

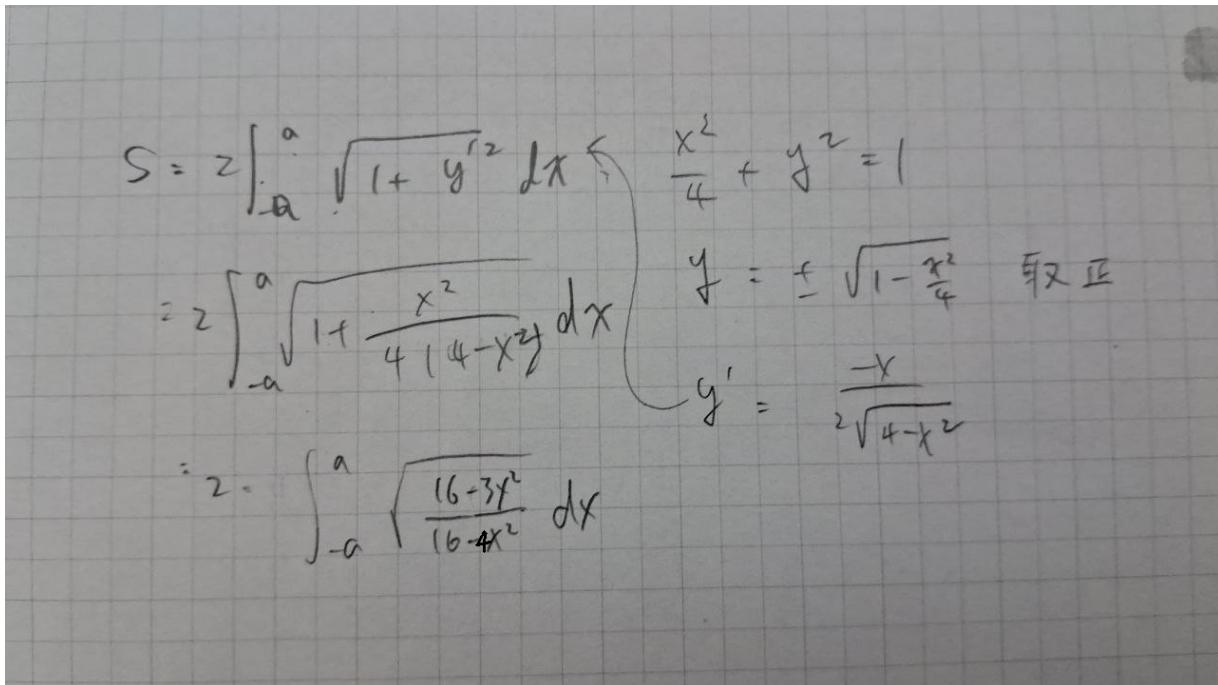
**Quiz:**

Problem Set 6.2

11. ■ The equation of an ellipse is  $x^2/a^2 + y^2/b^2 = 1$ . Write a program that computes the length

$$S = 2 \int_{-a}^a \sqrt{1 + (dy/dx)^2} dx$$

of the circumference to five decimal places for a given  $a$  and  $b$ . Test the program with  $a = 2$  and  $b = 1$ .



The image shows a handwritten derivation of the formula for the circumference of an ellipse. It starts with the formula for the arc length of a function  $y = f(x)$  from  $-a$  to  $a$ :

$$S = 2 \int_{-a}^a \sqrt{1 + y'^2} dx$$

For an ellipse, the equation is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . Solving for  $y$ , we get  $y = \pm \sqrt{1 - \frac{x^2}{a^2}}$ . Taking the positive root, we have  $y' = \frac{-x}{a^2 \sqrt{1 - x^2/a^2}}$ . Substituting this into the arc length formula, we get:

$$S = 2 \int_{-a}^a \sqrt{1 + \left(\frac{-x}{a^2 \sqrt{1 - x^2/a^2}}\right)^2} dx$$

After simplifying, the integral becomes:

$$S = 2 \int_{-a}^a \sqrt{\frac{16 - 3x^2}{16 - x^2}} dx$$

# trapezoid

trapezoid.py

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k = 6, Inew = 133.6265498552872, difference = 124.51974382305067
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算了七個小時都沒算完， $k = 36$  的一直算不出來

## gaussian

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gaussian.py

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