Assignment7

Q1.
Let's first define the following symbols:

- S: Sam plays baseball.
- P: Paul plays baseball.
- R: Ryan plays baseball.

Now we can express the knowledge base as:

1. Sam plays baseball or Paul plays baseball: $S \vee P$.
2. Sam plays baseball or Ryan doesn't play baseball: $S \vee \neg R$.

So, we can define the knowledge base as: KB: $\{S \vee P \ \wedge \ S \vee \neg R\}$.

Let's analyze the following sentences to see whether they are entailed by KB:

- Sam and Ryan both play baseball: this sentence can be expressed as $S \wedge R$. This is not entailed by KB since we can find a counterexample such that KB is true and the sentence is false. For instance, the model $S = True, P = False, R = True$ makes KB true ($True \vee False \ \wedge \ True \vee False = True$), but the sentence is not ($True \wedge False = False$).
- At least one among Sam, Paul and Ryan play baseball: this sentence can be expressed as $S \vee P \vee R$. This is entailed by KB for a simple reason: just looking at the first part of KB we can see that either Sam or Paul (or both) play baseball, which means that at least one of them plays baseball. It's clear that whenever KB is true also the sentence is true, since if at least Sam or Paul play baseball, at least one among Sam, Paul and Ryan play baseball.
  To further confirm this, we could compute the truth table of the sentence and of KB:

S	P	R	KB	$S \vee P \vee R$
True	True	True	True	True
True	True	False	True	True
True	False	True	True	True
True	False	False	True	True
False	True	True	False	True
False	False	True	False	True
False	True	False	True	True
False	False	False	False	False

  So $KB => S \vee P \vee R$, since when KB is true, $S \vee P \vee R$ is also true. When KB is false, $S \vee P \vee R$ can be whatever.

Q2.
Let's first define the following symbols:

- A: Ana eats.
- B: Bret eats.
- C: Charles eats.
- D: Derek eats.
- E: Earl eats.

- F: Fred eats.
- G: Gary eats.

Now we can express the following sentences as:

1. If Ana eats, Bret eats: $A => B$.
2. Charles eats and Derek doesn't eat: $C \wedge \neg D$.
3. Bret doesn't eat: $\neg B$.
4. If Derek doesn't eat at least one among Ana, Earl and Fred eats: $\neg D => (A \vee E \vee F)$.
5. If at least one of Charles and Gary eats, Earl doesn't eat: $(C \vee G) => \neg E$.

Q3.

We can derive that Fred eats (F) by observing that:

- Since $\neg D$ is true from $C \wedge \neg D$, from $\neg D => (A \vee E \vee F)$ we get that $A \vee E \vee F$ is true.
- From $A => B$ and from $\neg B$ we can derive that A is false.
- From $(C \vee G) => \neg E$, since we know from $C \wedge \neg D$ that C is true we can derive $True => \neg E$ which leads to E being false.
- Since E and A are false we can derive from $A \vee E \vee F$ that F is true, meaning that Fred eats.