```
# pip install arch
```

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import cvxpy as cp
from arch import arch_model
```

Skip this if already have the data file

```
# pip install wrds
```

```
import wrds
```

Double-click (or enter) to edit

```
# Connect to WRDS
db = wrds.Connection(wrds_username='simengut')
```

```
Enter your WRDS username [simengut]:simengut
Enter your password:··········
WRDS recommends setting up a .pgpass file.
Create .pgpass file now [y/n]?: y
Created .pgpass file successfully.
You can create this file yourself at any time with the create_pgpass_file() function.
Loading library list...
Done
```

```
renewwable_tickers = ['ICLN', 'PBW', 'QCLN']
nonrenewable_tickers = ['XLE']
benchmark = ["SPY"]

# SQL query to fetch the data
sql_query = """
SELECT
    a.permno,
    a.date,
    b.ticker,
    b.comnam,
    b.cusip,
    a.prc,
    a.vol,
    a.ret
FROM
    crsp.dsf AS a
JOIN
    crsp.dsenames AS b
ON
    a.permno = b.permno
WHERE
    b.ticker IN ('ICLN', 'PBW', 'QCLN', 'XLE', 'SPY')
    AND a.date BETWEEN '2007-01-01' AND '2024-12-31'
    AND a.date >= b.namedt
    AND a.date <= b.nameendt
ORDER BY
    b.ticker,
    a.date;
"""

# Execute query and get results
df = db.raw_sql(sql_query)

# Display the first few rows
df.head()
```

	permno	date	ticker	comnam	cusip	prc	vol	ret
0	92720	2008-06-25	ICLN	ISHARES TRUST	46428822	52.77	1935.0	<NA>
1	92720	2008-06-26	ICLN	ISHARES TRUST	46428822	51.06	8722.0	-0.032405
2	92720	2008-06-27	ICLN	ISHARES TRUST	46428822	50.157	7588.0	-0.017685
3	92720	2008-06-30	ICLN	ISHARES TRUST	46428822	50.25	16765.0	0.001854
4	92720	2008-07-01	ICLN	ISHARES TRUST	46428822	48.797	14393.0	-0.028915

```
# Pivot the DataFrame
returns_df = df.pivot(index='date', columns='ticker', values='ret')

# Sort the index (dates)
returns_df = returns_df.sort_index()

# Display the first few rows
print("Returns DataFrame (dates as index, tickers as columns):")
returns_df.head()
```

```
Returns DataFrame (dates as index, tickers as columns):
```

ticker	ICLN	PBW	QCLN	SPY	XLE
date					
2007-01-03	<NA>	-0.005774	<NA>	-0.001765	-0.034965
2007-01-04	<NA>	0.009292	<NA>	0.002122	-0.018204
2007-01-05	<NA>	-0.023015	<NA>	-0.007976	0.006661
2007-01-08	<NA>	0.006478	<NA>	0.004625	0.0
2007-01-09	<NA>	-0.005266	<NA>	-0.00085	-0.008584

```
# Find the latest start date among all ETFs
latest_start_date = df.groupby('ticker')['date'].min().max()
print(f"Latest start date among all ETFs: {latest_start_date}")

# Filter the data to start from the latest start date
df_filtered = df[df['date'] >= latest_start_date]

# Pivot the filtered data
returns_df = df_filtered.pivot(index='date', columns='ticker', values='ret')

# Sort the index (dates)
returns_df = returns_df.sort_index()

# Display the first few rows
print("\nReturns DataFrame (starting from latest common date):")
returns_df.head()
```

```
Latest start date among all ETFs: 2008-06-25

Returns DataFrame (starting from latest common date):
```

ticker	ICLN	PBW	QCLN	SPY	XLE
date					
2008-06-25	<NA>	0.008435	0.004221	0.004726	-0.003774
2008-06-26	-0.032405	-0.047398	-0.037065	-0.02716	-0.007231
2008-06-27	-0.017685	-0.015122	-0.012302	-0.005459	0.008787
2008-06-30	0.001854	-0.031699	-0.006669	0.003529	0.014098
2008-07-01	-0.028915	-0.014322	-0.021599	0.003126	0.004182

```
# check for null values
returns_df.isnull().sum()
returns_df.dropna(inplace=True)
returns_df.head()
returns_df.isnull().sum()
returns_df.dropna(inplace=True)
returns_df
```

ticker	ICLN	PBW	QCLN	SPY	XLE
date					
2008-06-26	-0.032405	-0.047398	-0.037065	-0.02716	-0.007231
2008-06-27	-0.017685	-0.015122	-0.012302	-0.005459	0.008787
2008-06-30	0.001854	-0.031699	-0.006669	0.003529	0.014098
2008-07-01	-0.028915	-0.014322	-0.021599	0.003126	0.004182
2008-07-02	-0.026375	-0.05397	-0.04382	-0.017137	-0.030163
...	...	...	...	...	...
2024-12-24	0.003448	0.026706	0.017939	0.011115	0.008459
2024-12-26	-0.003436	0.026012	-0.001399	0.000067	-0.000827
2024-12-27	-0.006035	-0.016901	-0.015966	-0.010527	-0.000118
2024-12-30	-0.008673	-0.021012	-0.018787	-0.011412	-0.000118
2024-12-31	-0.004374	-0.023902	-0.015666	-0.003638	0.013128

4157 rows × 5 columns

```
# Create separate DataFrames for renewable and non-renewable ETFs
nonrenewable_df = returns_df[['XLE', 'SPY']].copy()
renewable_df = returns_df.drop('XLE', axis=1).copy()
renewable_df = renewable_df.drop('SPY', axis=1).copy()

# Save both DataFrames to separate CSV files
nonrenewable_df.to_csv('nonrenewable_etf_returns.csv')
renewable_df.to_csv('renewable_etfs_returns.csv')

print("Non-renewable ETF (XLE) data:")
print(nonrenewable_df.head())
print("\nRenewable ETFs data:")
print(renewable_df.head())
```

```
Non-renewable ETF (XLE) data:
ticker           XLE       SPY
date
2008-06-26 -0.007231  -0.02716
2008-06-27  0.008787 -0.005459
2008-06-30  0.014098  0.003529
2008-07-01  0.004182  0.003126
2008-07-02 -0.030163 -0.017137

Renewable ETFs data:
ticker          ICLN       PBW      QCLN
date
2008-06-26 -0.032405 -0.047398 -0.037065
2008-06-27 -0.017685 -0.015122 -0.012302
2008-06-30  0.001854 -0.031699 -0.006669
2008-07-01 -0.028915 -0.014322 -0.021599
2008-07-02 -0.026375  -0.05397  -0.04382
```

```
# write the returns code to a file
nonrenewable_df.to_csv('data/nonrenewable_etfs_returns.csv')
renewable_df.to_csv('data/renewable_etfs_returns.csv')
```

Retrieve data from csv files

```
from google.colab import drive
drive.mount('/content/drive/')
```

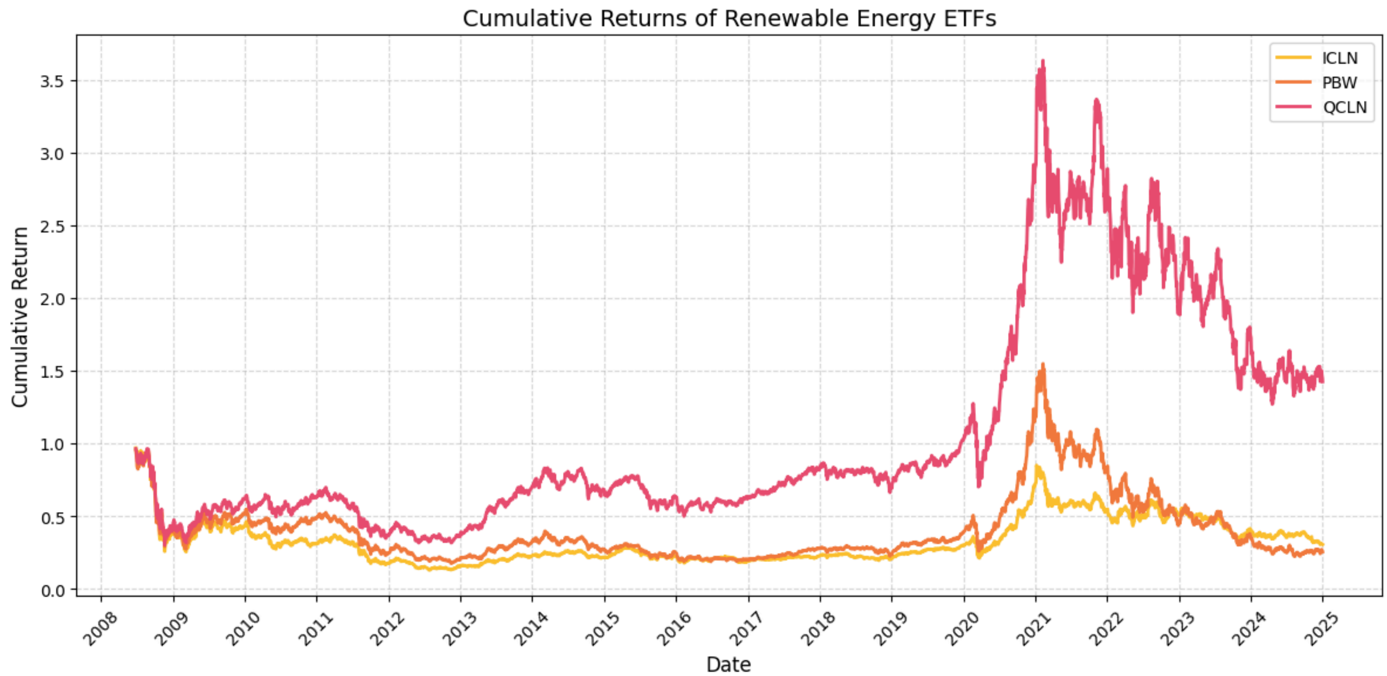
```
Mounted at /content/drive/
```

```
# simen location
# cd drive/MyDrive/Math583/FinalProject
```

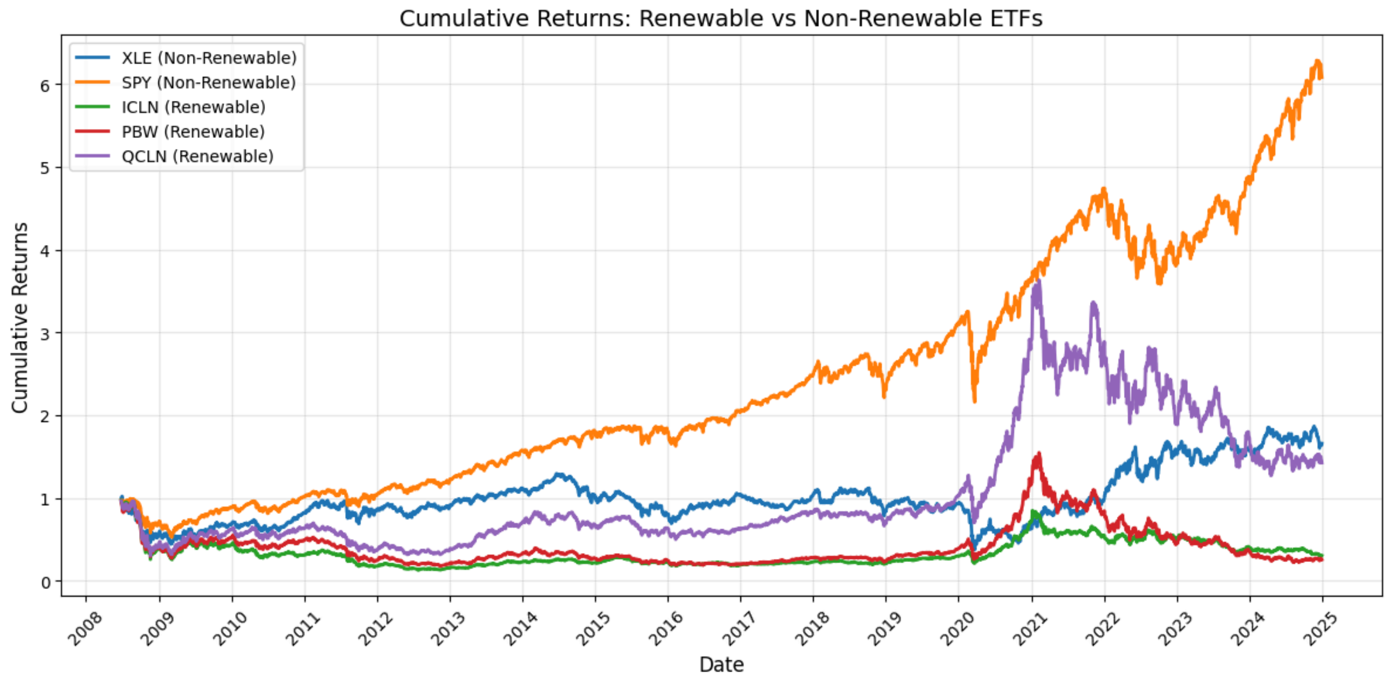
```
# nonrenewable_df = pd.read_csv('data/nonrenewable_etfs_returns.csv', index_col=0, parse_dates=True)
# renewable_df = pd.read_csv('data/renewable_etfs_returns.csv', index_col=0, parse_dates=True)
returns_df = pd.read_csv('daily_returns_df.csv', index_col=0, parse_dates=True)


renewable_df = returns_df[['ICLN', 'PBW', 'QCLN']]
nonrenewable_df = returns_df[['XLE', 'SPY']]
```

Exploratory analysis

```
import matplotlib.dates as mdates

# Compute cumulative returns
cum_returns_r = (1 + renewable_df).cumprod()
cum_returns_nr = (1 + nonrenewable_df).cumprod()


dates_nr = pd.to_datetime(cum_returns_nr.index)
dates_r = pd.to_datetime(cum_returns_r.index)

# Plot
plt.figure(figsize=(12, 6))

# Set custom colors
colors = {
    "ICLN": "#FDBF2D",  # golden
    "PBW": "#F47C3C",   # orange
    "QCLN": "#E94B6E",  # red-pink
}

for col in cum_returns_r.columns:
    plt.plot(cum_returns_r.index, cum_returns_r[col], label=col, color=colors.get(col, None), linewidth=2)

# Styling
plt.title("Cumulative Returns of Renewable Energy ETFs", fontsize=14)
plt.ylabel("Cumulative Return", fontsize=12)
plt.xlabel("Date", fontsize=12)
plt.grid(True, which='major', linestyle='--', alpha=0.5)

# Set x-ticks to show only every 100th date
ax = plt.gca()
ax.xaxis.set_major_locator(mdates.YearLocator())
ax.xaxis.set_major_formatter(mdates.DateFormatter('%Y'))
plt.xticks(rotation=45)


plt.legend()
plt.tight_layout()
plt.show()
```




```
plt.figure(figsize=(12, 6))

# Convert indices to datetime if they're not already
dates_nr = pd.to_datetime(cum_returns_nr.index)
dates_r = pd.to_datetime(cum_returns_r.index)

# Plot both cumulative returns series
plt.plot(dates_nr, cum_returns_nr['XLE'], label='XLE (Non-Renewable)', linewidth=2)
plt.plot(dates_nr, cum_returns_nr['SPY'], label='SPY (Non-Renewable)', linewidth=2)
plt.plot(dates_r, cum_returns_r['ICLN'], label='ICLN (Renewable)', linewidth=2)
plt.plot(dates_r, cum_returns_r['PBW'], label='PBW (Renewable)', linewidth=2)
plt.plot(dates_r, cum_returns_r['QCLN'], label='QCLN (Renewable)', linewidth=2)

# Customize the plot
plt.title('Cumulative Returns: Renewable vs Non-Renewable ETFs', fontsize=14)
plt.xlabel('Date', fontsize=12)
plt.ylabel('Cumulative Returns', fontsize=12)
plt.grid(True, alpha=0.3)

# Set x-ticks to show only every 100th date
ax = plt.gca()
ax.xaxis.set_major_locator(mdates.YearLocator())
ax.xaxis.set_major_formatter(mdates.DateFormatter('%Y'))
plt.xticks(rotation=45)

# Add legend
plt.legend(fontsize=10)

plt.tight_layout()
plt.show()
```

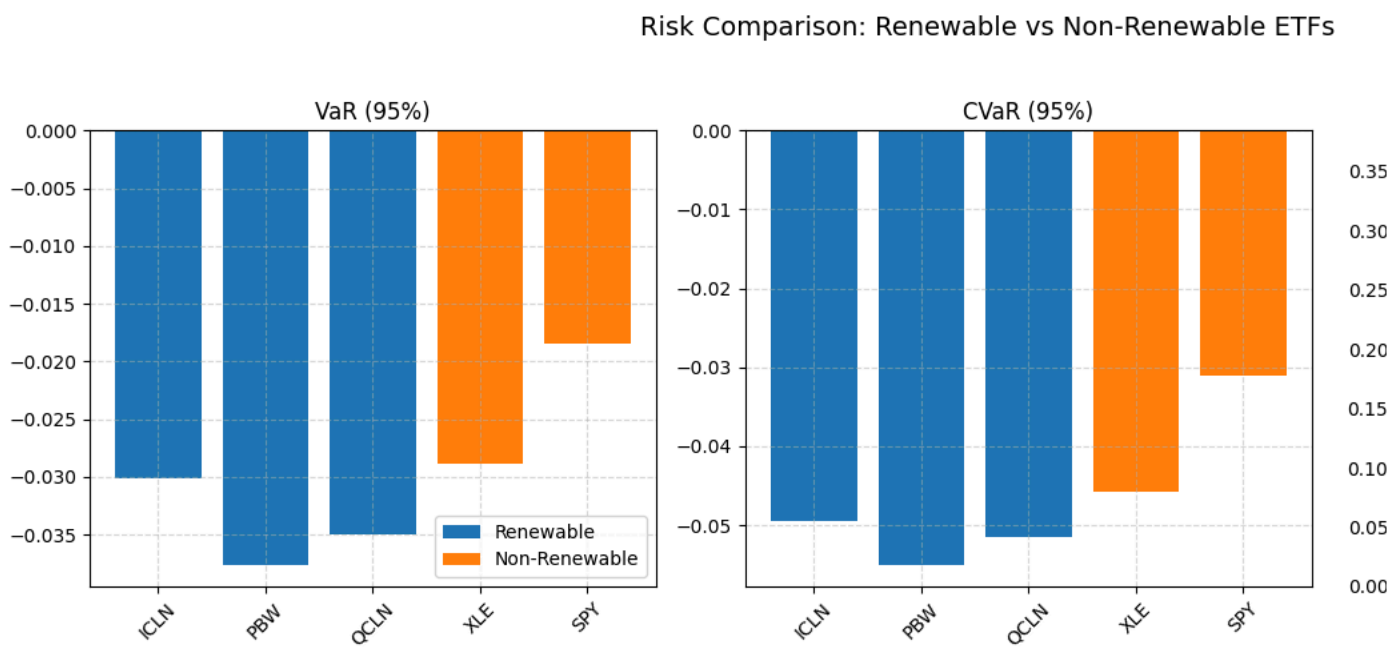


Compute VaR, CVaR and Max Drawdown for ETFs

```
import warnings
warnings.filterwarnings("ignore")

# 1. Compute risk metrics
def compute_risk_metrics(df):
    metrics = {}
    for col in df.columns:
        returns = df[col].dropna()
        var_95 = returns.quantile(0.05)
        cvar_95 = returns[returns <= var_95].mean()
        mdd = (returns.cummax() - returns).max()
        metrics[col] = {
            'VaR (95%)': round(var_95, 4),
            'CVaR (95%)': round(cvar_95, 4),
            'Max Drawdown': round(mdd, 4)
        }
    return pd.DataFrame(metrics).T

# 2. Generate risk metric tables
renewable_risk = compute_risk_metrics(renewable_df)
nonrenewable_risk = compute_risk_metrics(nonrenewable_df)

print(renewable_risk)
print(nonrenewable_risk)

# 3. Combine with group labels
combined_risk = pd.concat(
    [renewable_risk, nonrenewable_risk],
    keys=["Renewable", "Non-Renewable"]
)
combined_risk.index.names = ["Group", "ETF"]

# 4. Melt into tidy format for plotting
melted = combined_risk.reset_index().melt(
    id_vars=["Group", "ETF"],
    var_name="Metric",
    value_name="Value"
)

# 5. Plot
```

```
plt.figure(figsize=(15, 5))

metrics = melted["Metric"].unique()
for i, metric in enumerate(metrics):
    ax = plt.subplot(1, 3, i + 1)
    subset = melted[melted["Metric"] == metric]

    # Create ordered list of ETFs
    etf_order = ['ICLN', 'PBW', 'QCLN', 'XLE', 'SPY']

    # Filter and sort the data according to the ETF order
    subset = subset[subset['ETF'].isin(etf_order)]
    subset['ETF'] = pd.Categorical(subset['ETF'], categories=etf_order, ordered=True)
    subset = subset.sort_values('ETF')

    for group in subset["Group"].unique():
        group_data = subset[subset["Group"] == group]
        ax.bar(group_data["ETF"], group_data["Value"], label=group)

    ax.set_title(metric)
    ax.set_xticklabels(etf_order, rotation=45)
    ax.grid(True, linestyle="--", alpha=0.5)
    if i == 0:
        ax.legend()

plt.suptitle("Risk Comparison: Renewable vs Non-Renewable ETFs", fontsize=14)
plt.tight_layout(rect=[0, 0, 1, 0.95])
plt.show()
```

```
        VaR (95%)  CVaR (95%)  Max Drawdown
ICLN    -0.0301     -0.0495        0.3275
PBW     -0.0376     -0.0550        0.3162
QCLN    -0.0350     -0.0515        0.3040
      VaR (95%)  CVaR (95%)  Max Drawdown
XLE    -0.0288     -0.0458        0.3662
SPY    -0.0185     -0.0310        0.2546
```

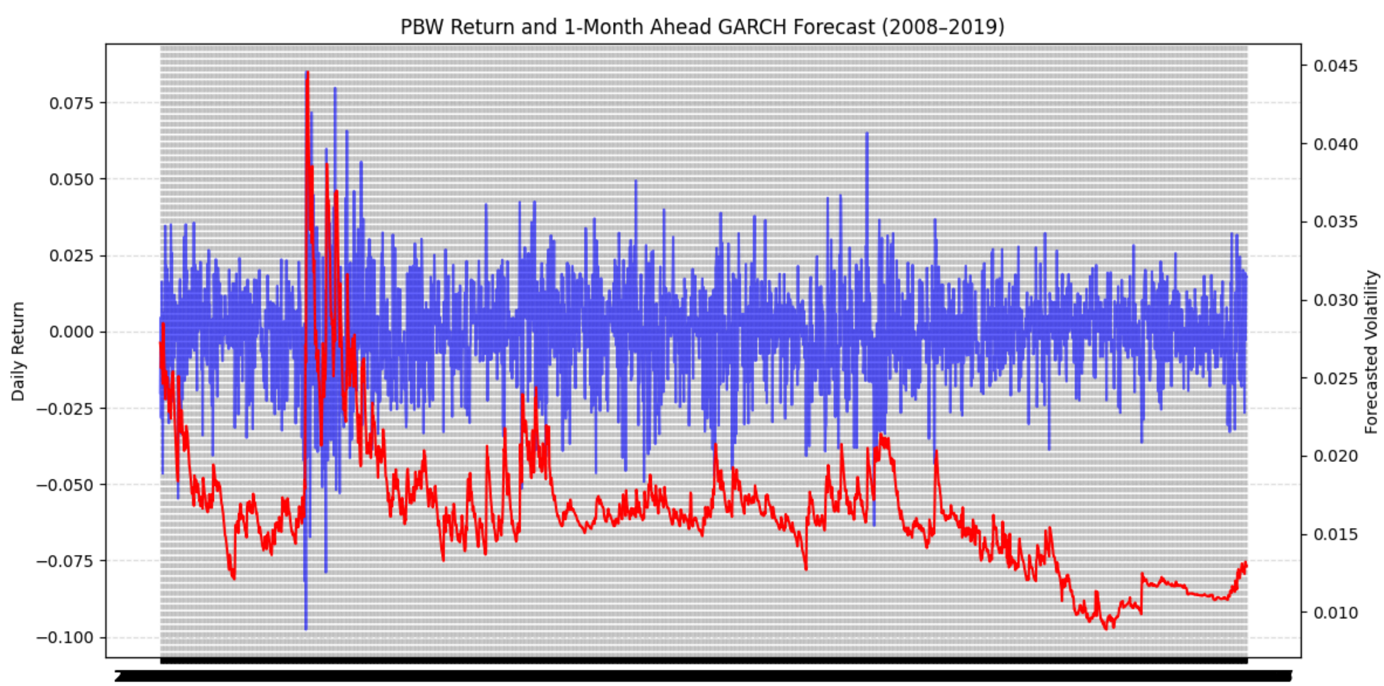


GARCH Volatility testing to get volatility signals

make training and test set to later test performance of volatility signals

```
# Step 1: Limit data to 2008–2019 (development phase)
subset_df = renewable_df.loc["2008":"2019"]
ticker = "PBW"
returns = subset_df[ticker].dropna() * 100  # GARCH expects percentage returns

# Step 2: Fit GARCH on full available history up to each point and forecast 1 month ahead
from arch import arch_model
```

```
import matplotlib.pyplot as plt
import numpy as np

# Define rolling forecast window and forecast horizon
rolling_window = 500  # use 500 days (~2 years) to fit GARCH
forecast_horizon = 22  # forecast 22 trading days (1 month)

# Store results
forecast_vols = []

# Only forecast when we have enough data
for i in range(rolling_window, len(returns) - forecast_horizon):
    window_data = returns.iloc[i - rolling_window:i]
    model = arch_model(window_data, vol='Garch', p=1, q=1)
    try:
        res = model.fit(disp="off")
        forecast = res.forecast(horizon=forecast_horizon)
        # Take average volatility over the 22-day forecast period
        avg_forecast_vol = np.mean(np.sqrt(forecast.variance.values[-1]) / 100)
        forecast_vols.append(avg_forecast_vol)
    except:
        forecast_vols.append(np.nan)

# Create time-aligned Series of forecasted volatilities
valid_index = returns.index[rolling_window:-forecast_horizon]
garch_vol_forecast_1M = pd.Series(forecast_vols, index=valid_index, name="1M GARCH Forecast Vol")

# Combine with actual returns
aligned_returns = returns.loc[valid_index] / 100  # convert back to decimal

# Plot
fig, ax1 = plt.subplots(figsize=(12, 6))
ax1.plot(aligned_returns.index, aligned_returns, label=f"{ticker} Return", color="blue", alpha=0.6)
ax2 = ax1.twinx()
ax2.plot(garch_vol_forecast_1M.index, garch_vol_forecast_1M, label="1M Forecasted Vol", color="red")
ax1.set_title(f"{ticker} Return and 1-Month Ahead GARCH Forecast (2008–2019)")
ax1.set_ylabel("Daily Return")
ax2.set_ylabel("Forecasted Volatility")
ax1.grid(True, linestyle="--", alpha=0.4)
plt.tight_layout()
plt.show()
```



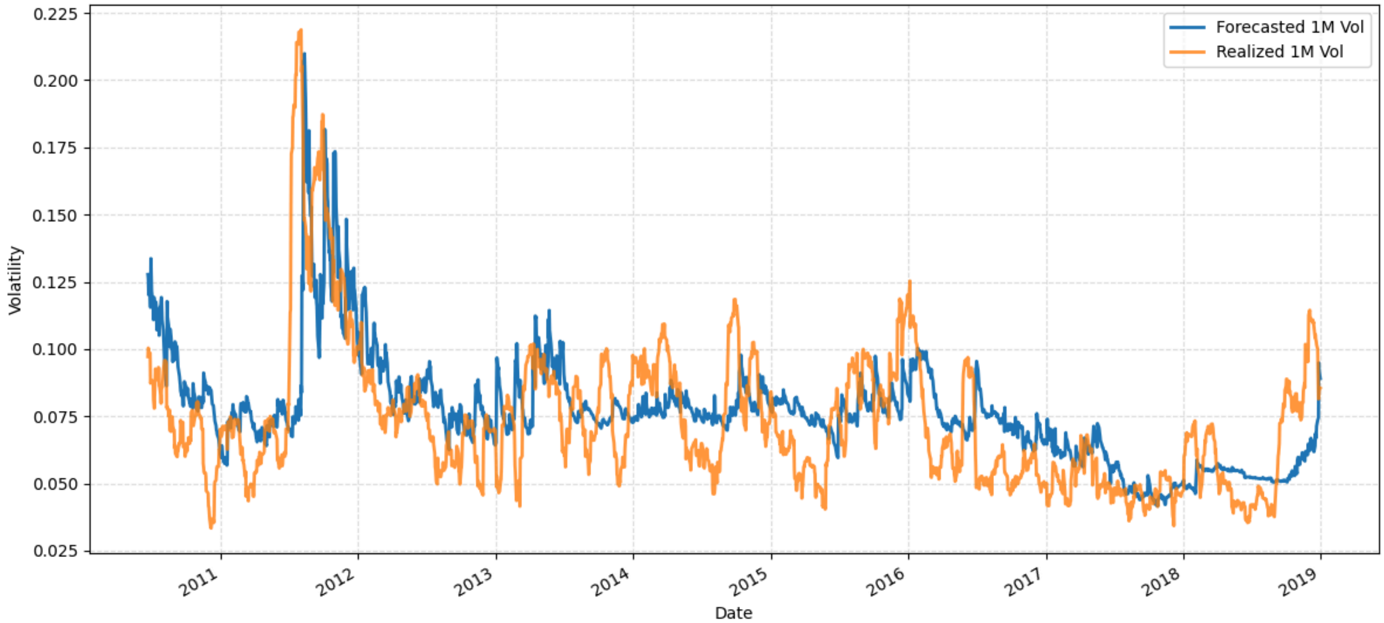
```python
def compute_forecast_and_realized_vol(
    returns_df: pd.DataFrame,
    ticker: str,
    start: str,
    end: str,
    rolling_window: int = 500,
    forecast_horizon: int = 22
) -> (pd.Series, pd.Series):
    """
    Compute 1-month ahead GARCH(1,1) forecasted volatility and realized volatility.

    Parameters:
    - returns_df: DataFrame of decimal returns, index is datetime, columns are tickers
    - ticker: the column name to analyze
    - start, end: date strings like '2008-01-01'
    - rolling_window: number of days for in-sample GARCH fit
    - forecast_horizon: days ahead to forecast & realized window (e.g. 22)

    Returns:
    - forecast_vol: Series of forecasted vol, indexed by forecast date
    - realized_vol: Series of realized vol, same index
    """
    # Ensure datetime index
    df = returns_df.copy()
    df.index = pd.to_datetime(df.index)

    # Subset and drop missing
    series = df[ticker].loc[start:end].dropna()

    # Convert to percent scale for GARCH
    pct_returns = series * 100

    forecast_dates = []
    forecast_vols = []
    # Rolling GARCH forecasts
    for i in range(rolling_window, len(pct_returns) - forecast_horizon):
        window = pct_returns.iloc[i - rolling_window : i]
        model = arch_model(window, vol='Garch', p=1, q=1)
        res = model.fit(disp='off')
        fc = res.forecast(horizon=forecast_horizon, reindex=False)
        # Sum daily variances (pct^2) and convert to decimal var
        var_fore = fc.variance.values[-1]
        monthly_var_dec = var_fore.sum() / 10000
        forecast_vols.append(np.sqrt(monthly_var_dec))
        forecast_dates.append(pct_returns.index[i])

    forecast_vol = pd.Series(forecast_vols, index=forecast_dates, name='Forecasted 1M Vol')

    # Compute realized vol by integer slicing
    realized_vols = []
    for dt in forecast_dates:
        pos = series.index.get_loc(dt)
        window = series.iloc[pos+1 : pos+1+forecast_horizon]
        realized_var = (window ** 2).sum()
        realized_vols.append(np.sqrt(realized_var))

    realized_vol = pd.Series(realized_vols, index=forecast_dates, name='Realized 1M Vol')
    return forecast_vol, realized_vol



def plot_forecast_vs_realized_vol(
    forecast_vol: pd.Series,
    realized_vol: pd.Series,
    ticker: str,
    start_date: str,
    end_date: str,
    figsize=(12, 6),
    linewidth=2,
    alpha=0.8):
    """
    Plot forecasted vs realized 1-month vol for a specific date range.

    Parameters:
    -----------
    forecast_vol : pd.Series
```

```
        Series of forecasted volatilities
    realized_vol : pd.Series
        Series of realized volatilities
    ticker : str
        ETF ticker symbol
    start_date : str
        Start date for the plot (format: 'YYYY-MM-DD')
    end_date : str
        End date for the plot (format: 'YYYY-MM-DD')
    figsize : tuple
        Figure size (width, height)
    linewidth : float
        Width of the lines
    alpha : float
        Transparency of the realized volatility line
    """
    # Convert dates to datetime if they aren't already
    start_date = pd.to_datetime(start_date)
    end_date = pd.to_datetime(end_date)

    # Filter data for the specified date range
    forecast_vol = forecast_vol.loc[start_date:end_date]
    realized_vol = realized_vol.loc[start_date:end_date]

    # Create the plot
    plt.figure(figsize=figsize)
    # Plot the lines
    plt.plot(forecast_vol.index, forecast_vol,
             label='Forecasted 1M Vol',
             linewidth=linewidth)
    plt.plot(realized_vol.index, realized_vol,
             label='Realized 1M Vol',
             linewidth=linewidth,
             alpha=alpha)

    # Add title and labels
    plt.title(f'{ticker}: Forecasted vs Realized 1-Month Volatility\n{start_date.strftime("%Y-%m-%d")} to {end_date.strftime("%Y
    plt.xlabel('Date')
    plt.ylabel('Volatility')
    # Add legend and grid
    plt.legend()
    plt.grid(True, linestyle='--', alpha=0.4)
    # Format x-axis dates
    plt.gcf().autofmt_xdate()
    plt.tight_layout()
    plt.show()
```

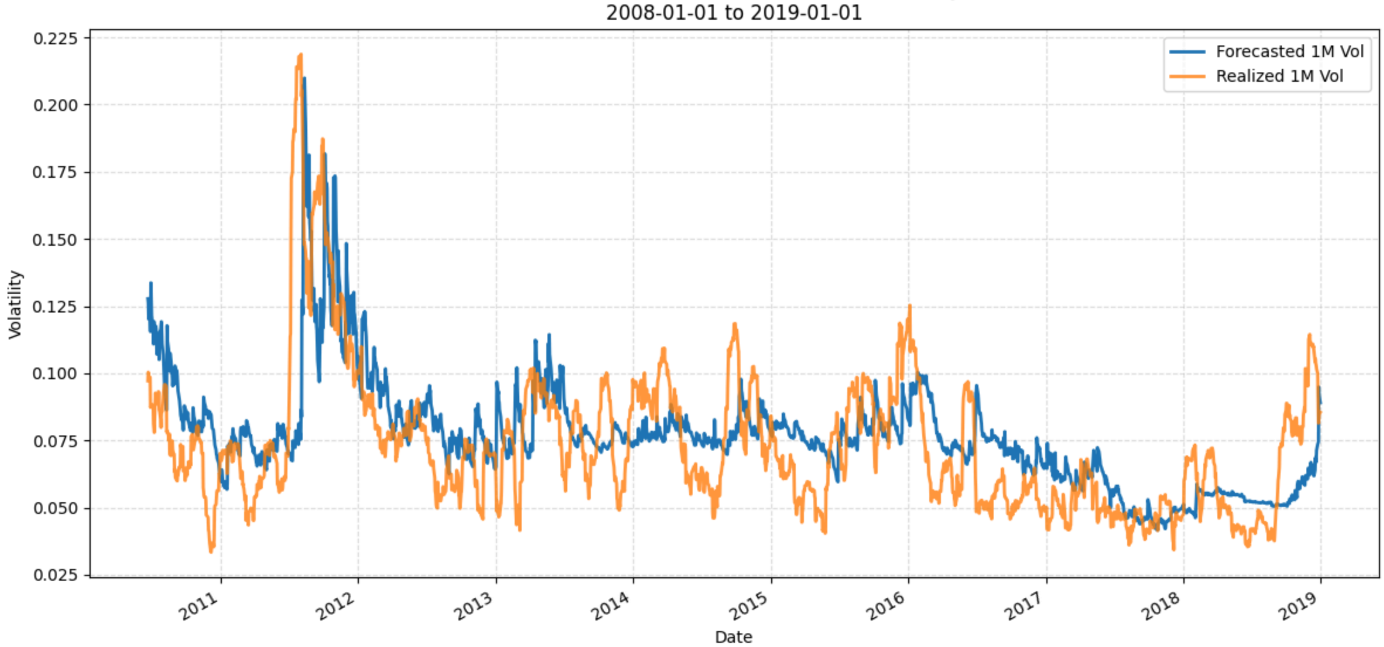
```
forecast_ICLN, realized_ICLN = compute_forecast_and_realized_vol(renewable_df, 'ICLN', '2008-01-01', '2019-12-31')
forecast_PBW, realized_PBW = compute_forecast_and_realized_vol(renewable_df, 'PBW', '2008-01-01', '2019-12-31')
forecast_QCLN, realized_QCLN = compute_forecast_and_realized_vol(renewable_df, 'QCLN', '2008-01-01', '2019-12-31')
```

```
plot_forecast_vs_realized_vol(forecast_PBW, realized_PBW, 'ICLN', '2008', '2019')
plot_forecast_vs_realized_vol(forecast_PBW, realized_PBW, 'PBW', '2008', '2019')
plot_forecast_vs_realized_vol(forecast_PBW, realized_PBW, 'QCLN', '2008', '2019')
```

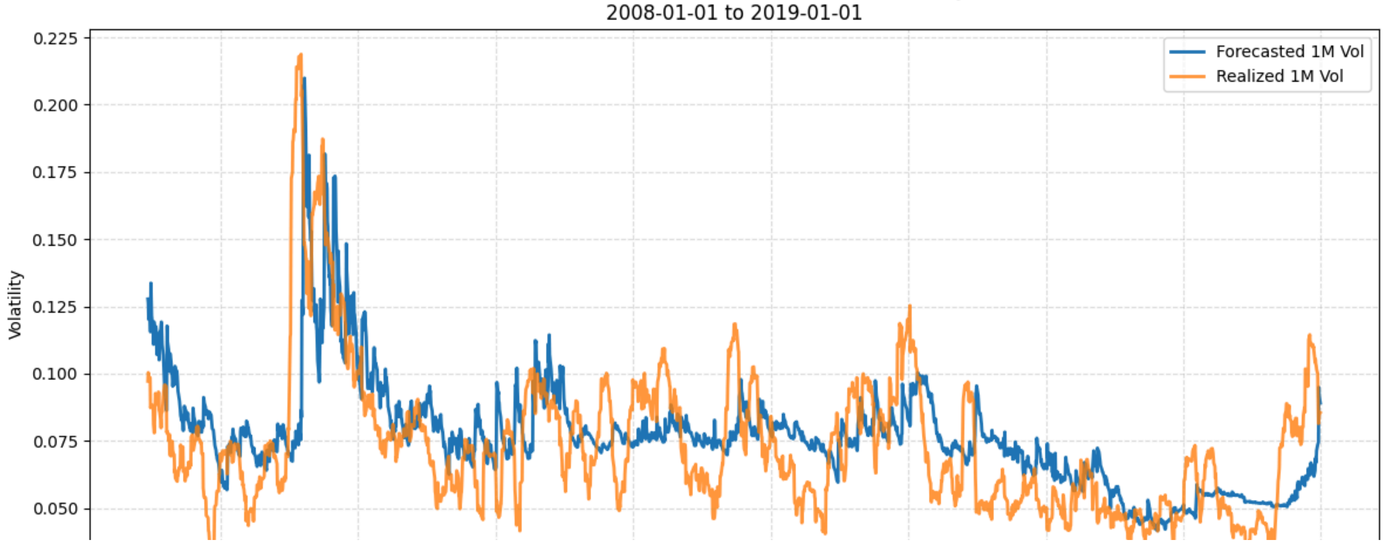
ICLN: Forecasted vs Realized 1-Month Volatility
2008-01-01 to 2019-01-01

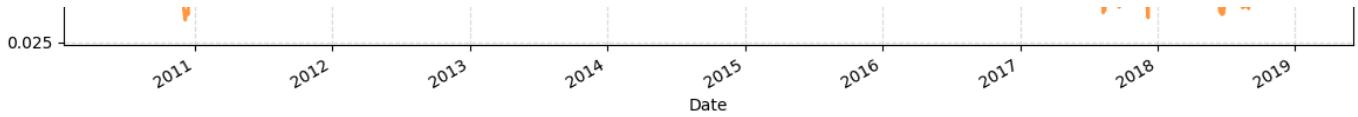


PBW: Forecasted vs Realized 1-Month Volatility
2008-01-01 to 2019-01-01



QCLN: Forecasted vs Realized 1-Month Volatility
2008-01-01 to 2019-01-01





GARCH Hedging

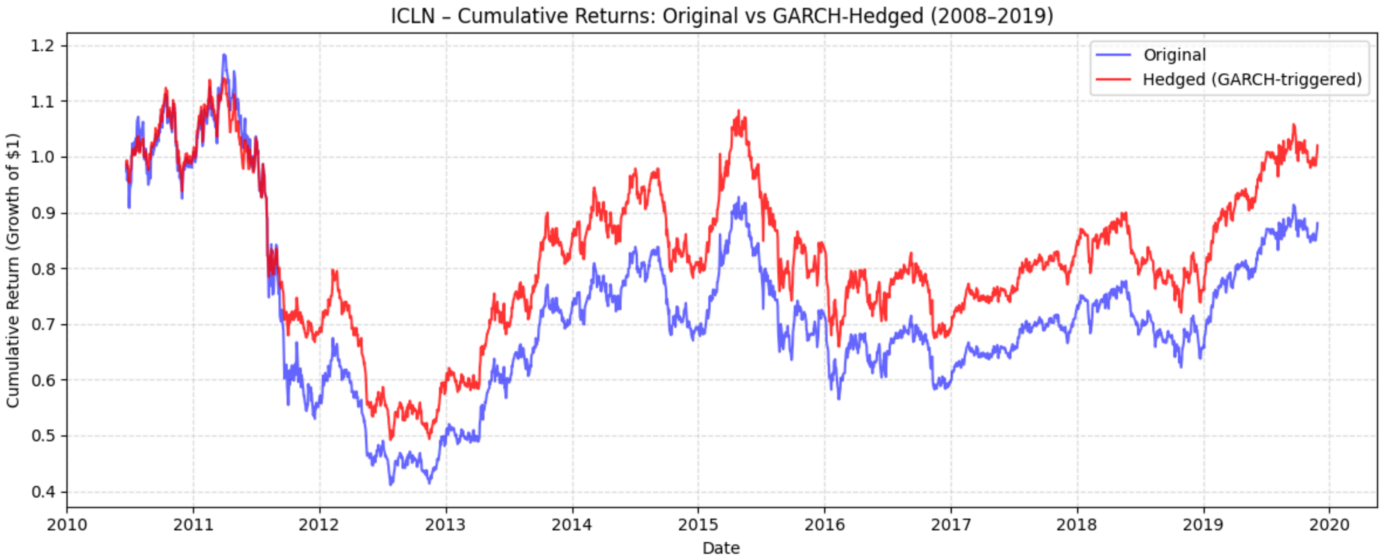
```
import pandas as pd
import numpy as np
from arch import arch_model
import matplotlib.pyplot as plt

# Parameters
rolling_window = 500
forecast_horizon = 22
de_risk_pct = 0.5
percentile_threshold = 0.9

# Restrict data to 2008–2019
renewable_df.index = pd.to_datetime(renewable_df.index)
renewable_subset = renewable_df.loc["2008":"2019"]

hedged_returns = {}
hedge_log = {}  # Store decision logs
hedge_trigger_dates_dict = {}  # NEW: stores dates with hedge activation

for col in renewable_subset.columns:
    print(f"Processing {col}...")
    returns = renewable_subset[col].dropna() * 100
    forecast_vols = []
    decisions = []
    hedge_dates = []

    for i in range(rolling_window, len(returns) - forecast_horizon):
        window = returns.iloc[i - rolling_window:i]
        model = arch_model(window, vol='Garch', p=1, q=1)
        try:
            res = model.fit(disp="off")
            forecast = res.forecast(horizon=forecast_horizon)
            avg_vol = np.mean(np.sqrt(forecast.variance.values[-1]) / 100)
            forecast_vols.append(avg_vol)
        except:
            forecast_vols.append(np.nan)

    valid_index = returns.index[rolling_window:-forecast_horizon]
    vol_signal = pd.Series(forecast_vols, index=valid_index)
    threshold = vol_signal.quantile(percentile_threshold)

    raw_returns = returns.loc[valid_index] / 100
    hedged = raw_returns.copy()

    for date in valid_index:
        vol = vol_signal.loc[date]
        if vol > threshold:
            decision = f"{date.date()}: De-risked (Vol={vol:.4f} > {threshold:.4f})"
            hedged.loc[date] *= (1 - de_risk_pct)
            hedge_dates.append(date)  # store hedge activation date
        else:
            decision = f"{date.date()}: Full exposure (Vol={vol:.4f} <= {threshold:.4f})"
        decisions.append(decision)

    hedged_returns[col] = hedged
```

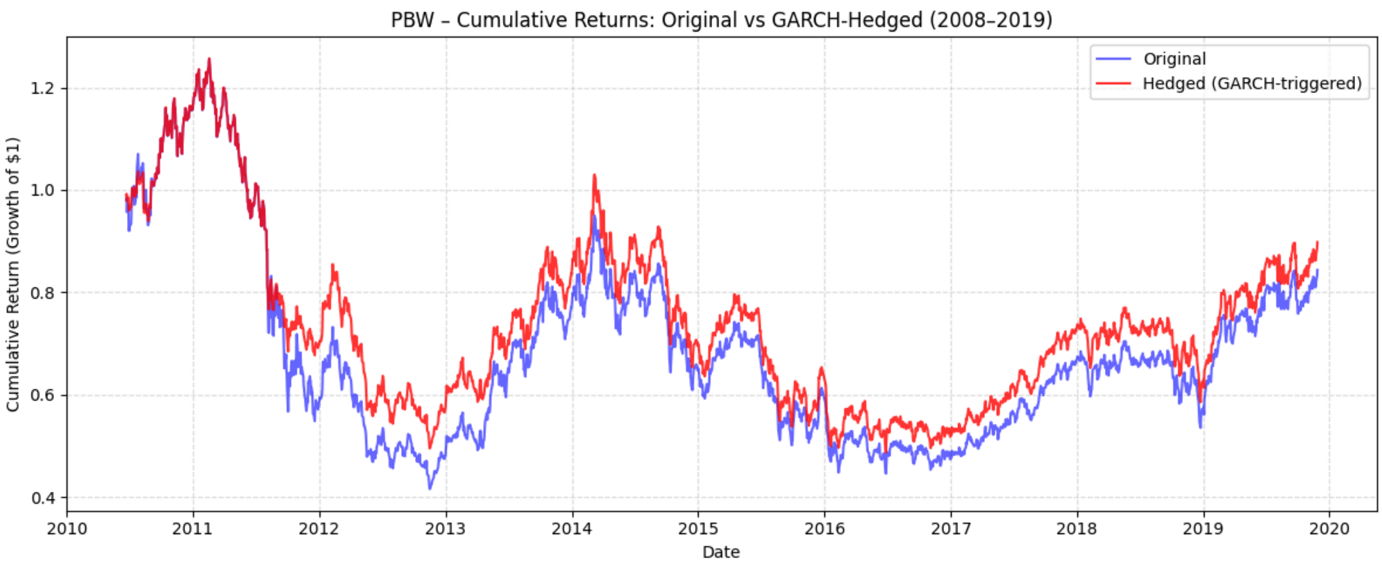
```
    hedge_log[col] = decisions
    hedge_trigger_dates_dict[col] = hedge_dates  # save hedge dates per ETF

# Save returns
hedged_renewable_df = pd.DataFrame(hedged_returns)
hedged_renewable_df.to_csv("renewable_hedged_returns.csv")

# Save logs
with open("hedge_decision_log.txt", "w") as f:
    for col, logs in hedge_log.items():
        f.write(f"\n=== {col} Hedge Decision Log ===\n")
        for entry in logs:
            f.write(entry + "\n")

# Save hedge trigger dates
with open("hedge_trigger_dates.txt", "w") as f:
    for col, dates in hedge_trigger_dates_dict.items():
        f.write(f"\n=== {col} Hedge Trigger Dates ===\n")
        for d in dates:
            f.write(str(d.date()) + "\n")

print("✅ Saved hedged returns, decision logs, and hedge trigger dates.")
```

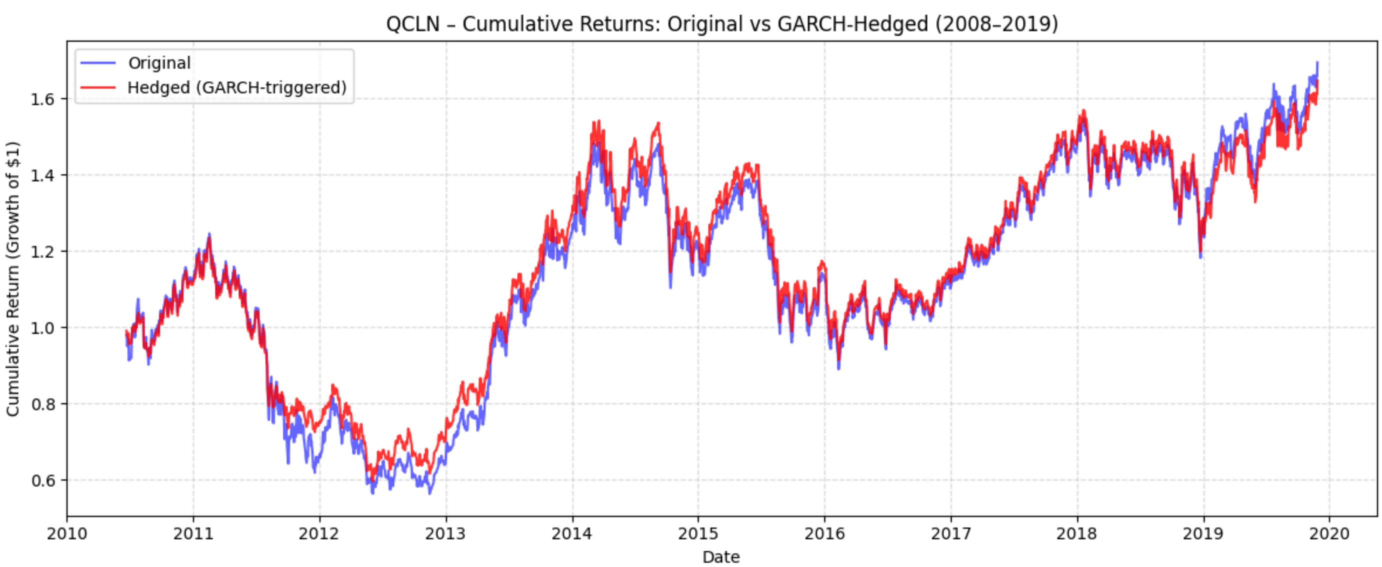
```
Processing ICLN...
Processing PBW...
Processing QCLN...
✅ Saved hedged returns, decision logs, and hedge trigger dates.
```

Start coding or generate with AI.

```
import pandas as pd
import matplotlib.pyplot as plt

# Load data
renewable_df = pd.read_csv("renewable_etfs_returns.csv", parse_dates=["date"], index_col="date")
hedged_renewable_df = pd.read_csv("renewable_hedged_returns.csv", parse_dates=["date"], index_col="date")

# Align date range
renewable_df = renewable_df.loc[hedged_renewable_df.index]

# Plot cumulative returns for each ETF
for col in hedged_renewable_df.columns:
    cum_orig = (1 + renewable_df[col]).cumprod()
    cum_hedged = (1 + hedged_renewable_df[col]).cumprod()

    plt.figure(figsize=(12, 5))
    plt.plot(cum_orig.index, cum_orig, label="Original", color="blue", alpha=0.6)
    plt.plot(cum_hedged.index, cum_hedged, label="Hedged (GARCH-triggered)", color="red", alpha=0.8)
    plt.title(f"{col} — Cumulative Returns: Original vs GARCH-Hedged (2008–2019)")
    plt.xlabel("Date")
    plt.ylabel("Cumulative Return (Growth of $1)")
    plt.grid(True, linestyle="--", alpha=0.4)
    plt.legend()
    plt.tight_layout()
    plt.show()
```


ICLN – Cumulative Returns: Original vs GARCH-Hedged (2008–2019)
Original
Hedged (GARCH-triggered)
Cumulative Return (Growth of $1)
Date
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020


PBW – Cumulative Returns: Original vs GARCH-Hedged (2008–2019)
Original
Hedged (GARCH-triggered)
Cumulative Return (Growth of $1)
Date
0.4
0.6
0.8
1.0
1.2
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020


QCLN – Cumulative Returns: Original vs GARCH-Hedged (2008–2019)
Original
Hedged (GARCH-triggered)
Cumulative Return (Growth of $1)
Date
0.6
0.8
1.0
1.2
1.4
1.6
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020

```
def run_garch_hedging_strategy(renewable_df,
                               etf_name,
                               rolling_window=500,
                               forecast_horizon=22,
                               de_risk_pct=0.5,
                               percentile_threshold=0.9,
                               start_date="2008",
                               end_date="2019"
                               ):
    """
    Run GARCH-based hedging strategy for a single ETF.

    Parameters:
    -----------
    renewable_df : pandas.DataFrame
        DataFrame containing ETF returns
    etf_name : str
        Name of the ETF to analyze
    rolling_window : int
        Window size for GARCH estimation
    forecast_horizon : int
        Number of days to forecast volatility
    de_risk_pct : float
        Percentage to de-risk when volatility is high
    percentile_threshold : float
        Percentile threshold for volatility (0-1)
    start_date : str
        Start date for analysis
    end_date : str
        End date for analysis
    Returns:
    --------
    dict
        Dictionary containing results and metrics
    """

    # Ensure datetime index
    renewable_df.index = pd.to_datetime(renewable_df.index)
    renewable_subset = renewable_df.loc[start_date:end_date]

    print(f"Processing {etf_name}...")
    returns = renewable_subset[etf_name].dropna() * 100
    forecast_vols = []
    decisions = []
    hedge_trigger_dates = pd.DataFrame(columns=['ETF', 'Date', 'Volatility', 'Threshold'])

    # Fit GARCH model and forecast volatility
    for i in range(rolling_window, len(returns) - forecast_horizon):
        window = returns.iloc[i - rolling_window:i]
        model = arch_model(window, vol='Garch', p=1, q=1)
        try:
            res = model.fit(disp="off")
            forecast = res.forecast(horizon=forecast_horizon)
            avg_vol = np.mean(np.sqrt(forecast.variance.values[-1]) / 100)
            forecast_vols.append(avg_vol)
        except:
            forecast_vols.append(np.nan)

    # Create volatility signal and determine threshold
    valid_index = returns.index[rolling_window:-forecast_horizon]
    vol_signal = pd.Series(forecast_vols, index=valid_index)
    threshold = vol_signal.quantile(percentile_threshold)

    # Apply hedging strategy
    raw_returns = returns.loc[valid_index] / 100
    hedged = raw_returns.copy()

    for date in valid_index:
        vol = vol_signal.loc[date]
        if vol > threshold:
            decision = f"{date.date()}: De-risked (Vol={vol:.4f} > {threshold:.4f})"
            hedged.loc[date] *= (1 - de_risk_pct)
            # Add to hedge trigger dates DataFrame
            hedge_trigger_dates = pd.concat([hedge_trigger_dates, pd.DataFrame({
                'ETF': [etf_name],
```

```
                'Date': [date],
                'Volatility': [vol],
                'Threshold': [threshold]
            })], ignore_index=True)
        else:
            decision = f"{date.date()}: Full exposure (Vol={vol:.4f} <= {threshold:.4f})"
        decisions.append(decision)

    # Create DataFrame of hedged returns
    hedged_returns = pd.DataFrame({
        'raw_returns': raw_returns,
        'hedged_returns': hedged
    })

    # Sort hedge trigger dates by date
    hedge_trigger_dates = hedge_trigger_dates.sort_values('Date')

    print(f"Finished Processing {etf_name}")
    # Return results dictionary
    return {
        'etf_name': etf_name,
        'hedged_returns': hedged_returns,
        'hedge_log': decisions,
        'hedge_trigger_dates': hedge_trigger_dates,
        'volatility_signal': vol_signal,
        'threshold': threshold,
        'parameters': {
            'rolling_window': rolling_window,
            'forecast_horizon': forecast_horizon,
            'de_risk_pct': de_risk_pct,
            'percentile_threshold': percentile_threshold,
            'start_date': start_date,
            'end_date': end_date
        }

    }


def compute_garch_volatility(renewable_df,
                             etf_name,
                             rolling_window=500,
                             forecast_horizon=22,
                             start_date="2008",
                             end_date="2019"):
    """
    Compute GARCH volatility forecasts for a single ETF.

    Parameters:
    -----------
    renewable_df : pandas.DataFrame
        DataFrame containing ETF returns
    etf_name : str
        Name of the ETF to analyze
    rolling_window : int
        Window size for GARCH estimation
    forecast_horizon : int
        Number of days to forecast volatility
    start_date : str
        Start date for analysis
    end_date : str
        End date for analysis

    Returns:
    --------
    dict
        Dictionary containing volatility forecasts and parameters
    """
    # Ensure datetime index
    renewable_df.index = pd.to_datetime(renewable_df.index)
    renewable_subset = renewable_df.loc[start_date:end_date]

    print(f"Computing GARCH volatility for {etf_name}...")
    returns = renewable_subset[etf_name].dropna() * 100
    forecast_vols = []

    # Fit GARCH model and forecast volatility
    for i in range(rolling_window, len(returns) - forecast_horizon):
```

```
        window = returns.iloc[i - rolling_window:i]
        model = arch_model(window, vol='Garch', p=1, q=1)
        try:
            res = model.fit(disp="off")
            forecast = res.forecast(horizon=forecast_horizon)
            avg_vol = np.mean(np.sqrt(forecast.variance.values[-1]) / 100)
            forecast_vols.append(avg_vol)
        except:
            forecast_vols.append(np.nan)

    # Create volatility signal
    valid_index = returns.index[rolling_window:-forecast_horizon]
    vol_signal = pd.Series(forecast_vols, index=valid_index)

    print(f"Finished computing GARCH volatility for {etf_name}")
    return {
        'etf_name': etf_name,
        'volatility_signal': vol_signal,
        'raw_returns': returns.loc[valid_index] / 100,
        'parameters': {
            'rolling_window': rolling_window,
            'forecast_horizon': forecast_horizon,
            'start_date': start_date,
            'end_date': end_date
        }
    }

def apply_hedging_strategy(garch_results,
                           de_risk_pct=0.5,
                           percentile_threshold=0.9):
    """
    Apply hedging strategy based on GARCH volatility forecasts.

    Parameters:
    -----------
    garch_results : dict
        Results from compute_garch_volatility function
    de_risk_pct : float
        Percentage to de-risk when volatility is high
    percentile_threshold : float
        Percentile threshold for volatility (0-1)

    Returns:
    --------
    dict
        Dictionary containing hedging results and metrics
    """
    etf_name = garch_results['etf_name']
    vol_signal = garch_results['volatility_signal']
    raw_returns = garch_results['raw_returns']

    print(f"Applying hedging strategy for {etf_name}...")

    # Determine threshold
    threshold = vol_signal.quantile(percentile_threshold)

    # Apply hedging strategy
    hedged = raw_returns.copy()
    decisions = []
    hedge_trigger_dates = pd.DataFrame(columns=['ETF', 'Date', 'Volatility', 'Threshold'])

    for date in vol_signal.index:
        vol = vol_signal.loc[date]
        if vol > threshold:
            decision = f"{date.date()}: De-risked (Vol={vol:.4f} > {threshold:.4f})"
            hedged.loc[date] *= (1 - de_risk_pct)
            # Add to hedge trigger dates DataFrame
            hedge_trigger_dates = pd.concat([hedge_trigger_dates, pd.DataFrame({
                'ETF': [etf_name],
                'Date': [date],
                'Volatility': [vol],
                'Threshold': [threshold]
            })], ignore_index=True)
        else:
            decision = f"{date.date()}: Full exposure (Vol={vol:.4f} <= {threshold:.4f})"
        decisions.append(decision)
```

```python
    # Create DataFrame of hedged returns
    hedged_returns = pd.DataFrame({
        'raw_returns': raw_returns,
        'hedged_returns': hedged
    })

    # Sort hedge trigger dates by date
    hedge_trigger_dates = hedge_trigger_dates.sort_values('Date')

    print(f"Finished applying hedging strategy for {etf_name}")
    return {
        'etf_name': etf_name,
        'hedged_returns': hedged_returns,
        'hedge_log': decisions,
        'hedge_trigger_dates': hedge_trigger_dates,
        'volatility_signal': vol_signal,
        'threshold': threshold,
        'parameters': {
            **garch_results['parameters'],
            'de_risk_pct': de_risk_pct,
            'percentile_threshold': percentile_threshold
        }
    }
```

```python
# Compute GARCH volatilities
# ICLN
garch_vol_ICLN = compute_garch_volatility(
     renewable_df=renewable_df,
     etf_name='ICLN',
     rolling_window=500,
     forecast_horizon=22,
     start_date='2008-01-01',
     end_date='2019-12-31'
)
# PBW
garch_vol_PBW = compute_garch_volatility(
     renewable_df=renewable_df,
     etf_name='PBW',
     rolling_window=500,
     forecast_horizon=22,
     start_date='2008-01-01',
     end_date='2019-12-31'
)
# QCLN
garch_vol_QCLN = compute_garch_volatility(
     renewable_df=renewable_df,
     etf_name='QCLN',
     rolling_window=500,
     forecast_horizon=22,
     start_date='2008-01-01',
     end_date='2019-12-31'
)
```

```
Computing GARCH volatility for ICLN...
Finished computing GARCH volatility for ICLN
Computing GARCH volatility for PBW...
Finished computing GARCH volatility for PBW
Computing GARCH volatility for QCLN...
Finished computing GARCH volatility for QCLN
```

```python
# Perform hedge strategies with 50% derisk and 90% threshold
garch_hedge_ICLN = apply_hedging_strategy(
     garch_results=garch_vol_ICLN,
     de_risk_pct=0.5,
     percentile_threshold=0.9
)

garch_hedge_PBW = apply_hedging_strategy(
     garch_results=garch_vol_PBW,
     de_risk_pct=0.5,
     percentile_threshold=0.9
)

garch_hedge_QCLN = apply_hedging_strategy(
     garch_results=garch_vol_QCLN,
     de_risk_pct=0.5,
```

```
    percentile_threshold=0.9
)
```

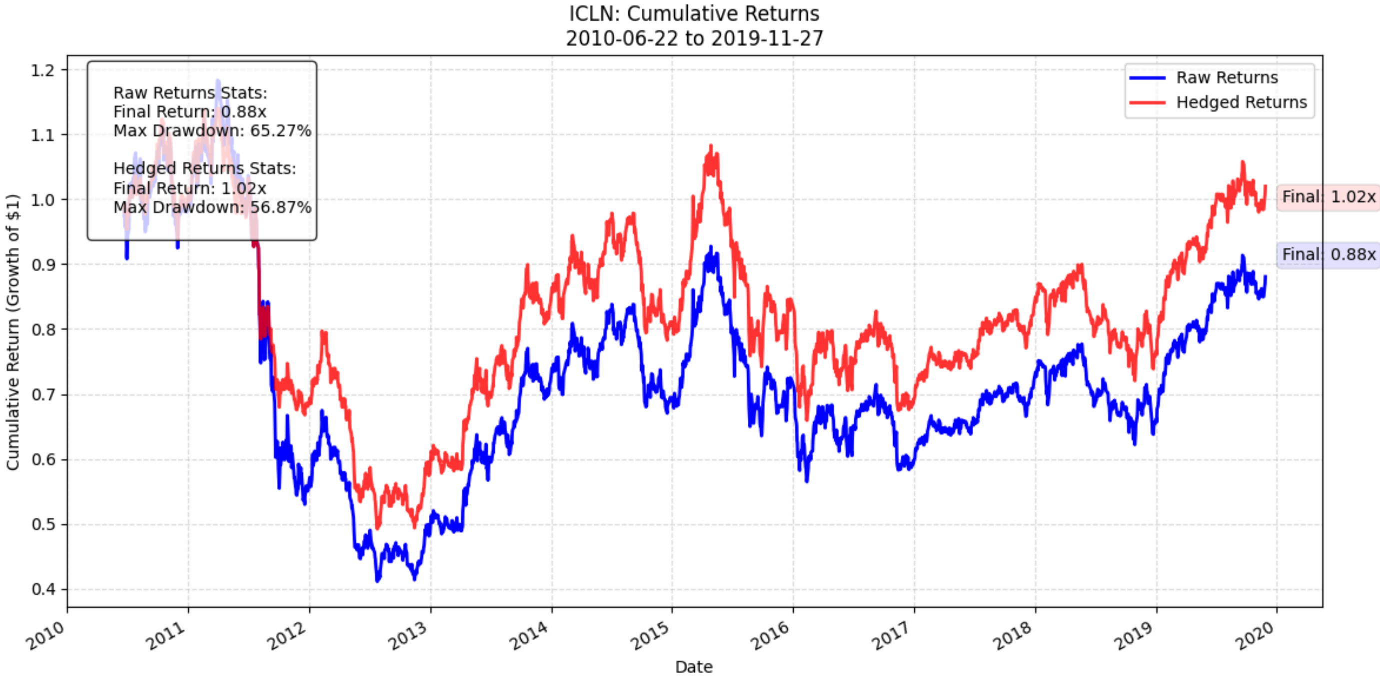
```
Applying hedging strategy for ICLN...
Finished applying hedging strategy for ICLN
Applying hedging strategy for PBW...
Finished applying hedging strategy for PBW
Applying hedging strategy for QCLN...
Finished applying hedging strategy for QCLN
```

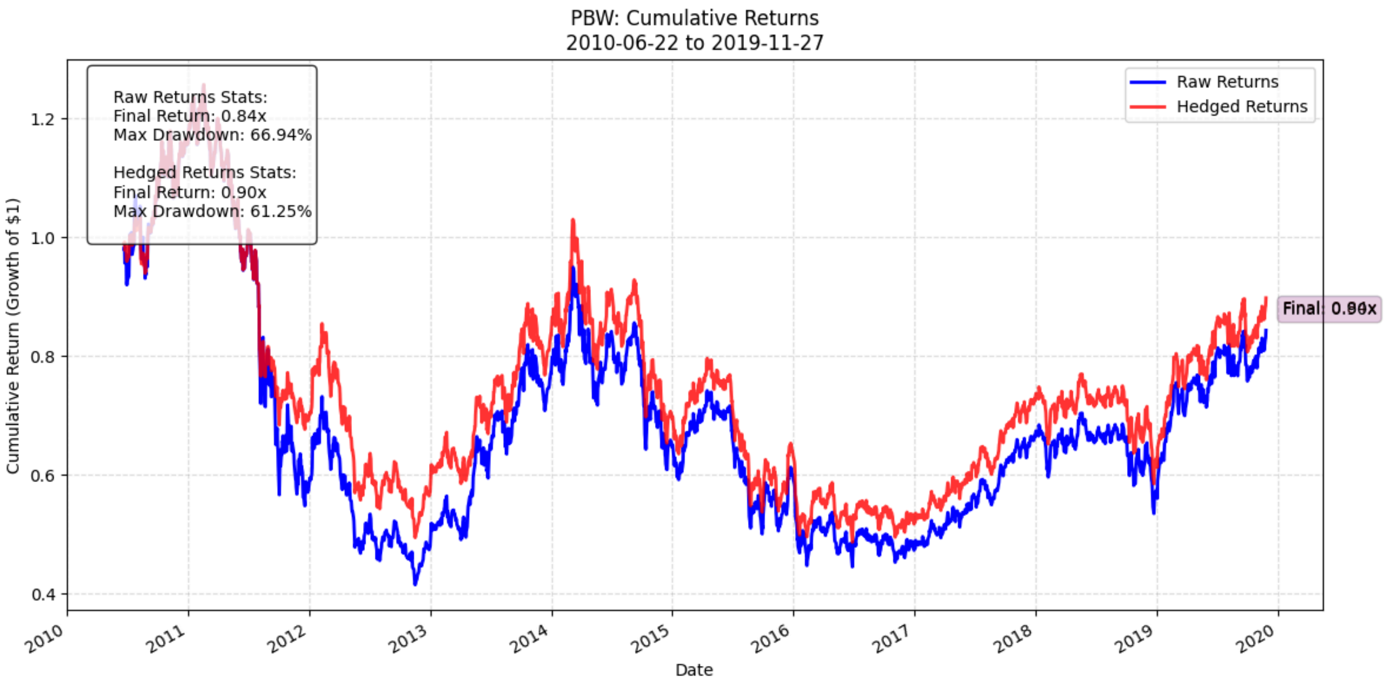
Start coding or generate with AI.

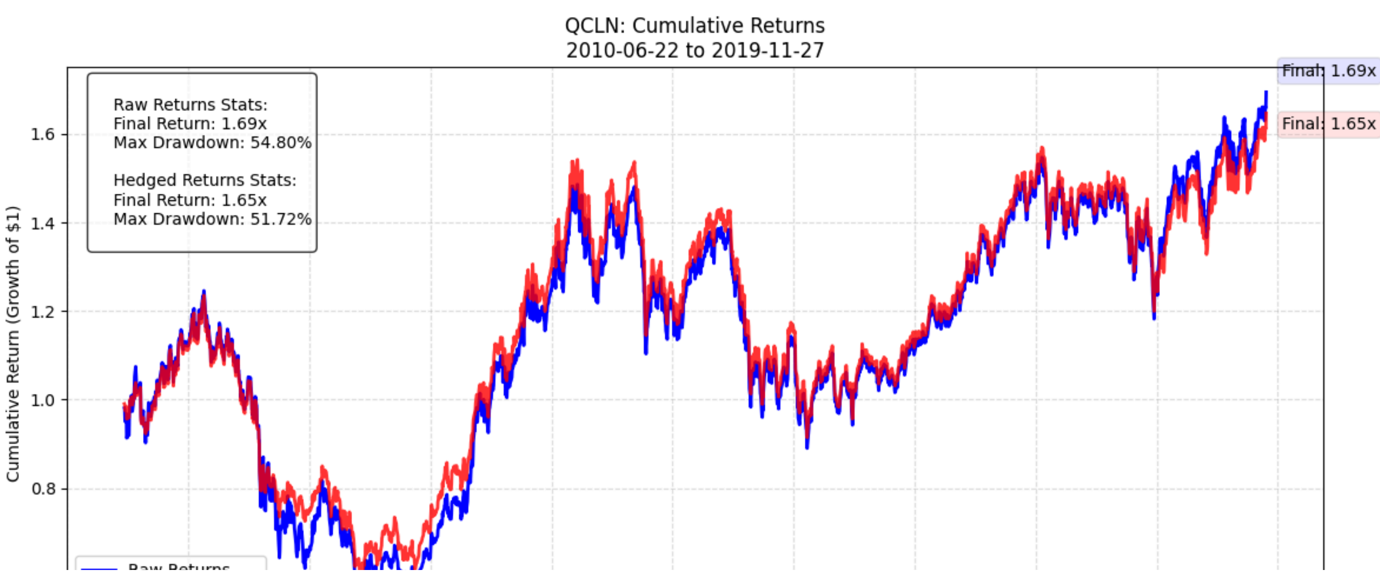
Need to experiment with de_risk_pct and percentile_threshold to minimize value at risk++ (while maintainint some sort of returns)

```
def plot_cumulative_returns(returns_df,
                          etf_name,
                          start_date=None,
                          end_date=None,
                          figsize=(12, 6),
                          linewidth=2,
                          alpha=0.8):
    """
    Plot cumulative returns for raw and hedged strategies.

    Parameters:
    -----------
    returns_df : pandas.DataFrame
        DataFrame with 'raw_returns' and 'hedged_returns' columns
    etf_name : str
        Name of the ETF for the plot title
    start_date : str, optional
        Start date for the plot (format: 'YYYY-MM-DD')
    end_date : str, optional
        End date for the plot (format: 'YYYY-MM-DD')
    figsize : tuple
        Figure size (width, height)
    linewidth : float
        Width of the lines
    alpha : float
        Transparency of the lines
    """
    # Filter by date range if provided
    if start_date:
        returns_df = returns_df.loc[start_date:]
    if end_date:
        returns_df = returns_df.loc[:end_date]

    # Calculate cumulative returns
    cum_raw = (1 + returns_df['raw_returns']).cumprod()
    cum_hedged = (1 + returns_df['hedged_returns']).cumprod()

    # Create the plot
    plt.figure(figsize=figsize)

    # Plot the lines
    plt.plot(cum_raw.index, cum_raw,
            label='Raw Returns',
            linewidth=linewidth,
            color='blue')

    plt.plot(cum_hedged.index, cum_hedged,
            label='Hedged Returns',
            linewidth=linewidth,
            alpha=alpha,
            color='red')

    # Add title and labels
    plt.title(f'{etf_name}: Cumulative Returns\n{returns_df.index[0].strftime("%Y-%m-%d")} to {returns_df.index[-1].strftime("%Y
    plt.xlabel('Date')
    plt.ylabel('Cumulative Return (Growth of $1)')

    # Add legend and grid
```

```python
    plt.legend()
    plt.grid(True, linestyle='--', alpha=0.4)

    # Format x-axis dates
    plt.gcf().autofmt_xdate()

    # Add final values as text annotations
    final_raw = cum_raw.iloc[-1]
    final_hedged = cum_hedged.iloc[-1]

    plt.annotate(f'Final: {final_raw:.2f}x',
                 xy=(cum_raw.index[-1], final_raw),
                 xytext=(10, 10), textcoords='offset points',
                 bbox=dict(boxstyle='round', facecolor='blue', alpha=0.1))

    plt.annotate(f'Final: {final_hedged:.2f}x',
                 xy=(cum_hedged.index[-1], final_hedged),
                 xytext=(10, -10), textcoords='offset points',
                 bbox=dict(boxstyle='round', facecolor='red', alpha=0.1))

    # Add some statistics as text
    stats_text = f"""
    Raw Returns Stats:
    Final Return: {final_raw:.2f}x
    Max Drawdown: {((cum_raw.cummax() - cum_raw) / cum_raw.cummax()).max():.2%}

    Hedged Returns Stats:
    Final Return: {final_hedged:.2f}x
    Max Drawdown: {((cum_hedged.cummax() - cum_hedged) / cum_hedged.cummax()).max():.2%}
    """

    # Add text box with statistics
    plt.text(0.02, 0.98, stats_text,
             transform=plt.gca().transAxes,
             verticalalignment='top',
             bbox=dict(boxstyle='round', facecolor='white', alpha=0.8))

    plt.tight_layout()
    plt.show()

    # Return the statistics
    return {
        'raw_stats': {
            'final_return': final_raw,
            'max_drawdown': ((cum_raw.cummax() - cum_raw) / cum_raw.cummax()).max()
        },
        'hedged_stats': {
            'final_return': final_hedged,
            'max_drawdown': ((cum_hedged.cummax() - cum_hedged) / cum_hedged.cummax()).max()
        }
    }



# Example usage:
plot_cumulative_returns(
    returns_df= garch_hedge_ICLN['hedged_returns'],
    etf_name='ICLN',
     start_date='2008-01-01',
     end_date='2019-12-31'
)
# Example usage:
plot_cumulative_returns(
    returns_df= garch_hedge_PBW['hedged_returns'],
    etf_name='PBW',
     start_date='2008-01-01',
     end_date='2019-12-31'
)
# Example usage:
plot_cumulative_returns(
    returns_df= garch_hedge_QCLN['hedged_returns'],
    etf_name='QCLN',
     start_date='2008-01-01',
     end_date='2019-12-31'
)
```


ICLN: Cumulative Returns
2010-06-22 to 2019-11-27
Raw Returns Stats:
Final Return: 0.88x
Max Drawdown: 65.27%
Hedged Returns Stats:
Final Return: 1.02x
Max Drawdown: 56.87%
Raw Returns
Hedged Returns
Final: 1.02x
Final: 0.88x
Cumulative Return (Growth of $1)
Date


PBW: Cumulative Returns
2010-06-22 to 2019-11-27
Raw Returns Stats:
Final Return: 0.84x
Max Drawdown: 66.94%
Hedged Returns Stats:
Final Return: 0.90x
Max Drawdown: 61.25%
Raw Returns
Hedged Returns
Cumulative Return (Growth of $1)
Date


QCLN: Cumulative Returns
2010-06-22 to 2019-11-27
Raw Returns Stats:
Final Return: 1.69x
Max Drawdown: 54.80%
Hedged Returns Stats:
Final Return: 1.65x
Max Drawdown: 51.72%
Final: 1.69x
Final: 1.65x
Cumulative Return (Growth of $1)


```
{'raw_stats': {'final_return': np.float64(1.6942908731104076),
  'max_drawdown': 0.5480178345644655},
 'hedged_stats': {'final_return': np.float64(1.6467953906668724),
  'max_drawdown': 0.517240033686768}}
```

```python
def risk_metrics(returns):
    """
    Calculate risk metrics for a returns series.

    Parameters:
    -----------
    returns : array-like
        Array or Series of returns

    Returns:
    --------
    dict
        Dictionary of risk metrics
    """
    # Convert to pandas Series if it's not already
    if not isinstance(returns, pd.Series):
        returns = pd.Series(returns)

    # Calculate metrics
    var = returns.quantile(0.05)
    cvar = returns[returns <= var].mean()
    mdd = (returns.cummax() - returns).max()

    # Return metrics dictionary
    return {
        "VaR (95%)": round(var, 4),
        "CVaR (95%)": round(cvar, 4),
        "Max Drawdown": round(mdd, 4)
    }


# Function to compare unhedged and hedged metrics
def compare_risk_metrics(returns_df):
    """
    Compare risk metrics between unhedged and hedged returns.

    Parameters:
    -----------
    returns_df : pandas.DataFrame
        DataFrame with 'raw_returns' and 'hedged_returns' columns

    Returns:
    --------
    pandas.DataFrame
        Comparison table of risk metrics
    """
    # Calculate metrics for both strategies
    unhedged_metrics = risk_metrics(returns_df['raw_returns'])
    hedged_metrics = risk_metrics(returns_df['hedged_returns'])

    # Create comparison DataFrame
    comparison = pd.DataFrame({
        'Unhedged': unhedged_metrics,
        'Hedged': hedged_metrics
    })
```

```
    # Calculate percent changes
    percent_changes = ((comparison['Hedged'] - comparison['Unhedged']) /
                       comparison['Unhedged'] * 100).round(2)
    comparison['Change (%)'] = percent_changes

    return comparison
```

```
compare_risk_metrics(garch_hedge_ICLN['hedged_returns'])
```

	Unhedged	Hedged	Change (%)
VaR (95%)	-0.0244	-0.0217	-11.0700
CVaR (95%)	-0.0344	-0.0296	-13.9500
Max Drawdown	0.1425	0.1222	-14.2500

```
compare_risk_metrics(garch_hedge_PBW['hedged_returns'])
```

	Unhedged	Hedged	Change (%)
VaR (95%)	-0.0278	-0.0255	-8.2700
CVaR (95%)	-0.0378	-0.0337	-10.8500
Max Drawdown	0.1639	0.1286	-21.5400

```
compare_risk_metrics(garch_hedge_QCLN['hedged_returns'])
```

	Unhedged	Hedged	Change (%)
VaR (95%)	-0.0275	-0.0252	-8.3600
CVaR (95%)	-0.0368	-0.0332	-9.7800
Max Drawdown	0.1291	0.1125	-12.8600

Hidden Markov Model on Realized Volatiliti

2-state Gaussigan HMM

```
pip install hmmlearn
```

```
Collecting hmmlearn
  Downloading hmmlearn-0.3.3-cp311-cp311-manylinux_2_17_x86_64.manylinux2014_x86_64.whl.metadata (3.0 kB)
Requirement already satisfied: numpy>=1.10 in /usr/local/lib/python3.11/dist-packages (from hmmlearn) (2.0.2)
Requirement already satisfied: scikit-learn!=0.22.0,>=0.16 in /usr/local/lib/python3.11/dist-packages (from hmmlearn) (1.6.1
Requirement already satisfied: scipy>=0.19 in /usr/local/lib/python3.11/dist-packages (from hmmlearn) (1.14.1)
Requirement already satisfied: joblib>=1.2.0 in /usr/local/lib/python3.11/dist-packages (from scikit-learn!=0.22.0,>=0.16->h
Requirement already satisfied: threadpoolctl>=3.1.0 in /usr/local/lib/python3.11/dist-packages (from scikit-learn!=0.22.0,>=
Downloading hmmlearn-0.3.3-cp311-cp311-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (165 kB)
   ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 165.9/165.9 kB 3.0 MB/s eta 0:00:00
Installing collected packages: hmmlearn
Successfully installed hmmlearn-0.3.3
```

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from hmmlearn.hmm import GaussianHMM

# 1) Prepare your realized volatility series
#    Use your daily PBW returns (decimal) to compute a 22-day rolling std
ticker = 'PBW'
ret = renewable_df[ticker].loc['2008':'2015'].dropna()
realized_vol = ret.rolling(window=22).std().dropna()

# 2) Reshape for HMM (n_samples, n_features)
X = realized_vol.values.reshape(-1, 1)

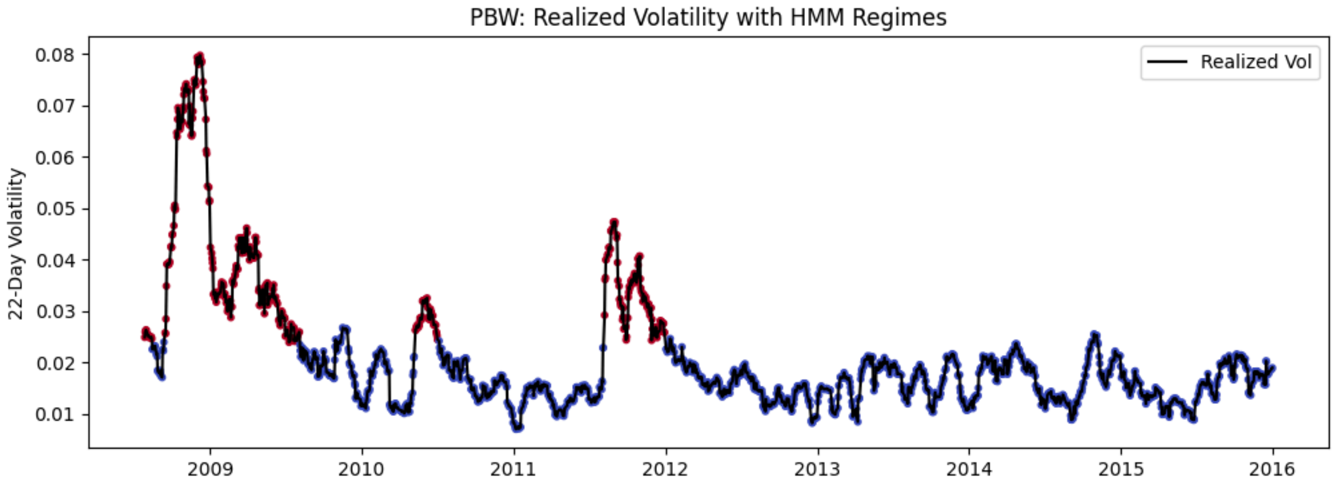
# 3) Fit a 2-state Gaussian HMM
model = GaussianHMM(n_components=2, covariance_type='full', n_iter=100, random_state=42)
model.fit(X)
```

```
# 4) Decode the most likely state sequence
states = model.predict(X)
state_series = pd.Series(states, index=realized_vol.index)

# 5) Inspect the regimes
for i in [0,1]:
    mu = model.means_[i][0]
    sigma = np.sqrt(model.covars_[i][0][0])
    print(f"State {i}: mean vol = {mu:.4f},   std vol = {sigma:.4f}")

# 6) Plot realized vol colored by regime
plt.figure(figsize=(12,4))
plt.plot(realized_vol.index, realized_vol, label='Realized Vol', color='black')
plt.scatter(realized_vol.index, realized_vol, c=states, cmap='coolwarm', s=10)
plt.title(f"{ticker}: Realized Volatility with HMM Regimes")
plt.ylabel("22-Day Volatility")
plt.legend()
plt.show()
```

```
State 0: mean vol = 0.0161,   std vol = 0.0046
State 1: mean vol = 0.0384,   std vol = 0.0154
```



3-state Gaussian HMM

```
# 1) Compute 22-day realized volatility for PBW (2008–2019)
ticker = 'PBW'
ret = renewable_df[ticker].loc['2008':'2015'].dropna()
realized_vol = ret.rolling(window=22).std().dropna()

# 2) Reshape for HMM
X = realized_vol.values.reshape(-1, 1)

# 3) Fit a 3-state Gaussian HMM
model3 = GaussianHMM(
    n_components=3,
    covariance_type='full',
    n_iter=200,
    random_state=42
)
model3.fit(X)

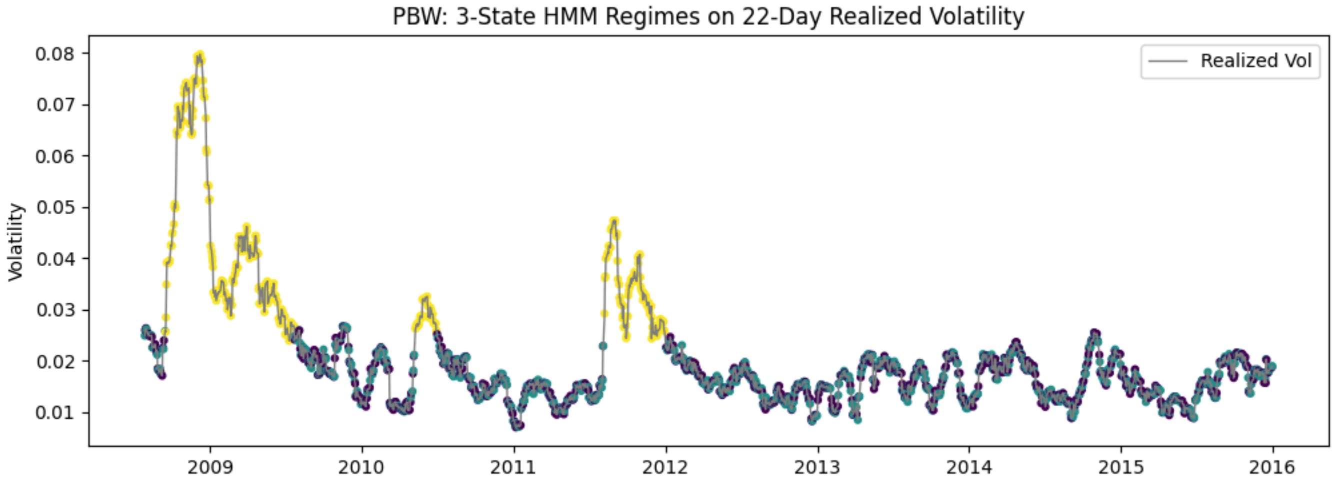
# 4) Decode the most likely state sequence
states3 = model3.predict(X)
state_series3 = pd.Series(states3, index=realized_vol.index)

# 5) Inspect each regime’s volatility characteristics
print("Regime means & std devs:")
for i in range(3):
    mu    = model3.means_[i][0]
    sigma = np.sqrt(model3.covars_[i][0][0])
    print(f" State {i}: mean = {mu:.4f},  std = {sigma:.4f}")

# 6) Plot realized vol colored by regime
plt.figure(figsize=(12,4))
plt.plot(realized_vol.index, realized_vol, color='grey', linewidth=1, label='Realized Vol')
```

```
plt.scatter(realized_vol.index, realized_vol, c=states3, cmap='viridis', s=12)
plt.title(f"{ticker}: 3-State HMM Regimes on 22-Day Realized Volatility")
plt.ylabel("Volatility")
plt.legend()
plt.show()
```

```
Regime means & std devs:
 State 0: mean = 0.0164,  std = 0.0054
 State 1: mean = 0.0164,  std = 0.0054
 State 2: mean = 0.0400,  std = 0.0155
```




```
# Fit both models on the 2008–2015 train set.
# Validate on 2016–2018:


# Try to import hmmlearn, else instruct user to install
try:
    from hmmlearn.hmm import GaussianHMM
except ImportError as e:
    raise ImportError("hmmlearn not installed. Install via 'pip install hmmlearn' to use HMM functionality.") from e

# Parameters
ticker = 'PBW'
start_train = '2008-01-01'
end_train = '2015-12-31'
end_full = '2019-12-31'
forecast_horizon = 22
de_risk_pct = 0.5  # 50% reduction in exposure in high-vol state

# Ensure datetime index
returns_df = renewable_df.copy()
returns_df.index = pd.to_datetime(returns_df.index)

# 1) Compute 22-day realized volatility series
ret_full = returns_df[ticker].loc['2008':end_full].dropna()
realized_vol_full = ret_full.rolling(window=forecast_horizon).std().dropna()

# 2) Split realized volatility into train and test
rv_train = realized_vol_full.loc[start_train:end_train]
rv_test = realized_vol_full.loc['2016-01-01':end_full]

# 3) Fit 2-state Gaussian HMM on training data
X_train = rv_train.values.reshape(-1, 1)
model_hmm = GaussianHMM(n_components=2, covariance_type='full', n_iter=200, random_state=42)
model_hmm.fit(X_train)

# Identify high-vol state (state with higher mean)
means = model_hmm.means_.flatten()
high_vol_state = np.argmax(means)

# 4) Decode states for full period
X_full = realized_vol_full.values.reshape(-1, 1)
states_full = model_hmm.predict(X_full)
state_series = pd.Series(states_full, index=realized_vol_full.index, name='Regime')

# 5) Plot realized volatility with regimes
plt.figure(figsize=(12,4))
plt.plot(realized_vol_full.index, realized_vol_full, color='grey', label='Realized Vol')
```

```
plt.plot(realized_vol_full.index, realized_vol_full, color='grey', label='Realized Vol')
plt.scatter(realized_vol_full.index, realized_vol_full, c=states_full, cmap='coolwarm', s=10)
plt.title(f"{ticker}: 2-State HMM Regimes on Realized Volatility")
plt.ylabel("22-day Volatility")
plt.legend()
plt.show()

# 6) Construct hedged returns: reduce exposure when in high-vol state
raw_returns = ret_full.loc[state_series.index]  # align dates
hedged_returns = raw_returns.copy()
hedged_returns[state_series == high_vol_state] *= (1 - de_risk_pct)

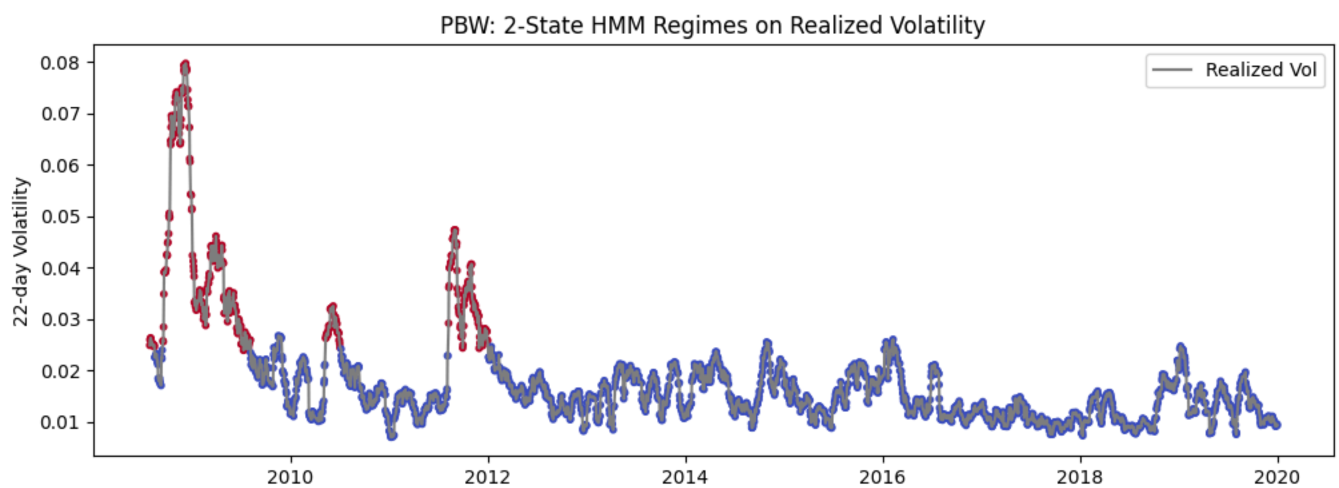
# 7) Evaluate risk metrics on test period
def compute_metrics(returns):
    var = returns.quantile(0.05)
    cvar = returns[returns <= var].mean()
    mdd = (returns.cummax() - returns).max()
    return var, cvar, mdd

# Align test returns
test_returns = raw_returns.loc[rv_test.index]
test_hedged = hedged_returns.loc[rv_test.index]

metrics_orig = compute_metrics(test_returns)
metrics_hedged = compute_metrics(test_hedged)

# Display metrics
metrics_df = pd.DataFrame({
    'Original': metrics_orig,
    'Hedged': metrics_hedged
}, index=['VaR (95%)','CVaR (95%)','Max Drawdown'])

metrics_df
```



	Original	Hedged
VaR (95%)	-0.0214	-0.0214
CVaR (95%)	-0.0299	-0.0299
Max Drawdown	0.1002	0.1002

Next steps: Generate code with `metrics_df` View recommended plots New interactive sheet

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

# Try to import hmmlearn, else instruct user to install
try:
    from hmmlearn.hmm import GaussianHMM
except ImportError as e:
    raise ImportError("hmmlearn not installed. Install via 'pip install hmmlearn' to use HMM functionality.") from e

# Parameters
ticker = 'PBW'
start_train = '2008-01-01'
```

```
start_train = '2008-01-01'
end_train = '2015-12-31'
end_full = '2019-12-31'
forecast_horizon = 22

# De-risking percentages for each state
de_risk_high = 0.50  # 50% reduction in high volatility state
de_risk_medium = 0.25  # 25% reduction in medium volatility state
de_risk_low = 0.00  # 0% reduction in low volatility state

# Ensure datetime index
returns_df = renewable_df.copy()
returns_df.index = pd.to_datetime(returns_df.index)

# 1) Compute 22-day realized volatility series
ret_full = returns_df[ticker].loc['2008':end_full].dropna()
realized_vol_full = ret_full.rolling(window=forecast_horizon).std().dropna()

# 2) Split realized volatility into train and test
rv_train = realized_vol_full.loc[start_train:end_train]
rv_test = realized_vol_full.loc['2016-01-01':end_full]

# 3) Fit 3-state Gaussian HMM on training data
X_train = rv_train.values.reshape(-1, 1)
model_hmm = GaussianHMM(n_components=3, covariance_type='full', n_iter=200, random_state=42)
model_hmm.fit(X_train)

# Identify states by their mean volatility (low to high)
means = model_hmm.means_.flatten()
sorted_states = np.argsort(means)  # States ordered from low to high volatility
low_vol_state = sorted_states[0]
medium_vol_state = sorted_states[1]
high_vol_state = sorted_states[2]

# 4) Decode states for full period
X_full = realized_vol_full.values.reshape(-1, 1)
states_full = model_hmm.predict(X_full)
state_series = pd.Series(states_full, index=realized_vol_full.index, name='Regime')

# 5) Plot realized volatility with regimes
# 5) Plot realized volatility with regimes
plt.figure(figsize=(12,4))
plt.plot(realized_vol_full.index, realized_vol_full, color='grey', label='Realized Vol')

# Create a custom colormap for the three states
colors = ['green', 'orange', 'red']  # Low, Medium, High volatility
state_colors = [colors[state] for state in states_full]

# Plot the scatter points with custom colors
scatter = plt.scatter(realized_vol_full.index, realized_vol_full,
                     c=state_colors, s=10)

# Create custom legend
from matplotlib.lines import Line2D
legend_elements = [
    Line2D([0], [0], marker='o', color='w', markerfacecolor='green', markersize=10, label='Low Volatility'),
    Line2D([0], [0], marker='o', color='w', markerfacecolor='orange', markersize=10, label='Medium Volatility'),
    Line2D([0], [0], marker='o', color='w', markerfacecolor='red', markersize=10, label='High Volatility'),
    Line2D([0], [0], color='grey', lw=2, label='Realized Volatility')
]

plt.title(f"{ticker}: 3-State HMM Regimes on Realized Volatility")
plt.ylabel("22-day Volatility")
plt.legend(handles=legend_elements)
plt.show()

# 6) Construct hedged returns with state-dependent de-risking
raw_returns = ret_full.loc[state_series.index]  # align dates
hedged_returns = raw_returns.copy()

# Apply different de-risking based on state
hedged_returns[state_series == high_vol_state] *= (1 - de_risk_high)
hedged_returns[state_series == medium_vol_state] *= (1 - de_risk_medium)
hedged_returns[state_series == low_vol_state] *= (1 - de_risk_low)

# 7) Evaluate risk metrics on test period
def compute_metrics(returns):
    var = returns.quantile(0.05)
```

```
    cvar = returns[returns <= var].mean()
    mdd = (returns.cummax() - returns).max()
    return var, cvar, mdd

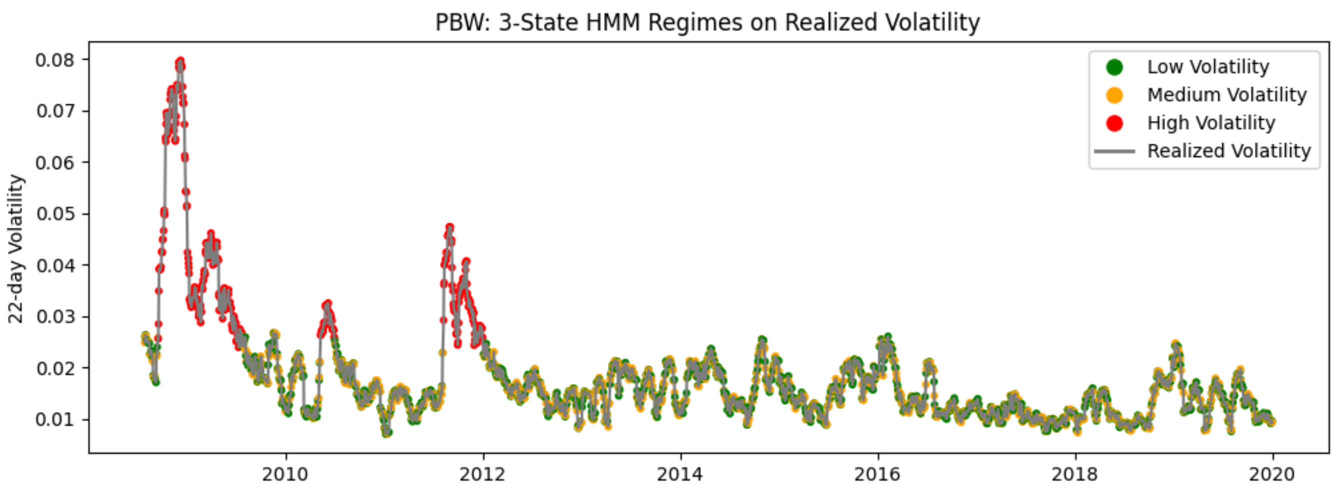
# Align test returns
test_returns = raw_returns.loc[rv_test.index]
test_hedged = hedged_returns.loc[rv_test.index]

metrics_orig = compute_metrics(test_returns)
metrics_hedged = compute_metrics(test_hedged)

# Display metrics
metrics_df = pd.DataFrame({
    'Original': metrics_orig,
    'Hedged': metrics_hedged
}, index=['VaR (95%)','CVaR (95%)','Max Drawdown'])

print("State Means (Low to High Volatility):")
print(f"Low Volatility State: {means[low_vol_state]:.4f}")
print(f"Medium Volatility State: {means[medium_vol_state]:.4f}")
print(f"High Volatility State: {means[high_vol_state]:.4f}")

print("\nRisk Metrics Comparison:")
metrics_df
```




```
State Means (Low to High Volatility):
Low Volatility State: 0.0164
Medium Volatility State: 0.0164
High Volatility State: 0.0400

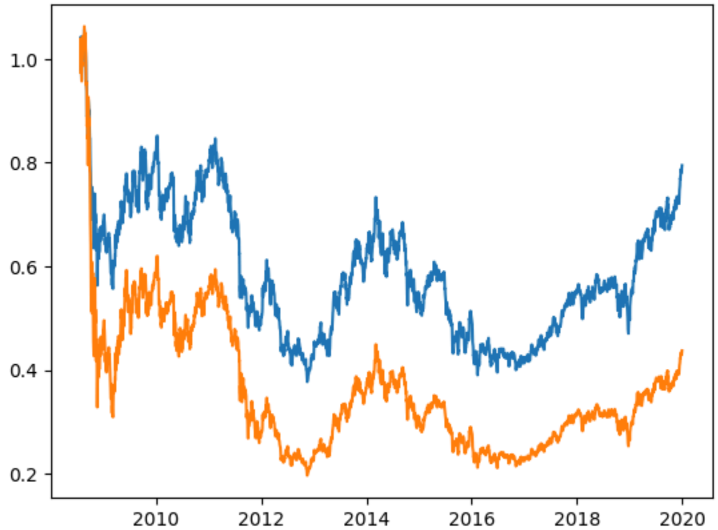
Risk Metrics Comparison:
```

	Original	Hedged
VaR (95%)	-0.0214	-0.0197
CVaR (95%)	-0.0299	-0.0267
Max Drawdown	0.1002	0.1002

Next steps: Generate code with metrics_df | View recommended plots | New interactive sheet

```
plt.plot((1 + hedged_returns).cumprod())
plt.plot((1 + raw_returns).cumprod())
```

[<matplotlib.lines.Line2D at 0x7cb378147350>]




```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from hmmlearn.hmm import GaussianHMM

# Parameters
ticker = 'QCLN'
start_train = '2010-01-01'
end_train = '2015-12-31'
start_test = '2019-01-01'
end_full = '2024-12-31'
forecast_horizon = 22

# De-risking percentages for each state
de_risk_high = 0.50  # 50% reduction in high volatility state
de_risk_medium = 0.25  # 25% reduction in medium volatility state
de_risk_low = 0.00  # 0% reduction in low volatility state

# Ensure datetime index
returns_df = renewable_df.copy()
returns_df.index = pd.to_datetime(returns_df.index)

# 1) Compute 22-day realized volatility series
ret_full = returns_df[ticker].loc[start_train:end_full].dropna()
realized_vol_full = ret_full.rolling(window=forecast_horizon).std().dropna()

def rolling_hmm_states(realized_vol, window=1000, n_states=3):
    states = np.full(len(realized_vol), np.nan)
    idx = realized_vol.index
    X = realized_vol.values.reshape(-1,1)

    # Clean the data - remove any remaining NaN or inf values
    valid_mask = ~np.isnan(X.flatten()) & ~np.isinf(X.flatten())
    X_clean = X[valid_mask]
    idx_clean = idx[valid_mask]

    for t in range(window, len(X_clean)):
        # Get the window of data
        window_data = X_clean[t-window:t]

        # Skip if window contains NaN or inf values
        if np.any(np.isnan(window_data)) or np.any(np.isinf(window_data)):
            continue

        try:
            model = GaussianHMM(n_components=n_states,
                              covariance_type='diag',  # Changed to diag for better stability
                              n_iter=100,
                              tol=1e-2,
                              random_state=0)
            model.fit(window_data)
            states[t] = model.predict(X_clean[t:t+1])[0]
        except:
```

```
            continue

    return pd.Series(states, index=idx_clean)

# Get rolling HMM states
states_series = rolling_hmm_states(realized_vol_full, window=750, n_states=3)

# Identify states by their mean volatility in the most recent window
def get_state_means(vol_series, states_series, window=750):
    X = vol_series.values.reshape(-1,1)
    valid_mask = ~np.isnan(X.flatten()) & ~np.isinf(X.flatten())
    X_clean = X[valid_mask]

    if len(X_clean) < window:
        window = len(X_clean)

    model = GaussianHMM(n_components=3,
                        covariance_type='diag',  # Changed to diag for better stability
                        n_iter=100,
                        tol=1e-2,
                        random_state=0)
    model.fit(X_clean[-window:])
    means = model.means_.flatten()
    sorted_states = np.argsort(means)  # States ordered from low to high volatility
    return sorted_states

# Get state means and identify high/medium/low states
sorted_states = get_state_means(realized_vol_full, states_series)
low_vol_state = sorted_states[0]
medium_vol_state = sorted_states[1]
high_vol_state = sorted_states[2]

# Plot the states
plt.figure(figsize=(12,4))
plt.plot(realized_vol_full.index, realized_vol_full, color='grey', label='Realized Vol')

# Create a custom colormap for the three states
colors = ['green', 'orange', 'red']  # Low, Medium, High volatility
state_colors = [colors[int(state)] if not np.isnan(state) else 'grey' for state in states_series]

# Plot the scatter points with custom colors
scatter = plt.scatter(states_series.index, realized_vol_full.loc[states_series.index],
                     c=state_colors, s=10)

# Create custom legend
from matplotlib.lines import Line2D
legend_elements = [
    Line2D([0], [0], marker='o', color='w', markerfacecolor='green', markersize=10, label='Low Volatility'),
    Line2D([0], [0], marker='o', color='w', markerfacecolor='orange', markersize=10, label='Medium Volatility'),
    Line2D([0], [0], marker='o', color='w', markerfacecolor='red', markersize=10, label='High Volatility'),
    Line2D([0], [0], color='grey', lw=2, label='Realized Volatility')
]

plt.title(f"{ticker}: Rolling 3-State HMM Regimes on Realized Volatility")
plt.ylabel("22-day Volatility")
plt.legend(handles=legend_elements)
plt.show()

# Construct hedged returns with state-dependent de-risking
raw_returns = ret_full.loc[states_series.index]  # align dates
hedged_returns = raw_returns.copy()

# Apply different de-risking based on state
hedged_returns[states_series == high_vol_state] *= (1 - de_risk_high)
hedged_returns[states_series == medium_vol_state] *= (1 - de_risk_medium)
hedged_returns[states_series == low_vol_state] *= (1 - de_risk_low)

# Evaluate risk metrics on test period
def compute_metrics(returns):
    var = returns.quantile(0.05)
    cvar = returns[returns <= var].mean()
    mdd = (returns.cummax() - returns).max()
    return var, cvar, mdd

# Align test returns
test_returns = raw_returns.loc[start_test:end_full]
test_hedged = hedged_returns.loc[start_test:end_full]
```

```
metrics_orig = compute_metrics(test_returns)
metrics_hedged = compute_metrics(test_hedged)


# Display metrics
metrics_df = pd.DataFrame({
    'Original': metrics_orig,
    'Hedged': metrics_hedged
}, index=['VaR (95%)','CVaR (95%)','Max Drawdown'])

print("State Means (Low to High Volatility):")
print(f"Low Volatility State: {sorted_states[0]}")
print(f"Medium Volatility State: {sorted_states[1]}")
print(f"High Volatility State: {sorted_states[2]}")

print("\nRisk Metrics Comparison:")
metrics_df
```

```
Model is not converging.  Current: 2664.650859185498 is not greater than 2664.6522742732336. Delta is -0.0014150877354950353
Model is not converging.  Current: 2663.9994175063694 is not greater than 2664.0007101904366. Delta is -0.001292684067266236
Model is not converging.  Current: 2663.3724187562393 is not greater than 2663.3734703173936. Delta is -0.0010515611543326918
Model is not converging.  Current: 2662.7017264495203 is not greater than 2662.7026225649074. Delta is -0.0008961153871149620
Model is not converging.  Current: 2662.091515100676 is not greater than 2662.091941352391. Delta is -0.0004262517145434685
Model is not converging.  Current: 2661.518601978697 is not greater than 2661.518744119714. Delta is -0.00014214101702236803
Model is not converging.  Current: 2661.4722656771864 is not greater than 2661.472532349826. Delta is -0.0002666726395545993
Model is not converging.  Current: 2662.0637508840136 is not greater than 2662.0644126845928. Delta is -0.0006618005791109694
Model is not converging.  Current: 2665.828234169133 is not greater than 2665.82858724485. Delta is -0.00035307571715748054
Model is not converging.  Current: 2666.478318288456 is not greater than 2666.4784575393983. Delta is -0.00013925094208838905
Model is not converging.  Current: 2666.7680534406522 is not greater than 2666.768178938376. Delta is -0.0001254977237294952
Model is not converging.  Current: 2666.953421596827 is not greater than 2666.9537071259774. Delta is -0.0002855291504602064
Model is not converging.  Current: 2667.13868111561 is not greater than 2667.1394014336975. Delta is -0.0007203180875876569
Model is not converging.  Current: 2667.23106650818 is not greater than 2667.231342570647. Delta is -0.00027606246703726356
Model is not converging.  Current: 2667.2279841195223 is not greater than 2667.2285648631514. Delta is -0.0005807436291433398
Model is not converging.  Current: 2667.276511593538 is not greater than 2667.2771005447516. Delta is -0.0005889512135581754
Model is not converging.  Current: 2667.4217918898335 is not greater than 2667.4223537391176. Delta is -0.0005618492841167692
Model is not converging.  Current: 2668.204109492316 is not greater than 2668.205375743872. Delta is -0.001266251555895837
Model is not converging.  Current: 2668.8947798955874 is not greater than 2668.8959891547875. Delta is -0.0012092592000095402
Model is not converging.  Current: 2669.6743600764053 is not greater than 2669.6757092999687. Delta is -0.0013492235633465784
Model is not converging.  Current: 2676.6558582634616 is not greater than 2676.656887370209. Delta is -0.001029106747409969
Model is not converging.  Current: 2677.3555452896685 is not greater than 2677.355562660422. Delta is -1.737075353958062e-05
Model is not converging.  Current: 2687.039216321803 is not greater than 2687.0392567132835. Delta is -4.039148052470409e-05
Model is not converging.  Current: 2689.428159986071 is not greater than 2689.4288786133716. Delta is -0.0007186273005557273
Model is not converging.  Current: 2693.1958957261845 is not greater than 2693.1961147677052. Delta is -0.000219041520722385
Model is not converging.  Current: 2816.8899580274683 is not greater than 2816.8931493066743. Delta is -0.003191279206021136
Model is not converging.  Current: 2860.922103798721 is not greater than 2860.9231751944035. Delta is -0.0010713956826293725
Model is not converging.  Current: 2870.1886212598247 is not greater than 2870.190029406528. Delta is -0.001408146703397506
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