The optical absorption length in the ice is typically 110 m at 400 nm with a strong wavelength dependence. The effective scattering length at 400 nm is on average $\simeq 20$ m. It is defined as $\lambda_s/(1 - \langle \cos\theta_s \rangle)$, where λ_s is the scattering length and θ_s is the scattering angle. The ice parameters vary strongly with depth due to horizontal ice layers, i.e., variations in the concentration of impurities which reflect past geological events and climate changes [13–19].

3. Reconstruction algorithms

The muon track reconstruction algorithm is a maximum likelihood procedure. Prior to reconstruction simple pattern recognition algorithms, discussed in Section 4, generate the initial estimates required by the maximum likelihood reconstructions.

3.1. Likelihood description

The reconstruction of an event can be generalized to the problem of estimating a set of unknown parameters $\{\mathbf{a}\}$, e.g. track parameters, given a set of experimentally measured values $\{\mathbf{x}\}$. The parameters, $\{\mathbf{a}\}$, are determined by maximizing the likelihood $\mathscr{L}(\mathbf{x}\,|\,\mathbf{a})$ which for independent components x_i of $\mathbf{x}$ reduces to

$$\mathscr{L}(\mathbf{x}|\mathbf{a}) = \prod_i p(x_i|\mathbf{a}) \tag{2}$$

where $p(x_i|\mathbf{a})$ is the probability density function (p.d.f.) of observing the measured value x_i for given values of the parameters $\{\mathbf{a}\}$ [20].

To simplify the discussion we assume that the Cherenkov radiation is generated by a single infinitely long muon track (with $\beta = 1$) and forms a cone. It is described by the following parameters:

$$\mathbf{a} = (\mathbf{r_0}, t_0, \hat{\mathbf{p}}, E_0) \tag{3}$$

and illustrated in Fig. 3. Here, $\mathbf{r_0}$ is an arbitrary point on the track. At time t_0, the muon passes $\mathbf{r_0}$ with energy E_0 along a direction $\hat{\mathbf{p}}$. The geometrical coordinates contain five degrees of freedom. Along this track, Cherenkov photons are emitted at a fixed angle θ_c relative to $\hat{\mathbf{p}}$. Within the reconstruction algorithm it is possible to use a different coordinate system, e.g. $\mathbf{a} = (d, \eta, \dots)$. The reconstruction is performed by minimizing $-\log(\mathscr{L})$ with respect to $\mathbf{a}$.

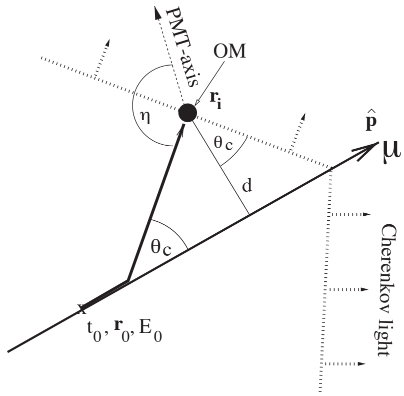

Fig. 3. Cherenkov light front: definition of variables.

The values $\{\mathbf{x}\}$ presently recorded by AMANDA are the time t_i and duration TOT_i (*Time Over Threshold*) of each PMT signal, as well as the peak amplitude A_i of the largest pulse in each PMT. PMTs with no signal above threshold are also accounted for in the likelihood function. The hit times give the most relevant information. Therefore we will first concentrate on $p(t|\mathbf{a})$.

3.1.1. Time likelihood

According to the geometry in Fig. 3, photons are expected to arrive at OM i (at $\mathbf{r_i}$) at time

$$t_{\text{geo}} = t_0 + \frac{\hat{\mathbf{p}} \cdot (\mathbf{r_i} - \mathbf{r_0}) + d \tan\theta_c}{c_{\text{vac}}} \tag{4}$$

with c_{vac} the vacuum speed of light.[3] It is convenient to define a relative arrival time, or *time residual*

$$t_{\text{res}} \equiv t_{\text{hit}} - t_{\text{geo}} \tag{5}$$

which is the difference between the observed hit time and the hit time expected for a "direct photon", a Cherenkov photon that travels undelayed directly from the muon to an OM without scattering.

[3] We note that Eq. (4) neglects the effect that Cherenkov light propagates with group velocity as pointed out in Ref. [21]. It was shown in Ref. [14] that for AMANDA this approximation is justified.