Homework

1. Consider
$$f(x, y) = x^2 + y^2 - 4x - 6y.$$
   (a) Find the critical points of f.
   (b) Classify them (minimum, maximum, or saddle point) using the Hessian.

2. Let
$$f(u, v) = u^2 + 3uv, \quad u(x, y) = x^2 - y, \quad v(x, y) = \sin(xy).$$
   Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ using the multivariable chain rule.

3. A rectangular box with square base and volume $V = 1\,\mathrm{m}^3$ is to be built with the least surface area.
   (a) Express the surface area $S(x, h)$ in terms of base side length x and height h.
   (b) Eliminate one variable using the volume constraint.
   (c) Use calculus to find the dimensions minimizing S.

4. Two products are manufactured in quantities q_1 and q_2 and sold at prices p_1 and p_2, respectively. The cost of producing them is given by
$$C = 2q_1^2 + 2q_2^2 + 10.$$
   (a) Find the maximum profit that can be made, assuming the prices are fixed.
   (b) Find the rate of change of that maximum profit as p_1 increases.

5. Consider the function
$$f(x, y) = |xy|.$$
   (a) Use a computer to draw the graph of f. Does the graph look like a plane when we zoom in on the origin?
   (b) Is f differentiable at $(x, y) \neq (0, 0)$?
   (c) Show that $f_x(0, 0)$ and $f_y(0, 0)$ exist.
   (d) Are f_x and f_y continuous at $(0, 0)$?
   (e) Is f differentiable at $(0, 0)$? **Hint:** Consider the directional derivative
$$f_{\mathbf{u}}(0, 0) \quad \text{for} \quad \mathbf{u} = \frac{\mathbf{i} + \mathbf{j}}{\sqrt{2}}.$$