Homework

1. Evaluate $\int_0^3 (3x+7)dx$ as a limit of Riemann sums.
2. Why does the Fundamental Theorem of Calculus not imply that
$$\int_{-1}^{1} \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_{-1}^{1} = -\frac{1}{1} - \left(-\frac{1}{-1}\right) = -2?$$
Why does this not make sense?
3. Find a function $g(x)$ such that $g'(x) = \sqrt{1+x^2}$ and $g(2) = 0$.
4. Find $\frac{d}{dx}\int_{x^2}^{3} \frac{\sin t}{t}\,dt$.
5. True or False? Give reasons for your answer.
   (a) The double integral $\int_R f\,dA$ is always positive.
   (b) If $f(x,y) = k$ for all points in a region R, then $\int_R f\,dA = k \cdot \text{Area}R$
   (c) If $\int_R f\,dA = 0$ then $f(x,y) = 0$ at all points of R.
6. Sketch the region of integration
   (a) $\displaystyle\int_0^2 \int_0^{y^2} y^2 x\,dx\,dy$
   (b) $\displaystyle\int_0^{\pi} \int_0^{x} y\sin x\,dy\,dx$
7. A city occupies a semicircular region of radius 3 km bordering on the ocean. Find the average distance from points in the city to the ocean.
8. Sketch the region of integration for the iterated integral
$$\int_0^6 \int_{x/3}^2 x\sqrt{y^3+1}\,dydx.$$
Then reverse the order of integration.
9. Find the area of the crescent-moon shape with circular arcs as edges and the dimensions shown in figure below.