ECN 1101 - Introductory Maths - Semester 1 2021

Lecture Notes 1 - Straight Line Geometry or Co-ordinate Geometry

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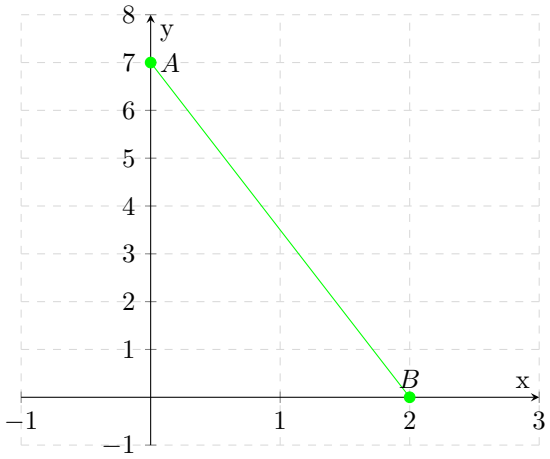
Here are two CXC questions:

Question 1



a. State the value of c.

$$c = 7$$

b. Determine the slope of AB. Given the points

$$\begin{matrix} A & B \\ \begin{pmatrix} 0 & 7 \\ x_1 & y_1 \end{pmatrix} & \begin{pmatrix} 2 & 0 \\ x_2 & y_2 \end{pmatrix} \end{matrix}$$

and the slope(m) of a straight line:

The equation of a straight line is $y = mx + c$ where $c =$ *y-intercept*, m = *slope* and x & $y = x$ *and* y *values*

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substituting the values we get:

$$m = \frac{0 - 7}{2 - 0}$$
$$m = \frac{-7}{2}$$
$$m = -3.5$$

c. Determine the mid-point of AB.

The midpoint of a straight line:

$$m = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

Therefore substituting the values into the equation we get.

$$m = (\frac{0 + 2}{2}, \frac{7 + 2}{2})$$
$$m = (1, 3.5)$$

Question 2

A straight line joins two points H(-4, 6) and G(5, 3) .i.e

$$\begin{matrix} H \\ \begin{pmatrix} -4 & 6 \\ x_1 & y_1 \end{pmatrix} \end{matrix} \quad \begin{matrix} G \\ \begin{pmatrix} 5 & 3 \\ x_2 & y_2 \end{pmatrix} \end{matrix}$$

a. Calculate the slope of HG.

The slope of a straight line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substituting the values given into the equation we get

$$m = \frac{3 - 6}{5 - (-4)}$$
$$m = \frac{3 - 6}{5 + 4)}$$
$$m = \frac{-3}{9}$$
$$m = -\frac{1}{3}$$

b. Determine the equation of HG.

The equation of straight line is

$$y = mx + c$$

Therefore choosing an ordered pair

$$\begin{matrix} & H \\ \begin{pmatrix} -4 & 6 \\ x_1 & y_1 \end{pmatrix} & \end{matrix}$$

and substituting the values $m = -\frac{1}{3}$ into the $y = mx + c$ to find c we get

$$\begin{aligned} y &= mx + c \\ 6 &= -\frac{1}{3}(-4) + c \\ 6 &= \frac{4}{3} + c \\ c &= 6 - \frac{4}{3} \\ c &= \frac{18}{3} - \frac{4}{3} \\ c &= \frac{14}{3} \end{aligned}$$

Then substituting $m = -\frac{1}{3}$ and $c = \frac{14}{3}$ into the line equation we get

$$y = -\frac{1}{3}x + \frac{14}{3}$$

OR

$$3y = -x + 14$$

c. Write the slope of a line $\perp$ to HG.

Given that

$$m_1 * m_2 = -1$$

Substituting $m_1 = -\frac{1}{3}$ we get

$$\begin{aligned} -\frac{1}{3} * m_2 &= -1 \\ m_2 &= -1 \div -\frac{1}{3} \\ m_2 &= -1 \times -\frac{3}{1} \\ m_2 &= 3 \end{aligned}$$

d. Determine the equation of the line which is $\perp$ to HG but passes through (7,2).

Given the equation of a line

$$y = mx + c$$

and $\perp$ line slope $m = 3$, $x = 7$ and $y = 2$, we substitute the values into the line equation to get the y-intercept c

$$\begin{aligned} y &= mx + c \\ mx + c &= y \\ 3(7) + c &= 2 \\ 21 + c &= 2 \\ c &= 2 - 21 \\ c &= -19 \end{aligned}$$

Therefore given than $m = 3$ and $c = -19$ the line equation is

$$\begin{aligned} y &= mx + c \\ y &= 3x + (-19) \\ y &= 3x - 19 \end{aligned}$$

e. Determine the equation of a line PQ which is parallel to HG and passes through (-4,2).

Given the equation of the line HG is

$$3y = -x + 14$$

finding for the y-intercept c and substituting the values $(-4, 2)$ we get

$$\begin{aligned} 3y &= -x + c \\ 3(2) &= -(-4) + c \\ 6 &= 4 + c \\ c &= 2 \end{aligned}$$

therefore the equation of the line is

$$3y = -x + 2$$

f. Calculate the mid-point of HG.

Given the points

$$\begin{matrix} H \\ \begin{pmatrix} -4 & 6 \\ x_1 & y_1 \end{pmatrix} \end{matrix} \quad \begin{matrix} G \\ \begin{pmatrix} 5 & 3 \\ x_2 & y_2 \end{pmatrix} \end{matrix}$$

the midpoint of the is

$$midpoint = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$
$$midpoint = (\frac{-4 + 5}{2}, \frac{6 + 3}{2})$$
$$midpoint = (\frac{1}{2}, \frac{9}{2})$$

g. Calculate the length of the line HG.

The length of a line is

$$\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Therefore using the points

$$\begin{matrix} H & G \\ \begin{pmatrix} -4 & 6 \\ x_1 & y_1 \end{pmatrix} & \begin{pmatrix} 5 & 3 \\ x_2 & y_2 \end{pmatrix} \end{matrix}$$

we get

$$\sqrt{(3 - 6)^2 + (5 - (-4))^2}$$
$$\sqrt{(-3)^2 + (9)^2}$$
$$\sqrt{9 + 81}$$
$$\sqrt{90}$$

Therefore the length of the line HG is 9.487 or ‘r sqrt(90)‘.

Question 3

a. Given $y = 5x + 2$ and $-5x + y - 3 = 0$, are these lines $\parallel$, $\perp$ or neither?

Writing both line equations in the form $y = mx + c$ we get:

$$y = 5x + 2$$

$$y = 5x + 3$$

Therefore, given $m = 5$ in both equations the lines are $\parallel$ to each other.

b. Given $3x + y = 4$ and $x - 3y + 1 = 0$ are these lines $\|$, $\perp$ or neither?

Writing both line equations in the form $y = mx + c$ we get:

$$y = -3x + 4$$

$$y = \frac{1}{3}x + \frac{1}{3}$$

Therefore $m_1 = -3$ and $m_2 = \frac{1}{3}$. A line is $\perp$ when $m_1 * m_2 = -1$. Checking to see if line is $\perp$.

$$\begin{aligned} -3 \times \frac{1}{3} &= -\frac{3}{3} \\ &= -1 \end{aligned}$$

Therfore the lines are $\perp$.

Homework

Find the midpoints, the length of the lines, the slopes and the equations of the straight lines passing through the points:

1. (-2, 10), (5, 3)

MIDPOINT

$$\begin{aligned} midpoint &= (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}) \\ &= (\frac{-2 + 5}{2}, \frac{10 + 3}{2}) \\ &= (\frac{3}{2}, \frac{13}{2}) \\ &= (1.5, 6.5) \end{aligned}$$

LENGTH OF LINE

$$\begin{aligned} length\ of\ line &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5 - (-2))^2 + (3 - 10)^2} \\ &= \sqrt{(7)^2 + (7)^2} \\ &= \sqrt{49 + 49} \\ &= \sqrt{98} \\ &= 9.90 \end{aligned}$$

SLOPE OF LINE

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - 10}{5 - (-2)} \\ &= -\frac{7}{7} \\ &= -1 \end{aligned}$$

EQUATION OF THE LINE

Using the points $(-2, 10)$ and $m = 1$ we find c

$$\begin{aligned} y &= mx + c \\ c &= y - mx \\ c &= 10 - (-1)(-2) \\ c &= 10 - 2 \\ c &= 8 \end{aligned}$$

Therefore the equation is

$$y = -x + 8$$

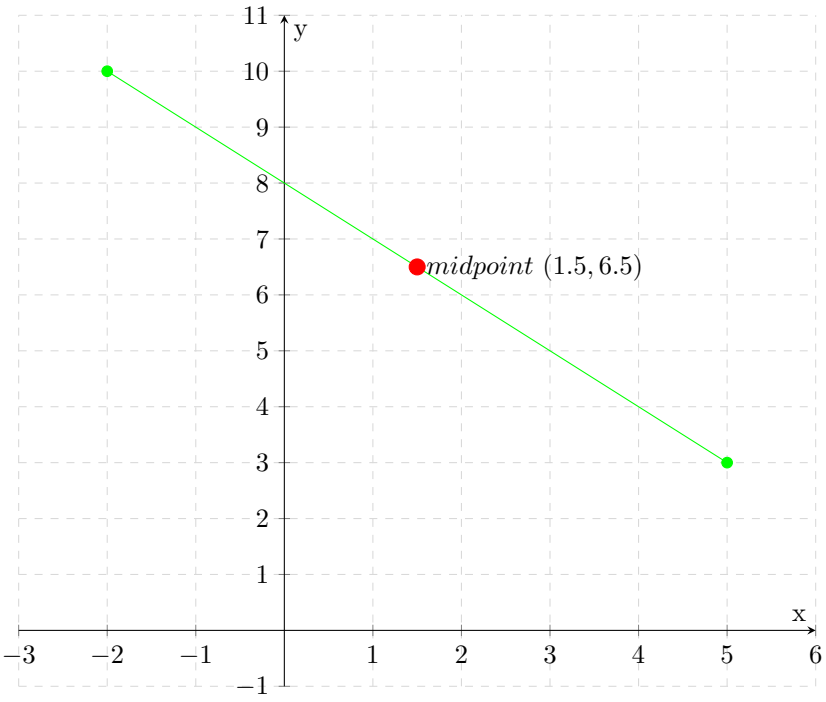


Figure 1: Graph showing $y = -x + 8$

2. (6, -2), (8, -3)

MIDPOINT

$$\begin{aligned} midpoint &= (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}) \\ &= (\frac{6+8}{2}, \frac{-2+(-3)}{2}) \\ &= (\frac{14}{2}, \frac{-5}{2}) \\ &= (7, -2.5) \end{aligned}$$

LENGTH OF LINE

$$\begin{aligned} length\ of\ line &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{8-6)^2 + (-3-(-2))^2} \\ &= \sqrt{(2)^2 + (-1)^2} \\ &= \sqrt{4+1} \\ &= \sqrt{5} \\ &= 2.24 \end{aligned}$$

SLOPE OF LINE

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3-(-2)}{8-6} \\ &= -\frac{1}{2} \\ &= -0.5 \end{aligned}$$

EQUATION OF THE LINE

Using the points $(6, -2)$ and $m = -0.5$ we find c

$$\begin{aligned} y &= mx + c \\ c &= y - mx \\ c &= -2 - (-0.5)(6) \\ c &= -2 + 3 \\ c &= 1 \end{aligned}$$

Therefore the equation is

$$y = -0.5x + 1$$

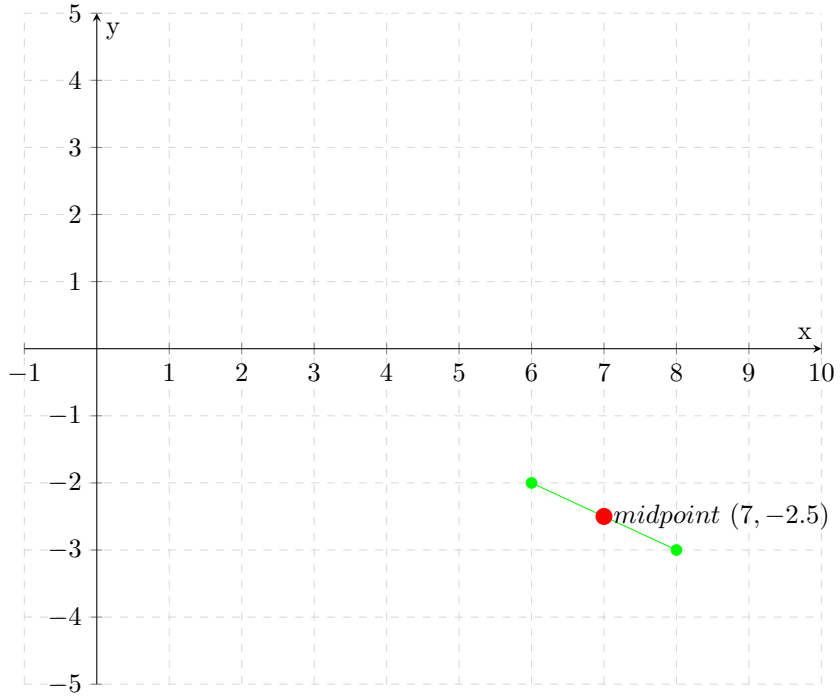


Figure 2: Graph showing $y = -0.5x + 1$

3. (0, -6), (3, 0)

MIDPOINT

$$\begin{aligned} midpoint &= (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}) \\ &= (\frac{0+3}{2}, \frac{-6+0}{2}) \\ &= (\frac{3}{2}, \frac{-6}{2}) \\ &= (1.5, -3) \end{aligned}$$

LENGTH OF LINE

$$\begin{aligned} length\ of\ line &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{3-0)^2 + (0-(-6))^2} \\ &= \sqrt{(3)^2 + (6)^2} \\ &= \sqrt{9 + 36} \\ &= \sqrt{45} \\ &= 6.71 \end{aligned}$$

SLOPE OF LINE

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0-(-6)}{3-0} \\ &= -\frac{6}{3} \\ &= 2 \end{aligned}$$

EQUATION OF THE LINE

Using the points $(0, -6)$ and $m = 2$ we find c

$$\begin{aligned} y &= mx + c \\ c &= y - mx \\ c &= (-6) - (2)(0) \\ c &= -6 - 0 \\ c &= -6 \end{aligned}$$

Therefore the equation is

$$y = 2x - 6$$

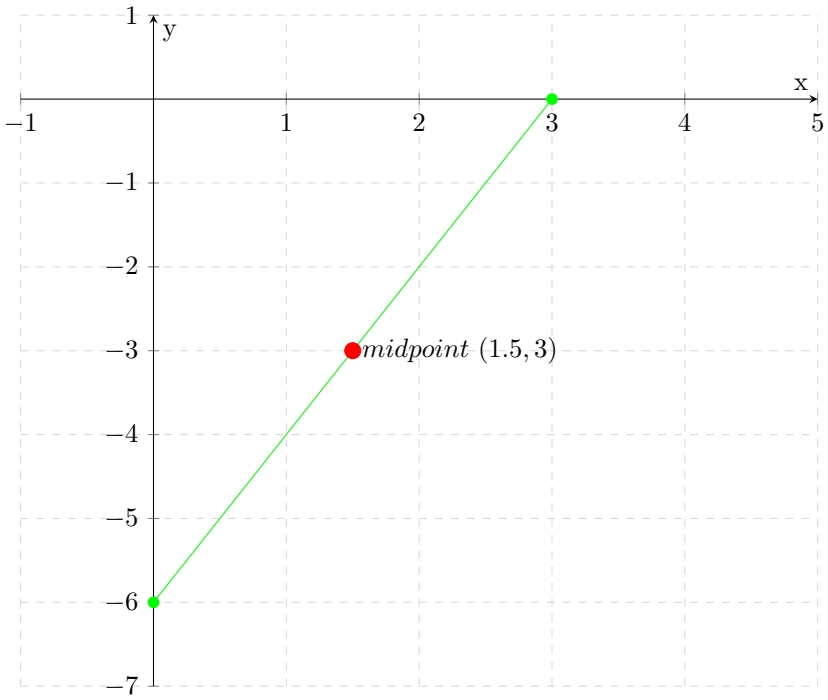


Figure 3: Graph showing $y = 2x - 6$

4. (1, -7), (9, 0)

MIDPOINT

$$\begin{aligned} midpoint &= (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}) \\ &= (\frac{1+9}{2}, \frac{-7+0}{2}) \\ &= (\frac{10}{2}, \frac{-7}{2}) \\ &= (5, -3.5) \end{aligned}$$

LENGTH OF LINE

$$\begin{aligned} length\ of\ line &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(9-1)^2+(0-(-7))^2} \\ &= \sqrt{(8)^2+(7)^2} \\ &= \sqrt{64+49} \\ &= \sqrt{113} \\ &= 10.63 \end{aligned}$$

SLOPE OF LINE

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{0-(-7)}{9-1} \\ &= -\frac{7}{8} \\ &= 0.88 \end{aligned}$$

EQUATION OF THE LINE

Using the points $(1, -7)$ and $m = 0.88$ we find c

$$\begin{aligned} y &= mx + c \\ c &= y - mx \\ c &= (-7) - (0.88)(1) \\ c &= -7 - 0.88 \\ c &= -7.88 \end{aligned}$$

Therefore the equation is

$$y = 0.88x - 7.88$$

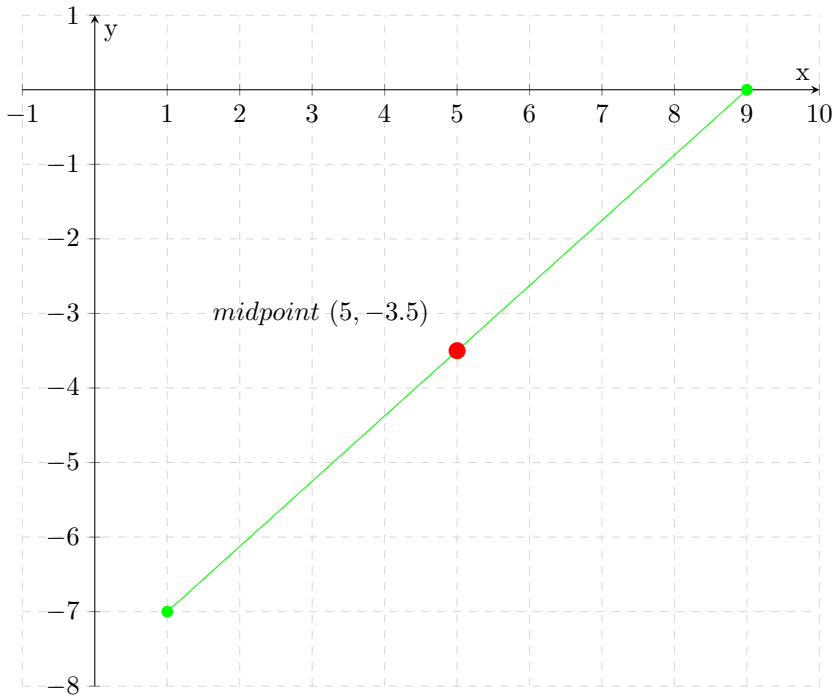


Figure 4: Graph showing $y = 0.88x - 7.88$