Notes for "Calculus"

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AI503: Calculus

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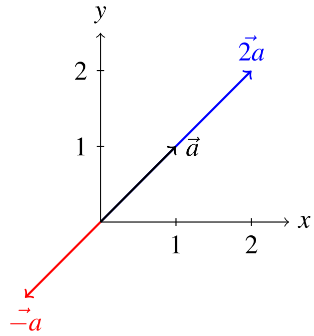
1 Introduktion

Notes and exercises for lectures and TA-sessions. Note that mistakes in note and/or exercises may occur.

2 Notes

2.1 Exercises

2.2 Vectors



What number can you multiply $\vec{-a}$ with to get a vector in the opposite direction?

$$\left\|c\vec{-a}\right\| = c \cdot \sqrt{2} \quad c = \frac{1}{\sqrt{2}}$$

2.3 Dot Product

The dot product of two vectors $\vec{a}$ and $\vec{b}$ is defined as:

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \left\|\vec{b}\right\| \cos(\theta)$$

where θ is the angle between the two vectors.

2.3.1 Example

Let

$$\mathbf{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

1 Compute $\mathbf{v} \cdot \mathbf{w}$

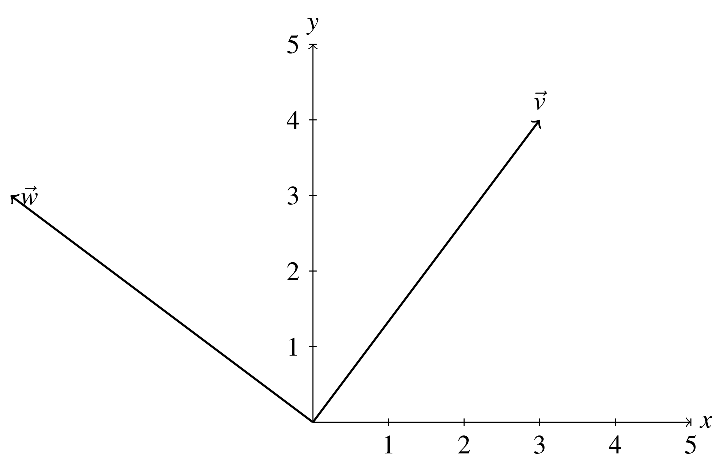
2 Use (1) to deduce that the angle between $\mathbf{v}$ and $\mathbf{w}$ is a right angle

3 Draw the vectors to confirm.

Using the definition:

$\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\| \|\mathbf{w}\| \cos(\theta)$

$v \cdot w = 25\cos(0) = 0$



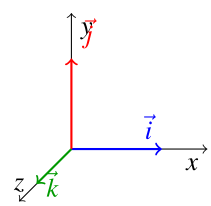
2.4 Standard Basis Vectors in $\mathbb{R}^3$

The standard basis vectors in $\mathbb{R}^3$ are:

$$\vec{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Note that: any vector $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ can be written as:

$$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$$



2.5 Cross Product

2.6 Partial Derivatives

2.7 Examples

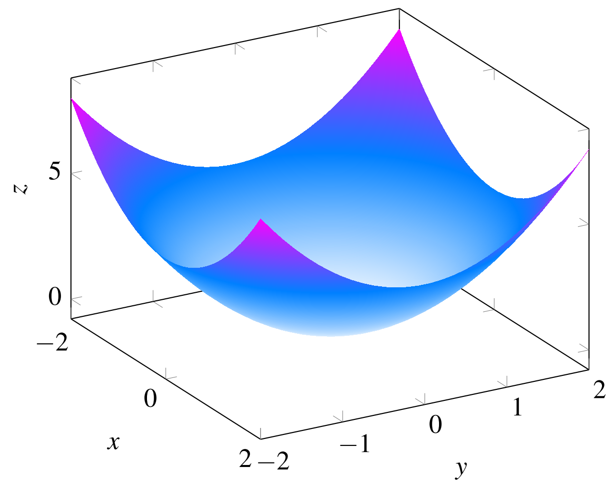
Use the second derivative test to classify the critical points of the function

$$f(x,y) = x^3 - 3x + 4.$$

2.8 3dplots

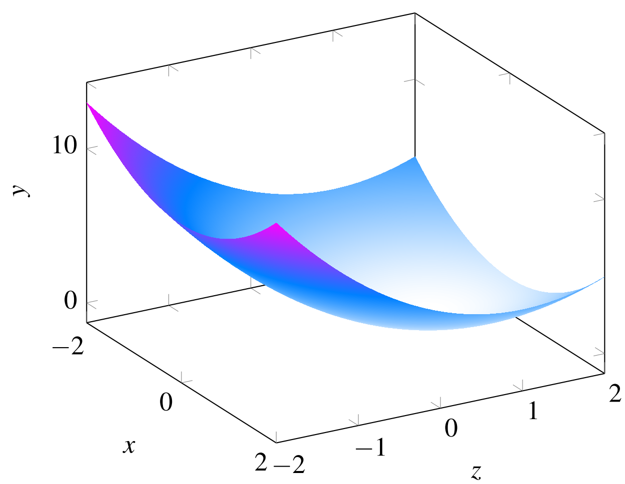
For the function

$$f(x,y) = x^2 + y^2$$



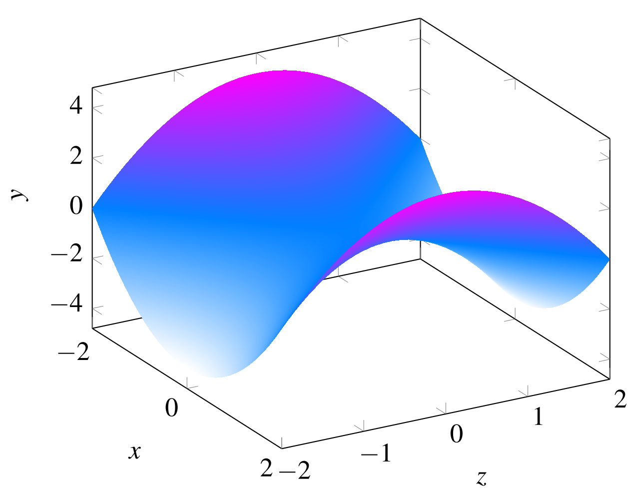
For the function

$$f(x,z) = x^2 + (z-1)^2$$



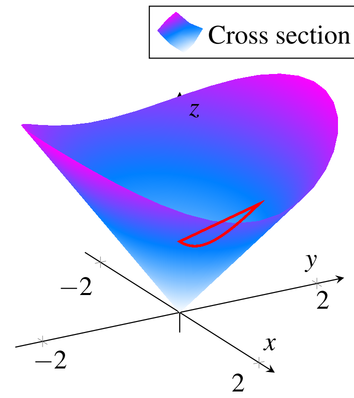
For the function

$$f(x,z) = x^2 - z^2$$



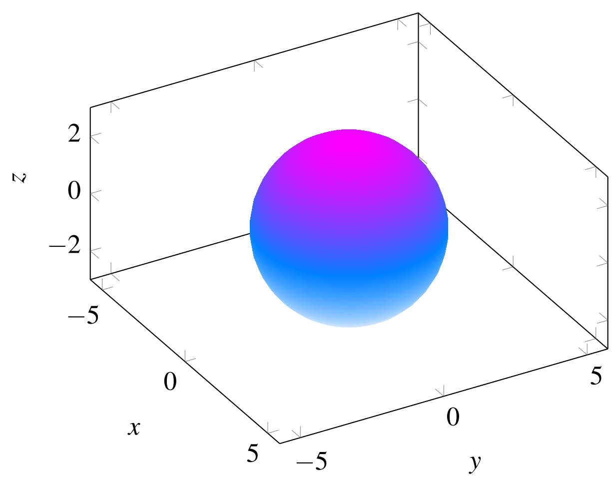
For the function

$$f(x,y) = \sqrt{x^2+y^2}$$



For the function

$$f(x,y,z) = \sqrt{x^2+y^2+z^2}$$



For the function

$$f(x,y,z) = \sqrt{x^2+y^2-z^2}$$