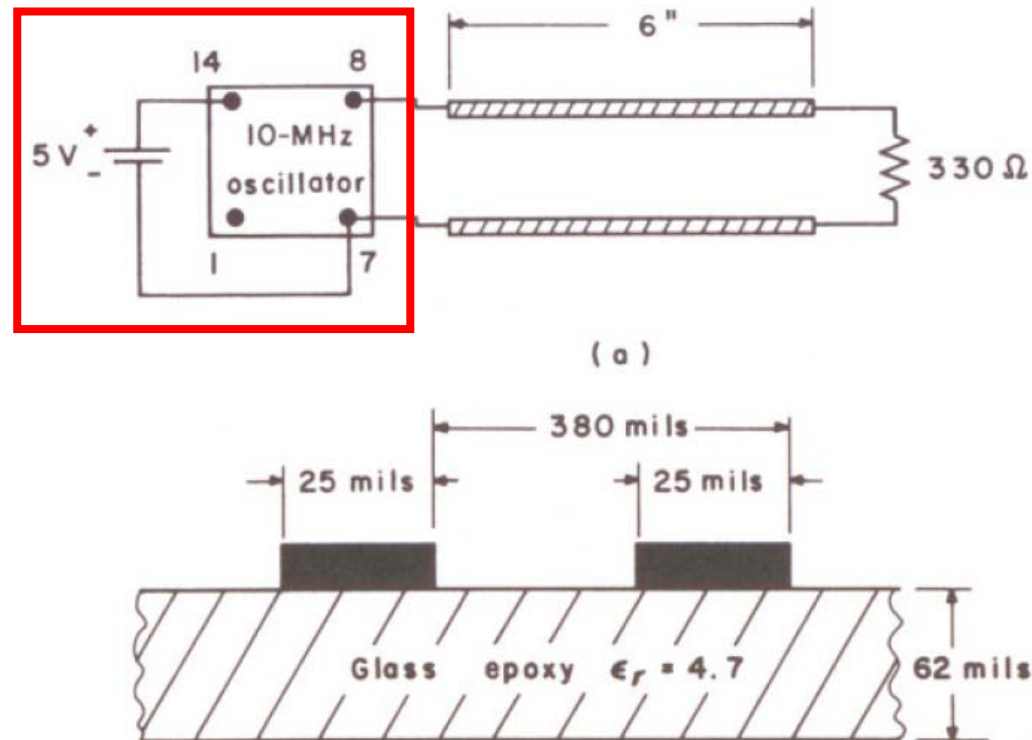
common-mode emission is reduced.



# Experimental Results

- PCB Lands Emissions
  - Circuit

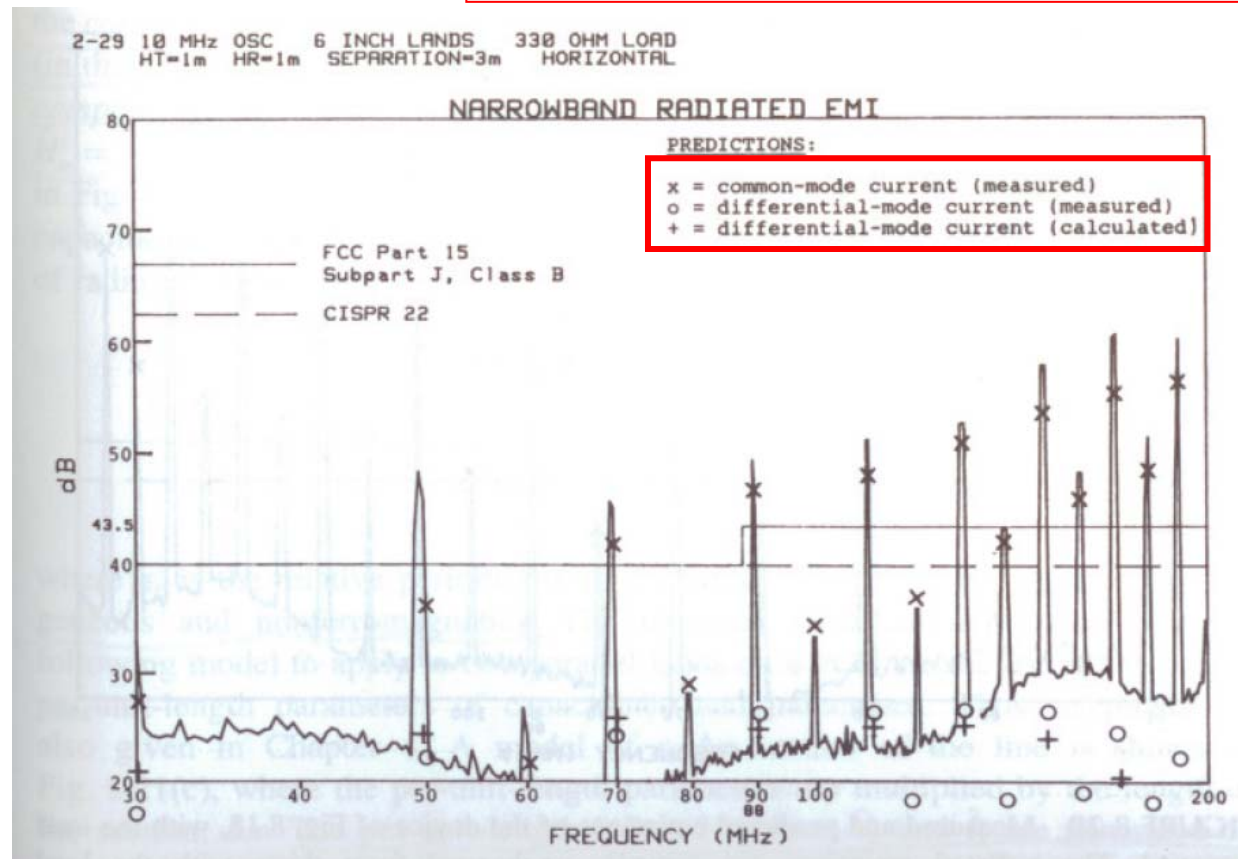
Noiseless



# Experimental Results

- PCB Lands Emissions
  - Prediction and Measurement Results – with Load

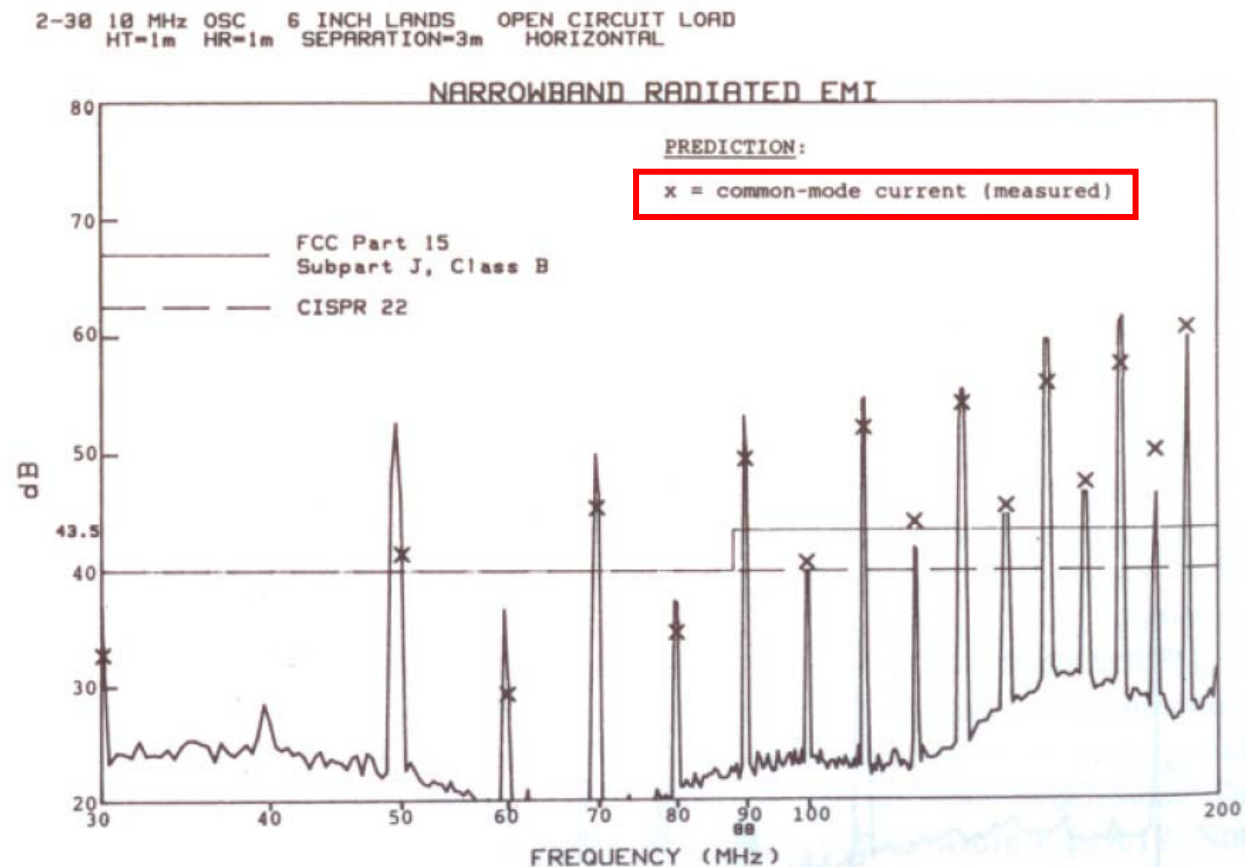
Common-mode emission dominates



# Experimental Results

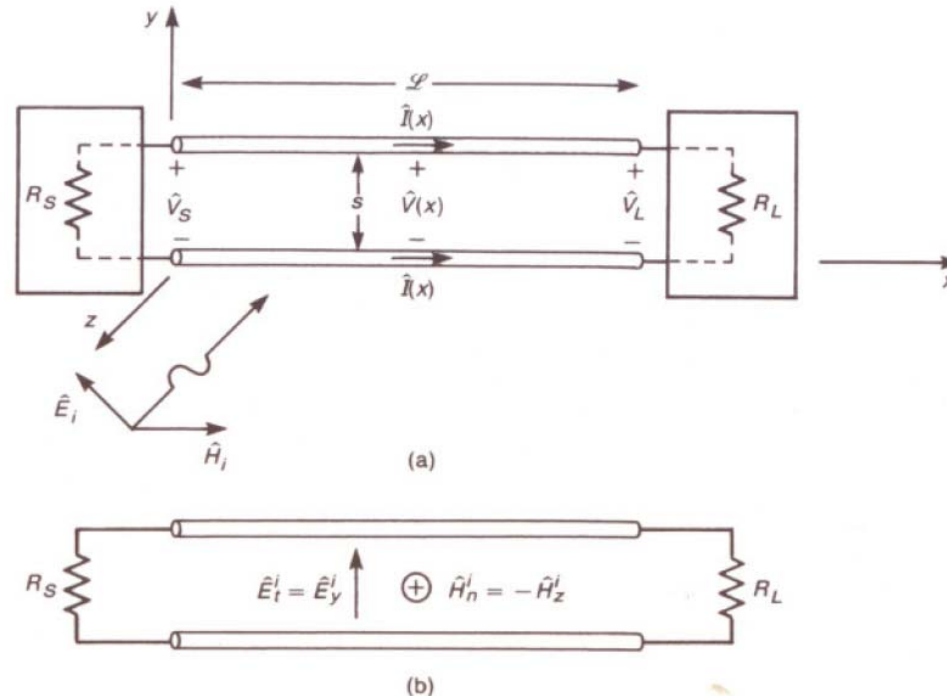
- PCB Lands Emissions
  - Prediction and Measurement Results – without Load

Only common-mode current is excited.



# Simple Susceptibility Models for Wires and PCB Lands

- Two-Conductor Line
  - General Susceptibility Model
    - The two effective components of the incident wave are (1) transverse electric field  $E_y$  (2) perpendicular magnetic field  $-H_z$





# Simple Susceptibility Models for Wires and PCB Lands

- Two-Conductor Line
  - General Susceptibility Model

- The per-unit-length inductance and capacitance are

$$l = \frac{\mu_0}{\pi} \ln\left(\frac{S}{r_w}\right) \quad (\text{in H/m})$$

$$c = \pi\epsilon_0\epsilon_r / \ln\left(\frac{S}{r_w}\right) \quad (\text{in F/m})$$

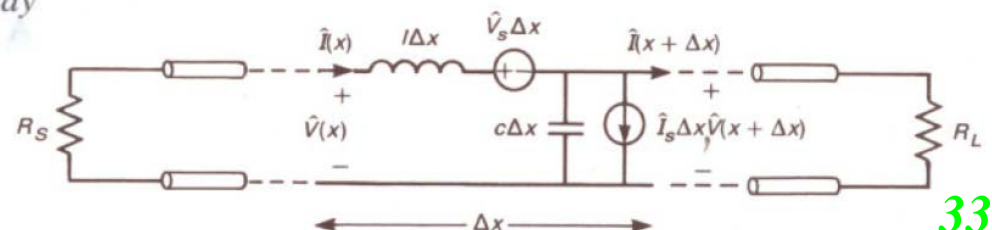
- According Faraday's law, the induced emf is

$$\text{emf} = j\omega \int_S \hat{B}_n^i ds \quad \longrightarrow \quad \hat{V}_s(x) = j\omega\mu_0 \int_{y=0}^S \hat{H}_n^i dy$$

$$= j\omega\mu_0 \int_S \hat{H}_n^i ds$$

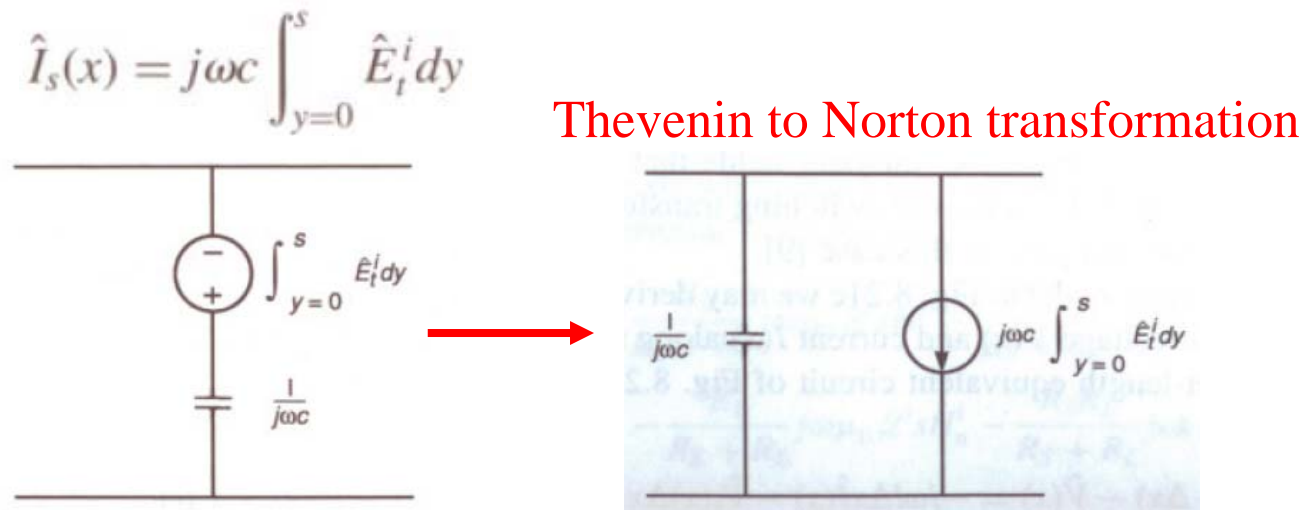
$$= j\omega\mu_0 \Delta x \int_{y=0}^S \hat{H}_n^i dy$$

The per-unit-length induced voltage source



# Simple Susceptibility Models for Wires and PCB Lands

- Two-Conductor Line
  - General Susceptibility Model
    - The per-unit-length induced current source is



- The incident electric field is

$$|\hat{E}^i| = \frac{\sqrt{60P_T G}}{d} \longrightarrow |\hat{H}^i| = \frac{|\hat{E}^i|}{n_0} \text{ the incident magnetic field}$$

# Simple Susceptibility Models for Wires and PCB Lands

- Two-Conductor Line
  - General Susceptibility Model - TX Line Equations

- Using KCL and KVL, we have

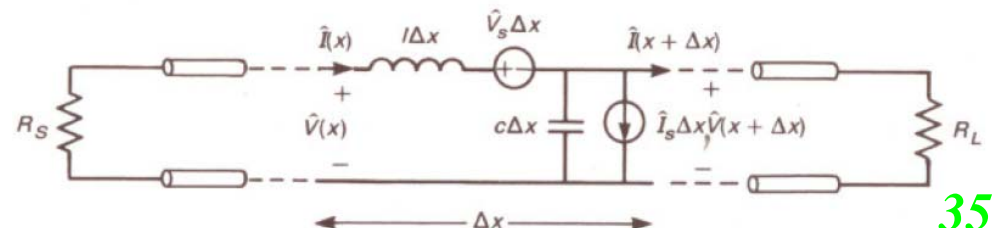
$$\hat{V}(x + \Delta x) - \hat{V}(x) = -j\omega l \Delta x \hat{I}(x) - \hat{V}_s(x) \Delta x$$

$$\hat{I}(x + \Delta x) - \hat{I}(x) = -j\omega c \Delta x \hat{V}(x + \Delta x) - \hat{I}_s(x) \Delta x$$

$$\frac{d\hat{V}(x)}{dx} + j\omega l \hat{I}(x) = -\hat{V}_s(x) = -j\omega \mu_0 \int_{y=0}^S \hat{H}_n^i dy$$



$$\frac{d\hat{I}(x)}{dx} + j\omega c \hat{V}(x) = -\hat{I}_s(x) = -j\omega c \int_{y=0}^S \hat{E}_t^i dy$$



# Simple Susceptibility Models for Wires and PCB Lands

- Two-Conductor Line

- General Susceptibility Model - TX Line Equations

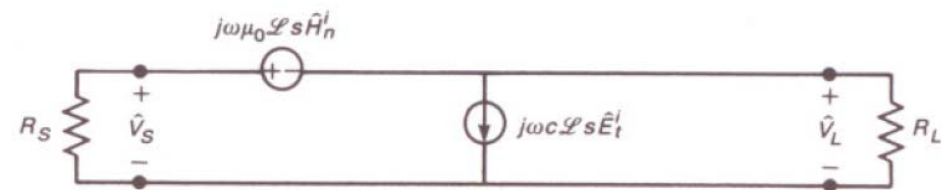
- When the line is electrically short,  $L \ll \lambda_0$  and the line inductance and capacitance are neglected, we have the simplified model

If loads not approaching open or short circuit.

$$\hat{V}_s \mathcal{L} \cong j\omega\mu_0 \hat{H}_n^i A$$

$$\hat{I}_s \mathcal{L} \cong j\omega c \hat{E}_t^i A$$

$$A = s \mathcal{L}$$



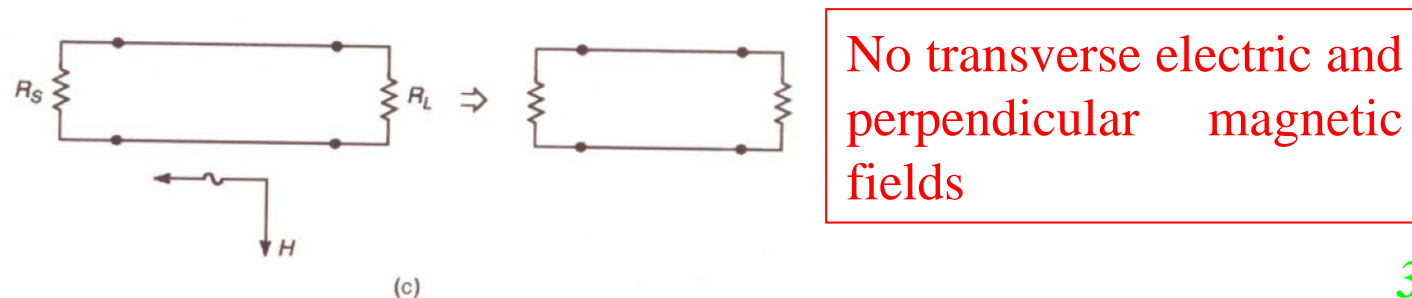
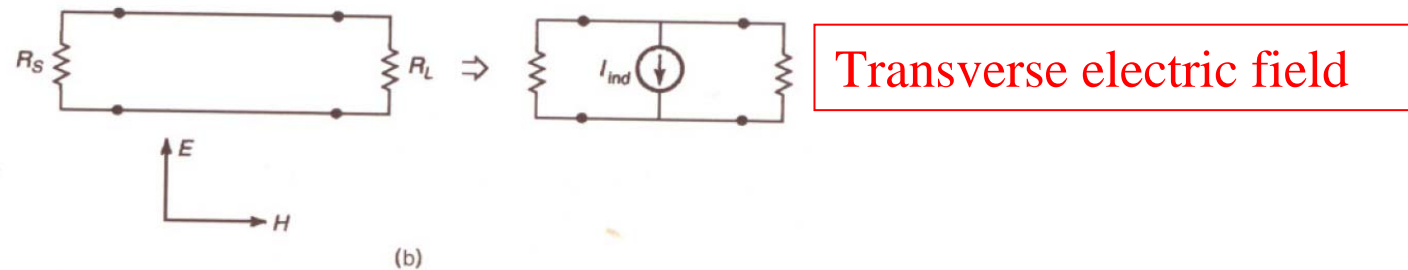
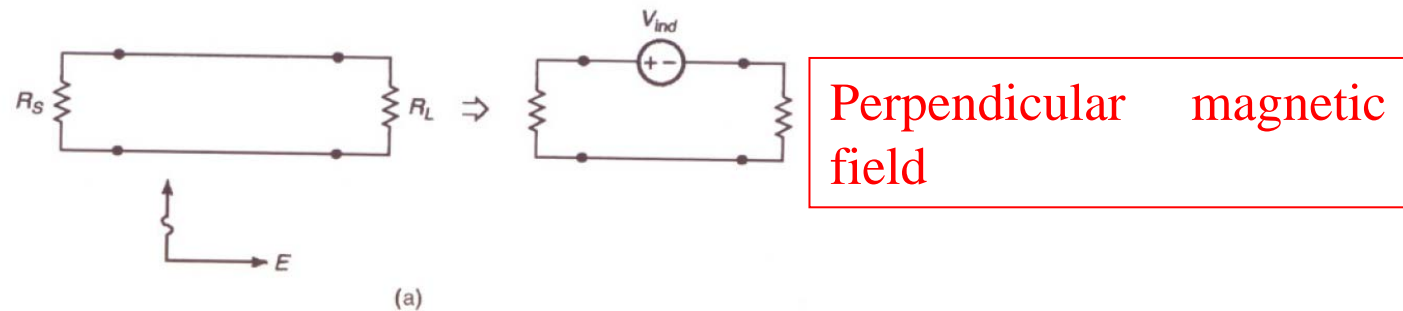
- The induced terminal voltages are (superposition)

$$\hat{V}_S = \frac{R_S}{R_S + R_L} j\omega\mu_0 \mathcal{L} s \hat{H}_n^i - \frac{R_S R_L}{R_S + R_L} j\omega c \mathcal{L} s \hat{E}_t^i$$

$$\hat{V}_L = -\frac{R_L}{R_S + R_L} j\omega\mu_0 \mathcal{L} s \hat{H}_n^i - \frac{R_S R_L}{R_S + R_L} j\omega c \mathcal{L} s \hat{E}_t^i$$

# Simple Susceptibility Models for Wires and PCB Lands

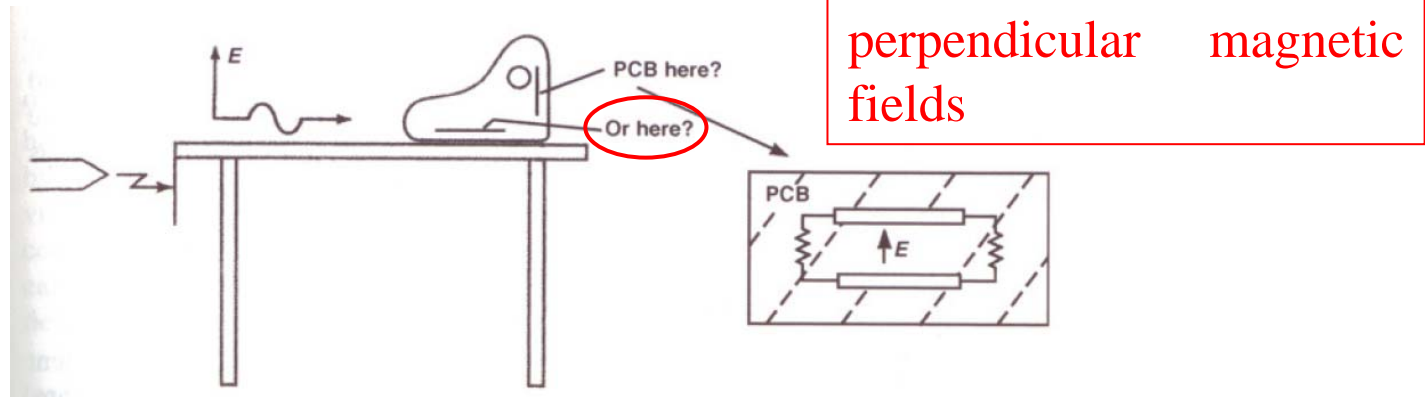
- Effect of Polarization of Incident Wave
  - Various Incident Waves



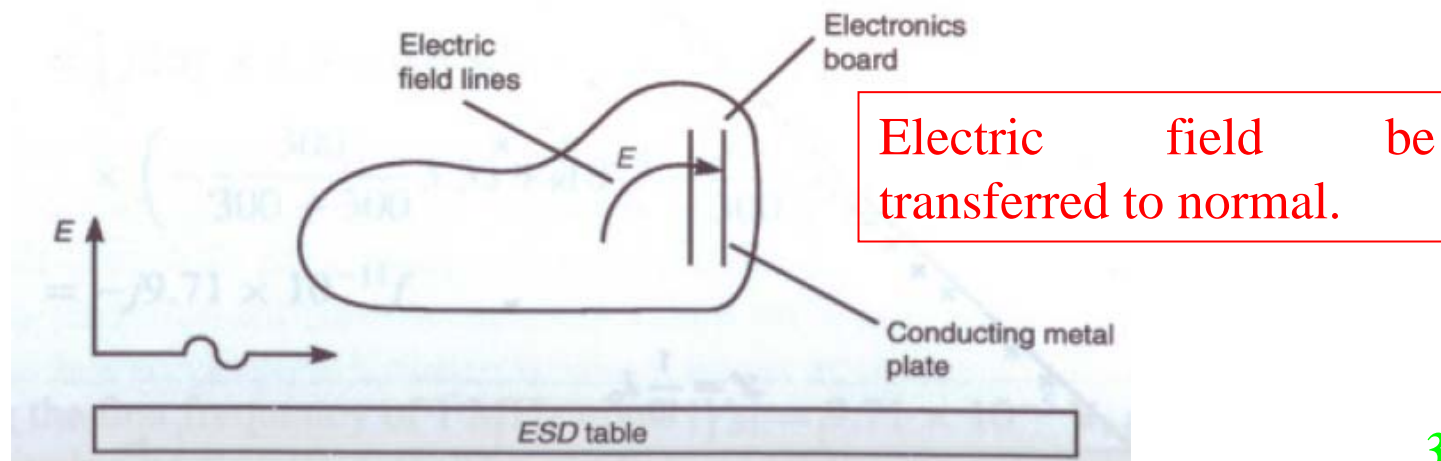
# Simple Susceptibility Models for Wires and PCB Lands

- Two Ways to Preventing from ESD

- PCB Placed Horizontal

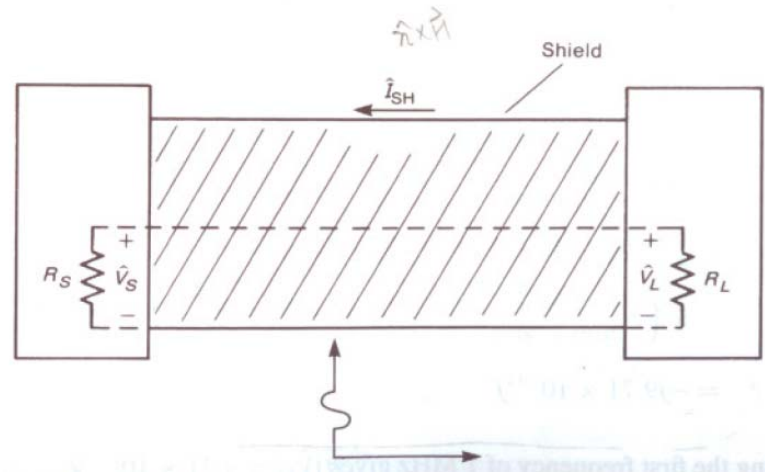


- Placing Another PEC at the Back of PCB



# Simple Susceptibility Models for Wires and PCB Lands

- Shielded Cables and Surface Transfer Impedance
  - Incident Field Pickup for a Shielded Cable



- The shield current **diffuses through** the shield wall to give a **voltage drop** on the **interior surface of the shield** of

$$d\hat{V} = \hat{Z}_T \hat{I}_{SH} dx$$

# Simple Susceptibility Models for Wires and PCB Lands

- Shielded Cables and Surface Transfer Impedance

- Incident Field Pickup for a Shielded Cable

- where the surface transfer impedance of the shield is given by

$$\hat{Z}_T = \frac{1}{\sigma 2\pi r_{sh} t_{sh}} \frac{\gamma t_{sh}}{\sinh \gamma t_{sh}} \quad (\text{in } \Omega/\text{m})$$

$$\gamma = \frac{1 + j1}{\delta}$$

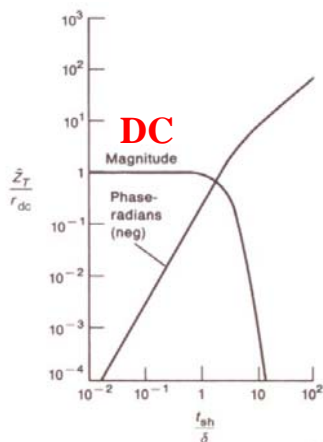
$$\delta = 1/\sqrt{\pi f \mu_0 \sigma}$$

$r_{dc}$

- and the per-unit-length dc resistance of the shield is

$$r_{dc} = \frac{1}{\sigma 2\pi r_{sh} t_{sh}} \quad (\text{in } \Omega/\text{m}) \quad \text{for } t_{sh} \ll \delta$$

- The shield inner radius is denoted by  $r_{sh}$  and the shield thickness is by  $t_{sh}$ .





# Simple Susceptibility Models for Wires and PCB Lands

- Shielded Cables and Surface Transfer Impedance

- Incident Field Pickup for a Shielded Cable

- For braided shields, this becomes

$$\hat{Z}_T = \frac{1}{\sigma \pi r_{bw}^2 BW \cos \theta_w} \frac{\gamma 2 r_{bw}}{\sinh(\gamma 2 r_{bw})} \quad (\text{in } \Omega/\text{m})$$

- and the per-unit-length **dc resistance of the shield** is

$$r_{dc} = \frac{r_b}{BW \cos \theta_w} \quad (\text{in } \Omega/\text{m})$$

- and the per-unit-length **dc resistance of the braid wires** is

$$r_b = \frac{1}{\sigma \pi r_{bw}^2} \quad (\text{in } \Omega/\text{m}) \quad \text{for } r_{bw} \ll \delta$$

$r_{bw}$ : radius of the wire

- where  $B$  is the **number of belts** in the shield braid,  $W$  is the **number of the braid wires** per belt,  $\theta_w$  is the **weave angle of these belts**.

# Simple Susceptibility Models for Wires and PCB Lands

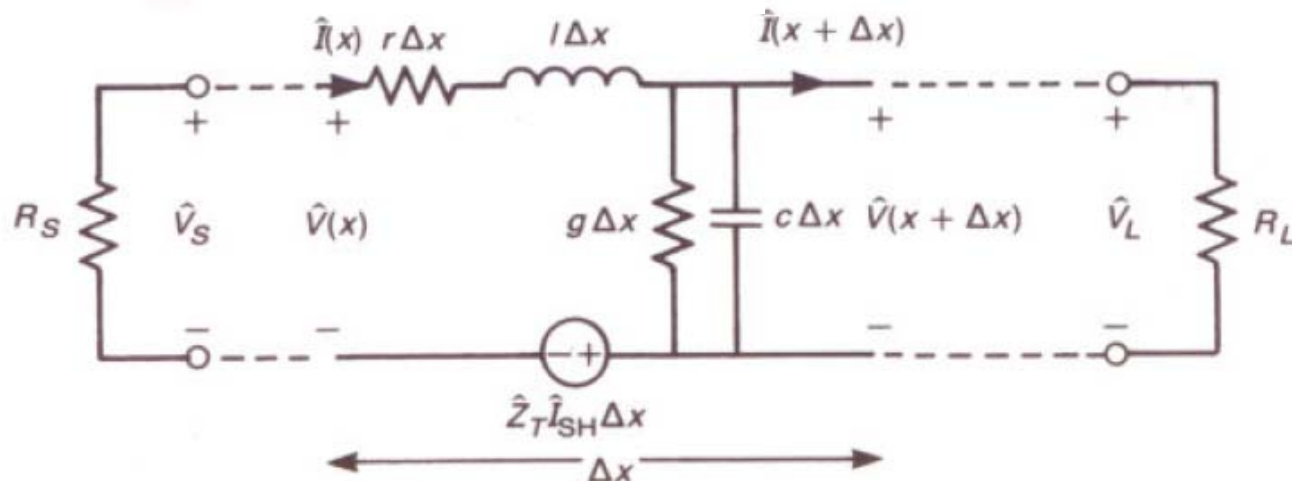
- Shielded Cables and Surface Transfer Impedance

- Equivalent Circuit

- The transmission lines equations are

$$\frac{d\hat{V}(x)}{dx} + (r + j\omega l)\hat{I}(x) = -\hat{Z}_T\hat{I}_{SH}$$

$$\frac{d\hat{I}(x)}{dx} + (g + j\omega c)\hat{V}(x) = 0$$



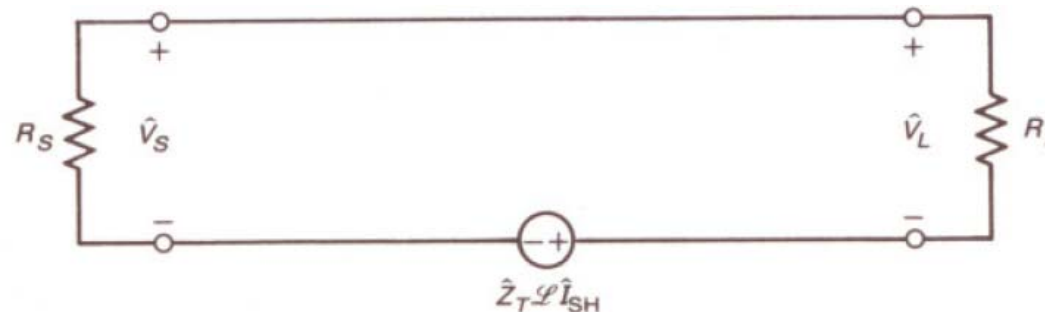
# Simple Susceptibility Models for Wires and PCB Lands

- Shielded Cables and Surface Transfer Impedance

- Equivalent Circuit

- Assuming the line is electrically short and neglecting the inductance and capacitance, the equivalent circuit simplifies to

$$\hat{V}_S = \frac{R_S}{R_S + R_L} \hat{Z}_T \hat{I}_{SH} \mathcal{L}$$
$$\hat{V}_L = -\frac{R_L}{R_S + R_L} \hat{Z}_T \hat{I}_{SH} \mathcal{L}$$



# Simple Susceptibility Models for Wires and PCB Lands

- Shielded Cables and Surface Transfer Impedance

- Equivalent Circuit – Braided Shield

- For magnetic field penetrating the holes in the braided shields, the surface transfer impedance becomes

$$\hat{Z}_T = \frac{1}{\sigma \pi r_{bw}^2 B W \cos \theta_w} \frac{\gamma 2 r_{bw}}{\sinh(\gamma 2 r_{bw})} + j\omega m_{12} \quad (\text{in } \Omega/\text{m})$$

- For electric field penetrating the holes in the braided shields, a current source  $Y_T V_{SG}$  must be added in parallel in the equivalent circuit.

$$\hat{Y}_T = \frac{1}{\hat{V}_{SG}} \frac{d\hat{I}}{dx}$$

$V_{SG}$ : voltage between the shield and the ground plane