

Properties of Arithmetical and Boolean Expressions

Free Variables

The set of free variables of an arithmetical or a Boolean expression is defined to be the set of variables occurring in it.

Exercise 30 (Locating free variables) Indicate the free variables in the following arithmetical expressions.

1. $x + 1$
2. $3 * x + y$

Exercise 31 (Free variables of arithmetical expressions)

1. Define, in a compositional manner, a function $FV : \mathbf{Aexp} \rightarrow 2^{\mathbf{Var}}$ that computes the *free variables* for an arithmetic expression.
2. Prove that only the free variables of an arithmetical expression may influence its value.

Exercise 32 (Free variables of Boolean expressions)

1. Define, in a compositional manner, a function $FV : \mathbf{Bexp} \rightarrow 2^{\mathbf{Var}}$ that computes the *free variables* of Boolean expressions.
2. Prove that only the free variables of a Boolean expression may influence its value.

Substitution

In the lecture, we define the notion of substitution for a state.

Exercise 33 (Computing some substitutions) Indicate the values of variables in the following states, where $\sigma = [x \mapsto 1, y \mapsto 2, z \mapsto 3]$:

1. $\sigma[x \mapsto y + z]$
2. $\sigma[x \mapsto 2 * x + y, y \mapsto x]$

Substituting a_0 to y in an arithmetical expressions a consists in replacing every occurrence of y by a_0 , and is noted $a[y \mapsto a_0]$.

Exercise 34 (Defining Substitution for arithmetical expressions) We consider the arithmetical expressions defined in the course lecture.

1. Define formally substitution for arithmetical expressions.
2. Prove that the defined substitution is correct.

Exercise 35 (Defining substitution for Boolean expressions) Define substitution for Boolean expressions where variables are replaced by arithmetical expressions.