YIELD CURVES

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Yield_Curve.doc	July, 2006	From Yield_Curves_Rev_D.pdf
Yield_Curve.doc	July, 2011	Removed Section 8.0, Smoothing Option for Forward Rate Calculations

CONTENTS

Section **Page**

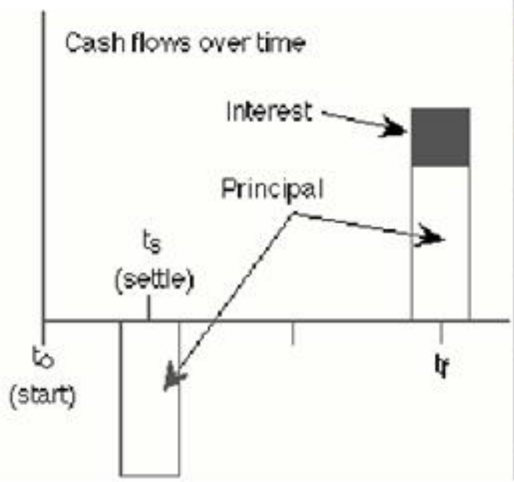
1.0 OVERVIEW

This document is intended to provide information on some of the methodology used to build the yield curves. Each section consists of a brief description followed by an example.

2.0 YIELD CURVE CALCULATIONS – CASH GRID POINTS

2.1 Discount Factor Calculations

Consider a grid point based on a cash interest rate. At the settlement date, there will be a cash flow equal to the principal. At the end of the time period specified, there will be a cash flow consisting of the principal and the interest earned during the period. If the **Index Format**, as specified on the *Index Definition* screen is entered as **DF**, the discount factor for the end date (t_f) is calculated as described below.



The basis for the calculation is that the sum of the discounted cash flows must equal zero:

$$\sum_{i=1}^{n} df_i c_i = 0 \quad (1)$$

$$df_i = \text{discount factors}$$

$$c_i = \text{cash flows}$$

Note that the cash flows include both principal and interest payments.

The discount factor for the settlement date is determined as follows:

The present value, PV, of a cash flow of N occurring on the settlement date, t_s, is given by:

$$PV = \frac{N}{(1 + r(t_s - t_o))} \quad (2)$$

where

r = interest rate (between starting date and settlement date)

t_s = time (day count factor) of settlement date

t_o = time (day count factor) of starting date (usually zero)

The discount factor, df_s, is simply equal to PV divided by N, or

$$df_s = \frac{1}{1 + r(t_s - t_o)} \quad (3)$$

Solving the summation equation, $\sum_{i=1}^{n} df_i c_i = 0$, the discount factor for the final payment is then given by:

$$df_f = \frac{df_s}{1 + r_f(t_f - t_s)} \quad (4)$$

where

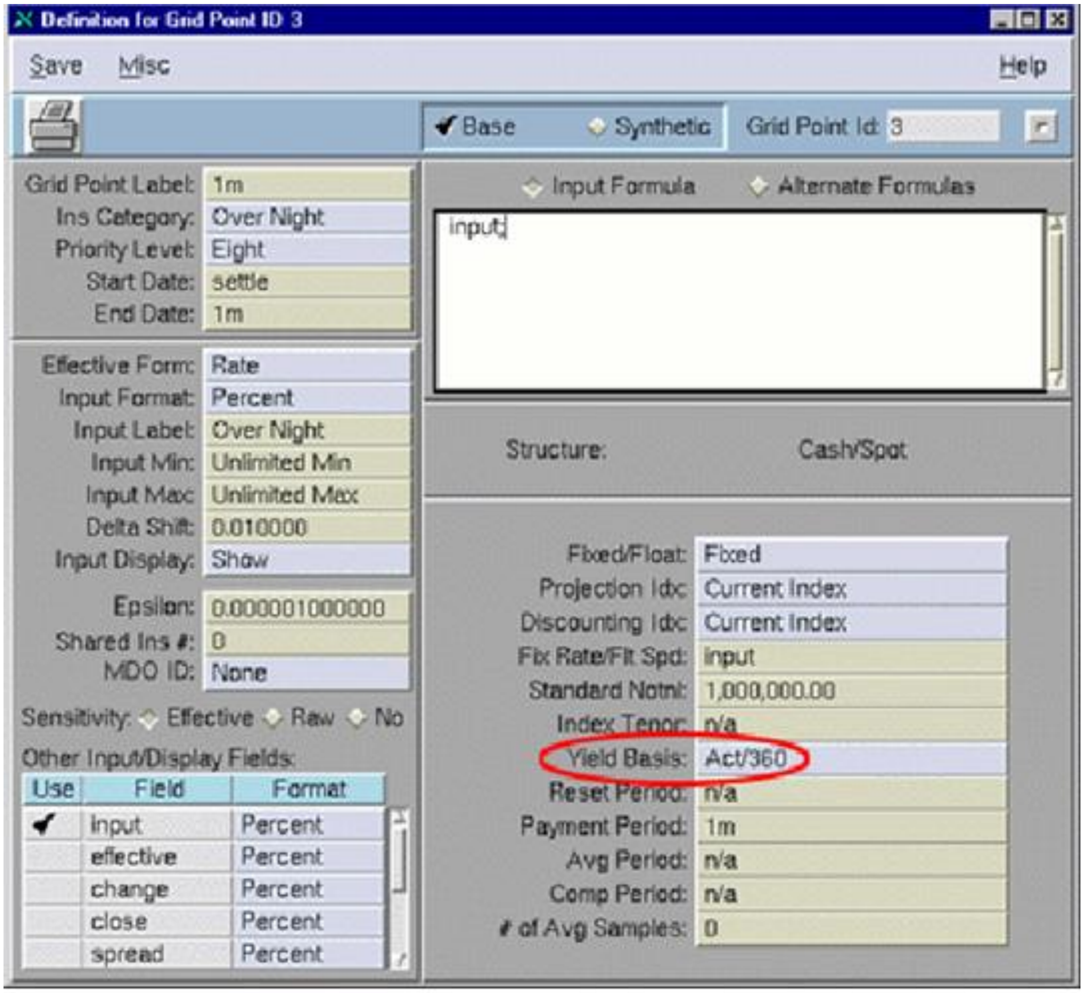
df_f = discount factor for final date

r_f = interest rate between settlement date and final date

t_f = time (day count factor) of final date

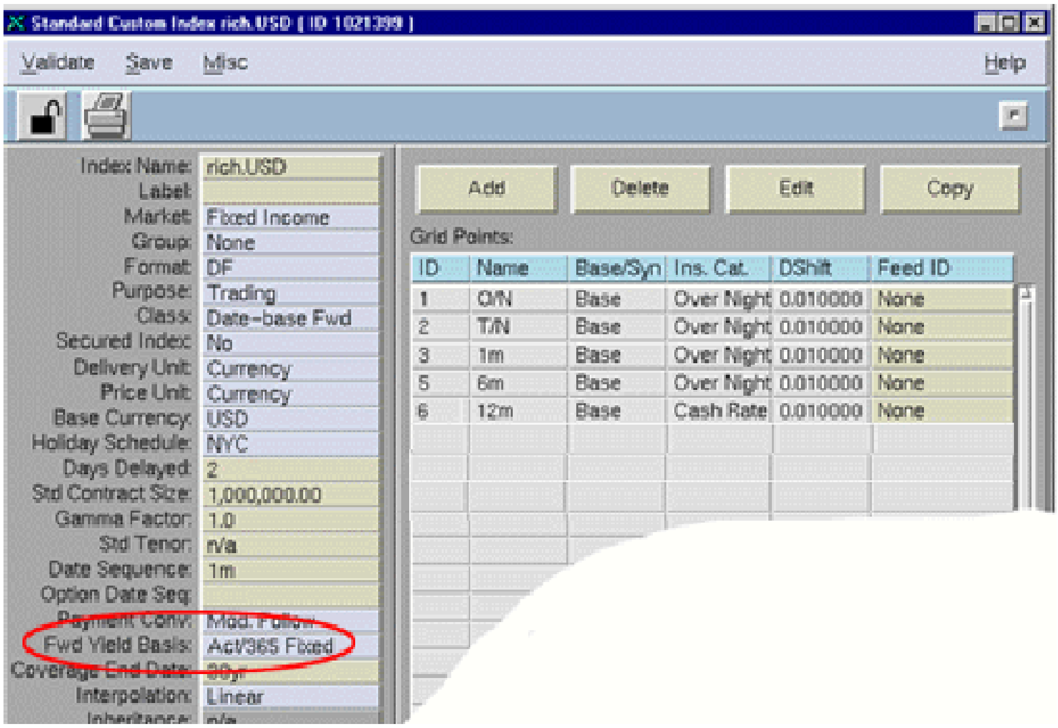
t_s = time (day count factor) of settlement

Note that the day count factor used to calculate the discount factor must be based on the same date convention as that specified for the interest rate data in the Yield Basis field (see below). If the interest rate data is on a Act/360 basis, the day count factor used to calculate the discount factor is also on an Act/360 basis.



Notes:

For cash rates, discount factors are calculated independently for each grid point. For example, the 6m discount factor is calculated directly from the 6m input and is not affected by the 3m discount factor. However, the derived forward rate is determined by both the 3m and 6m discount factors.

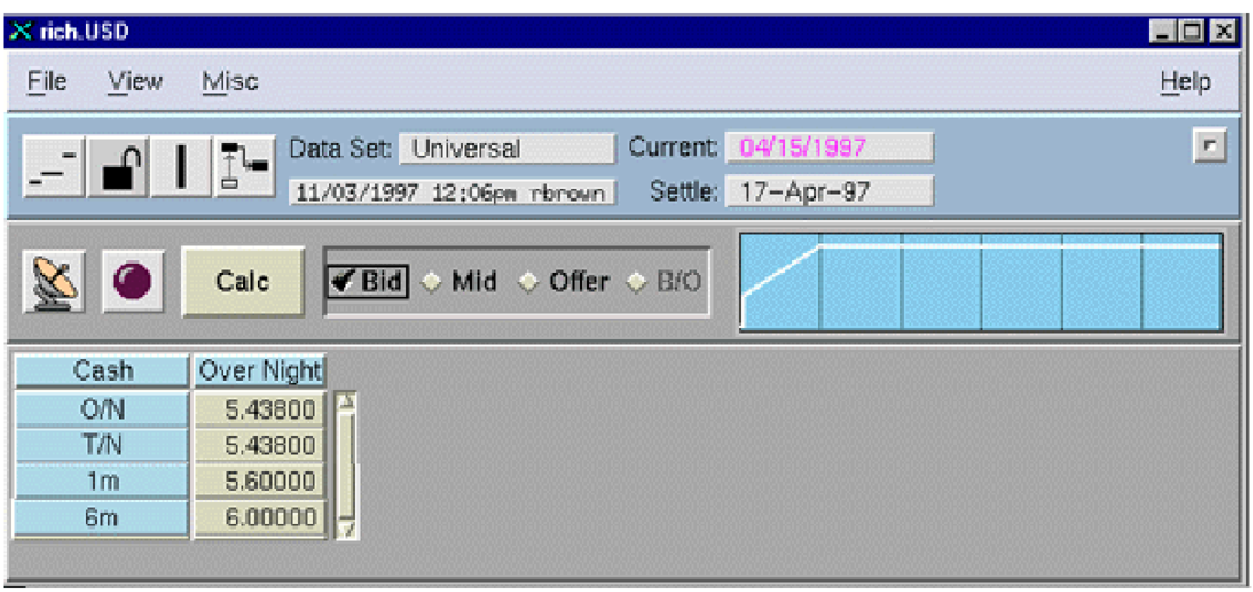


2.2 Zero and Forward Rate Calculations

- Description of Zero Rate Calculations
- Description of Forward Rate Calculations

2.3 Example

Calculate the discount factors for the T/N, 1m, and 6m grid points based on Cash. Assume the grid points have been created and the interest rates assigned as shown below.



The **Yield Basis** (date convention) for each grid point is set as Act/360, while the index **Yield Basis** is Act/365 Fixed. The day count factors (DCF) for each convention are given below for reference. The current date is 4/15/97 and the settlement date is 4/17/97. Smoothing is turned off.

Grid Point	Date	DCF (Act/360)	DCF (Act/365)
O/N	4/16/97	0.002778	0.0027397
T/N	4/17/97	0.005556	0.0054795
1m	5/19/97	0.094444	0.0821918
6m	10/15/97	0.508333	0.501370

The discount factors are calculated as follows:

O/N:

The discount factor is found from the present value of a payment one day in the future, given by equation (3) as

$$df_{O/N} = \frac{1}{1 + r_{O/N}\left(t_{O/N}\right)} = \frac{1}{1 + 0.5438(0.002778)} = 0.99984897$$

T/N:

The discount factor is found from the present value of a payment two days in the future, or

$$df_{T/N} = \frac{1}{1 + r_{T/N}\left(t_{T/N}\right)} = \frac{1}{1 + 0.5438(0.005556)} = 0.9996979$$

1m

The 1m discount factor is obtained from Equation (4)

$$df_{1m} = \frac{df_{T/N}}{1 + r_{1m}\left(t_{1m} - t_{T/N}\right)} = \frac{0.9996979}{1 + 0.0560(0.094444 - 0.005556)} = 0.994746$$

6m

The 6m discount factor is obtained from Equation (4)

$$df_{6m} = \frac{df_{T/N}}{1 + r_{6m}\left(t_{6m} - t_{T/N}\right)} = \frac{0.9996979}{1 + 0.060(0.50137 - 0.005479)} = 0.970424$$

Once the discount factors have been calculated, the zero and forward rates can also be calculated as described in Zero Rate Calculation and Forward Rate Calculation.

The day count factor based on the Date Sequence on the Index Definition screen, Act/365 in this example, is used for the zero and forward rate calculations.

$$azr = \left(df_t^{\left(-1/t\right)} - 1\right);\quad ccr = \left(-1/t\right)\cdot \ln\left(df_t\right);\quad f_{1-2} = \left(\frac{1}{\left(t_2 - t_1\right)}\right) * \left(df_1/df_2 - 1\right)$$

For the 1m grid point,

Annualized Zero Rate

$$azr_{1m} = \left(df_{1m}^{\left(-1/t_m\right)} - 1\right) = \left(0.994746^{\left(-1/0.093151\right)} - 1\right) = 0.058178$$

Continuously Compounded Zero Rate

$$ccr_{1m} = \left(-1/t_{1m}\right)\cdot \ln\left(df_{1m}\right) = \left(-1/0.093151\right)\cdot \ln\left(0.994746\right) = 0.056548$$

Forward Rate from 1m to 6m

$$f_{1m-6m} = \left(\frac{1}{\left(t_{6m} - t_{1m}\right)}\right) * \left(df_{1m}/df_{6m} - 1\right) = \left(\frac{1}{\left(0.506849 - 0.093151\right)}\right) * \left(0.994746/0.97011 - 1\right) = 0.061387$$

The figure below shows the System output for the all of the grid points in this example. Note that the forward rate reported in a given row is the forward rate between the grid point on that row and the grid int on the next row.

rich.USD Output (Mid)

File Edit Format Configure Reporting View Search Help

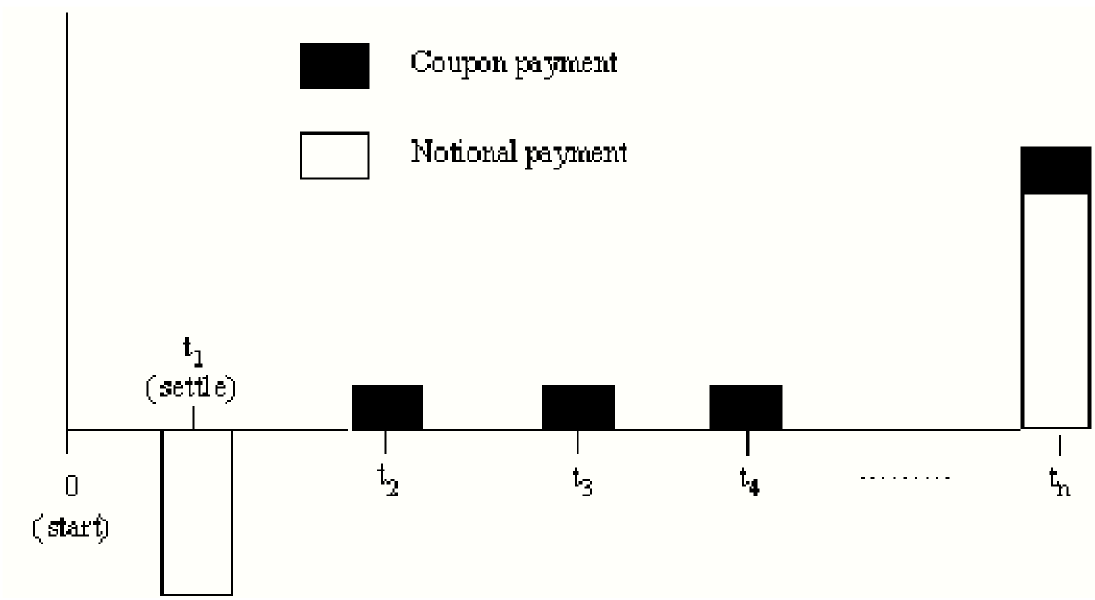
Date Seq input

Date	Disc. Factor	Ann Zero Rate	CC Zero Rate	Forward Rate	DCF(Act/365 Fixed)	Grid Point
04/15/1997	1.000000	0.000000	0.000000	5.513528	0.000000	
04/16/1997	0.999849	5.667915	5.513111	5.514361	0.002740	O/N
04/17/1997	0.999698	5.668355	5.513528	5.677778	0.005479	T/N
05/19/1997	0.994746	5.817804	5.654860	6.139870	0.093151	1m
10/15/1997	0.970423	6.171058	5.988137	6.139870	0.501370	6m

3.0 YIELD CURVE CALCULATIONS – BOND GRID POINTS

3.1 Discount Factor Calculation

Consider a bond structure as illustrated below, an initial notional payment on the settlement date, periodic coupon payments, and final notional and coupon payments on the end date.



A grid point constructed on a bond interest rate is based on the relation that the sum of the discounted cash flows (notional and coupon payments) must equal zero.

$$\sum_{i=1}^{n} df_i c_i = 0 \qquad (1)$$

df_i = discount factors

c_i = cash flows

The cash flows are given by:

for $i = 1$: $\quad c_i = -N$

(i is the settlement date)

for $1 < i \leq n-1$: $\quad c_i = N r_i (t_i - t_{i-1})$

for $i = n$: $\quad c_n = N[r_n(t_n - t_{n-1}) + 1]$

where,

N = principal (notional)

r_i = rate for ith coupon payment

t_i = time of cash flow i (day count factor)

Substituting the cash flows into Eqn. (1) results in (the N terms will cancel):

$$-df_1 + \sum_{i=2}^{n-1} r_i (t_i - t_{i-1}) df_i + \left[r_n (t_n - t_{n-1}) \right] df_n = 0 \qquad (2)$$

- The discount factors (df_i) from i = 1 to n-k are obtained from interpolation of the surrounding grid points, where k is the # of unknown discount factors that must be interpolated
- The discount factors from n-k+1 to n-1 are interpolated between df_{n-k} and df_n

If linear Interpolation is used, the interpolated discount factors can be written as

$$df_i = df_{n-k} + \left(\frac{df_n - df_{n-k}}{t_n - t_{n-k}} \right) (t_i - t_{n-k})$$

and the resulting equation for the nth discount factor is

$$df_n = \frac{df_1 - \sum_{i=2}^{n-k} r_i (t_i - t_{i-1}) df_i - \sum_{i=n-k+1}^{n-1} r_i (t_i - t_{i-1}) df_{n-k} + \frac{df_{n-k}}{(t_n - t_{n-k})} \sum_{i=n-k+1}^{n-1} r_i (t_i - t_{i-1})(t_i - t_{n-k})}{\left\{ r_n (t_{ni} - t_{n-1}) + 1 + \left[\frac{\sum_{i=n-k+1}^{n-1} r_i (t_i - t_{i-1})(t_i - t_{n-k})}{(t_n - t_{n-k})} \right] \right\}}$$

If log-linear Interpolation is used, the resulting implicit equation for the nth discount factor is

$$df_n \left\{ r_n (dcf_n - dcf_{n-1}) + 1 \right\} + \left\{ \sum_{i=n-k+1}^{n-1} \left[(dcf_i - dcf_{i-1}) df_{n-k}^{\left[1 - \frac{(dcf_i - dcf_{n-i})}{(dcf_n - dcf_{n-i})} \right]} \right] \right\} df_n^{\left[\frac{(dcf_i - dcf_{n-i})}{(dcf_n - dcf_{n-i})} \right]}$$
$$= df_1 - \sum_{i=2}^{n-k} r_i (dcf_i - dcf_{i-1}) df_i$$

This equation must be solved implicitly for df_n. If it is necessary to find more than one discount factor, than a series of simultaneous equations must be solved.

3.2 Zero and Forward Rate Calculations

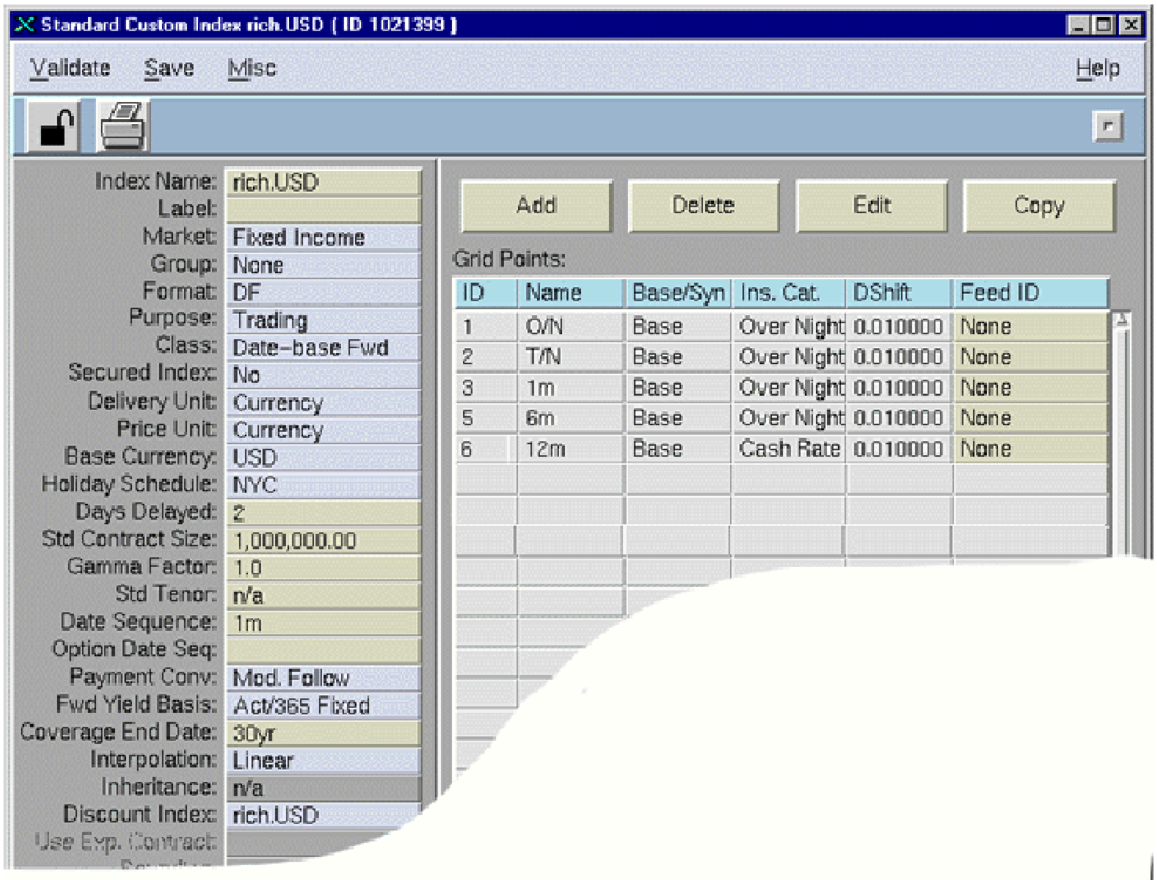
- Description of Zero Rate Calculations

- Description of Forward Rate Calculations

3.3 Example – Grid Points from Bonds

Consider the following example, in which grid points will be calculated based on a bond structure.

The figure below shows the existing grid points for the sample index.

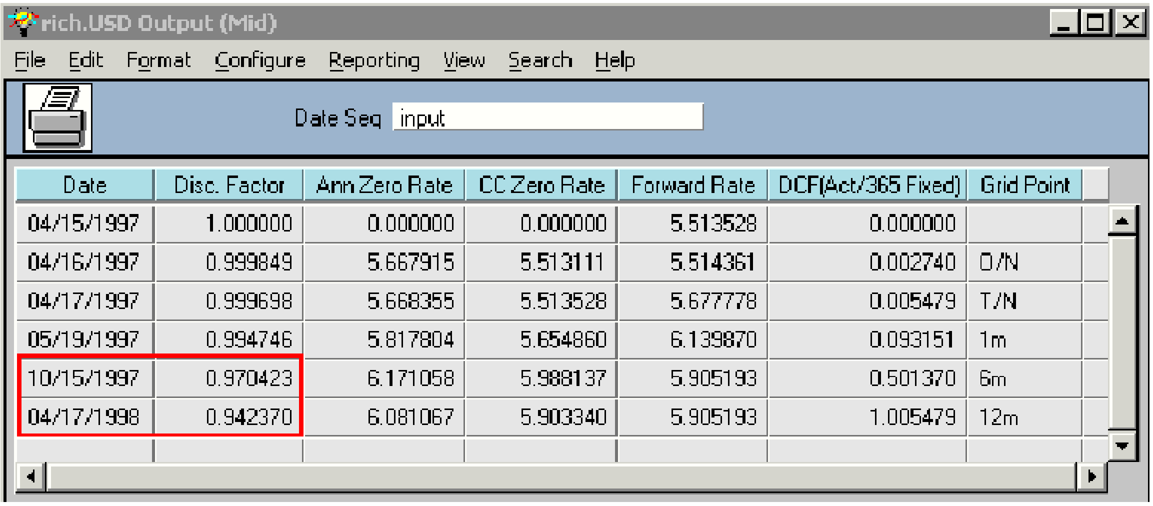


To this index, we wish to add two grid points based on a two-year bond having coupon payments every six months. The timeline for this bond is as follows:

Settlement Date	6m Payment Date	1y Payment Date	18m Payment Date	2y Payment Date
4/17/97	10/17/97	4/17/98	10/19/98	4/19/99

In this example, all grid points have the same priority, so the T/N, 6m and 12m cash points will be used, and the Bond cash flows will be used to calculate the discount factors at 18m and 2y. As discussed above, these points will be based on equation stating that the sum of the discounted cash flows for the bond must equal zero.

The *Index Output* screen for the pre-existing grid points is shown below. The discount factor for the settlement date (4/17/97) is equal to 0.9996979. The next discount factor in the Output Table is for 10/15/97, since that is the date of the 6m cash grid point. However, the bond has a payment due on 10/17/97, so the discount factor for that date needs to be calculated. To do this, we interpolate between the 1 year (12m) grid point (4/17/98, DF=0.9423704) and the 10/15/97 grid point. Using log-linear interpolation, as specified on the *Index Definition* screen, we get a value of 0.97011412. If linear interpolation is used, the results will differ.



Date	Disc. Factor	Ann Zero Rate	CC Zero Rate	Forward Rate	DCF(Act/365 Fixed)	Grid Point
04/15/1997	1.000000	0.000000	0.000000	5.513528	0.000000	
04/16/1997	0.999849	5.667915	5.513111	5.514361	0.002740	O/N
04/17/1997	0.999698	5.668355	5.513528	5.677778	0.005479	T/N
05/19/1997	0.994746	5.817804	5.654860	6.139870	0.093151	1m
10/15/1997	0.970423	6.171058	5.988137	5.905193	0.501370	6m
04/17/1998	0.942370	6.081067	5.903340	5.905193	1.005479	12m

Next, we need to use the bond information to calculate the 18m and 2y gridpoints. The applicable equations are below.

$$\sum_{i=1}^{n} df_i c_i = 0 \qquad (1)$$

df_i = discount factors

c_i = cash flows

Cash flows

for $i = 1$: $\qquad c_i = -N$

(i is the settlement date)

for $1 < i \le n-1$: $\qquad c_i = N r_i (t_i - t_{i-1})$

for $i = n$: $\qquad c_n = N\left[r_n (t_n - t_{n-1}) + 1\right]$

Here, n=5, for the 5 cash flows occurring at settlement, 6m, 1y, 18m, and 2y. The interest rate, r_i, is constant for all of the bond payments and is specified as 6.2489%. Note that the principal (N) will drop out of the equations, and so a value need not be specified. The cash flows can then be written as:

c_1 = - N

c_2 = N *0.062489 * (t_2 - t_1)

c_3 = N *0.062489 * (t_3 - t_2)

c_4 = N *0.062489 * (t_4 - t_3)

c_5 = N *{0.062489 * (t_5 - t_4) + 1}

with the "t" terms being the day count factors from the start date, based on the **Yield Basis** specified for the bond on the **Grid Point Definition** screen. In this example, the yield basis is Act/365, providing the following day count factors

t_1 = 0.00547945

t_2 = 0.506849

t_3 = 1.00547945

t_4 = 1.512329

t_5 = 2.0109589

The discount factors for the settlement date, 6m, and 1y are obtained from the existing grid points. The discount factors for 18m and 2y are interpolated using log-linear interpolation, as given below.

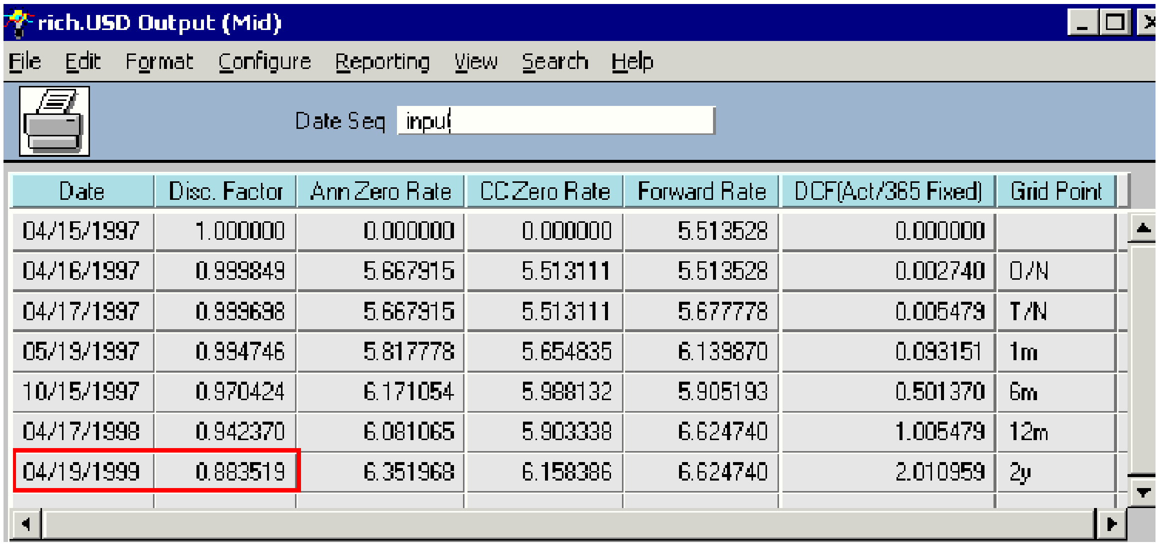
$$df_{18m} = \exp\left\{\left(\frac{\{\ln(df_{2y}) - \ln(df_{1y})\}\, t_{18m}}{(t_{2y} - t_{1y})}\right) + \left(\frac{-\ln(df_{2y}) \cdot t_{1y} + \ln(df_{1y}) \cdot t_{2y}}{(t_{2y} - t_{1y})}\right)\right\}$$

$$df_{2y} = \exp\left\{\left(\frac{\{\ln(df_{18m}) - \ln(df_{1y})\} \cdot t_{2y}}{(t_{18m} - t_{1y})}\right) + \left(\frac{-\ln(df_{18m}) \cdot t_{1y} + \ln(df_{1y}) \cdot t_{18m}}{(t_{18m} - t_{1y})}\right)\right\}$$

The equations are solved simultaneously for the 18m and 2y discount factors (df_{18m} and df_{2y}) to yield the following results:

df_{18m} = o.911223

df_{2y} = 0.883519 (see Output Table below)



Date	Disc. Factor	Ann Zero Rate	CC Zero Rate	Forward Rate	DCF(Act/365 Fixed)	Grid Point
04/15/1997	1.000000	0.000000	0.000000	5.513528	0.000000	
04/16/1997	0.999849	5.667915	5.513111	5.513528	0.002740	O/N
04/17/1997	0.999698	5.667915	5.513111	5.677778	0.005479	T/N
05/19/1997	0.994746	5.817778	5.654835	6.139870	0.093151	1m
10/15/1997	0.970424	6.171054	5.988132	5.905193	0.501370	6m
04/17/1998	0.942370	6.081065	5.903338	6.624740	1.005479	12m
04/19/1999	0.883519	6.351968	6.158386	6.624740	2.010959	2y

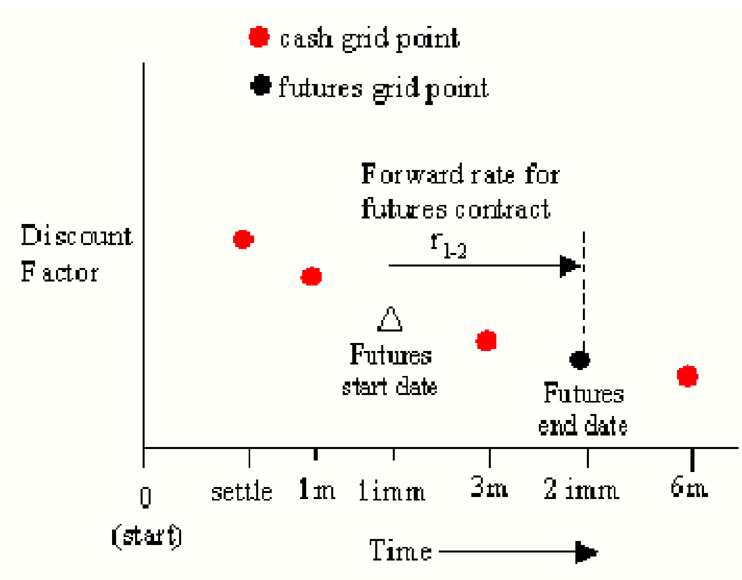
4.0 YIELD CURVE CALCULATIONS FROM FUTURES INSTRUMENTS

4.1 Discount Factor Calculations

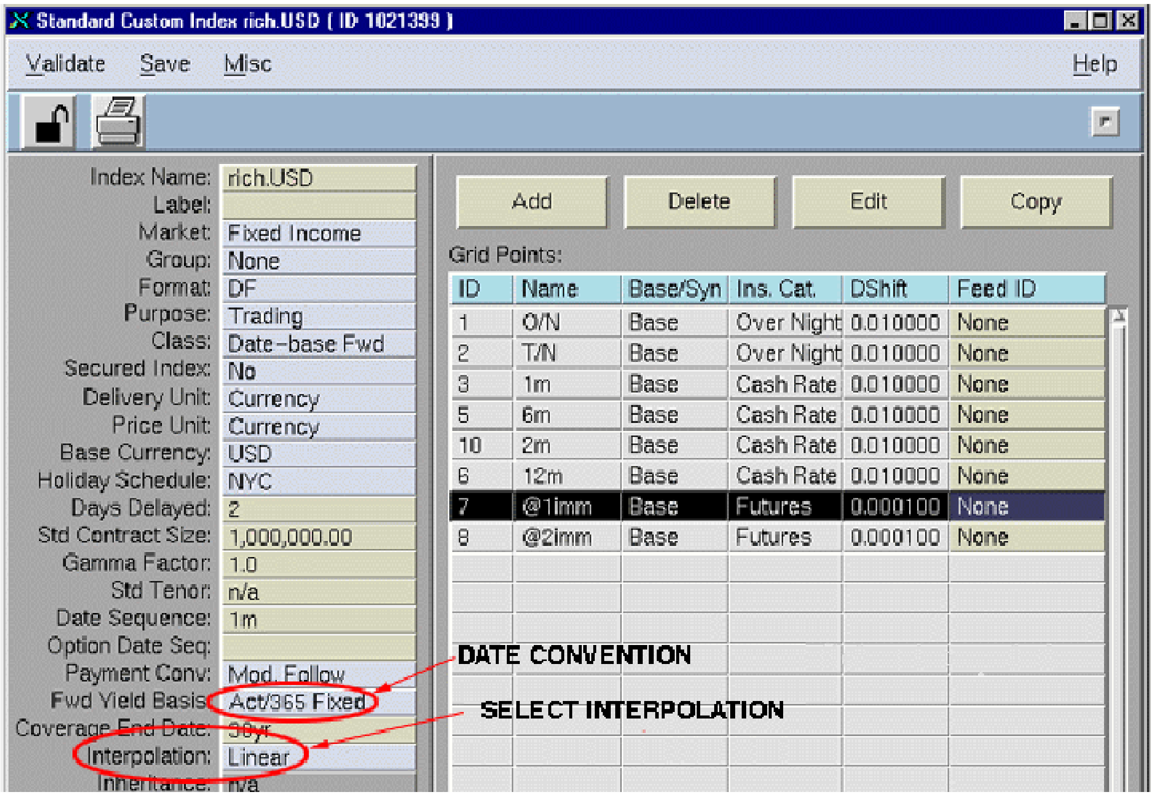
A grid point can be calculated for the end date of a futures contract. The methods for calculating the discount factors are discussed below.

Futures contract starting between two cash grid points

Consider an index consisting of cash grid points, to which we wish to add a new grid point. The new point will be defined on the basis of a futures contract which starts at time 1imm, ends at time 2imm, and has a forward rate of $r_{1\text{-}2}$. The date for the grid point is defined as the end date of the futures contract. Note that following this convention, it is typical to use this end date as the Grid Point Label for futures contracts.

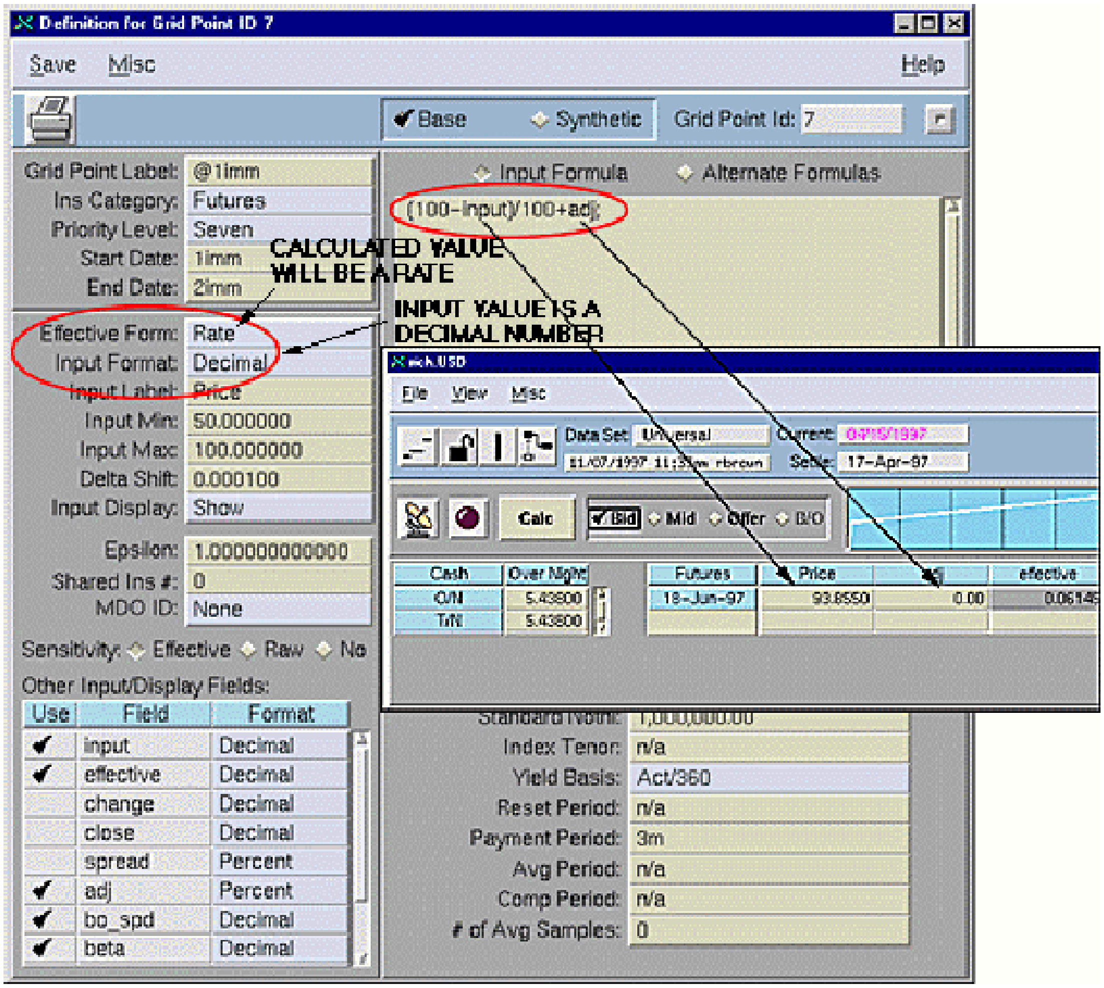


First, a discount factor at time 1imm (the start of the futures contract) is calculated by interpolating between the discount factors of the two surrounding grid points (1m and 3m) in the above figure. The interpolation method and date convention are specified on the *Index Definition* screen, shown below. Note that days are counted from the start date, to be consistent with the basis for the surrounding points.



The futures rate between the start and end dates, r_{1-2} is determined by the input formula given on the *Grid Point Definition* screen, and the user input supplied on the *Main Index* screen (see below). The input value is a price, from which the input formula calculates a rate. In the screen shown below, the effective rate (r_{1-2}) is given by

$$\text{EffectiveRate} = \frac{(100 - \text{input})}{100} + \text{adj} \qquad \text{(F1)}$$



Once the effective rate is determined, the discount factor for the grid point at 2imm, the end of the futures contract, is calculated by discounting back from 2imm to 1imm based on the $r_{1\text{-}2}$ rate. Starting with the relationship that the sum of the cash flows multiplied by the discount factors must equal zero ($\sum_{i=1}^{n} df_i c_i$), the following equation for the discount factor at time 2imm can be derived.

$$df_{2imm} = \frac{df_{1imm}}{1 + r\left(t_{2imm} - t_{1imm}\right)} \quad \text{(F2)}$$

where

df_{2imm} = discount factor for 2imm date

r = effective rate between 1imm and 2imm

t_{2imm} = time (day count factor in years) of 2imm date

t_{1imm} = time (day count factor in years)of 1imm date

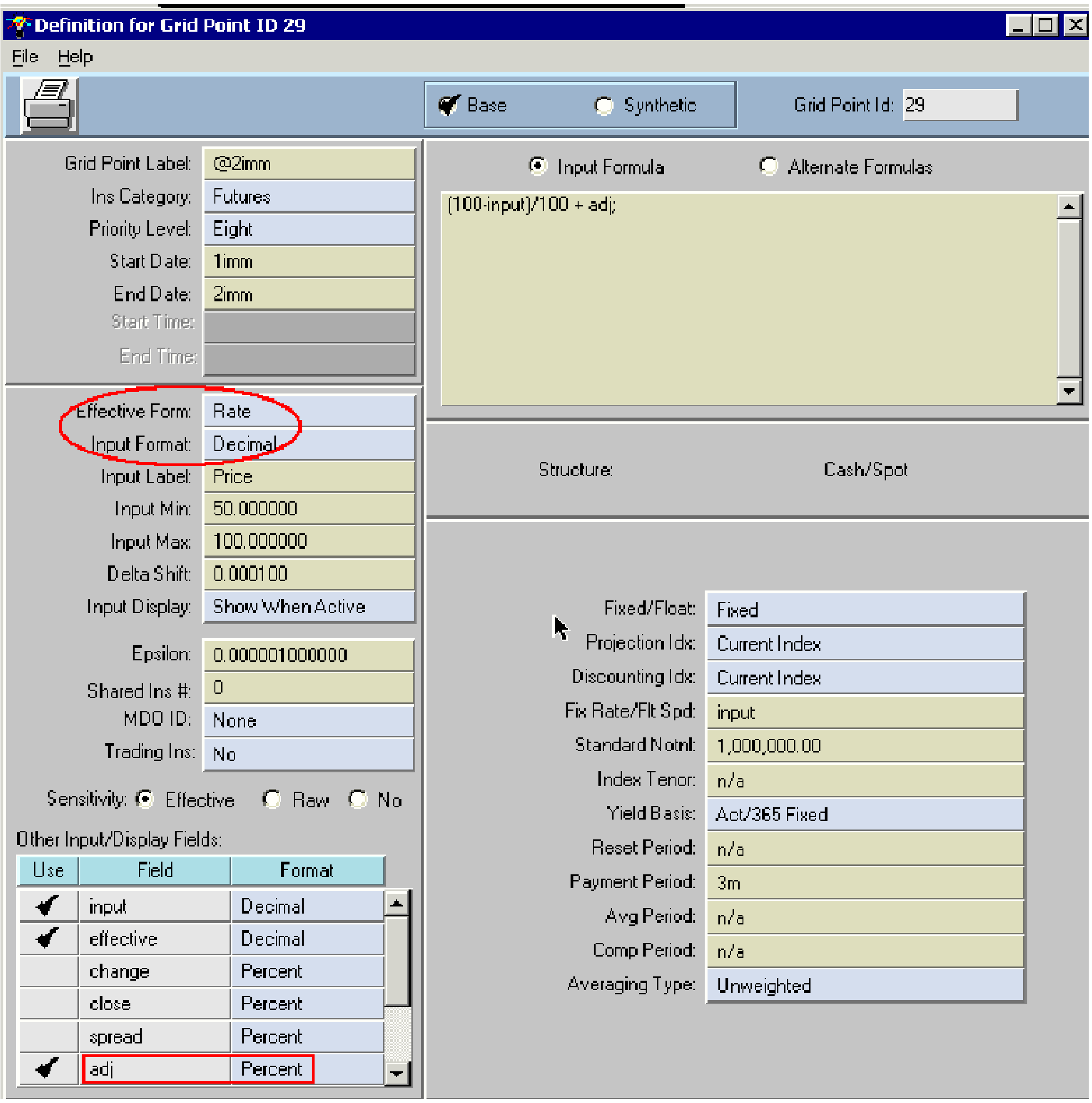
The day count factors must be based on the convention defined for the grid points and must be calculated from the starting date, not the settlement date.

4.2 Zero and Forward Rate Calculations

- Description of Zero Rate Calculations
- Description of Forward Rate Calculation

4.3 Example – Futures Grid Point

Assume we wish to construct a grid points based on a futures contract that starts on 1imm (6/18/97), and ends on 2imm (9/17/97). The interpolation method is log-linear, and the date convention is Act/365 Fixed. Both are specified on the *Index Definition* screen. The *Grid Point Definition* screen is shown below. Note the formats that are specified for the **Input** and **adj** parameters.



Further assume that the following grid points already exist

rich.USD Output (Offer)

File Edit Format Configure Reporting View Search Help

Date Seq input

Date	Disc. Factor	Ann Zero Rate	CC Zero Rate	Forward Rate	DCF(Act/365 Fixed)	Grid Point
04/15/1997	1.000000	0.000000	0.000000	5.513528	0.000000	
04/16/1997	0.999849	5.667915	5.513111	5.514361	0.002740	O/N
04/17/1997	0.999698	5.668355	5.513528	5.677778	0.005479	T/N
05/19/1997	0.994746	5.817804	5.654860	5.960865	0.093151	1m
07/17/1997	0.985253	6.004239	5.830890	6.197502	0.254795	3m
10/15/1997	0.970423	6.171058	5.988137	6.197502	0.501370	6m

The discount factor for 1imm (6/18/97) is obtained from log-linear interpolation between the surrounding grid points (5/19/97 and 7/17/97). The resulting value is 0.989908.

The grid point at the end of the futures contract, 2imm, is calculated as follows. Using the input formula shown on the above Grid Point Definition screen, with a price of 93.30 and an adj of 0.0%, yields an effective rate of 0.067. This value represents the rate over the time period of the futures contract, in this case from 1imm to 2imm. These values are displayed on the main index screen.

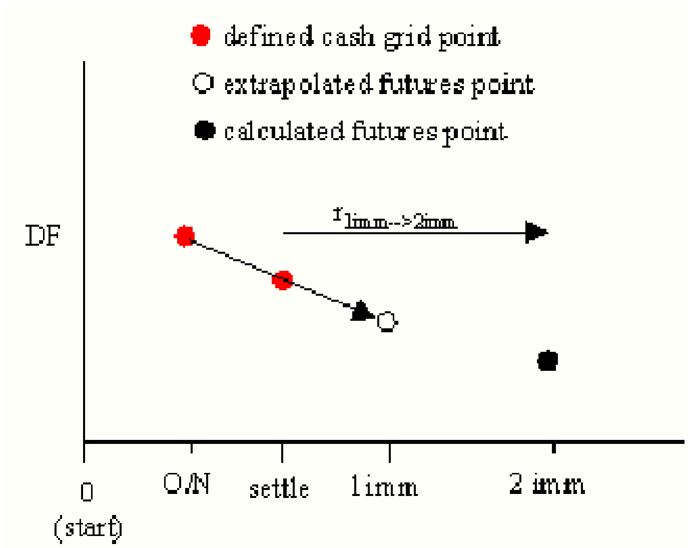
$$\text{Effective Rate} = \frac{(100 - input)}{100} + adj = \frac{100 - 93.3}{100} + 0 = 0.067$$

Using Equation (F2), the discount factor for 2imm is now obtained.

$$df_{2imm} = \frac{df_{1imm}}{1 + r_{1-2}\left(t_{2imm} - t_{1imm}\right)} = \frac{0.989908}{1 + 0.067\left(0.424658 - 0.180822\right)} = 0.973644$$

Futures Grid Point with No Cash Grid Point Between Start and End Dates

If no cash grid point exists during the time period of the futures contract, the discount factor for the first futures point is extrapolated from the previous cash points. In the figure below, the 1imm discount factor is extrapolated from the O/N and settle date cash grid points. The extrapolation method is selected on the *Index Definition* screen. The discount factor for the end time (2imm) is calculated in the same manner as described previously for the futures point surrounded by cash points.

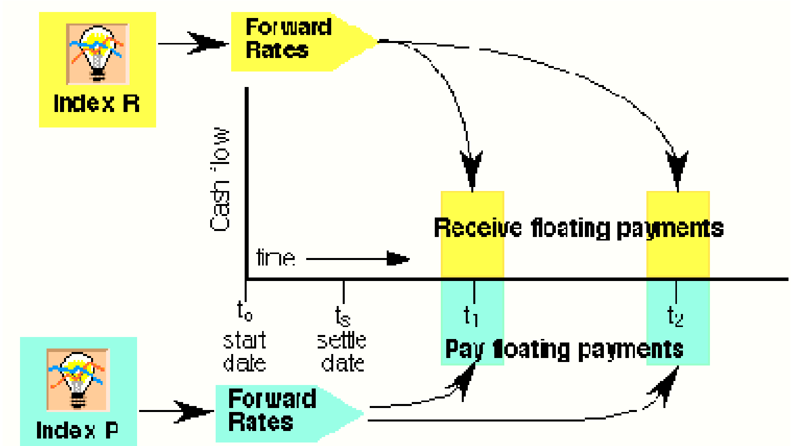


5.0 YIELD CURVE CALCULATIONS FROM SWAPS

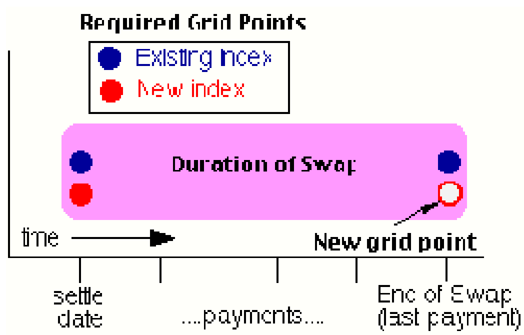
5.1 Discount Factor Calculations

Swaps are essentially a set of forward contracts used to exchange cash flows. Payments are exchanged on pre-determined days, with the amount of the payment determined by a forward interest rate for each side of the swap. Both the payment and receive side of a swap can be based on either floating or fixed interest rates. Typically, each side of the swap will be based on a different index, with the applicable forward rates calculated from the index and the specified spread. In versions prior to 4.0, the System only supports floating/floating swaps for grid point calculation. Fixed/floating swaps are supported in versions 4.0 and later.

The diagram below illustrates the cash flows for a floating/floating swap. Index R is the basis of the receive side, and Index P is the basis of the pay side.



In order to use a swap to define a grid point, the index used for one of the two sides must contain a grid point for the start (settlement) and end dates of the swap, and the other index must contain a grid point at the settlement date. A grid point for the end of the swap will be calculated. The diagram to the right illustrates this. The "New Index", the one for which the gridpoint will be calculated only requires a pre-defined gridpoint at the settlement date. The "Existing Index", the one used for the other side of the swap, must as a minimum contain a grid point at both the settlement and end dates of the swap. The indices can also contain additional gridpoints, but the ones shown on the diagram are the minimum required.



The following procedure can then be used to calculate the discount factor for a swap-based grid point:

For the swap, the sum of the discount factors times the payments must equal zero. This can be written as:

$$\sum_{i=1}^{n} df_{pi} c_{pi} + \sum_{i=1}^{n} df_{ri} c_{ri} = 0 \qquad (1)$$

where

df_{pi} = discount factor for pay side

c_{pi} = cash flows for pay side

df_{ri} = discount factor for receive side

c_{ri} = cash flows for receive side

If the swap has two payments, the cash flows are given by

Payment side	**Receive side**
$c_{p1} = N \cdot f_{p1} \cdot (t_1 - t_s)$	$c_{r1} = N \cdot f_{r1} \cdot (t_1 - t_s)$
$c_{p2} = N \cdot f_{p2} \cdot (t_2 - t_1)$	$c_{r2} = N \cdot f_{r2} \cdot (t_2 - t_1)$

f_{p1} = forward rate from the settlement date to the first payment date on the Pay side

f_{p2} = forward rate from the first payment date to the second payment date on the Pay side

f_{r1} = forward rate from the settlement date to the first payment date on the Receive side

f_{r2} = forward rate from the first payment date to the second payment date on the Receive side

t_s = day count factor for settlement day

t_1 = day count factor for payment 1

t_2 = day count factor for payment 2

Assume the following inputs are known:

t_1 = day count factor for payment 1

t_2 = day count factor for payment 2

df_{p1} = discount factor for t_1 for the index used on the Pay side of the swap (if this is not known it can be calculated by interpolating the surrounding grid points

df_{p2} = discount factor for t_2 for the index used on the Pay side of the swap

df_{rs} = discount factor for the settlement date on the Receive side of the swap

N = notional

The forward rates are typically calculated from the discount factors by (for the receive side)

$$f_{r1} = \left(\frac{1}{(t_1 - t_s)}\right) * \left(\frac{df_{rs}}{df_{r1}} - 1\right) \quad (2) \quad \text{and}$$

$$f_{r2} = \left(\frac{1}{(t_2 - t_1)}\right) * \left(\frac{df_{r1}}{df_{r2}} - 1\right) \quad (3)$$

The equations for the pay side forward rates are similar.

If an additional assumption is made regarding the relationship between the discount factors df_{r1} and df_{r2}, equations (1), (2), and (3) can be solved simultaneously to obtain the discount factors and forward rates for the receive side of the swap. The required assumption is that df_{r1} is interpolated between df_{rs} and df_{r2}. The interpolation can be either linear or log-linear.

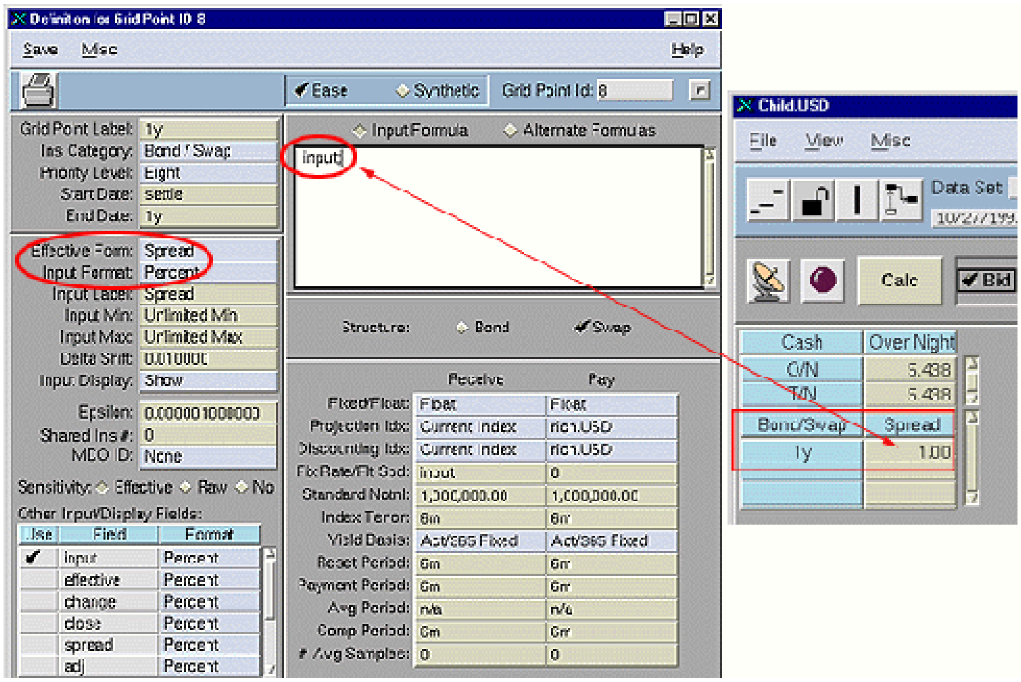
5.2 Zero and Forward Rate Calculations

- Description of Zero Rate Calculations
- Description of Forward Rate Calculation

5.3 Example – Swap Grid Point

In this example we will calculate the discount factors for the receive-side payments of a two-payment float/float swap.

Assume that we wish to construct a grid point for the last payment date on the receive side of the swap. The interest rate on which the swap payments are based is the forward rate from the start of each payment period until the payment date. Consider the swap grid point definition shown below. The *Grid Point Definition* screen is on the left, and a portion of the *Index Input* screen is on the right. The swap parameters are defined on the *Grid Point Definition* screen. The Receive side of the swap is based on the current index (Child.USD in this case), the one for which we will define the gridpoint. In this example, the spread is applied to the receive side, and the value of the spread is input by the user as a percent on the *Index Input* screen as indicated.



The information needed to create the swap grid point is summarized below. The day count factors are based on the **Yield Basis** convention specified on the *Grid Point Definition* screen (Act/365 here).

	Receive Side	**Pay Side (using O/N, T/N and 12m grid points)**
Index	rich-child.USD	rich.USD
Settle date	4/17/97 (day count factor = 0.00547952)	
First Payment	10/17/97 (day count factor = 0.506849)	
Last Payment	4/17/98 (day count factor = 1.005479452)	
Spread	1%	N/A
Discount Factor for Settle Date	0.999702	0.999698
Discount Factor for End Date	TBD	0.942370

Substituting the known quantities into the applicable equations (1) - (3) results in the following equations:

$$\sum_{i=1}^{n} df_{pi} c_{pi} + \sum_{i=1}^{n} df_{ri} c_{ri} = 0$$

$$df_{p1} f_{p1}(t_1 - t_s) + (0.943111) f_{p2}(t_2 - t_1) = df_{r1}(f_{r1} + s)(t_1 - t_s) + df_{r2}(f_{r2} + s)(t_2 - t_1) \qquad (4)$$

where

s – interest rate spread

and the forward rates are

$$f_{r1} = \left(\frac{1}{(t_1 - t_s)}\right) * \left(\frac{df_{rs}}{df_{r1}} - 1\right) = \left(\frac{1}{(0.506849 - 0.00547952)}\right) * \left(\frac{0.999698}{df_{r1}} - 1\right) \qquad (5)$$

$$f_{r2} = \left(\frac{1}{(t_2 - t_1)}\right) * \left(\frac{df_{r1}}{df_{r2}} - 1\right) = \left(\frac{1}{(1.00547952 - 0.506849)}\right) * \left(\frac{df_{r1}}{df_{r2}} - 1\right) \qquad (6)$$

$$f_{p1} = \left(\frac{1}{(t_1 - t_s)}\right) * \left(\frac{df_{ps}}{df_{p1}} - 1\right) = \left(\frac{1}{(0.506849 - 0.00547952)}\right) * \left(\frac{0.999702}{df_{p1}} - 1\right) \qquad (7)$$

$$f_{p2} = \left(\frac{1}{(t_2 - t_1)}\right) * \left(\frac{df_{p1}}{df_{p2}} - 1\right) = \left(\frac{1}{(1.00547952 - 0.506849)}\right) * \left(\frac{df_{p1}}{0.942370} - 1\right) \qquad (8)$$

How do I solve these equations?

Step 1 - Calculate the discount factor for the pay side of the first payment

Since in this example the discount factor on the payment side for the first payment is not known, its value must be obtained by interpolation of the discount factor for the settlement date (df_s) and the second payment date (df_{p2}). Using linear interpolation, as specified on the Index Definition screen, we obtain

df_{p1} = 0.91327

Step 2 - Calculate the forward rates for the pay side of the swap

Substituting this value into Equations (7) and (8), we can solve for the forward rates on the payment side of the swap.

f_{p1} = 5.9042%

f_{fp2} = 6.0833%

Step 3 - Calculate the receive side discount factors and forward rates

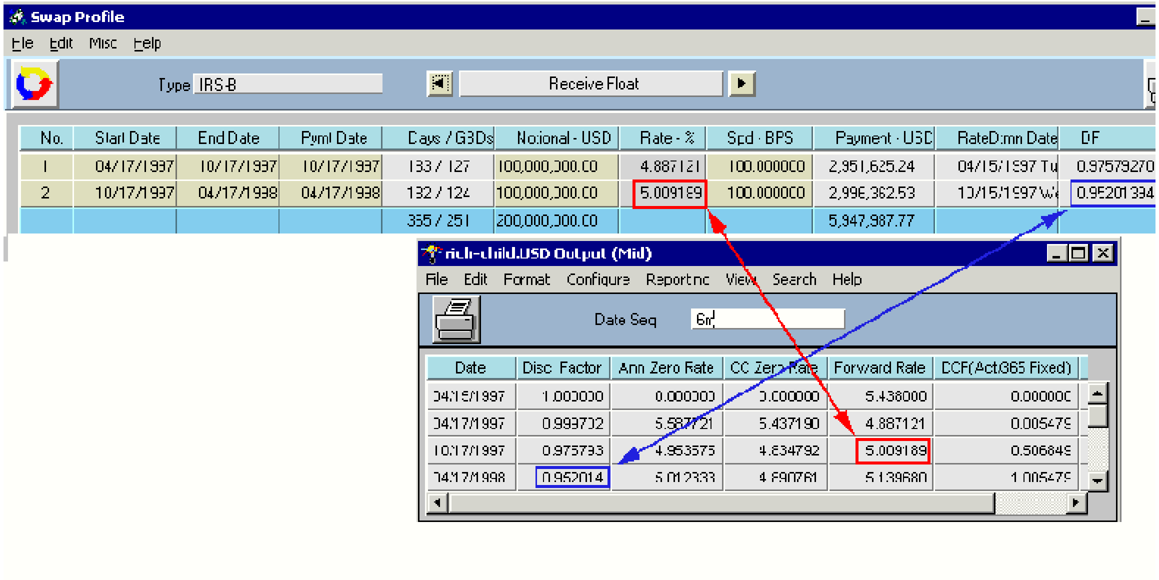
We are now left with three equations (4, 5, and 6) containing the following four unknowns: f_{r1}, f_{r2}, df_{r1}, and df_{r2}. In order to find a solution, we once again assume that the discount factor for the first payment is interpolated from the discount factors for the settlement date and the second payment. Equations (4), (5), and (6), along with the interpolation equation (linear interpolation in this case) can be solved simultaneously to yield the following solution:

df_{r1} = 0.975793; df_{r2}= 0.952014

f_{r1} = 4.8871%; f_{r2} = 5.0092%

df_{r2} is the discount factor for the grid point coinciding with the end of the swap for the receive-side index.

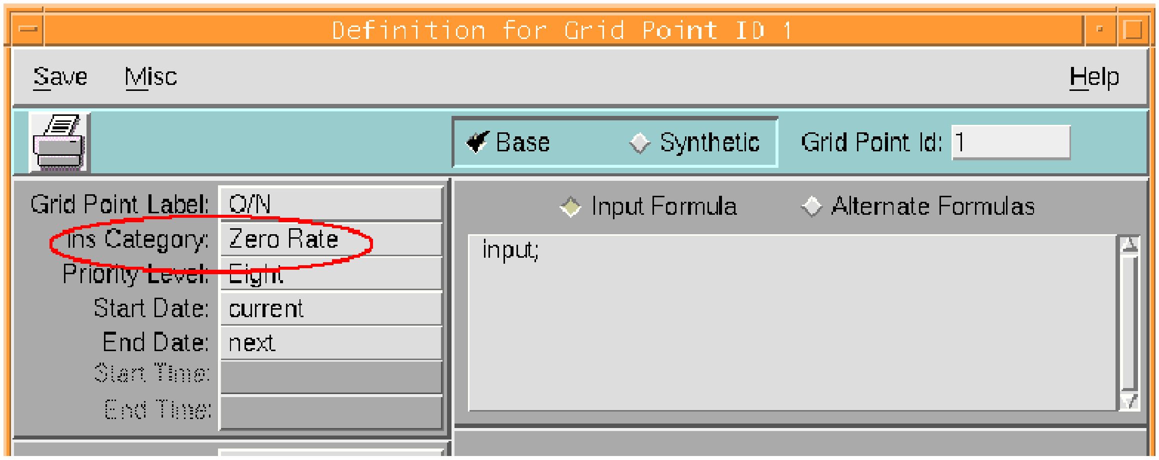
This result can be confirmed by comparing the swap profile table for the receive side of the swap and the index output table, as shown below.



6.0 YIELD CURVE CALCULATIONS – ZERO RATE GRID POINTS

6.1 Discount Factor Calculations

Consider a grid point based on the zero rate category, specified on the *Grid Point Definition* grid point definition screen (see below).



The discount factor for a zero rate grid point is calculated using the following equation:

Note that the cash flows include both principal and interest payments.

The discount factor for the settlement date is determined as follows:

$$\text{Discount Factor(grid point)} = DF_g = \exp\left[-(\text{rate})(\text{dcf}\{\text{end}, \text{start}\})\right]$$

$$\text{rate} = \text{input interest rate for the grid point}$$

$$\text{dcf}\{\text{end}, \text{start}\} = \text{day count factor based on the end and start dates of the grid point}$$

Note that the day count factor is based on the start date of the grid point, not the current date. Therefore, the day count factor used to calculate the discount factor may be different than the day count factor displayed in the Output window.

Now, the above is the discount factor between the start and end dates of the grid point. The yield curve requires the discount factor between the current date and the end date of the grid point. This is obtained as follows:

$$\text{DiscountFactor(curve)} = \text{DF(current date} \rightarrow \text{grid pt. start date}) * DF_g$$

6.2 Example

Calculate the discount factors for the O/N, T/N (settle), 1m, and 2m grid points based on Zero Rates. Assume the grid points have been created and the interest rates assigned as shown below.

Grid Point	Date	Rate
O/N	10/29/97	5.4375%
T/N	10/30/97	5.450%
1m	11/28/97	5.500%
2m	12/30/97	5.700%

The **Yield Basis** (date convention) for each grid point is set as Act/365 Fixed, while the index **Yield Basis** is Act/365 Fixed. The current date is 10/28/97 and the settlement date is 10/30/97. Smoothing is turned off.

Using the equations presented above, the discount factors are calculated as:

O/N Zero Rate Grid Point Discount Factor

$$\begin{aligned} DF &= \exp[-(rate)(dcf\{10/29/97, 10/28/97\})] \\ &= \exp[-(0.054375)(0.0027397)] \\ &= 0.999851 \end{aligned}$$

T/N (settlement) Zero Rate Grid Point Discount Factor

$$\begin{aligned} DF &= DF_{O/N} \exp[-(rate)(dcf\{10/30/97, 10/28/97\})] \\ &= 0.999851 * \exp[-(0.054375)(0.0027397)] \\ &= 0.9997018 \end{aligned}$$

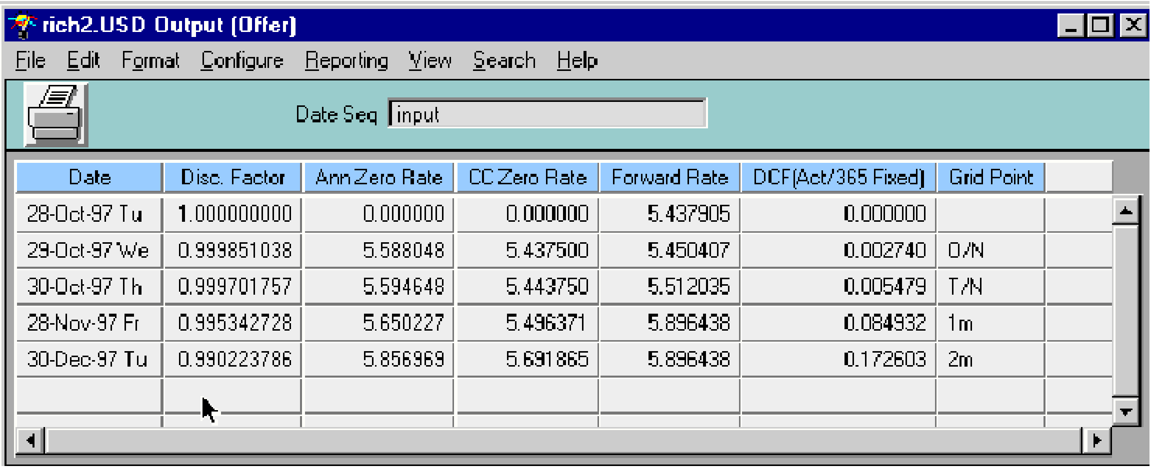
1m Zero Rate Grid Point Discount Factor

$$\begin{aligned} DF &= DF_{T/N} \exp[-(rate)(dcf\{11/28/97, 10/30/97\})] \\ &= 0.9997018 * \exp[-(0.0550)(0.079)]4521 \\ &= 0.9953427 \end{aligned}$$

2m Zero Rate Grid Point Discount Factor

$$
\begin{aligned}
DF &= DF_{T/N}\exp\left[-(rate)\left(dcf\{12/30/97,\ 10/30/97\}\right)\right] \\
&= 0.997018 * \exp\left[-(0.0570)(0.01671233)\right] \\
&= 0.9902238
\end{aligned}
$$

The image below shows the results obtained in The System. The discount factors match, but note that the day count factors differ. This is because the *Output* window displays the day count factor for the grid point date, which is different than the one used to calculate the discount factor from the zero rate.



Date	Disc. Factor	Ann Zero Rate	CC Zero Rate	Forward Rate	DCF(Act/365 Fixed)	Grid Point
28-Oct-97 Tu	1.000000000	0.000000	0.000000	5.437905	0.000000	
29-Oct-97 We	0.999851038	5.588048	5.437500	5.450407	0.002740	O/N
30-Oct-97 Th	0.999701757	5.594648	5.443750	5.512035	0.005479	T/N
28-Nov-97 Fr	0.995342728	5.650227	5.496371	5.896438	0.084932	1m
30-Dec-97 Tu	0.990223786	5.856969	5.691865	5.896438	0.172603	2m

7.0 ZERO RATE AND FORWARD RATE CALCULATIONS

7.1 Zero Rate Calculations

Once the discount factors have been calculated, the zero rates can then be calculated from the discount factors as follows:

$$\text{Annualized zero rate} = \text{azr} = \left(\text{df}_t^{\left(-1/t\right)} - 1\right)$$

where,

t = day count factor, given in the format specified on the Index Definition screen.

$$\text{Continuously compounded zero rate} = \text{ccr} = \left(-1/t\right) \cdot \ln\left(\text{df}_t\right)$$

7.2 Forward Rate Calculations

The forward rate is calculated from the current grid point to the next grid point displayed in the **Output Table**. The calculation is based on the following relationship:

(Discounted return between time 1 and time 2) = (discounted notional at time 1) - (discounted notional at time 2)

Or

$$N * \text{df}_2 * f_{1-2} * \left(t_2 - t_1\right) = \left(N * \text{df}_1 - N * \text{df}_2\right)$$

where,

$N =$ principal

$f_{1-2} =$ forward rate from time 1 to time 2

$t_1 =$ time 1 (day count factor of current grid point)

$t_2 =$ time 2 (day count factor of next grid point)

$\text{df}_1 =$ discount factor at time 1

$\text{df}_2 =$ discount factor at time 2

This equation can be solved for the forward rate to yield;

$$f_{1-2} = \left(\frac{1}{\left(t_2 - t_1\right)}\right) * \left(\text{df}_1 / \text{df}_2 - 1\right)$$

Note that the day count factor convention is based on the **Date Sequence** defined in the *Index Definition* screen, and whether or not smoothing has been enabled. With smoothing disabled, the time interval between consecutive grid points is used.

Notes:

For cash rates, discount factors are calculated independently for each grid point. For example, the 6m discount factor is calculated directly from the 6m input and is not affected by the 3m discount factor. However, the derived forward rate is determined by both the 3m and 6m discount factors.

8.0 INTERPOLATION SCHEMES

8.1 Linear Interpolation

Linear interpolation is based on fitting a curve of the form

$$\mathrm{df} = \mathrm{A} \cdot \mathrm{dcf} + \mathrm{B}$$

$$\mathrm{df} = \text{discount factor}$$

$$\mathrm{dcf} = \text{day count factor}$$

$$\text{A and B are constants}$$

to the existing discount factors.

If the discount factors are known at times ti+1 and ti-1, then the interpolated discount factor at time i can be written as

$$df_i = df_{i-1} + \left(\frac{df_{i+1} - df_{i-1}}{t_{i+1} - t_{i-1}}\right)(t_i - t_{i-1})$$

8.2 Log-linear Interpolation

Log-linear interpolation is based on fitting a curve of the form

$$\ln \mathrm{df} = \mathrm{A} \cdot \mathrm{dcf} + \mathrm{B}$$

$$\mathrm{df} = \text{discount factor}$$

$$\mathrm{dcf} = \text{day count factor}$$

$$\text{A and B are constants}$$

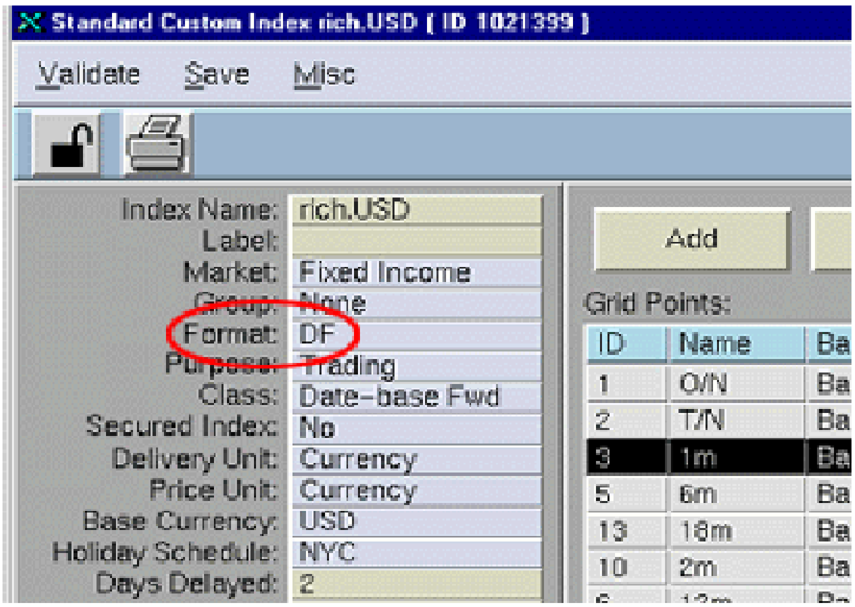
to the existing discount factors.

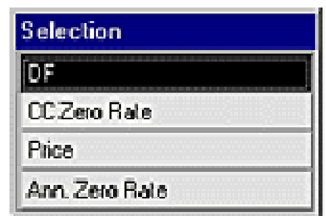
If the discount factors are known at times i+1 and i-1, then the interpolated discount factor at time it can be written as

$$df_i = \exp\left\{\left(\frac{\{\ln(df_{i+1}) - \ln(df_{i-1})\} \cdot t_i}{(t_{i+1} - t_{i-1})}\right) + \left(\frac{-\ln(df_{i+1}) \cdot t_{i-1} + \ln(df_{i-1}) \cdot t_{i+1}}{(t_{i+1} - t_{i-1})}\right)\right\}$$

9.0 INDEX FORMAT

One of the options available on the Index Definition screen is the Format option. The user has four choices for this option: DF, CC Zero Rate, Ann. Zero Rate and Price. The DF, CC Zero Rate, and Ann. Zero Rate options will calculate discount factors between grid points in a slightly different manner, as discussed below.





9.1 DF Format

If the **DF** option is selected, values for the discount factor between grid points are interpolated from the discount factor values at the surrounding gridpoints, based on the interpolation scheme selected in the *Index Definition* screen.

9.2 Ann. Zero Rate Format

If the **Ann. Zero Rate** (AZR) option is selected, the discount factors and rates between grid points are calculated as follows:

1. Calculate the annualized zero rates at the surrounding grid points from the discount factors

$$\text{Annualized zero rate} = \text{azr} = \left(\text{df}^{\left(-1/t\right)} - 1\right)$$

$$\text{Annualized zero rate} = \text{azr} = \left(\text{df}^{\left(-1/t\right)} - 1\right)$$ (Eqn. 1)

t = day count factor, given in the format specified on the Index Definition screen.

df = discount factor

2. Interpolate a value for the annualized zero rate on the desired date from the annualized zero rates values at the grid points, based on the interpolation scheme selected in the Index Definition screen.
3. Calculate the discount factor using the following equation:

$$\text{discount factor} = (1 + \text{azr})^{-1/t} \quad \text{(Eqn. 2)}$$

9.3 CC Zero Rate Format

If the **CC Zero Rate** (CCZR) option is selected, the discount factors and rates between grid points are calculated as follows:

1. Calculate the continuously compounded zero rates at the surrounding grid points from the discount factors

$$\text{Continuously compounded zero rate} = \text{ccr} = \left(-\frac{1}{t}\right) \cdot \ln(\text{df}) \quad \text{(Eqn. 3)}$$

   t = day count factor, given in the format specified on the *Index Definition* screen.

   df = discount factor

2. Interpolate a value for the continuously compounded zero rate on the desired date from the CCZR values at the surrounding gridpoints, based on the interpolation scheme selected in the *Index Definition* screen.
3. Calculate the discount factor using the following equation:

$$\text{discount factor} = \exp(\text{cczr} \cdot t) \quad \text{(Eqn. 4)}$$

   t = day count factor, given in the format specified on the *Index Definition* screen

   cczr = continuously compounded zero rate at time t

9.4 Example

Assume we wish to calculate the discount factor for a 6 month point, and that the surrounding grid points are the 1m and the 12m points. Day count factors, discount factors, annualized zero rates and continuously compounded zero rates are defined below.

Grid Point	Day Count Factor	Discount Factor	AZR	CCZR
1m	0.09315068	0.994860959	5.686963	5.531136
12m	1.00547945	0.941927863	6.130635	5.950055

Assume further that linear interpolation is being used.

9.4.1 DF Format

The 6m discount factor is directly interpolated from the 1m and 12m points to obtain 0.970858264

9.4.2 AZR Format

1. Use linear interpolation of the 1m and 12m AZR's to get a 6m AZR of 5.888148%.
2. Use Eqn. 2 above to calculate the 6m discount factor of 0.9714179788

9.4.3 CCZR Format

1. Use linear interpolation of the 1m and 12m CCZR's to get a 6m CCZR of 5.721097%.
2. Use Eqn. 4 above to calculate the 6m discount factor of 0.971419049

As can be seen, the discount factors obtained from the AZR and CCZR formats are similar, while the discount factor obtained with the DF format is somewhat lower.

