Exercise – stabilization of uncertain/time varying systems

a) Consider a discrete time system with the time varying dynamics

$$x_{k+1} = Ax_k + Bu_k$$

where

$$B = \begin{bmatrix} 0 \\ 0.0787 \end{bmatrix}$$

and A can vary in the range

$$\begin{bmatrix} 1 & 0.1 \\ 0 & 0 \end{bmatrix} \text{ to } \begin{bmatrix} 1 & 0.1 \\ 0.05 & 0.99 \end{bmatrix}$$

Design a controller to stabilize the system by state feedback.

b) For the state constraints

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix} \leq x_k \leq \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

And input constraint

$$-5 \leq u \leq 5$$

Find, for the controller found in a), the largest (positive) invariant ellipsoid such that for any initial state inside the ellipsoid, the closed loop trajectory does not violate state or input constraints (considering linear feedback only – i.e., without input saturation).

Hint: Handling state and input constraints can be found on slide 32, while the last LMI on slide 21 ensures that the ellipsoid is positive invariant. However, note that the matrices P on slide 21 is the inverse of the matrix P on slide 32. For P defined as in slide 32, the objective function in YALMIP can be defined as '-geomean(P)'.

Note that in order to specify the input constraint, you should use Y = KP in the lower LMI on slide 32.

c) With the same state and input constraints as in b), find a controller K, matrix P, and auxiliary matrix variable H (or Y) to maximize the size of the robustly stable ellipsoid given by $x_k^T P^{-1} x_k \leq 1$.

Here Y must be as defined on slide 32.