Quiz 1: Use arctan to calculate π

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

(1) $\tan \dfrac{\pi}{4} = 1 \quad \Rightarrow \quad \dfrac{\pi}{4} = \tan^{-1} 1 \quad \Rightarrow \quad \pi = 4 \tan^{-1} 1$

(2) $\tan \dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}} \quad \Rightarrow \quad \dfrac{\pi}{6} = \tan^{-1} \dfrac{1}{\sqrt{3}} \quad \Rightarrow \quad \pi = 6 \tan^{-1} \dfrac{1}{\sqrt{3}}$

Write a code by using the expansion formulation to approach π with both (1) & (2)'s settings and make the errors to be less than 10^{-4}. Which case converges faster? Why?

Execute Result

```
pi = 0.785319645811445 , n = 3184 , error = 0.7500249929238642
pi = 0.5235514642438139 , n = 6 , error = 0.8333483930051946
```

Questions

Which converges faster?

The second, $\frac{\pi}{6} = \tan^{-1} \frac{1}{\sqrt{3}}$

Why?

Because $\frac{1}{\sqrt{3}} < 1$, for all integer $n > 0$, $\frac{\left(\frac{1}{\sqrt{3}}\right)^n}{n}$ converge faster than $\frac{1^n}{n}$