ECN 1101 - Introductory Maths - Semester 1 2021

Worksheet 2

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Simplify

1. $\frac{x^2+3x+2}{x^2+4x+3}$

$$\begin{aligned}
&= \frac{x^2+2x+x+2}{x^2+3x+x+3} \\
&= \frac{x(x+2)+1(x+2)}{x(x+3)+1(x+3)} \\
&= \frac{(x+1)(x+2) \div (x+1)}{(x+1)(x+3) \div (x+1)} \\
&= \frac{x+2}{x+3}
\end{aligned}$$

2. $\frac{3x^2-14x+8}{x^2-16}$

$$\begin{aligned}
&= \frac{3x^2-12x-2x+8}{(x)^2-(4)^2} \\
&= \frac{3x(x-4)-2(x-4)}{(x+4)(x-4)} \\
&= \frac{(3x-2)(x-4)}{(x+4)(x-4)} \\
&= \frac{3x-2}{x+4}
\end{aligned}$$

3. $\frac{15a^4b^5}{30a^3b}$

$$= \frac{15}{30}(a^4 - a^3)(b^5 - b^1)$$
$$= \frac{1}{2}ab^4$$

4. $\frac{6x^3y^3-15x^2y}{3x^2y^2+9x^2y}$

$$= \frac{6x^3y^3 - 15x^2y \ \div 3}{3x^2y^2 + 9x^2y \ \div 3}$$
$$= \frac{2x^3y^3 - 5x^2y}{x^2y^2 + 3x^2y}$$
$$= \frac{2x^3y^3 - 5x^2y \ \div x^2y}{x^2y^2 + 3x^2y \ \div x^2y}$$
$$= \frac{2xy^2 - 5}{y + 3}$$

5. $\frac{15ac^2}{7bd} \div \frac{4a}{14b^2d}$

$$= \frac{15ac^2}{7bd} \times \frac{14b^2d}{4a}$$
$$= \frac{(15 \times 14)ab^2c^2d}{(7 \times 4)abd}$$
$$= \frac{210ab^2c^2d}{28abd}$$
$$= \frac{15bc^2}{2}$$

6. $\frac{x}{x+1} + x + 1$

$$
\begin{aligned}
&= \frac{x}{x+1} + \frac{(x+1)(x+1)}{x+1} \\
&= \frac{x + x(x+1) + 1(x+1)}{x+1} \\
&= \frac{x + x^2 + x + x + 1}{x+1} \\
&= \frac{x^2 + 3x + 1}{x+1}
\end{aligned}
$$

7. $x - \frac{2}{x-1}$

$$
\begin{aligned}
&= \frac{(x-1)(x) - 2}{x-1} \\
&= \frac{x^2 - x - 2}{x-1} \\
&= \frac{x^2 - 2x + x - 2}{x-1} \\
&= \frac{x(x-2) + 1(x-2)}{x-1} \\
&= \frac{(x+1)(x-2)}{x-1}
\end{aligned}
$$

8. $\frac{49}{3x+6} + \frac{5a^2}{4x+8}$

$$
\begin{aligned}
&= \frac{(4x+8)(49) + (3x+6)(5a^2)}{(3x+6)(4x+8)} \\
&= \frac{196x + 392 + 15a^2x + 30a^2}{(3x+6)(4x+8)} \\
&= \frac{15a^2x + 30a^2 + 196x + 392}{3x(4x+8) + 6(4x+8)} \\
&= \frac{15a^2x + 30a^2 + 196x + 392}{12x^2 + 24x + 24x + 48} \\
&= \frac{15a^2x + 30a^2 + 196x + 392}{12x^2 + 48x + 48} \\
&= \frac{15a^2(x+2) + 196(x+2)}{12(x^2 + 4x + 4)} \\
&= \frac{15a^2(x+2) + 196(x+2)}{12(x^2 + 2x + 2x + 4)} \\
&= \frac{15a^2(x+2) + 196(x+2)}{12(x(x+2) + 2(x+2))} \\
&= \frac{15a^2(x+2) + 196(x+2) \ \div (x+2)}{12((x+2)(x+2)) \ \div (x+2)} \\
&= \frac{15a^2 + 196}{12(x+2)}
\end{aligned}
$$

Solve

1. $\sqrt{x+5} = 4$

$$\begin{aligned}
\sqrt{x} + \sqrt{5} &= 4 \\
\sqrt{x} &= 4 - \sqrt{5} \\
x &= (4 - \sqrt{5})^2 \\
x &= 16 - 5 \\
x &= 11
\end{aligned}$$

2. $\sqrt{3x-4} - 8 = 0$

$$\begin{aligned}
\sqrt{3x-4} &= 8 \\
\sqrt{3x} - \sqrt{4} &= 8 \\
\sqrt{3x} &= 8 + \sqrt{4} \\
3x &= (8 + \sqrt{4})^2 \\
3x &= 64 + 4 \\
3x &= 68 \\
x &= \frac{68}{3} \; OR \; 22.6
\end{aligned}$$

3. $\sqrt{4x-6} = \sqrt{x}$

$$\begin{aligned}
\sqrt{4x-6} - \sqrt{x} &= 0 \\
\sqrt{4x} - \sqrt{6} - \sqrt{x} &= 0 \\
\sqrt{4x} - \sqrt{x} &= \sqrt{6} \\
(\sqrt{4x} - \sqrt{x})^2 &= (\sqrt{6})^2 \\
4x - x &= 6 \\
3x &= 6 \\
x &= \frac{6}{3} \\
x &= 2
\end{aligned}$$

4. $\sqrt{-2x+3} \leq 6$

$$\begin{aligned}
-2x + 3 &\leq 36 \\
-2x &\leq 36 - 3 \\
x &\geq -\frac{33}{2}
\end{aligned}$$

5. $3(2-3x) > 4(1-4x)$

$$\begin{aligned}
6 - 9x &> 4 - 16x \\
16x - 9x &> 4 - 6 \\
7x &> -2 \\
x &> -\frac{2}{7}
\end{aligned}$$

6. $\frac{5}{6}x < 40$

$$\begin{aligned}
x &< 40 \times \frac{6}{5} \\
x &< \frac{240}{5} \\
x &< 48
\end{aligned}$$

Exponents and Radicals

Rules

1. $x^m \times x^n = x^{m+n}$

$$2^5 \times 2^3 = 2^8$$

2. $x^0 = 1$

$$2^0 = 1$$

3. $x^{-n} = \frac{1}{x^n}$

$$2^{-3} = \frac{1}{2^3}$$

4. $\frac{1}{x^{-n}} = x^n$

$$\frac{1}{2^{-3}} = 2^3$$

5. $\frac{x^m}{x^n} = x^{m-n} = \frac{1}{x^{n-m}}$

$$\frac{2^{12}}{2^8} = 2^4 \; OR \; \frac{x^8}{x^{12}} = \frac{1}{x^4}$$

6. $(x^m)^n = x^{mn}$

$$(2^3)^5 = 2^{15} \; OR \; (x^2)^3 = x^6$$

7. $(xy)^n = x^n y^n$

$$(2 \times 4)^3 = 2^3 \times 4^3$$

8. $(\frac{x}{y})^n = \frac{x^n}{y^n}$

$$(\frac{2}{3})^3 = \frac{2^3}{3^3}$$

9. $(\frac{x}{y})^{-n} = (\frac{y}{x})^n$

$$(\frac{3}{4})^{-2} = (\frac{4}{3})^2$$

10. $x^{\frac{1}{n}} = \sqrt[n]{x}$

$$3^{\frac{1}{5}} = \sqrt[5]{3} \;\; OR \;\sqrt{a} = a^{\frac{1}{2}}$$

11. $x^{\frac{2}{3}} = \sqrt[3]{x^2}$

12. $x^{-\frac{1}{n}} = \frac{1}{x^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{x}}$

$$4^{-}\frac{1}{2} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt{4}}$$

13. $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$

14. $(\sqrt[m]{x})^m = x$

To do

Simplify

1. $\sqrt{4x}$

$$= 2\sqrt{x}$$

2. $\sqrt{16x^4}$

$$= 4x^2$$

3. $\sqrt[4]{\frac{x}{16}}$

$$= \frac{\sqrt[4]{x}}{2}$$

4. $(9z^4)^{\frac{1}{2}}$

$$= 9^{\frac{1}{2}} z^2$$
$$= 3z^2$$

5. $(16y^8)^{\frac{3}{4}}$

$$= (2^4 y^8)^{\frac{3}{4}}$$
$$= 2^3 y^6$$
$$= 8y^6$$

6. $(\frac{27t^3}{8})^{\frac{2}{3}}$

$$= (\frac{3^3 t^3}{2^3})^{\frac{2}{3}}$$
$$= \frac{3^2 t^2}{2^2}$$
$$= \frac{9t^2}{4}$$

Avoid radicals and express in positive exponents

1. $y^{-1}\sqrt{x}$

$$= \frac{1}{y}x^{\frac{1}{2}}$$
$$= \frac{x^{\frac{1}{2}}}{y}$$

2. $\frac{a^5b^{-3}}{c^{-2}}$

$$= a^5 \times \frac{1}{b^3} \div \frac{1}{c^2}$$
$$= a^5 \times \frac{1}{b^3} \times c^2$$
$$= a^5 \times \frac{c^2}{b^3}$$
$$= \frac{a^5c^2}{b^3}$$

3. $(3t)^{-2}$

$$= \frac{1}{(3t)^2}$$
$$= \frac{1}{9t^2}$$

4. $\sqrt[5]{5x^2}$

$$= 5^{\frac{1}{5}}x^{\frac{2}{5}}$$
$$= (5x^2)^{\frac{1}{5}}$$

5. $\sqrt{x} - \sqrt{y}$

$$= x^{\frac{1}{2}} - y^{\frac{1}{2}}$$