Algebra Worksheet

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A. Simplify

1. $(6x^2 - 10xy + 2) + (2z - xy + 4)$

$$\begin{aligned} &= 6x^2 - 10xy + 2 + 2z - xy + 4 \\ &= 6x^2 - 10xy - xy + 2z + 2 + 4 \\ &= 6x^2 - 11xy + 2z + 6 \end{aligned} \tag{1}$$

2. $4(2z - w) - 3(w - 2z)$

$$\begin{aligned} &= 8z - 4w - 3w - 6z \\ &= -4w - 3w + 8z - 6z \\ &= -7w + 2z \end{aligned} \tag{2}$$

3. $(8t^2 - 6x^2) + (4s^2 - 2t^2 + 6)$

$$\begin{aligned} &= 8t^2 - 6x^2 + 4s^2 - 2t^2 + 6 \\ &= 4s^2 + 8t^2 - 2t^2 - 6x^2 + 6 \\ &= 4s^2 - 6t^2 - 6x + 6 \end{aligned} \tag{3}$$

4. $(3a - 7b - 9) - (5a + 9b + 21)$

$$\begin{aligned} &= 3a - 7b - 9 - 5a - 9b - 21 \\ &= 3a - 5a - 7b - 9b - 9 - 21 \\ &= -2a - 16b - 30 \end{aligned} \tag{4}$$

5. $2 - [3 + 4(p - 3)]$

$$\begin{aligned} &= 2 - [3 + 4p - 12] \\ &= 2 - 3 - 4p + 12 \\ &= -1 - 4p + 12 \\ &= -4p - 1 + 12 \\ &= -4p + 11 \end{aligned} \tag{5}$$

6. $(2p-1)(2p+1)$

$$\begin{aligned} &= 2p(2p+1) - 1(2p+1) \\ &= 4p^2 + 2p - 2p - 1 \\ &= 4p^2 - 1 \end{aligned} \tag{6}$$

7. $(x^2+x+1)^2$

$$\begin{aligned} &= (x^2+x+1)(x^2+x+1) \\ &= x^2(x^2+x+1) + x(x^2+x+1) + 1(x^2+x+1) \\ &= x^4 + x^3 + x^2 + x^3 + x^2 + x + x^2 + x + 1 \\ &= x^4 + x^3 + x^3 + x^2 + x^2 + x^2 + x + x + 1 \\ &= x^4 + 2x^3 + 3x^2 + 2x + 1 \end{aligned} \tag{7}$$

8. $\frac{x^3}{(x+2)}$

$$\begin{aligned} &= \frac{(x^3) \times (x-2)}{(x+2) \times (x-2)} \\ &= \frac{x^4 - 2x^3}{x(x-2) + 2(x-2)} \\ &= \frac{x^4 - 2x^3}{x^2 - 2x + 2x - 4} \\ &= \frac{x^4 - 2x^3}{x^2 - 4} \end{aligned} \tag{8}$$

9. $\frac{2x^3-7x+4}{x}$

(9)

10. $\frac{6x^5+4x^3-1}{2x^2}$

(10)

B. Factor Completely

1. $2ax - 2b$

$$= 2(ax - b) \tag{11}$$

2. $z^2 - 49$

$$\begin{aligned} &= (z)^2 - (7)^2 \\ Since(a+b)(a-b) &= a^2 - b^2 \\ &= (z+7)(z-7) \end{aligned} \tag{12}$$

3. $16x^2 - 9$

$$\begin{aligned} &= (4x)^2 - (3)^2 \\ Since(a+b)(a-b) = a^2 - b^2 \\ &= (4x-3)(4x+3) \end{aligned} \tag{13}$$

4. $3x^2 - 3$

$$\begin{aligned} &= 3(x^2 - 1) \\ &= 3((x)^2 - (1)^2) \\ Since(a+b)(a-b) = a^2 - b^2 \\ &= 3((x-1)(x+1)) \end{aligned} \tag{14}$$

5. $x^2 + 2x - 24$

$$\begin{aligned} &= x^2 + 6x - 4x - 24 \\ &= x(x+6) - 4(x+6) \\ &= (x-4)(x+6) \end{aligned} \tag{15}$$

6. $4x^2 - x - 3$

$$\begin{aligned} &= 4x^2 - 4x + 3x - 3 \\ &= 4x(x-1) + 3(x-1) \\ &= (4x+3)(x-1) \end{aligned} \tag{16}$$

7. $(4x+2)^2$

$$\begin{aligned} &= (4x+2)(4x+2) \\ &= 4x(4x+2) + 2(4x+2) \\ &= 16x^2 + 8x + 8x + 4 \\ &= 16x^2 + 16x + 4 \end{aligned} \tag{17}$$

8. $2x^2(2x - 4x^2)^2$

$$\begin{aligned} &= 2x^2(4x^2 - 16x^4) \\ &= 8x^4 - 32x^6 \end{aligned} \tag{18}$$

C. Simplify

1. $\frac{a^2-9}{a^3-3a}$

$$\begin{aligned} &= \frac{(a)^2 - (3)^2}{a(a^2-3)} \\ &= \frac{(a+3)(a-3)}{a(a^2-3)} \end{aligned} \tag{19}$$

2. $\frac{x^2-3x-10}{x^2-4}$

$$\begin{aligned}
&= \frac{x^2-5x+2x-10}{(x)^2-(2)^2} \\
&= \frac{x(x-5)+2(x-5)}{(x+2)(x-2)} \\
&= \frac{(x+2)(x-5)}{(x+2)(x-2)} \\
&= \frac{\frac{(x+2)(x-5)}{x+2}}{\frac{(x+2)(x-2)}{x+2}} \\
&= \frac{x-5}{x-2}
\end{aligned} \tag{20}$$

3. $\frac{6x^2+x-2}{2x^2+3x-2}$

$$\begin{aligned}
&= \frac{6x^2+4x-3x-2}{2x^2+4x-x-2} \\
&= \frac{2x(3x+2)-1(3x+2)}{2x(x+2)-1(x+2)} \\
&= \frac{(2x-1)(3x+2)}{(2x-1)(x+2)} \\
&= \frac{\frac{(2x-1)(3x+2)}{2x-1}}{\frac{(2x-1)(x+2)}{2x-1}} \\
&= \frac{3x+2}{x+2}
\end{aligned} \tag{21}$$

4. $(\frac{y^2}{y-3})(\frac{-1}{y+2})$

$$\begin{aligned}
&= \frac{(y^2)(-1)}{(y-3)(y+2)}) \\
&= \frac{y^2}{y(y+2)-3(y+2)} \\
&= \frac{y^2}{y^2+2y-3y-6} \\
&= \frac{y^2}{y^2-y-6} \\
&= \frac{\frac{y^2}{y^2}}{\frac{y^2-y-6}{y^2}} \\
&= \frac{1}{-y-6}
\end{aligned} \tag{22}$$

5. $(\frac{ax-b}{x-c})(\frac{c-x}{ax+b})$

$$\begin{aligned}
&= \frac{(ax-b)(c-x)}{(x-c)(ax+b)} \\
&= \frac{ax(c-x)-b(c-x)}{x(ax+b)-c(ax+b)} \\
&= \frac{acx-ax^2-bc+bx}{ax^2+bx-acx-bc} \\
&= \frac{-ax^2+acx-bc+bx}{ax^2-acx+bx-bc} \\
&=
\end{aligned} \tag{23}$$

6. $\frac{4}{a+4}+a$

$$\begin{aligned}
&= \frac{4}{a+4}+\frac{a}{1} \\
&= \frac{4+a(a+4)}{a+4} \\
&= \frac{4+a^2+4a}{a+4} \\
&= \frac{a^2+4a+4}{a+4}
\end{aligned} \tag{24}$$

7. $\frac{x^2}{x+3}+\frac{5x+6}{x+3}$

$$\begin{aligned}
&= \frac{x^2+5x+6}{x+3} \\
&= \frac{x^2+3x+2x+6}{x+3} \\
&= \frac{x(x+3)+2(x+3)}{x+3} \\
&= \frac{\frac{x(x+3)}{x+3}+\frac{2(x+3)}{x+3}}{\frac{x+3}{x+3}} \\
&= \frac{x+2}{1} \\
&= x+2
\end{aligned} \tag{25}$$

8. $\frac{\frac{x^2+6x+9}{x}}{x+3}$

$$\begin{aligned}
&= \frac{x^2+6x+9}{x} \div \frac{x+3}{1} \\
&= \frac{x^2+6x+9}{x} \times \frac{1}{x+3} \\
&= \frac{x^2+6x+9}{x^2+3x} \\
&= \frac{x^2+3x+3x+9}{x^2+3x} \\
&= \frac{x(x+3)+3(x+3)}{x(x+3)} \\
&= \frac{(x+3)(x+3)}{x(x+3)} \\
&= \frac{\frac{(x+3)(x+3)}{x+3}}{\frac{x(x+3)}{x+3}} \\
&= \frac{x+3}{x}
\end{aligned} \tag{26}$$

9. $\frac{\frac{4x}{3}}{2x}$

$$\begin{aligned}
&= \frac{4x}{3} \div \frac{2x}{1} \\
&= \frac{4x}{3} \times \frac{1}{2x} \\
&= \frac{4x}{6x} \\
&= \frac{\frac{4x}{2}}{\frac{6x}{2}} \\
&= \frac{2x}{3x} \\
&= \frac{2}{3}
\end{aligned} \tag{27}$$

10. $\frac{7+\frac{1}{x}}{5}$

$$\begin{aligned}
&= (7+\frac{1}{x}) \div \frac{5}{1} \\
&= (\frac{7x}{x}+\frac{1}{x}) \times \frac{1}{5} \\
&= \frac{7x+1}{x} \times \frac{1}{5} \\
&= \frac{7x+1}{5x}
\end{aligned} \tag{28}$$

D. Solve for x

1. $7x + 7 = 2(x + 1)$

$$\begin{aligned} 7x + 7 &= 2x + 1 \\ 7x - 2x &= 1 - 7 \\ 5x &= -6 \\ x &= \frac{-6}{5} \end{aligned} \tag{29}$$

2. $5(p - 7) - 2(3p - 4) = 3p$

$$\begin{aligned} 5p - 35 - 6p + 8 &= 3p \\ 5p - 6p - 3p &= 35 - 8 \\ -4p &= 27 \\ p &= -\frac{27}{4} \end{aligned} \tag{30}$$

3. $\frac{5}{x} = 25$

$$\begin{aligned} 5 &= 25x \\ 25x &= 5 \\ x &= \frac{5}{25} \\ x &= \frac{1}{5} \end{aligned} \tag{31}$$

4. $\frac{5}{3-x} = 0$

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5. $\frac{x+3}{x} = \frac{2}{5}$

$$\begin{aligned} 5(x + 3) &= 2(x) \\ 5x + 15 &= 2x \\ 5x - 2x &= -15 \\ 3x &= -15 \\ x &= -\frac{15}{3} \\ x &= -5 \end{aligned} \tag{33}$$

6. $\frac{3}{5-x} = \frac{7}{2}$

$$\begin{aligned} 2(3) &= 7(5-x) \\ 6 &= 35 - 7x \\ 7x &= 35 - 6 \\ 7x &= 29 \\ x &= \frac{29}{7} \end{aligned} \tag{34}$$

7. $\frac{2x-3}{4x-5} = 6$

$$\begin{aligned} 6(4x-5) &= 1(2x-3) \\ 24x - 30 &= 2x - 3 \\ 24x - 2x &= 30 - 3 \\ 22x &= 27 \\ x &= \frac{27}{22} \end{aligned} \tag{35}$$

8. $\frac{1}{x} + \frac{1}{7} = \frac{3}{7}$

$$\begin{aligned} \frac{1}{x} &= \frac{3}{7} - \frac{1}{7} \\ \frac{1}{x} &= \frac{2}{7} \\ 2(x) &= 1(7) \\ 2x &= 7 \\ x &= \frac{7}{2} \end{aligned} \tag{36}$$

9. $\frac{2}{x-1} = \frac{3}{x-2}$

$$\begin{aligned} 2(x-2) &= 3(x-1) \\ 2x - 4 &= 3x - 3 \\ 2x - 3x &= -3 + 4 \\ -x &= 1 \\ -x \times -1 &= 1 \times -1 \\ x &= -1 \end{aligned} \tag{37}$$

10. $\sqrt{x+5} = 4$

$$\begin{aligned} \sqrt{x+5}^2 &= 4^2 \\ x + 5 &= 16 \\ x &= 16 - 5 \\ x &= 11 \end{aligned} \tag{38}$$

11. $(x+6)^{\frac{1}{2}} = 7$

$$\begin{aligned} ((x+6)^{\frac{1}{2}})^2 &= 7^2 \\ x+6 &= 49 \\ x &= 49-6 \\ x &= 43 \end{aligned} \tag{39}$$

E. Express the indicated symbol in terms of the remaining symbols.
Example: If $s = \frac{u}{au+v}$, express u in terms of the others, i.e. find for u.
by cross multiplying:

$$\begin{aligned} s(au+v) &= u \\ sau + sv &= u \\ sau - u &= -sv \\ u(sa-1) &= -sv \\ u &= \frac{-sv}{sa-1} \end{aligned} \tag{40}$$

To do

1. $p = -3q + 6$, find q

$$\begin{aligned} p &= -3q+6 \\ 3q &= -p+6 \\ q &= \frac{-p+6}{3} \end{aligned} \tag{41}$$

2. $s = P(1+rt)$, find r

$$\begin{aligned} s &= P(1+rt) \\ s &= P+Prt \\ -Prt &= P-s \\ r &= \frac{P-s}{-Pt} \end{aligned} \tag{42}$$

3. $\frac{2mI}{B(n+1)}$, find I

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4. $\frac{d}{1+dt}$, find t

F. Solve by factoring.

1. $t^2 - 8t + 15 = 0$

$$\begin{aligned} t^2 - 8t + 15 &= 0 \\ t^2 - 5t - 3t + 15 &= 0 \\ t(t-5) - 3(t-5) &= 0 \\ (t-3)(t-5) &= 0 \end{aligned} \tag{44}$$

2. $-x^2 + 3x + 10 = 0$

$$\begin{aligned} -x^2 + 3x + 10 &= 0 \\ -x^2 + 5x - 2x + 10 &= 0 \\ -x(x-5) - 2(x-5) &= 0 \\ (-x-2)(x-5) &= 0 \end{aligned} \tag{45}$$

3. $2b^2 + 9b = 5$

$$\begin{aligned} 2b^2 + 9b - 5 &= 0 \\ 2b^2 + 10b - b - 5 &= 0 \\ 2b(b+5) - 1(b+5) &= 0 \\ (2b-1)(b+5) &= 0 \end{aligned} \tag{46}$$

G. Solve by using the quadratic formula.

1. $x^2 + 2x - 24 = 0$

$$\begin{aligned} a = 1, \quad b = 5, \quad c = 6& \\ x =& \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x =& \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)} \\ x =& \frac{-5 \pm \sqrt{25 - 24}}{2} \\ x =& \frac{-5 \pm \sqrt{1}}{2} \end{aligned} \tag{47}$$

Therefore x = -2 OR x = -3

2. $q^2 - 5q = 0$

$$\begin{aligned} q^2 - 5q - 0 &= 0 \\ a = 1, \quad b = 5, \quad c &= 0 \\ x =& \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x =& \frac{-5 \pm \sqrt{5^2 - 4(1)(0)}}{2(1)} \\ x =& \frac{-5 \pm \sqrt{25 - 0}}{2} \\ x =& \frac{-5 \pm \sqrt{25}}{2} \end{aligned} \tag{48}$$

Therefore x = 0 OR x = -5

3. $-2x^2 - 6x + 5 = 0$

$$\begin{aligned} a = 2, \quad b = 6, \quad c = 5 \\ x = &\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x = &\frac{-6 \pm \sqrt{6^2 - 4(2)(5)}}{2(2)} \\ x = &\frac{-6 \pm \sqrt{36 - 40}}{4} \\ x = &\frac{-6 \pm \sqrt{-4}}{4} \end{aligned} \tag{49}$$

4. $2 - 2x + x^2 = 0$

$$\begin{aligned} x^2 - 2x + 2 &= 0 \\ a = 1, \quad b = 2, \quad c &= 2 \\ x = &\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x = &\frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)} \\ x = &\frac{-2 \pm \sqrt{4 - 8}}{2} \\ x = &\frac{-2 \pm \sqrt{-4}}{2} \end{aligned} \tag{50}$$

H. Solve the systems algebraically.

1.

$$\begin{aligned} x + 4y &= 3 \quad (eq.\ 1) \\ 3x - 2y &= -5 \quad (eq.\ 2) \end{aligned}$$

Finding x in equation 1:

$$\begin{aligned} x + 4y &= 3 \\ x &= 3 - 4y \end{aligned} \tag{51}$$

Substituting $x = 3 - 4y$ in equation 2:

$$\begin{aligned} 3x - 2y &= -5 \\ 3(3 - 4y) - 2y &= -5 \\ 9 - 12y - 2y &= -5 \\ 9 - 14y &= -5 \\ -14y &= -5 - 9 \\ -14y &= -14 \\ y &= \frac{-14}{-14} \\ y &= 1 \end{aligned} \tag{52}$$

Substituting $y = 1$ in equation 1:

$$\begin{aligned} x + 4y &= 3 \\ x + 4(1) &= 3 \\ x + 4 &= 3 \\ x &= 3 - 4 \\ x &= -1 \end{aligned} \tag{53}$$

Therefore $x = -1$ and $y = 1$.

2.

$$\begin{aligned} x - 2y &= -7 \quad (eq.\ 1) \\ 5x + 3y &= -9 \quad (eq.\ 2) \end{aligned}$$

Finding x in equation 1:

$$\begin{aligned} x - 2y &= -7 \\ x &= 2y - 7 \end{aligned} \tag{54}$$

Substituting $x = 2y - 7$ in equation 2:

$$\begin{aligned} 5x + 3y &= -9 \\ 5(2y - 7) + 3y &= -9 \\ 10y - 35 + 3y &= -9 \\ 10y + 3y - 35 &= -9 \\ 13y &= -9 + 35 \\ 13y &= 26 \\ y &= \frac{26}{13} \\ y &= 2 \end{aligned} \tag{55}$$

Substituting $y = 2$ in equation 1:

$$\begin{aligned} x - 2y &= -7 \\ x - 2(2) &= -7 \\ x - 4 &= -7 \\ x &= -7 + 4 \\ x &= -3 \end{aligned} \tag{56}$$

Therefore $x = -3$ and $y = 2$.

3.

$$\begin{aligned} 4x - 3y - 2 &= 3x - 7y \quad (eq.\ 1) \\ x + 5y - 2 &= y + 4 \quad (eq.\ 2) \end{aligned}$$

Finding x in equation 1:

$$\begin{aligned} 4x - 3y - 2 &= 3x - 7y \\ 4x - 3x &= -7y + 3y + 2 \\ x &= -4y + 2 \end{aligned} \tag{57}$$

Substituting $x = -4y + 2$ in equation 2:

$$\begin{aligned} x + 5y - 2 &= y + 4 \\ (-4y + 2) + 5y - 2 &= y + 4 \\ -4y + 2 + 5y - 2 &= y + 4 \\ -4y + 5y + 2 - 2 &= y + 4 \\ -4y + 5y + y &= 4 - 2 + 2 \\ 2y &= 4 \\ y &= \frac{4}{2} \\ y &= 2 \end{aligned} \tag{58}$$

Substituting $y = 2$ in equation 1:

$$\begin{aligned} 4x - 3y - 2 &= 3x - 7y \\ 4x - 3(2) - 2 &= 3x - 7(2) \\ 4x - 6 - 2 &= 3x - 14 \\ 4x - 3x &= -14 + 6 + 2 \\ x &= -8 \end{aligned} \tag{59}$$

Therefore $x = -8$ and $y = 2$.

4.

$$\begin{aligned} \frac{1}{2}z - \frac{1}{4}w &= \frac{1}{6} \quad (eq.\ 1) \\ \frac{1}{2}z + \frac{1}{4}w &= \frac{1}{6} \quad (eq.\ 2) \end{aligned}$$

Finding z in equation 1:

$$\begin{aligned}
\frac{1}{2}z - \frac{1}{4}w &= \frac{1}{6} \\
\frac{1}{2}z &= \frac{1}{6} + \frac{1}{4}w \\
2(\frac{1}{2}z) &= 2(\frac{1}{6} + \frac{1}{4}w) \\
z &= \frac{2}{6} + \frac{2}{4}w \\
z &= \frac{1}{3} + \frac{1}{2}w \\
z &= \frac{1}{2}w + \frac{1}{3}
\end{aligned} \tag{60}$$

Substituting $z = \frac{1}{2}w + \frac{1}{3}$ in equation 2:

$$\begin{aligned}
\frac{1}{2}z + \frac{1}{4}w &= \frac{1}{6} \\
\frac{1}{2}(\frac{1}{2}w + \frac{1}{3}) + \frac{1}{4}w &= \frac{1}{6} \\
\frac{1}{4}w + \frac{1}{6} + \frac{1}{4}w &= \frac{1}{6} \\
\frac{1}{4}w + \frac{1}{4}w + \frac{1}{6} &= \frac{1}{6} \\
\frac{1}{4}w + \frac{1}{4}w &= \frac{1}{6} - \frac{1}{6} \\
\frac{1}{2}w &= 0 \\
w &= 0
\end{aligned} \tag{61}$$

Substituting $w = 0$ in equation 1:

$$\begin{aligned}
\frac{1}{2}z - \frac{1}{4}w &= \frac{1}{6} \\
\frac{1}{2}z - \frac{1}{4}(0) &= \frac{1}{6} \\
\frac{1}{2}z - 0 &= \frac{1}{6} \\
\frac{1}{2}z &= \frac{1}{6} + 0 \\
2(\frac{1}{2}z) &= 2(\frac{1}{6}) \\
z &= \frac{2}{6} \\
z &= \frac{1}{3}
\end{aligned} \tag{62}$$

Therefore $w = 0$ and $z = \frac{1}{3}$.