Fibonacci system number

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Home Work **Maths for Computer Science** – *MOSIG 1 – 2015*

Write on your homework: I understand what plagiarism entails and I declare that this report is my own, original work.
Name, date and signature.

The firm deadline is November 18, midnight.

- The homework should be maximum 3 pages in the pdf format (scanned manuscripts in pdf are allowed)
- The file should be named: HW2FamilyName.pdf
- send with your official mail at Denis.Trystram@imag.fr with a clear subject *Homework2*

Fibonacci system number

Let us study the way the Fibonacci numbers can be used for representing integers.

Let us write $j \gg k$ iff $j \geq k+2$.

We will first prove the *Zeckendorf's theorem* which states that:
every positive integer n has a unique representation of the form:
$n = F_{k_1} + F_{k_2} + ... + F_{k_r}$ where $k_1 \gg k_2 \gg ...k_r \gg k_r$ and $k_r \geq 2$.

For instance, the representation of 12345 turns out to be:
$12345 = 10946 + 987 + 377 + 34 + 1 = F_{21} + F_{16} + F_{14} + F_9 + F_2$

Figure 1 shows the decomposition for the first integers where each color corresponds to a Fibonacci number and its height corresponds with the number's value

- Show the existence by an induction on n.
- Show that this representation is unique.

Any unique system of representation is a number system.

The previous theorem ensures that any non-negative integer can be written as a sequence of bits b_i, in other words,
$n = (b_m b_{m-1} ... b_2)_F$ iff $n = \Sigma_{k=2}^{m} b_k F_k$.

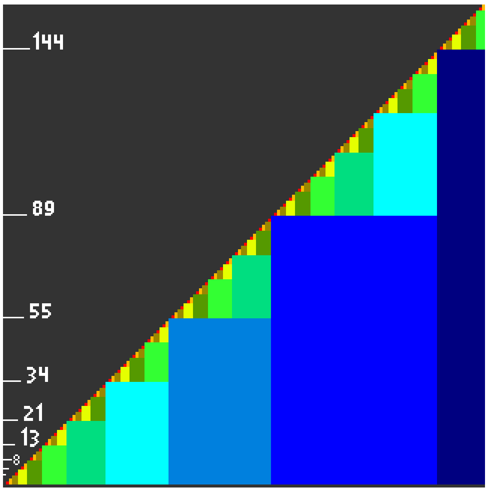

Figure 1: The first 160 integers (on the X-axis) broken down into the Zeckendorf representation.

- Write the Fibonacci representation of 12345 and compare it to the usual binary representation.

  Recall that $12345 = 2^{13} + 2^{12} + 2^5 + 2^4 + 2^3 + 2^0 = (1100000111001)_2$.

- Conclude about their respective features.

  In particular, write the decomposition in the Fibonacci basis for the first 8 integers (starting from $1 = (00001)_F$).

- Prove that there is no consecutive digits equal to 1 in such representations.

Let us now study how to perform basic arithmetic operations within this system.

We will focus on the increment (addition of 1): obtaining $n + 1$ from n.

- Detail first this operation when the last digit is 0 and justify it.

- Give a process to obtain the *increment* of n when its two last digits are 01.

Finally, extend the previous analysis for computing the sum of two integers using this representation.