HW 5 (due March 28)

1. Consider a ball with a radius R, mass M, and a uniform density $\rho \equiv \frac{M}{\frac{4\pi}{3}R^3}$. Please calculate it's self gravitational potential.

2. Demonstrate that the derived form of the Kepler's 1st law,

$$r = \frac{r_0}{(1+e\cos\phi)},$$

where e is ellipticity, ϕ is the angle in the polar coordinate, and r_0 is a characteristic radius, is a ellipse that can be described as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

3. Given the equation in Problem 2, what is the averaged distance between the star and the orbiting planet? That's, derive

$$\langle r \rangle \equiv \frac{1}{2\pi}\int_{\phi=0}^{2\pi} r(\phi)\,d\phi.$$

Hint: $\int_0^{2\pi} \frac{d\theta}{1+e\cos\theta} = \frac{2\pi}{\sqrt{1-e^2}}$.