Pattern Recognition Letters 19 (1998) 315–318

A note on the structured light of three-dimensional imaging systems

Yi-Chih Hsieh *

Department of Industrial Engineering, National Huwei Institute of Technology, Huwei, Yunlin 63208, Taiwan

Received 10 July 1997

Abstract

Griffin et al. (1992) introduced a new structured light pattern which can acquire the range data of an object with the use of single camera for three-dimensional imaging systems. This note points out that the sequences used to construct the structured light pattern are indeed special cases of de Bruijn sequences. Additionally, this note also proposes a new simple approach to generate various de Bruijn sequences. As shown, this new approach has several advantages over other typical approaches. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: De Bruijn sequence; Three-dimensional imaging; Light pattern

1. Introduction

Three-dimensional (3-D) range data are spatial coordinates for surface points of an object and they are useful for 3-D object matching, object recognition, and dimensional measurement (Yee and Griffin, 1994). Griffin et al. (1992) introduced a special encoded light pattern which may be used for a structured lighting system to acquire 3-D range data with the use of a single camera. Since only a single camera is required, unlike the approaches of Altschuler et al. (1981) and Posdamer and Altschuler (1981), their approach can be used in a dynamic environment (i.e., a scene with moving objects) for 3-D imaging systems. We refer the interested readers to Griffin et al. (1992) and Yee and Griffin (1994) for details of the use of the encoded light pattern.

In this note we concern the generation of encoded light pattern for a structured lighting system used by Griffin et al. (1992) in 3-D imaging systems. The aim of this note is twofold. Firstly, we point out that the sequences used by Griffin et al. to generate the encoded light pattern are indeed special cases of de Bruijn sequences. Secondly, we present a simple algorithm for the generation of various de Bruijn sequences. As shown, the new algorithm has several computational advantages over other typical approaches. Note that, except for the generation of encoded light pattern, there are several practical applications of de Bruijn sequences, e.g., telecommunications problems (Chartrand and Oellermann, 1993) and reaction-time experiment problems (Emerson and Tobias, 1995; Sohn et al., 1996), etc.

* E-mail: yhsieh@sparc.nhit.edu.tw.

PII S0167-8655(97)00159-1

2. De Bruijn sequences

Given m symbols which, without loss of generality, it is assumed that $1,2,\ldots,m-1,m$, with the nature order $1<\cdots<m-1<m$. An m-symbol n-tuple de Bruijn sequence ((m,n) de Bruijn sequence) is a string of m^n symbols, $s_0 s_1 \ldots s_{m^n-1}$, such that each substring of length n,

$$s_{i+1} s_{i+2} \ldots s_{i+n}, \tag{1}$$

is unique with subscripts in (1) taken modulo m^n. For example, sequence 123133221 is a valid (3,2) de Bruijn sequence over the symbol set {1,2,3}, since each substring of length 2, namely, 12, 23, 31, 13, 33, 32, 22, 21, 11, is unique over the symbol set {1,2,3}; and sequence 11222121 is a valid (2,3) de Bruijn sequence over the symbol set {1,2}, since each substring of length 3, namely, 112, 122, 222, 221, 212, 121, 211, 111 is unique over the symbol set {1,2}. Obviously, the sequences developed by Griffin et al. (1992) (pp. 611–612) for the generation of the encoded light pattern are a (4,3) de Bruijn sequence and a (4,2) de Bruijn sequence, respectively.

In the 1970–80s, de Bruijn sequences had been well studied and several algorithms had been proposed for the generation of sequences (e.g., Fredricksen and Kessler, 1981; Fredricksen and Maiorana, 1978; Ralston, 1981). It is well known that each (m,n) de Bruijn sequence corresponds to an Eulerian circuit in the so-called de Bruijn digraph $D_{m,n}$, in which the vertex set of $D_{m,n}$ is the set of all distinct m^{n-1} words of length $n-1$ (Chartrand and Oellermann, 1993). However, as Ralston asserted: ''*the graphical interpretation is not useful in finding algorithms for generating de Bruijn sequences*'' and ''*all published algorithms use either an approach based upon the theory of finite field or a direct combinatorial approach*'' (Ralston, 1981, p. 50).

Next a new simple algorithm directly based upon de Bruijn digraphs is proposed to generate (m,n) de Bruijn sequences for $m \geqslant 2$ and $n \geqslant 2$. As shown, this new approach is much simpler than the other aforementioned approaches.

3. The new algorithm

Given m and n, finding an (m,n) de Bruijn sequence is equivalent to finding an Eulerian circuit in its corresponding digraph $D_{m,n}$, in which the vertex set of $D_{m,n}$ is the set of all distinct m_{n-1} words of length $n-1$ over the symbol set $\{1,2,\ldots,m\}$ (Chartrand and Oellermann, 1993). Based upon the set of vertices and the set of arcs, one may represent the de Bruijn digraph $D_{m,n}$ by an $m^{n-1} \times m^{n-1}$ adjacency matrix $\boldsymbol{A}$ (Emerson and Tobias, 1995), where

$$A_{ij} = \begin{cases} 1 \text{ if } 1 \leqslant j - \left[(i-1)\, m \bmod m^{n-1}\right] \leqslant m \\ 0 \text{ otherwise} \end{cases} \quad \text{for } i,j \in \{1,2,\ldots,m^{n-1}\}. \tag{2}$$

Thus, finding an Eulerian circuit in the de Bruijn digraph $D_{m,n}$ is equivalent to assigning numbers of $1,2,3,\ldots,m^n$ (in a particular way) to m^n cells $(i,j) \in T \equiv \{(i,j) | A_{ij}=1\}$ of the adjacency matrix $\boldsymbol{A}$, where $1,2,3,\ldots,m^n$ indicating the ordering of arcs to be travelled in the de Bruijn digraph $D_{m,n}$. In general, if t is assigned to cell (i,j), then the next positive integer $t+1$ must be assigned to an unassigned cell of row j. Next, throughout the paper, the value of arc(i,j) (from the ith vertex to the jth vertex in $D_{m,n}$) is defined as $r_{ij} \equiv 1 + ([(m-1)+j \bmod m^{\mathrm{n}-1}] \bmod m)$.

Denote $L(S)$ the length of sequence S. The new algorithm is shown below.

Algorithm A. (Input: m and n; Output: S, an (m,n) de Bruijn sequence)

1. $i \leftarrow 1,\ I \leftarrow (m,m,\ldots,m) \in R^{m^{n-1}}$, and S is an empty sequence.

2. While $L(S) < m^n$ do
   begin

$$j \leftarrow I[i] + \left[(i-1)\,m \bmod m^{n-1}\right] \tag{3a}$$

$$\text{Append } 1 + \left(\left[(m-1+j) \bmod m^{n-1}\right] \bmod m\right) \text{ to } S \tag{3b}$$

$$I[i] \leftarrow I[i] - 1 \tag{3c}$$

$$i \leftarrow j \tag{3d}$$

   end (while)

For a given vertex i, (3a) is used to find the next vertex j to be travelled, (3b) is to compute the arc value r_{ij} and append it to the sequence S, (3c) is to exclude the travelled vertex j, and (3d) indicates the next starting vertex j. This new algorithm has several computational advantages over typical approaches. For example:
1. it does not require to generate additional necklaces (substrings) (Fredricksen and Kessler, 1981; Fredricksen and Maiorana, 1978);
2. it does not require to test whether additional necklaces had been previously generated or not (Fredricksen and Kessler, 1981; Fredricksen and Maiorana, 1978);
3. it does not require to test that additional necklaces are periodic or aperiodic (Ralston, 1981; Fredricksen and Maiorana, 1978).

A brief proof of the algorithm is shown in Appendix A. Following the similar procedure of Griffin et al. (1992) (p. 612), interested readers may use the (4,3) de Bruijn sequence (i.e., 444344244143343243142342242 1413412411333233132232131231122212111) and the (4,2) de Bruijn sequence (i.e., 4434241332312211) generated by the new algorithm to construct the distinct encoded light pattern for 3-D imaging systems as below.

4443442441433432431423422421413412411333233132232131231122212111
4443442441433432431423422421413412411333233132232131231122212111
4443442441433432431423422421413412411333233132232131231122212111
3332331334322321324312311314342341344222122421121424124411141444
3332331334322321324312311314342341344222122421121424124411141444
1114113112144143142134133132124123122444344243343242342233323222
1114113112144143142134133132124123122444344243343242342233323222
2221224223211214213241244243231234233111411314414313413344434333
1114113112144143142134133132124123122444344243343242342233323222
4443442441433432431423422421413412411333233132232131231122212111
2221224223211214213241244243231234233111411314414313413344434333
1114113112144143142134133132124123122444344243343242342233323222
2221224223211214213241244243231234233111411314414313413344414333
4443442441433432431423422421413412411333233132232131231122232111
2221224223211214213241244243231234233111411314414313413344414333
3332331334322321324312311314342341344222122421121424124411121444
4443442441433432431423422421413412411333233132232131231122232111

Acknowledgements

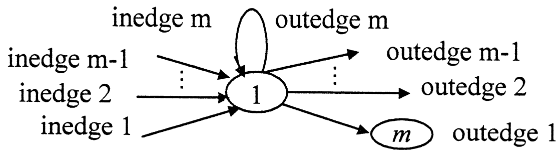
This research is partially supported by National Science Council, Taiwan, under grant NSC 87-2213-C-150-002.

Appendix A

Theorem. *Algorithm A can generate an (m,n) de Bruijn sequence, $m \geqslant 2$ and $n \geqslant 2$.*

Proof (outline). Since (1) each (m,n) de Bruijn sequence can be represented by a de Bruijn digraph $D_{m,n}$, (2) this de Bruijn digraph is Eulerian (Chartrand and Oellermann, 1993), and (3) Algorithm A starts assignment from cell $(1,m)$ in the adjacency matrix $\boldsymbol{A}$, there exists a circuit C starting from the 1st vertex to itself and it cannot extend further (Chartrand and Oellermann, 1993). It is sufficient to show that this circuit C is an Eulerian circuit of de Bruijn digraph $D_{m,n}$.

Firstly, one finds that for each vertex of digraph $D_{m,n}$ the number of indegrees (number of inedges to a vertex)) equals to number of outdegrees (number of outedges from a vertex) and $A_{1,1} = 1$ (by Eq. (2)) in the adjacency matrix $\boldsymbol{A}$, thus the 1st vertex of $D_{m,n}$ is as below.



Hence once circuit C is constructed, all edges to the 1st vertex must have been travelled. In other words, all cells $(i,j) \in T$ in column 1 of adjacency matrix $\boldsymbol{A}$ must have been assigned. Since these cells are last assigned in their corresponding rows (by (3a) and (3c)), namely, rows $1 + k_1$m, $0 \leqslant k_1 \leqslant m-1$, it implies that all cells in these rows must also have been assigned (since the number of indegrees equals to number of outdegrees for each vertex of digraph $D_{m,n}$). Now one observes that, if $I[i] = 1$ and $i = 1 + k_1 + k_2 m^{n-2}$, $0 \leqslant k_2 \leqslant m-2$, then $j = 1 + [(k_1 + k_2 m^{n-1})m \bmod m^{n-2}] = 1 + k_1 m$ in (3b). This implies that cells in columns $1 + k_1 m$ are last assigned for their corresponding rows by Algorithm A. Hence cells in rows $i = 1 + k_1 + k_2 m^{n-2}$ must have been assigned. Repeat this similar procedure, it is concluded that all cells in adjacency matrix A have been completely assigned and circuit C is an Eulerian circuit of de Bruijn digraph $D_{m,n}$. □

References

Altschuler, M.D., Altschuler, B.R., Taboada, J., 1981. Topographic mapping of surfaces. Optical Engineering 20, 953–961.

Chartrand, G., Oellermann, O.R., 1993. Applied and algorithmic graph theory. McGraw-Hill, Singapore.

Emerson, P.L., Tobias, R.D., 1995. Computer program for quasi-random stimulus sequences with equal transition frequencies. Behavior Research Methods, Instruments and Computers 27, 88–98.

Fredricksen, H., Kessler, I., 1981. Lexicographic compositions and de Bruijn sequences. Journal of Combinatorial Theory 2, 63–76.

Fredricksen, H., Maiorana, J., 1978. Necklaces of beads in k colors and k-ary de Bruijn sequences. Discrete Mathematics 23, 207–210.

Griffin, P.M., Narasimhan, L.S., Yee, S.R., 1992. Generation of uniquely encoded light patterns for range data acquisition. Pattern Recognition 25, 609–616.

Posdamer, J., Altschuler, M.D., 1981. Surface measurement by space-encoded projected beam system. Computer Vision Graphics and Image Processing 18, 1–7.

Ralston, A., 1981. A new memoryless algorithm for de Bruijn sequences. Journal of Algorithms 2, 50–62.

Sohn, H.S., Bricker, D.L., Simon, J.R., Hsieh, Y.C., 1996. Optimal sequences of trials for balancing practice and repetition effects. Behavior Research Methods, Instruments and Computers, to appear.

Yee, S.R., Griffin, P.M., 1994. Three-dimensional imaging system. Optical Engineering 33, 2070–2075.