Week 1: Complex numbers and differential equations

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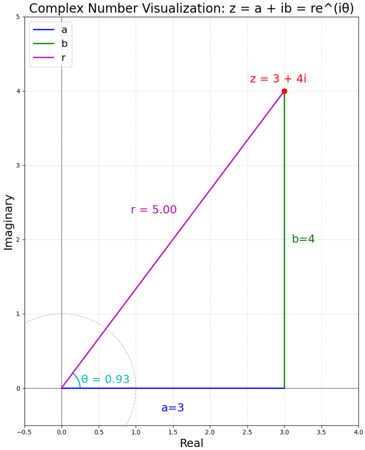
- Q1.12 Why is a diffraction pattern generated by an electron gun formed by electrons interfering with themselves rather than with one another?

Some repetition on complex numbers

- Every complex number $z \in \mathbb{C}$ can be written as $z = a + ib$, for $a, b \in \mathbb{R}$.
- We can also write z as $z = re^{i\theta}$, where $r, \theta \in \mathbb{R}$ and $r \geq 0$, $\theta \in [0, 2\pi)$.



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- A note on notation: Sometimes, z^* is written as $\bar{z}$. We have $a = \mathrm{Re}(z) = \Re(z)$, $b = \mathrm{Im}(z) = \Im(z)$.

Example exercises

- Calculate $z_3 = z_1/z_2$, where $z_1 = r_1 e^{i\theta_1}$,$z_2 = r_2 e^{i\theta_2}$.
- Write $z = 3 + 4i$ in the form $re^{i\theta}$.
- Write the following number in the form $a + ib$:

$$z = \frac{3 + 5i}{2 - 4i}$$

- For two complex numbers z_1, z_2, show that

$$(z_1 z_2)^* = z_1^* z_2^*$$

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- Consider now the partial differential equation

$$\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2} + f(x,y) = 0$$

How do you solve it?