

天文學一期中考

宇宙學

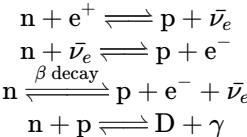
亂七八糟的宇宙常數

$$\begin{aligned} u &\propto T^4 \\ H &= \frac{\dot{a}}{a} \\ H^2 &\propto \rho(\text{宇宙物質密度}) \\ \rho_m &\propto a^{-3} \\ \rightarrow H &\propto a^{-3/2} \\ \rightarrow a &\propto t^{2/3} \\ \frac{a_0}{a} &= 1 + z \end{aligned}$$

溫度與能量

from $E = k_B T, 1eV \approx 10^4 K$

一些簡單的反應式



物質組成

$$\begin{aligned} \frac{N_n}{N_p} &= \frac{1}{7} \\ 2\text{ n} + 14\text{ p} &\longrightarrow (2\text{ n} + 2\text{ p}) + 12\text{ p} \longrightarrow \text{He} + 12\text{ H} \\ \frac{m_{He}}{n_H} &= \frac{4 \times 1}{1 \times 12} = \frac{1}{3} \end{aligned}$$

天球座標

| 英文 | 中文 | 單位 |
|-----------------|----|------|
| Right Ascension | 經度 | hour |

| 英文 | 中文 | 單位 |
|-------------|----|--------|
| Declination | 緯度 | degree |

潮汐力

$$F_g = \frac{GMm}{L^2} \left[1 + \left(\frac{2r}{L} \right) \cos \theta \right]$$

θ : 物體的軸與中心點和大質量星體連線的夾角

solid angle

$$\begin{aligned} &A(\theta_1, \phi_1) \\ &B(\theta_2, \phi_2) \\ \overline{AB} &= 2 - 2 \cos \theta_1 \cos \theta_1 \cos(\phi_1 - \phi_2) - 2 \sin \theta_1 \sin \theta_2 \\ \delta(A, B \text{ 夾角}) &= \cos^{-1} [\cos \theta_1 \cos \theta_1 \cos(\phi_1 - \phi_2) + \sin \theta_1 \sin \theta_2] \approx \sqrt{(\Delta \theta)^2 + \cos^2 \theta (\Delta \phi)^2} \end{aligned}$$

角動量

$$\frac{1}{r_0} = \frac{GMm^2}{L^2}$$

The Virial Theorem

in a gravitational bound system

$$\begin{aligned} K &= -\frac{1}{2}U \\ \text{let } Q &:= \sum_i m_i \vec{v}_i \cdot \vec{r}_i = \frac{1}{2} \frac{dI}{dt} \\ \frac{dQ}{dt} &= 2K + \sum_i \vec{f}_i \cdot \vec{r}_i = 2K + U \end{aligned}$$

星等/Luminosity/color

| name | symbol | dimension |
|------------|--------|------------------------------------|
| Luminosity | L | $J \cdot s^{-1} = W$ |
| flux | F | $W \cdot m^{-2}$ |
| intensity | I | $W \cdot m^{-2} \cdot \Omega^{-1}$ |

$$F(\text{flux}) = \frac{L}{4\pi D^2}$$

magnitude

$$m = -2.5 \log F + ZeroPoint$$

AB magnitude

$$m_{AB} = -2.5 \log \frac{F}{erg \cdot cm^{-2} \cdot s^{-1}} - 48.6$$

Vega magnitude

$$m_{vega} = -2.5 \log \frac{F}{F_{vega}}$$

Absolute magnitude

$$M = -2.5 \log F_{10pc} = -2.5 \log F - \underbrace{5 \log \frac{D}{10pc}}_{DM: \text{ distance modulus}}$$

black body radiation

$$n = \frac{2L}{\lambda}, N = \frac{4\pi}{3} n^3 \frac{1}{8_{polorization}} = \frac{4\pi}{3} \frac{L^3}{\lambda^3}$$

$$\frac{dN_{\lambda}}{d\lambda} = -\frac{4\pi L^3}{\lambda^4}, \frac{dn_{\lambda}}{d\lambda} = \frac{\widehat{2}}{\lambda^4} \times 4\pi$$

Planck Law

$$\nu = \frac{c}{\lambda}$$

$$u_{\lambda} = \frac{8\pi \hbar c}{\lambda^5} \left(\frac{1}{e^{\frac{\hbar c}{k_B T \lambda}} - 1} \right)$$

$$u_{\nu} = \frac{8\pi \nu^2}{c^3} \left(\frac{h\nu}{e^{\frac{\hbar \nu}{k_B T}} - 1} \right)$$

Wien’s Displacement Law

$$\lambda_{max} \cdot T = 2.89 \times 10^{-3}$$

$$u = \int u_{\lambda} d\lambda = \frac{8\pi^5 K_B^4}{15h^3 c^3} T^4 \approx 10^{-15} T^4$$

$$flux = uc = 4\pi I$$

Stefan-Boltzmann

$$f(\text{星體表面的通量}) = \int f \cos \theta d\Omega = I\pi$$

$$f_B = 10^{-7} T^4 (\frac{J}{m^2 s K^4})$$

$$L_{*} = \int f_B dA = 4\pi R_{*}^2 \sigma T^4$$

$$F_{*}(\text{距離 D 的通量}) = \frac{L_{*}}{4\pi D^2}$$