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A note on ''A fresh look at the Hough transform''

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In a recent paper, Agrawal et al. showed that if
1. $\Delta m \leqslant 1$, and
2. $\Delta c/h \leqslant 1$

for the (m,c) parameterization of the Hough Transform, members of the parameter set are subsets of Digital Straight Lines (DSLs) (Agrawal et al., 1996). For conditions 1 and 2, Δm and Δc are the parameterization steps, and h is the size of the pixel. Their proof makes use of the equation of a DSL provided by Bongiovanni et al. (1975). In this paper we use a lemma provided by Kim and Rosenfeld to show that the second condition provides a necessary and sufficient condition for the members of the parameter space to represent subsets of DSLs.

Lemma 1. (Kim and Rosenfeld, 1982) *Let (m_1,c_1) and (m_2,c_2) be two parallel lines $(m_1 = m_2)$ such that the smaller of the two distances, the horizontal and the vertical, between them is equal to h. S is the set of lattice points that lie between the two lines and on (m_1,c_1) but not on (m_2,c_2). If u and w are two points of S and L is the set of points that belong to S and lie between u and w, u and w included, then L is a digital straight line segment.*

Fig. 1 illustrates the ideas of Lemma 1.

Obviously, each member of the parameter space (m_i,c_j) represents a strip in the image space. This strip is defined by two parallel lines $(m_i,c_j - \Delta c/2)$ and $(m_i,c_j + \Delta c/2)$. For $m_i \in [0,1]$, if

$$\Delta c = h, \tag{1}$$

the object within this strip is a DSL. This is the case because of Lemma 1. For $m_i \in [0,1]$ the largest of the two distances (horizontal, vertical) for this strip is the vertical. The vertical distance between the two strips is equal to Δc. Therefore, if Eq. (1) holds the members of the parameter space represent DSLs. Obviously, if $\Delta c < h$ the members of the parameter space are subsets of DSLs. This is the case even if $m_i \notin \mathbb{Q}$, where $\mathbb{Q}$ is the set of rational numbers.

For $m_i \in (1, +\infty)$, one can rotate the image by $\pi/2$ and get similar results. This rotation is equivalent to the Twin-parameterization discussed by Risse (1989). As we can see, the Twin-parameterization not only overcomes the difficulty of searching over an infinite domain, but also forces all the members of the parameter space, for $\Delta c = h$, to represent DSLs.

Our proof is simpler than the one provided by Agrawal et al., yet more general. It does not pose any requirements on m and on $h/\Delta c$ ($m \in \mathbb{R}$, $h/\Delta c \in \mathbb{R}$ where $\mathbb{R}$ is the set of real numbers). On the other hand the proof in the above mentioned work is based on the assumption that $h/\Delta c \in \mathbb{Z}$, where $\mathbb{Z}$ is the set of integer numbers. But if $h/\Delta c \in \mathbb{Z}$, it is obvious that $h/\Delta c \geqslant 1$ (notice that $\Delta c > 0$, $h > 0$). Hence, the proof provided by Agrawal et al.

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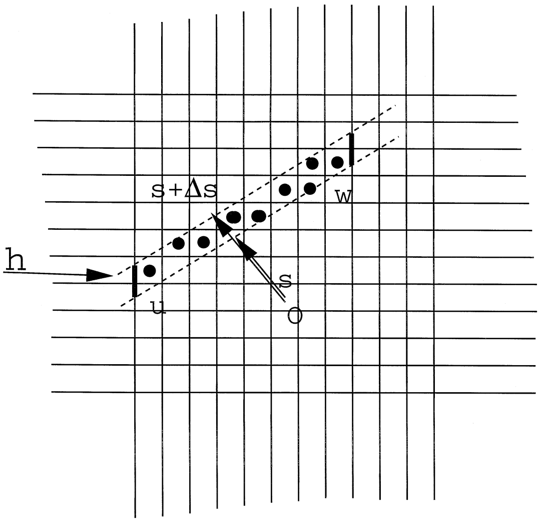

Fig. 1. A digital straight line segment. The vertical distance between the two bounding lines is equal to h. The distances of the two lines from the point of origin are equal to s and $s+\Delta s$.

is actually a verification that conditions 1 and 2 for $h/\Delta c \in \mathbb{Z}$ guarantee that the members of the parameter space are subsets of DSLs.

Digital Topology theory provides the theoretical framework for the mapping of the continuous arcs (i.e., straight lines) to digital images. An in-depth analysis of the effects of the discretization on the Hough Transform from the Digital Topological point of view can be found in the work of Ioannou (Thesis) and Ioannou and Dugan (1997).

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