天文學一期中考

宇宙學

亂七八糟的宇宙常數

$$u \propto T^4$$
 $H = \frac{\dot{a}}{a}$
 $H^2 \propto
ho$ (宇宙物質密度)
 $ho_m \propto a^{-3}$
 $ho H \propto a^{-3/2}$
 $ho a \propto t^{2/3}$
 $ho a = 1 + z$

溫度與能量

from $E=k_BT$, $1eV pprox 10^4K$

一些簡單的反應式

$$egin{aligned} \mathbf{n} + \mathbf{e}^+ & \Longrightarrow \mathbf{p} + ar{
u_e} \\ \mathbf{n} + ar{
u_e} & \Longrightarrow \mathbf{p} + \mathbf{e}^- \\ \mathbf{n} & \stackrel{eta \, \mathrm{decay}}{\longleftrightarrow} \mathbf{p} + \mathbf{e}^- + ar{
u_e} \\ \mathbf{n} + \mathbf{p} & \Longrightarrow \mathbf{D} + \gamma \end{aligned}$$

物質組成

$$egin{aligned} rac{N_n}{N_p} &= rac{1}{7} \ 2\,\mathrm{n} + 14\mathrm{p} &\longrightarrow (2\,\mathrm{n} + 2\mathrm{p}) + 12\mathrm{p} &\longrightarrow \mathrm{He} + 12\,\mathrm{H} \ rac{m_{He}}{n_H} &= rac{4 imes 1}{1 imes 12} = rac{1}{3} \end{aligned}$$

天球座標

英文	中文	單位
Right Ascension	經度	hour

英文	中文	單位
Declination	緯度	degree

潮汐力

$$F_g=rac{GMm}{L^2}\left[1+\left(rac{2r}{L}
ight)\cos heta
ight] \ heta$$
:物體的輔與中心點和大質量早體連線的來角

solid angle

$$A(heta_1,\phi_1)$$
 $B(heta_2,\phi_2)$
$$\overline{AB} = 2 - 2\cos heta_1\cos heta_1\cos(\phi_1-\phi_2) - 2\sin heta_1\sin heta_2$$
 $\delta(A,B$ 夾角) $=\cos^{-1}\left[\cos heta_1\cos heta_1\cos(\phi_1-\phi_2) + \sin heta_1\sin heta_2
ight] pprox \sqrt{(\Delta heta)^2 + \cos^2 heta(\Delta\phi)^2}$

角動量

$$rac{1}{r_0}=rac{GMm^2}{L^2}$$

The Virial Theorem

in a gravitational bound system

$$K = -rac{1}{2}U$$
 $\operatorname{let} Q := \sum_i m_i ec{v_i} \cdot ec{r_i} = rac{1}{2}rac{dI}{dt}$ $rac{dQ}{dt} = 2K + \sum_i ec{f_i} \cdot ec{r_i} = 2K + U$

星等/Luminosity/color

name	symbol	dimension
Luminosity	L	$J\cdot s^{-1}=W$
flux	F	$W\cdot m^{-2}$
intensity	I	$W \cdot m^{-2} \cdot \Omega^{-1}$

$$F(\mathrm{flux}) = rac{L}{4\pi D^2}$$

magnitude

$$m = -2.5 \log F + ZeroPoint$$

AB magnitude

$$m_{AB} = -2.5 \log rac{F}{erq \cdot cm^{-2} \cdot s^{-1}} - 48.6$$

Vega magnitude

$$m_{vega} = -2.5 \log rac{F}{F_{vega}}$$

Absolute magnitude

$$M = -2.5 \log F_{10pc} = -2.5 \log F - \underbrace{5 \log rac{D}{10pc}}_{DM: ext{ distance modulus}}$$

black body radiation

$$n=rac{2L}{\lambda}, N=rac{4\pi}{3}n^3rac{1}{8}=rac{4\pi}{3}rac{L^3}{\lambda^3} \ rac{dN_\lambda}{d\lambda}=-rac{4\pi L^3}{\lambda^4}, rac{dn_\lambda}{d\lambda}=rac{2}{\lambda^4} imes 4\pi$$

Planck Law

$$egin{aligned}
u &= rac{c}{\lambda} \ u_{\lambda} &= rac{8\pi hc}{\lambda^5} \left(rac{1}{e^{rac{hc}{k_BT\lambda}}-1}
ight) \ u_{
u} &= rac{8\pi
u^2}{c^3} \left(rac{h
u}{e^{rac{h
u}{k_BT}}-1}
ight) \end{aligned}$$

Wien's Displacement Law

$$\lambda_{max} \cdot T = 2.89 imes 10^{-3} \ u = \int u_{\lambda} d\lambda = rac{8\pi^5 K_B^4}{15 h^3 c^3} T^4 pprox 10^{-15} T^4$$

$$flux=uc=4\pi I$$

Stefan-Boltzmann

$$f(星體表面的通量) = \int f\cos\theta d\Omega = I\pi$$
 $f_B = 10^{-7}T^4(rac{J}{m^2sK^4})$
 $L_* = \int f_B dA = 4\pi R_*^2 \sigma T^4$
 $F_*(距離 D 的通量) = rac{L_*}{4\pi D^2}$