

A real bounce in your step

Obligatory Assignment 2

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1 Abstract

In this article we'll show and weigh different solutions to storing potential energy in a shoe. We'll also calculate these, and discuss how certain real results don't correspond with their theoretical results.

2 Introduction

Ever since the dawn of time, man's main method of transportation has been to use his feet. And although we no longer solely rely on our feet to travel, other activities have ensured that our feet haven't become entirely obsolete. I am of course referring to sport, more precisely, I'll be talking about running, and even more precisely, the energy conversation you can attain using polymers and the right kind of shoes.

In mankind's never-ending quest to optimize just about everything, not even your shoes are safe. Nike's Vapourfly model of shoe boasts about a whopping 87% energy return when rebounding off of the ground. Now that's a pretty good deal if you ask me, and they're doing it by using their new ZoomX foam material, a kind of polymer, which we'll be discussing here.

3 Theory and Method

Energy storage and the idea of storing energy in compressed objects isn't exactly a new concept. Springs, for example metal ones, also store energy.

You might recall, that the force on a spring can be expressed as:

$$F = kx$$

where F is the force exerted on the spring to deform it by a length of x . k is the characteristic factor and a constant of the rigidity of the spring. This is Hookes law, and from this we can integrate this relation to find:

$$PE = \int F dx = \int kx dx = \frac{1}{2}kx^2 \quad (1)$$

Of course, the corresponding kinetic energy in such a system would be:

$$KE = \frac{1}{2}mv^2 \quad (2)$$

where m is the mass of the object connected to the spring, and v is it's velocity.

If we then assume a downwards motion, like for a shoe with springs hitting the ground, we'd get the relation:

$$KE_1 + \cancel{PE_1} = \cancel{KE_2} + PE_2 - LE_2 = KE_3 + \cancel{PE_3} - LE_2 \quad (3)$$

where 1 is on impact with the ground, 2 is at max compression of the spring, and 3 is when the spring is back in equilibrium. LE is an unknown variable representing the energy lost to lower forms of energy, like heat or sound. It's worth mentioning that there is probably LE_1 and LE_3 also, but, in my opinion, the chief contributor to energy loss happens when compressing the spring, and as such I'm assuming LE_1 and LE_3 are minimal or close to zero.

Obviously, we'd ideally like for LE_2 to be as small as possible.

However, there's another option. Like previously discussed in the introduction, we can alternatively use polymers as a method of retaining the energy we have on impact. The closest thing we can relate this to would be a rubber band. Now the interesting thing about rubber bands, or rather polymers, is that they experience a decrease in entropy when stretched. This is the driving force behind the idea of using polymers for cushioning and storing energy in running shoes.

You see, as opposed to a regular spring, when compressing (stretching) a polymer all the work is immediately thermal energy, same as for an ideal gas compressed in a piston. We can derive this from:

$$\Delta G = \Delta H - T\Delta S$$

where G is free energy, H the enthalpy and S is the entropy. T is temperature. Since the act of stretching, or compressing, is not spontaneous, $T\Delta S$ must be negative. Remind yourself that T is given in Kelvin, which is only ever positive, as such, ΔS must be negative, which means that when we stop compressing the polymers, the resulting reaction will be spontaneous, given us a negative ΔG , and consequently a positive ΔH and ΔS .

Such a system is driven entirely by entropy changes at a given temperature, and because the free energy derives entirely from the entropy change, the work done is entirely from thermal energy in the polymer, with a 100% efficiency of conversion of thermal energy to work. Do note that this is also the case for ideal gases.

4 Results

According to the NYTimes [3], whilst jogging we take an average of 140 steps per minute. That means we step $\frac{7}{3}$ times a second. Knowing this, we can approximate the acceleration (and velocity) of our feet coming down and touching the ground. This means that an average step takes about 0.45s. Let's assume we spend about 0.25s on the ground+raising our foot, meaning our time in the air is 0.2s. Know also that according to The Guardian [4], the average weight of an Olympic runner is 69.2kg

$$v = v_0 + at = 0 + -9.81m/s^2 \cdot 0.2s = -1.962m/s$$

Knowing this, we can approximate our kinetic energy as we hit the ground to be:

$$KE_1 = \frac{1}{2}mv^2 = \frac{1}{2}69.2kg \cdot -1.962^2m/s = 133J$$

This means that $LE_2 = \frac{1}{2}KE$, assuming 50% loss from [5], and:

$$KE_3 = 66.6J$$

Now, I can't provide any calculations in regards to the efficiency of a polymer solution to energy retention, but we can probably deduce the reasoning as to why we get a 86% return on energy as opposed to the 100% theoretical return. I'm guessing this is excess energy getting lost as sound energy. We know that, unlike springs, all thermal energy is being converted 100% back to kinetic energy when we stop compressing and stretching the polymers. There will also likely be some thermal energy lost as a result of friction against the surface we're running on.

5 Discussion

While we weren't able to precisely calculate the conservation of energy in polymer-soled shoes, we atleast managed to deduce a possible reasoning as to why it deviates from the theoretical 100% return when converting kinetic energy to thermal in a polymer.

I'm also not so sure what kind of calculations or plots you'd like for me to do. I debated having a plot where I looked at the optimal k given a weight m when the sole is l tall, but decided against it as it wasn't very relevant to the question.

I'd also like to point out that I felt the values when calculating KE_3 were a bit large, seeing as 66.6 J means an upwards velocity of 1.39. That being said, it probably does not matter as it was only to illustrate in practice.

6 Conclusion

We've shown and discussed the different properties of a spring-loaded shoe vs a polymer-loaded(?) shoe. We've additionally looked at it's theoretical outputs, and weighed them against reality as a way to explain why we got the results we got. We've also shown in practice what a 50% return on energy would look like.

References

- [1] Dag Kristian Dysthe. *Oblig 2 FYS2160 2020*. UiO, 2020.
- [2] Veronica Bessone. *Materials for Shoes That Helped to Break The World Marathon Record*. matmatch.com, 2020.
- [3] Gretchen Reynolds. *Walk Briskly for Your Health. About 100 Steps a Minute*. NYTimes, 2018.
- [4] Unknown Author. *Could you be an athlete? Olympics 2012 by age, weight and height*. The Guardian, 2012.
- [5] Z-Coil Footwear. *Z-CoiL Footwear Explained*. Youtube, 2010.
- [6] Israel Urieli. *Specific Heat Capacities of Air*. Ohio University, 2020.
- [7] Aldo da Rosa. *Fundamentals of Renewable Energy Processes*. Elsevier, 2013.

Appendices

Appendix A - Calculations: all answers to part one of the assignment

- a) **If you study a single cycle, what is the change in the entropy of the gas?**
It is specified that the cycle is reversible, and as such, the entropy must be unchanged after one whole cycle.
- b) **Calculate ΔS_{23} and ΔS_{41} and show that this leads to $T_3^\gamma T_1 = T_2^\gamma T_4$.**
Knowing what we know from a), we can deduce that the entropy in ΔS_{23} and ΔS_{41} must be equal but opposite. Therefore, we can calculate:

$$\Delta S_{23} = \Delta S_{41}$$
$$C_p \ln \frac{T_3}{T_2} = -C_v \ln \frac{T_4}{T_1}$$

We can then get rid of the natural logarithms, and we find using logarithm rules that:

$$T_3^\gamma T_1 = T_2^\gamma T_4$$

- c) **Show that $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ and $T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1}$.**
Using the first law of adiabatic compression, we can find that:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

which can easily be rewritten as:

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

The same is true for the other equation, as it is the same process, just the other way.

$$\frac{T_3}{T_4} = \left(\frac{V_1}{V_3} \right)^{\gamma-1}$$
$$T_3 V_3^{\gamma-1} = T_4 V_1^{\gamma-1}$$

- d) **Find Q_{41} and Q_{23} .**

$$Q_{23} = \Delta h = C_p \Delta T = C_p (T_3 - T_2)$$
$$Q_{41} = -\Delta U = -C_V (T_4 - T_1)$$

Note that there is no work being done in this process, and W is therefore 0 and crossed out. We find Q_{41} in much the same way:

$$Q_{41} - \cancel{W} = \Delta U = C_V \Delta T = C_V (T_1 - T_4)$$

- e) **Find the ideal efficiency as function of the adiabatic constant γ , the compression ratio $r_k = V_1/V_2$ and the expansion ratio $r_l = V_1/V_3$.**

The ideal efficiency of a diesel engine is:

$$\eta = 1 - r_k^{1-\gamma} \left(\frac{r_l^\gamma - 1}{\gamma(r_l - 1)} \right) \quad (4)$$

This is not yet relevant, but do note that for the Otto Cycle, $r_k = r_l$.

- f) **Find some reasonable values of the volume and pressure of an engine and calculate the heat and work done in each step. Verify your answer in the previous exercise by calculating the efficiency of the motor you have envisioned.**

First, we'll need to define some initial variables here. For simplicities sake, let's assume that we're operating at $T_1 = 300K$ (room temperature). C_V, C_p and γ are all known at this temperature, and are given as: $C_V = 0.718 \frac{kJ}{kgK}$, $C_p = 1.005 \frac{kJ}{kgK}$ and $\gamma = 1.4$ [6]. Let's also assume a standard ammount of pressure, $P_1 = 100kPa$. We also need to assume by what factor the volume changes, let's assume it increases by a factor of 6 between $1 \rightarrow 2$ and a factor 2 between $2 \rightarrow 3$.

Knowing this, we can start:

$$\begin{aligned} T_2 &= T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} \\ T_2 &= 300K \cdot 6^{0.4} = 614K \\ P_2 &= P_1 \left(\frac{V_1 T_2}{V_2 T_1} \right) \\ P_2 &= 100kPa \left(6 \frac{614}{300} \right) = 1228kPa \\ T_3 &= T_2 \frac{V_3}{V_2} \\ T_3 &= 614K \cdot 2 = 1228K \\ P_3 &= P_2 \\ P_3 &= 1228kPa \\ Q_{23} &= \Delta h = C_p(T_3 - T_2) \\ Q_{23} &= 1.005(1228 - 614)K = 617 \\ T_4 &= T_3 \left(\frac{V_3}{V_4} \right)^{\gamma-1} \\ T_4 &= 1228K \left(\frac{2}{6} \right)^{0.4} = 791K \\ P_4 &= P_3 \frac{V_3 T_4}{V_4 T_3} \\ P_4 &= 1228kPa \frac{2 \cdot 791K}{8 \cdot 1228K} = 263kPa \\ Q_{41} &= C_V(T_4 - T_1) \\ Q_{41} &= 0.718(791 - 300)K = 353kJ/kg \end{aligned}$$

For clarities sake, let's sort this into a table:

	T [K]	P [kPa]
1	300	100
2	614	1228
3	1228	1228
4	791	263

Table 1: All temperatures and pressures

and

	Q [kJ/kg]
2 → 3	617
4 → 1	353

Table 2: Heat in and out

Using this, we can find our efficiency with the formula:

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{617kJ/kg - 353kJ/kg}{617kJ/kg} = 43\%$$

- g) **How could you in practice produce the highest possible efficiency?**

No. You would not be able to produce the maximum possible efficiency, which in this case would be the Carnot efficiency. In an absolutely ideal scenario, that's to say, not possible in the real world, according to [7]:

"The ideal Otto cycle achieves the Carnot efficiency of an engine working between the maximum, pre-combustion, temperature and the intake temperature. This means that the ideal Otto cycle cannot achieve the Carnot efficiency determined by the highest and lowest temperature during the cycle."

It is worth mentioning that this is regarding the standard Otto Cycle, not the Diesel Cycle with which we're working with here.

Out of curiosity I decided to try and plot efficiency as a function of temperature, in addition to varying volume factors, shown on the next page below:

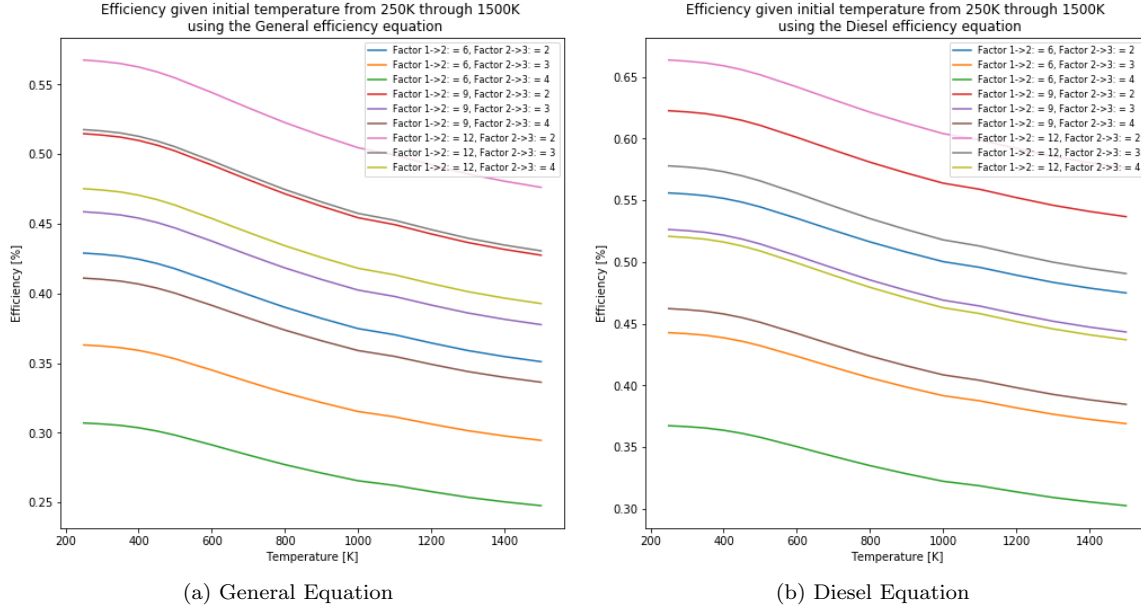


Figure 1: Two different plots showing the efficiency given a temperature and varying volume factors. Factor 1 is the change in volume from $1 \rightarrow 2$ while Factor 2 is the change in volume from $2 \rightarrow 3$. I retrieved the values for C_V , C_p and γ from [6].

It is worth mentioning however that there are likely some issues with this model, as I'm having trouble believing that a diesel engine would be more efficient at -27°C versus 23°C . What we are seeing that I could believe however, is that bigger discrepancies in volume between 1 and 2, or rather, ΔV for $1 \rightarrow 2$. What this means essentially, is that as we compress and apply more force onto our fuel, we'll experience an increase in efficiency accordingly. This is however, probably pointless in reality, as the extra force we'd need to apply to get any noticeable increase in effect would likely far exceed whatever extra mileage we'd get.

Appendix B: Python Code for plots in 1g)

```

import numpy as np
import matplotlib.pyplot as plt

Cp = np.array([1.003,1.005,1.008,1.013,1.020,1.029,1.040,1.051,1.063,1.075,
               1.087,1.099,1.121,1.142,1.155,1.173,1.190,1.204,1.216])
Cv = np.array([0.716,0.718,0.721,0.726,0.733,0.742,0.753,0.764,0.776,0.788,
               0.8,0.812,0.834,0.855,0.868,0.886,0.903,0.917,0.929])
vol1 = [6,9,12]; vol2 = []
for k in vol1:
    vol2.append(k/3)
T = np.array([250,300,350,400,450,500,550,600,650,700,750,800,900,
              1000,1100,1200,1300,1400,1500])
def eff(T1, vol1, vol2, Cv,Cp):
    g = Cp/Cv
    T2 = T1*vol1**(g-1)
    T3 = T2*vol2
    T4 = T3*(vol2/vol1)**(g-1)
    Q1 = Cp*(T3-T2)
    Q2 = Cv*(T4-T1)
    netW = Q1-Q2
    eff = netW/Q1
    return eff
def eff2(T1, vol1, vol2, Cv,Cp):
    g = Cp/Cv
    eff = 1- 1/vol1**(g-1) * ((vol2)**g - 1)/(g*(vol2)-1)
    return eff
p = str(input("General_Efficiency_or_Diesel_Efficiency?[G/D]\n"))
p = p.lower()
if p == 'g':
    mode = eff; s = 'General'
elif p == 'd':
    mode = eff2; s = 'Diesel'
else:
    raise ValueError('Not_recognized_input._Please_enter_G_or_D_only')
plt.figure(figsize=(8,8))
for i in range(len(vol1)):
    for j in range(len(vol2)):
        plt.plot(T, mode(T, vol1[i], vol2[j], Cv, Cp),
                 label='Factor_1->2:=%i, _Factor_2->3:=%i' % (vol1[i], vol2[j]))
plt.title("""Efficiency given initial temperature from 250K through 1500K
           using the %s efficiency equation""" % (s))
plt.xlabel('Temperature [K]'); plt.ylabel('Efficiency [%]')
plt.legend(fontsize=8)
plt.show()

```