CS230: Deep Learning

Spring Quarter 2018
Stanford University

Midterm Examination

180 minutes

	Problem	Full Points	Your Score
1	Multiple Choice	14	
2	Short Answers	28	
3	Loss functions	17	
4	Batch Norm	15	
5	Numpy coding	10	
6	”Who let the cars in?”	4	
7	X-Network	23	
8	Practical case Study	11	
Total		122	

The exam contains 34 pages including this cover page.

- This exam is **closed book i.e. no laptops, notes, textbooks, etc. during the exam**. However, you may use one A4 sheet (front and back) of notes as reference.
- In all cases, and especially if you're stuck or unsure of your answers, **explain your work, including showing your calculations and derivations!** We'll give partial credit for good explanations of what you were trying to do.

Name: ____

SUNETID: ________________________________@stanford.edu

The Stanford University Honor Code:
I attest that I have not given or received aid in this examination, and that I have done my share and taken an active part in seeing to it that others as well as myself uphold the spirit and letter of the Honor Code.

Signature: ________________________________

Question 1 (Multiple Choice Questions, 14 points)

For each of the following questions, circle the letter of your choice. There is only ONE correct choice unless explicitly mentioned. No explanation is required.

(a) **(2 points)** Which of the following is true about max-pooling?

(i) It allows a neuron in a network to have information about features in a larger part of the image, compared to a neuron at the same depth in a network without max pooling.

(ii) It increases the number of parameters when compared to a similar network without max pooling.

(iii) It increases the sensitivity of the network towards the position of features within an image.

Solution: (i)

(b) **(2 points)** In order to backpropagate through a max-pool layer, you need to pass information about the positions of the max values from the forward pass.

(i) True

(ii) False

Solution: (i)

(c) **(2 points)** Consider a simple convolutional neural network with one convolutional layer. Which of the following statements is true about this network? (Check all that apply.)

(i) It is scale invariant.

(ii) It is rotation invariant.

(iii) It is translation invariant.

(iv) All of the above.

Solution: (iii)

(d) **(2 points)**You are training a Generative Adversarial Network to generate nice iguana images, with mini-batch gradient descent. The generator cost $\mathcal{J}^{(G)}$ is extremely low, but the generator is not generating meaningful output images. What could be the reason? (Circle all that apply.)

(i) The discriminator has poor performance.

(ii) Your generator is overfitting.

(iii) Your optimizer is stuck in a saddle point.

(iv) None of the above.

Solution: (i)

(e) **(2 points)** Mini-batch gradient descent is a better optimizer than full-batch gradient descent to avoid getting stuck in saddle points.

(i) True

(ii) False

Solution: (i)

(f) **(2 points)** Using ”neural style transfer” (as seen in class), you want to generate an RGB image of the Great Wall of China that looks like it was painted by Picasso. The size of your image is 100x100x3 and you are using a pretrained network with 1,000,000 parameters. At every iteration of gradient descent, how many updates do you perform?

(i) 10,000

(ii) 30,000

(iii) 1,000,000

(iv) 1,030,000

Solution: (ii) You only update the image, i.e. 10,000 pixels and each pixel has 3 channels.

(g) **(2 points)** You are building a model to predict the presence (labeled 1) or absence (labeled 0) of a tumor in a brain scan. The goal is to ultimately deploy the model to help doctors in hospitals. Which of these two metrics would you choose to use?

(i) Precision $= \dfrac{\text{True positive examples}}{\text{Total predicted positive examples}}$

(ii) Recall $= \dfrac{\text{True positive examples}}{\text{Total positive examples}}$.

Solution: (ii) Increase recall, because we don't want false negatives.

Question 2 (Short Answers, 28 points)

The questions in this section can be answered in 3-4 sentences. Please be short and concise in your responses.

(a) Consider an input image of shape $500 \times 500 \times 3$. You flatten this image and use a fully connected layer with 100 hidden units.

(i) **(1 point)** What is the shape of the weight matrix of this layer?

Solution: Weight matrix which is $750,000 \times 100$, giving 75 million.

(ii) **(1 point)** What is the shape of the corresponding bias vector?

Solution: 100×1

(b) **(2 points)** Consider an input image of shape $500 \times 500 \times 3$. You run this image in a convolutional layer with 10 filters, of kernel size 5×5. How many parameters does this layer have?

Solution: $5 \times 5 \times 3 \times 10$ and a bias value for each of the 10 filters, giving 760 parameters.

(c) **(2 points)** You forward propagate an input x in your neural network. The output probability is $\hat{y}$. Explain briefly what $\frac{\partial \hat{y}}{\partial x}$ is.

Solution: The derivative represents how much the output changes when the input is changed. In other words, how much the input has influenced the output.

(d) **(2 points)** Why is it necessary to include non-linearities in a neural network?

Solution: Without nonlinear activation functions, each layer simply performs a linear mapping of the input to the output of the layer. Because linear functions are closed under composition, this is equivalent to having a single (linear) layer. Thus, no matter how many such layers exist, the network can only learn linear functions.

(e) **(2 points)** The universal approximation theorem states that a neural network with a single hidden layer can approximate any continuous function (with some assumptions on the activation). Give one reason why you would use deep networks with multiple layers.

Solution: While a neural network with a single hidden layer can represent any continuous function, the size of the hidden layer required to do so is prohibitively large for most problems. Also, having multiple layers allows the network to represent highly nonlinear (e.g. more than what a single sigmoid can represent) with fewer number of parameters than a shallow network can.

(f) **(3 points)** Consider a dataset $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$ where each example $x^{(i)}$ contains a sequence of 100 numbers:

$$x^{(i)} = (x^{(i)<1>}, x^{(i)<2>}, \dots, x^{(i)<100>})$$

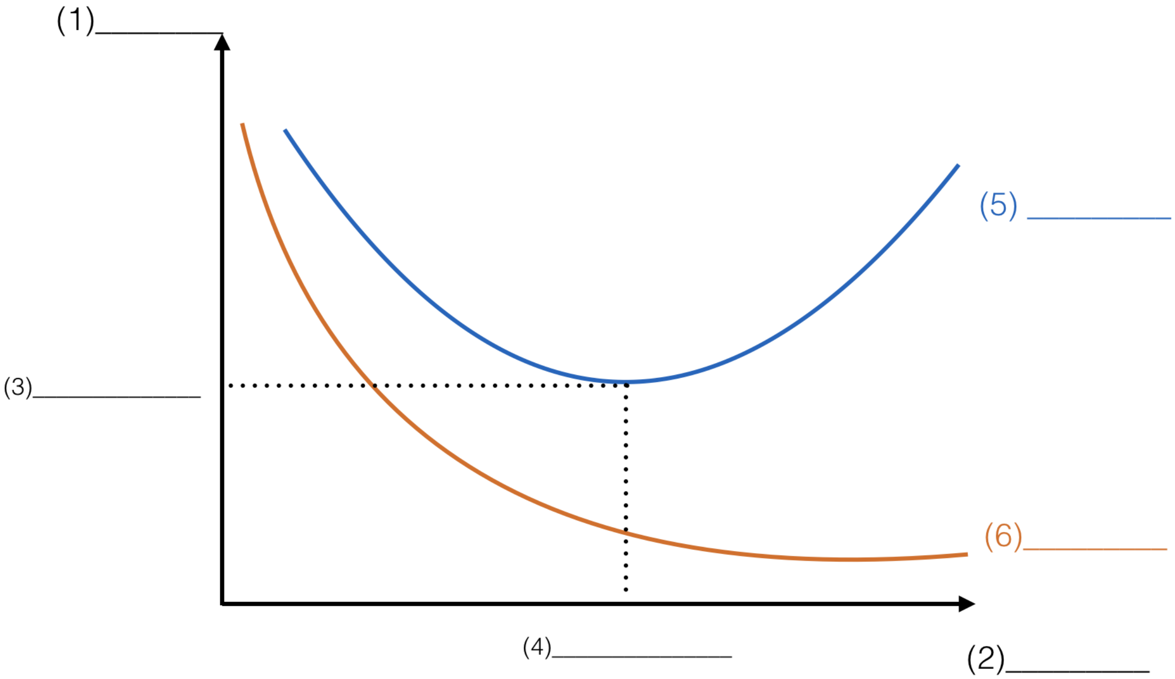
You have a very accurate (but not perfect) model that predicts $x^{<t+1>}$ from $(x^{<1>}, \dots, x^{<t>})$. Given $x^{<1>}$, you want to generate $(x^{<2>}, \dots, x^{<100>})$ by repeatedly running your model. Your method is:

1. Predict $\hat{x}^{<2>}$ from $x^{<1>}$
2. Predict $\hat{x}^{<3>}$ from $(x^{<1>}, \hat{x}^{<2>})$
3. ...
4. Predict $\hat{x}^{<100>}$ from $(x^{<1>}, \hat{x}^{<2>}, ..., \hat{x}^{<99>})$

The resulting $\hat{x}^{<100>}$ turns out to be very different from the true $x^{<100>}$. Explain why.

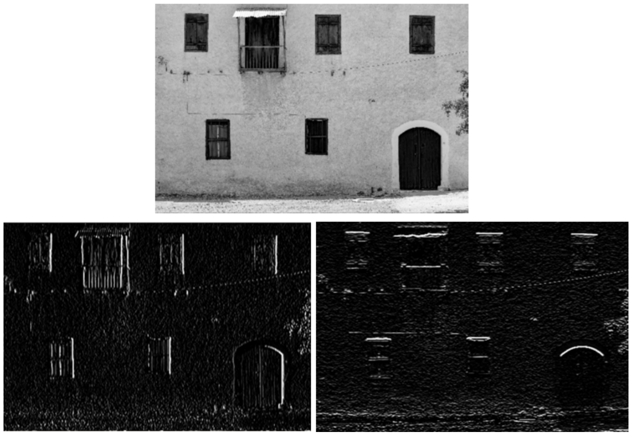
Solution: (Training/Test mismatch on sequence prediction)
There is a mismatch between training data and test data. During training, the network took in $\langle x_1, \dots, x_t \rangle$ as input to predict x_{t+1}. In test time, however, most of the input $(x_2, \dots, x_t)$ are what the model generated. And because the model is not perfect, the input distribution changes from the real distribution. And this drift from the training data distribution becomes worse because the error compounds over 100 steps.

(g) **(3 points)** You want to use the figure below to explain the concept of early stopping to a friend. Fill-in the blanks. (1) and (2) describe the axes. (3) and (4) describe values on the vertical and horizontal axis. (5) and (6) describe the curves. Be precise.



Solution:
(1) Loss value (or error)
(2) Number of iterations
(3) Best validation loss value (or best validation error)
(4) Optimal stopping point (or early stopping point)
(5) Validation loss (or validation error)
(6) Training loss (or training error)

(h) **(2 point)** Look at the grayscale image at the top of the collection of images below. Deduce what type of convolutional filter was used to get each of the lower images. Explain briefly and include the values of these filters. The filters have a shape of (3,3).



Solution: Left image: Vertical edge detector. Filter: [[1,0,-1][1,0,-1][1,0,-1]] Right image: Horizontal edge detector. Filter: [[1,1,1][0,0,0][-1,-1,-1]]

(i) **(1 point)**When the input is 2-dimensional, you can plot the decision boundary of your neural network and clearly see if there is overfitting.

How do you check overfitting if the input is 10-dimensional?

Solution: Compute cost function in the dev and training set. If there is a significant difference, then you have a variance problem.

(j) **(2 points)**What is the advantage of using Inverted Dropout compared to its older version ("Normal" dropout)? Mention the difference of implementation.

Solution: Implementation difference: Add line of code, $a^{[L]} / = keep_{prob}$ The expected value of the activation doesn't decrease with this extra line of code, so the activations do not need to be rescaled at test time.

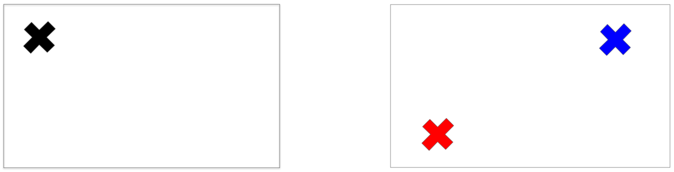
(k) **(1 point)** Give a method to fight exploding gradient in fully-connected neural networks.

Solution: Gradient clipping.

(l) **(2 points)** You are using an Adam optimizer. Show why the bias correction naturally disappears when the numbers of steps to compute the exponential moving averages gets large.

Solution: $\frac{V_t}{1-\beta^t} \to V_t$ when $t \to \infty$

(m) Consider the two images below:



(A) Image 1 (B) Image 2

Figure 1: (A) Image 1 is a binary image. Each pixel has value 1 (white) or 0 (black). There's only a single channel. (B) Image 2 is a color image. Each pixel has value 0 (absence of that color) or 1 (presence of that color), but there are 3 color channels (RGB). The cross on the left is red, and the cross on the right is blue.

You have access to an algorithm ($\star$) that can efficiently find the position of the maximum value in a matrix.

(i) **(2 points)** You would like to detect the 3 × 3 cross in Image 1. Hand-engineer a filter to be convolved over Image 1. Given the output of this "convolved" operation, the algorithm ($\star$) should give you the position of the cross.

Solution: $\begin{pmatrix} -1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & -1 \end{pmatrix}$ bias is anything really

(ii) **(2 points)** You would like to detect the 3 × 3 red cross in Image 2 but not the blue cross. Hand-engineer a filter to be convolved over Image 2. Given the output of this "convolved" operation, the algorithm ($\star$) should give you the position of the red cross.

Solution: $\begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$ for red channel and previous part's answer everywhere else

Question 3 (Loss Functions, 17 points + 3 bonus points)

Equipped with cutting-edge Deep Learning knowledge, you are working with a biology lab. Specifically, you're asked to build a classifier that predicts the animal type from a given image into four ($n_y = 4$) classes: `dog, cat, iguana, mouse`. There's always exactly one animal per image. You decide to use cross-entropy loss to train your network. Recall that the cross-entropy (CE) loss for a single example is defined as follows:

$$\mathcal{L}_{CE}(\hat{y}, y) = -\sum_{i=1}^{n_y} y_i \log \hat{y}_i$$

where $\hat{y} = (\hat{y}_1, \ldots, \hat{y}_{n_y})^\top$ represents the predicted probability distribution over the classes and $y = (y_1, \ldots, y_{n_y})^\top$ is the ground truth vector, which is zero everywhere except for the correct class (e.g. $y = (1, 0, 0, 0)^\top$ for `dog`, and $y = (0, 0, 1, 0)^\top$ for `iguana`).

(a) **(2 points)** Suppose you're given an example image of an iguana. If the model correctly predicts the resulting probability distribution as $\hat{y} = (0.25, 0.25, 0.3, 0.2)^\top$, what is the value of the cross-entropy loss? You can give an answer in terms of logarithms.

Solution: $-\log 0.3$

(b) **(2 points)** After some training, the model now incorrectly predicts `mouse` with distribution $\langle 0.0, 0.0, 0.4, 0.6 \rangle$ for the same image. What is the new value of the cross-entropy loss for this example?

Solution: $-\log 0.4$

(c) **(2 points)** Suprisingly, the model achieves lower loss for a misprediction than for a correct prediction. Explain what implementation choices led to this phenomenon.

Solution: This is because our objective is to minimize CE-loss, rather than to directly maximize accuracy. While CE-loss is a reasonable proxy to accuracy, there is no guarantee that a lower CE loss will lead to higher accuracy.

(d) **(2 points)** Given your observation from question (c), you decide to train your neural network with the accuracy as the objective instead of the cross-entropy loss. Is this a good idea? Give one reason. Note that the accuracy of a model is defined as

$$\text{Accuracy} = \frac{\text{(Number of correctly-classified examples)}}{\text{(Total number of examples)}}$$

Solution: It's difficult to directly optimize the accuracy because

- it depends on the entire training data, making it impossible to use stochastic gradient descent.
- the classification accuracy of a neural network is not differentiable with respect to its parameters.

(e) **(3 points)** After tuning the model architecture, you find that *softmax* classifier works well. Specifically, the last layer of your network computes logits $z = (z_1, \ldots, z_{n_y})^\top$, which are then fed into the softmax activation. The model achieves 100% accuracy on the training data. However, you observe that the training loss doesn't quite reach zero. Show why the cross-entropy loss can never be zero if you are using a softmax activation.

Solution: Assume the correct class is c and let $\sum\limits_{i=1}^{n_y} e^{z_i}$ be the normalization term for softmax. Then $\hat{y} = \left\langle \frac{e^{z_1}}{\sum\limits_{i=1}^{n_y} e^{z_i}}, \ldots, \frac{e^{z_{n_y}}}{\sum\limits_{i=1}^{n_y} e^{z_i}} \right\rangle$.

Then the CE-loss reduces to the following term:

$$-\log \hat{y}_c = -\log \frac{e^{z_c}}{\sum\limits_{i=1}^{n_y} e^{z_i}} = \log \sum_{i=1}^{n_y} e^{z_i} - \log e^{z_c}$$

And because $\sum\limits_{i=1}^{n_y} e^{z_i} \neq e^{z_c}$, the two terms are never equal and the loss will never reach zero, although it will get very close to zero at the end of training.

(f) **(2 points)** The classifier you trained worked well for a while, but its performance suddenly dropped. It turns out that the biology lab started producing chimeras (creatures that consist of body parts of different animals) by combining different animals together. Now each image can have multiple classes associated with them; for example, it could be a picture of a dog with mouse whiskers, cat ears and an iguana tail! Propose a way to label new images, where each example can simultaneously belong to multiple classes.

Solution: Use multi-hot encoding, e.g. $(1, 0, 0, 1)$ would be dog and mouse.

(g) **(2 points)** The lab asks you to build a new classifier that will work on chimeras as well as normal animals. To avoid extra work, you decide to retrain a new model with the same architecture (softmax output activation with cross-entropy loss). Explain why this is problematic.

Solution: This is problematic because softmax activation with CE-loss unfairly penalizes examples with many labels. In the extreme case, if the example belongs to *all* classes and the model correctly predicts $\langle \frac{1}{n_y}, \ldots, \frac{1}{n_y} \rangle$, then the CE-loss becomes $-\sum_{i=1}^{n_y} \log \hat{y}_i = n_y \log n_y$, which will be far bigger than the loss for most single-class examples.

(h) **(2 points)** Propose a different activation function for the last layer and a loss function that are better suited for this multi-class labeling task.

Solution: We can formulate this as n_y independent logistic regression tasks, each trying to predict whether the example belongs to the corresponding class or not. Then the loss can simply be the average of n_y logistic losses over all classes.

(i) **(Bonus: 3 points extra credit)** Prove the following lower bound on the cross-entropy loss for an example with L correct classes:

$$\mathcal{L}_{CE}(\hat{y}, y) \geq L \log L$$

Assume you are still using softmax activation with cross-entropy loss with ground truth vector $y \in \{0, 1\}^{n_y}$ with L nonzero components.

Solution: Let S denote the set of classes the given example belongs to (note $|S| = L$. Then,

$$\begin{aligned}
\mathcal{L}_{CE}(\hat{y}, y) &= -\sum_{i \in S}^{n_y} \log \hat{y}_i \\
&= (-L) \sum_{i \in S}^{n_y} \frac{1}{L} \log \hat{y}_i \\
&\geq (-L) \log \left(\sum_{i \in S}^{n_y} \frac{1}{L} \hat{y}_i \right) && \text{(by Jensen's Inequality)} \\
&= (-L) \log \frac{1}{L} && \text{(softmax sums to 1)} \\
&= L \log L
\end{aligned}$$

Question 4 (Batch Norm, 15 points)

In this question, you will explore the importance of good balance between positive and negative labeled examples within a mini-batch, especially when your network includes batch normalization layers.

Recall the batch normalization layer takes values $z = (z^{(1)}, \ldots, z^{(m)})$ as input and computes $z_{\text{norm}} = (z_{\text{norm}}^{(1)}, \ldots, z_{\text{norm}}^{(m)})$ according to:

$$z_{\text{norm}}^{(i)} = \frac{z^{(i)} - \mu}{\sqrt{(\sigma^2) + \epsilon}} \quad \text{where} \quad \mu = \frac{1}{m}\sum_{i=1}^{m} z^{(i)} \quad (\sigma^2) = \frac{1}{m}\sum_{i=1}^{m}(z^{(i)} - \mu)^2$$

It then applies a second transformation to get $\widetilde{z} = (\widetilde{z}^{(1)}, \ldots, \widetilde{z}^{(m)})$ using learned parameters γ and β:

$$\widetilde{z}^{(i)} = \gamma z_{\text{norm}}^{(i)} + \beta$$

(a) **(1 point)** Explain the purpose of ϵ in the expression for $z_{\text{norm}}^{(i)}$.

Solution: It prevents division by 0 for features with variance 0.

You want to use a neural network with batch normalization layers to build a classifier that can distinguish a hot dog from not a hot dog. (For the rest of this question, assume $\epsilon = 0$.)

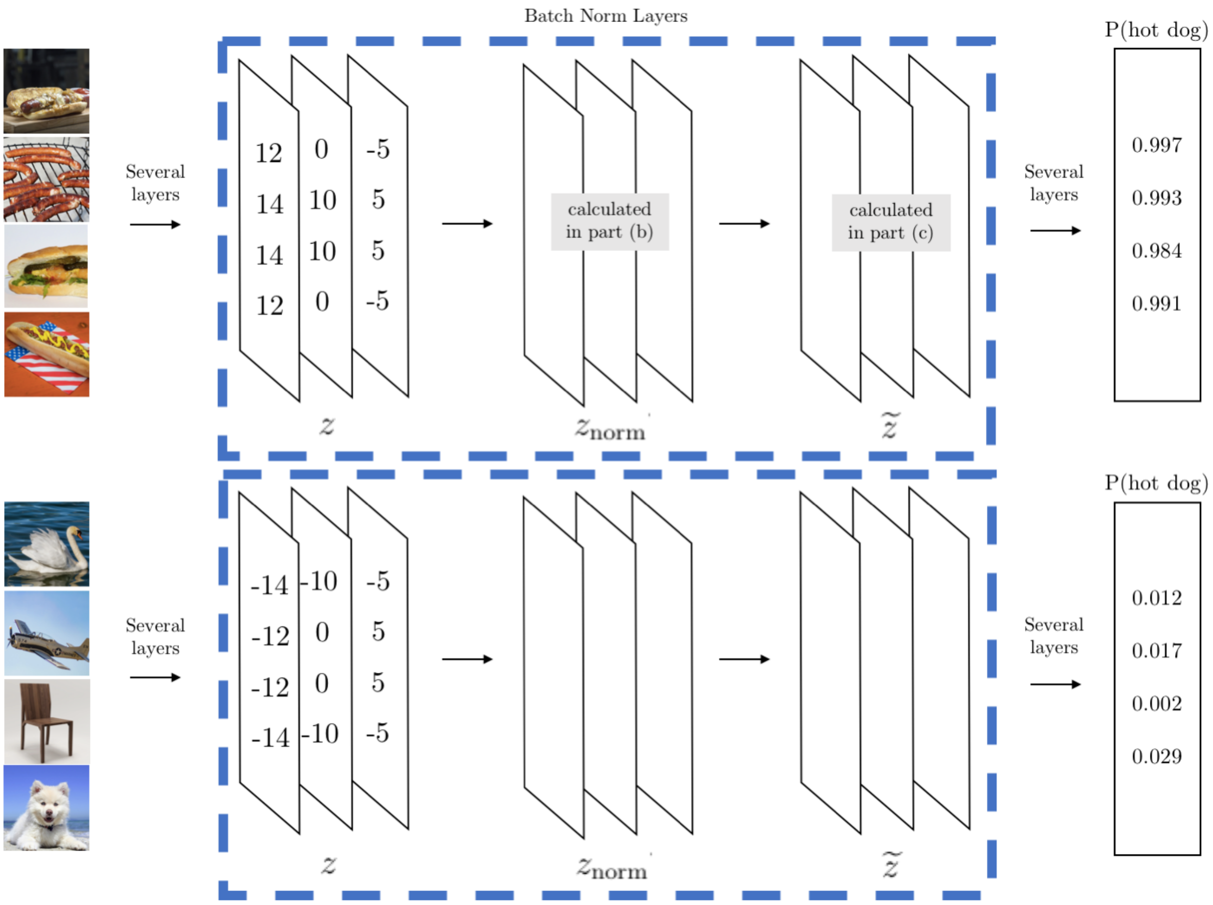


Figure 2: Hot Dog Network

(b) **(2 points)** You forward propagate a batch of m examples in your network. The input $z = (z^{(1)}, \dots, z^{(m)})$ of the batch normalization layer has shape $(n_h = 3, m = 4)$, where n_h represents the number of neurons in the pre-batchnorm layer:

$$\begin{pmatrix} 12 & 14 & 14 & 12 \\ 0 & 10 & 10 & 0 \\ -5 & 5 & 5 & -5 \end{pmatrix}$$

What is z_{norm}? Express your answer as a matrix with shape (3, 4).

Solution:

$$\begin{pmatrix} -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \end{pmatrix}$$

(c) **(2 points)** Suppose $\gamma = (1, 1, 1)$ and $\beta = (0, -10, 10)$. What is $\widetilde{z}$? Express your answer as a matrix with shape (3, 4).

Solution:

$$\begin{pmatrix} -1 & 1 & 1 & -1 \\ -11 & -9 & -9 & -11 \\ 9 & 11 & 11 & 9 \end{pmatrix}$$

(d) **(2 points)** Give 2 benefits of using a batch normalization layer.

Solution: We accept any of these: (i) accelerates learning by reducing covariate shift, decoupling dependence of layers, and/or allowing for higher learning rates/ deeper networks, (ii) accelerates learning by normalizing contours of output distribution to be more uniform across dimensions, (iii) Regularizes by using batch statistics as noisy estimates of the mean and variance for normalization (reducing likelihood of overfitting), (iv) mitigates poor weights initialization and/or variability in scale of weights, (v) mitigates vanishing/exploding gradient problems, (vi) constrains output of each layer to relevant regions of an activation function, and/or stabilizes optimization process, (vii) mitigates linear discrepancies between batches, (viii) improves expressiveness of the model by including additional learned parameters, γ and β, producing improved loss.

Partial credit was awarded to responses which cited multiple benefits from the same category.

Some responses that were intentionally not accepted: (i) symmetry-breaking, (ii) adds noise (without mentioning regularization), (iii) normalizes input distribution (without mentioning accelerated training), (iv) adds stability (without mentioning optimization process), (v) faster (without mentioning learning or training) (vi) reduces variance (without mentioning of what)

(e) **(2 points)** Explain what would go wrong if the batch normalization layer only applied the first transformation (z_{norm}).

Solution: Normalizing each input of a layer may change what the layer can represent. For instance, normalizing the inputs of a sigmoid would constrain them to the linear regime of the nonlinearity. $\widetilde{z}$ makes sure that the transformation inserted in the network can represent the identity transform.

Recall that during training time, the batch normalization layer uses the mini-batch statistics to estimate μ and (σ^2). However, at test time, it uses the moving averages of the mean and variance tracked (but not used) during training time.

(f) **(2 points)** Why is this approach preferred over using the mini-batch statistics during training *and* at test time?

Solution: There were two correct answers:

(1) Moving averages of the mean and variance produce a normalization that's more consistent with the transformation the network used to learn during training than the mini-batch statistics. You need to support variable batch sizes at test time, which includes small batch sizes (as small as a single example). The variability/noisiness between input images means batches with small batch sizes at test time will be less likely to have the same mini-batch statistics that produce the normalized activations trained on at training time. Using the moving averages of mean and variance as estimates of the population statistics addresses this issue.

To receive full credit, these responses included three components: (i) Mini-batches might be small at test time. (ii) Smaller mini-batches mean the mini-batch statistics are more likely to differ from the mini-batch statistics used at training. (iii) Moving averages are better estimates.

(2) Moving averages of the mean and variance produce a consistent normalization for an example, that's independent of the other examples in the batch.

To receive full credit, these responses included three components: (i) An single example might be part of different batches at test time. (ii) That example should receive the same prediction at test time, independent of the other examples in its batch. (iii) Mini-batch statistics vary per batch but moving averages do not.

Suppose you have the following dataset:

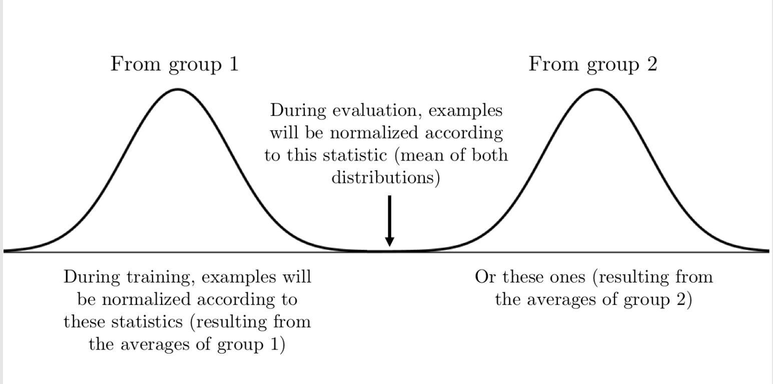
Description	Number of images	Example of an image
Photos of hot dogs	100k	
Photos of not hot dogs	100k	

You make the mistake of constructing each mini-batch entirely out of one of the two groups of data, instead of mixing both groups into each mini-batch. As a result, even though your training loss is low, when you start using the trained model at test time in your new mobile app, it performs very poorly.

(g) **(4 points)** Explain why inclusion of the batch normalization layer causes a discrepancy between training error and testing error when mini-batches are constructed with poor mixing. *Hint: Consider the example values given in Figure 2. What are the moving-averages over the two batches depicted and how do they compare to the batch statistics?*

Solution: First, consider the *distributions* for the mini-batch statistics μ and σ over the mini-batches used during training. If batches were constructed IID, both of the distributions would be normal centered on the population statistic (for which the moving averages are accurate estimates). However, since the batches were constructed as described, both the distributions would be compositions of two normal distributions (i.e. bimodal)—one per group of data.

During training, batches on average get normalized according to a statistic drawn from each of the distributions; however, at test time, batches get normalized according to the mean of both of the distributions, which never occurred during training. The following figure demonstrates this point in one dimension.


From group 1
From group 2
During evaluation, examples will be normalized according to this statistic (mean of both distributions)
During training, examples will be normalized according to these statistics (resulting from the averages of group 1)
Or these ones (resulting from the averages of group 2)

This effect is observable given the values in Figure 2. Consider the mini-batch means μ for the hot dogs $(13, 5, 0)$ and not hot dogs $(-13, -5, 0)$. During training, a hot dog will be normalized according to these (and similar) statistics to produce normalized values similar to the ones calculated in part (b). Now consider the moving means for the training set. Activations from hot dogs and not hot dogs are all grouped together into a single moving statistic $(0, 0, 0)$. At test time, normalizing with this statistic will produced normalized values very different from the ones calculated in part (b), which were unseen during training; therefore, the testing error is much higher than the training error.

The same argument applies to the mini-batch variances.

Rubric +1 for mentioning the moving averages are different from the mini-batch statistics. +1 for describing how the moving averages are different from the mini-batch statistics (as in the figure above or using the example numbers provided). +1 for describing this is caused by the way we constructed the mini-batches. +1 for describing how this causes a disparity in training/testing.

Question 5 (Numpy coding, 10 points)

In this question, you will implement a fully-connected network. The architecture is *LINEAR* → *RELU* → *DROPOUT* → *BATCHNORM*. This is a dummy architecture that has been made up for this exercise.

The code below implements the forward propagation, but some parts are missing. You will need to fill the parts between the tags (START CODE HERE) and (END CODE HERE) **based on the given instructions**. Note that we are using only numpy (not tensorflow), and the relu function has been imported for you.

```
import numpy as np
from utils import relu

def forward_propagation_with_dropout_and_batch_norm(X, parameters, keep_prob = 0.5):
    """
    Implements the forward propagation: LINEAR -> RELU -> DROPOUT -> BATCHNORM.

    Arguments:
    X -- input data of shape (n_x, m)
    parameters -- python dictionary containing your parameters:
         W -- weight matrix of shape (n_y, n_x)
         b -- bias vector of shape (n_y, 1)
         keep_prob -- probability of keeping a neuron active during drop-out, scalar
         gamma -- shifting parameter of the batch normalization layer, scalar
         beta -- scaling parameter of the batch normalization layer, scalar
         epsilon -- stability parameter of the batch normalization layer, scalar

    Returns:
    A2 -- output of the forward propagation
    cache -- tuple, information stored for computing the backward propagation
    """

    # retrieve parameters
    W = parameters["W"]
    b = parameters["b"]
    gamma = parameters["gamma"]
    beta = parameters["beta"]
    epsilon = parameters["epsilon"]

    ### START CODE HERE ###
    # LINEAR
    Z1 =

    # RELU
    A1 =
    # DROPOUT
    # Step 1: initialize matrix D1 = np.random.rand(..., ...)
    D1 =
    # Step 2: convert entries of D1 to 0 or 1 (using keep_prob as the threshold)
    D1 =
    # Step 3: shut down some neurons of A1
    A1 =
    # Step 4: scale the value of neurons that haven't been shut down
    A1 =

    # BATCHNORM
    # Step 1: compute the vector of means; you can use np.mean(..., axis=...)
    mu =
    # Step 2: compute the vector of variances; you can use np.var(..., axis=...)
    var =
    # Step 3: normalize the activations (don't forget about stability)
    A1_norm =
    # Step 4: scale and shift
    A2 =
    ### END CODE HERE ###

    cache = (Z1, D1, A1, W, b, A1_norm, mu, var, gamma, beta, epsilon)

    return A2, cache
```

Solution:

```
import numpy as np
from utils import relu

def forward_propagation_with_dropout_and_batch_norm(X, parameters, keep_prob = 0.5):
    """
    Implements the forward propagation: LINEAR -> RELU -> DROPOUT -> BATCHNORM.

    Arguments:
    X -- input data of shape (n_x, m)
    parameters -- python dictionary containing your parameters:
         W -- weight matrix of shape (n_y, n_x)
         b -- bias vector of shape (n_y, 1)
         keep_prob -- probability of keeping a neuron active during drop-out, scalar
         gamma -- shifting parameter of the batch normalization layer, scalar
         beta -- scaling parameter of the batch normalization layer, scalar
         epsilon -- stability parameter of the batch normalization layer, scalar

    Returns:
    A2 -- output of the forward propagation
    cache -- tuple, information stored for computing the backward propagation
    """

    # retrieve parameters
    W = parameters["W"]
    b = parameters["b"]
    keep_prob = parameters["W2"]
    gamma = parameters["gamma"]
    beta = parameters["beta"]
    epsilon = parameters["epsilon"]

    ### START CODE HERE ###
    # LINEAR
    Z1 = np.dot(W, X) + b

    # RELU
    A1 = relu(Z1)

    # DROPOUT
    # Step 1: initialize matrix D1 = np.random.rand(..., ...)
    D1 = np.random.rand(A1.shape[0], A1.shape[1])
    # Step 2: convert entries of D1 to 0 or 1 (using keep_prob as the threshold)
    D1 =  D1 <= keep_prob
    # Step 3: shut down some neurons of A1
    A1 =  D1 * A1
    # Step 4: scale the value of neurons that haven't been shut down
    A1 =  A1 / keep_prob

    # BATCHNORM
    # Step 1: compute the vector of means
    mu = np.mean(A1, axis=1)
    # Step 2: compute the vector of variances
    var = np.var(A1, axis=1)
    # Step 3: normalize the activations (don't forget about stability)
    A1_norm = (A1 - mu) / np.sqrt(var + epsilon)
    # Step 4: scale and shift
    A2 = gamma * A1_norm + beta
    ### END CODE HERE ###

    cache = (Z1, D1, A1, W, b, A1_norm, mu, var, gamma, beta, epsilon)
    return A2, cache
```

Question 6 ("Who let the cars in?", 4 points)

You have just moved into your new holiday mansion Mar-a-Yolo and would like to build a car detection model to use at your front gate. You have a database $\mathcal{D} = \{x_1, ..., x_N\}$ of the N cars allowed to enter your mansion. A training example for your network is a pair of images $(x^{(1)}, x^{(2)})$ where $x^{(1)}$ and $x^{(2)}$ are different images of the **same** car. You forward propagate $(x^{(1)}, x^{(2)})$ through your network to get encodings $(e^{(1)}, e^{(2)})$.

Using these encodings, you compute the loss (distance between encodings):

$$\mathcal{L} = ||e^{(1)} - e^{(2)}||^2 \tag{1}$$

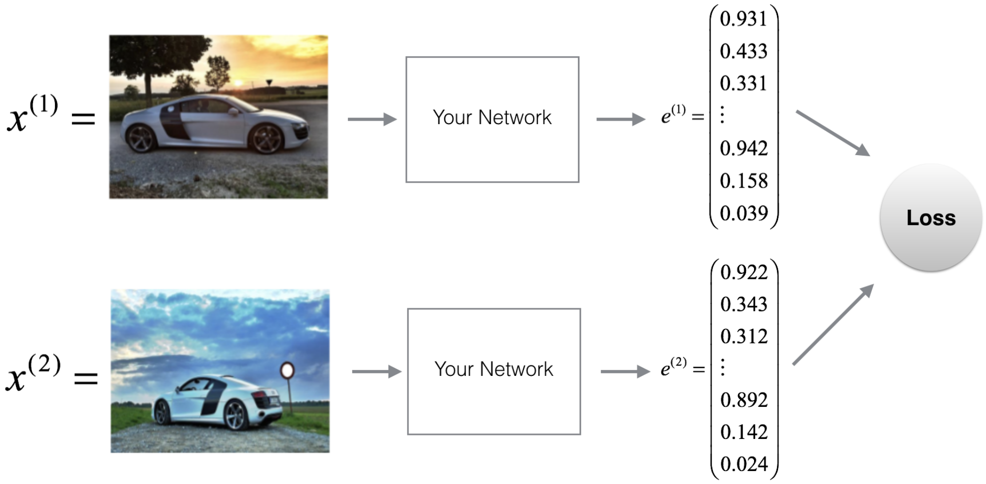


Figure 3: Car detection model

You've chosen your loss such that pictures of the same car have similar encodings. You hope that your network learned to output meaningful encodings to identify cars.

(a) **(2 points)** You pick a pair of (different) unseen images of the same car and compute the loss $\mathcal{L}$. The loss is extremely low, but your model performs poorly when deployed. What could be a possible problem with your approach?

> **Solution:** Using pairs causes all encodings to be clustered tightly in the embedding space. Cannot differentiate between different objects. Having the same encoding for all objects gives minimum MSE.

You decide to implement the triplet training scheme, as used in class for face recognition and retrain your network using this scheme.

(b) **(2 points)** Explain how you would you use this newly trained network to decide whether a car in front of your mansion should be allowed to enter.

Solution: Encode all examples in the database $\mathcal{D}$ using the newly trained network. Encode the image of the car waiting in front of the mansion. Use the Nearest Neighbors algorithm with a thresholding to decide if this car is in the database or not.

Question 7 (X-Network, 23 points)

An X-neuron, as opposed to a neuron, takes vectors as input, and outputs vectors. There are three stages in the forward propagation for an X-neuron.

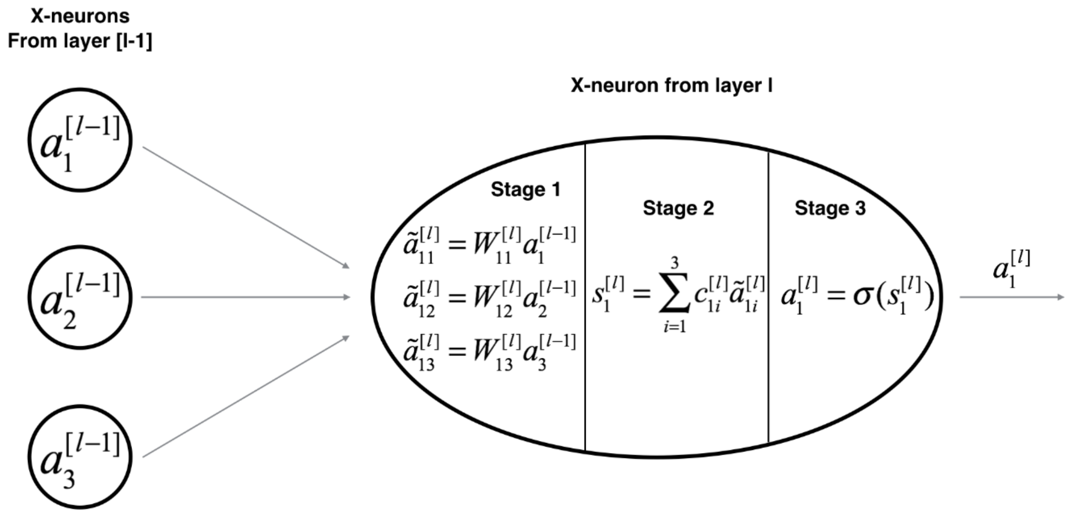

X-neurons
From layer [l-1]
$a_1^{[l-1]}$
$a_2^{[l-1]}$
$a_3^{[l-1]}$
X-neuron from layer l
Stage 1
$\tilde{a}_{11}^{[l]} = W_{11}^{[l]} a_1^{[l-1]}$
$\tilde{a}_{12}^{[l]} = W_{12}^{[l]} a_2^{[l-1]}$
$\tilde{a}_{13}^{[l]} = W_{13}^{[l]} a_3^{[l-1]}$
Stage 2
$s_1^{[l]} = \sum_{i=1}^{3} c_{1i}^{[l]} \tilde{a}_{1i}^{[l]}$
Stage 3
$a_1^{[l]} = \sigma(s_1^{[l]})$
$a_1^{[l]}$

Figure 4: $(a_1^{[l-1]}, a_2^{[l-1]}, a_3^{[l-1]})$ are vector outputs of the previous layer's X-neurons (assuming there are only 3 X-neurons in the previous layer). The forward propagation in an X-neuron is split into 3 stages. $a_1^{[l]}$ is the vector output of layer l's X-neuron.

Stage 1 is the linear transformation. For a given X-neuron j with inputs from X-neurons in the previous layer indexed with i:

$$\widetilde{a}_{ji}^{[l]} = W_{ji}^{[l]} a_i^{[l-1]}$$

Stage 2 is the linear combination of the outputs of stage 1.

$$s_j^{[l]} = \sum_{i=1}^{3} c_{ji}^{[l]} \widetilde{a}_{ji}^{[l]}$$

Stage 3 is the non-linear sigmoid activation of the X-neuron. It is applied element-wise to its input vector $s_j^{[l]}$.

$$a_j^{[l]} = \sigma(s_j^{[l]})$$

Note that with the above scheme, every X-neuron of a layer is connected to every X-neuron of the previous layer, just like in classic fully-connected networks. The difference is that each X-neuron outputs a vector.

Consider the following 3-layer X-network, made of X-neurons exclusively.

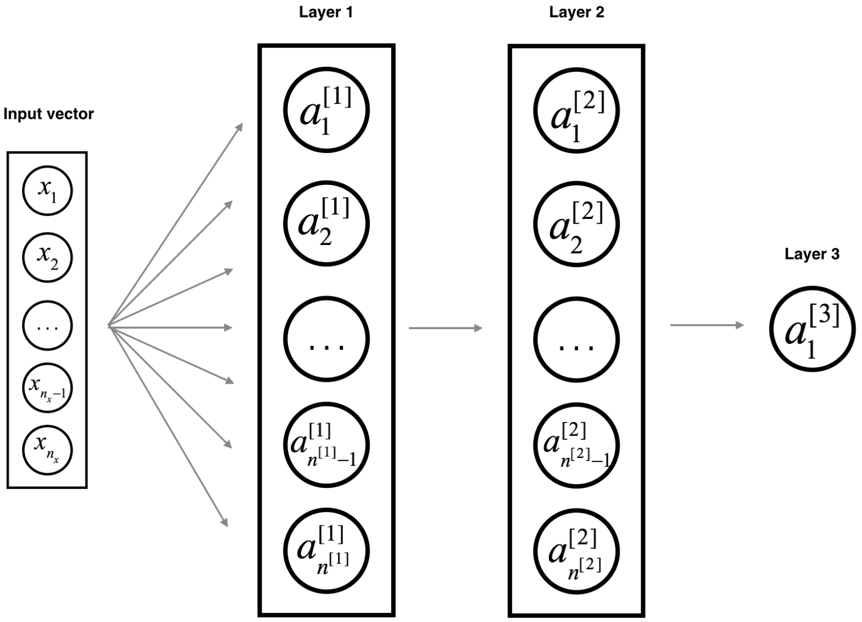


Figure 5: The input is a vector x of shape $(n_x, 1)$, which is duplicated and given to every X-neuron of the first layer. The X-network consists of three layers in which layer 1 has $n^{[1]}$ X-neurons, layer 2 has $n^{[2]}$ X-neurons, and layer 3 has a single X-neuron. Although for simplicity we did not draw all arrows, every layer is fully-connected to the next one.

(a) Given that all X-neurons in your network output a vector of shape $(D, 1)$:

(i) **(2 points)** What is the dimension of $W_{ji}^{[2]}$, for a single X-neuron in layer 2 of your X-network?

Solution: $D \times D$

(ii) **(2 points)** What is the dimension of $W_{ji}^{[1]}$, for a single X-neuron in layer 1 of your network?

Solution: $D \times n_x$

(b) **(2 points)** For the j^{th} X-neuron of the 2^{nd} layer, how many weights ($c_{ij}^{[l]}$) does it have in its stage 2?

Solution: $n^{[1]} or n^{[l-1]} both acceptable$

(c) Consider the Stage 2 computation of the j^{th} X-neuron in layer 2:

(i) **(2 points)** How would you vectorize this sum as a matrix product?

Solution: Stack all $\hat{u}_{j|i}$ in matrix that is $D \times n^{[1]}$ multiply with vector $c_i^{[2]}$ which is $n^{[1]} \times 1$

(ii) **(1 point)**What is the dimension of the output of stage 2 of the j^{th} X-neuron?

Solution: $D \times 1$.

(d) The loss function $\mathcal{L}$ is a scalar that has been calculated from the output $a_1^{[3]}$ from the final layer's X-neuron. You are given the derivative $\frac{\partial \mathcal{L}}{\partial a_1^{[2]}}$.

(i) **(2 points)** What is the dimension of this gradient?

Solution: Dimension (D, 1)

(ii) **(3 points)** Given this derivative, write the derivative $\frac{\partial \mathcal{L}}{\partial s_1^{[2]}}$ in terms of $\frac{\partial \mathcal{L}}{\partial a_1^{[2]}}$ and $a_1^{[2]}$?

Solution: $\frac{\partial \mathcal{L}}{\partial a_1^{[2]}} \bigodot a_1^{[2]} \bigodot (\mathbf{1} - a_1^{[2]})$

(e) **(3 points)** Using the derivative calculated in question (d), write down the gradient $\frac{\partial \mathcal{L}}{\partial W_{1i}^{[2]}}$.

Solution: $c_{1i} \left(\frac{\partial \mathcal{L}}{\partial a_1^{[2]}} \bigodot (1 - a_1^{[2]}) \bigodot a_1^{[2]} \right) a_i^{[1]T}$

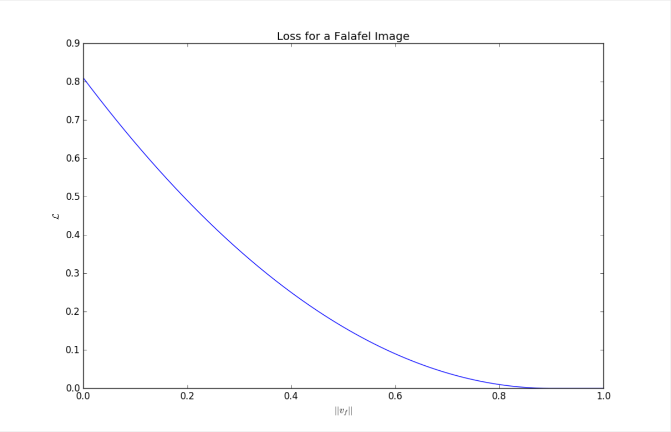
You decide to use your X-network to predict if an image contains a falafel or not. The falafel is a type of food, and this is a binary classification application.
Given an image, you run it through your X-network, and use the output activation $a_1^{[3]}$ to make a prediction. You do this by using the $L2$ norm of the output as a measure of probability of the presence of a falafel. You define the loss function for one training example as follows:

$$\mathcal{L} = y \max(0, m^+ - ||a_1^{[3]}||)^2 + \lambda(1-y) \max(0, ||a_1^{[3]}|| - m^-)^2$$

You have set $\lambda = 1$, $m^- = 0.1$ and $m^+ = 0.9$ (they are hyperparameters). Here, y is the label which tells you whether the image truly has a falafel ($y = 1$) or not ($y = 0$).

(f) **(2 point)** Draw the graph of $\mathcal{L}$ against $||a_1^{[3]}||$ assuming the picture contains a falafel.

Solution:



(g) Recall that your output activation is sigmoid, and the output of every X-neuron is of dimension D. Considering an image that is **not** a falafel:

(i) **(2 points)** What is the maximum value the loss can have?

Solution: $\left(\sqrt{D} - 0.1\right)^2$

(ii) **(2 points)** What is the minimum value of the loss and when is this minimum reached?

Solution: minimum is reached when $||a_1^{[3]}||$ is less than 0.1. Value is 0.

Question 8 (Practical case study: Online advertising, 11 points)

Learning and predicting user response plays a crucial role in online advertising.

(a) **(2 points)** Let's say you have a website with a lot of traffic. You would like to build a network that computes the probability of a user clicking on a given advertisement on your website. This is a supervised learning setting. What dataset do you need to train such a network?

Solution: Pairs $(\vec{x}, y)$ such that:

- $\vec{x}$: Features of the user + advertisement
- y: Binary label (0/1)

(b) **(2 points)** Choose an appropriate cost function for the problem and give the formula of the cost.

Solution: $-\sum_{i=1}^{m}(y_i log(\hat{y}_i) + (1 - y_i)log(1 - \hat{y}_i))$ Summation over all training examples.

(c) **(2 points)** Your website sells sport footwear and you have already collected a dataset of 1 million examples from past visits. Your friend, who works in the high fashion industry, offers to let you use their dataset as it has similar descriptors. However, you are concerned about the impact of this different distribution dataset in the performance of your predictive system. Explain how you would use your friend's dataset.

Solution: Training set. Reasons include:

- Neural networks are very data intensive. Therefore, opportunities to increase the dataset should not be missed.
- The new data in the training set could help the neural network to learn better lower level features (as you have more data)
- Using the new data in the dev/test set would change the goal/target of the optimization process. Thus, the optimized system would not perform well with real world data.

(d) **(1 point)** How would you assess the impact of the new dataset?

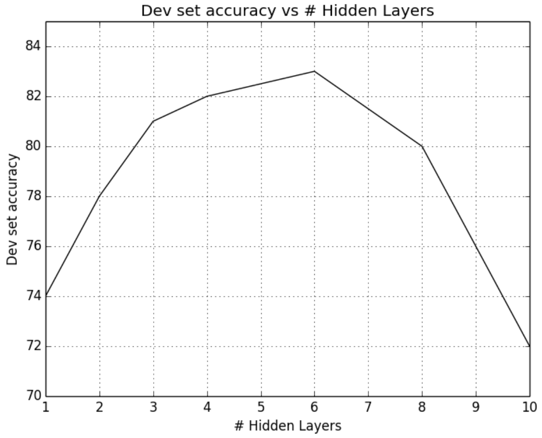
Solution: Split the training set in a new training set + train-dev set (made exclusively of old training examples). Measure the difference of accuracy between the new training set and train-dev set. If there is a significant difference, then the distribution mismatch between the old and new images is a problem.

(e) **(2 points)** Initially, you decide to build a fully connected neural network for the problem. This baseline model would have L hidden layers, one input layer and an output layer. The number of neurons in the l^{th} layer is $n^{[l]}$. Write down the number of parameters of this model as a function of L and $n^{[l]}$ for $l = 0...L$.

Note: The input layer is considered to be layer number 0

Solution: $\sum_{i=0}^{L}(n^{[l+1]} * n^{[l]} + n^{[l+1]}) = \sum_{i=0}^{L}(n^{[l+1]} * (n^{[l]} + 1))$

(f) **(2 points)** Based on the information provided in the graph below, what type of problem will you encounter as the number of hidden layer approaches 10? Mention possible solutions to this problem.



Solution: Overfitting problem. Possible solutions are:

- Regularization: Parameter norm penalties (L2/L1), Dropout.
- Reduce complexity of the neural network (decrease number of neurons and hidden layers)
- Increase the training dataset. Data Augmentation/GANs

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END OF PAPER