POLITECNICO
MILANO 1863

Reliability Risk & Safety Analysis

Flipped Classroom 2

Bayesian Networks

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1.1 Model the system reliability with a Bayesian network

The earthquake is modelled via a top node with a probability of being true $P_E = 10^{-4}$. It is linked to the generators and the transmission lines, whose state it influences (conditional probability of failure equal to one half). The states 'connected' or 'disconnected' of the two communities $L1$ and $L2$ depend on the state of the generators and lines that should be powering them: each community should be at least connected to a working generator via a working transmission line. Finally the system is in its 'working' state if both communities are electrically satisfied.

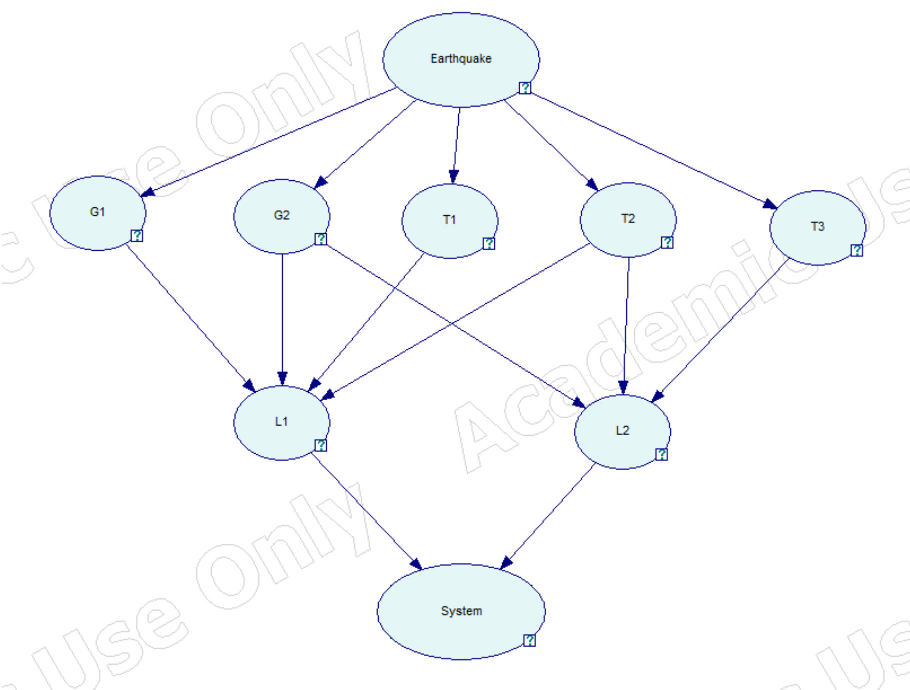

Figure 1: Bayesian network for the electric grid

1.2 Is the system reliable if an earthquake occurs?

Setting as evidence that the earthquake has occurred, the system is updated. It yields a working probability of the system after an earthquake : $P(Working) = 0.281$. Hence, it is safe to say that the system is not reliable if an earthquake occurs.

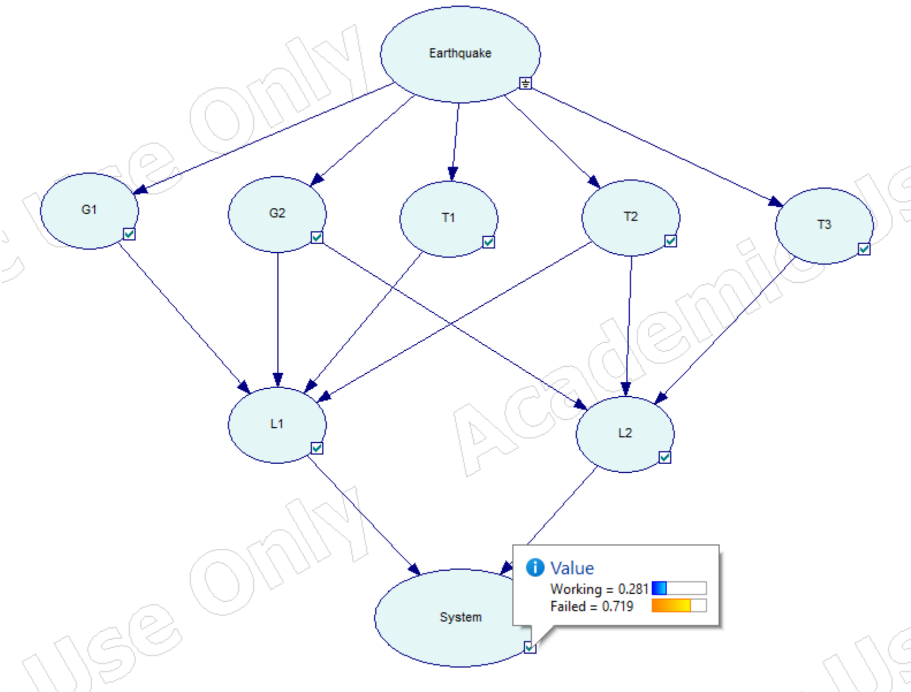

Figure 2: Results for the system state after the earthquake

1.3 What is the set of actions that guarantees the maximum resilience of the system?

It results from the available budget of 250 M€ that sixteen different strategies could be implemented. Only the strategies with a cost equal to the budget have been considered. For each one of them, the resilience, that is, the probability that the system is still working after an earthquake, are summarized in the following table.

Strategy	G1	G2	T1	T2	T3	Resilience
I	A2	-	-	-	-	0.306
II	-	A2	-	-	-	0.506
III	-	-	A1	A1	A2	0.397
IV	-	-	A1	A2	A1	0.462
V	-	-	A2	A1	A1	0.397
VI	A1	-	A2	-	-	0.329
VII	A1	-	-	A2	-	0.459
VIII	A1	-	-	-	A2	0.329
IX	-	A1	A2	-	-	0.429
X	-	A1	-	A2	-	0.639
XI	-	A1	-	-	A2	0.429
XII	A1	A1	A1	-	-	0.436
XIII	A1	A1	-	A1	-	0.527
XIV	A1	A1	-	-	A1	0.436
XV	A1	-	A1	A1	A1	0.401
XVI	-	A1	A1	A1	A1	0.541

Hence, the best course of action is strategy X: performing action A1 on generator G1 and action A2 on transmission line T2. In this way, there is a 0.639 probability that the two communities are still electrically satisfied after an earthquake.

2 Exercise 4

2.1 Posterior distribution using discrete distribution as prior

The evidence provided by the test is $E : \{1 \text{ out of } 6 \text{ is defective}\}$. For each value of p_i we have :

$$P(E|p_i) = \binom{6}{1} p^1 (1-p)^{6-1} = 6p(1-p)^5 \tag{1}$$

Using the Bayes theorem we get the posterior distribution of p :

$$P''(p_i) = P(p_i|E) = \frac{P(E|p_i)P'(p_i)}{\sum_{j=1}^{3} P(E|p_j)P'(p_j)} \tag{2}$$

p_i	$P'(p_i)$	$P(E\|p_i)$	$P''(p_i)$
0.2	0.1	0.393	0.158
0.3	0.6	0.303	0.729
0.5	0.3	0.094	0.113

2.2 Posterior distribution using Beta distribution as prior

Since the binomial distribution and the Beta distribution are conjuguate distributions, the posterior is also a Beta distribution :

$$P''(p_i) = \frac{\Gamma(q''+r'')}{\Gamma(q'')\Gamma(r'')} p^{q''-1}(1-p)^{r''-1} \tag{3}$$

Since :

- $q' = r' = 2$
- $x = 1$
- $n = 6$

The posterior is a Beta distribution with parameters $q'' = 3$ and $r'' = 7$:

$$P''(p_i) = \frac{9!}{2!\,9!} p^2(1-p)^6 = 252p^2(1-p)^6 \tag{4}$$

2.3 Bayesian network model

The tank is protected by two main systems : the safety valve that can preserve it from all fire damage, and the mitigation system (comprising the fireproof coating and the two sprinklers) that can mitigate the fire damage. Hence three final states are possible for the tank in case of fire: no damage (working safety valve), minor damage (failed safety valve and working mitigation system) and major damage (failure of both protection systems).

Each sprinkler has a failure probability which is the expected value of the parameter p in question 2.1. Thus $p_{prior} = 0.35$ and $p_{posterior} = 0.3068$. The failure probabilities of the fireproof coating and safety valve are unknown and not needed.

2.4 Tank damage probability assuming that the safety valve and the fireproof coating are both failed

Let us compute the tank damage probabilities, knowing that the safety valve and the fireproof coating are not working. Because of the failed safety valve the tank can experience either minor or major damage.

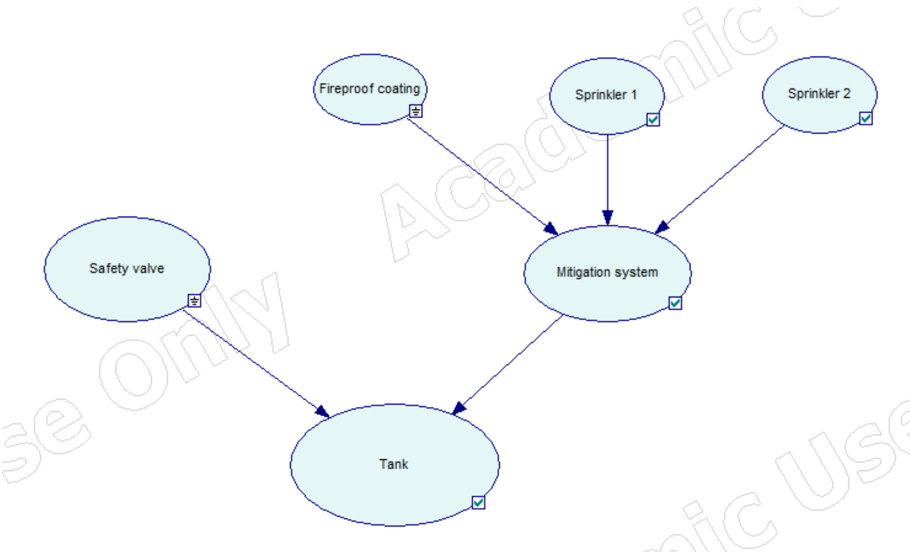

Figure 3: Architecture of the Bayesian network

2.4.1 With the prior distribution

With a sprinkler failure probability $p = 0.35$ from the prior distribution, the tank has a 57.7% probability of experiencing major damage.

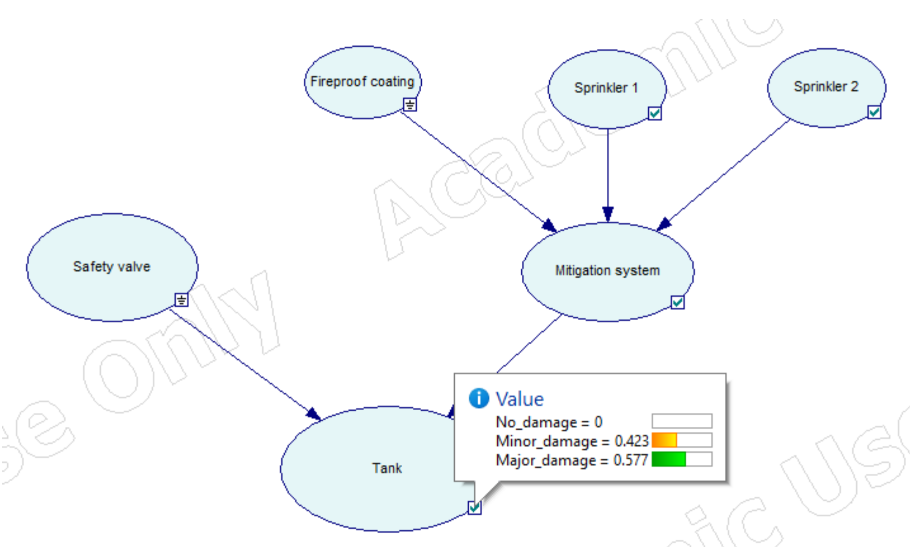

Figure 4: Tank damage probabilities with the prior distribution

Since the two sprinkler can be considered independent, the major damage probability can also be computed easily in an analytical way as $P_{maj} = 1 - (1 - p_{prior})^2 = 0.5775$. The two value are in agreement, so this likely indicates that the model is correct.

2.4.2 With the posterior distribution

With a sprinkler failure probability $p = 0.3068$ from the prior distribution, the tank has a 51.9% probability of experiencing major damage. As expected, this is slightly lower than the value given by the prior distribution.

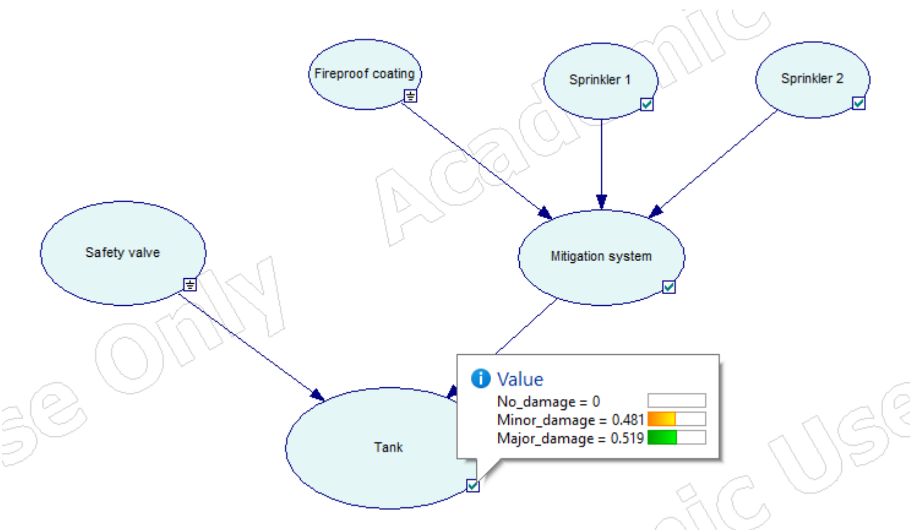

Figure 5: Tank damage probabilities with the posterior distribution

Again, the major damage probability can also be computed as $P_{maj} = 1 - (1 - p_{posterior})^2 = 0.51947$.