Exercise 1 (Disjoint sets (24 points)) We want to implement a disjoint set data structure with union and find operations. The template for this program is available on the course website and named `DisjointSets.java`.

In this question, we model a partition of n elements with distinct integers ranging from 0 to $n-1$ (i.e. each element is represented by an integer in $[0, \cdots, n-1]$, and each integer in $[0, \cdots, n-1]$ represent one element). We choose to represent the disjoint sets with trees, and to implement the forest of trees with an array named `par`. More precisely, the value stored in `par[i]` is parent of the element `i`, and `par[i]==i` when `i` is the root of the tree and thus the representative of the disjoint set.

You will implement union by rank and the *path compression* technique seen in class. The rank is an integer associated with each node. Initially (i.e. when the set contains one single object) its value is 0. Union operations link the root of the tree with smaller rank to the root of the tree with larger rank. In the case where the rank of both trees is the same, the rank of the new root increases by 1. You can implement the rank with a specific array (called `rank`) that has been added to the template, or use the array `par` (this is tricky). Note that path compression does not change the rank of a node.

Download the file `DisjointSets.java`, and complete the methods `find(int i)` as well as `union(int i, int j)`. The constructor takes one argument n (a strictly positive integer) that indicates the number of elements in the partition, and initializes it by assigning a separate set to each element.

The method `find(int i)` will return the representative of the disjoint set that contains `i` (do not forget to implement path compression here!). The method `union(int i, int j)` will merge the set with smaller rank (for instance `i`) in the disjoint set with larger rank (in that case `j`). In that case, the root of the tree containing `i` will become a child of the root of the tree containing `j`), and return the representative (as an integer) of the new merged set. Do not forget to update the ranks. In the case where the ranks are identical, you will merge `i` into `j`.

Once completed, compile and run the file `DisjointSets.java`. It should produce the output available in the file `unionfind.txt` available on the course website.

Exercise 2 (Minimum Spanning trees (24 points)) We will implement the Kruskal algorithm to calculate the minimum spanning tree (MST) of an undirected weighted graph. Here you will use the file `DisjointSets.java` completed in the previous question and two other files: `WGraph.java` and `Kruskal.java` available on the course website. You will need the classes `DisjointSets` and `WGraph` to execute `Kruskal.java`. Your role will be to complete two methods in the template `Kruskal.java`.

The file `WGraph.java` implements two classes `WGraph` and `Edge`. An `Edge` object stores all informations about edges (i.e. the two vertices and the weight of the edge), which are used to build graphs.

The class `WGraph` has two constructors `WGraph()` and `WGraph(String file)`. The first one creates an empty graph and the second uses a file to initialize a graph. Graphs are encoded using the following format. The first line of this file is a single integer n that indicates the number of nodes in the graph. Each vertex is labelled with an number in $[0, \cdots, n-1]$, and each integer in $[0, \cdots, n-1]$ represents one and only one vertex. The following lines respect the syntax "n_1 n_2 w", where n_1 and n_2 are integers representing the nodes connected by an edge, and w the weight of this edge. n_1, n_2, and w must be separated by space(s). An example of such file can be found on the course website with the file `g1.txt`. These files will be used as an input in the program `Kruskal.java` to initialize the graphs. Thus, an example of a command line is `java Kruskal g1.txt`.

The class `WGraph` also provide a method `addEdge(Edge e)` that adds an edge to a graph (i.e. an object of this class). Another method `listOfEdgesSorted()` allows you to access the list of edges

of a graph in the increasing order of their weight.
You task will be to complete the two static methods `isSafe(DisjointSets p, Edge e)` and `kruskal(WGraph g)` in `Kruskal.java`. The method `isSafe` considers a partition of the nodes p and an edge e, and should return `True` if it is safe to add the edge e to the MST, and `False` otherwise. The method `kruskal` will take a graph object of the class `WGraph` as an input, and return another `WGraph` object which will be the MST of the input graph.
Once completed, compile all the java files and run the command line `java Kruskal g1.txt`. An example of the expected output is available in the file `mst1.txt`. You are invited to run other examples of your own to verify that your program is correct. Note that you need to compile it with the two other files for it to work.

Exercise 3 (Greedy algorithms (52 points)) In this exercise, you will plan your homework with a greedy algorithm. The input is a list of homeworks defined by two arrays: deadlines and weights (the relative importance of the homework towards your final grade). These arrays have the same size and they contain integers between 1 and 100. The index of each entry in the arrays represents a single homework, for example, `Homework 2` is defined as a homework with deadline `deadlines[2]` and weight `weights[2]`. Each homework takes exactly one hour to complete.

Your task is to output a `homeworkPlan`: an array of length equal to the last deadline. Each entry in the array represents a one-hour timeslot, starting at 0 and ending at 'last deadline - 1'. For each time slot, `homeworkPlan` indicates the homework which you plan to do during that slot. You can only complete a single homework in one 1-hour slot. The homeworks are due at the beginning of a time slot, in other words if an assignment's deadline is `x`, then the last time slot when you can do it is `x - 1`. For example, if the homework is due at t=14, then you can complete it before or during the slot t=13. If your solution plans to do `Homework 2` first, then you should have `homeworkPlan[0]=2` in the output. Note that sometimes you will be given too much homework to complete in time, and that is okay.

Your homework plan should maximize the sum of the weights of completed assignments.

To organize your schedule, we give you a class `HW_Sched.java`, which defines an `Assignment` object, with a number (its index in the input array), a weight and a deadline.

The input arrays are unsorted. As part of the greedy algorithm, the template we provide sorts the homeworks using Java's `Collections.sort()`. This sort function uses Java's `compare()` method, which takes two objects as input, compares them, and outputs the order they should appear in. The template will ask you to override this `compare()` method, which will alter the way in which Assignments will be ordered. You have to determine what comparison criterion you want to use. Given two assignments A1 and A2, the method should output:

- 0, if the two items are equivalent
- 1, if a1 should appear after a2 in the sorted list
- -1, if a2 should appear after a1 in the sorted list

The `compare` method (called by `Collections.sort()`) should be the only tool you use to re-organize lists and arrays in this problem. You will then implement the rest of the `SelectAssignments()` method.