Plan

General modeling techniques

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Integer variables are used

- to describe discrete structures

  subset of $S \subseteq \{1, \ldots, n\} \Rightarrow$ vector $(x_1, \ldots, x_n) \in \{0, 1\}^n$

- to linearize non linear expressions

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$$\begin{cases} \displaystyle\sum_{i=1}^{k} y_i = 1 \\ x = \displaystyle\sum_{i=1}^{k} p_i y_i \\ y_i \in \{0,1\} \quad \text{for } i = 1, 2 \ldots k \end{cases}$$

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$$x \leq y$$

OR: x or y must be $\textsc{True}$

x and y being two boolean variables $\{0, 1\}$

$$x + y \geq 1$$

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x: a decision variable

Objective with fixed cost (affine function): min $f\mathbf{1}_{\{x>0\}} + cx$

- The cost is composed of a unitary cost c and a fixed cost f paid only if $x > 0$

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$$\begin{cases} \min fy + cx \\ x \leq My \\ y \in \{0,1\} \end{cases}$$

where M is a constant $\geq x$

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Disjunctive constraints:

- Two tasks of duration d_i and d_j must be processed on the same machine

$$\begin{cases} t_i + d_i \leq t_j & \text{if } i \text{ is scheduled before } j \\ t_j + d_j \leq t_i & \text{if } j \text{ is scheduled before } i \end{cases}$$

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$$\begin{cases} t_i + d_i \leq t_j + M(1 - y_{ij}) \\ t_j + d_j \leq t_i + My_{ij} \\ y_{ij} \in \{0, 1\} \end{cases}$$

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  We must express $y = 1$ iff ($x = 1$ and $x' = 1$)

$$\begin{cases} y \leq x \\ y \leq x' \\ x + x' - 1 \leq y \\ y \in \{0,1\} \end{cases}$$