Heron's Formula

Area of a Triangle

Triangle

The plane closed figure, with three sides and three angles is called as a triangle.

Types of triangles:
Based on **sides** - a) Equilateral b) Isosceles c) Scalene
Based on **angles** - a) Acute angled triangle b) Right- angled triangle c) Obtuse angled triangle

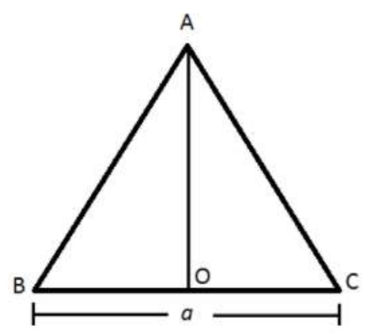
Area of a triangle

$$Area = \frac{1}{2} \times base \times height$$

In case of equilateral and isosceles triangles, if the length of the sides of triangles are given then,
we use Pythagoras theorem in order to find the height of a triangle.

Area of an equilateral triangle

Consider an equilateral ΔABC, with each side as a units. Let AO be perpendicular bisector of BC. In order to derive the formula for the area of equilateral triangle, we need to find height AO.



Equilateral triangle ABC

Using Pythagoras theorem,
$AC^2 = OA^2 + OC^2$
$OA^2 = AC^2 - OC^2$
Substitute $AC = a, OC = \frac{a}{2}$ to find OA

$OA^2 = a^2 - \frac{a^2}{4}$

$OA = \frac{\sqrt{3}a}{2}$

We know the area of triangle is

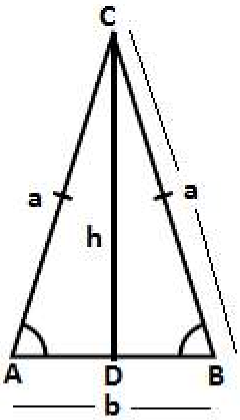
$A = \frac{1}{2} \times base \times height,$

$A = \frac{1}{2} \times a \times \frac{\sqrt{3}a}{2}$

$$\therefore Area\ of\ Equilateral\ triangle = \frac{\sqrt{3}a^2}{4}$$

Area of an isosceles triangle

Consider an isosceles ΔABC with equal sides as a units and base as b unit.



Isosceles triangle ABC

The height of the triangle can be found by Pythagoras' Theorem :

$CD^2 = AC^2 - AD^2$

$\Rightarrow h = a^2 - \frac{b^2}{4} = \frac{4a^2 - b^2}{4}$

$\Rightarrow h = \frac{1}{2}\sqrt{4a^2 - b^2}$

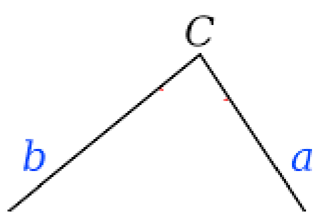
Area of triangle is $A = \frac{1}{2}bh$

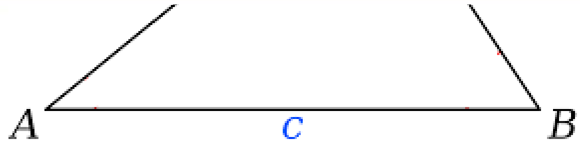
$\therefore A = \frac{1}{2} \times b \times \frac{1}{2}\sqrt{4a^2 - b^2}$

$$\therefore A = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

Area of a triangle - By Heron's formula

Area of a ΔABC, given sides a, b, c by **Heron's formula** (Also known as Hero's Formula) :





Triangle ABC

Find semi perimeter (s) = $\frac{a+b+c}{2}$

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

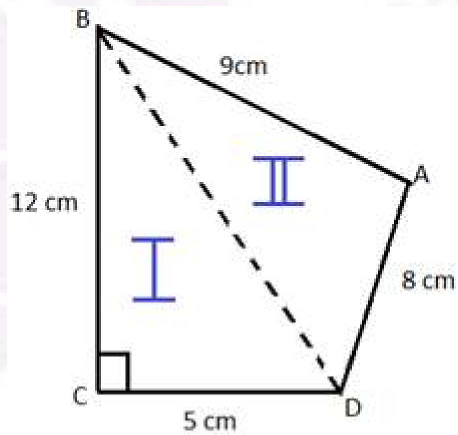
This formula is helpful to find area of a scalene triangle, given the lengths of all its sides.

Area of any polygon - By Heron's formula

Area of a quadrilateral whose sides and one diagonal are given, can be calculated by **dividing the quadrilateral into two triangles** and using the **Heron’s formula.**

Example :A park, in the shape of a quadrilateral ABCD, has $\angle C = 90°$, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?

⇒We draw the figure according to the information given.



Quadrilateral ABCD

The figure can be split into 2 triangles $\Delta BCD \ and \ \Delta ABD$
From ΔBCD, we can find BD (Using Pythagoras’ Theorem)
$BD^2 = 12^2 + 5^2 = 169$
$BD = 13cm$
Semi-perimeter for $\Delta BCD \ S_1 = \frac{12+5+13}{2} = 15$
Semi-perimeter $\Delta ABD \ S_2 = \frac{9+8+13}{2} = 15$
Using Heron's formula we find $A_1 \ and \ A_2$
$A_1 = \sqrt{15(15-12)(15-5)(15-13)} = \sqrt{15 \times 3 \times 10 \times 2}$

$A_1 = \sqrt{900} = 30cm^2$

Similarly we find A_2 to be $35.49cm^2$.

The area of the quadrilateral $ABCD = A_1 + A_2 = 65.49\ cm^2$