Homework #5 (Due 06/07 10:10 a.m.)

1. (10%) In Fig. 5-1, observer S detects two flashes of light. A big flash occurs at x_1 = 1200 m and, 5.00 μs later, a small flash occurs at x_2 = 700 m. As detected by observer S′, the two flashes occur at a single coordinate x′. (a) What is the speed parameter of S′, and (b) is S′ moving in the positive or negative direction of the x axis? To S′, (c) which flash occurs first and (d) what is the time interval between the flashes?

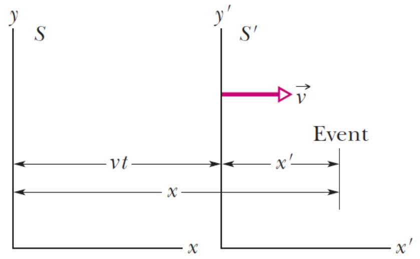

Figure 5-1

2. (10%) The center of our Milky Way galaxy is about 23000 ly away. (a) To eight significant figures, at what constant speed parameter would you need to travel exactly 23000 ly (measured in the Galaxy frame) in exactly 40 y (measured in your frame)? (b) Measured in your frame and in lightyears, what length of the Galaxy would pass by you during the trip?

3. (10%) An alpha particle with kinetic energy 7.70 MeV collides with an ^{14}N nucleus at rest, and the two transform into an ^{17}O nucleus and a proton. The proton is emitted at 90° to the direction of the incident alpha particle and has a kinetic energy of 4.44 MeV. The masses of the various particles are alpha particle, 4.00260 u; ^{14}N, 14.00307 u; proton, 1.007825 u; and ^{17}O, 16.99914 u. In MeV, what are (a) the kinetic energy of the oxygen nucleus and (b) the Q of the reaction? (*Hint*: The speeds of the particles are much less than c.)

4. (10%) Show that when a photon of energy E is scattered from a free electron at rest, the maximum kinetic energy of the recoiling electron is given by:

$$K_{max} = \frac{E^2}{E + mc^2/2}$$

5. (10%) An electron with total energy E = 5.1 eV approaches a barrier of height U_b = 6.8 eV and thickness L = 750 pm. What percentage change in the transmission coefficient T occurs for a 1.0% change in (a) the barrier height, (b) the barrier thickness, and (c) the kinetic energy of the incident electron?

6. (10%) In a photoelectric experiment using a sodium surface, you find a stopping potential of 1.85 V for a wavelength of 300 nm and a stopping potential of 0.820 V for a wavelength of 400 nm.

From these data find (a) a value for the Planck constant, (b) the work function Φ for sodium, and (c) the cutoff wavelength λ_0 for sodium.

7. (10%) What are (a) the energy of a photon corresponding to wavelength 1.00 nm, (b) the kinetic energy of an electron with de Broglie wavelength 1.00 nm, (c) the energy of a photon corresponding to wavelength 1.00 fm, and (d) the kinetic energy of an electron with de Broglie wavelength 1.00 fm?

8. (10%) An electron is in a certain energy state in a one-dimensional, infinite potential well from x = 0 to x = L = 180 pm. The electron's probability density is zero at x = 0.300L, and x = 0.400L; it is not zero at intermediate values of x. The electron then jumps to the next lower energy level by emitting light. What is the change in the electron's energy?

9. (10%) The wave function for the hydrogen-atom quantum state represented by the dot plot shown in Fig. 5-9, which has $n = 2$ and $l = m_l = 0$, is

$$\psi_{200}(r) = \frac{1}{4\sqrt{2\pi}} a^{-\frac{3}{2}} \left(2 - \frac{r}{a}\right) e^{-r/2a}$$

in which a is the Bohr radius and the subscript on $\psi(r)$ gives the values of the quantum numbers n, l, m_l. (a) Plot $\psi_{200}^2(r)$ and show that your plot is consistent with the dot plot of Fig. 5-9. (b) Show analytically that $\psi_{200}^2(r)$ has a maximum at r = 4a. (c) Find the radial probability density $P_{200}(r)$ for this state. (d) Show that

$$\int_0^\infty P_{200}(r)dr = 1$$

and thus that the expression above for the wave function $\psi_{200}(r)$ has been properly normalized.

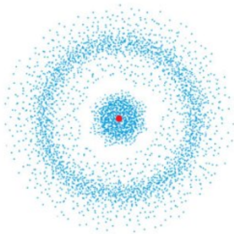

Figure 5-9

10. (10%) An electron (mass m) is contained in a rectangular corral of widths $L_x = L$ and $L_y = 2L$. (a) How many different frequencies of light could the electron emit or absorb if it makes a transition between a pair of the lowest five energy levels? What multiple of $h/8mL^2$ gives the (b) lowest, (c) second lowest, (d) third lowest, (e) highest, (f) second highest, and (g) third highest frequency?

11. (10%) If orbital angular momentum $\vec{L}$ is measured along, say, a z axis to obtain a value for L_z, show that

$$\left(L_x^2 + L_y^2\right)^{1/2} = [l(l+1) - m_l^2]^{1/2}\hbar$$

is the most that can be said about the other two components of the orbital angular momentum.

12. (10%) An electron is in a state with $n = 4$. What are (a) the number of possible values of l, (b) the number of possible values of m_l, (c) the number of possible values of m_s, (d) the number of states in the n = 4 shell, and (e) the number of subshells in the n = 3 shell?

13. (20%) A rocket that has a proper length of 700 m is moving to the right at a speed of $0.900c$. It has two clocks—one in the nose and one in the tail—that have been synchronized in the frame of the rocket. A clock on the ground and the clock in the nose of the rocket both read zero as they pass by each other. (a) At the instant the clock on the ground reads zero, what does the clock in the tail of the rocket read according to observers on the ground? When the clock in the tail of the rocket passes the clock on the ground, (b) what does the clock in the tail read according to observers on the ground, and (c) what does the clock in the nose read according to observers on the ground, and (d) what does the clock in the nose read according to observers on the rocket? (e) At the instant the clock in the nose of the rocket reads 1.00 h, a light signal is sent from the nose of the rocket to an observer standing by the clock on the ground. What does the clock on the ground read when the observer on the ground receives the signal? (f) When the observer on the ground receives the signal, he immediately sends a return signal to the nose of the rocket. What is the reading of the clock in the nose of the rocket when that signal is received at the nose of the rocket?

14. (20%) Assume a particle in the infinite square well has as its initial wave function an even superposition of the first two stationary states:

$$\Psi(x,0) = A[\psi_1(x) + \psi_2(x)]$$

(a) Normalize the wave function $\Psi(x,0)$, i.e., find A. (b) Find $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Note that in class we learnt $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$. Also, you can express the latter in terms of sinusoidal functions of time, eliminating the exponentials with the help of $e^{i\theta} = \cos\theta + i\sin\theta$. (c) If you measured the energy of this particle, what values might you get, and what is the probability of getting each of them?

Now if we add a phase into this superposition wave function, which becomes

$$\Psi(x,0) = A\left[\psi_1(x) + e^{i\phi}\psi_2(x)\right]$$

Find $\Psi(x,t)$ and $|\Psi(x,t)|^2$ for (d) $\phi = \pi/2$ and (e) $\phi = \pi$. This problem shows you an important concept that the relative phase of a superposition state is important.