Image and Signal Processing

December 16, 2010

MOSIG
M1

Duration: 3 Hours
Lecture notes and Computer Exercises allowed
Calculators allowed
All answers must be given on the answers sheet

1 Sharpening Filters in the Spatial Domain *(7 points)*

The main objective of sharpening is to highlight fine details in an image or to enhance details that has been blurred, either in error or as a natural effect of a particular method of image acquisition.

In the following, we consider first- and second-order derivative functions. The derivatives of a digital function are defined in terms of differences. There are various ways to define these differences.

However, for our system to be able to enhance fine details, we want the differences to verify the following properties:

- the first-derivative:
  1. must be zero in flat segments (areas of constant gray-level values)
  2. must be nonzero at the onset of a gray-level step or ramp and
  3. must be nonzero along ramps.
- the second-derivative
  1. must be zero in flat areas
  2. must be nonzero at the onset and end of gray-level steps or ramps and
  3. must be zero along ramps of constant slope.

1.1 First and second-derivatives of 1D signals

A basic definition of the first-order derivative of a one-dimensional a signal$f[x]$ is the difference:

$$\frac{\partial f}{\partial x} = f[x-1] - f[x] \tag{1}$$

Similarly, we define a second-order derivative as the difference:

$$\frac{\partial^2 f}{\partial x^2} = f[x+1] + f[x-1] - 2f[x] \tag{2}$$

Figure 5 (a) and (b) shows a simple 1D gray-level profile along a line in a 2D image (the points are joined by dashed lines to simplify interpretation).

Question 1.1 (1 point(s))
Calculate the first-oder derivative (using equation 1) of this signal (*i.e. complete table (c) of Figure 5*) and draw its 1D representation on Figure 5 (e).

Question 1.2 (1 point(s))
What do you conclude?

Question 1.3 (1 point(s))
Calculate the second-order derivative (using equation 2) of this signal *i.e. complete table (d) of Figure 5*) and draw its 1D representation on Figure 5 (e).

Question 1.4 (1 point(s))
What do you conclude?

1.2 Second-order derivative of a 2D signal

We consider the *Laplacian* operator, which for an image f of two variables is defined as:

$$\begin{aligned} \nabla^2 f &= \tfrac{\partial^2 f}{\partial x^2} + \tfrac{\partial^2 f}{\partial y^2} \\ &= f[x+1,y] + f[x-1,y] + f[x,y+1] + f[x,y-1] - 4f[x,y] \end{aligned} \tag{3}$$

For a discrete image, we consider the following system:

$$f[x,y] \longrightarrow \boxed{\begin{matrix} LaplacianSystem \\ h[x,y] \end{matrix}} \longrightarrow g[x,y] = \nabla^2 f[x,y]$$

Figure 1: Laplacian system

Question 1.5 (1 point(s))
Express $g[x,y]$ with respect to $f[x,y]$ and $h[x,y]$
(suppose $x \in]-\infty,\infty[$ and $y \in]-\infty,\infty[$).

Question 1.6 (0.5 point(s))
How is called h?

Question 1.7 (1 point(s))
Draw h for a $2D$ 3×3 impulse.

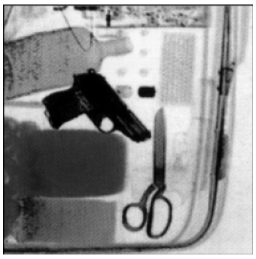


Figure 2: Airport luggage scan.

Question 1.8 (0.5 point(s))
Which figure 3 (a), (b), (c) or (d) is the system answer to the image Figure 2?

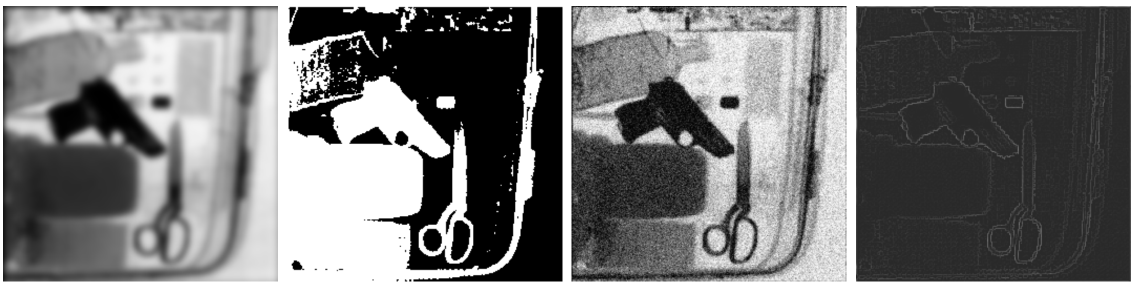
2 Sharpening Filters in the Frequency Domain *(7 points)*

We consider now the Frequency version of the discrete *Laplacian* operator represented in the system of Figure 1. To simplify the problem, we consider a 2D discrete image $f[x, y]$ of $M \times M$ points.

Question 2.1 (1 point(s))
Prove that $\mathcal{F}\left(f[x+1, y]\right) = \mathcal{F}f[u, v]e^{i2\pi\frac{u}{M}}$

Question 2.2 (1.5 point(s))
In the same way, what are:

- $\mathcal{F}\left(f[x-1, y]\right)$
- $\mathcal{F}\left(f[x, y+1]\right)$
- $\mathcal{F}\left(f[x, y-1]\right)$



(a) (b) (c) (d)

Figure 3: Figure 2 after processing

> *Question 2.3 (2 point(s))*
> Show that $\mathcal{F}g[u,v] = \mathcal{F}h[u,v] \times \mathcal{F}f[u,v]$ where $\mathcal{F}h[u,v] = -2\left((sin(\frac{\pi u}{M}))^2 + (sin(\frac{\pi v}{M}))^2\right)$

Note: In addition to Euler's relations, you may use the fact that $1 - cos(2a) = 2(sin(a))^2$.

> *Question 2.4 (0.5 point(s))*
> How is called $\mathcal{F}h$?

In the case where M is large, we can approximate $(sin(\frac{\pi u}{M}))^2$ by $\left(\frac{\pi u}{M}\right)^2$

> *Question 2.5 (0.5 point(s))*
> What becomes $\mathcal{F}h$?

To actually compute $\mathcal{F}g$ from $\mathcal{F}f$ and $\mathcal{F}h$, $\mathcal{F}f$ and $\mathcal{F}h$ are generally centered.

> *Question 2.6 (1 point(s))*
> What does it mean?
> What has to be done to f and h before computing Fourier Transform so that the result is centered?

> *Question 2.7 (0.5 point(s))*
> The *Laplacian* filter is also called high-pass filter. Why?

3 Mickey Mouse Processing *(6 points)*

Figure 4 represents an image in 256 gray levels. Mickey's face is symmetric along x-axis and the gray values are indicated on the figure. The dimensions of the image are $195 = 15 \times 13$ pixels.

3.1 Histogram

> *Question 3.1 (2 point(s))*
> Draw the histogram of this image on Figure 6.

3.2 Otsu Thresholding

Thresholding is a way to classify the pixels of an image in several classes. For example, in Figure 4, if we consider the threshold $T = 65$, we can classify the pixels of the image into pixels with values below T (actually those represented in grey on the image) and those with values above T (actually the white ones in the image). Table 7 gives for each value T of threshold, for each class C_{below}, the class of pixels whose values are below T and C_{above} the class of pixels whose values are above T:

- the mean value μ of the class
- the variance σ^2 of the class
- the mean of the variances of each class.

> *Question 3.2 (1 point(s))*
> Complete table 7

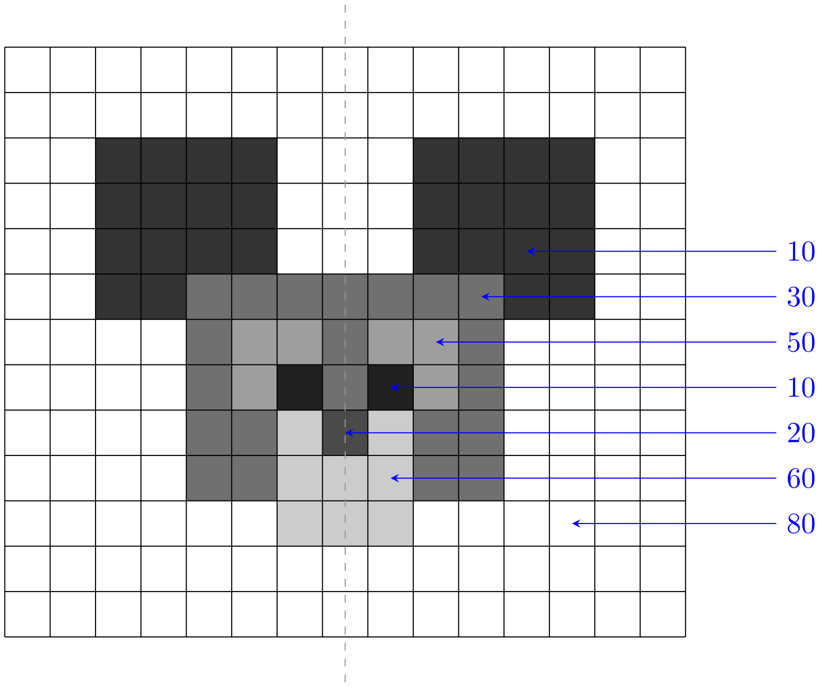


Figure 4: Mickey Mouse

Otsu's method is to determine automatically the threshold for an image by taking the threshold value that minimises the mean of the variances of each class.

Question 3.3 (0.5 point(s))
Explain why.

Question 3.4 (0.5 point(s))
What would be the Otsu threshold in the case of Figure 4?

3.3 Contrast

The actual image representation of Figure 4 is much less contrasted.

Question 3.5 (0.5 point(s))
Why?

Question 3.6 (1.5 point(s))
What would you do to enhance its contrast?

Name:_____________ First name: _____________

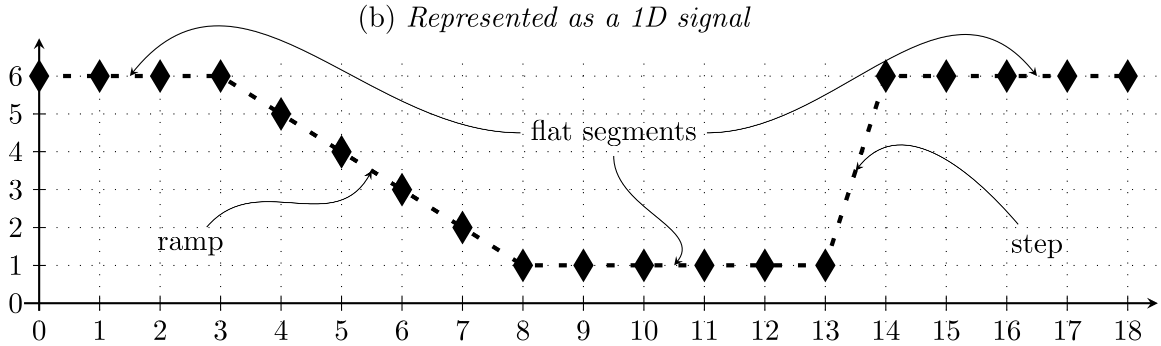
Answering Sheet

All answers must be given on THIS sheet

Answers to Exercise 1

(a) *Profile of an Image Line*

(b) *Represented as a 1D signal*



6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6

(c) *first-order derivative* (Question 1.1)

X																		

(d) *second-order derivative* (Question 1.3)

X																		

(e) *Graphical representation of the first and second derivatives* (Questions 1.1 and 1.3)

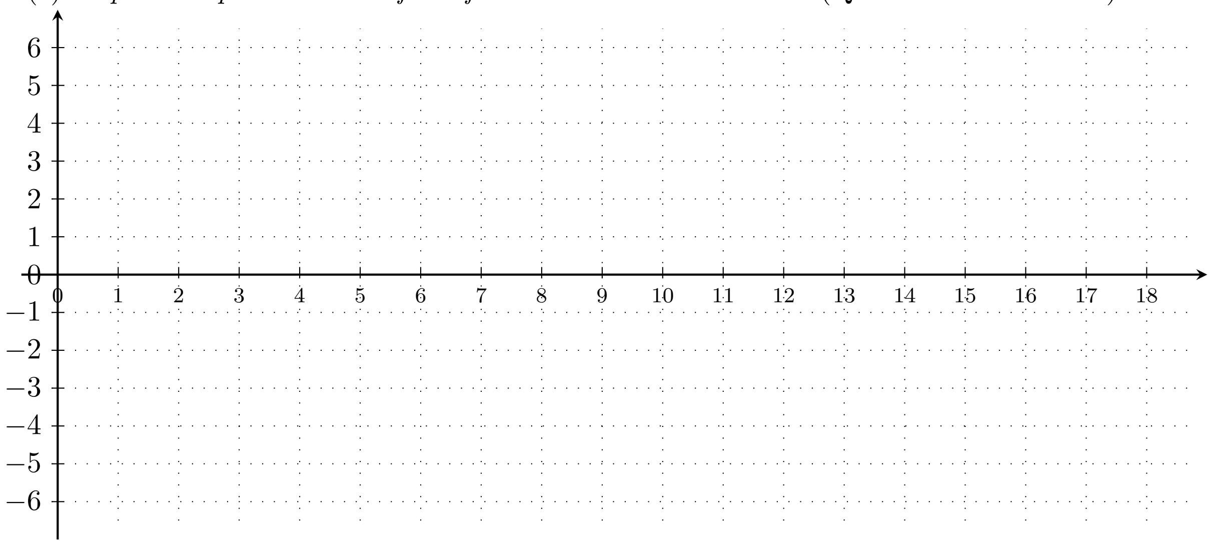


Figure 5: First and Second derivative of a 1D signal.

Name :

Answer to Question **1.2**

Answer to Question **1.4**

Answer to Question **1.5**

$img_2[j][i] =$

Answer to Question **1.6**

Answer to Question **1.6**

0	0	0
0	1	0
0	0	0

(a) 3×3 2D impulse.

(b) h

Answer to Question **1.7**

☐(a) ☐(b) ☐(c) ☐(d)

Name :

Answers to Exercise 2

Answer to Question **2.1**

Answer to Question **2.2**

$\mathcal{F}\left(f[x-1,y]\right) =$

$\mathcal{F}\left(f[x,y+1]\right) =$

$\mathcal{F}\left(f[x,y-1]\right) =$

Name :

Answer to Question **2.3**

Answer to Question **2.4**

Answer to Question **2.5**

$$\mathcal{F}h =$$

Answer to Question **2.6**

Answer to Question **2.7**

Answers to Exercise 3

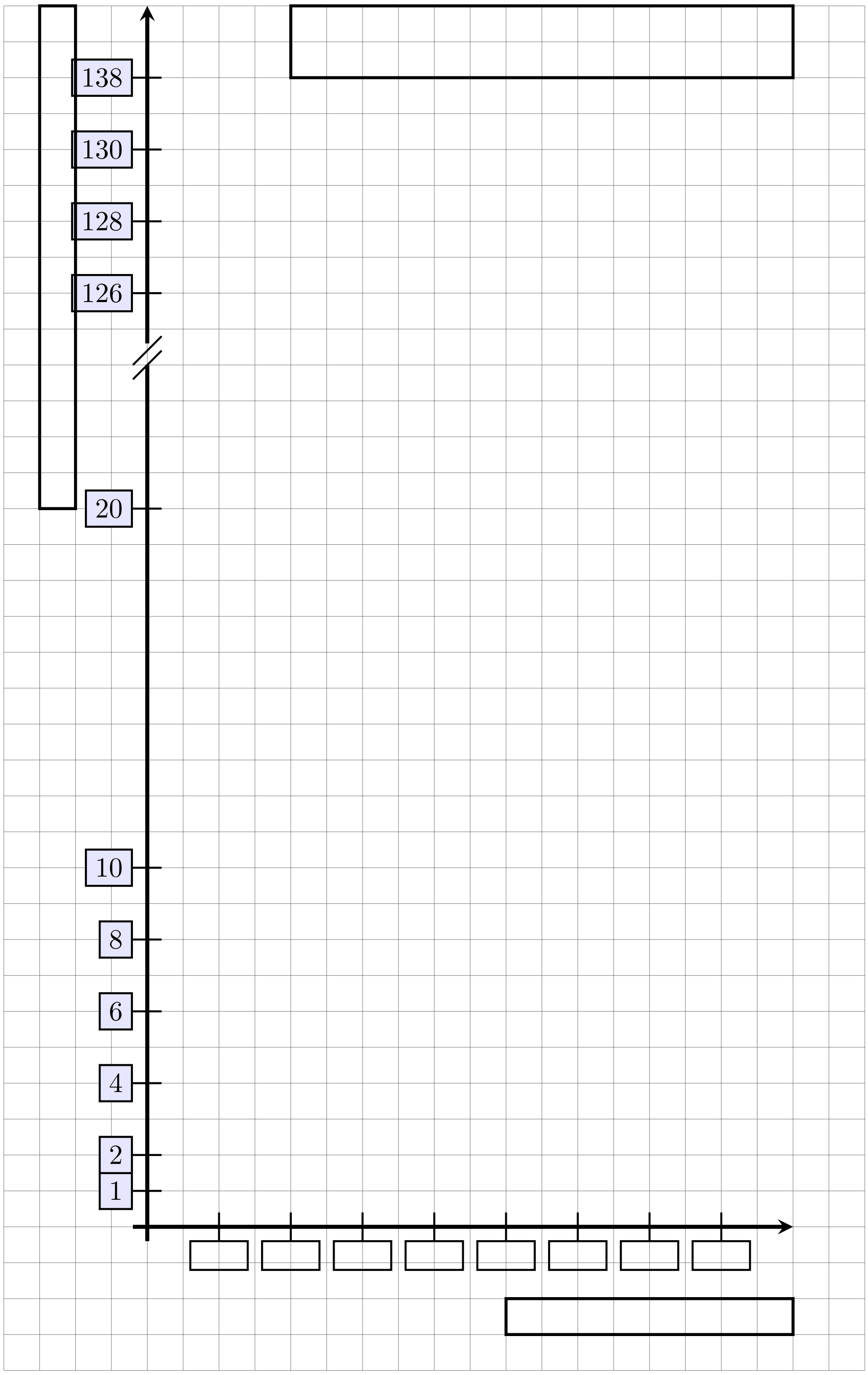


Figure 6: *Answer to Question* **3.1**

Answer to Question **3.2**

Threshold T	5	15	25	35	45	55	65
Mean of C_{below}: μ_{below}	X		10.3		18.27	21.6	
Variance of C_{below}: σ^2_{below}	0.0		96.8		4944.2	10 360.3	
Mean of C_{above}: μ_{above}	61.8		71.5		77.6	78.8	
Variance of C_{above}: σ^2_{above}	147 071.8		49 318.9		7 791.6	3 013.1	
$\frac{\sigma^2_{below}+\sigma^2_{above}}{2}$	73 535.9		24 707.8		6 367.9	6 686.7	

Figure 7: Means and Variances

Answer to Question **3.3**

Answer to Question **3.4**

Answer to Question **3.5**

Name :

Answer to Question **3.6**