# Bayesian estimation of ETAS models using Rstan (applied to seismic recurrence in Ecuador 2016)

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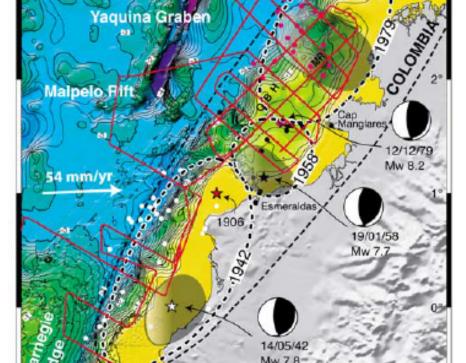
StanCon2018

#### Introduction

There were four mega earthquakes in the Ecuadorian Coast in the past century:

- ▶ 1906(magnitude 8.8)
- ▶ 1942(magnitude 7.8)
- ▶ 1958(magnitude 7.7)
- ▶ 1979(magnitude 8.2)

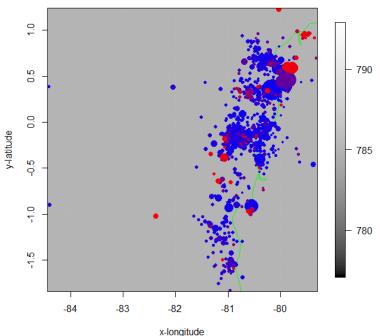
And then... April 16th, 2016 a quake of magnitude 7.8 according to USGS (magnitude 7.4 according to Seismology Institute - EPN)





Total Intensity with observed points

Circles area proportional to magnitude; red: recent, blu:older



#### Omori law

The empirical law of Omori and the law of Omori-Utsu (also called Modified Law of Omori) Utsu1995 describe the decreasing frequency of aftershocks over time after an earthquake:

$$N(t) = \frac{K}{(t+c)} \tag{1}$$

$$N(t) = \frac{K}{(t+c)^p} \tag{2}$$

Where N(t) is the occurrence rate of events, t is the time since the earthquake and K, c, p are constants.

#### Gutenberg-Ritcher law

 $\dots$  relates the magnitude to the number of earthquakes with magnitudes greater than M:

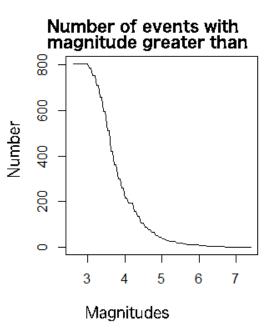
$$log_{10}N(\geq M) = a - bM \tag{3}$$

That is, the number of events of magnitude greater than a threshold, decreases exponentially with the increase of that threshold value by a power law.

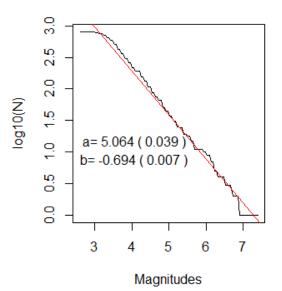
So, the distribution of magnitudes by the Gutenberg-Richter law  $s(m) = \beta e^{-\beta m}$ 

## Gutenberg-Richter law

Gutenberg-Richter law: linear regression log(N) = a + bM and then a = 5.064(sd = 0.039) and -0.694(sd = 0.007). From these values, b is 0.694 and  $\beta = bln(10) = 1.598$ .



# Log base 10 of number of events with magnitude greater than



An earthquake T is represented by a tuple  $(x_i, y_i, z_i, M_i)$ 

#### Temporal ETAS model

The simplest ETAS model is the temporal model with constant background seismicity:  $\lambda(t|H_t) = \mu + \sum_{j:t_j < t} \frac{K(p-1)c^{p-1}}{(t-t_j+c)^p}$  where  $\mu$  is the background intensity that is assumed to be constant (measured in events / day) and

$$g(t) = \frac{(p-1)c^{p-1}}{(t+c)^p} \tag{4}$$

is the probability density function of the occurrence times of the events triggered by previous earthquakes.



#### Temporal ETAS model

If the background intensity is not constant, but depends only on the longitude and latitude x, y we have:

$$\lambda(t|H_t) = \mu(x,y) + \sum_{j:t_j < t} \frac{K(p-1)c^{p-1}}{(t-t_j+c)^p}$$

where  $\mu(x,y)$  is now measured in events per day per unit of longitude and per unit of latitude. In this case, it is generally assumed that  $\mu(x,y) = \mu u(x,y)$  where  $\mu$  on the right side is a constant.

#### ETAS model with magnitudes

Considering the magnitudes of earthquakes we have the model

$$\lambda(t|H_t) = \mu + \sum_{j:t_j < t} \frac{K(p-1)c^{p-1}Ae^{\alpha(M_j - M_0)}}{(t-t_j + c)^p}$$

#### ETAS models stability

The event rate in the ETAS models may explode. Stability depends on the branching ratio n= expected number of descendants of a parent event. We have  $n=\int_0^\infty dt \int_{M_0}^{M_{max}} s(m)\lambda(t)dm$ 

with  $s(m) = \beta e^{-\beta m}$  the distribution of magnitudes by the Gutenberg-Richter law and  $\lambda(t)$  the branching term.

For the temporal ETAS model with magnitudes and p>1 we have  $n=\frac{Kc^{1-p}\beta}{(p-1)(\beta-\alpha)}\frac{1-e^{-(\beta-\alpha)(M_{max}-M_0)}}{1-e^{-\beta(M_{max}-M_0)}}$ 

For the temporal ETAS model with magnitudes and p>1 we have  $n=\frac{Kc^{1-p}\beta}{(p-1)(\beta-\alpha)}\frac{1-e^{-(\beta-\alpha)(M_{max}-M_0)}}{1-e^{-\beta(M_{max}-M_0)}}$  Assuming  $M_{max}=\infty$  the previous formula can be reduced to:  $n=\frac{Kc^{1-p}\beta}{(p-1)(\beta-\alpha)}$  Sornette2005 y Touati2011

and *n* is infinite if p < 1 or if  $\alpha > \beta$ .

#### ETAS models stability

If each event induces another event: n=1 then the process propagates indefinitely. This justifies normalizing the functions that appear in the sum over the preceding events. For example  $\int_0^\infty \frac{K}{(t+c)^p} dt = 1 \text{ implies that we need to add } (p-1)c^{p-1} \text{ to the constant } K \text{ and similarly } \int_{M_0}^{M_{max}} \beta e^{-\beta(m)} dm = 1 \text{ implies we need to add}$ 

 $1/(exp(-\beta M_0) - exp(-\beta M_{max}))$  to the constant  $\beta$  .

For temporal models:  $\log L(\theta) = \sum_{j} \log \lambda(t_{j}|H_{t}) - \int_{0}^{T_{max}} \lambda(t) dt$  The closed forms for temporal and temporal with magnitudes models:  $\log L(\mu, k, p, c) = \sum_{i=1}^{N} \log(\lambda(t_{i})) - \mu T_{max} - k \sum_{i=1}^{N} (1 - \frac{c^{p-1}}{(T_{max} - t_{i} + c)^{p-1}}) \log L(\mu, k, p, c, A, \alpha) = \sum_{i=1}^{N} \log(\lambda(t_{i})) - \mu T_{max} - k A \sum_{i=1}^{N} e^{\alpha(M_{i} - M_{0})} (1 - \frac{c^{p-1}}{(T_{max} - t_{i} + c)^{p-1}})$ 

```
Listing 1: Temporal Rstan model
    with constant background seiss
                                            26
    micity
                                            27
                                             28
                                            29
1
    functions {
2
       real loglikelihood (int N,
3
                           real mu,
                                             30
                                            31
                           real k.
                                            32
                           real p.
6
                                            33
                           real c.
7
                           vector times diff34
                                             35
8
                           real tmax){
                                             36
9
         real seismic_rate[N];
                                            37
10
                                             38
         real integral_of_rate[N];
11
         real integral_mu;
                                            39
12
         real log_likelihood;
                                            40
13
         seismic_rate[N]=log(mu);
                                            41
                                            42
14
         integral_of_rate[N]=0;
15
         for (i in 1:(N-1)) {
                                            43
16
           vector[N-i] v:
                                            44
                                            45
17
           int start:
18
           int end:
                                            46
           start =N*(i-1)-(i*(i-1))/2 + 1;47
19
20
           end = j*N-(j*(j+1))/2;
                                            48
21
           y=times_diff[start:end];
                                            49
22
           y=(k*(p-1) * c^(p-1))*exp(-p*
                 log (y+c));
                                            50
23
           seismic_rate[j]=log(mu+sum(y));51
```

```
integral_of_rate[j]=(k)*(1-c^(p
         ((times_diff[j]+c)^(p-1)));
    integral_mu=mu*tmax:
    log_likelihood=sum(seismic_rate)-
        integral_mu -sum (
             integral_of_rate);
    return (log_likelihood);
data{
  int < lower = 0 > N;
  real < lower=0> max_time;
  vector[N*(N-1)/2] times_diff;
parameters {
  real < lower=0> mu:
  real<lower=0> k:
  real < lower = 1.000005 > p;
  real < lower = 0.00005 > c;
model {
 mu~exponential(2.8);
k~exponential(2.8):
p~exponential(0.3);
c~exponential(2.8);
 increment_log_prob (loglikelihood (N, mu
      . k .
   p,c,times_diff,max_time));
```

# Listing 2: Temporal with magning tudes Rstan model with constant background seismicity

```
functions {
                                               34
 2
       real loglikelihood (int N,
                                               35
                            real mu.
                                               36
                            real k.
                                              37
 5
                            real p.
                             real c.
7
                            real alpha.
                                               39
8
                            real A.
                                               40
9
                            vector t,
                                               41
10
                            vector magnitudes12
11
                            vector times_diff44
12
                            real max_time.
                                              46
13
                            real magnitude0)47
14
         real seismic_rate[N]:
                                              48
15
         real integral_of_rate[N];
                                              49
16
         real integral_mu;
                                              50
17
         real log_likelihood;
                                              51
18
         seismic_rate[N]= log(mu);
                                              52
19
         integral_of_rate[N]=0;
                                               53
20
         for (j in 1:(N-1)) {
                                               54
21
            vector[N-j] y;
                                              55
22
            int start;
                                              56
23
            int end;
                                              57
            start = N*(j-1)-(j*(j-1))/2 +58
24
                 1:
                                              59
25
           end = i*N-(i*(i+1))/2;
                                              60
26
           v=times_diff[start:end];
                                              61
           v=(k*A*(p-1)*c^(p-1))*exp(alpha62
27
28
                 (magnitudes[(i+1):]-
                                              63
                       magnitude())
```

```
.* exp(-p*log(y+c));
      seismic_rate[j]=log(mu+sum(y));
     integral_of_rate[j]=(k*A)*exp(
           alpha*
          (magnitudes[j+1]-
          magnitude0))*(1-c^(p-1)/
          ((times_diff[i]+c)^(p-1)));
    integral_mu =mu*max_time;
    log_likelihood=sum(seismic_rate)-
      integral_mu -sum(integral_of_rate
    return (log_likelihood);
data{
  int < lower = 0 > N;
  vector[N] times:
  real < lower=0> max_time:
  vector[N] magnitudes;
  vector [N*(N-1)/2] times_diff;
  real < lower = 0 > threshold_magnitude:
parameters {
  real<lower=0> mu:
  real<lower=0> k:
  real < lower = 1.000005 > p;
  real < lower = 0.00005 > c:
  real<lower=0> alpha:
  real < lower=0 > A;
model {
 mu~exponential(2.8);
 k~exponential(2.8);
 p^{\sim} exponential (0.3):
 c~exponential(2.8);
  alpha exponential (2.8);
             4 D > 4 P > 4 B > 4 B >
```

```
64 A exponential (2.8);
65 increment_log_prob(loglikelihood(N,67 mu,k,p,c 68 }
66 ,alpha,A,times,magnitudes,times_diff
```

#### Isotropic spatio temporal ETAS model

In 1998 Ogata proposed a modified version of the spatio-temporal ETAS model:

$$\lambda(t,x,y) = \mu + \sum_{j:t_j < t} g(t-t_j,x-x_j,y-y_j,M_j|H_t)$$
 where

 $g(t,x,y,M) \frac{K e^{\alpha(M-M_0)}}{(t+c)^p} \{ \frac{x^2+y^2}{e^{\gamma(M-M_0)}} + d \}^{-q}$  and the background intensity  $\mu$  is constant. We can normalize:

$$g(t, x, y, M) = \frac{K(p-1)c^{p-1}(q-1)d^{q-1}\alpha e^{(\alpha-\gamma)(M-M_0)}}{\pi(t+c)^p} \left\{ \frac{x^2+y^2}{e^{\gamma(M-M_0)}} + d \right\}^{-q}$$

#### Anisotropic spatio temporal ETAS model

$$\lambda(t, x, y) = \mu + \sum_{j:t_i < t} g(t - t_j, x - x_j, y - y_j, M_j | H_t)$$

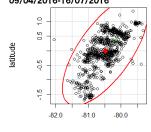
where 
$$g(t, x, y, M) = \frac{Ke^{\alpha(M-M_0)}}{(t+c)^p} \{ \frac{pS_jp^T}{e^{\alpha(M_j-M_0)}} + d \}^{-q}$$

and  $p=(x-x_j,y-y_j)$  is a row vector,  $x_j$ ,  $y_j$  are the coordinates of the earthquake j preceding the earthquake with epicenter x, y (both in the same cluster) and  $S_j$  (j=1,2,...) are positive definite symmetric matrices representing the normalized covariance matrix of the earthquake cluster obtained by applying MBC or Magnitude Based Cluster algorithm

#### Cluster of earthquakes Manabí-Esmeraldas



## Cluster earthquakes Manabí- Esmeraldas 09/04/2016-16/07/2016



longitude

#### Magnitude Based Cluster MBC

This method is based on selecting the greatest magnitude earth-quake(with magnitude  $M_j$ ) between those that are not in any cluster yet (if there are two with equal magnitude the oldest one is chosen) and then the earthquakes of the cluster associated with the previous selected earthquake, are those with latitude and longitude  $\pm 3.33 * 10^{0.5 M_j - 2}$  km (Utsu spatial distance) from the latitude and longitude of the selected earthquake and with a time difference therefrom (towards the future) of

 $\max(100,10^{0.5M_j-1})$  days Ogata1998. Then the process is repeated with earthquakes that are not yet in any cluster until all earthquakes belong to a cluster.

#### Bivariate normal distribution

For the anisotropic model, four normal bivariate models

Ogata1998: 
$$N\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} \tilde{\sigma}^2 & 0 \\ 0 & \tilde{\sigma}^2 \end{pmatrix})$$

$$N\begin{pmatrix} \left(\bar{x} \\ \bar{y}\right), \begin{pmatrix} \hat{\sigma}^2 & 0 \\ 0 & \hat{\sigma}^2 \end{pmatrix})$$

$$N\begin{pmatrix} \left(x_1 \\ y_1\right), \begin{pmatrix} \tilde{\sigma}_1^2 & \tilde{\rho}\tilde{\sigma}_1\tilde{\sigma}_2 \\ \tilde{\rho}\tilde{\sigma}_1\tilde{\sigma}_2 & \tilde{\sigma}_2^2 \end{pmatrix})$$

$$N\begin{pmatrix} \left(\bar{x} \\ \bar{y}\right), \begin{pmatrix} \hat{\sigma}_1^2 & \hat{\rho}\hat{\sigma}_1\hat{\sigma}_2 \\ \hat{\rho}\hat{\sigma}_1\hat{\sigma}_2 & \hat{\sigma}_2^2 \end{pmatrix})$$

$$N\begin{pmatrix} \left(x_1 \\ y_1\right), \begin{pmatrix} \tilde{\sigma}_1^2 & \tilde{\rho}\tilde{\sigma}_1\tilde{\sigma}_2 \\ \tilde{\rho}\tilde{\sigma}_1\tilde{\sigma}_2 & \tilde{\sigma}_2^2 \end{pmatrix})$$

$$N\begin{pmatrix} \left(\bar{x} \\ y_1\right), \begin{pmatrix} \hat{\sigma}_1^2 & \tilde{\rho}\hat{\sigma}_1\hat{\sigma}_2 \\ \tilde{\rho}\tilde{\sigma}_1\tilde{\sigma}_2 & \hat{\sigma}_2^2 \end{pmatrix})$$

$$N\begin{pmatrix} \left(\bar{x} \\ \bar{y}\right), \begin{pmatrix} \hat{\sigma}_1^2 & \hat{\rho}\hat{\sigma}_1\hat{\sigma}_2 \\ \hat{\rho}\hat{\sigma}_1\hat{\sigma}_2 & \hat{\sigma}_2^2 \end{pmatrix})$$

where  $(x_1, y_1)$  is the position of the cluster's main earthquake,  $(\bar{x}, \bar{y})$  and centroid of the cluster. (Ogata 1998)

#### Bivariate normal distribution

And 
$$\tilde{\sigma}^2 = [\sum_j (x_j - x_1)^2 + \sum_j (y_j - y_1)^2]/(2n)$$
 $\hat{\sigma}^2 = [\sum_j (x_j - \bar{x})^2 + \sum_j (y_j - \bar{y})^2]/(2n)$ 
 $\tilde{\sigma}_1^2 = [\sum_j (x_j - x_1)^2]/n \ \tilde{\sigma}_2^2 = [\sum_j (y_j - y_1)^2]/n$ 
 $\tilde{\rho} = [\sum_j (x_j - x_1)(y_j - y_1)]/(n\tilde{\sigma}_1\tilde{\sigma}_2)$ 
 $\tilde{\sigma}_1^2 = [\sum_j (x_j - x_1)^2]/n$ 
 $\tilde{\sigma}_2^2 = [\sum_j (y_j - y_1)^2]/n \ \tilde{\rho} = [\sum_j (x_j - x_1)(y_j - y_1)]/(n\tilde{\sigma}_1\tilde{\sigma}_2)$ 
 $\hat{\sigma}_1^2 = [\sum_j (x_j - \bar{x})^2]/n$ 
 $\hat{\sigma}_2^2 = [\sum_j (y_j - \bar{y})^2]/n$ 
 $\hat{\rho} = [\sum_j (x_j - \bar{x})(y_j - \bar{y})]/(n\hat{\sigma}_1\hat{\sigma}_2)$ 
(Ogata 1998)

#### Bivariate normal

We select the model with the lowest AIC = -nln(det(S)) + 2k where S is the variance covariance matrix for each of the four models and k is the corresponding number of parameters. Ogata1998.

Then the selected matrix is normalized:

$$\left(\frac{1}{\sqrt{(1-\rho^2)}}\right) \begin{pmatrix} \sigma_2/\sigma_1 & -\rho \\ -\rho & \sigma_1/\sigma_2 \end{pmatrix}$$

#### Hypocentral ETAS model

Guo2015 introduces a modification of the spatio temporal ETAS model that includes the depths of earthquakes

$$\lambda(t, x, y) = \mu + \sum_{j:t_j < t} g(t - t_j, x - x_j, y - y_j, M_j | H_t)$$

where 
$$g(t, x, y, M) = \frac{Ke^{\alpha(M-M_0)}}{(t+c)^p} \{ \frac{pS_j p^T}{e^{\alpha(M_j-M_0)}} + d \}^{-q} h(z-z_i, z_i)$$
 and

$$h(z,z') = \frac{(\frac{z}{Z})^{\eta \frac{z}{Z}} (1 - \frac{z}{Z})^{\eta(1 - \frac{z}{Z})}}{ZB(\eta \frac{z'}{Z} + 1, \eta(1 - \frac{z'}{Z}) + 1)}$$

with Z the thickness of the seismogenic layer and

$$B(p,q) = \int_0^1 t^{p-1} (1-t)^{q-1}$$
 is the Beta function.

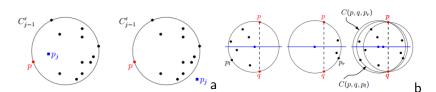
For spatio temporal models  $log L(\theta) = \sum_{j} log \lambda(t_j, x_j, y_j | H_t) - \int_0^{T_{max}} \int \int_{S} \lambda(t, x, y) dx dy dt$ 

For the hypocentral ETAS model, the logarithm of the likelihood is

$$logL(\theta) = \sum_{j} log \lambda(t_j, x_j, y_j | H_t) - \int_0^{T_{max}} \int \int_{S} \int_0^{Z} \lambda(t, x, y) dz dx dy dt$$

In spatio temporals models, the log likelihood does not have a closed form.

We approximate the log likelihood for spatio temporals models using polar coordinates and S the minimum covering circle Minimum Covering Circle Welzl 1991



## Methods(Cont)

#### Approximation

$$\begin{split} \log & \mathsf{L}(\mu, k, p, c, d, \alpha, \gamma) \\ &\approx \sum_{i=1}^{N} \log(\lambda(t_i)) - \mu T_{\mathsf{max}} \pi r^2 - \\ & k\alpha \sum_{i=1}^{N} \mathsf{e}^{\alpha(M_i - M_0)} (1 - \frac{\mathsf{c}^{p-1}}{(T_{\mathsf{max}} - t_i + c)^{p-1}}) (1 - \frac{\mathsf{d}^{q-1}}{(\frac{r_i^2}{\mathsf{e}^{\alpha(M_i - M_0)}} + d)^{q-1}}) \end{split}$$

where r is the radius of the minimum covering circle,  $r_i$  is the greatest distance between earthquake i and the previous earthquakes according to the metric defined in the cluster.

For the anisotropic model, the Euclidean distance is changed by the metric defined by the standardized variance covariance matrix  $S_j$  where now  $r_i$  is calculated as the maximum distance (according to the previous metric) between each earthquake and the previous earthquakes in the cluster.

#### Pre-processing

We sorted the data in reverse chronological time and calculated the time, latitude, and longitude differences for each earthquake j for which we know the initial and final positions where the differences of its time, latitude, and longitude with respect to the previous earthquakes are:

$$start = N * (j - 1) - (j * (j - 1))/2 + 1$$
  
 $end = j * N - (j * (j + 1))/2$ 

	Listing 3: Anisotropic spatio <sub>6</sub>	real seismic_rate[N];
		<pre>real integral_of_rate[N];</pre>
	temporal Rstan model with cons	<b>real</b> integral_mu;
	29	<b>real</b> log_likelihood;
	stant background seismicity 30	seismic_rate [N] <b>=log</b> (mu);
	31	integral_of_rate [N]=0;
1	functions { 32	for $(j in 1: (N-1))$ {
2	real loglikelihood(int N, 33	vector[N-j] y;
3	real mu, 34	vector[N-j] z;
4	real k, 35	vector $[N-1] \times$ ;
5	real p, 36	real temp;
6	real c, 37	real temp1;
7	real q, 38	real temp2;
8	real d, 39	int start;
9	real alpha, 40	int end;
.0	real gamma, 41	start = $N*(j-1)-(j*(j-1))/2 + 1$ ;
.1	vector t, 42	end = $j * N - (j * (j+1)) / 2;$
2	vector magnitudes43	y=times_diff[start:end];
_	, 44	z <b>=exp</b> (-q* <b>log</b> ( quadratic_factorAni
.3	vector times_diff	[ start : end ] . /
	, 45	(exp(gamma*(magnitudes[(j+1):]-
.4	vector latitudes,	magnitude0)))+d));
.5	vector longitudes46	y=( k * a l p h a * ( p - 1) * c ^ ( p - 1) * ( q - 1) * d
_	,	^(q-1)*(1/pi()))
.6	vector 47	*exp((alpha—gamma)*(magnitudes
	quadratic_facto	[(3, )]
_	, 48	magnitude0))
.7	vector 49	.*exp(-p*log(y+c));
	quadratic_f <b>s</b> @to	, , ,
_	, 51	seismic_rate[j] <b>=log</b> (mu+ <b>sum</b> (y));
.8	real tmax, 52	temp=exp(alpha*(magnitudes[j+1]-
9	real magnitude0,	magnitude0));
0	real lat_min, 53	temp1 <b>=exp</b> (gamma * ( m a g n i t u d es [ j
1	real lat_max,	+1]- magnitude $0));$
2	real long_min, 54	temp2 <b>=max</b> (quadratic_factorAni[
:3	real long_max,	start:end]);
4	real radius 55	integral_of_rate[j]=k*alpha*temp
25	){	(4日)(御)(き)(き) きょり

```
*(1-c^{(p-1)}/
                                             82
                                                  parameters {
            ((times_diff[j]+c)^(p-1))*(1-83)
56
                                                     real < lower=0> mu:
                  ^(a-1)/
                                             84
                                                     real<lower=0> k:
            ((temp2/(temp1)+d)^(q-1));
57
                                             85
                                                     real < lower = 1.000005 > p;
58
                                             86
                                                     real<lower=0> c:
59
         integral_mu=mu*tmax*pi()*radius^287
                                                     real<lower=0> d:
60
         log_likelihood=sum(seismic_rate)-88
                                                     real < lower = 1.00005 > q;
61
           integral_mu -sum(integral_of_rat&9
                                                     real < lower=0> alpha;
                                             90
                                                     real<lower=0> gamma:
         return (log_likelihood);
62
                                             91
63
                                             92
                                                  model{
64
                                             93
                                                    mu~exponential(2.8):
65
     data{
                                             94
                                                     k~exponential(2.8);
66
       int < lower=0> N:
                                             95
                                                    p~exponential(2.8);
67
       vector[N] times:
                                             96
                                                    c~exponential(2.8):
68
       real < lower=0> max_time:
                                             97
                                                    d~exponential(2.8):
69
       vector[N] magnitudes;
                                             98
                                                    q~exponential(2.8);
70
       vector[N*(N-1)/2]
                                             99
                                                    gamma exponential (2.8):
             quadratic_factorAni:
                                             100
                                                     alpha -gamma exponential (5);
71
       vector[N*(N-1)/2]
                                             101
                                                     increment_log_prob(loglikelihood(N,
             quadratic_factorIso;
                                                          mu, k, p, c, q, d,
72
       vector[N*(N-1)/2] times_diff;
                                             102
                                                     alpha, gamma, times, magnitudes,
73
       real < lower = 0 > threshold_magnitude;
                                                           times_diff,
74
       vector[N] latitudes;
                                             103
                                                     latitudes, longitudes,
75
       vector[N] longitudes:
                                                           quadratic_factorAni .
76
       real lat_min:
                                             104
                                                     quadratic_factorIso, max_time,
77
       real lat_max;
                                                           threshold_magnitude,
78
       real long_min:
                                             105
                                                     lat_min .lat_max .long_min .long_max .
79
       real long_max:
                                                           radius)):
80
       real<lower=0> radius:
                                             106
81
```

```
Listing 4:
                  Anisotropic
                                                                  real lat_min,
                                                                  real lat_max.
tio temporal hypocentral Rstans
                                                                  real long_min,
                                                                  real long_max,
model with constant backgroung
                                                                  real radius.
                                       28
                                                                  real layer_depth
seismicity
                                       29
                                                                  ) {
                                       30
functions {
                                       31
                                                real seismic_rate[N];
  real loglikelihood (int N,
                                       32
                                                real integral_of_rate[N];
                      real mu,
                                       33
                                                real integral_mu;
                      real k.
                                       34
                                                real log_likelihood:
                      real p.
                                       35
                                                seismic_rate[N]=log(mu);
                                       36
                                                integral_of_rate[N]=0;
                      real c.
                      real q.
                                       37
                                                for (j in 1: (N-1)) {
                      real d.
                                       38
                                                  vector[N-j] y;
                      real alpha,
                                       39
                                                  vector[N-i] z;
                      real gamma,
                                       40
                                                  vector[N-i] x:
                      real eta.
                                       41
                                                  vector[N-j] w;
                      vector t.
                                                  real temp;
                      vector magnitudes43
                                                  real temp1;
                                                  real temp2:
                      vector times_diff45
                                                  real temp3:
                                       46
                                                  int start:
                      vector
                                                  int end:
                            depths_diff 48
                                                  start =N*(j-1)-(j*(j-1))/2 + 1;
                      vector latitudes 49
                                                  end = i*N-(i*(i+1))/2;
                      vector longitudesso
                                                  y=times_diff[start:end];
                                       51
                                                  z=exp(-q*log(quadratic_factorAni
                      vector depths,
                                                         start:end
                      vector
                                       52
                                                     ./(exp(gamma*(magnitudes[(j+1)
                            quadratic_factorAni
                                                          :]-
                                       53
                                                     magnitude()))+d));
                                       54
                                                  y=(k*alpha*(p-1)*c^(p-1)*(q-1)*d
                      vector
                            quadratic_factorIso
                                                        ^(a-1)*
                                       55
                                                     (1/pi()))*exp((alpha—gamma)*
                      real tmax,
                                                     (magnitudes [(j+1):] - magnitude0
                      real magnitude0,
                                                         ⟨□⟩ ⟨□⟩ ⟨∃⟩ ⟨∃⟩
```

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```
82
    .* exp(-p*log(y+c));
                                    83
                                    84
                                         data{
  x=v .* z :
  z=(1/layer_depth) *
                                    85
                                           int < lower = 0 > N;
                                    86
     depths_diff[start:end]:
                                           vector[N] times:
  y=(1/layer_depth) *
                                    87
                                           real < lower=0> max_time:
     depths[(i+1):N];
                                    88
                                           vector[N] magnitudes;
  for(i in 1:(N-j)){
                                    89
                                           vector [N*(N-1)/2]
     w[i]=(1/(layer\_depth * exp(
                                                 quadratic_factorAni:
           Ibeta (eta *
                                    90
                                           vector[N*(N-1)/2]
      (y[i]) + 1,eta - eta * (
                                                 quadratic_factorIso;
                                           vector[N*(N-1)/2] times_diff;
            i])+ 1)))) *
                                    91
                                           vector[N*(N-1)/2] depths_diff;
            ^ (eta * (v[i]))) *
                                    92
                                    93
                                           real < lower = 0> threshold_magnitude;
            ((1 - z[i])
      (eta-eta * ( y[i])));
                                    94
                                           vector[N] latitudes:
                                    95
                                           vector[N] longitudes;
  seismic_rate[j]=log(mu+sum(x .* 96
                                           vector[N] depths;
       w)):
                                    97
                                           real lat_min:
                                           real lat_max;
  temp=exp(alpha*(magnitudes[j+1]-98
        magnitude());
                                    99
                                           real long_min;
  temp1=exp(gamma*(magnitudes[i
                                   100
                                           real long_max;
       +1] - magnitude 0));
                                   101
                                           real < lower = 0 > radius:
  temp2=max(quadratic_factorAni[102
                                           real < lower=0> layer_depth;
                                   103
        start:end]);
  integral_of_rate[j]=k*alpha*tem1694
                                         parameters {
        *(1-c^{(p-1)})
                                   105
                                           real<lower=0> mu:
   /((times_diff[j]+c)^(p-1))*(1106)
                                           real < lower=0> k;
        d^{(a-1)}
                                   107
                                           real < lower = 1.000005 > p:
   /((temp2/(temp1)+d)^(q-1));
                                   108
                                           real<lower=0> c:
                                   109
                                           real<lower=0> d:
                                   110
integral_mu=mu*tmax*pi()*
                                           real < lower=1.00005 > q;
     layer_depth*
                                   111
                                           real<lower=0> alpha:
                                   112
                                           real < lower=0> gamma;
    (radius^2);
log_likelihood=sum(seismic_rate)-113
                                           real < lower=0> eta;
  integral_mu -sum(integral_of_ratld4
                                   115
                                         model{
return (log_likelihood);
                                   116
                                           mu~exponential(2);
                                                       4日 > 4周 > 4 至 > 4 至 >
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117 118	k~exponential(2); p~exponential(2);	126		alpha,gamma,eta,times,magnitudes
119	c~exponential(2);	127		depths_diff, latitudes, longitudes
120	d~exponential(2);	128		depths , quadratic_factorAni ,
121	q~exponential(2);	129		quadratic_factorlso , max_time ,
122	eta~exponential(2);	130		threshold_magnitude,lat_min,
123	gamma~exponential(2);			lat_max , long_min ,
124	alpha—gamma~exponential(5);	131		long_max , radius , layer_depth ) ) ;
125	increment_log_prob (loglikeliho	od (N 132	}	
	mu, k, p, c, q, d,			

Listing 5: Anisoti	ropic Rsta <del>n</del>	real seismic_rate[N];
model with variable	hackground	<pre>real integral_of_rate[N];</pre>
model with variable		real integral_mu;
coicmicity	30	real log_likelihood;
seismicity	31	seismic_rate [N] <b>=log</b> (mu);
	32	integral_of_rate [N]=0;
functions {	33	for(j in 1:(N-1)){
real loglikelihood(int N		vector[N-j] y;
real mu,	35	vector[N-j] z;
real k,	36	vector $[N-1] \times$ ;
real p,	37	real temp;
real c,	38	real temp1;
real q,	39	real temp2;
real d,	40	int start;
real alph		int end;
<b>real</b> gamr		start = $N*(j-1)-(j*(j-1))/2 + 1$ ;
vector t	, 43	end = $j*N-(j*(j+1))/2$ ;
	agnitudes, 44	y=times_diff[start:end];
	imes_diff, 45	z=
	atitudes, 46	exp(−q∗log ( quadratic_factorAni
vector lo	ongitudes ,	[ start : end ] . /
vector	47	(exp(gamma*(magnitudes[(j+1)
bac	kground_seismic_rates	:] — magnitude0 ) ) )+d ) ;
1	48	y=(k*alpha*(p-1)*c^(p-1)*(q-1)*d
vector		^(q-1)*
qua	ıdratic_factorA49ai	(1/pi()))*exp((alpha—gamma)*(
,		magnitudes $[(j+1):]$
vector	50	-magnitude0)).* <b>exp</b> (-p* <b>log</b> (y+c)
q u a	ndratic_factorIso	);
,	51	y=y .∗ z;
real tmax	×, 52	seismic_rate[j]=
real mag		log (mu∗
real lat_		background_seismic_rates[
<b>real</b> lat_		j +1]+ <b>sum</b> ( y ) ) ;
<b>real</b> long	g_min, 54	temp= $exp(alpha*(magnitudes[j+1]-$
real long		magnitude());
<b>real</b> radi	ius 55	temp1=exp(gamma*(magnitudes[j
){		(日本) (日本) (日本) (日本) (日本) (日本) (日本) (日本)

```
+1]-magnitude0));
                                              83
                                                     real long_max;
           temp2=max(quadratic_factorAni[
56
                                              84
                                                     real < lower = 0 > radius:
                                              85
                 start:endl):
57
           integral_of_rate[i]=k*alpha*tem86
                                                   parameters {
                                              87
                                                     real < lower=0> mu;
                 *(1-c^{(p-1)})
                                                     real < lower=0> k;
58
            ((times_diff[j]+c)^(p-1))*
                                              88
59
            (1-d^{(q-1)}/((temp2/(temp1)+d)
                                              89
                                                     real < lower = 1.000005 > p;
                  q-1)));
                                              90
                                                     real < lower=0 > c;
60
                                              91
                                                     real<lower=0> d:
61
         integral_mu=mu*sum(
                                              92
                                                     real < lower = 1.00005 > q;
               background_seismic_rates)*
                                              93
                                                     real < lower=0> alpha;
62
           tmax*pi()*radius^2:
                                              94
                                                     real<lower=0> gamma:
63
         log_likelihood=sum(seismic_rate)-95
64
           integral_mu -sum(integral_of_rate6
                                                   model {
                                              97
                                                     mu~exponential(2.8):
65
         return (log_likelihood);
                                              98
                                                     k~exponential(2.8):
66
                                              99
                                                     p~exponential(2.8);
67
                                             100
                                                     c~exponential(2.8):
68
     data{
                                             101
                                                    d~exponential(2.8);
69
       int < lower = 0 > N;
                                             102
                                                     q~exponential(2.8);
70
       vector[N] times;
                                             103
                                                     gamma~exponential(2.8);
71
       real < lower=0> max_time:
                                             104
                                                     alpha - gamma ~ exponential(5):
72
       vector[N] magnitudes;
                                             105
                                                     increment_log_prob(loglikelihood(N,
73
       vector [N*(N-1)/2]
                                                           mu, k, p, c, q, d,
             quadratic_factorAni:
                                             106
                                                       alpha .gamma .times .magnitudes .
       vector[N*(N-1)/2]
74
                                                             times_diff.
             quadratic_factorIso;
                                             107
                                                        latitudes, longitudes,
75
       vector[N*(N-1)/2] times_diff:
                                                             background_seismic_rates .
76
       real < lower = 0 > threshold_magnitude; 108
                                                       quadratic_factorAni .
77
       vector[N] latitudes:
                                                             quadratic_factorIso,
78
       vector[N] longitudes;
                                             109
                                                       max_time, threshold_magnitude,
79
       vector[N] background_seismic_rates 110
                                                       lat_min , lat_max , long_min , long_max ,
80
       real lat_min:
                                                             radius));
81
       real lat max:
                                             111
82
       real long_min:
```

#### Methods

We can estimate the probability that a given event is spontaneous or is triggered by others

Kagan1980Zhuang2002. The contribution of the spontaneous seismicity rate to the occurrence of an event i can be taken as the probability that the event i is spontaneousZhuang2008:

$$\phi(i) = \frac{\mu(x_i, y_i)}{\lambda(t_i, x_i, y_i)}$$

Similarly, the probability that the event j is produced by the event i is  $\rho_{ij} = \frac{\kappa(M)g(t_j - t_i)f(x_j - x_i, y_j - y_i, m_i)}{\lambda(t_i, x_i, y_i)}$ 

We can also obtain the expected number of direct aftershocks from the earthquake i as  $\sum_{j} \rho_{ij}$  Zhuang2008

#### Methods

#### Inter-event times

The probability density function of the recurrence times (time between two successive events) au

$$H(\tau) \approx \lambda f(\lambda \tau)$$

where the function f(x) has been found practically the same in different regions and  $\lambda$  is the average rate of events observed in the analyzed region. Saichev2007

The scaling factor of times between earthquakes is taken as the inverse of its mean.

The form of the function f(x) which is demonstrated in Saichev2007 is

$$f(x) = (n\epsilon^{\theta}x^{-1-\theta} + [1 - n + n\epsilon^{\theta}x^{-\theta}]^2) *$$
  
 $\varphi(x,\epsilon)$ 

Closest March April May June July Total province Azuay 1 Bolivar

Chinchipe Total

10

Table 1: Earthquake distribution in Ecuador 18/03/2016-16/07/2016

	4				4
1	4				5
1	1				2
1	1				2
2	3	4	2		11
	171	47	16	35	269
	3				3
	3	5			8
	1 1 1 2	1 4 1 1 1 1 2 3 1771 3	1 4 1 1 1 1 2 3 4 4 171 47 3	1 4 1 1 1 1 1 2 3 4 2 171 47 16 3 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 4

Galapagos		3				3
Guayas		3	5			8
Imbabura	1	1				2
Loja	1	3	1	1		6
Los Rios		1		1		2
Manabi	1	399	92	27	22	541
Los Rios		1		1		2
Morona		6	7	2	1	16
Santiago		0	<b>'</b>		1	10
Napo		1	1			2

Guayas		J 0	0			0
Imbabura	1	1				2
Loja	1	3	1	1		6
Los Rios		1		1		2
Manabi	1	399	92	27	22	541
Los Rios		1		1		2
Morona		6	7	2	1	16
Santiago		0	'	4	1	10
Napo		1	1			2
Pastaza		4	1	1		6
Pichincha	2	5				7
Santa Elena		3	3	1		7
Sto Domingo de		4				4
los Tsachilas		4				4
Tungurahua		5	1			6
Zamora		2	2			4
Chinchine		4	4			4

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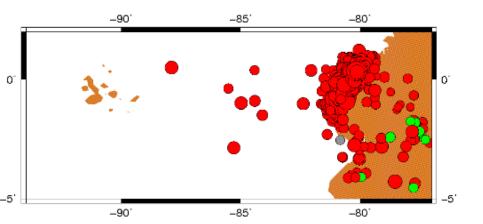
58

908

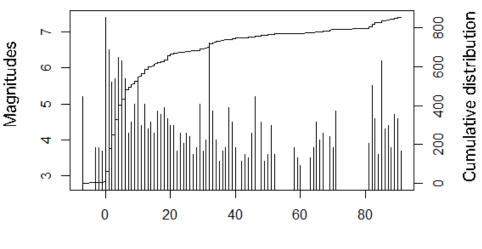
165

Los Rios		1		1		2
Manabi	1	399	92	27	22	541
Los Rios		1		1		2
Morona Santiago		6	7	2	1	16
Napo		1	1			2
Pastaza		4	1	1		6
Pichincha	2	5				7
C / T31		0	0	-1		-

624

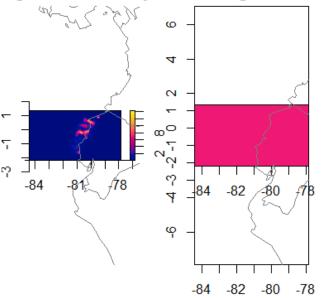


## Earthquake magnitudes 09/04/2016-16/07/2016

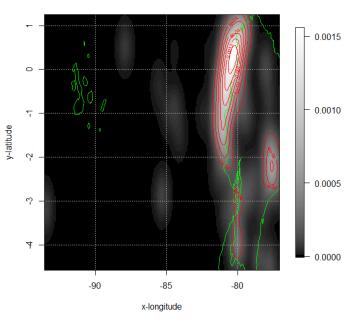


Time(days)

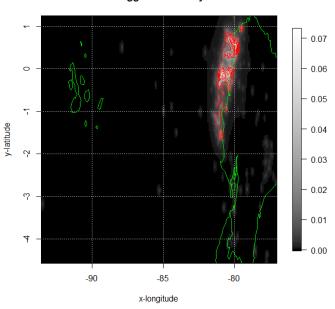
### kground seismicity raclustering coefficient



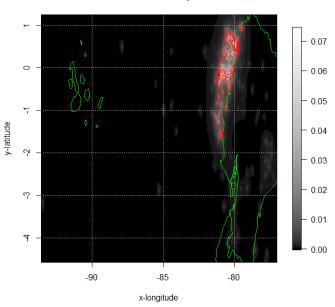
#### **Background Intensity**



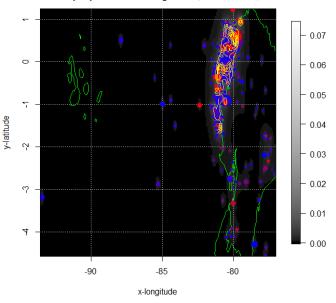
#### **Triggered Intensity**



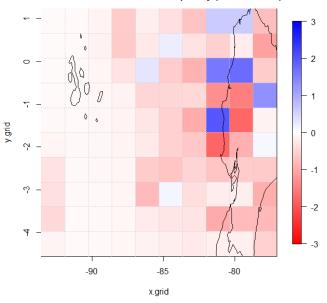
#### **Total Intensity**



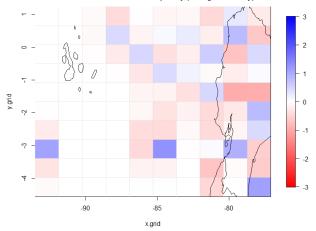
Total Intensity with observed points
Circles area proportional to magnitude; red: recent, blu:older



## Standardized differences between theoretical and observed frequency (whole model)



## Standardized differences between theoretical and observed frequency (background only)



#### Anisotropic Distribution

The covariance matrix for the cluster associated with the April 16th big earthquake (804 earthquakes in the cluster) is

$$\begin{pmatrix} 1.469 & -0.696 \\ -0.696 & 0.709 \end{pmatrix}$$

and corresponds to the fourth bivariate model fitted.

Table 12: Etas models comparison ln(L) AIC Int. 95% ln(L) # param.

1801.98

1977.0 9

1926.1 9 int.95% AIC

-3566.5 -3531.4

-3366.5

-3582.7

-2758.7

-3689.4

-3573.3

-3920.8

-3821.4

-3577.6

-3542.0-3673.7

-3593.8

-2780.1

-3703.2

-3936.8

-3936.1

-3834.2

-3583.8

-3930.2

-3828.6

remp. sis. cst	1790.7	1787.3	1792.8	4	-3573.5	
Temp. seis. var.	1773.3	1769.7	1775.0	4	-3538.6	
Magn. seis. cst	1832.8	1689.2	1842.8	6	-3653.7	Г
Magn. seis.	1800.8	1797.4	1802.9	6	-3589.5	Г

Temp. seis. var.	1773.3	1769.7	1775.0	4	
Magn seis est	1832.8	1689.2	1842.8	6	Г

var.	21100	210011	211010	-	
Magn. seis. cst	1832.8	1689.2	1842.8	6	ŀ
Magn. seis.	1800.8	1797.4	1802.9	6	٦.

Magn. seis. cst	1832.8	1689.2	1842.8	6	Г
Magn. seis. var.	1800.8	1797.4	1802.9	6	Г

Magn. seis. cst	1832.8	1689.2	1842.8	6	I
Magn. seis. var.	1800.8	1797.4	1802.9	6	

Etas

Model

ani. seis. var. Нуро.

ani. seis. cst. Нуро.

ani. seis. var.

Magn. seis. est	1002.0	1009.2	1042.0	0	L
Magn. seis. var.	1800.8	1797.4	1802.9	6	
Sna temp					Γ

						_
	Magn. seis. var.	1800.8	1797.4	1802.9	6	
	Spa. temp.	10044	100= 1	1000 1		Г

Magn. seis. var.	1800.8	1797.4	1802.9	6	
C					

1969.4

1919.4

iso. seis. cst	1394.4	1387.4	1398.1	8	
Spa. temp.				_	

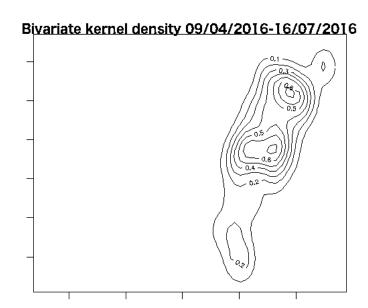
1799.9

1974.1

1923.3.1

#### Kernel bivariate density

Kernel bivariate density using the R package kde.



#### **Priors**

### Weakly informative priors

For the ETAS parameters(must be positive) we used exponential priors and for p and q we use a minimum value close to 1:1.000005.

The radius of the minimum covering circle using Welzl algorithm was, r = 1.70479.

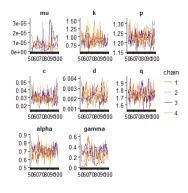


Figure: Chains for anisotropic ETAS model with constant background seismicity

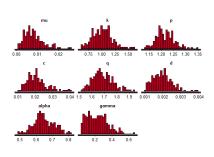


Figure: A posteriori parameter distributions for anisotropic ETAS model with constant background seismicity

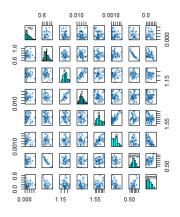


Figure: Correlation of parameter values in the chains for anisotropic ETAS model with constant background seismicity

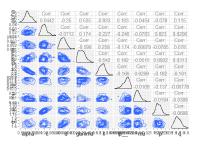


Figure: Correlation of parameter values in the chains for anisotropic ETAS model with constant background seismicity

Residuals

# Temporal ETAS(constant background seismicity)

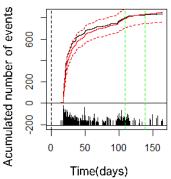


Figure: Acumulated number of events for temporal Etas model with constant background seismicity

#### Residuals temporal ETAS( constant background seismicity)

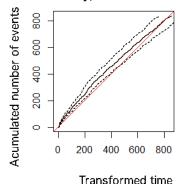


Figure: Residuals for Etas temporal temporal Etas model with constant background seismicity

# Temporal ETAS(variable background seismicity)

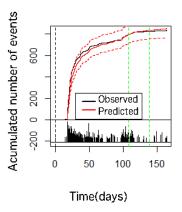


Figure: Accumulated number of events for temporal ETAS model with variable background seismicity

# Residuals temporal ETAS (variable background seismicity)

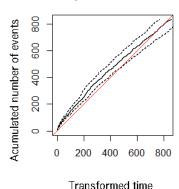


Figure: Residuals for temporal ETAS model with variable background seismicity

## Temporal ETAS: constant vs variable background seismicity)

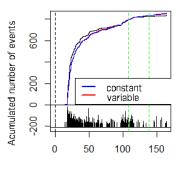
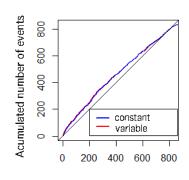


Figure: Accumulated number of events for temporal ETAS model with constant background seismicity vs variable background seismicity

Time(days)

## Residuals temporal ETAS: constant vs variable background seismicity



Transformed time

Figure: Residuals for temporal ETAS model with constant background seismicity vs variable background seismicity

# Temporal ETAS with magnitudes and constant background seismicity

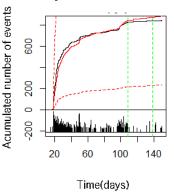


Figure: Accumulated number of events for temporal ETAS with magnitudes model and constant background seismicity

# Temporal ETAS with magnitudes and constant background seismicity

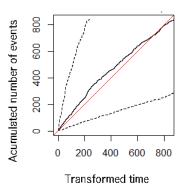
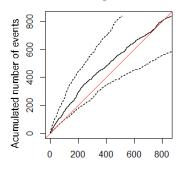


Figure: Residuals for temporal ETAS with magnitudes model and constant background seismicity

### Anisotropic ETAS with variable background seismicity 800 Acumulated number of events 900 400 200 0 200 120 Time(days)

Figure: Accumulated number of events for spatio temporal anisotropic ETAS model and variable background seismicity

#### Residuals anisotropic ETAS with variable background seismicity



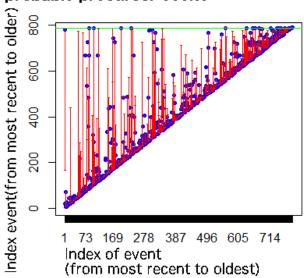
Transformed time

Figure: Residuals for spatio temporal anisotropic ETAS model and variable background seismicity

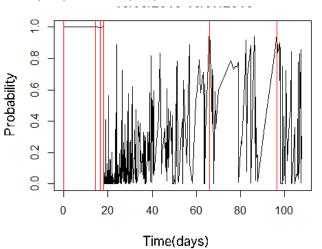
#### Parent earthquakes

95% credibility intervals for the index of the most probable parent earthquake with 1000 draws from the posterior distributions. The green line is the big earthquake on April 16 and blue points are the medians of the credibility intervals using the anisotropic spatio temporal model with variable background seismicity.

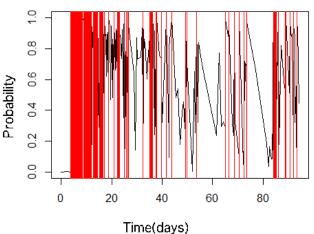
# Confidence intervals for indexes of most probable precursor event



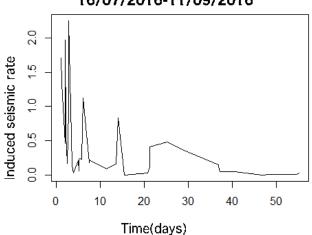
# Probability of being background earthquake 16/07/2016-11/09/2017



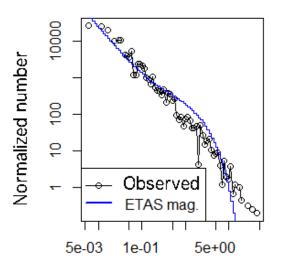
# Probability of being spontaneous earthquake 13/04/2016-16/07/2016



# Induced seismic rate 16/07/2016-11/09/2016

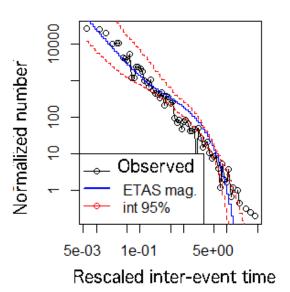


### Inter-event times 18/03/2016-16/07/2016



Rescaled inter-event time

### Inter-event times 18/03/2016-16/07/2016



### **Thanks**