# Univariate Descriptive Statistics

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Scientific Methodology and Performance Evaluation
ENS Lyon, November 2016

# Outline

# Motivation

We have set up a world where we keep collecting data, huge amount of data...

Sweet, what knowledge can we exctract from such data? How do we summarize a data set?

With a few numbers, some graphics? How? Why is this difficult?

> *There are three kinds of lies: lies, damned lies and statistics*
>
> – Mark Twain's Autobiography
>
> *Statistical thinking will one day be as necessary for efficient citizenship as the ability to read or write*
>
> – Attributed to H. G. Wells
>
> *The only statistics you can trust are those you falsified yourself*
>
> – Winston Churchill

# Outline

# I just got new Tees!

- A series of measurements (one value per measurement)
- Nature of the measurements
  - Factors (nominal data)

```
 [1] Red   Red   Black Green Blue  Black White Black Blue
[10] White Black White Red   Black Black Red   Red   Black
[19] Black Black
Levels: Black Blue Green Red White
```

  - Ordered factors (ordinal data)

```
 [1] XL M  S  XL M  M  M  XL M  L  M  L  M  M  M  L  M
[18] M  XL M
Levels: S < M < L < XL
```

  - Numbers (e.g., price, duration, . . . ) (numerical data)

```
 [1]  9.1  4.7  9.5 13.6 15.7  8.7  9.2  4.7 11.4  8.1
[11] 11.4 12.1 13.1  8.2 11.5  4.8  7.6  7.4  2.8 10.1
```

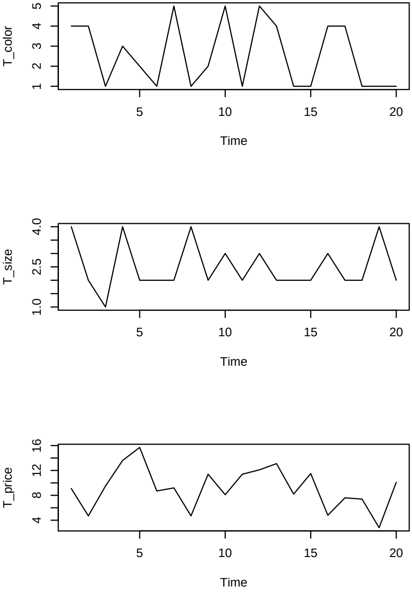
```
str(T_size); # May want to use the str function
```

```
Ord.factor w/ 4 levels "S"<"M"<"L"<"XL": 4 2 1 4 2 2 2 4 2 3 ...
```

# Are these sample "structured"?

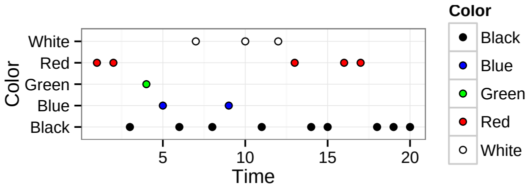
Use `plot.ts` (for time series)

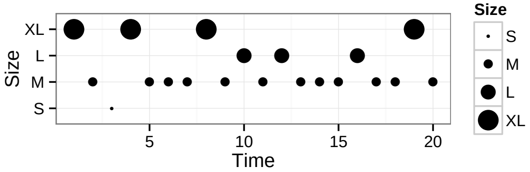
```
par(mfrow=c(3,1));
plot.ts(T_color,xy.lines=F);
plot.ts(T_size,xy.lines=F);
plot.ts(T_price,xy.lines=F);
```

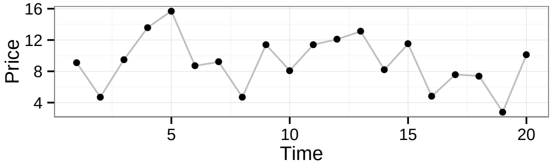


# Are these sample "structured"?

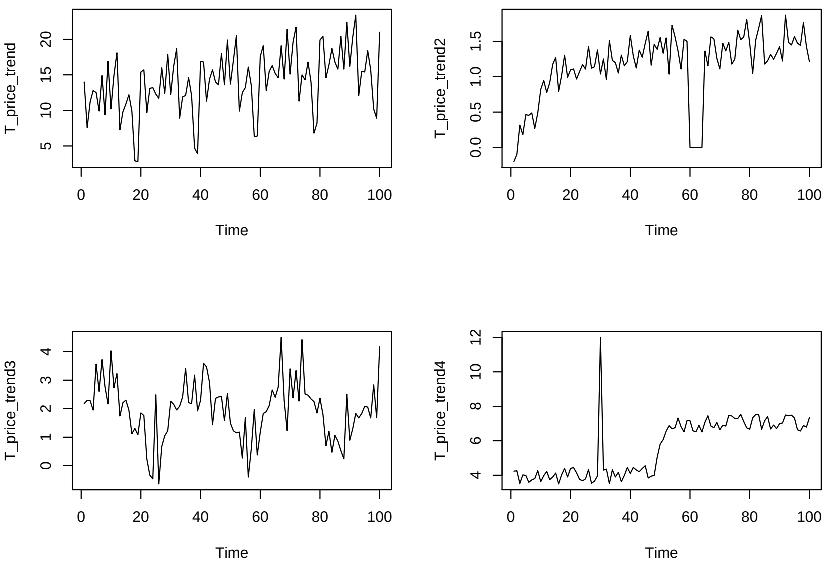
Fancier output can be built using `ggplot2`



# There could indeed be "trends"



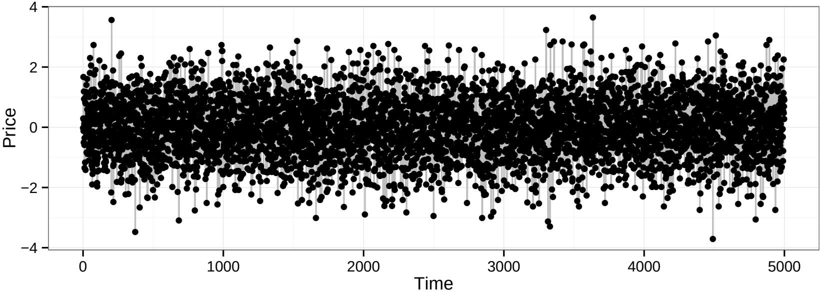
# What should we look for?

- Structured/unstructured
- Trend, evolution
- Localization/order of magnitude
- Outliers, aberrant values

This preliminary study will:

- guide your analysis
- provide feedback on your experimental setup

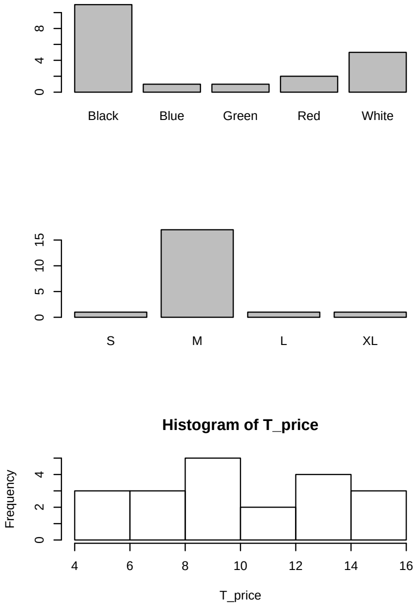
This may be harder to do than it looks...



# Outline

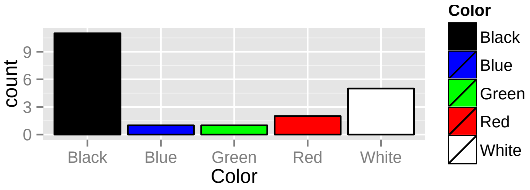
# Bar charts vs. Histograms

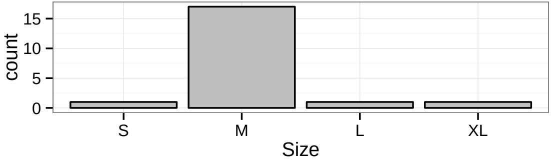
```
par(mfrow=c(3,1));
plot(T_color,xy.lines=F);
plot(T_size,xy.lines=F);
hist(T_price,xy.lines=F);
```

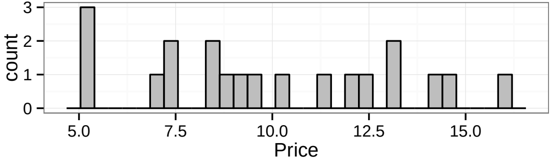


# Bar charts vs. Histograms

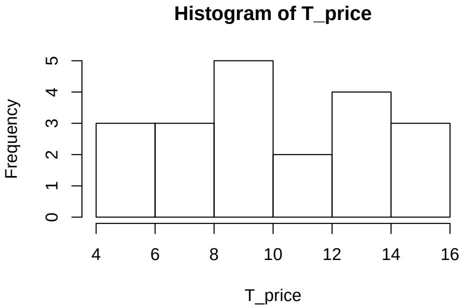
Again, fancier output can be built using ggplot2



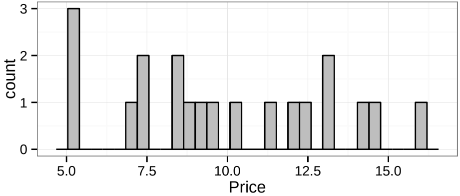



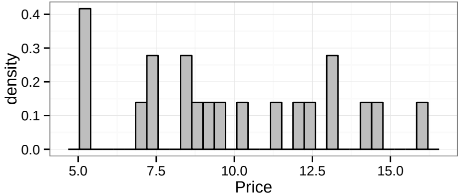



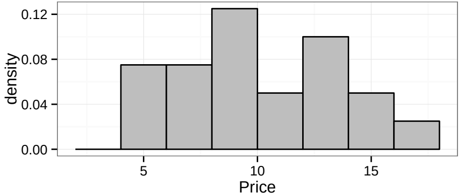
# Wait, why are these histograms so different?







# Beware of histograms

**Rather indicate density than count**

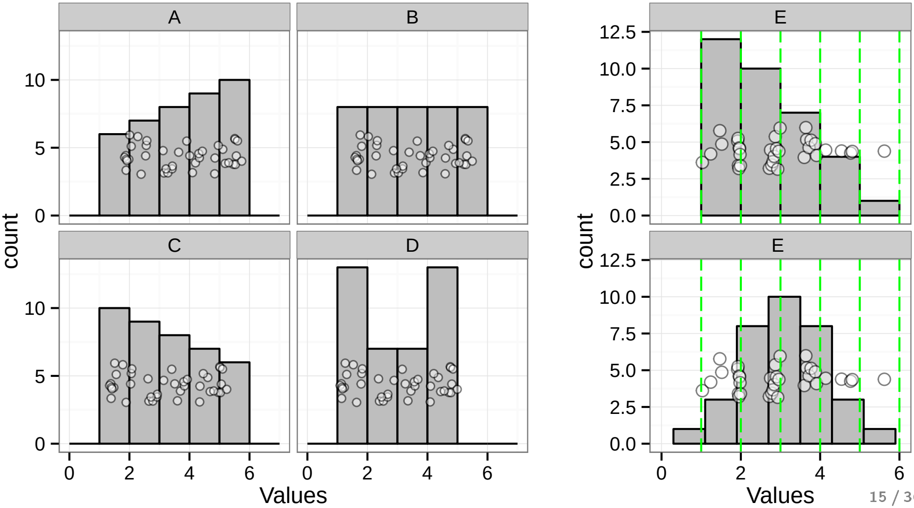
How many bins? Which binwidth?

- `ggplot` defaults to $k = 30$ bins of width $h = \texttt{range}/30$ $\ddot{\frown}$
- Square-root choice: $k = \sqrt{n}$ (Excel, $\ddot{\frown}$)
- Sturges: $k = \lceil \log_2 n + 1 \rceil$ (default for `hist` in R)
- Rice: $k = \lceil 2n^{1/3} \rceil$
- Scott: $k = \left\lceil \frac{\max x - \min x}{h} \right\rceil$, where: $h = \frac{3.5\hat{\sigma}}{n^{1/3}}$ (equivalent to Rice under some conditions)
- ...

# Beware of Histograms

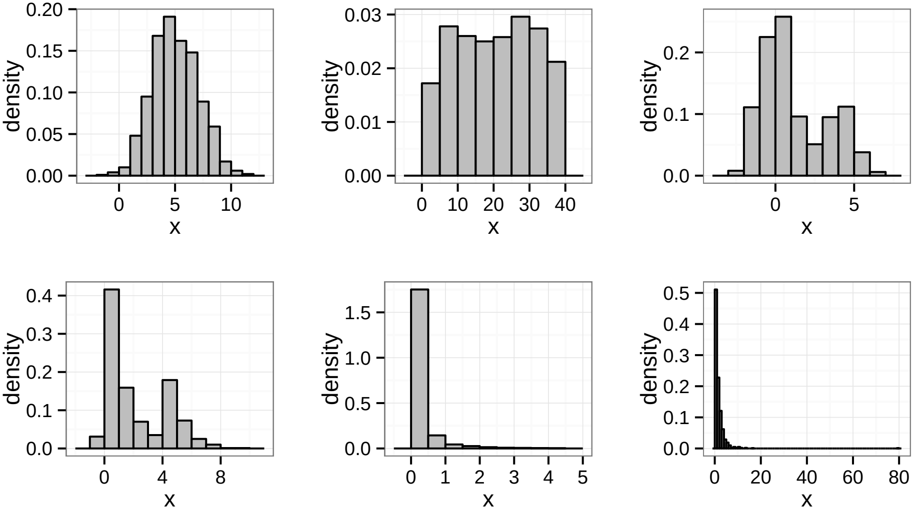
## At which value should the bin start?

- In most cases, the binning is aligned on human readable values, which can create nasty artifacts (nice illustration from *stackexchange*)



# What should we look for?

Shape: flat? symmetrical? multi-modal? Play with `binwidth` (and `origin` if you have few samples) to uncover the full story behind your data...



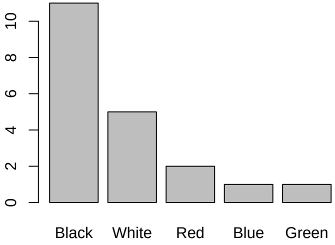
# Outline

# Nominal Values

- What is the mode (most frequent value)?
- Sort values according to their frequency...

```r
summary(T_color)
```

```
Black  Blue Green   Red White
   11     1     1     2     5
```




```r
col_freq=table(T_color);
T_color <- factor(T_color,
    levels = names(col_freq[order(col_freq, decreasing = TRUE)]));
plot(T_color);
```

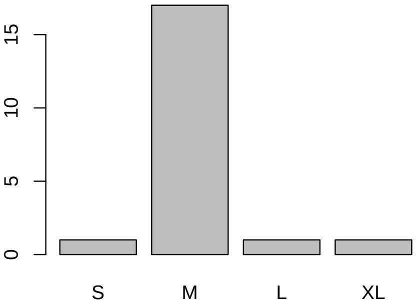
# Ordinal Values

- What is the mode (most frequent value)?

```
summary(T_size)
```

```
S   M   L  XL
1  17   1   1
```

- May still want to sort values according to their frequency...
- Median: not implemented in standard R for ordinal values, as it's not well defined




```
median(T_size)
library(DescTools)
median(T_size) # :(
```

```
Error in median.default(T_size) : requires numerical data
[1] NA
```

# Numerical Values

```
str(T_price);
```

```
num [1:20] 14.5 13.1 9.3 6.9 8.6 7.2 7.3 12.4 13.1 16 ...
```

```
summary(T_price);
```

```
 Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
5.200   7.275   9.500   9.960  12.580  16.000
```

- `min`, `max`, `median` in R
- Median: 50% of values are smaller than 9.5\ quad (a possible measure of central tendency)

# Numerical Values

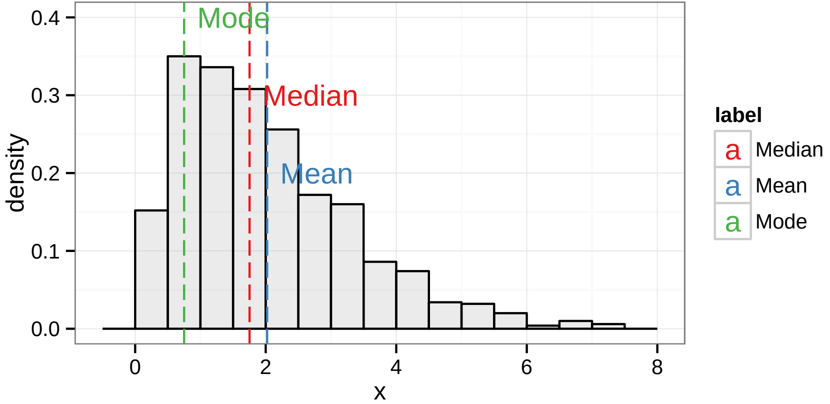
The mode and the median are measures of central tendency (typical value)

- Note: There may be several modes and it depends on binning...

There is also the (arithmetic) mean: $A = \overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$

```
mean(T_price)
```

```
[1] 9.96
```



# Things to know about the mean

- This measure is sensitive to "outliers".
  - One aberrant (say very large) value will drag the mean to the right while it would not change the median
- The key question is what makes sense?
  - Your favorite pair has been added a +20% mark-up in August but you have a -20% discount as a regular customer. Is the price the same?
    - No, you actually saved 4% of the original price ($1.2 \times .8 = .96$).
  - You drove half the way at 50mph and half of the way at 100mph. Did you drive on average at 75mph?
    - Obviously not...
  - Although you can compute the average of gains/loss, it is not at all what you would consider as the average gain.
  - May want to consider the geometric or the harmonic mean... $G = \sqrt[n]{\prod_{i=1}^{N} x_i}$ or $H = \frac{1}{\frac{1}{N}\sum_{i=1}^{N}\frac{1}{x_i}}$

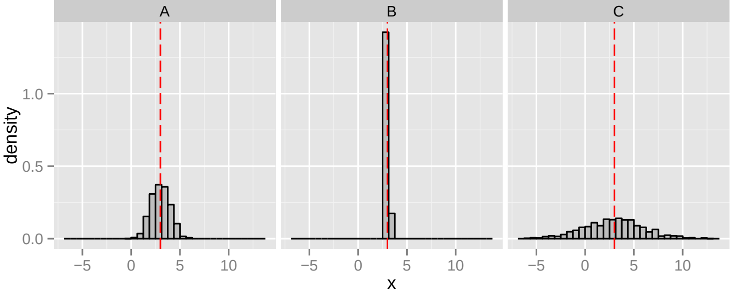
# What should I look for?

- If the distribution is unimodal and symmetrical, then
$$\text{mean} = \text{mode} = \text{median}$$
- Depending on the problem, one or the other may be more relevant
- Anyway, reporting such measure with no indication about variability is generally useless

# Outline

# Variance

We expect most values to be "around" the mean




Departure from the mean:

- Mean absolute deviation: $\frac{1}{N}\sum_{i=1}^{N}|x_i - A|$
  - Rarely used
- Variance: $V = \frac{1}{N}\sum_{i=1}^{N}(x_i - A)^2$
  - only positive values and gives more importance to large deviations $\ddot\smile$
  - not homogeneous to the mean (units) $\ddot\frown$
- Standard deviation: $SD = \sqrt{V}$

# Quantile

```
quantile(T_price,c(.05,.25,.5,.75,.95))
```

```
    5%     25%     50%     75%     95%
 4.605   7.550   9.150  11.425  13.705
```

Inter-Quantile Range:

- Inter-quartile range: $IQR = Q_{75} - Q_{25}$
- But other values are possible, e.g., $Q_{95} - Q_5$
- Range: max − min (may grow unbounded)
    - $\rightsquigarrow$ quite difficult to use

# What about nominal or ordinal values?

There is for example the notion of Entropy: how many bits are required to encode the sample?
Say there is a fraction $f_v$ of items with value $v$.
$H = -\sum_{v \in V} f_v \log_2(f_v)$
$-(x+y)\log_2(x+y) < -x\log_2(x) - y\log_2(y)$ so the smaller the entropy, the more condensed/predictable the sample distribution

- $H([0,1,0,0]) = 0$
- $H([.25,.25,.25,.25]) = 2$
- $H([1/n,\ldots,1/n]) = \log_2(n)$ so you generally normalize $H$ by $\log_2(n)$

This notion can be extended to numerical values (but the computation is complex as it depends on the binning...)

# Outline

# Skewness

Remember the mean and the variance:

- $A = \overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$
- $V = \frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2$

Could we measure the asymmetry of the samples around the mean?

- Proposal 1: $\frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})$ (always 0... $\ddot\frown$)
- Proposal 2: $\frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^3$ (not well normalized... $\ddot\frown$)

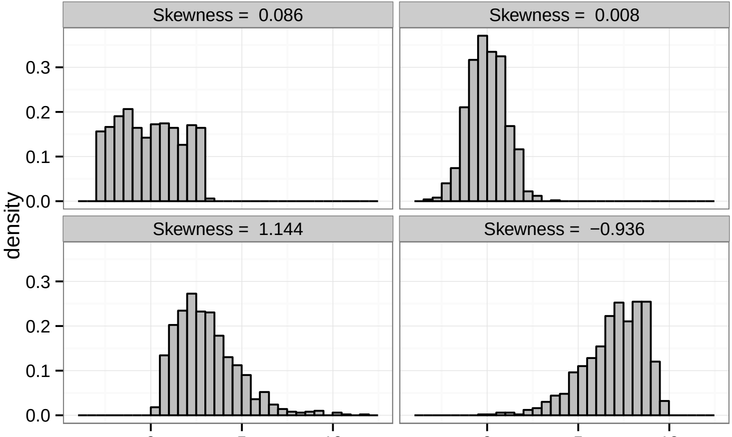
$$S = \frac{\displaystyle\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^3}{\underbrace{\left[\displaystyle\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2\right]}_{\text{variance}}{}^{3/2}}$$

# Skewness

Could we illustrate this a bit?

```r
library(moments)
skewness(runif(1000))
```

```
[1] 0.04626483
```



# Kurtosis

- peakedness (width of peak), tail weight, lack of shoulders...
- measure infrequent extreme deviations, as opposed to frequent modestly sized deviations

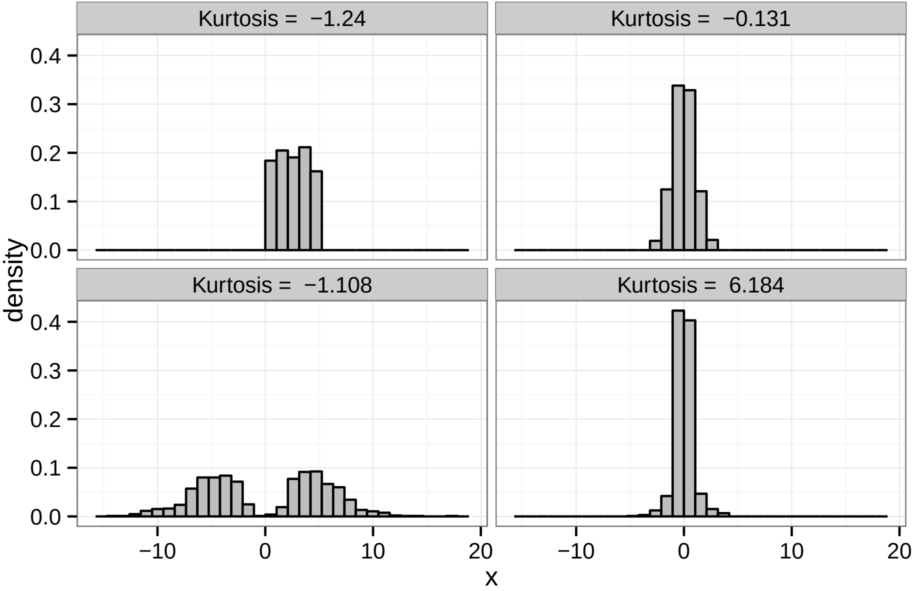
$$K = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\overline{x})^4}{\underbrace{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i-\overline{x})^2\right]}_{\text{variance}}^2} - 3$$

The -3 is here so that normal distribution have a Kurtosis of 0

```r
library(moments)
x = rnorm(1000) ; var(x);
kurtosis(x)-3
```

```
[1] 1.039743
[1] 0.01825114
```

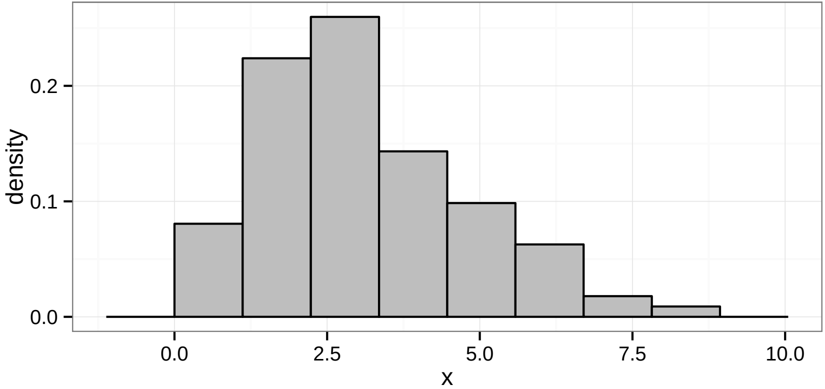
# Kurtosis



Kurtosis = −1.24
Kurtosis = −0.131
Kurtosis = −1.108
Kurtosis = 6.184
density
0.4
0.3
0.2
0.1
0.0
−10
0
10
20
x

# Outline

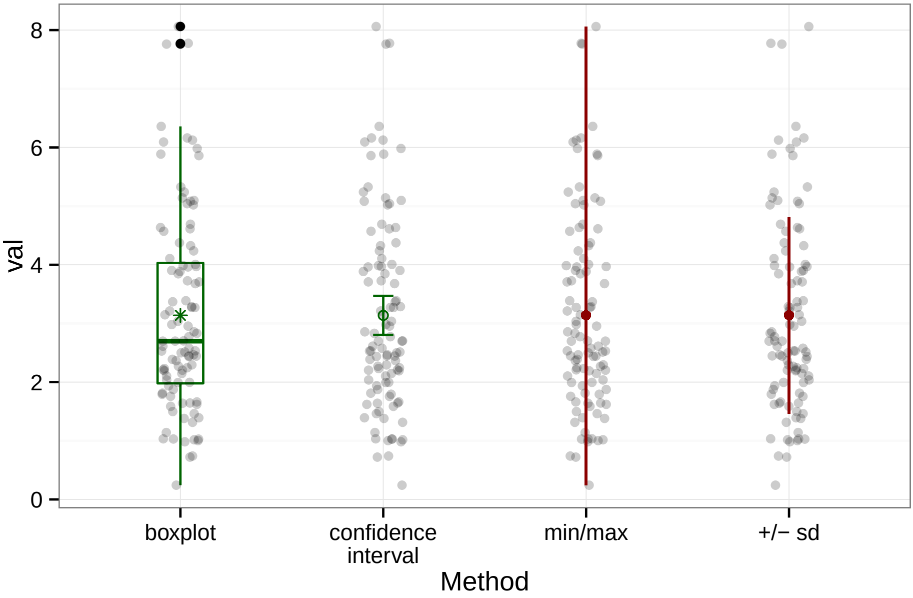
# Classical information



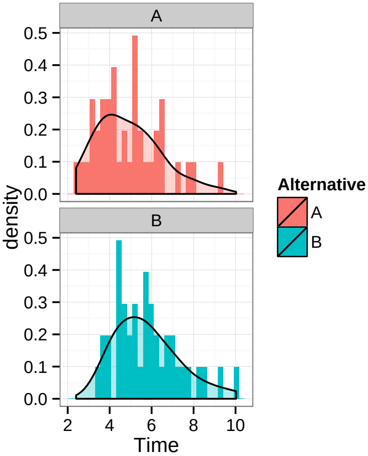

```r
summary(x)
var(x)
```

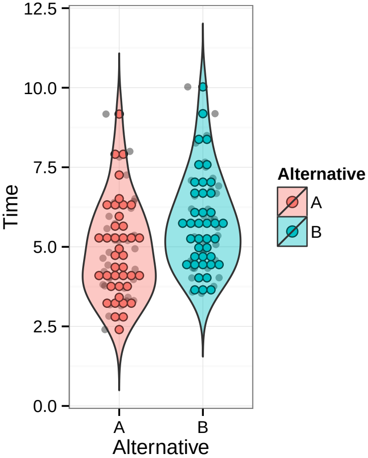
```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.4065  1.8430  2.5020  2.8660  3.6310  7.0220
[1] 2.117541
```

# Good and bad summaries



# Be careful with fancy plots you do not fully understand!




*The average human has one breast and one testicle*

– Des McHale

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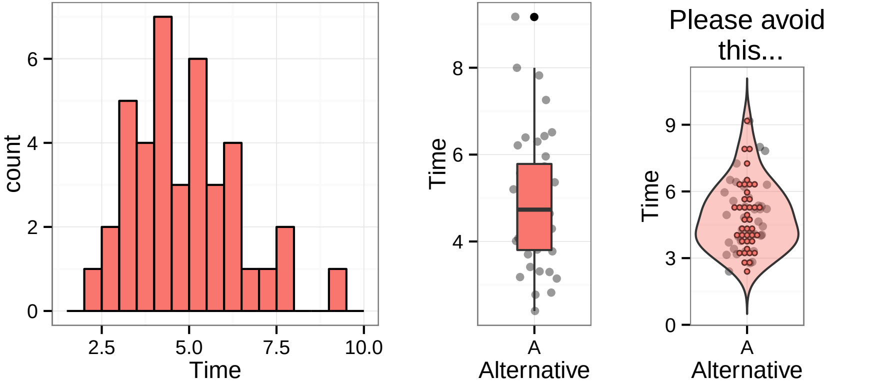


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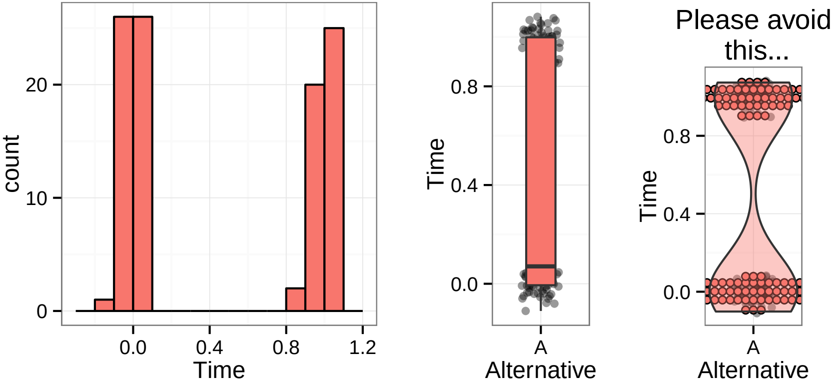


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