Homework #2 (Due 03/31 at the beginning of lecture)

1. (10%) Figure 2-1 shows a thin plastic rod of length $L = 13.5$ cm and uniform charge 43.6 fC. (a) In terms of distance d, find an expression for the electric potential at point $P1$. (b) Next, substitute variable x for d and find an expression for the magnitude of the component Ex of the electric field at $P1$. (c) What is the direction of Ex relative to the positive direction of the x axis? (d) What is the value of Ex at $P1$ for $x = d = 6.20$ cm? (e) From the symmetry in Fig. 2-1, determine Ey at $P1$.
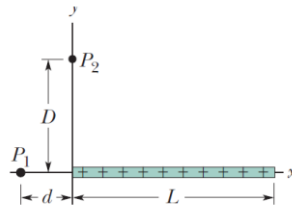

Figure 2-1

2. (10%) In the rectangle of Fig. 2-2, the sides have lengths 5.0 cm and 15 cm, $q_1 = -5.0$ μC, and $q_2 = +2.0$ μC. With $V = 0$ at infinity, what is the electric potential at (a) corner A and (b) corner B? (c) How much work is required to move a charge $q_3 = +3.0$ μC from B to A along a diagonal of the rectangle? (d) Does this work increase or decrease the electric potential energy of the three-charge system? Is more, less, or the same work required if q_3 is moved along a path that is (e) inside the rectangle but not on a diagonal and (f) outside the rectangle?

Figure 2-2

3. (10%)The smiling face of Fig. 2-3 consists of three items:

    I. a thin rod of charge -3.0 μC that forms a full circle of radius 6.0 cm;

    II. a second thin rod of charge 1.0 μC that forms a circular arc of radius 4.0 cm, subtending an angle of 90% about the center of the full circle;

    III. an electric dipole with a dipole moment that is perpendicular to a radial line and has a magnitude of 1.28×10^{-21} C · m.
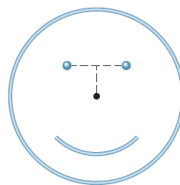

Figure 2-3

4. (10%) Two uniformly charged, infinite, nonconducting planes are parallel to a yz plane and positioned at $x = -50$ cm and $x = +50$ cm. The charge densities on the planes are -50 nC/m² and +25 nC/m², respectively. What is the magnitude of the potential difference between the origin and the point on the x axis at $x = +100$ cm? (*Hint:* Use Gauss' law.)

5. (10%) In Fig. 2-5a, we move an electron from an infinite distance to a point at distance $R = 8.00$ cm from a tiny charged ball. The move requires work $W = 5.32 \times 10^{-13}$ J by us. (a) What is the charge Q on the ball? In Fig. 2-5b, the ball has been sliced up and the slices spread out so that an equal amount of charge is at the hour positions on a circular clock face of radius $R = 8.00$ cm. Now the electron is brought from an infinite distance to the center of the circle. (b) With that addition of the electron to the system of 12 charged particles, what is the change in the electric potential energy of the system?
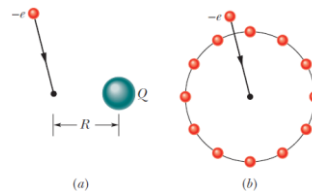

(a)   (b)

Figure 2-5

6. (10%) As a safety engineer, you must evaluate the practice of storing flammable conducting liquids in nonconducting containers. The company supplying a certain liquid has been using a squat, cylindrical plastic container of radius $r = 0.20$ m and filling it to height $h = 10$ cm, which is not the container's full interior height (Fig. 2-6). Your investigation reveals that during handling at the company, the exterior surface of the container commonly acquires a negative charge density of magnitude 2.0 $\mu C/m^2$ (approximately uniform). Because the liquid is a conducting material, the charge on the container induces charge separation within the liquid. (a) How much negative charge is induced in the center of the liquid's bulk? (b) Assume the capacitance of the central portion of the liquid relative to ground is 35 pF. What is the potential energy associated with the negative charge in that effective capacitor? (c) If a spark occurs between the ground and the central portion of the liquid (through the venting port), the potential energy can be fed into the spark. The minimum spark energy needed to ignite the liquid is 10 mJ. In this situation, can a spark ignite the liquid?
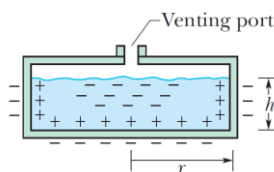

Figure 2-6

7. (10%) Figure 2-7 shows a parallel plate capacitor of plate area $A = 12.5$ cm^2 and plate separation $2d = 7.12$ mm. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 21.0$; the top of the right half is filled with material of dielectric constant $\kappa_2 = 42.0$; the bottom of the right half is filled with material of dielectric constant $\kappa_3 = 58.0$. What is the capacitance?
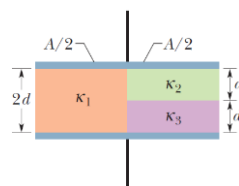

Figure 2-7

8. (10%) Figure 2-8 shows a variable "air gap" capacitor for manual tuning. Alternate plates are connected together; one group of plates is fixed in position, and the other group is capable of rotation. Consider a capacitor of $n = 8$ plates of alternating polarity, each plate having area $A = 1.50$ cm^2 and separated from adjacent plates by distance $d = 3.40$ mm. What is the maximum capacitance of the device?
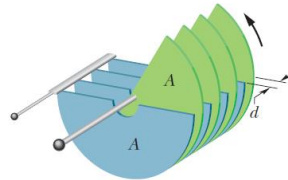

Figure 2-8

9. (10%) In Fig. 2-9, the capacitances are $C_1 = 1.0$ μF and $C_2 = 3.0$ μF, and both capacitors are charged to a potential difference of $V = 100$ V but with opposite polarity as shown. Switches S_1 and S_2 are now closed. (a) What is now the potential difference between points a and b? What now is the charge on capacitor (b) 1 and (c) 2?
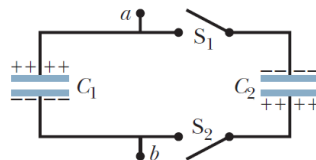

Figure 2-9

10. (10%) Earth's lower atmosphere contains negative and positive ions that are produced by radioactive elements in the soil and cosmic rays from space. In a certain region, the atmospheric electric field strength is 120 V/m and the field is directed vertically down. This field causes singly charged positive ions, at a density of 640 cm^{-3}, to drift downward and singly charged negative ions, at a density of 550 cm^{-3}, to drift upward (Fig. 2-10). The measured conductivity of the air in that region is 2.70×10^{-14} (Ω · m)$^{-1}$. Calculate (a) the magnitude of the current density and (b) the ion drift speed, assumed to be the same for positive and negative ions.
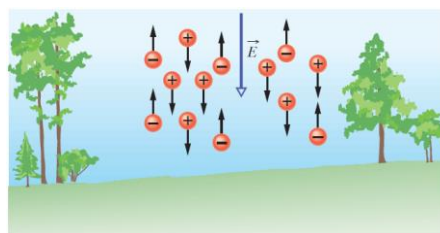

Figure 2-10

11. (10%) *Swimming during a storm*. Figure 2-11 shows a swimmer at distance $D = 38.0$ m from a lightning strike to the water, with current $I = 78$ kA. The water has resistivity 30 Ω · m, the width of the swimmer along a radial line from the strike is 0.70 m, and his resistance across that width is 4.00 kΩ. Assume that the current spreads through the water over a hemisphere centered on the strike point. What is the current through the swimmer?
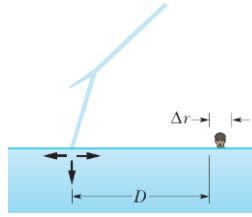
Figure 2-11

12. (10%) In Fig. 2-12, a voltmeter of resistance $R_V = 300\ \Omega$ and an ammeter of resistance $R_A = 3.00\ \Omega$ are being used to measure a resistance R in a circuit that also contains a resistance $R_0 = 100\ \Omega$ and an ideal battery with an emf of $\varepsilon = 12.0$ V. Resistance R is given by $R = V/i$, where V is the potential across R and i is the ammeter reading. The voltmeter reading is V', which is V plus the potential difference across the ammeter. Thus, the ratio of the two meter readings is not R but only an *apparent* resistance $R' = V'/i$. If $R = 85.0\ \Omega$, what are (a) the ammeter reading, (b) the voltmeter reading, and (c) R'? (d) If R_A is decreased, does the difference between R' and R increase, decrease, or remain the same?
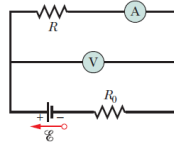

Figure 2-12

13. (10%) Figure 2-13 shows a battery connected across a uniform resistor R_0. A sliding contact can move across the resistor from $x = 0$ at the left to $x = 15$ cm at the right. Moving the contact changes how much resistance is to the left of the contact and how much is to the right. Find the rate at which energy is dissipated in resistor R as a function of x. Plot the function for $\varepsilon = 50$ V, $R = 2000\ \Omega$, and $R_0 = 100\ \Omega$.
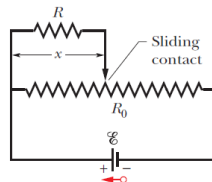

Figure 2-13

14. (10%) In the circuit of Fig. 2-14, $\varepsilon = 1.2$ kV, $C = 6.5\ \mu$F, $R_1 = R_2 = R_3 = 0.73$ MΩ. With C completely uncharged, switch S is suddenly closed (at $t = 0$). At $t = 0$, what are (a) current i_1 in resistor 1, (b) current i_2 in resistor 2, and (c) current i_3 in resistor 3? At $t = \infty$, (that is, after many time constants), what are (d) i_1, (e) i_2, and (f) i_3? What is the potential difference V_2 across resistor 2 at (g) $t = 0$ and (h) $t = \infty$? (i) Sketch V_2 versus t between these two extreme times.
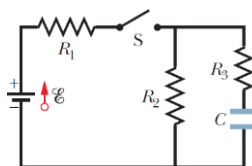

Figure 2-14

15. (10%) *Side flash*. Figure 2-15 indicates one reason no one should stand under a tree during a lightning storm. If lightning comes down the side of the tree, a portion can jump over to the person, especially if the current on the tree reaches a dry region on the bark and thereafter must travel through air to reach the ground. In the figure, part of the lightning jumps through distance d in air and then travels through the person (who has negligible resistance relative to that of air because of the highly conducting salty fluids within the body).The rest of the current travels through air alongside the tree, for a distance h. If $d/h = 0.400$ and the total current is $I = 5000$ A, what is the current through the person?
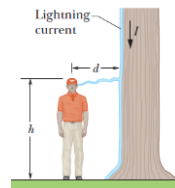

Figure 2-15

16. (20%) When an electric field line passes through a surface separating two dielectrics of dielectric constants κ_1 and κ_2, it makes angles θ_1 and θ_2 with the normal to the surface in the two media. Please show that the relation between the two angles will be given by $\kappa_1 \cot\theta_1 = \kappa_2 \cot\theta_2$. Hint: Assume there are no free charges on the surface.