Assignment8

Q1.

Propositional logic is limited in what it can express using only facts, so first order logic overcomes these shortcomings by adding variables (objects), predicates (relations and functions) and quantifiers. In this way first order logic is able to express more complex statements.

In first order logic we can have two quantifiers that increase the expressiveness of the logic: universal quantifier (means "for all") and existential quantifier (means "there exists"). We can also use predicates that allow us to specify the "type" of each variable, like Train(x) to say that x is a train, but also some relationships between objects, like Parent(x, y) to say that x is parent of y.

Q2.

Universal and existence quantifiers are closely related to each other. In fact, we can define one in terms of the other: $\forall x\ P(x) = \neg\exists x\ \neg P(x)$. This is quite intuitive since if we try to "read it in English" we get: all x are such that $P(x)$ is true and this statement is equal to saying there exists no x such that $P(x)$ is false.

Q3.

Let's first define the following predicates:

- $S(x)$: x is a student
- $T(x, CS411, Spring2020)$: x took the course CS411 in Spring 2020
- $W(x, y)$: x wears y
- $H(x)$: x is a hoodie
- $UIC(x)$: x has the UIC logo on it
- $G(x)$: x glitters
- $Au(x)$: x is gold
- $Win(x, y)$: x wins against y
- $CS(x)$: x is a CS course
- $D(x)$: x is difficult

We can define the following sentences as:

- Some students took CS411 in Spring2020:
$$\exists x\ \big(S(x) \wedge\ T(x, CS411, Spring2020)\big)$$
- Some students wear a hoodie with UIC logo on it:
$$\exists x\ \big(S(x) \wedge H(Hoodie) \wedge UIC(Hoodie) \wedge W(x, Hoodie)\big)$$
- Something that glitters is not always gold, whereas gold always glitters:
$$\exists x\ \big(G(x) \wedge \neg Au(x)\big) \wedge \forall x\big(Au(x) \Rightarrow G(x)\big)$$
- No one can win with everyone all the time:
$$\forall x\ \neg\Big(\forall y\ \big((x \neq y) \Rightarrow Win(x, y)\big)\Big)$$
- All CS courses are difficult, except two:
$$\exists x \exists y\ \Big(CS(x) \wedge CS(y) \wedge \neg D(x) \wedge \neg D(y) \wedge (x \neq y) \wedge \forall z\ \Big(\big(CS(z) \wedge (z \neq x) \wedge (z \neq y)\big) \Rightarrow D(z)\Big)\Big)$$