Formulari Càlcul 2

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Corbes

•Long. i param. de l'arc

$Ex.: r(t) = (\cos t, \sin t, t)$

$Ex.: r'(t) = (-\sin t, cost, 1)$

$Longitud = s(t) = \int_a^t ||r'(t)||dt$

$t(s), u(s) = r(t(s)) \to ||u'(s)|| = 1$

•Triede de Frenet

$r(t) \to r'(t) \to r''(t) \to r'''(t)$

$\vec{T} = \frac{r'(t)}{||r'(t)||}$

$\vec{N} = \frac{(r'(t)\times r''(t))\times r'(t)}{||(r'(t)\times r''(t))\times r'(t)||}$

$\vec{B} = \frac{r'(t)\times r''(t)}{||r'(t)\times r''(t)||}$

•Curvatura i Torsio

$\kappa(t) = \frac{||r'(t)\times r''(t)||}{||r'(t)||^3}$

$\tau(t) = \frac{r'(t)\cdot(r''(t)\times r'''(t))}{||r'(t)\times r''(t)||^2}$

•Pla que conte una corba

$r(t), t \to P$

$(x, y, z) = P + \lambda\vec{T}(t) + \mu\vec{N}(t)$

•Recta normal

$r(t), t \to P$

$(x, y, z) = P + \lambda\vec{N}(t)$

n-variables

•Linealització, pla tangent

$$\nabla f(x, y) = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{pmatrix}$$

$$z = f(\alpha) + \nabla f(\alpha) \begin{pmatrix} x - \alpha_x \\ y - \alpha_y \end{pmatrix}$$

Ex.: $f(x, y) = x^2y - x, \alpha = (1, 2)$

$$\nabla f(x, y) = \begin{pmatrix} 2xy - 1 & x^2 \end{pmatrix}$$

$$z = 1 + \begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} x - 1 \\ y - 2 \end{pmatrix}$$

$z = 1 + 3(x - 1) + (y - 2)$

$z = 3x + y - 4 \leftarrow lineal!$

Es equacio del pla tangent a la grafica de f al punt (1,2)

•Derivades Direccionals

$\frac{\partial f}{\partial v}(a) = \nabla f(a) \cdot v$

•Regla de la cadena

Ex.:

$f(x, y) = (xy, x^2, x - y)$

$g(x, y, z) = (xyz, \sin y)$

$g \circ f(x, y) = g(f(x, y))$

$g(f(x, y)) = (xyx^2(x - y), \sin x^2)$

n-variables

•Punts critics

Ex.:

$f(x, y) = x^2 - xy + y^2 + 3x - 2y + 1$

$\frac{\partial f}{\partial x}(x, y) = 2x - y + 3 = 0$

$\frac{\partial f}{\partial y}(x, y) = -x + 2y - 2 = 0$

$$Hf(a) = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2}(a) & \frac{\partial^2 f}{\partial y \partial x}(a) \\ \frac{\partial^2 f}{\partial x \partial y}(a) & \frac{\partial^2 f}{\partial y^2}(a) \end{pmatrix}$$

$det(Hf(a)) > 0, \frac{\partial^2 f}{\partial x^2}(a) > 0 \to$ $f(a)$ min. relatiu

$det(Hf(a)) > 0, \frac{\partial^2 f}{\partial x^2}(a) < 0 \to$ $f(a)$ max. relatiu

$det(Hf(a)) < 0 \to f(a)$ punt sella

$det(Hf(a)) = 0 \to$ cal estudi

Derivades

•Basic

$y = ku \to y' = ku', k \in \mathbb{R}$

$y = u \pm v \to y' = u' \pm v'$

$y = u \cdot v \to y' = u'v + uv'$

$y = \frac{u}{v} \to y' = \frac{u'v - uv'}{v^2}$

$[u(v)]' \to u'(v)v'$

•Constant

$y = k \to y' = 0$

•Identitat

$y = x \to y' = 1$

•Potencials

$y = u^n \to y' = nu^{n-1}u'$

$y = \sqrt{u} \to y' = \frac{u'}{2\sqrt{u}}$

$y = \sqrt[n]{u} \to y' = \frac{u'}{n\sqrt[n]{u^{n-1}}}$

•Exponencials

$y = e^u \to y' = u'e^u$

$y = a^u \to y' = u'a^u \ln a$

•Logaritmiques

$y = \ln u \to y' = \frac{u'}{u}$

$y = \log_a u \to y' = \frac{u'}{u}\frac{1}{\ln a}$

•Trigonometriques

$y = \sin u \to y' = u' \cos u$

$y = \cos u \to y' = -u' \sin u$

$y = \tan u \to y' = \frac{u'}{\cos^2 u} = u'(1 + \tan^2 u)$