TMA4135 Exercise 4 Simen Sandhaug

$\boxed{1}$ b) $f(x) = (1-3^x)x^2 + 4(x-1)3^x + 4(1-x)$, $\underline{f(0)=0}$

1: $a,b = -2, 3 \Rightarrow c = 0.5$, $f(c) = -1.64711$

$f(a) = 14.2222$

$f(b) = -26$

2: $a,b = -2, 0.5 \Rightarrow c = -0.75$

$f(c) = 4.245$

$f(a) = 14.2222$

$f(b) = -1.64711$

3: $a,b = -0.75, 0.5 \Rightarrow c = -0.125$

$f(c) = 0.5794$

$f(a) = 4.245$

$f(b) = -1.64711$

4: $a,b = -0.125, 0.5 \Rightarrow c = 0.1875$

$f(c) = -0.7514$

$f(a) = 0.5794$

$f(b) = -1.64711$

5: $a,b = -0.125, 0.1875 \Rightarrow c = 0.03125$

$\tilde{x} = f(c) = -0.13538$ $\quad E_k = |x_k - r| \leq \frac{b-a}{2^{k+1}} \Rightarrow e = |\tilde{x} - r| \leq \frac{3-(-2)}{2^5} = \underline{0.15625}$

c)

$$e_k = |c_k - r| \leq \frac{b-a}{2^{k+1}}$$

$$\frac{b-a}{2^{k+1}} = tol \implies k = \left\lceil \log_2\left(\frac{b-a}{2 \cdot tol}\right)\right\rceil$$

$$k = \left\lceil \log_2\left(\frac{3+2}{2 \cdot 10^{-3}}\right)\right\rceil = \underline{\underline{12}}$$

At most 12 iter.

$\boxed{2}$ a)

If:

1) $|g'(x)| \leqslant L < 1 \quad \forall \ x \in [a,b]$

and

2) $g(x) \in [a,b] \quad \forall \ x \in [a,b]$

then:

$x = g(x)$ has a unique root $r \in (a,b)$

---

$$g(x) = \frac{\cos^2(e^{-x})}{4} \quad , \quad r = \tilde{x} \geqslant 0. \quad ([0,\infty))$$

$$g'(x) = \frac{1}{4}\frac{d}{dx}\cos^2(e^{-x}) = \frac{1}{2}e^{-x}\sin(e^{-x})\cos(e^{-x}) \quad \left(\frac{du^2}{dx} \, , \quad u = \cos(e^{-x})\right)$$

Since $\cos(e^{-x}) \leqslant 1$ , $\sin(e^{-x}) \leqslant 1 \quad \forall \ x$ and $\max\limits_{x \geqslant 0} e^{-x} = e^0 = 1$

$\Rightarrow \quad |g'(x)| < 1 \quad \forall \ \tilde{x} \geqslant 0 \quad \checkmark$

Since $\cos^2(e^{-x}) \geqslant 0 \quad \forall x$

$\Rightarrow \quad g(x) \geqslant 0 \quad \forall \ x \quad \checkmark$

$\Rightarrow \quad x = g(x)$ has a unique root $\tilde{x} \geqslant 0.$

$\boxed{3}$ a)

$$f(x) = \cos(x) - \sqrt{x} \quad , \quad f'(x) = -\sin(x) - \frac{1}{2\sqrt{x}}$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$x_0 = 1$ :

$$x_1 = 1 - \frac{\cos(1) - \sqrt{1}}{-\sin(1) - \frac{1}{2\sqrt{1}}} \approx 0.657.$$

$$x_2 \approx 0.657 - \frac{\cos(0.657) - \sqrt{0.657}}{-\sin(0.657) - \frac{1}{2\sqrt{0.657}}} \approx 0.642.$$