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State augmentation method for buffeting analysis of structures subjected to non-stationary wind

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ABSTRACT: Extreme winds such as hurricanes and thunderstorms often present non-stationary characteristics, having time-varying mean wind speeds and non-stationary wind fluctuations. When concerning the wind-induced vibrations under non-stationary winds, the excitation will be a non-stationary process, and the wind-structure coupled system can be represented by a linear time-varying (LTV) system. The aim of this study is to present a state augmentation method to investigate the non-stationary buffeting of a model bridge tower subjected to a non-stationary wind with consideration of the aeroelastic damping. Based on the theory of stochastic differential equations and Itô's lemma, the statistical moments of the non-stationary buffeting response are derived through solving a first-order ordinary differential equation system. The proposed method is validated by comparisons with Monte Carlo simulations. The result shows that the state augmentation method has higher accuracy and efficiency than the well-known Monte Carlo method.

Keywords: non-stationary winds; aerodynamic damping; buffeting response; Itô's lemma.

1. INTRODUCTION

In contrast with the stationary synoptic winds, extreme wind events such as hurricanes and thunderstorms always exhibit considerable non-stationary characteristics (G. Huang et al., 2015), having time-varying mean wind speeds and non-stationary wind fluctuations. The rapid changes in the kinematics and dynamics of these flow fields can potentially amplify aerodynamic loads on structures and result in higher non-stationary buffeting responses. When considering aeroelastic effects, the aerodynamic damping will be time-dependent due to the time-varying mean wind speed, and the wind-structure coupled system can be thus represented as a linear time-varying (LTV) system (L. Hu et al., 2013). These facts lead to difficulties in the calculation of the structural response by using the conventional buffeting analysis method.

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In view of these non-stationary effects, many attempts have been made to develop random vibration theory for non-stationary buffeting, including the Monte Carlo method, generalized frequency-domain method, and pseudo excitation method. However, some methods may need intensive calculations due to time-integration process, and some may be difficult to consider time-dependent system properties.

Based on the theory of Itô's stochastic differential equation, Grigoriu (M. Grigoriu and S. Ariaratnam, 1988) proposed the state augmentation method to calculate the stochastic response of linear systems subjected to stationary excitations. With this method, the statistical moments of any order of the response can be directly obtained by solving a system of linear differential equations with high efficiency. Although this method has been applied in several wind engineering problems, such as non-Gaussian turbulence (W. Cui et al., 2022), it has not been reported for non-stationary buffeting analysis. The aim of this paper is to extend the state augmentation method to investigate the non-stationary along-wind buffeting of a bridge tower subjected to non-stationary wind loads.

2. METHODS

2.1. Non-stationary wind model

The velocity of a non-stationary extreme wind $U(t)$ can be usually characterized as the summation of a deterministic time-varying mean $\overline{U}(t)$ and uniformly modulated (G. Huang et al., 2015) non-stationary wind fluctuations $\beta(t)u_s(t)$

$$U(t) = \overline{U}(t) + \beta(t)u_s(t) \tag{1}$$

in which $\beta(t)$ is the deterministic time-modulation function, and $u_s(t)$ is a stationary Gaussian process with zero mean.

2.2. Steady-state equation of wind-induced vibrations

As an example of application of the present method, the bridge tower during construction stage is considered, as shown in Fig. 1. Only the vibration in the along-wind direction is considered for the sake of simplicity.

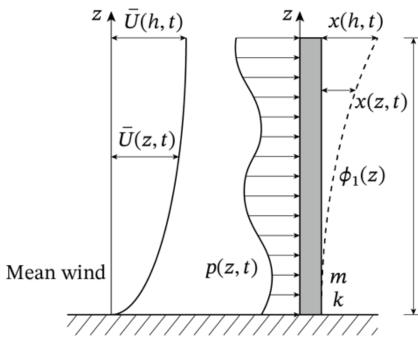


Figure 1. Schematic of along-wind buffeting of bridge tower

To formulate the non-stationary buffeting forces, the strip and quasi-steady theories are invoked. The aeroelastic term is included by considering the relative velocity of the structural velocity and the total wind speed. With the application of the modal-superposition method, and the Modal Correlation Length (MCL) (L. Caracoglia, 2014) to consider the spatial coherence of buffeting forces, the dynamic equation of the along-wind buffeting of the bridge tower can be written in its state-space form

$$\mathrm{d}\begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & -2\xi_1\omega_1 - M_1^{-1}\rho C_D D_y \gamma_1 \overline{U}_h \end{bmatrix}\begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix}\mathrm{d}t + \begin{bmatrix} 0 \\ M_1^{-1}\rho C_D D_y \overline{U}_h h \Lambda_{1u}\beta \end{bmatrix} u_{0.6h}^s \mathrm{d}t \tag{2}$$

in which q_1 is the generalized coordinate of the buffeting response; ω_1 is the first modal circular frequency, and ξ_1 is the first modal damping ratio, M_1 is the corresponding generalized mass; ρ is the air density, D_y is the tower width orthogonal to the wind direction, and C_D is the drag coefficient. γ_1 is a constant depending on wind profile and modal shape. $\bar{U}_h$ is the mean wind speed referenced at the top of the tower, $z = h$, and Λ_{1u} is the along-wind MCL.

2.3. Moments equations of the response

The stationary Gaussian process $u_{0.6h}^s(t)$ can be approximated by an Ornstein-Uhlenbeck (OU) process, i.e., $Z(t) \approx u_{0.6h}^s(t)$, which satisfies the stochastic differential equation

$$\mathrm{d}Z(t) = -\alpha Z(t)\mathrm{d}t + \sigma\sqrt{2\alpha}\mathrm{d}W(t) \tag{3}$$

in which $1/\alpha$ is the time relaxing coefficient, σ is the standard deviation of $Z(t)$, and $W(t)$ is a standard Wiener process. The parameters α and σ can be found through fitting the single-side power spectral density function $S_{ZZ}(\omega) = 4\alpha\sigma^2/(\alpha^2 + \omega^2)$ with the one of $u_{0.6h}^s(t)$. By substituting Eq. (3) into Eq. (2), the states of the system and the excitation can be written as an Itô-type stochastic differential equation of the form (S. Karlin and H.E. Taylor, 1981)

$$\mathrm{d}\mathbf{Y}(\mathrm{t}) = \mathbf{g}(\mathbf{Y}(\mathrm{t}), t)\mathrm{d}t + \mathbf{h}(\mathbf{Y}(\mathrm{t}), t)\mathrm{d}W(\mathrm{t}) \tag{4}$$

in which $\mathbf{Y}(\mathrm{t}) = [q_1 \quad \dot{q}_1 \quad Z]^{\mathrm{T}}$ is the augmented state vector, $\mathbf{h}(\mathbf{Y}(\mathrm{t}), t) = [0 \quad 0 \quad \sigma\sqrt{2\alpha}]^{\mathrm{T}}$, and

$$\mathbf{g}(\mathbf{Y}(\mathrm{t}), t) = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_1^2 & -2\xi_1\omega_1 - M_1^{-1}\rho C_D D_y \gamma_1 \bar{U}_h & M_1^{-1}\rho C_D D_y \bar{U}_h h \Lambda_{1u}\beta \\ 0 & 0 & -\alpha \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \\ Z \end{bmatrix} \tag{5}$$

Assume $\xi(\mathbf{Y})$ to be the moments function of $\mathbf{Y}$, i.e., $\xi(\mathbf{Y}) = q_1^a \dot{q}_1{}^b Z^f$, in which the superscripts a, b and f are the non-negative integer power indices. According to Itô's lemma (K. Itô, 1944)

$$\frac{\mathrm{dE}[\xi]}{\mathrm{d}t} = \mathrm{E}\left[\frac{\partial \xi}{\partial t}\right] + \sum_i^3 \mathrm{E}\left[\frac{\partial \xi}{\partial Y_i} g_i\right] + \frac{1}{2}\sum_{i,j}^3 \mathrm{E}\left[h_i h_j \frac{\partial^2 \xi}{\partial Y_i \partial Y_j}\right] \tag{6}$$

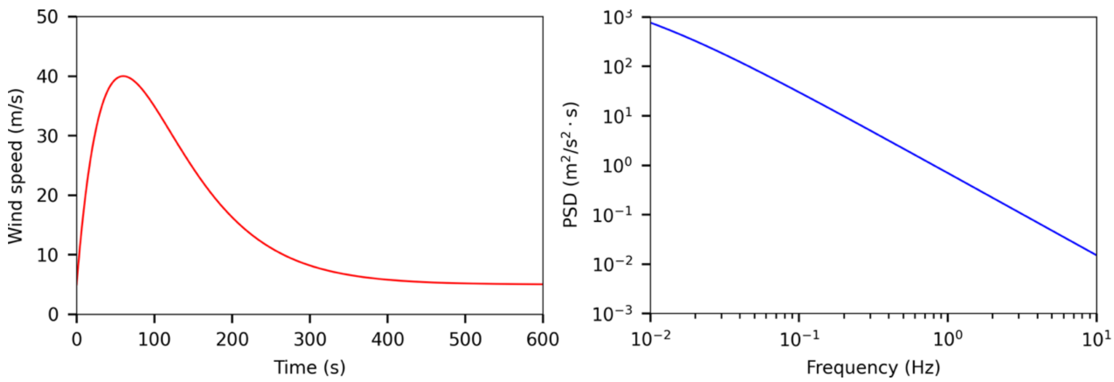
in which g_i is the i-th element of the vector $\mathbf{g}(\mathbf{Y}(\mathrm{t}), t)$, and h_j is the j-th element of the vector $\mathbf{h}(\mathbf{Y}(\mathrm{t}), t)$. $\mathrm{E}[\cdot]$ indicates the expectation operator. By substituting $\xi = q_1^a \dot{q}_1{}^b Z^f$ and then expanding Eq. (6), the moments equation of the non-stationary response is derived as a system of first-order ordinary differential equations

$$\dot{\mathbf{m}} = \mathbf{Pm} + \mathbf{Q} \tag{7}$$

in which $\mathbf{m}$ is the vector of the response variances; $\mathbf{P}$ and $\mathbf{Q}$ are the deterministic time-depending coefficients matrixes corresponding to the structure and wind field properties.

3. VALIDATION

To illustrate the reliability of the state augmentation method, the proposed method is applied to calculate the buffeting response of a bridge tower subject to a non-stationary wind field consisting of a time-varying mean and a stationary wind fluctuation. Fig. 2 shows the time-varying mean wind speed and the Simiu's spectrum for the stationary wind fluctuations.



(a) (b)

Figure 2. Adopted non-stationary wind speed model: (a) time-varying mean wind speed; (b) PSD of the stationary fluctuating wind component based on Simiu's spectrum.

The obtained results are validated by comparing them with those obtained using Monte Carlo simulations. The stationary wind fluctuation samples for the Monte Carlo simulation are generated from the Simiu's spectrum given by Fig. 2 (b). A fourth-order Runge-Kutta method with an error estimator of fifth order is used to calculate the time history responses, with integration step $\Delta t = 0.01$ s, and the statistical characteristics of the responses at each instant of time are estimated over 1000 random samples. Fig. 3 shows the RMS at each instant of time for the along-wind displacement at the top of the bridge tower, given by the proposed state augmentation (SA) method and by the Monte Carlo (MC) simulation. These results are calculated from 0 s to 600 s.

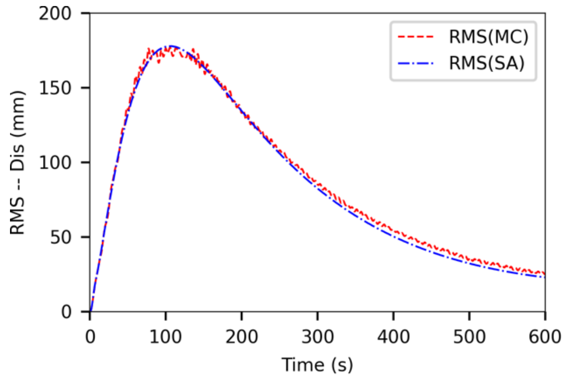


Figure 3. Time-varying RMS of the displacement response at the top of the bridge tower

As shown in Fig. 3, the results calculated by the proposed state augmentation method and by using the Mente Carlo method are in very good agreement, which shows the reliability of the proposed method for determining the non-stationary buffeting response. Moreover, the computational efficiency of the proposed method is much higher than that of the Monte Carlo simulations. The computation time cost by the proposed method is around 0.87 s, whereas the time taken by the Monte Carlo simulations with 1000 samples is nearly 2.6 h.

4. CONCLUSIONS

This study investigated the non-stationary buffeting of a bridge tower subjected to non-stationary wind loads. The strip and quasi-steady assumptions are adopted to formulate the buffeting forces and taking the motion-induced force into account. Based on the stochastic differential equation theory and Itô's lemma, a state augmentation method has been presented to calculate the statistical moments of the non-stationary buffeting response. The proposed state augmentation method is validated by comparisons with Monte Carlo simulations. In the Monte Carlo method, intensive simulations are needed due to the non-stationary characteristics of the response, whereas the proposed method is far more efficient.

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