ECN 1101 - Introductory Maths - Semester 1 2021

Lecture Notes 2 - Application of Straight Line Geometry

Lecturer: Anand Persaud
"Transliterator": Simeon Chester

2021-11-06

Objectives

To determine

1. the linear demand equation or demand function
2. the linear supply equation or supply function
3. the equilibrium point/position
4. other outcomes: interpreting the slopes etc.

Consider the following schedules:

The highlighted row shows equilibirum

Prices	**Demand Units**	**Supply Units**
10	1400	200
20	1200	300
30	1000	400
40	800	600
50	600	600
60	400	700
70	200	800

Table 1: Table showing the demand and supply units along with corresponding prices for an unknown product

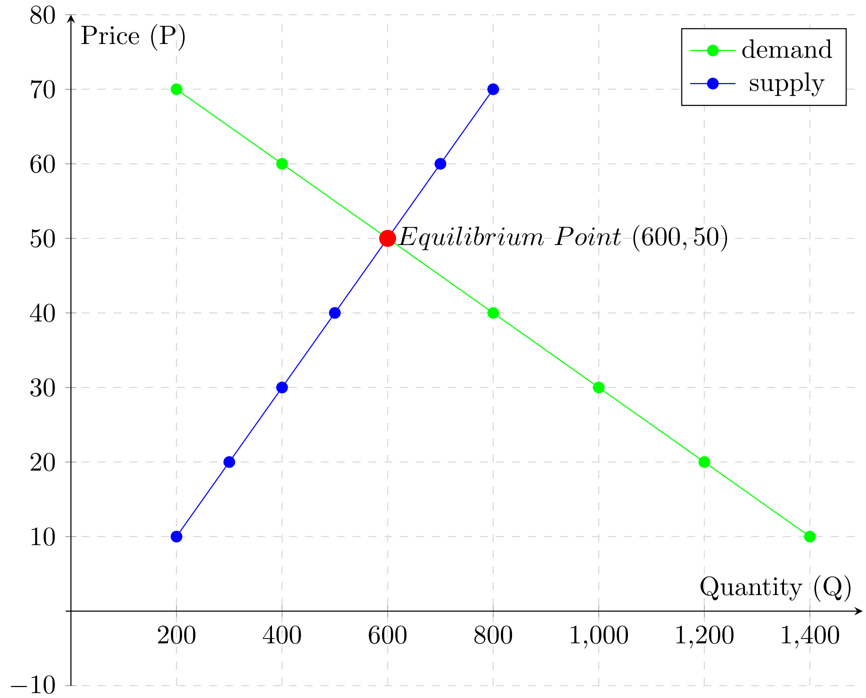

Figure 1: Graph showing demand and supply units as a factor of price along with the market equilibrium point

We should notice that as price increases, quantity demanded falls, while quantity supplied, increases. At 10$ per unit, 1400 are demanded while 200 units are supplied, at a higher price, say $60 per unit, 400 units are demanded and supply increases.

1. Determining the linear demand equation

Using:

$$Q_d = mP + c$$

and

$$m = \frac{Q_2 - Q1}{P_2 - P_1}$$

and choose any two ordered pairs from the demand schedule. Let us choose:

$$\begin{pmatrix} 20 & 1200 \\ P_1 & Q_1 \end{pmatrix} and \begin{pmatrix} 70 & 200 \\ P_2 & Q_2 \end{pmatrix}$$

and properly assign each.

$$m = \frac{200 - 1200}{70 - 20} = -20$$

$$Q_d = -20P + c$$

To find c, simply use any any P and Q pair from the schedule and plug it into the aforementioned equation found.

Using

$$\begin{pmatrix} 20 & 1200 \\ P_1 & Q_1 \end{pmatrix}$$

we get

$$\begin{aligned} Q_d &= -20P + c \\ 1200 &= -20(20) + c \\ -20(20) + c &= 1200 \\ -400 + c &= 1200 \\ c &= 1200 + 400 \\ c &= 1600 \end{aligned}$$

Using $m = -20$ and $c = 1600$ in the equation

$$Q_d = mP + c$$

we get

$$Q_d = -20P + 1600$$

2. Determing the linear supply equation

Let us use the ordered pairs with labels

$$\begin{pmatrix} 20 & 300 \\ P_1 & Q_1 \end{pmatrix} and \begin{pmatrix} 70 & 800 \\ P_2 & Q_2 \end{pmatrix}$$

and substitute into

$$m = \frac{Q_2 - Q_1}{P2 - P1}$$
$$m = \frac{800 - 300}{70 - 20} = 10$$

With $m = 10$, the supply equation can now be written as:

$$Q_s = 10P + c$$

To find c, simply use any any P and Q pair from the schedule and plug it into the aforementioned equation found.
Using

$$\begin{pmatrix} 20 & 300 \\ P_1 & Q_1 \end{pmatrix}$$

we get

$$\begin{aligned} Q_s &= 10P + c \\ 300 &= 10(20) + c \\ 10(20) + c &= 300 \\ 200 + c &= 300 \\ c &= 300 - 200 \\ c &= 100 \end{aligned}$$

Using $m = 10$ and $c = 100$ in the equation

$$Q_s = mP + c$$

we get

$$Q_s = 10P + 100$$

3. Determining the equilibrium point/position

Rewrite the demand and supply equations:

$$Q_d = -20p + 1600$$
$$Q_s = 10p + 100$$

then set

$$Q_d = Q_s$$

Therefore

$$\begin{aligned}
-20p + 1600 &= 10p + 100 \\
-20p - 10p &= 100 - 1600 \\
-30p &= -1500 \\
p &= \frac{-1500}{-30} \\
p = 50
\end{aligned}$$

Substituting $p = 50$ into

$$Q_d = -20p + 1600$$

OR

$$Q_s = 10p + 100$$

we get

$$Q_d = 600$$

Therefore the equilibrium point position is $(50, 600)$ or $p = \$50$ and $Q = 600$ *units*.

4. Interpreting both slopes

The slope of the demand equation is 20 or $\frac{20(quantity)}{1(price)}$ therefore a \$1 increase in price is likely to result in a fall in quantity demanded by 20 *units* or vice versa i.e. if P falls by \$1, Q is likely to increase by 20 *units*. The slope of the supply equation is 10 or $\frac{10(quanity)}{1(price)}$. Therefore as P increase by \$1, 10 additional units are likely to be supplied and vice versa.

5. Others

Using the equations derived:

a. Determine Q_d if $p = \$30$.

Substituting $c = 1600$, $m = -20$ and $P = \$30$ in the equation:

$$Q_d = mP + c$$

we get:

$$\begin{aligned} Q_d &= -20(30) + 1600 \\ &= -600 + 1600 \\ &= 1000 \ units \end{aligned}$$

b. Determine Q_s if $p = \$30$.

Substituting $c = 100$, $m = 10$ and $P = \$30$ in the equation:

$$Q_s = mP + c$$

we get:

$$\begin{aligned} Q_s &= mP + c \\ &= 10(30) + 100 \\ &= 300 + 100 \\ &= 400 \ units \end{aligned}$$

c. If $Q_d = 100$ and $Q_s = 400$, is there a surplus or demand?

Since both demand and supply are at the same price but $Q_d > Q_s$ the market is at shortage.

d. Suppose $Q_d = 800$, find P.

Substituting $m = -20$, $c = 1600$ and $Q_d = 800$ in the equation:

$$Q_d = mP + c$$

we get:

$$\begin{aligned} 800 &= -20P + 1600 \\ 20P &= 1600 - 800 \\ 20P &= 800 \\ P &= \frac{800}{20} \\ &= \$40 \end{aligned}$$

e. Suppose $Q_s = 1000$, find P.

Substituting $m = 10$, $c = 100$ and $Q_s = 1000$ in the equation:

$$Q_s = mP + c$$

we get:

$$\begin{aligned}
1000 &= 10P + 100 \\
10P &= 1000 - 100 \\
10P &= 900 \\
P &= \frac{900}{10} \\
P &= \$90
\end{aligned}$$