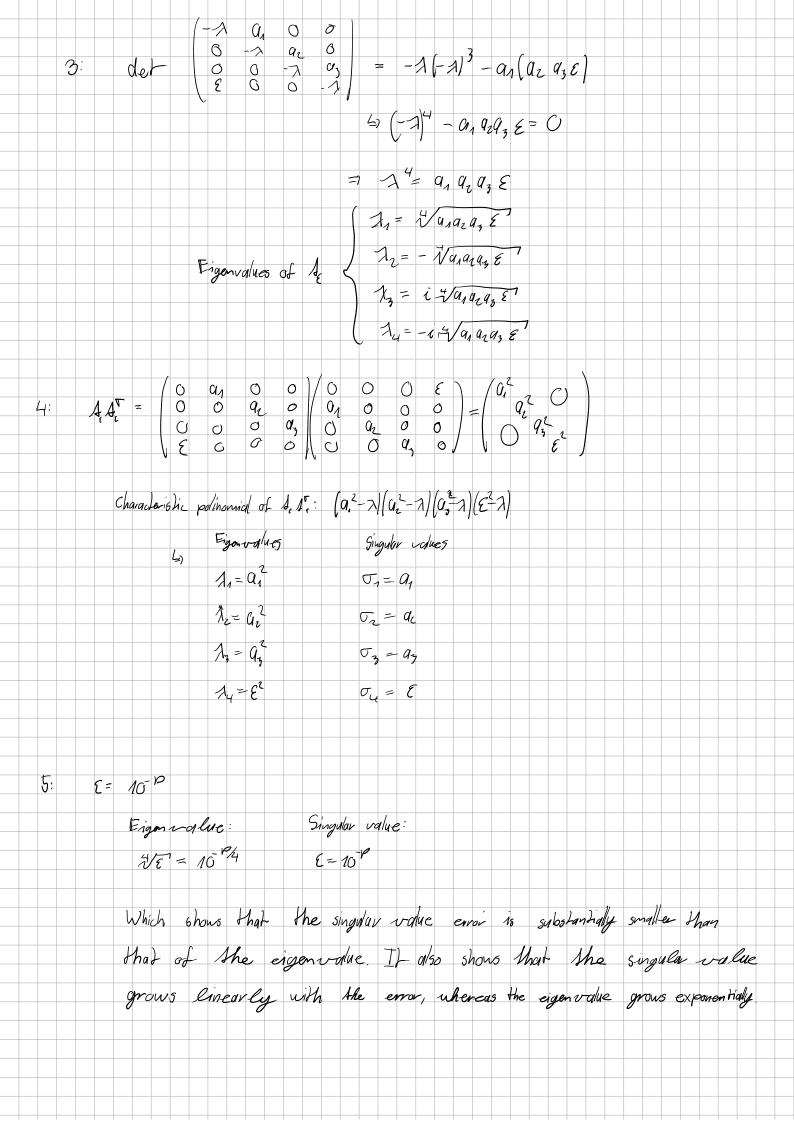


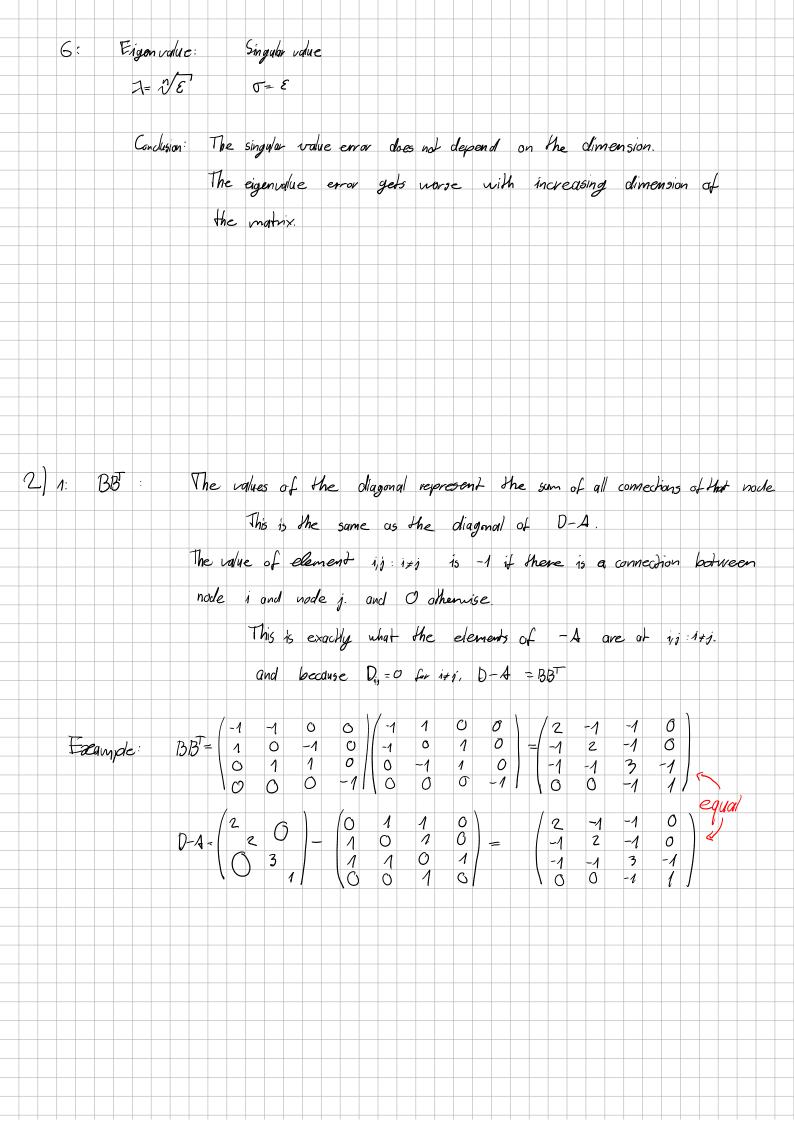
Figure vectors of AAT (AAT = A,T)e; = 0

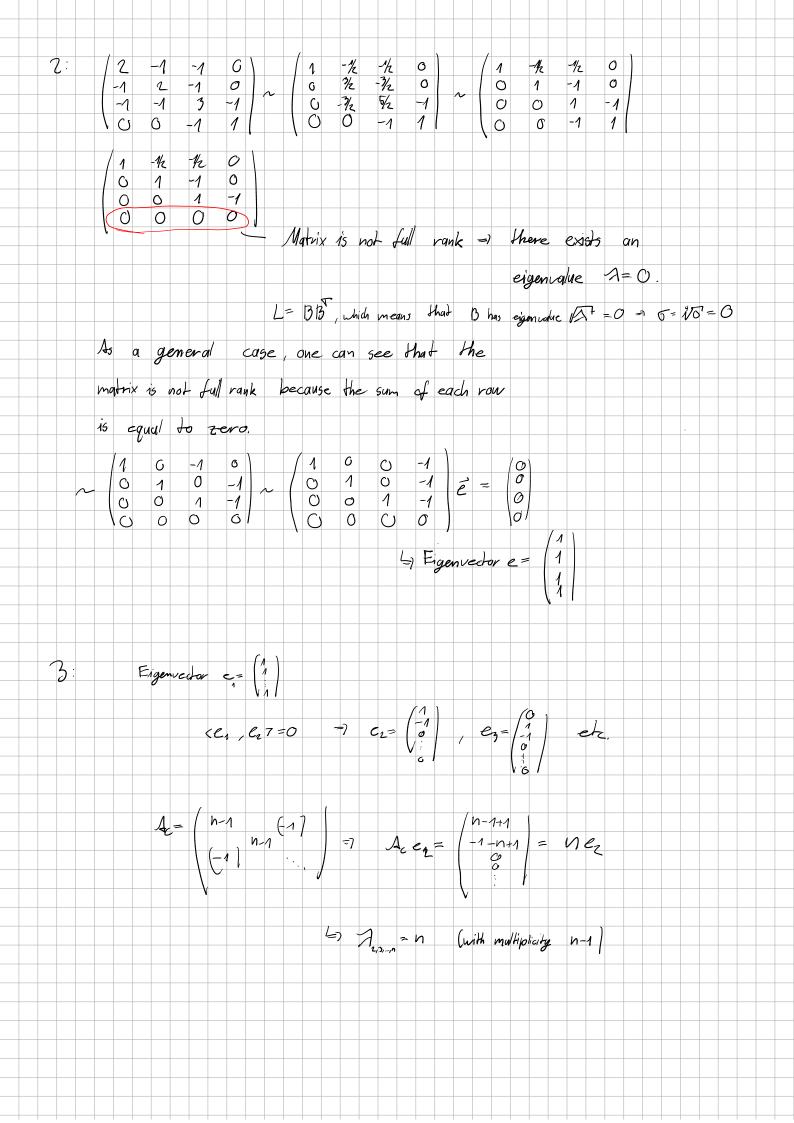
$$\cos A \left(\begin{array}{c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \left(\begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} \right) = 0$$

$$\cos A \left(\begin{array}{c} 0 & 0 \\ 0 & 0 \end{array} \right) \left(\begin{array}{c} c_2 \\ c_3 \\ c_4 \end{array} \right) \left(\begin{array}{c} c_3 \\ c_4 \\ c_5 \\ c_6 \end{array} \right) = 0$$

$$\frac{1}{2} \cos \frac{1}{2} \sin \frac{1$$







 $x^{T} \angle x = x^{T} B B^{T} x = (B^{T} x)^{T} B^{T} x$ Because BT has an equal number of +1 and -1 per row, at positions i,j per connection of the node k, (i,j E Ez) (Bx) Bx = < Bx, Bx > where the kth element of Bx = xi-xi, where i, i are the kth connection of = { {u, v}, {u, w}, ...} $=7 < 65 \times , 67 \times 7 = \sum_{i,j \in E} (x_i - x_j)^2 = x^T Z \times$ Connection 2 5: Let $x \in \mathbb{R}^n$ and $\|x\|_{custom}$ be defined as $= |x_i|$ (the custom norm is the sum of the absolute value of each etement of x) Because the 1-4 element of BTIs, is equal to the number of connections coming in, minus the connections going to nodes in S. This corresponds to the number of councerion of the node to the outside of S, and by summing over all the values in 131, we get the total number of connections to the outside of s. because all elements of BT 1s are O or 11 we can define cut(5) = 1 BT 1/2 which is equal to < B 1, B 1, > = (B 1) B 1, = 1, B B 1, = 15 L 1 = cw (5) 6: cut(5)=0 implies that the subset S is complety isolated from the remaining graph. If S + V, then the graph is not connected 7: $x^T \angle x = x^T / 3 B^T x = (B^T x)^T B^T x = \|B^T x\|_2^2 \ge 0$

(Assuming we're discussing L) Each connected component has its own block matrix B, where 2=0 has multiplicity greater or equal to 1. There being k connected components det (L) = T det (Bi) which impoles that the multiplicity of 2=0 must be at least k It cannot have a multiplicity of 2=0 grader than 1. u be the normalized eigenvector s.t. Lu=0, $u=\frac{1}{n}\left(\frac{1}{n}\right)$. For A=0 to have multiplicity greater than 1, there must exist another eigenvector x such that Lx=0 and x + k u We know that for u $u^{T} L u = \underbrace{\left\{ \left(u_{i} - u_{j} \right)^{2} = 0 \right\}}_{i,j \in \mathbb{Z}}$ By extension, if Lx-0, then $x^{T} \angle x - Z \left(\chi_{i} - \chi_{j} \right)^{2} = 0$ But since x + 6 u, there exists at least one relation x # X6 $\Rightarrow (x_k - x_k)^2 \neq 0$. This mease that $(A_k) \notin E$ Knowing that the grouph is connected, any connection k, i EE = { EB} which means that any nocles i connected to k will satisfy xi-xk The same can be shown for all x; connected to xi. So for any i which has a path to k has the same value xi=xx however, xx xx which means xx is not connected to the graph of k. I we know that the graph is connected => there oxists only and exactly 1 eigenvertor, => 1=0 has a multiplicity of exactly 1.