ECN 1101 - Introductory Maths - Semester 1 2021

Worksheet 3

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This worksheet accompanies lecture notes 1 on straight line geometry/coordinate geometry

1 For each of the following points find:

a. the midpoint

b. the length of the line

c. the equation of the line

1.1   (-2, 10),(5, 3)

<u>MIDPOINT</u>

$$midpoint = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$
$$= (\frac{-2 + 5}{2}, \frac{10 + 3}{2})$$
$$= (\frac{3}{2}, \frac{13}{2})$$
$$= (1.5, 6.5)$$

<u>LENGTH OF LINE</u>

$$length\ of\ line = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5 - (-2))^2 + (3 - 10)^2}$$
$$= \sqrt{(7)^2 + (-7)^2}$$
$$= \sqrt{49 + 49}$$
$$= \sqrt{98}$$
$$= 9.90$$

<u>SLOPE OF LINE</u>

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3 - 10}{5 - (-2)}$$
$$= \frac{-7}{7}$$
$$= -1.00$$

<u>EQUATION OF THE LINE</u>

Using the points $(-2, 10)$ and $m = -1$ we find c

$$y = mx + c$$
$$c = y - mx$$
$$c = 10 - (-1)(-2)$$
$$c = 10 - 2$$
$$c = 8$$

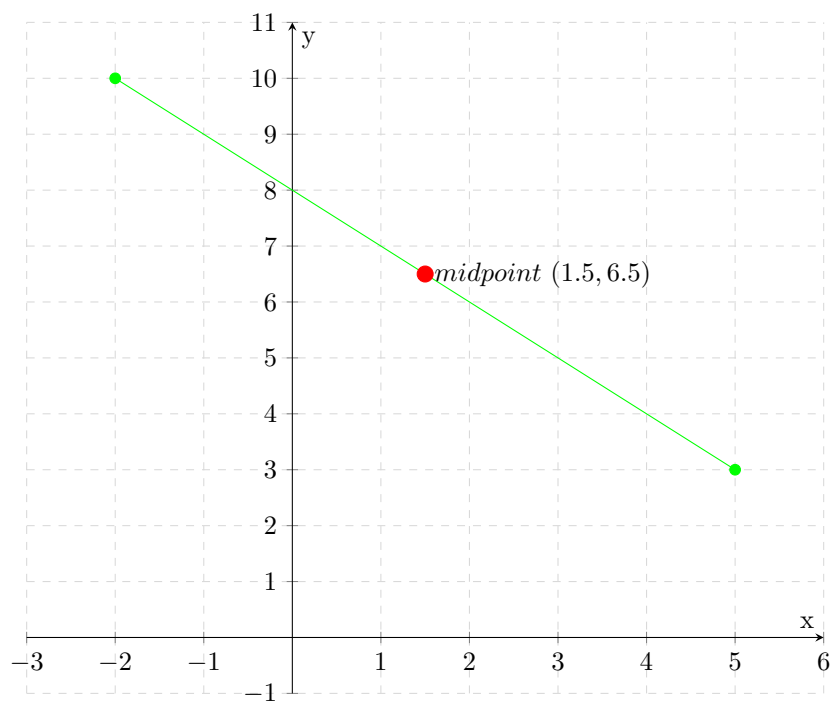Therefore the equation is

$$y = -x + 8$$

Figure 1: Graph showing $y = -x + 8$

1.2   (6, -2),(8, -3)

MIDPOINT

$$midpoint = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$
$$= (\frac{6 + 8}{2}, \frac{-2 - 3}{2})$$
$$= (\frac{14}{2}, \frac{-5}{2})$$
$$= (7, -2.5)$$

LENGTH OF LINE

$$length\ of\ line = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(8 - 6)^2 + (-3 - (-2))^2}$$
$$= \sqrt{(2)^2 + (-1)^2}$$
$$= \sqrt{4 + 1}$$
$$= \sqrt{5}$$
$$= 2.24$$

SLOPE OF LINE

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-3 - (-2)}{8 - 6}$$
$$= \frac{-1}{2}$$
$$= -0.5$$

EQUATION OF THE LINE

Using the points $(6, -2)$ and $m = -0.5$ we find c

$$y = mx + c$$
$$c = y - mx$$
$$c = -2 - (-0.5)(6)$$
$$c = -2 + 3$$
$$c = 1$$

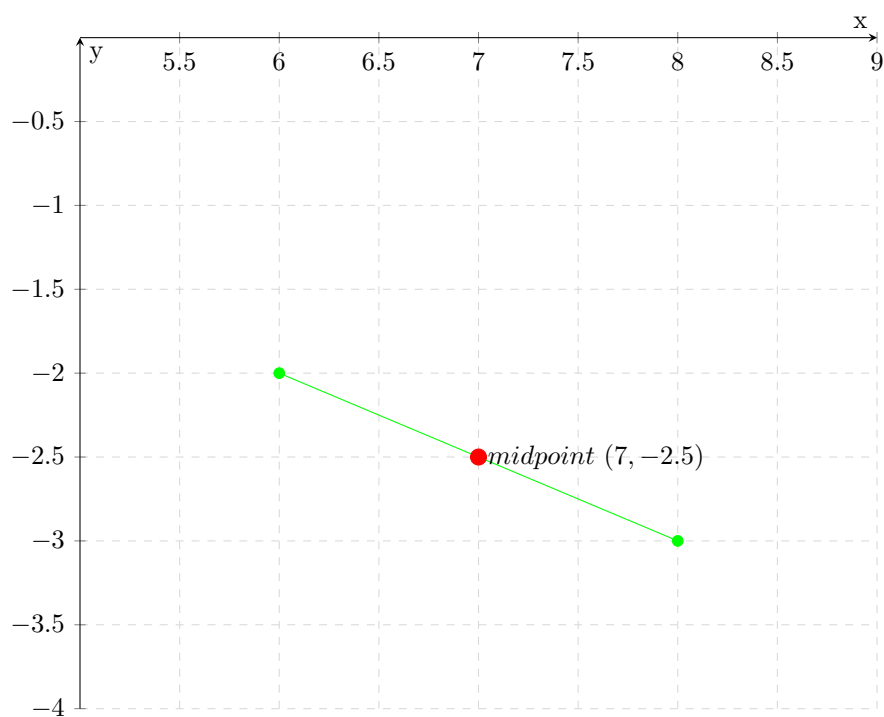Therefore the equation is

$$y = -0.5x + 1$$

Figure 2: Graph showing $y = -0.5x + 1$

1.3 (0, -6),(3, 0)

<u>MIDPOINT</u>

$$midpoint = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$
$$= (\frac{0 + 3}{2}, \frac{-6 + 0}{2})$$
$$= (\frac{3}{2}, \frac{-6}{2})$$
$$= (1.5, -3)$$

<u>LENGTH OF LINE</u>

$$length\ of\ line = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3 - 0)^2 + (0 - (-6))^2}$$
$$= \sqrt{(3)^2 + (6)^2}$$
$$= \sqrt{9 + 36}$$
$$= \sqrt{45}$$
$$= 6.71$$

<u>SLOPE OF LINE</u>

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0 - (-6)}{3 - 0}$$
$$= \frac{6}{3}$$
$$= 2$$

<u>EQUATION OF THE LINE</u>

Using the points $(0, -6)$ and $m = 2$ we find c

$$y = mx + c$$
$$c = y - mx$$
$$c = -6 - 2(0)$$
$$c = -6 - 0$$
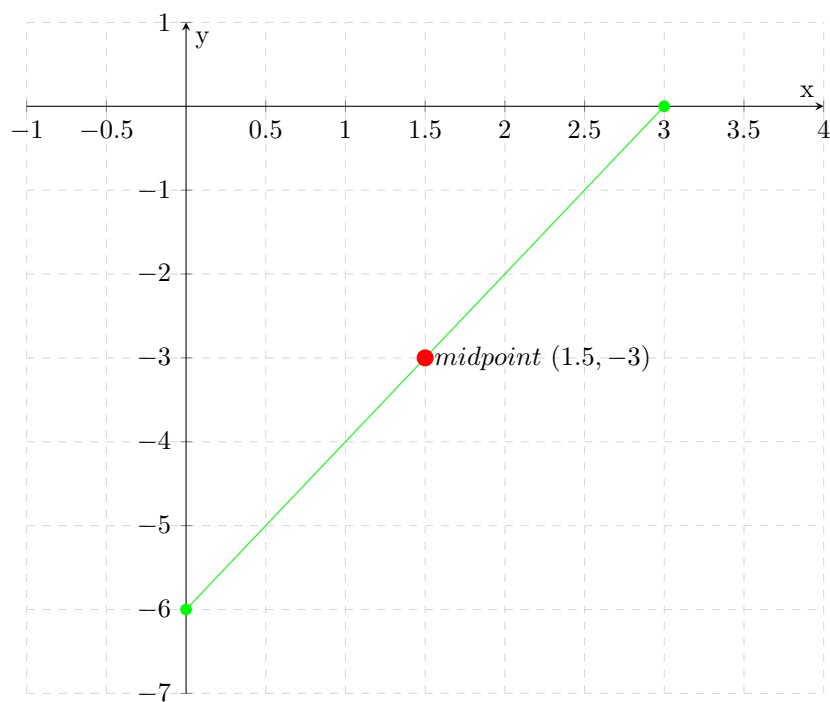$$c = -6$$

Therefore the equation is

$$y = 2x - 6$$

Figure 3: Graph showing $y = 2x - 6$

1.4   (1, -7),(9, 0)

<u>MIDPOINT</u>

$$midpoint = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$
$$= (\frac{1 + 9}{2}, \frac{-7 + 0}{2})$$
$$= (\frac{10}{2}, \frac{-7}{2})$$
$$= (5, -3.5)$$

<u>LENGTH OF LINE</u>

$$length\ of\ line = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(9 - 1)^2 + (0 - (-7))^2}$$
$$= \sqrt{(8)^2 + (7)^2}$$
$$= \sqrt{64 + 49}$$
$$= \sqrt{113}$$
$$= 10.63$$

<u>SLOPE OF LINE</u>

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0 - (-7)}{9 - 1}$$
$$= \frac{7}{8}$$
$$= 0.88$$

<u>EQUATION OF THE LINE</u>

Using the points $(1, -7)$ and $m = 0.88$ we find c

$$y = mx + c$$
$$c = y - mx$$
$$c = -7 - 0.88(1)$$
$$c = -7 - 0.88$$
$$c = -7.88$$
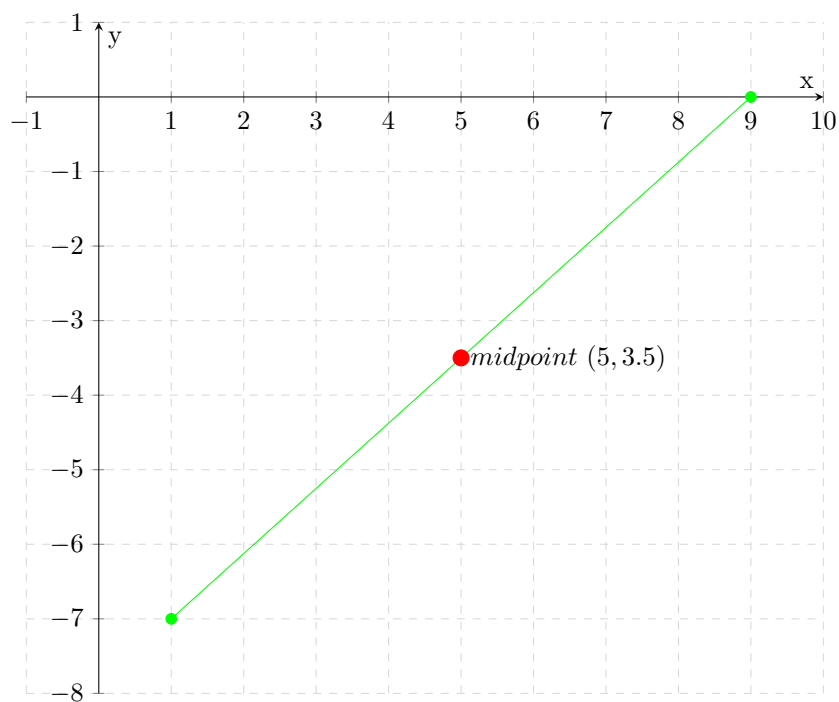
Therefore the equation is

$$y = 0.88x - 7.88$$

Figure 4: Graph showing $y = 0.88x - 7.88$

1.5   (0, 9),(5, -6)

MIDPOINT

$$midpoint = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$
$$= (\frac{0 + 5}{2}, \frac{9 - 6}{2})$$
$$= (\frac{5}{2}, \frac{3}{2})$$
$$= (2.5, 1.5)$$

LENGTH OF LINE

$$length \; of \; line = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5 - 0)^2 + (-6 - 9)^2}$$
$$= \sqrt{(5)^2 + (-15)^2}$$
$$= \sqrt{25 + 225}$$
$$= \sqrt{250}$$
$$= 15.81$$

SLOPE OF LINE

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-6 - 9}{5 - 0}$$
$$= \frac{-15}{5}$$
$$= -3$$

EQUATION OF THE LINE

Using the points $(0, 9)$ and $m = -3$ we find c

$$y = mx + c$$
$$c = y - mx$$
$$c = 9 - (-3)(0)$$
$$c = 9 - 0$$
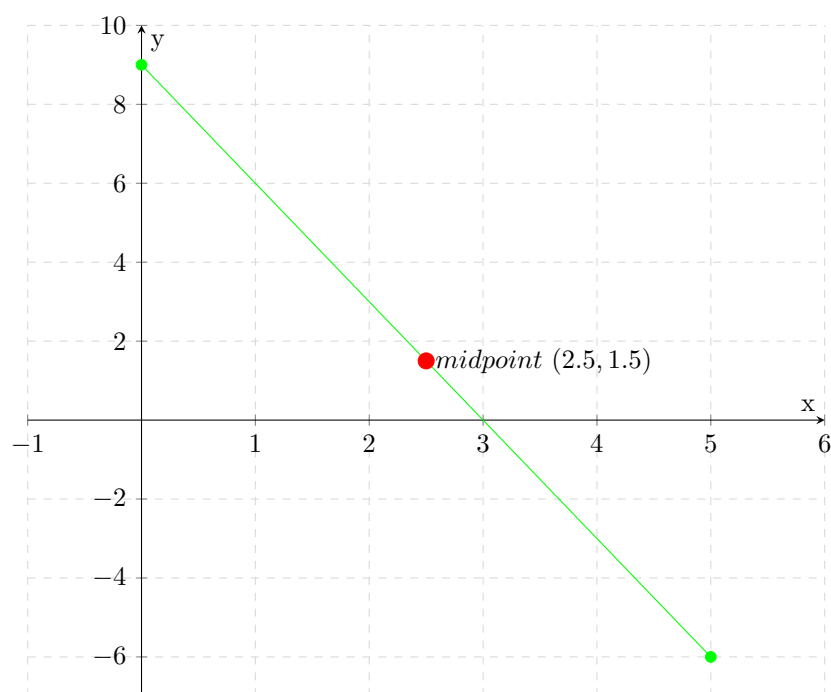$$c = 9$$

Therefore the equation is

$$y = -3x + 9$$

Figure 5: Graph showing $y = -3x + 9$

1.6   (5, -2),(4,-2)

MIDPOINT

$$midpoint = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$
$$= (\frac{5 + 4}{2}, \frac{-2 - 2}{2})$$
$$= (\frac{9}{2}, \frac{-4}{2})$$
$$= (4, 5, -2)$$

LENGTH OF LINE

$$length\ of\ line = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - 5)^2 + (-2 - (-2))^2}$$
$$= \sqrt{(-1)^2 + (0)^2}$$
$$= \sqrt{1 + 0}$$
$$= \sqrt{1}$$
$$= 1$$

SLOPE OF LINE

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-2 - (-2)}{4 - 5}$$
$$= \frac{0}{-1}$$
$$= 0$$

EQUATION OF THE LINE

Using the points $(5, -2)$ and $m = 0$ we find c

$$y = mx + c$$
$$c = y - mx$$
$$c = -2 - (0)(5)$$
$$c = -2 - 0$$
$$c = -2$$

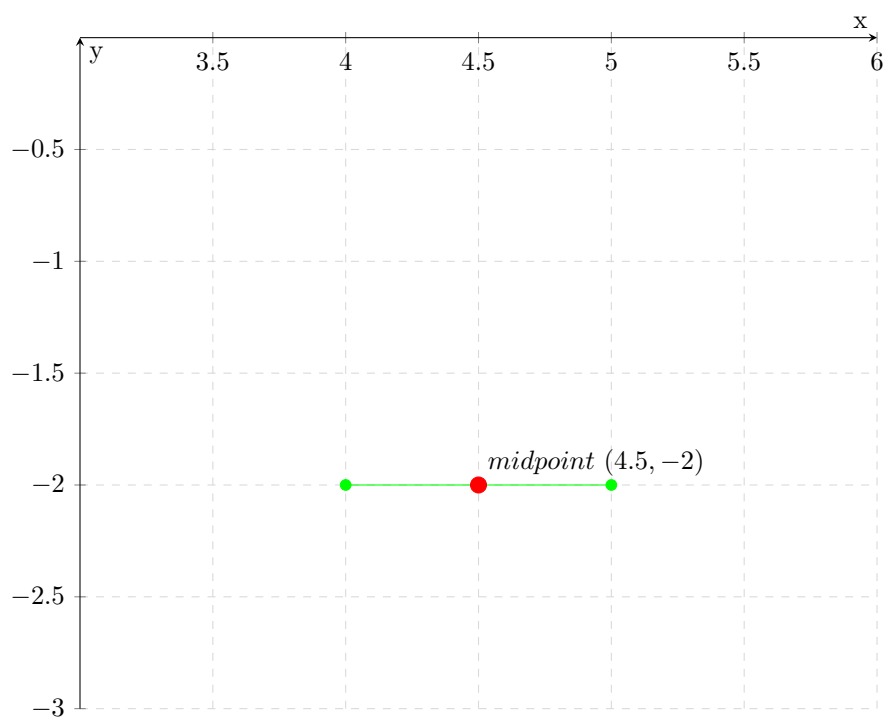Therefore the equation is

$$y = -2$$

Figure 6: Graph showing $y = -2$

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2.1 Refer to the above, determine the equation of the line that is parallel to (1.1) but passes through (3, -2)

Given the equation of line with points $(-2, 10), (5, 3)$ i.e. A is

$$y = -x + 8$$

The $y - intercept$ i.e. c of line parallel to A is

$$-2 = -3 + c$$
$$c = -2 + 3$$
$$c = 1$$

Therefore the equation of the parallel line B is

$$y = -x + 1$$

2.2 Determine the equation of the line that is perpendicular to (1.1) bit passes through (10, 3)

Given the equation of line A with points $(-2, 10), (5, 3)$ is

$$y = -x + 8$$

and

$$m_1 = -1$$

and two lines are $\perp$ when

$$m_1 * m_2 = -1$$

m_2 is therefore

$$-1 * m_2 = -1$$

$$m_2 = \frac{-1}{-1}$$

$$m_2 = 1$$

The $y - intercept$ i.e. c of $\perp$ line C using points $(10, 3)$ and $m = 1$ is

$$y = mx + c$$

$$3 = 1(10) + c$$

$$c = -7$$

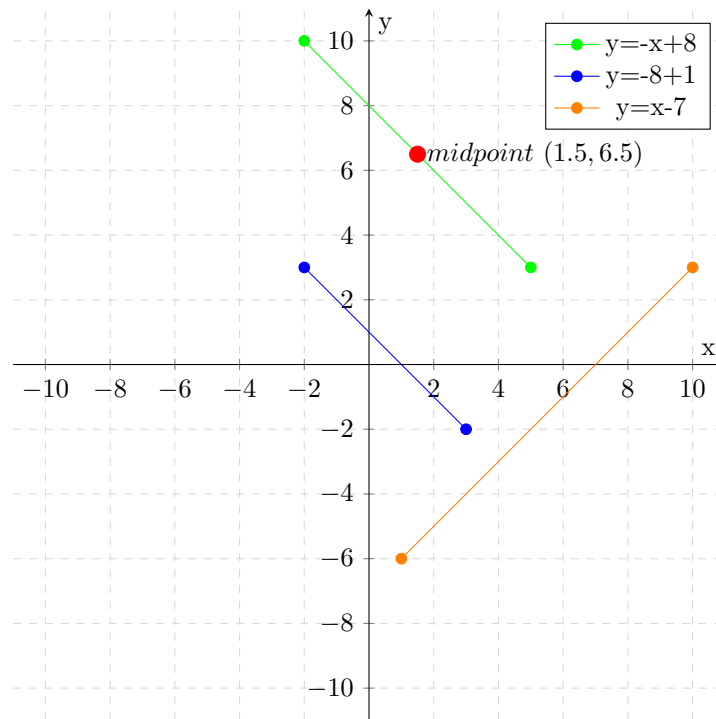Therefore the equation of $\perp$ line C is

$$y = x - 7$$

Figure 7:

15

3 Two points on a straight line are $P(6, 10)$ and $Q(-4, -6)$. Determine:

a. the length of the line PQ

$$= sqrt(x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= sqrt(-4 - 6)^2 + (-6 - 10)^2$$
$$= sqrt100 + 156$$
$$= sqrt256$$
$$= 18.87$$

b. the midpoint of PQ

$$= (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$
$$= (\frac{6 - 4}{2}, \frac{10 - 6}{2})$$
$$= (\frac{2}{2}, \frac{2}{2})$$
$$= (1, 2)$$

c. the equation of PQ The equation of a line is

$$y = mx + c$$

The slope of the line PQ is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-6 - 10}{-4 - 6}$$
$$= \frac{-16}{-10}$$
$$= 1.6$$

The $y-intercept$ of the line is

$$c = y - mx$$
$$c = 10 - (1.6)(6)$$
$$c = 10 - 9.6$$
$$c = 0.4$$

Therfore the equation of the line is

$$y = 1.6x + 0.4$$

d. the equation of the line MN that is $\parallel$ to PQ and passes through $(0, -8)$ Given that MN $\parallel$ to PQ

$$m = 1.6$$

The intercept of the line MN is using the points $(0, -8)$ is

$$y = 1.6x + c$$
$$-8 = 1.6(0) + c$$
$$c = -8$$

Therefore the equation of the line MN is

$$y = 1.6x - 8$$

e. the equation of the line AB that is $\perp$ to PQ but passes through $(10, 4)$ Given $AB \perp PQ$, the slope of AB is

$$m_1 * m_2 = -1$$
$$1.6 * m_2 = -1$$
$$m_2 = \frac{-1}{1.6}$$
$$m_2 = -0.625$$

The intercept of the line using the points $(10, 4)$ is

$$c = y - mx$$
$$c = 4 - (-0.625)(10)$$
$$c = 4 - (-0.625)(10)$$
$$c = 4 + 6.25)$$
$$c = 10.25)$$

Therefore the equation of the line is

$$y = -0.625x + 10.25$$

Figure 8:

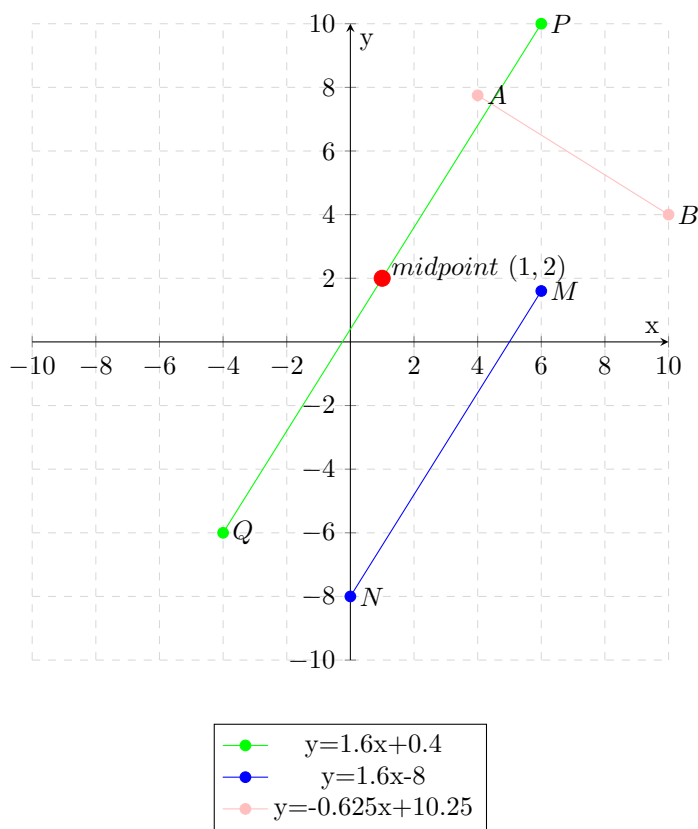## 4 For each of the following, if possible, find the slope, the y-intercept and the x-intercept

4.1 $y = 4x - 6$

Slope

$$m = 4$$

y-intercept

$$c = -6$$

x-intercept

$$y = 4x - 6$$
$$0 = 4x - 6$$
$$4x = 6$$
$$x = \frac{6}{4}$$
$$x = 1.5$$

4.2 $3x + 5y - 9 = 0$

Writing equation in the form $y = mx + c$:

$$y = -\frac{3}{5}x + \frac{9}{5}$$
$$y = -0.6x + 1.8$$

Slope

$$m = -0.6$$

y-intercept

$$c = 1.8$$

<u>x-intercept</u>

$$y = -0.6x + 1.8$$
$$0 = -0.6x + 1.8$$
$$-0.6x = -1.8$$
$$x = \frac{1.8}{0.6}$$
$$x = 1.125$$

4.3 $y = 3x$

Writing equation in the form $y = mx + c$:

$$y = 3x + 0$$

<u>Slope</u>

$$m = 3$$

<u>y-intercept</u>

$$c = 0$$

<u>x-intercept</u>

$$y = 3x + 0$$
$$0 = 3x + 0$$
$$3x = 0$$
$$x = 0$$

4.4 $y - 7 = 3(x - 4)$

Writing equation in the form $y = mx + c$:

$$y - 7 = 3(x - 4)$$
$$y - 7 = 3x - 12$$
$$y = 3x - 12 + 7$$
$$y = 3x - 5$$

<u>Slope</u>

$$m = 3$$

<u>y-intercept</u>

$$c = -5$$

<u>x-intercept</u>

$$y = 3x - 5$$
$$0 = 3x - 5$$
$$3x = 5$$
$$x = \frac{5}{3}$$
$$= 1.67$$

4.5 $y + 4 = 7$

Writing equation in the form $y = mx + c$:

$$y + 4 = 7$$
$$y = 3$$

<u>Slope</u>
  Probably

$$m = 0$$

<u>y-intercept</u>

$$c = 7$$

<u>x-intercept</u>

$$impossible$$

4.6 $y = 3x$

Writing equation in the form $y = mx + c$:

$$y = 3x$$
$$y = 3x + 0$$

Slope

$$m = 3$$

y-intercept

$$c = 0$$

x-intercept

$$y = 3x$$
$$3x = 0$$
$$x = \frac{0}{3}$$
$$x = 0$$

4.7 $6y - 24 = 0$

writing equation in the form $y = mx + c$:

$$6y - 24 = 0$$
$$6y = 24$$
$$y = \frac{24}{6}$$
$$y = 4$$

slope
probably

$$impossible$$

y-intercept

$$c = 4$$

x-intercept

$$impossible$$

4.8 $2x = 5 - 3y$

writing equation in the form $y = mx + c$:

$$2x = 5 - 3y$$
$$3y = -2x + 5$$
$$y = -\frac{2}{3}x + \frac{5}{3}$$
$$y = -0.67x + 1.67$$

slope

$$m = -0.67$$

y-intercept

$$c = 1.67$$

x-intercept

$$2x = 5 - 3y$$
$$2x = 5 - 3(0)$$
$$2x = 5$$
$$x = \frac{5}{3}$$
$$x = 1.67$$

4.9 $-\frac{x}{2} + \frac{2y}{3} = -4\frac{3}{3}$

writing equation in the form $y = mx + c$:

$$-\frac{x}{2} + \frac{2y}{3} = -4\frac{3}{3}$$

$$-\frac{x}{2} + \frac{2y}{3} = -5$$

$$-\frac{x}{2} + \frac{2y}{3} + \frac{5}{1} = 0$$

$$\frac{-3x + 4y + 30}{6} = 0$$

$$-3x + 4y + 30 = 0$$

$$4y = 3x - 30$$

$$y = \frac{3}{4}x - \frac{30}{4}$$

$$y = 0.75x - 7.5$$

slope

$$m = 0.75$$

y-intercept

$$c = -7.5$$

x-intercept

$$y = 0.75x - 7.5$$

$$0 = 0.75x - 7.5$$

$$0.75x = 7.5$$

$$x = \frac{7.5}{0.75}$$

$$x = 10$$

4.10 $y = \frac{1}{300}x + 8$

writing equation in the form $y = mx + c$:

$$y = \frac{1}{300}x + 8$$
$$y = 0.003x + 8$$

slope

$$m = 0.003$$

y-intercept

$$c = 8$$

x-intercept

$$y = 0.003x + 8$$
$$0 = 0.003x + 8$$
$$0.003x = -8$$
$$x = \frac{-8}{0.003}$$
$$x = -2666.67$$