天文學一期中考

宇宙學

亂七八糟的宇宙常數

$$
\begin{gathered}
u \propto T^4 \\
H = \frac{\dot{a}}{a} \\
H^2 \propto \rho(\text{宇宙物質密度}) \\
\rho_m \propto a^{-3} \\
\rightarrow H \propto a^{-3/2} \\
\rightarrow a \propto t^{2/3} \\
\frac{a_0}{a} = 1 + z
\end{gathered}
$$

溫度與能量

from $E = k_B T$, $1eV \approx 10^4 K$

一些簡單的反應式

$$
\begin{gathered}
\mathrm{n} + \mathrm{e}^+ \rightleftharpoons \mathrm{p} + \bar{\nu}_e \\
\mathrm{n} + \bar{\nu}_e \rightleftharpoons \mathrm{p} + \mathrm{e}^- \\
\mathrm{n} \xrightleftharpoons{\beta\ \text{decay}} \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e \\
\mathrm{n} + \mathrm{p} \rightleftharpoons \mathrm{D} + \gamma
\end{gathered}
$$

物質組成

$$
\begin{gathered}
\frac{N_n}{N_p} = \frac{1}{7} \\
2\,\mathrm{n} + 14\mathrm{p} \longrightarrow (2\,\mathrm{n} + 2\mathrm{p}) + 12\mathrm{p} \longrightarrow \mathrm{He} + 12\,\mathrm{H} \\
\frac{m_{He}}{n_H} = \frac{4 \times 1}{1 \times 12} = \frac{1}{3}
\end{gathered}
$$

天球座標

英文	中文	單位
Right Ascension	經度	hour
Declination	緯度	degree

潮汐力

$$
F_g = \frac{GMm}{L^2}\left[1 + \left(\frac{2r}{L}\right)\cos\theta\right]
$$

θ：物體的軸與中心點和大質量星體連線的夾角

solid angle

$$
\begin{gathered}
A(\theta_1, \phi_1) \\
B(\theta_2, \phi_2) \\
\overline{AB} = 2 - 2\cos\theta_1\cos\theta_1\cos(\phi_1 - \phi_2) - 2\sin\theta_1\sin\theta_2 \\
\delta(A, B\ \text{夾角}) = \cos^{-1}\left[\cos\theta_1\cos\theta_1\cos(\phi_1 - \phi_2) + \sin\theta_1\sin\theta_2\right] \approx \sqrt{(\Delta\theta)^2 + \cos^2\theta(\Delta\phi)^2}
\end{gathered}
$$

角動量

$$
\frac{1}{r_0} = \frac{GMm^2}{L^2}
$$

The Virial Theorem

in a gravitational bound system

$$
\begin{gathered}
K = -\frac{1}{2}U \\
\text{let } Q := \sum_i m_i \vec{v}_i \cdot \vec{r}_i = \frac{1}{2}\frac{dI}{dt} \\
\frac{dQ}{dt} = 2K + \sum_i \vec{f}_i \cdot \vec{r}_i = 2K + U
\end{gathered}
$$

星等/Luminosity/color

name	symbol	dimension
Luminosity	L	$J \cdot s^{-1} = W$
flux	F	$W \cdot m^{-2}$
intensity	I	$W \cdot m^{-2} \cdot \Omega^{-1}$

$$F(\text{flux}) = \frac{L}{4\pi D^2}$$

magnitude

$$m = -2.5\log F + ZeroPoint$$

AB magnitude

$$m_{AB} = -2.5\log\frac{F}{erg\cdot cm^{-2}\cdot s^{-1}} - 48.6$$

Vega magnitude

$$m_{vega} = -2.5\log\frac{F}{F_{vega}}$$

Absolute magnitude

$$M = -2.5\log F_{10pc} = -2.5\log F - \underbrace{5\log\frac{D}{10pc}}_{DM\text{: distance modulus}}$$

black body radiation

$$n = \frac{2L}{\lambda}, N = \frac{4\pi}{3}n^3\frac{1}{8} = \frac{4\pi}{3}\frac{L^3}{\lambda^3}$$
$$\frac{dN_\lambda}{d\lambda} = -\frac{4\pi L^3}{\lambda^4}, \frac{dn_\lambda}{d\lambda} = \frac{\overbrace{2}^{polorization}\times 4\pi}{\lambda^4}$$

Planck Law

$$\nu = \frac{c}{\lambda}$$
$$u_\lambda = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{k_BT\lambda}}-1}\right)$$
$$u_\nu = \frac{8\pi\nu^2}{c^3}\left(\frac{h\nu}{e^{\frac{h\nu}{k_BT}}-1}\right)$$

Wien’s Displacement Law

$$\lambda_{max}\cdot T = 2.89\times 10^{-3}$$

$$u = \int u_\lambda d\lambda = \frac{8\pi^5K_B^4}{15h^3c^3}T^4 \approx 10^{-15}T^4$$

$$flux = uc = 4\pi I$$

Stefan-Boltzmann

$$f(\text{星體表面的通量}) = \int f\cos\theta d\Omega = I\pi$$
$$f_B = 10^{-7}T^4(\frac{J}{m^2sK^4})$$
$$L_* = \int f_B dA = 4\pi R_*^2\sigma T^4$$
$$F_*(\text{距離 D 的通量}) = \frac{L_*}{4\pi D^2}$$