

1. (10 pts) Let  $f(x) = \sqrt{x^2 - 1}$ . Please answer the following questions.

- (a) (4 pts) Write down the domain and range of  $f$ .
- (b) (4 pts) Write out expressions for  $f(x + 2)$  and  $f(x) + 2$ .
- (c) (2 pts) Does  $f(x)$  have an inverse function? Explain.

2. (10 pts) For  $t$  in months, a population, in thousands, is approximated by a **continuous** function

$$P(t) = \begin{cases} e^{kt} & 0 \leq t \leq 12 \\ 100 & t > 12 \end{cases}$$

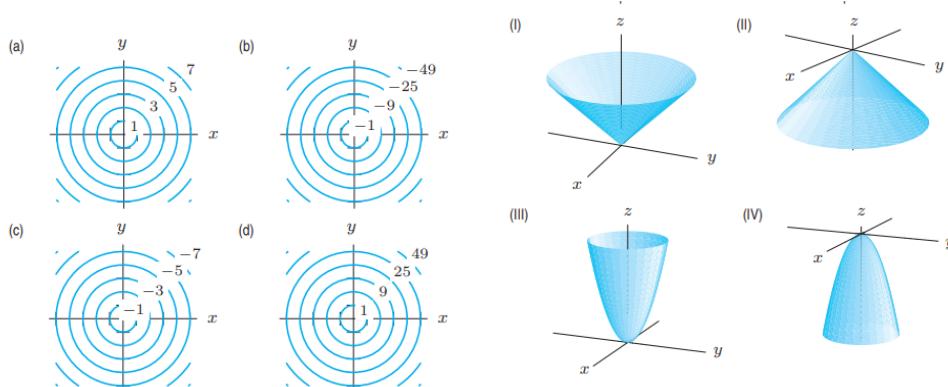
- (a) (3 pts) What is the initial value of the population?
- (b) (7 pts) What must be the value of  $k$ ?

3. (10 pts) Use the following table to answer the questions below

$x$	0	0.2	0.4	0.6	0.8	1.0
$f(x)$	3.7	3.5	3.5	3.9	4.0	3.9

- (a) (4 pts) Using the table, estimate  $f'(0.6)$  and  $f'(0.5)$ .
- (b) (3 pts) Estimate  $f''(0.6)$
- (c) (3 pts) Where do you think the maximum and minimum values of  $f$  occur in the interval  $0 \leq x \leq 1$ ?

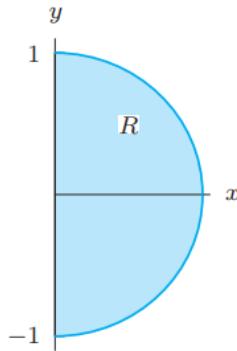
4. (10 pts) Match the contour diagrams (a)–(d) with the surfaces (I)–(IV).



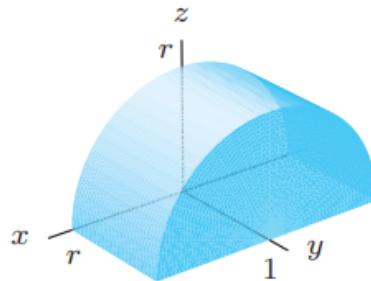
5. (10 pts) Your house lies on the surface  $z = f(x, y) = 2x^2 - y^2$  directly above the point  $(4, 3)$  in the  $xy$  plane.

- (a) (2 pts) How high above the  $xy$  plane do you live?
- (b) (4 pts) What is the slope of your lawn as you look from your house directly toward the  $z$ -axis (that is along the vector  $(-4, -3)$ )?

- (c) (2 pts) When you wash your car in the driveway, on this surface above the point  $(4, 3)$ , which way does the water run off? (Give your answer as a two-dimensional vector.)
- (d) (2 pts) What is the equation of the tangent plane to this surface at your house?
6. (10 pts) Use polar coordinate to integrate  $f(x, y) = \sqrt{\frac{1}{x^2+y^2}}$  over the shaded region in the figure below.



7. (10 pts) We want to express the volume of the half cylinder below as an integral of the cylindrical coordinate  $\int_W f dV$  where  $W$  is the half cylinder and  $dV = r dr d\theta dy$ .



- (a) (2 pts) What must the function  $f$  be?
- (b) (5 pts) Write the limits of the integration.
- (c) (3 pts) Evaluate the integral.
8. (10 pts) Let  $C$  be the line segment connecting the points  $\mathbf{p} = (1, 2, 0)$  and  $q = (0, 1, -1)$
- Find a curve  $\mathbf{c}(t) : [a, b] \rightarrow \mathbb{R}^3$  that traces out  $C$ .
  - Find the arc length of  $\mathbf{c}(t)$ .
  - Find  $\|\mathbf{p} - \mathbf{q}\|$ .
9. (10 pts) Let  $\mathbf{c}(t) = (\sin t, \cos t, t)$  with  $0 \leq t \leq 2\pi$ . Let  $\mathbf{F}$  be defined by  $\mathbf{F}(x, y, z) = (x, y, z)$ . Compute  $\int_C \mathbf{F} \cdot d\mathbf{s}$ .

10. (10 pts) Decide whether the following statements are true or false.
- (a) If a function is continuous, it must be differentiable.
  - (b) The curl of a vector field is a vector.
  - (c) The divergence of a vector field is a vector.
  - (d) The function  $f(x, y) = x^3 + y^3 - 3xy$  has two critical points.
  - (e) A function having no critical points in a region  $R$  cannot have a global maximum in the region.