Mobile antenna model description

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Abstract

This document outlines a model that describes the spread of cell phone signal for the purposes of mapping cell phone antenna coverage areas and detecting the user's location somewhat more accurately.

1 Model description

1.1 Assumptions

The connection of the cell phone to the mobile network is facilitated through a system of antennas. Usually, the operator shares very little information about these antennas; nevertheless, some parameters, such as antenna location, are almost always available, sometimes through crowdsourced registers. We thus assume the antenna locations $\vec{x_0} = (x_0, y_0)$ are fixed and known; we also assume a planar coordinate system for simplicity.

1.2 Signal spread model

We assume three orthogonal effects on the signal strength $S_i(\vec{x})$ of antenna i for a given location $\vec{x} = (x, y)$, that is:

- antenna power P_i,
- distance from the antenna $d_i(\vec{x}) = ||\vec{x} - \vec{x_i}||$,
- azimuth of the vector from the antenna to the location $\varphi_i(\vec{x}) = \arctan \frac{y-y_i}{x-x_i}$.

We can employ a probabilistic approach to the problem: we first model the probability of a user being at a given location $\vec{x}$ if we know that he or she is connected to an antenna i and then use the Bayesian rule to determine signal strength as the inverse conditional probability:

$$S_i(\vec{x}) = P(i|\vec{x}) = P(\vec{x}|i)\frac{P(i)}{P(\vec{x})} \tag{1}$$

We can isolate the $P(i)$ as the antenna power P_i and amalgamate the remaining two terms to make the model agnostic to an apriori distribution of users (i.e. population/settlement density):

$$S_i(\vec{x}) = P_i\frac{P(\vec{x}|i)}{P(\vec{x})} = P_i \cdot L_i(\vec{x}) \tag{2}$$

The resulting term $L_i(\vec{x})$ should capture the remaining two effects of distance and azimuth. As we assume the effects to be orthogonal, we can express it as

$$L_i(\vec{x}) = f_d(d_i(\vec{x})) \cdot f_a(\varphi_i(\vec{x})) \tag{3}$$

1.2.1 Distance decay

As we operate on macroscopic scales, the distance decay of an EM signal is governed by an inverse square law:

$$S_i(\vec{x}) \propto d_i(\vec{x})^{-2} \tag{4}$$

This, however, would imply the signal strength goes to infinity near the location of the antenna. Therefore, it is useful to employ a small correcting factor. Using this, we arrive at a probabilistic description by a Cauchy distribution, which is useful:

$$d_i(\vec{x}) \sim \text{Cauchy}(0, \gamma_i) \tag{5}$$

where γ_i is the distance decay scale factor. We can use the probability density function of the Cauchy distribution as our distance decay function:

$$f_d(d_i(\vec{x})) = \frac{1}{\pi\gamma_i \left[1 + \left(\frac{d_i(\vec{x})}{\gamma_i}\right)^2\right]} = \frac{\gamma_i}{\pi[d_i(\vec{x})^2 + \gamma_i^2]} \tag{6}$$

1.2.2 Angular distribution

To model the angular dispersion of the signal, we can use the von Mises distribution in a manner similar to the Cauchy distribution for distance decay. The von Mises distribution is a circular analogue of the normal distribution:

$$f(\varphi) = \frac{\exp\left[\kappa \cos\left(\varphi - \alpha\right)\right]}{2\pi I_0(\kappa)} \tag{7}$$

where $I_0(\kappa)$ is the modified Bessel function of order zero, α is the principal angle and κ is the directional concentration.

Using this, we define

$$f_a(\varphi_i(\vec{x})) = \frac{\exp\left[\kappa_i \cos\left(\varphi_i(\vec{x}) - \alpha_i\right)\right]}{2\pi I_0(\kappa_i)} \tag{8}$$

which leaves us with two parameters for the antenna: orientation (principal angle) α and angular signal concentration (narrowness) κ.

1.2.3 Final signal strength equation

Combining the above results, we arrive at the final signal model result

$$\begin{aligned}
S_i(\vec{x}) &= P_i \cdot f_d(d_i(\vec{x})) \cdot f_a(\varphi_i(\vec{x})) && (9)\\
&= P_i \frac{\gamma_i \exp\left[\kappa_i \cos\left(\varphi_i(\vec{x}) - \alpha_i\right)\right]}{2\pi^2 I_0(\kappa_i)[d_i(\vec{x})^2 + \gamma_i^2]} && (10)\\
&= \frac{P_i\gamma_i}{2\pi^2 I_0(\kappa_i)} \cdot \frac{\exp\left[\kappa_i \cos\left(\varphi_i(\vec{x}) - \alpha_i\right)\right]}{d_i(\vec{x})^2 + \gamma_i^2} && (11)\\
&= K_i \cdot \frac{\exp\left[\kappa_i \cos\left(\varphi_i(\vec{x}) - \alpha_i\right)\right]}{d_i(\vec{x})^2 + \gamma_i^2} && (12)
\end{aligned}$$

Since the signal strengths are relative to each other, we should specify a condition on their absolute values; this can be formulated e.g. as

$$\sum_{a=1}^{n_a} P_a \gamma_a = 1 \tag{13}$$

1.3 Antenna parameters

The signal strength model uses four additional parameters (in addition to location) to describe the antennas:

- Antenna orientation azimuth $\alpha \in [0; 2\pi)$ (with zero pointing north).
- Antenna power or overall signal strength $P \in \mathbb{R}^+$. This is somehow related to the power of the antenna transmitter.
- Distance decay scale parameter $\gamma \in \mathbb{R}^+$, defining the rate of signal strength diminishing with increasing distance from the antenna. This can be related e.g. to the vertical orientation of the antenna with respect to the surface.
- Angular concentration of antenna signal $\kappa \in \mathbb{R}_0^+$. The higher the concentration (narrowness), the more the signal power is concentrated along the direction of the antenna orientation. $\kappa = 0$ means the antenna is isotropic and radiates the signal uniformly in all directions.

Sometimes, some of the parameters may be known a priori, such as the antenna azimuths. The rest of the parameters need to be estimated from the antenna connection data, which should be the next large task.

2 Model parameter estimation

To estimate the antenna parameters, several variants of data sources may be used. In this estimation scenario, we consider a set of n_p places p with locations $\vec{x}_p$. We denote the signal strength of a given antenna a at that place as S_{ap}, the distance d_{ap} and angle φ_{ap} of a given place from it as in 1.2.

In addition to the place data, we enter data about a set of n_u users and their simultaneous connections to antennas (that is, how often they are connected to what antennas while not moving themselves). Therefore, we propose using night-time signalling (SS.7 logs) or CDR data where antenna connection variability is most likely to come from antenna signal strength variation, not from user movements. Given this, we may derive for each user a set of connection time fractions f_{ua} denoting the likelihood of the connection of the user to the respective antennas, satisfying $\sum_{a=1}^{n_a} f_{ua} = 1 \, \forall u$.

2.1 Estimation with known locations

If we have an additional data source that allows us to determine the actual night-time positions of the users (such as independent mobility tracker app logs), the model estimation becomes simpler because we can compute the user location probabilities with respect to the places p directly as p_{up}.

The algorithm works as an EM procedure, trying to estimate the signal strengths for all places and antennas with antenna parameters as proxy variables. The E step estimates the antenna parameters from signal strengths using OLS, with the initial values obtained from location averaging, and the M step estimates the signal strengths from the antenna parameters using the equations from 1.

2.1.1 Antenna dominance fractions

Dominance fractions denote how much the given place is dominated by a signal from a given antenna, an therefore, the probability of a user at a given location being connected to the antenna:

$$\Psi_{ap} = \frac{\sum_{u=1}^{n_u} p_{up} f_{ua}}{\sum_{u=1}^{n_u} p_{up}} \tag{14}$$

This ensures that $\sum_{a=1}^{n_a} \Psi_{ap} = 1 \, \forall p$.

2.1.2 Expectation step

In this step, we compute the optimal antenna parameters using OLS on a linearized signal strength function (with parameters from the previous round) set as equal to the E-computed signal strength:

$$
\begin{aligned}
&\frac{1}{P_a^{(i)}} \cdot P_a^{(i+1)} \\
&+\frac{d_{ap}^2 - {\gamma_a^{(i)}}^2}{\gamma_a^{(i)}(d_{ap}^2 + {\gamma_a^{(i)}}^2)} \cdot \gamma_a^{(i+1)} \\
&+\kappa_a^{(i)} \sin\left(\varphi_{ap} - \alpha_a^{(i)}\right) \cdot \alpha_a^{(i+1)} \\
&+\left[\cos\left(\varphi_{ap} - \alpha_a^{(i)}\right) - \frac{I_1(\kappa_a^{(i)})}{I_0(\kappa_a^{(i)})}\right] \cdot \kappa_a^{(i+1)} = \\
&\frac{S_{ap}^{(i+1)}}{S_{ap}^{(i)}} + 1 + \frac{d_{ap}^2 - {\gamma_a^{(i)}}^2}{d_{ap}^2 + {\gamma_a^{(i)}}^2} + \kappa_a^{(i)} \alpha_a^{(i)} \sin\left(\varphi_{ap} - \alpha_a^{(i)}\right) + \kappa_a^{(i)} \left[\cos\left(\varphi_{ap} - \alpha_a^{(i)}\right) - \frac{I_1(\kappa_a^{(i)})}{I_0(\kappa_a^{(i)})}\right]
\end{aligned}
\tag{15}
$$

2.1.3 Maximization step

In this step, we estimate the signal strengths for places from their antenna dominances and antenna parameters:

$$
S_{ap}^{(i+1)} = \Psi_{ap} \sum_{b=1}^{n_a} \frac{P_b^{(i)} \gamma_b^{(i)} \exp\left[\kappa_b^{(i)} \cos\left(\varphi_{bp} - \alpha_b^{(i)}\right)\right]}{2\pi^2 I_0(\kappa_b^{(i)})\left[d_{bp}^2 + {\gamma_b^{(i)}}^2\right]} \tag{16}
$$

2.1.4 Initial antenna parameter estimations

The initial values can be obtained as follows:

Antenna principal angles α_a – if not known – can be determined as weighted circular means

$$
\alpha_a = \arctan \frac{\sum_{p=1}^{n_p} \Psi_{ap} \sin\varphi_{ap}}{\sum_{p=1}^{n_p} \Psi_{ap} \cos\varphi_{ap}} \tag{17}
$$

Antenna angular concentrations κ_a can be determined using the von Mises-Fisher iterative estimation procedure from the parameter $\bar{R}_a$:

$$
\bar{R}_a = \frac{\left(\sum_{p=1}^{n_p} \Psi_{ap} \sin\varphi_{ap}\right)^2 + \left(\sum_{p=1}^{n_p} \Psi_{ap} \cos\varphi_{ap}\right)^2}{\left(\sum_{p=1}^{n_p} \Psi_{ap}\right)^2} \tag{18}
$$

With this parameter, we can produce the initial estimate for κ_a as

$$
\kappa_a^{(0)} = \frac{\bar{R}_a(2 - \bar{R}_a^2)}{1 - \bar{R}_a^2} \tag{19}
$$

and then repeat the following equation until convergence

$$
\kappa_a^{(i+1)} = \kappa_a^{(i)} - \frac{A_p(\kappa_a^{(i)}) - \bar{R}_a}{1 - A_p(\kappa_a^{(i)})^2 - \frac{A_p(\kappa_a^{(i)})}{\kappa_a^{(i)}}} \tag{20}
$$

where $A_p(x) = \frac{I_1(x)}{I_0(x)}$ is the ratio of modified Bessel functions of the first and zeroth order respectively. Three iterations are usually sufficient.

Antenna distance decay parameter γ_a may be estimated as the weighted mean distance of a dominated place from the antenna:

$$\gamma_a = \frac{\sum_{p=1}^{n_p} \Psi_{ap} d_{ap}}{\sum_{p=1}^{n_p} \Psi_{ap}} \tag{21}$$

Strengths of the antennas P_a, to honor the condition from (13), can be estimated as the relative size of the antenna's area of dominance to the total area of study. We need to factor out the general influence of the distance decay parameter, too:

$$P_a = \frac{\sum_{p=1}^{n_p} \Psi_{ap} A_p}{\gamma_a \sum_{p=1}^{n_p} A_p} \tag{22}$$

where A_p is the area corresponding to place p – if the places are only given as points, the areas may be computed e. g. using Voronoi polygons.

2.2 Estimation with unknown locations

If we do not have data about real user locations p_{up}, we can supply them by employing an EM estimation procedure over the proposed approach.

To do this, it is useful to know at least roughly the expected user densities over the places used w_p. These densities may be derived from an apriori population distribution measure such as a census grid or, alternatively, a land use layer such as GHSL aggregated to a reasonable level.

2.2.1 Expectation step

This step of the procedure estimates the user locations from signal strengths in those places by computing place affinities as

$$a_{up}^{(i+1)} = \left(1 + \frac{w_p - \sum_{u=1}^{n_u} p_{up}^{(i)}}{w_p + \sum_{u=1}^{n_u} p_{up}^{(i)}}\right)^{\frac{1}{k_c}} \sqrt{\frac{1}{n_a} \sum_{a=1}^{n_a} \left(f_{ua} - \frac{S_{ap}^{(i)}}{\sum_{b=1}^{n_a} S_{bp}^{(i)}}\right)^2} \tag{23}$$

where k_c is the crowding tolerance coefficient specifying the degree of adherence to the apriori place weights w_p. Its higher values mean a more different spatial distribution of users can arise.

Then the actual localization probabilities are produced by normalizing the affinities to sum to one:

$$p_{up}^{(i+1)} = \frac{a_{up}^{(i+1)}}{\sum_{q=1}^{n_p} a_{uq}^{(i+1)}} \tag{24}$$

2.2.2 Maximization step

This step of the procedure uses the method from 2.1 to derive $S_{ap}^{(i)}$ from $p_{up}^{(i)}$.