Homework #4 (Due 05/17 at the beginning of lecture)

1. (10%) Figure 4-1*a* shows, in cross section, two wires that are straight, parallel, and very long. The ratio i_1/i_2 of the current carried by wire 1 to that carried by wire 2 is 1/3. Wire 1 is fixed in place. Wire 2 can be moved along the positive side of the x axis so as to change the magnetic energy density u_B set up by the two currents at the origin. Figure 4-1*b* gives u_B as a function of the position x of wire 2. The curve has an asymptote of $u_B = 1.96$ nJ/m^3 as $x \to \infty$, and the horizontal axis scale is set by $x_s = 60.0$ cm. What is the value of (a) i_1 and (b) i_2?

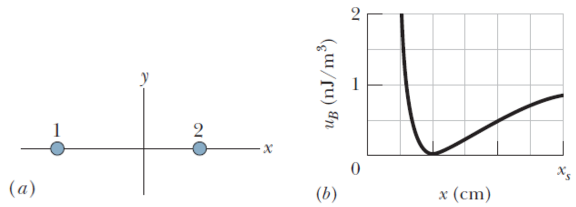

Figure 4-1

2. (10%) In Fig. 4-2, after switch S is closed at time $t = 0$, the emf of the source is automatically adjusted to maintain a constant current i through S. (a) Find the current through the inductor as a function of time. (b) At what time is the current through the resistor equal to the current through the inductor?

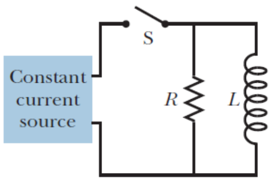

Figure 4-2

3. (10%) Two coils connected as shown in Fig. 4-3 separately have inductances L_1 and L_2. Their mutual inductance is M. (a) Show that this combination can be replaced by a single coil of equivalent inductance given by $L_{eq} = L_1 + L_2 + 2M$. (b) How could the coils in Fig. 4-3 be reconnected to yield an equivalent inductance of $L_{eq} = L_1 + L_2 - 2M$?

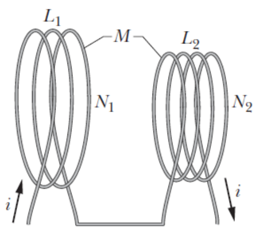

Figure 4-3

4. (10%) Figure 4-4 shows a rod of length $L = 10.0$ cm that is forced to move at constant speed $v = 5.00$ m/s along horizontal rails. The rod, rails, and connecting strip at the right form a conducting loop. The rod has resistance 0.400 Ω; the rest of the loop has negligible resistance. A current $i = 100$ A through the long straight wire at distance $a = 10.0$ mm from the loop sets up a (nonuniform) magnetic field through the loop. Find the (a) emf and (b) current induced in the loop. (c) At what

rate is thermal energy generated in the rod? (d) What is the magnitude of the force that must be applied to the rod to make it move at constant speed? (e) At what rate does this force do work on the rod?

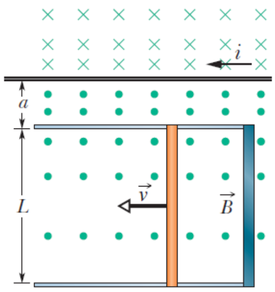

Figure 30-54 Problem 33.

Figure 4-4

5. (10%) Two long, parallel copper wires of diameter 4.0 mm carry currents of 7.0 A in opposite directions. (a) Assuming that their central axes are 20 mm apart, calculate the magnetic flux per meter of wire that exists in the space between those axes. (b) What percentage of this flux lies inside the wires? (c) Repeat part (a) for parallel currents.

6. (10%) An ac generator provides emf to a resistive load in a remote factory over a two-cable transmission line. At the factory a stepdown transformer reduces the voltage from its (rms) transmission value V_t to a much lower value that is safe and convenient for use in the factory. The transmission line resistance is 0.30 Ω/cable, and the power of the generator is 300 kW. If V_t = 80 kV, what are (a) the voltage decrease ΔV along the transmission line and (b) the rate P_d at which energy is dissipated in the line as thermal energy? If V_t = 8.0 kV, what are (c) ΔV and (d) P_d? If Vt = 0.80 kV, what are (e) ΔV and (f) P_d?

7. (10%) Figure 4-7shows an ac generator connected to a "black box" through a pair of terminals. The box contains an *RLC* circuit, possibly even a multiloop circuit, whose elements and connections we do not know. Measurements outside the box reveal that $\varepsilon(t) = (75.0\text{ V})\sin(\omega_d t)$ and $i(t) = (1.20\text{ A})\sin(\omega_d t + 42.0°)$. (a) What is the power factor? (b) Does the current lead or lag the emf? (c) Is the circuit in the box largely inductive or largely capacitive? (d) Is the circuit in the box in resonance? (e) Must there be a capacitor in the box? (f) An inductor? (g) A resistor? (h) At what average rate is energy delivered to the box by the generator? (i)Why don't you need to know ω_d to answer all these questions?

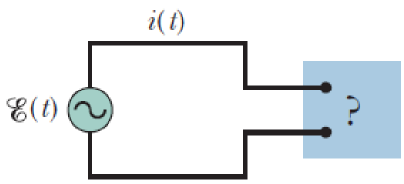

Figure 4-7

8. (10%) Figure 4-8 shows a driven *RLC* circuit that contains two identical capacitors and two switches. The emf amplitude is set at 12.0 V, and the driving frequency is set at 60.0 Hz. With both switches open, the current leads the emf by 30.9°. With switch S_1 closed and switch S_2 still open, the emf leads the current by 15.0°. With both switches closed, the current amplitude is 447 mA. What are (a) *R*, (b) *C*, and (c) *L*?

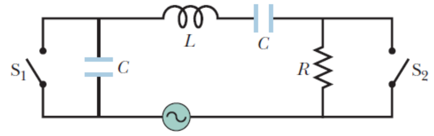

Figure 4-8

9. (10%) In Fig. 4-9, R = 14.0 Ω, C = 6.20 μF, and L = 54.0 mH, and the ideal battery has emf ε = 34.0 V. The switch is kept at position *a* for a long time and then thrown to position *b*. What are the (a) frequency and (b) current amplitude of the resulting oscillations?

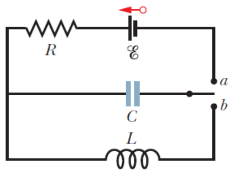

Figure 4-9

10. (10%) *Nonuniform displacement-current density.* Figure 4-10 shows a circular region of radius R = 4.00 cm in which a displacement current is directed out of the page. The magnitude of the density of this displacement current is $J_d = (4.00\ \text{A/m}^2)(1 - r/R)$, where r is the radial distance ($r \leq R$).What is the magnitude of the magnetic field due to the displacement current at (a) r = 2.00 cm and (b) r = 5.00 cm (c)At what radial distance is the field magnitude maximun?

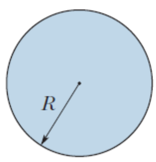

Figure 4-10

11. (10%) In Fig. 4-11, a capacitor with circular plates of radius R = 18.0 cm is connected to a source of emf $\varepsilon = \varepsilon_m \sin(\omega t)$, where ε_m = 220 V and ω = 130 rad/s. The maximum value of the displacement current is i_d = 7.60 μA. Neglect fringing of the electric field at the edges of the plates. (a) What is the maximum value of the current i in the circuit? (b) What is the maximum value of $d\Phi_E/dt$, where Φ_E is the electric flux through the region between the plates? (c) What is the separation d between the plates? (d) Find the maximum value of the magnitude of $\vec{B}$ between the plates at a distance r = 11.0 cm from the center.

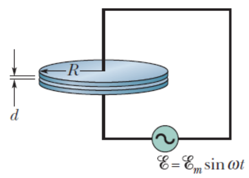

Figure 4-11

12. (10%) In Fig. 4-12, a ray is incident on one face of a triangular glass prism in air. The angle of incidence θ is chosen so that the emerging ray also makes the same angle θ with the normal to the other face. Show that the index of refraction n of the glass prism is given by $n = \frac{\sin\frac{1}{2}(\psi+\phi)}{\sin\frac{1}{2}\phi}$, where ϕ is the vertex angle of the prism and ψ is the *deviation angle,* the total angle through which the beam is turned in passing through the prism. (Under these conditions the deviation angle ψ has the smallest possible value, which is called the *angle of minimum deviation.*)

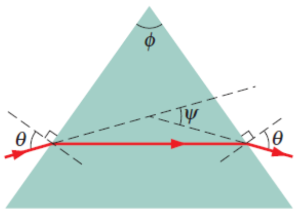

Figure 4-12

13. (10%) As a comet swings around the Sun, ice on the comet's surface vaporizes, releasing trapped dust particles and ions. The ions, because they are electrically charged, are forced by the electrically charged *solar wind* into a straight *ion tail* that points radially away from the Sun (Fig. 4-13). The (electrically neutral) dust particles are pushed radially outward from the Sun by the radiation force on them from sunlight. Assume that the dust particles are spherical, have density 3.5×10^3 kg/m^3, and are totally absorbing. (a) What radius must a particle have in order to follow a straight path, like path 2 in the figure? (b) If its radius is larger, does its path curve away from the Sun (like path 1) or toward the Sun (like path 3)?

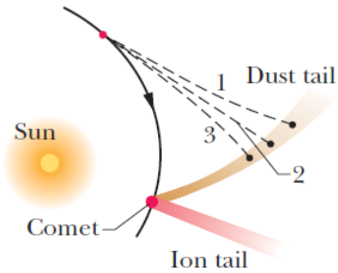

Figure 4-13

14. (10%) At a beach the light is generally partially polarized due to reflections off sand and water. At a particular beach on a particular day near sundown, the horizontal component of the electric field vector is 2.0 times the vertical component. A standing sunbather puts on polarizing sunglasses; the glasses eliminate the horizontal field component. (a) What fraction of the light intensity received before the glasses were put on now reaches the sunbather's eyes? (b) The sunbather, still wearing the glasses, lies on his side. What fraction of the light intensity received before the glasses were put on now reaches his eyes?

15. (10%) In Fig. 4-15, a light ray in air is incident on a flat layer of material 2 that has an index of refraction $n_2 = 1.5$. Beneath material 2 is material 3 with an index of refraction n_3. The ray is incident on the air–material 2 interface at the Brewster angle for that interface. The ray of light

refracted into material 3 happens to be incident on the material 2 – material 3 interface at the Brewster angle for that interface. What is the value of n_3?

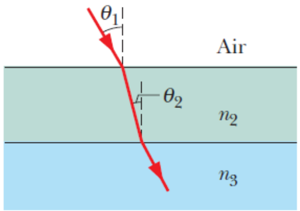

Figure 4-15

16. (20%) Assume an alternating current $I = I_0 \cos(\omega t)$ flows down a long straight wire and return back along a coaxial conducting cylinder of radius R. (a) In what direction does the induced electric field point (radial, circumferential, or longitudinal)? (b) Find the electric field E as a function of r (the distance from the axis). (c) What is the total displacement current I_d? Note that you can integrate the displacement current density to obtain the total current. (d) What is the ratio between I_d and I? (e) Assume the radius R = 1 mm, how large does the frequency need to be if one is going to observe the displacement current I_d to be more than 10% of I? Note that your answer may explain why the displacement current is difficult to be discovered by Faraday during the 1800s.