August 27-31, 2023
FLORENCE - ITALY
ICWE 16
16th International Conference on Wind Engineering

State augmentation method for non-stationary vibrations of long-span bridges with multi-degree of freedom

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SUMMARY:
Extreme winds such as hurricanes and thunderstorms often present non-stationary characteristics, having time-varying mean wind speeds and non-stationary wind fluctuations. When concerning wind-induced vibrations under non-stationary wind, the excitation will be a non-stationary process, and the wind-structure coupled system can be represented by a linear time-varying system. The study aims to present a state augmentation method to investigate the non-stationary buffeting of a long-span bridge subjected to non-stationary wind. The unsteady self-excited forces and the non-stationary turbulence-induced forces, i.e., buffeting forces, are included both in this study to predict the flutter and buffeting response. Based on Itô's Formula, the statistical moments of the response are derived through solving a first-order ordinary differential equation system. This study is regarded as subsequent research by the author in order to generalize the previous one in a single degree of freedom case to a multiple degree of freedom case and take the unsteady aerodynamic effects into consideration.

Keywords: non-stationary wind, buffeting response, Itô's Formula

1. INTRODUCTION

In contrast with the stationary synoptic wind, extreme wind events such as hurricanes and thunderstorms always exhibit considerable non-stationary characteristics. When considering aeroelastic effects, the aerodynamic damping and stiffness will be time-dependent due to the time-varying mean wind speed, and the wind-structure coupled system will be a linear time-varying (LTV) system (L. Hu et al., 2013). In view of these non-stationary effects, many attempts have been made to develop random vibration theory for non-stationary buffeting, including the Monte Carlo method, generalized frequency-domain method, and pseudo excitation method. However, some methods may need intensive calculations due to time-integration process, and the others may be difficult to consider time-dependent system properties.

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This is the accepted manuscript version of the peer-reviewed extended abstract for conference.

Based on the theory of Itô's Stochastic Differential Equation, Grigoriu (M. Grigoriu and S. Ariaratnam, 1988) proposed the state augmentation method to calculate the stochastic response of linear systems subjected to stationary excitations. With this method, the statistical moments of any order of the response can be directly obtained by solving a system of linear differential equations with high efficiency. Although this method has been applied in several wind engineering problems, such as non-Gaussian turbulence (W. Cui et al., 2022), it has been rarely reported for non-stationary buffeting analysis. This paper aims to extend the state augmentation method to investigate the non-stationary buffeting of a long-span bridge subjected to non-stationary wind loads. This study is regarded as subsequent research by the author (Lei et al., 2022) in order to generalize the previous one in a single degree of freedom case to a multiple degree of freedom case and take the unsteady aerodynamic effects into consideration.

2. NON-STATIONARY WIND-INDUCED VIBRATIONS OF BRIDGES

2.1. Dynamics of Bridge Deck Motions

The motions $\mathbf{X}_d$ of the full-order bridge deck subjected to self-excited forces $\mathbf{F}^{se}$ and non-stationary buffeting forces $\mathbf{F}^{b}$ are determined by solving the dynamic equation

$$\mathbf{M}_d\ddot{\mathbf{X}}_d + \mathbf{C}_d\dot{\mathbf{X}}_d + \mathbf{K}_d\mathbf{X}_d = \mathbf{F}^{se} + \mathbf{F}^{b} \tag{1}$$

in which $\mathbf{M}_d$, $\mathbf{C}_d$ and $\mathbf{K}_d$ are the mass, damping and stiffness matrices, respectively.

2.2. Unsteady Aerodynamic Forces in Time domain

The flutter derivatives are approximated in terms of rational functions known as Roger's approximation. In this case, the unsteady self-excited forces in the time domain are obtained as

$$\mathbf{F}^{se} = \frac{1}{2}\rho U^2 \Delta L \left(\mathbf{A}_1^{se}\mathbf{X}_d + \frac{B}{U}\mathbf{A}_2^{se}\dot{\mathbf{X}}_d + \frac{B^2}{U^2}\mathbf{A}_3^{se}\ddot{\mathbf{X}}_d + \sum_{l=1}^{m_{se}} \boldsymbol{\phi}_l^{se}\right) \tag{2a}$$

$$\dot{\boldsymbol{\phi}}_l^{se} = -\frac{d_l^{se}U}{B}\boldsymbol{\phi}_l^{se} + \mathbf{A}_{l+3}^{se}\dot{\mathbf{X}}_d \tag{2b}$$

where $\mathbf{A}_1^{se}$, $\mathbf{A}_2^{se}$, $\mathbf{A}_3^{se}$, $\mathbf{A}_{l+3}^{se}$ and $d_l^{se} \geq 0$ are frequency-independent coefficients. $\boldsymbol{\phi}_l^{se}$ are additional variables introduced to consider the unsteady effect. Analogous to the self-excited forces, the non-stationary buffeting forces in the time domain can be expressed as

$$\mathbf{F}^{b} = \beta^{u}(t)\mathbf{F}^{bu} + \beta^{w}(t)\mathbf{F}^{bw} \tag{3a}$$

$$\mathbf{F}^{bu} = \frac{1}{2}\rho U \Delta L \sum_{l=1}^{m_u+1} \mathbf{A}_l^{u}\,\mathbf{u}^{s} - \frac{1}{2}\rho U \Delta L \sum_{l=1}^{m_u} \frac{d_l^{u}U}{B}\boldsymbol{\phi}_l^{u} \tag{3b}$$

$$\dot{\boldsymbol{\phi}}_l^{u} = -\frac{d_l^{u}U}{B}\boldsymbol{\phi}_l^{u} + \mathbf{A}_{l+1}^{u}\mathbf{u}^{s} \tag{3c}$$

in which $\mathbf{A}_1^{u}$, $\mathbf{A}_{l+1}^{u}$ and $d_l^{u} \geq 0$ are frequency-independent coefficients. $\boldsymbol{\phi}_l^{u}$ are additional variables. $\beta_u(t)$ and $\beta_w(t)$ are the slowly varying time-modulation functions. $\mathbf{u}^{s}$ are the derived stationary Gaussian processes with zero means. Similar formulas for buffeting forces induced by vertical wind fluctuations are omitted here for the sake of brevity.

2.3. Augmented States of the System and Excitation

With the help of the modal-superposition method, the generalized dynamic equation is derived as

$$\mathbf{M}_q\ddot{\mathbf{q}} + \mathbf{C}_q\dot{\mathbf{q}} + \mathbf{K}_q\mathbf{q} = \mathbf{Q}^{se} + \beta^u\mathbf{Q}^{bu} + \beta^w\mathbf{Q}^{bw} \tag{4}$$

in which $\mathbf{M}_q$, $\mathbf{C}_q$ and $\mathbf{K}_q$ are the generalized modal mass, damping and stiffness, respectively. $\mathbf{q}$ is the modal coordinate vector. $\mathbf{Q}^{se} = \mathbf{\Psi}^{\mathrm{T}}\mathbf{F}^{se}$ is the generalized self-excited force vector. $\mathbf{Q}^{bu} = \mathbf{\Psi}^{\mathrm{T}}\mathbf{F}^{bu}$ and $\mathbf{Q}^{bw} = \mathbf{\Psi}^{\mathrm{T}}\mathbf{F}^{bw}$ are the generalized buffeting force vectors. $\mathbf{\Psi}$ is the matrix of mode shapes. The generalized dynamic equation is transformed to its state-space form

$$\dot{\mathbf{Y}} = \boldsymbol{\mathcal{A}}(t)\mathbf{Y} + \boldsymbol{\mathcal{B}}_u(t)\mathbf{u}^{s,q} + \boldsymbol{\mathcal{B}}_w(t)\mathbf{w}^{s,q} \tag{5}$$

in which $\mathbf{Y}$ is the state vector. $\boldsymbol{\mathcal{A}}$, $\boldsymbol{\mathcal{B}}_u$ and $\boldsymbol{\mathcal{B}}_w$ are matrices of deterministic time-varying coefficients. $\mathbf{u}^{s,q}$ and $\mathbf{w}^{s,q}$ are the generalized stationary Gaussian pseudo-fluctuations which can be approximated individually by multivariate Ornstein-Uhlenbeck (OU) processes, i.e., $\mathbf{u}^{s,q} \approx \mathbf{Z}^u(t)$ and $\mathbf{w}^{s,q} \approx \mathbf{Z}^w(t)$, which satisfy the stochastic differential equation

$$\mathrm{d}\mathbf{Z}^u(t) = -\boldsymbol{\alpha}^u\mathbf{Z}^u(t)\mathrm{d}t + \boldsymbol{\Theta}^u\mathrm{d}\mathbf{W}^u(t) \tag{6a}$$
$$\mathrm{d}\mathbf{Z}^w(t) = -\boldsymbol{\alpha}^w\mathbf{Z}^w(t)\mathrm{d}t + \boldsymbol{\Theta}^w\mathrm{d}\mathbf{W}^w(t) \tag{6b}$$

in which $\boldsymbol{\alpha}^u$, $\boldsymbol{\Theta}^u$, $\boldsymbol{\alpha}^w$ and $\boldsymbol{\Theta}^w$ are the corresponding coefficient matrices, and $\mathbf{W}^u$ and $\mathbf{W}^w$ are the corresponding Wiener processes which are mutually independent. By substituting Eq. (6) into Eq. (5), the augmented states of the system and the excitations are written as

$$\mathrm{d}\begin{bmatrix}\mathbf{Y}\\ \mathbf{Z}^u\\ \mathbf{Z}^w\end{bmatrix} = \begin{bmatrix}\boldsymbol{\mathcal{A}} & \boldsymbol{\mathcal{B}}_u & \boldsymbol{\mathcal{B}}_w\\ \mathbf{0} & -\boldsymbol{\alpha}^u & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & -\boldsymbol{\alpha}^w\end{bmatrix}\begin{bmatrix}\mathbf{Y}\\ \mathbf{Z}^u\\ \mathbf{Z}^w\end{bmatrix}\mathrm{d}t + \begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \boldsymbol{\Theta}^u & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\Theta}^w\end{bmatrix}\mathrm{d}\begin{bmatrix}\mathbf{W}^u\\ \mathbf{W}^w\end{bmatrix} \tag{7}$$

Eq. (7) is usually recognized as an Itô-type stochastic differential equation of the form

$$\mathrm{d}\mathbb{Y}(t) = \mathbf{g}(\mathbb{Y}(t), t)\mathrm{d}t + \mathbf{h}(\mathbb{Y}(t), t)\mathrm{d}\mathbf{W} \tag{8}$$

in which $\mathbb{Y}$ is the augmented state vector. $\mathbf{g}$ and $\mathbf{h}$ are explicit functions of $\mathbb{Y}$ and time t.

2.4. Differential Equations Satisfied by Statistical Moments

Assume $\xi(\mathbb{Y})$ as a scalar-valued function of $\mathbb{Y}$, i.e., $\xi(\mathbb{Y}) = \prod_i^N \mathbb{Y}_i^{a_i}$ where the superscript a_i is the non-negative integer exponent. According to Itô's lemma

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{E}[\xi] = \mathrm{E}\left[\frac{\partial \xi}{\partial t}\right] + \sum_i^N \mathrm{E}\left[\frac{\partial \xi}{\partial Y_i}g_i\right] + \frac{1}{2}\sum_i^N\sum_j^N\left[(\mathbf{h}\mathbf{h}^{\mathrm{T}})_{ij}\frac{\partial^2\xi}{\partial Y_j \partial Y_i}\right] \tag{9}$$

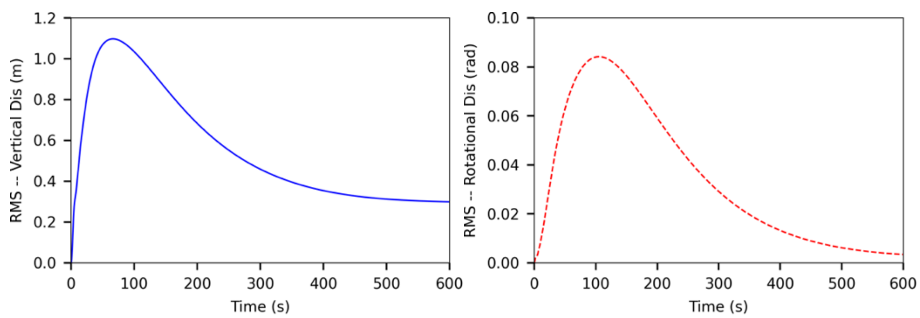
For a prescribed order of $s = \sum_{i=1}^N a_i$, substituting all the combinations of a_i into Eq. (9) gives

$$\dot{\mathbf{m}}_s = \mathbf{P}_1(t)\mathbf{m}_s + \mathbf{Q}_1(t) + \mathbf{P}_2\mathbf{m}_{s-2} + \mathbf{Q}_2 \tag{10}$$

in which $\mathbf{m}_s = \mathrm{E}\left[\prod_i^N \mathbb{Y}_i^{a_i}\right]$ is the vector of all the statistical moments in the order of s. $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are the coefficient matrices corresponding to the system properties. The above formulas are the basis of the state augmentation method.

3. NUMERICAL APPLICATIONS

As an example of application of the SAM, the proposed method is applied to calculate the non-stationary vibrations of the bridge deck of a long-span bridge (Shenzhen–Zhongshan Bridge in China) subjected to non-stationary wind. The first symmetric torsional mode together with three prior symmetric vertical modes is considered in this study, with the modal frequencies $n_1 = 0.1049\ \text{Hz}$, $n_2 = 0.1366\text{Hz}$, $n_3 = 0.2256\ \text{Hz}$ and $n_4 = 0.2987\ \text{Hz}$ (torsional frequency), respectively. Simiu and Panofsky type spectra are adopted to represent the along-wind and vertical wind turbulence, with $u_* = 1.6\ \text{m/s}$. The time-varying wind speed model is given by $\bar{U}(t) = (\bar{U}_{\max} - \bar{U}_{\min})(t/t_0)\exp(1 - t/t_0) + \bar{U}_{\min}$ where $\bar{U}_{\max} = 40\ \text{m/s}$, $\bar{U}_{\min} = 5\ \text{m/s}$ and $t_0 = 60\ \text{s}$. Fig. 1 shows the RMS responses for the vertical and rotational displacements at the mid-span. The results are calculated from 0 s to 600 s.



(a) (b)

Figure 1. Time-varying RMS response at the mid-span: (a) vertical displacement; (b) rotational displacement.

4. CONCLUSIONS

This study investigated the non-stationary vibrations of the bridge deck of a long-span bridge subjected to non-stationary unsteady wind loads. Based on Itô's Formula, a state augmentation method is presented to calculate the statistical moments of the non-stationary buffeting response. It is seen that the statistical moments can be directly obtained by solving the first-order ordinary differential equation without going through an intermediate step, e.g., the response spectra.

ACKNOWLEDGEMENTS

The study was supported by the National Natural Science Foundation of China (grant No. 52278530, 52008314 and 51978527) and the China Scholarship Council (No.202106260170).

REFERENCES

K. Itô (1944). 109. Stochastic integral, Proceedings of the Imperial Academy, 20, 519-524.

Lei, S., Cui, W., Patruno, L., de Miranda, S., Zhao, L., and Ge, Y., 2022. Improved state augmentation method for buffeting analysis of structures subjected to non-stationary wind. Probabilistic Engineering Mechanics, 69, 103309.

L. Hu, Y.-L. Xu, W.-F. Huang (2013). Typhoon-induced non-stationary buffeting response of long-span bridges in complex terrain. Engineering Structures, 57, 406-415.

M. Grigoriu, S. Ariaratnam (1988). Response of linear systems to polynomials of gaussian processes.

W. Cui, L. Zhao, Y. Ge (2022). Non-gaussian turbulence induced buffeting responses of long-span bridges based on state augmentation method, Engineering Structures, 254, 113774.