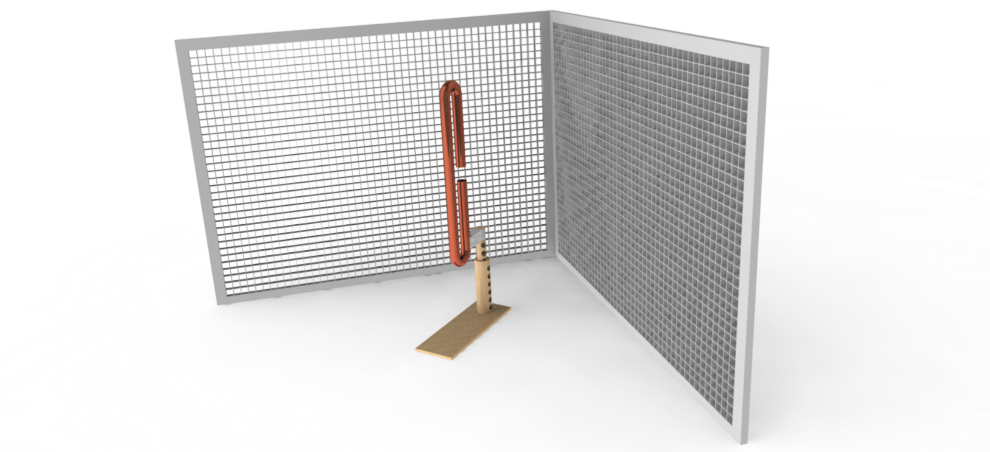
University of Twente

Group Fleming

Advanced Technology

Antenna design



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1 Introduction

An antenna is used to convert an electrical signal into an electric field, but it can also be used the other way around. This depends of course on whether you want to receive or transmit a signal. A large range of antenna designs can be found over the world, and every design has its certain advantages in terms of gain, directivity and feasibility in the fabrication process.
In the previous project plan the design of a folded dipole antenna was selected for its high feasibility. A folded dipole antenna is basically joining two dipoles at the ends and attaching the transmission lines at the center of one side. Furthermore, a corner reflector was also included in the design to increase the gain and directivity. Basically, a corner reflector consists of two intersecting sheets of a conducting material placed near the dipole.
The antenna that will be constructed at the end of this project has to meet some requirements and specifications as well. The requirements can be found in the project plan [1]. These requirements include a maximum cost of €25, an operating frequency of 433 MHz which is in the High to Ultra High Frequency range. Other specifications include the dimension that are found using calculations and simulations, respectively in sections 3 and 4.
This report focuses on the antenna design. A more in-depth knowledge can be acquired on the theory behind the folded dipole antenna and on topics such as electrodynamics. Based on this theory, a suitable antenna design is chosen and its design is validated using calculations to illustrate the main working principles of the antenna. In the last step of validating some simulations were done to verify the theory.

2 Theory

2.1 Folded Dipole

To find the electric field, magnetic field, poynting vector of the folded dipole, first the hertzian dipole is solved. This is used as a basis and adapted to a linear dipole. This linear dipole is then adjusted to a half wave dipole and with some assumptions adjusted to a folded dipole. The derivations can be read in appendix A. The electric field($\vec{E}$), magnetic field($\vec{B}$) and poynting vector($\vec{S}$) become:

$$\vec{E} = \left(0, \frac{jk}{\pi R}\sqrt{\frac{\mu_0}{\epsilon_0}} e^{-jkR}\, \frac{\cos((\pi/2)\cos(\theta))}{k\sin(\theta)} \left(\frac{\lambda - a}{\lambda}\right), 0\right) \quad \text{for } 0 < a < \lambda \tag{1}$$

$$\vec{B} = \left(0, 0, \frac{jk}{\pi R} e^{-jkR}\, \frac{\cos((\pi/2)\cos(\theta))}{k\sin(\theta)} \left(\frac{\lambda - a}{\lambda}\right)\right) \quad \text{for } 0 < a < \lambda \tag{2}$$

$$\vec{S} = \left(-\frac{(a-\lambda)^2 \cos^2\left(\frac{1}{2}\pi\cos(\theta)\right) e^{-2ikR}\sqrt{\frac{\mu}{\epsilon}}}{\pi^2\lambda^2\mu R^2 \sin^2(\theta)}, 0, 0\right) \tag{3}$$

Where the coordinate system is spherical (r, θ, ϕ), with θ as inclination and ϕ as the azimuthal angle.

2.2 Corner Reflector

A corner reflector consists of two conducting metal sheets connected at an angle, placed behind a dipole antenna. [2] On its own it can be used to reflect radio waves back to their source, but when combined with a dipole, it becomes a corner reflector antenna. [3]

The addition of a corner reflector to a dipole results in an increased gain and directivity.
Figure 2 (left) provides an overview of several parameters for such an antenna. The distance s is approximately half of the wavelength. l is approximately one whole wavelength, while the distance D is kept between one and two wavelengths. [2] The height of the two conducting plates is 1.2 - 1.5 times the length of the dipole. The effect of the corner reflector on the antenna can be explained using image theory. [4] The two conducting plates effectively work as mirrors, as can be seen in figure 2 (right). If the angle α is 180^o, there would only be one image of the feed #1. Decreasing the angle would result in two images. At 90^o there will be three images, as can be seen in the figure. Image #4 would be in phase with the feed #1, while image #2 and #3 would both be 180^o out of phase. This is indicated by the crosses and dots in the figure. The three images and the feed add up to a total of four antenna's and thus an intensity which will be four times larger than that of the feed. Decreasing the angle even further than 90^o will result in more than three images, however it does not result in a significant increase in gain.

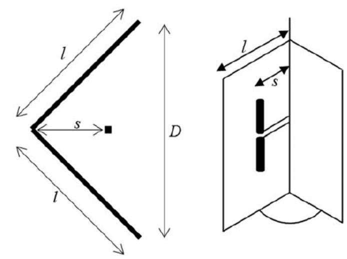

Figure 1: Parameters of a corner reflector

Therefore, the most often used angle is 90^o. An exact analytical solution for the values of the corner reflector parameters cannot be derived from this image theory, therefore these dimensions are often used as guidelines. [4] The exact values of the corner reflector are not as important as for the image theory, infinite conducting plates are assumed. As long as the dimensions are large enough, the theory holds true.
Rather than using two conducting plates, also a conducting wire screen can be used. As long as the distance of the wires is small enough such that the waves cannot pass trough, this should provide sufficient shielding to prevent the waves from going in the backward direction. The derivation for the required spacing can be found in appendix D Since the waves cannot go into the opposite direction, the directivity of the antenna is increased too.

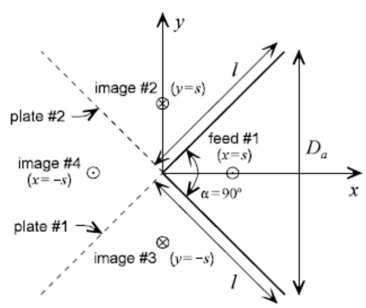

Figure 2: mirrored images of the feed created by the corner reflector

The array factor is used to describe the radiation pattern of an array of antennas, in this case the corner reflector and its three reflections. [5]. The array factor for the corner reflector antenna with a 90^o angle is:

$$AF = 2[\cos(ks\cos\phi) - \cos(ks\sin\phi)] \tag{4}$$

The derivation of this formula can be found in appendix D.

2.3 Impedance

The impedance depends on several factors such as the antenna design, the proximity to other dipoles and specially the corner reflector. For the purpose of this section, the theory will be derived only for

a folded dipole without the inclusion of other external factors. Further on the project, simulations will be made to analyze how the impedance changes by adding the corner reflector.
Essentially, the complex antenna impedance is defined as: $Z = R + jX$, where R is the resistance of the antenna and X is the reactance [6]. It is important to match the input impedance to the output impedance to minimize the reflection of the signal and to maximize the power transfer [7]. From the voltage transfer coefficient [8]:

$$\Gamma = \frac{Z_i - Z_o}{Z_i + Z_o} \tag{5}$$

it is seen that if $Z_i = Z_o$ then Γ is zero, which ensures that the signal is not reflected back into the transmission line.
As seen from figure 3, the system is composed of a transmission, a coax cable and an antenna, where the input impedance is composed of the transmission and the coax cable and the output impedance the the folded dipole.

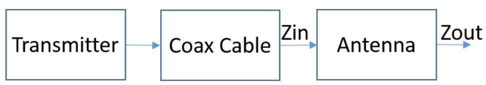

Figure 3: Input impedance (Z_i) is the connection from the coax cable to the antenna and the output impedance (Z_o) is the impedance of the folded dipole.

For simplification, an assumption is made that the impedance of the transmission is comparably lower, therefore it can be neglected. Thus, to match the impedance, the first step is to calculate the impedance of a folded dipole antenna and the coax cable.

Folded Dipole From the theory of the folded dipole, 2.1, it is known that the dipole is radiating, which implies that Kirchoff's current law does not hold. Therefore, the folded dipole can be analyzed by decomposing the current into a transmission mode and an antenna mode [9], as seen from figure 14. The transmission mode describes the current flowing in opposite directions, which yields little radiation. Now Kirchoff's current law is satisfied, and calculations are simpler because the input impedance of a transmission line is well known. In the antenna mode the current flows in the same direction, yielding radiation. This mode represents the input impedance of an ordinary dipole of the same wire size.
Thus, by combining the expressions of current for the transmission and antenna modes I_a and I_t, the impedance of the folded dipole can be described as:

$$Z_{fd} = \frac{4Z_tZ_a}{Z_t + 2Z_a} \tag{6}$$

by taking the limit of Z_t to infinity the expression can be simplified to: $Z_{fd} = 4Z_a$, where the full derivation of these expressions can be found in the appendix C.

Coax Cable The coax cable consists of two cylindrical conductors with radii a and b respectively. The input impedance of the coax cable is given by [10]:

$$Z_{cc} = \sqrt{\frac{L}{C}} = \left(\frac{\mu_0}{4\pi^2\epsilon_0}\right)^{(1/2)} \frac{ln\left(\frac{b}{a}\right)}{\sqrt{\epsilon}} \tag{7}$$

μ_0 being the permeability and ϵ_0 the permittivity and ϵ the dielectric constant of the material in between the two conductors. The derivation for this equation can be found in the appendix C.

3 Calculations

In the design of antenna systems, one must consider important requirements such as the power radiated, the impedances, phase change and the distance between the corner reflector and the antenna. All of which are calculated here.

3.1 Power

The total raidated power $P_R\,|W|$, can be calculated by integrating the radial component $\langle P_r \rangle$ of $\left\langle \mathbf{P}(\vec{T}, \theta) \right\rangle$ over all directions

$$P_R = \int_0^{2\pi} d\phi \int_0^{\pi} \langle P_r \rangle\, r^2 sin^2\theta d\theta, \tag{8}$$

$$P_R = \pi\eta_0 \left| \frac{kId}{4\pi} \right|^2 \int_0^{\pi} sin^3\theta d\theta = \frac{\eta_0}{12\pi} |kId|^2 \cong 395 \left(\frac{Id}{\lambda} \right)^2. \tag{9}$$

3.2 Directivity and Gain

Directivity is the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.

$$D(\theta, \phi) = \frac{P(r, \theta, \phi)}{P_R/4\pi r^2} \tag{10}$$

$$D(\theta, \phi) = \frac{\frac{\eta_0}{2} \left| \frac{kId}{4\pi r} \right|^2 sin^2\theta}{\frac{\eta_0}{12\pi} \frac{|kId|^2}{(4\pi r)^2}} = 6\pi sin^2\theta \tag{11}$$

Gain is the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically. The maximum directive gain of an antenna is called the directivity of the antenna. Directivity the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. Begin by calculating the radiation intensity produced by the dipole

$$U(\theta) = \frac{1}{2} r^2 |E_\theta|^2 = 36.5640 I_m^2 \tag{12}$$

The maximum effective area is A_{em} and the maximum directivity relative to an isotopic radiator is then calculated as

$$D_m = \frac{4\pi U_m}{P_{rad}} \approx 1.64 \approx 2.15 dBI \tag{13}$$

3.3 Impedance

The impedance is important for calculating the distance separating the two dipoles. As stated above in the theory, the input impedance should equal the output impedance to minimize the transfer coefficient, $\Gamma = 0$. The input impedance is the impedance of the coaxial cable which is 75Ω [10]. This can be calculated using equation 7. Filling in the values, $\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$,

$\epsilon_0 = 8.854 \times 10^{-12} Fm^{-1}$, $a = 0.0285cm$, $b = 0.332cm$ [10]. We therefore get that the Z_{in} is 75Ω. The output impedance, or the impedance of the half wave folded dipole, can be calculated using equation 6. An ordinary dipole has an impedance of 70Ω. Therefore, the impedance of the folded dipole is

$$Z_{fd} = 4Z_a = 4(70) = 280\Omega \tag{14}$$

3.4 Phase

The phase change is dependent on the separation of the two dipoles. To maximize the signal, we want the two dipole to have a phase difference of either 0 or π. If we have a phase difference of π then the antenna will radiate in two directions. However, we need the antenna to radiate in one direction. Therefore a phase difference of 0 is preferable which practically is not possible. Therefore, as a rule of thrumb, a separation distance of 0.05λ [11] is used. This means that the separation distance from the center of the pipe is $3.45cm$.

3.5 Corner Reflector

From the theory of the corner reflector it is known that by placing the two conducting plates at an angle of 90 degrees, three images are created and together with the dipole, it forms four sources. This is equivalent to a special case of spatial array. [12]
To calculate the gain first the field intensity of the dipole can be described as:

$$F = \sqrt{\frac{P}{R_{00} + R_{0L}}} k \tag{15}$$

where k is the current distribution, R_{00} is the self-resistance and R_{0L} is the loss resistance of the dipole. By placing a corner reflector at a distance S from the dipole, the field intensity is given by: $F' = 2kI\left[\cos(S\cos\phi) - \cos(S\sin\phi)\right]$. The current I can be described in terms of the voltage and the power.

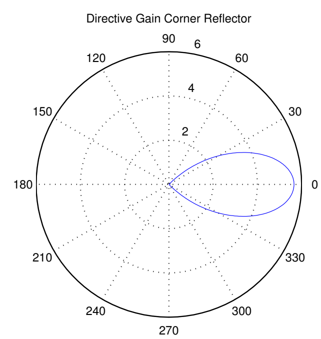

Figure 4

$$I = \sqrt{\frac{P}{R_{11} + R_{1L} + R_{14} - 2R_{12}}} \tag{16}$$

Substituting the previous equations the expression for the gain is:

$$G = \frac{F'}{F} = 2\left[\cos(S\cos\phi) - \cos(S\sin\phi)\right]\sqrt{\frac{R_{00} + R_{0L}}{R_{11} + R_{1L} + R_{14} - 2R_{12}}} \tag{17}$$

Figure 4, represents the gain in a polar plot.

4 Simulations

4.1 Folded Dipole

To pursue earlier simulations 4NEC2 software was used, this is to be exchanged with a more elaborate Comsol Model for the final report. A standard folded dipole antenna was modeled and experimentation with the wire's diameters was conducted. Data regarding the gain and source impedance was collected and can be found in Appendix E.1. In addition to altering the diameters, the ratios of segments along the long elements corresponding to the main wires were also changed. From all simulations in 4NEC2 the lowest source impedance of the folded dipole antenna was $164.5 + j0.6\Omega$ while the highest found source impedance was $329 - j2.33\Omega$. A figure of how the model was set up in 4NEC2 as well as the 3-Dimensional and E-plane radiation patterns can also be found in Appendix E.1.

4.2 Dipole

To get acquainted with modeling antennas in Comsol a model of a standard dipole was set up. The investigation into this model was restricted to a parametric sweep to find the optimal source impedance of a standard dipole antenna. It was found to be $120.9 + j28\Omega$, an elaboration as well as several radiation patterns can be found in Appendix E.2. A full-fledged simulation of a folded dipole antenna and corner reflector of our specific dimensions will be conducted for the final report.

5 Proposed Construction

As proposed from the project plan the chosen antenna was the folded dipole with a corner reflector. Below is an elaboration on design and material choices

5.1 Specification of design

A Solidworks design of the proposed design was made. The 3D view can be seen on the title page. The exact dimensions can be found in figure 5.

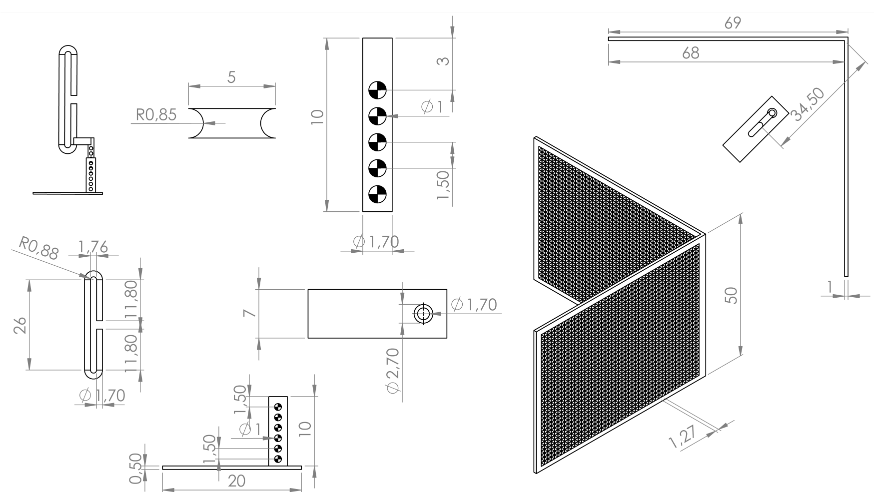

Figure 5: Exact dimensions of the antenna design

The height adjustment system in this design is chosen this way as the dipole needs to be movable in the y-axis. The corner reflector consists of a metal frame with chicken wire in between. Validation for the chicken wire can be found in appendix D.4 The exact dimensions and distances in figure 5 come from sections 3 & 4.

5.2 Material Choice

The material of the folded dipole is required to have a good conductivity at a reasonable cost, this is fulfilled optimally by copper.[13] The corner reflector will be made from a wire mesh, since it only has to be made from a conductive material to be able to reflect waves. [14] The choice for the coax cable type is the RG59 with its characteristic impedance of 75Ω. Ideally, a short cable is preferred to prevent a large signal loss.[15] This coax also has to be transformed to match the impedance of the folded dipole to work properly through a balun transformer.
The balun transformer will be made from a ferrite toroid core with copper wire wound around it. The ferrite is chosen because of its nice magnetic permeability, while the copper wiring is needed to conduct the signal.[16] The stand of the antenna will be made from wood, because this is a cheap material which insulates electrical waves. This will prevent stray signals to go through the stand. The mount between the dipole and the stand will be made of perspex as this material insulates very good and is strong as well.

6 Summary and Conclusions

This midterm report had its main goal to focus on the antenna design specifications of the folded dipole antenna with a reflector. In order to get to these design specifications several calculations were done. After giving a minor section on related theory that was used to come to these specifications, various expressions have been found for the impedance as used for impedance matching, phase change of the dipole as used for maximizing the signal and the exact position and dimensions for the corner reflector. The initial thought was to use these expressions in a model in order to do simulations on how our proposed antenna design would work.
In order to reach the goal of the best working antenna in this module, a closer look was taken on how to maximize the power output, directivity and gain. It was found that to maximize the power output the impedances of the folded dipole and coax cable should be matched. The value of 280Ω was found and this is the value that has to be matched to. For the optimal directivity calculations on the exact dimensions and positioning of the corner reflector were done and can be found in the calculations section. The maximal gain was found as well and can be found under calculations.
The next steps that have to be taken are in order to confirm the design found in this report with the last simulations using the COMSOL software. After these simulations the design can be further improved and the construction process can begin.

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Appendices

A Maxwell's Equations

The four Maxwell's equations in derivative form are described as

Gauss's law	$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$
Gauss's law for magnetism	$\nabla \cdot \vec{B} = 0$
Maxwell-Faraday equation	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Ampere's circuital law	$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$

Table 1: Maxwell's Equations

From here the radiation emitted by some time varying charges and currents can be found. Starting with the curl of the curl of the electric field that depends on the distance and time. Using the identity that $c^2 = (\mu_0 \epsilon_0)^{-1/2}$ [17]

$$\nabla \times \nabla \times \vec{E}(\vec{r}, t) = -\frac{\partial \nabla \times \vec{B}}{\partial t} = -\frac{\partial \mu_0 \, \vec{J}(\vec{r}, t)}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \vec{E}(\vec{r}, t)}{\partial t^2} \tag{18}$$

The divergence of the magnetic field is always zero, giving the relation between the electric field and vector potential as followed:

$$\nabla \cdot \vec{B}(\vec{r}, t) = 0 \quad \text{such that } \vec{B}(\vec{r}, t) = \nabla \times \vec{A}(\vec{r}, t) \tag{19}$$

$$\nabla \times \vec{E}(\vec{r}, t) = -\frac{\partial \nabla \times \vec{A}(\vec{r}, t)}{\partial t} \tag{20}$$

$$\nabla \times \left\{ \vec{E}(\vec{r}, t) + \frac{\partial \vec{A}(\vec{r}, t)}{\partial t} \right\} = \vec{0} \tag{21}$$

Because the curl of electric field and the time derivative of the vector potential is zero, one can always introduce a potential.

$$\vec{E}(\vec{r}, t) = -\frac{\partial \vec{A}(\vec{r}, t)}{\partial t} - \nabla \phi(\vec{r}, t) \tag{22}$$

Now fixing the gauge with the Lorentz gauge, which gives the relation between the vector potential and the scalar potential.

$$\nabla \cdot \vec{A}(\vec{r}, t) = -\frac{1}{c^2} \frac{\partial \phi(\vec{r}, t)}{\partial t} \tag{23}$$

To find the vector potential wave equation, Ampere's law is used.

$$\nabla \times \vec{B}(\vec{r}, t) = \mu_0 \vec{J}(\vec{r}, t) + \frac{1}{c^2} \frac{\partial \vec{E}(\vec{r}, t)}{\partial t}$$

$$\nabla \times \nabla \times \vec{A}(\vec{r}, t) = \mu_0 \vec{J}(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2 \vec{A}(\vec{r}, t)}{\partial t^2} - \frac{1}{c^2} \nabla \left[\frac{\partial \phi(\vec{r}, t)}{\partial t} \right] \tag{24}$$

Remembering that $\nabla \times \nabla \times \vec{A}(\vec{r},t) = \nabla \left[\nabla \cdot \vec{A}(\vec{r},t)\right] - \nabla^2 \vec{A}(\vec{r},t)$ and the Lorentz gauge.

$$\nabla^2 \vec{A}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2 \vec{A}(\vec{r},t)}{\partial t^2} = -\mu_0 \vec{J}(\vec{r},t) \tag{25}$$

Which is the vector potential wave equation. The same can be done with the scalar potential. Remembering that $\vec{E}(\vec{r},t) = -\frac{\partial \vec{A}(\vec{r},t)}{\partial t} - \nabla\phi(\vec{r},t)$.

$$\nabla \cdot \vec{E}(\vec{r},t) = \frac{\rho(\vec{r},t)}{\epsilon_0}$$

$$-\frac{\partial \nabla \cdot \vec{A}(\vec{r},t)}{\partial t} - \nabla^2\phi(\vec{r},t) = \frac{\rho(\vec{r},t)}{\epsilon_0} \tag{26}$$

Using the gauge from earlier, the scalar potential becomes as followed

$$\nabla^2\phi(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2\phi(\vec{r},t)}{\partial t^2} = \frac{\rho(\vec{r},t)}{\epsilon_0} \tag{27}$$

A.1 Solutions to the wave equations

In electrostatics, the gauge is fixed at zero ($\nabla \cdot \vec{A}(\vec{r},t) = 0$) which gives the Poisson equation

$$\nabla^2\phi(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0} \tag{28}$$

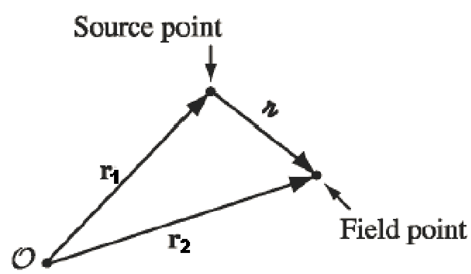


Figure 6: Direction of r_2 and r_1 [18].

With $\vec{r}_2$ as the distance from the origin to the observer, $\vec{r}_1$ the distance from the origin to the 'source', such that $r = \vec{r}_2 - \vec{r}_1$ is the distance between the source and the observer.
The solution of the poisson equation is

$$\phi(\vec{r}_2) = \iiint \frac{\rho(\vec{r}_1)}{4\pi\epsilon_0|\vec{r}_2 - \vec{r}_1|} dv_1 \tag{29}$$

The wave equation has also a time dependency in it

$$\nabla^2\phi(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2\phi(\vec{r},t)}{\partial t^2} = \frac{\rho(\vec{r},t)}{\epsilon_0} \tag{30}$$

Which has the solution

$$\phi(\vec{r}_2, t) = \iiint \frac{\rho(\vec{r}_1, t - |\vec{r}_2 - \vec{r}_1|/c)}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|} dv_1 \tag{31}$$

Which is the retarded potential. It is the potential of a charge at the source at an earlier time, the retarded time. That is because electromagnetic waves travel at the speed c.

The vector Poisson equation is

$$\nabla^2 \vec{A}(\vec{r}, t) = -\mu_0 \vec{J}(\vec{r}) \tag{32}$$

And has the solution

$$\vec{A}(\vec{r}_2) = \iiint \frac{\mu_0 \vec{J}(\vec{r}_1)}{4\pi |\vec{r}_2 - \vec{r}_1|} dv_1 \tag{33}$$

The vector potential wave equation was

$$\nabla^2 \vec{A}(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2 \vec{A}(\vec{r}, t)}{\partial t^2} = -\mu_0 \vec{J}(\vec{r}, t) \tag{34}$$

Which then has the solution of the retarded vector potential

$$\vec{A}(\vec{r}_2, t) = \iiint \frac{\mu_0 \vec{J}(\vec{r}_1, t - |\vec{r}_2 - \vec{r}_1|/c)}{4\pi |\vec{r}_2 - \vec{r}_1|} dv_1 \tag{35}$$

To solve the integral, the radial dependence and the time dependence is split.

$$\begin{aligned}
\vec{E}(\vec{r}, t) &= \mathrm{Re}\left[\vec{E}(\vec{r}) e^{j\omega t}\right] & \vec{B}(\vec{r}, t) &= \mathrm{Re}\left[\vec{B}(\vec{r}) e^{j\omega t}\right] \\
\phi(\vec{r}, t) &= \mathrm{Re}\left[\phi(\vec{r}) e^{j\omega t}\right] & \vec{A}(\vec{r}, t) &= \mathrm{Re}\left[\vec{A}(\vec{r}) e^{j\omega t}\right] \\
\rho(\vec{r}, t) &= \mathrm{Re}\left[\rho(\vec{r}) e^{j\omega t}\right] & \vec{J}(\vec{r}, t) &= \mathrm{Re}\left[\vec{J}(\vec{r}) e^{j\omega t}\right]
\end{aligned} \tag{36}$$

Filling these separation by variables into the wave equations 30 and 34 will give

$$\nabla^2 \phi \vec{r} + k^2 \phi(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0} \tag{37}$$

$$\nabla^2 \vec{A}(\vec{r}) + k^2 \vec{A}(\vec{r}) = -\mu_0 \vec{J}(\vec{r}) \tag{38}$$

Where $k = \frac{\omega}{c}$.

Using the fact that $\rho(\vec{r}_1, t - |\vec{r}_2 - \vec{r}_1|/c) = \mathrm{Re}\left[\rho(\vec{r}_1) e^{j\omega(t - |\vec{r}_2 - \vec{r}_1|/c)}\right] = \mathrm{Re}\left[\rho(\vec{r}_1) e^{-j\omega|\vec{r}_2 - \vec{r}_1|/c)}\ e^{j\omega t}\right]$ and $\vec{J}(\vec{r}_1, t - |\vec{r}_2 - \vec{r}_1|/c) = \mathrm{Re}\left[\vec{J}(\vec{r}) e^{j\,\omega\,(t - |\vec{r}_2 - \vec{r}_1|/c)}\right] = \mathrm{Re}\left[\vec{J}(\vec{r}) e^{-j\,\omega|\vec{r}_2 - \vec{r}_1|/c}\ e^{j\omega t}\right]$.

Thus the scalar potential and the vector potential becomes.

$$\phi(\vec{r}_2) = \iiint \frac{\rho}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|} e^{-j\,k\,|\vec{r}_2 - \vec{r}_1|} dv_1 \tag{39}$$

$$\vec{A}(\vec{r}_2) = \iiint \frac{\mu_0 \vec{J}}{4\pi|\vec{r}_2 - \vec{r}_1|} e^{-j\,k\,|\vec{r}_2-\vec{r}_1|} dv_1 \tag{40}$$

Now there are two methods to solve the fields, the first method is solving equation 37 and 38 and find the $\vec{E}$ and $\vec{B}$ fields by equations 19 and 22 where $\frac{\partial \vec{A}(\vec{r},t)}{\partial t} = j\omega\vec{A}(\vec{r})$.

The second method is slightly faster. First solve equation 38 and then use 19 and Ampere's law($\nabla \times \vec{B}(\vec{r}) = \mu_0\vec{J}(\vec{r}) + \frac{j\omega}{c^2}\vec{E}(\vec{r})$).

A.2 Herzian Dipole Antenna

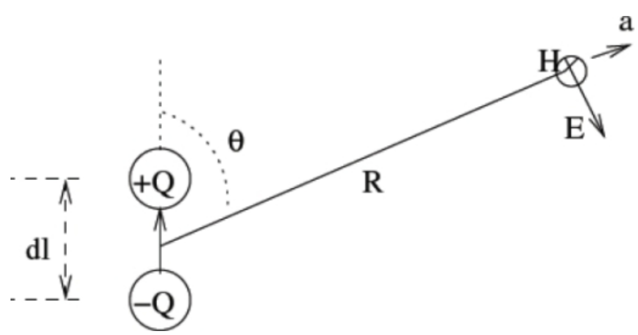


Figure 7: Hertzian dipole [19].

The Herzian dipole is a simple radiating element where analytical solutions for the field can be obtained. Therefore the Herzian dipole will be used for the basis of the antenna theory. [20]

A Herzian dipole has a positive and a negative charge with the same magnitude located at a distance δl from each other. When these charges move, a current will flow.

Suppose the Herzian dipole has charges that go up and down in the z-direction.
$\iiint \vec{J} dx\, dy\, dz = Id\ \hat{z}$. The current oscillating is $i(t) = \mathrm{Re}[Ie^{j\omega t}]$, with $i(t) = \pm\frac{dq(t)}{dt} = \pm\frac{\mathrm{Re}[Qe^{j\omega t}]}{dt} = \pm j\omega Q e^{j\omega t}$, such that $I = \pm j\omega Q$. If the size of the dipole is much smaller than the wavelength, it can be rewritten as $\vec{J}(\vec{r}) = Id\delta^3(\vec{r})$.

What is the radiation emitted by a Hertzian dipole? First solve equation 38 for that the integral of the $\vec{A}(r,\vec{t})$ is used. Because the charges move only in the z-direction a choice for the A field can be that $A_x = A_y = 0$ and only A_z has a value. When $r >> d$, $|\vec{r}_2 - z_1\hat{z}| \approx r_2$.

$$\vec{A}(\vec{r}_2) = \iiint \frac{\mu_0 \vec{J}}{4\pi\vec{r}_2} e^{-j\,k\,\vec{r}_2} dv_1 = \frac{\mu_0\, I\, \delta l}{4\pi}\, \frac{e^{-j\,k\,r_2}}{r}\hat{z} \tag{41}$$

With $k = \omega/c = 2\pi/\lambda$.

In antenna theory, the coordinate system is often spherical. Therefore the coordination change becomes, see figure 8

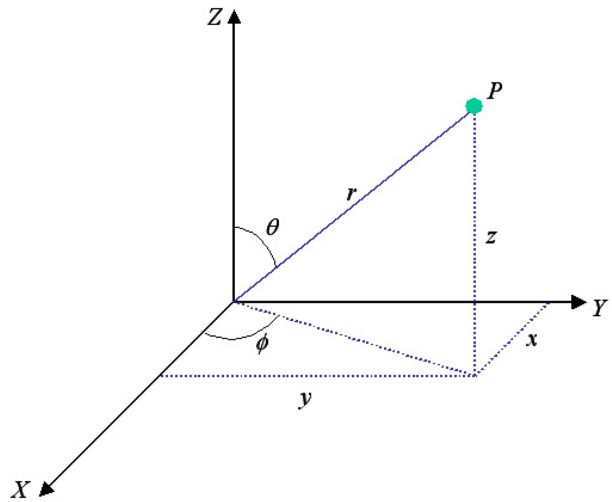


Figure 8: Spherical coordinate system [21].

$$
\begin{aligned}
\hat{z} &= \cos\theta\ \hat{r} - \sin\theta\ \hat{\theta} \\
A_r = A_z \cos\theta &= \frac{\mu_0\ I\ \delta l}{4\pi} \left(\frac{e - jkr_2}{r_2} \right) \cos\theta \\
A_\theta = -A_z \sin\theta &= -\frac{\mu_0\ I\ \delta l}{4\pi} \left(\frac{e - jkr_2}{r_2} \right) \sin\theta \\
A_\phi &= 0
\end{aligned} \tag{42}
$$

Because of the symmetry seen in figure 7, changing ϕ does not make any difference for the A field. Now equation 19 can be used to find the magnetic field.

$$\vec{B} = \nabla \times \vec{A} = \frac{1}{\mu_0 r_2} \left(\frac{\partial}{\partial r_2}(r_2 A_\theta - \frac{\partial A_r}{\partial \theta} \right)\ \hat{\phi} = -\frac{I\delta l}{4\pi} k^2 \sin\theta \left(\frac{1}{jkr_2} + \frac{1}{(jkr_2)^2} \right) e^{-jkr_2}\ \hat{\phi} \tag{43}$$

Ampere's law tells that if no current is present at the observation point

$$
\begin{aligned}
\nabla \times \vec{B} &= \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \\
\vec{E} = \frac{1}{j\omega\epsilon_0\mu_0} \nabla \times \vec{B} &= \frac{1}{j\omega\epsilon_0\mu_0} \left(\frac{1}{r_2 \sin\theta} \frac{\partial}{\partial\theta}(B_\phi \sin\theta)\ \hat{r} - \frac{1}{r_2} \frac{\partial}{\partial r_2}(r_2 B_\phi)\ \hat{\theta} \right)
\end{aligned} \tag{44}
$$

Which results into

$$E_r = -\frac{I\delta l}{4\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} k^2 2 \cos\theta \left(\frac{1}{(jkr_2)^2)} + \frac{1}{(jkr_2)^3} \right) e^{-jkr_2} \tag{45}$$

$$E_\theta = -\frac{I\delta l}{4\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} k^2 \sin\theta \left(\frac{1}{jkr_2} + \frac{1}{(jkr_2)^2)} + \frac{1}{(jkr_2)^3} \right) e^{-jkr_2} \tag{46}$$

$$E_\phi = 0 \tag{47}$$

A.3 Near field

In the near field ($k\, r_2 = 2\pi r_2/\lambda << 1$), the driving terms of the magnetic field become

$$B_\phi = \frac{I\,\delta l}{4\pi(r_2)^2}\sin(\theta)\, e^{-jkr_2} \approx \frac{I\,\delta l}{4\pi(r_2)^2}\sin(\theta) \tag{48}$$

Where for $e^{-jkr_2} \approx 1$ a Taylor expansion is used.
For the electric field, the same procedure is followed

$$E_r = \frac{I\,\delta l}{4\pi}\sqrt{\frac{\mu_0}{\epsilon_0}}\frac{2\cos\theta}{jkr_2^3} \tag{49}$$

This can be rewritten if $i(t) = \mathrm{Re}(Ie^{j\omega t}) = \frac{dq(t)}{dt}$ as is mentioned earlier. If $q(t) = \mathrm{Re}(Qe^{j\omega t})$, $I = j\omega Q$. Using that the dipole moment in this case is $\vec{p} = Q\delta l\, \hat{z}$ and $\mu_0\epsilon_0 = \frac{1}{c^2}$ and $k = \omega/c$, E_r can be rewritten as followed

$$E_r = \frac{p}{4\pi\epsilon_0 r_2^3}2\cos\theta \tag{50}$$

The same can be done for E_θ such that in near field the following fields are present:

$$B_r = 0 \qquad E_r = \frac{p}{4\pi\epsilon_0 r_2^3}2\cos\theta \tag{51}$$

$$B_\theta = 0 \qquad E_\theta = \frac{p}{4\pi\epsilon_0 r_2^3}\sin\theta \tag{52}$$

$$B_\phi = \frac{I\,\delta l}{4\pi(r_2)^2}\sin(\theta) \qquad E_\phi = 0 \tag{53}$$

A.4 Far field

In far field $k\, r_2 = 2\pi r_2/\lambda >> 1$. The magnetic field becomes

$$B_\phi = -\frac{I\,\delta l}{4\pi j}\left(\frac{e^{-jkr_2}}{r_2}\right)k\sin(\theta) \tag{54}$$

$$E_\theta = -\frac{I\,\delta l}{4\pi j}\left(\frac{e^{-jkr_2}}{r_2}\right)\sqrt{\frac{\mu_0}{\epsilon_0}}\,k\sin(\theta) \tag{55}$$

The rest of the terms go to zero very quickly if '$r_2\, k$' increases. One important remark for far field is that far field is further away at lower frequencies, because $k\, r_2 >> 1 \rightarrow r_2 >> \lambda/(2\pi)$ because $\lambda = c/f = 2\pi/k$.

A.5 Radiation pattern

An antenna pattern is here plotted for the magnitude of the normalized field strength versus θ with a constant ϕ(E-plane pattern) and with a ϕ with a constant $\theta(=\pi/2)$ (H-plane pattern).
For the Hertzian dipole the normalized magnitude of E_θ is $|\sin\theta|$. The E-plane pattern then becomes
The H-plane pattern gives $\theta = \pi/2$, $|\sin\theta| = 1$. Which means that changing over ϕ gives a circle of unity.

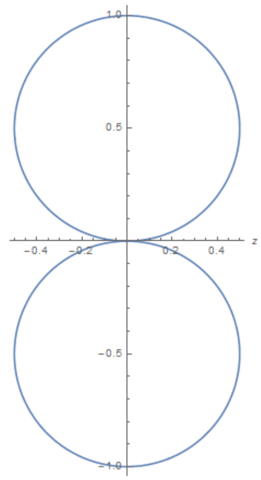


Figure 9: E-plane pattern of a Hertzian dipole

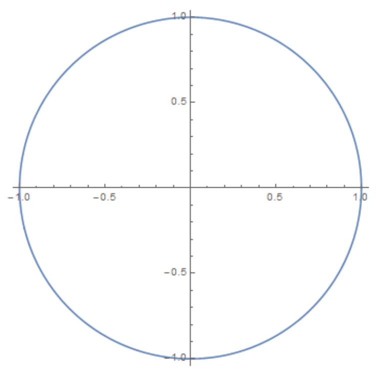


Figure 10: H-plane pattern of a Hertzian dipole

It is not necessary but to visualize this in 3d, because that can be seen from figures 9 and 10. The 3d plot can be seen in figure 11.

Because E_θ and B_ϕ are proportional to each other($E_\theta = \sqrt{\frac{\mu_0}{\epsilon_0}} B_\theta$), only the electric field is given in picture 9, 10 and 11.

A.6 Linear dipole antenna

A linear dipole antenna is an antanna that has a length comparable to the wavelength. The radiating field can be found by integrating over the entire length of the antenna where determining the current distribution is a very difficult task. An exact solution of the integral does not exist.
The current distribution is from the middle such that the current can be described as:

$$\begin{aligned} I(z) &= I_0 \sin k(h - |z|) \\ I(z) &= I_0 \sin k(h - z) \qquad z > 0 \\ I(z) &= I_0 \sin k(h + z) \qquad z < 0 \end{aligned} \tag{56}$$

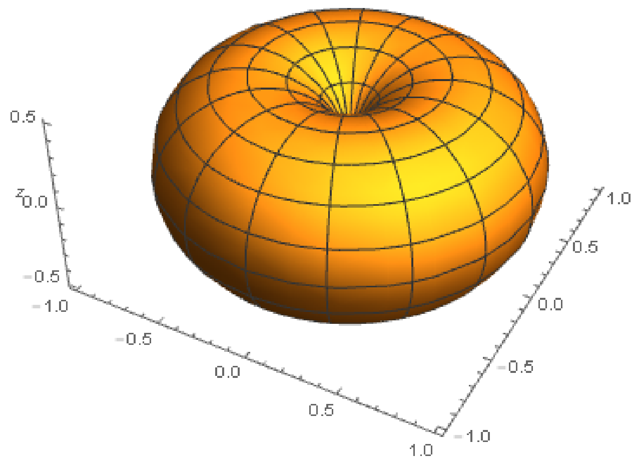


Figure 11: 3d radiation pattern of a Hertzian dipole with the hertzian dipole on the z-axis

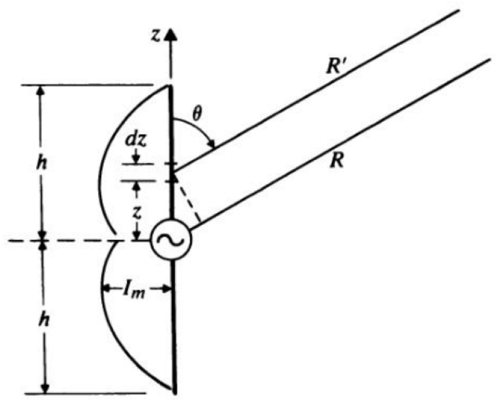


Figure 12: A center fed linear dipole from [20].

Using equation 54 or 55, the electric field is

$$\delta E_\theta = -\frac{I\,\delta l}{4\pi j}\left(\frac{e^{-jkr_2}}{r_2}\right)\sqrt{\frac{\mu_0}{\epsilon_0}}\,k\sin(\theta) \tag{57}$$

From figure 12, $(R' = R^2 + z^2 - 2Rz\cos\theta)^{1/2} \approx R - z\cos\theta$. Thus the electric field will become

$$E_\theta = \frac{jk\sin\theta}{4\pi R}\sqrt{\frac{\mu_0}{\epsilon_0}}e^{-jkR}\int_{-h}^{h}\sin\left(k(h-|z|)\right)e^{jkz\cos\theta}dz \tag{58}$$

Because of symmetry, that a $\sin\theta$ is a odd function and $\cos\theta$ is a even function, integrating between $-h$ and h gives zero for odd functions. A $\sin\cos\theta$ is an even function and $\cos\cos\theta$ is also an even function. $\sin k(h-|z|)$ is even giving:

$$\int_{-h}^{h} \sin\left(k(h-|z|)\right)\cos\left(kz\cos\theta\right) + j\sin\left(k\cos\theta\right)\sin\left(k(h-|z|)\right) \tag{59}$$

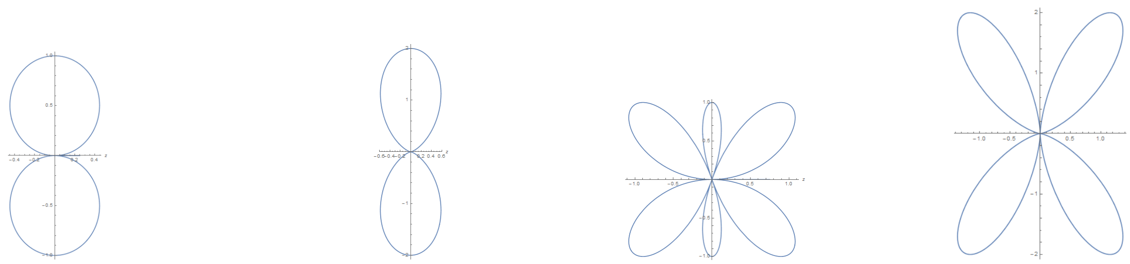
$$\int_{-h}^{h} f_{\text{even}} \cdot g_{\text{even}} + k_{\text{odd}} \cdot l_{\text{even}} dz = 2\int_{0}^{h} f_{\text{even}} \cdot g_{\text{even}} dz$$

And thus the integral simplifies to

$$E_\theta = \frac{jk\sin\theta}{2\pi R}\sqrt{\frac{\mu_0}{\epsilon_0}}e^{-jkR}\int_{0}^{h} \sin\left(k(h-|z|)\right)\cos\left(kz\cos\theta\right)dz \tag{60}$$

$$E_\theta = \frac{jk}{2\pi R}\sqrt{\frac{\mu_0}{\epsilon_0}}e^{-jkR}\ \frac{\cos(kh\cos(\theta)) - \cos(\mathrm{k}h)}{k\sin(\theta)} \tag{61}$$

For the E-plane patterns, the E_θ is only changing in ϕ. The pattern functions can be seen for different lengths of the dipole (2 h) in figure 13a, 13b, 13c and 13d.



(a) $2h/\lambda = 1/2$ (b) $2h/\lambda = 1$ (c) $2h/\lambda = 3/2$ (d) $2h/\lambda = 2$

Figure 13: E-plane radiation pattern with the dipole on the z-axis(horizontal) and having a height of different wavelengths.

A.7 Half wave dipole

In a folded dipole not a full wave dipole is used, but a two half wave dipoles. Therefore $kh = 2\pi h/\lambda = \pi/2$ because $2h = \lambda/2$. Using that, equation 61 changes to

$$E_\theta = \frac{jk}{2\pi R}\sqrt{\frac{\mu_0}{\epsilon_0}}e^{-jkR}\ \frac{\cos((\pi/2)\cos(\theta))}{k\sin(\theta)} \tag{62}$$

$$H_\phi = \frac{jk}{2\pi R}e^{-jkR}\ \frac{\cos((\pi/2)\cos(\theta))}{k\sin(\theta)} \tag{63}$$

A.8 Folded dipole

To make a complete model of the folded dipole without any assumptions is very hard. Theory from literature could not be found. A few assumptions were made:

1. The side parts where the dipole is 'folded' are neglected
2. The separation distance is not larger than 1 wavelength

3. The electric and magnetic field decrease linearly with the separation distance

If these assumptions are well made can later be checked using simulations. Using these assumptions, the folded dipole is a actually two linear dipoles with a slight misalignment due to the separation distance between them. Using equation 62 and 63, the electric field can be multiplied by 2, but also have a term that takes the separation distance into account.

$$E_\theta = \frac{jk}{\pi R}\sqrt{\frac{\mu_0}{\epsilon_0}}e^{-jkR}\,\frac{\cos((\pi/2)\cos(\theta))}{k\sin(\theta)}\left(\frac{\lambda - a}{\lambda}\right) \quad \text{for } 0 < a < \lambda \tag{64}$$

$$B_\phi = \frac{jk}{\pi R}e^{-jkR}\,\frac{\cos((\pi/2)\cos(\theta))}{k\sin(\theta)}\left(\frac{\lambda - a}{\lambda}\right) \quad \text{for } 0 < a < \lambda \tag{65}$$

Where a is the separation distance between the dipoles. The poynting vector is defined as

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \tag{66}$$

Which will result in

$$\vec{S} = \left(-\frac{(a-\lambda)^2\cos^2\left(\frac{1}{2}\pi\cos(\theta)\right)e^{-2ikR}\sqrt{\frac{\mu}{\epsilon}}}{\pi^2\lambda^2\mu R^2\sin^2(\theta)}, 0, 0\right) \tag{67}$$

A.9 Far Field Folded Dipole

For the far field of the folded dipole, the vector potential A, magnetic B and electric E, poynting vector S and Power were found.

Vector Potential Calculating the vector potential of the folded dipole we first start with a half wave antenna as the folded dipole is a combination of two half wave antennas. The half wave antenna can be seen as a finite length wire. Therefore, using the following equation we can find the vector potential of a finite wire for retarded time.

$$\vec{A}(\vec{r}_2, t) = \iiint \frac{\mu_0\vec{J}(\vec{r}_1, t - |\vec{r}_2 - \vec{r}_1|/c)}{4\pi|\vec{r}_2 - \vec{r}_1|}dv_1 \tag{68}$$

Here the current density ($\vec{J}$) for the retarded time is dependent on $\vec{v}\rho$. We are assuming that the velocity of the flow of current through the half wave dipole is constant for all charged particles in spacial coordinates. ρ represents the charge density. When integrated over volume, the charge density for retarded time turns into the total charged enclosed. When viewed from a large distance, $|\vec{r}_2 - \vec{r}_1| = r$. This is represented as below.

$$\vec{A}(r, t) = \frac{\mu_0\vec{v}q}{4\pi r} \tag{69}$$

Here the velocity can be represented as

$$\vec{v} = (0, 0, \frac{\omega L}{2}cos(\omega t')) \tag{70}$$

where, $t' = t - \frac{r}{c}$. Therefore, the vector potential becomes,

$$\vec{A}(r,t) = (0, 0, \frac{\mu_0 q\omega L}{8\pi r} cos(\omega t')) \tag{71}$$

Magnetic Field From the vector potential we can find the magnetic field (B). This is using the equation from the appendix

$$\vec{B}(r,t) = \nabla \times \vec{A}(r,t) \tag{72}$$

Therefore, taking the curl of the vector potential will give us the magnetic field.

$$B_x = \frac{\mu_0 q\omega^2 yL}{8\pi r^2 c} sin(\omega t') - \frac{\mu_0 q\omega Ly}{8\pi r^3} cos(\omega t') \tag{73}$$

$$B_y = -\frac{\mu_0 q\omega^2 xL}{8\pi r^2 c} sin(\omega t') + \frac{\mu_0 q\omega Lx}{8\pi r^3} cos(\omega t') \tag{74}$$

$$B_z = 0 \tag{75}$$

Electric Field Using the relation between the electric potential and the vector potential we can calculate the electric field.

$$\vec{E}(r,t) = -\nabla\phi(r,t) - \frac{\partial \vec{A}(r,t)}{\partial t} \tag{76}$$

Calculating the time derivative of the vector potential.

$$\frac{\partial \vec{A}(r,t)}{\partial t} = (0, 0, -\frac{\mu_0 q\omega^2 L}{8\pi r} sin(\omega t')) \tag{77}$$

The electric potential is represented by equation 31 found in appendix A.1. You can clearly see that the the electric potential is dependent on the integral of the charge density over volume. However, outside the half wave dipole, the charge density is zero.

$$\phi(r,t) = \iiint \frac{\rho(r, t - r/c)}{4\pi\epsilon_0 r} dv_1 \tag{78}$$

Therefore, the electric potential outside is zero. We can then calculate the electric field and it is given as

$$\vec{E}(r,t) = (0, 0, \frac{\mu_0 q\omega^2 L}{8\pi r} sin(\omega t')) \tag{79}$$

Therefore, the electric field of the half wave dipole is directed in the z direction, or the direction of the vector potential but in the negative z direction.

Poynting Vector The poynting vector determines the direction of propagation of the fields in time. It is determined by taking the cross product between the electric field and the magnetic field.

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \tag{80}$$

Solving the cross product we get,

$$S_x = -\frac{\mu_0 q^2 \omega^4 L^2 x}{64\pi^2 r^3 c} sin^2(\omega t') + \frac{\mu_0 q^2 \omega^3 L^2 x}{64\pi^2 r^4} sin(\omega t')cos(\omega t') \tag{81}$$

$$S_y = -\frac{\mu_0 q^2 \omega^4 L^2 y}{64\pi^2 r^3 c} sin^2(\omega t') + \frac{\mu_0 q^2 \omega^3 L^2 y}{64\pi^2 r^4} sin(\omega t')cos(\omega t') \tag{82}$$

$$S_z = 0 \tag{83}$$

We can see from this that the poynting vector is directed in the xy-plane.

Power The power of the half wave dipole can be calculated using the following formula,

$$P = \frac{\partial U}{\partial t} = \int_S \vec{S} \cdot d\vec{a} \tag{84}$$

Since the area of the finite wire or the half wave dipole is perpendicular to the poynting vector, the integral becomes,

$$P = \frac{\partial U}{\partial t} = |\vec{S}|A \tag{85}$$

where A is the area where the power is delivered. The surface area of the wire.

$$P = |\vec{S}|2\pi r L \tag{86}$$

B Gain and Directivity

B.1 Directivity

The directivity is defined as

$$D = \frac{P(\theta,\phi)_{max}}{P(\theta,\phi)_{av}} \tag{87}$$

The directivity is a dimensionless ratio ≥ 1
The total power is obtained by integrating the radiation intensity over the entire solid angle of 4π. Thus

$$P(\theta,\phi)_{av} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} P(\theta,\phi) sin\theta d\theta d\phi = \frac{1}{4\pi}\int\int_{4\pi} P(\theta,\phi) d\Omega \tag{88}$$

$$(W sr^{-1})$$

Therefore , the directivity

$$D = \frac{P(\theta,\phi)_{max}}{(1/4\pi)\int\int_{4\pi} P(\theta,\phi)d\Omega} = \frac{1}{(1/4\pi)\int\int_{4\pi}[P(\theta,\phi)/P(\theta,\phi)_{max}]d\Omega} \tag{89}$$

Directivity from beam area Ω_AThe beam solid angle Ω_A is defined as the solid angle through which all the power of the antenna would flow if its radiation intensity is constant (and equal to the maximum value of U) for all angles within Ω_A.

$$D = \frac{4\pi}{\int\int_{4\pi} P_n(\theta,\phi)d\Omega} = \frac{4\pi}{\Omega_A} \tag{90}$$

$$D(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{rad}}$$

$$U(\theta) = \frac{1}{2}r^2|E_\theta|^2 = \frac{1}{2}\frac{\eta I_m^2}{2(2\pi)^2}\frac{cos^2(\pi/2cos\theta)}{sin^2\theta} \tag{91}$$

$$\frac{1}{2}(2\pi)\frac{\eta I_m^2}{(2\pi)^2}\int_0^\pi \frac{cos^2\left(\frac{\pi}{2}cos\theta\right)}{sin\theta}d\theta \tag{92}$$

$$\int_0^\pi \frac{cos^2\left(\frac{\pi}{2}cos\theta\right)}{sin\theta}d\theta \approx 1.21888$$

$$30(1.2188)I_m^2 = 36.5640I_m^2$$

B.2 Radiated Power

The time average power radiated can be expressed as

$$P_r = \oint \vec{S}_{av} \cdot d\vec{l} = \oint U d\Omega \tag{93}$$

With $U = R^2\, \vec{S}_{av}$ and $d\Omega = \sin\theta d\theta d\phi$, where $\vec{S}_{av}$ is the time average Poynting vector.

The Poynting vector is defined as

$$\begin{aligned}\vec{S}(t) &= \vec{E}(t) \times \vec{B}(t)\\ &= \frac{1}{2}Re[\vec{E}_m e^{j\omega t}] \times Re[\vec{B}_m e^{j\omega t}]\\ &= \frac{1}{2}\left(\vec{E}_m e^{j\omega t} + \vec{E}_m^* e^{-j\omega t}\right) \times \left(\vec{B}_m e^{j\omega t} + \vec{B}_m^* e^{-j\omega t}\right)\\ &= \frac{1}{4}\left(\vec{E}_m \times \vec{B}_m^* + \vec{E}_m^* \times \vec{B}_m + \vec{E}_m \times \vec{B}_m e^{2j\omega t} + \vec{E}_m^* \times \vec{B}_m^* e^{2j\omega t}\right)\\ &= \frac{1}{2}\left(\vec{E}_m \times \vec{B}_m^*\right) + \frac{1}{2}\left(\vec{E}_m \times \vec{B}_m e^{2j\omega t}\right)\end{aligned} \tag{94}$$

Where the time average poynting vector then becomes:

$$< \vec{S} > = \frac{1}{T}\int_0^T \vec{S}(t)\, dt = \frac{1}{2}\left(\vec{E}_m \times \vec{B}_m^*\right) \tag{95}$$

Taking the curl of equation 54 and 55 gives for U

$$U = \frac{I^2 dl^2}{32\pi}\sqrt{\frac{\mu_0}{\epsilon_0}}\, k^2 \sin\theta^2 \tag{96}$$

Then do the integration

$$P = \int_0^{2\pi} \int_0^{\pi} \frac{I^2 dl^2}{32\pi} \sqrt{\frac{\mu_0}{\epsilon_0}}\, k^2 \sin\theta^2 \sin\theta \, d\theta \, d\phi \tag{97}$$

The gain can be found by

$$G(\theta, \phi) = \frac{4\pi U}{P} = \frac{3}{2} \sin^2\theta \tag{98}$$

Where the directivity is the maximum of the gain, which is at $\theta = \pi/2$, and thus gives a value of 1.76dB.

C Impedance

Folded Dipole As discussed in section 2.3, the folded dipole can be modeled as a superposition of a transmission line mode and an antenna mode [9]. This way the total current of the folded dipole

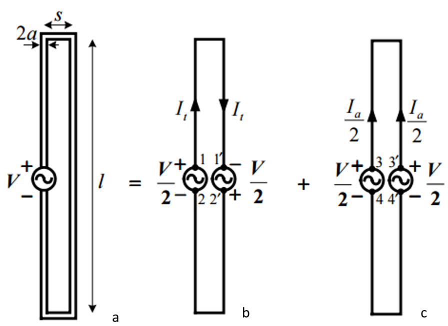


Figure 14: Representation of a folded dipole (a) divided into an antenna mode (b) and a transmission line mode (c) [22]

can be described as:

$$I = I_t + \frac{I_a}{2} \tag{99}$$

Therefore, using the basic definition that the impedance is the ratio of voltage to current, Z_{fd} can be described as:

$$Z_{fd} = \frac{V}{I_t + \frac{I_a}{2}} \tag{100}$$

In order to derive the full expression for the impedance of the folded dipole, I_t and I_a have to be expressed in terms of V and the respective Z_t and Z_a. The voltage and current of the transmission line are related by [22]:

$$I_t = \frac{V}{2Z_t} \tag{101}$$

since a voltage over 1 and 1' (from figure 14) is V/2. The input impedance Z_t for a transmission line is:

$$Z_t = jZ_0 \tan\left(\frac{\beta L}{2}\right) \tag{102}$$

which is derived by computing I(x) and V(x) by expressing the input impedance in terms of the load impedance. [23] For the antenna mode, the model can be interpreted as an equivalent dipole where there are two parallel currents 14, therefore:

$$I_a = \frac{V}{2Z_a} \tag{103}$$

where Z_a is assumed to equal the input impedance of an ordinary dipole of the same wire size. The expression for Z_a is given by:
Combining 7, 6 and 103 the impedance of the folded dipole is:

$$Z_{fd} = \frac{V}{\frac{V}{2Z_t} + \frac{V}{4Z_a}} = \frac{4Z_t Z_a}{Z_t + 2Z_a} \tag{104}$$

If we take the limit as Z_t goes to infinity:

$$Z_{fd} = 4Z_a \tag{105}$$

Coax Cable The most common type of transmission line is a coax cable. The impedance for the transmission line is given by [24]:

$$Z_0 = \sqrt{\frac{L}{C}} \tag{106}$$

The capacitance per unit length is given by:

$$C = \frac{2\pi\epsilon_0}{ln\left(\frac{b}{a}\right)} \tag{107}$$

by knowing that $C = \frac{q}{V}$ and V is calculated by:

$$V = -\int_{outer}^{inner} \vec{E} \cdot d\vec{l} = \int_a^b E dr = \frac{q}{2\pi\epsilon_0}\int_a^b \frac{dr}{r} = \frac{q}{2\pi\epsilon_0} ln\left(\frac{b}{a}\right) \tag{108}$$

To calculate the inductance per unit length we know that according to Ampère's law the magnetic field in between the two conductors is $B = \frac{\mu_0 I}{2\pi r}$, thus by integrating to find the flux:

$$\Phi = \int_a^b B dr = \frac{\mu_0 I}{2\pi} ln\left(\frac{b}{a}\right) \tag{109}$$

and diving by the current:

$$L = \frac{\mu_0}{2\pi} ln\left(\frac{b}{a}\right) \tag{110}$$

By combining equation 107 and 110 into 106, the final expression for the impedance of the cable is:

$$Z_0 = \left(\frac{\mu_0}{4\pi^2\epsilon_0}\right)^{\frac{1}{2}} ln\left(\frac{b}{a}\right) \tag{111}$$

If we take into account that the medium in between the two conductors is filled with a certain material with dielectric ϵ then:

$$Z_0 = \left(\frac{\mu_0}{4\pi^2\epsilon_0}\right)^{\frac{1}{2}} \frac{ln\left(\frac{b}{a}\right)}{\sqrt{\epsilon}} \tag{112}$$

D Corner Reflector

D.1 Image theory

The image theory discussed in 2.2 follows from figure 2, where #2, #3 and #4 are the mirrored images from the dipole source #1. Gobi Vetharatnam and Ali Rashidifar derive equation 4 in their 2015 paper "Comparison between microstrip and dipole antenna backed by a corner reflector". [2] This image theory is used before by Kraus in his paper on the corner reflector antenna [25] where he based his paper on the mathematics found in "Elements of the mathematical theory of electricity and magnetism", written by James Jeans in 1909. [26]
The latter states that the equation of any surface can be written in the form:

$$\frac{e^{-jkr}}{r} + \frac{e^{-jkr_1}}{r_1} + \frac{e^{-jkr_2}}{r_2} + \frac{e^{-jkr_3}}{r_3} + ... = 0 \tag{113}$$

Where e^{-jkr} is a charge at a distance r on the outside of the surface. Then e^{-jkr_1}, e^{-jkr_2}, etc. are the images of the charge at a distance r_1, r_2, etc. This is also what Vetharatnam and Rashidifar have applied this to the corner reflector with the use of figure 15.

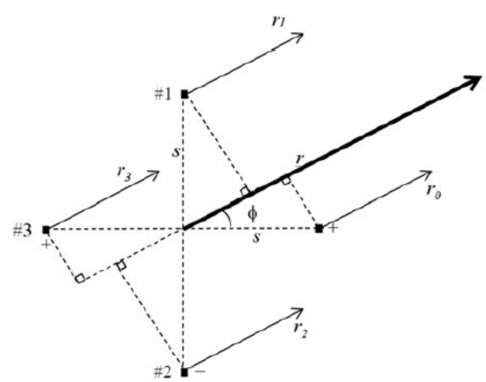


Figure 15: The field emitted from the source and the images can be added to find the total field. [2]

Using basic trigometry, expressions for r_1 to r_3 can be obtained:

$$r_0 = r - s\cos\phi \tag{114}$$

$$r_1 = r - s\sin\phi \tag{115}$$

$$r_2 = r + s\sin\phi \tag{116}$$

$$r_3 = r + s\cos\phi \tag{117}$$

The total electric field can be determined by adding the fields from the images and the source:

$$\vec{E} = \vec{E_0} + \vec{E_1} + \vec{E_2} + \vec{E_3} \tag{118}$$

The electric field can be determined by multiplying the wave pattern with equation 113 [2][26]:

$$\vec{E} = f(\theta,\phi)\frac{e^{-jkr_0}}{r_0} - f(\theta,\phi)\frac{e^{-jkr_1}}{r_1} - f(\theta,\phi)\frac{e^{-jkr_2}}{r_2} + f(\theta,\phi)\frac{e^{-jkr_3}}{r_3} \tag{119}$$

Substituting equation 114 to 117 into this expression results in the following equation:

$$\vec{E} \approx f(\theta,\phi)\frac{e^{-jkr}}{r}[e^{jks\cos\phi} - e^{jks\sin\phi} - e^{-jks\sin\phi} + e^{-jks\cos\phi}] \tag{120}$$

Which can be simplified to:

$$\vec{E} = f(\theta,\phi)\frac{e^{-jkr}}{r}[2\cos(ks\cos\phi) - 2\cos(ks\sin\phi)] \tag{121}$$

This means the array factor will be:

$$AF = [2\cos(ks\cos\phi) - 2\cos(ks\sin\phi)] \tag{122}$$

So if one assumes that the source has a radiation pattern $f(\theta,\phi)$, the total radiation pattern of the corner reflector antenna, R, becomes:

$$R(\theta,\phi) = f(\theta,\phi)[2\cos(ks\cos\phi) - 2\cos(ks\sin\phi)] \tag{123}$$

D.2 Shielding

Rather than using a solid conducting plate for the corner reflector, also a wiring sheet can be used to prevent the radiation from going into the backward direction. As mentioned in section 2.2 This shielding is comparable to the shielding used un microwaves. The spacing between the wires plays a major role in determining if the wire frame provides sufficient shielding. This spacing can be determined using the wave guide theory. A clear explanation is provided by Richard P. Feynman [27].

Assume a rectangular waveguide with the waves propagating in the positive z direction. One can assume that the wave in the waveguide looks like:

$$E_y = E_0 e^{i(\omega t - k_z z)} \sin k_x x \tag{124}$$

The field should be perpendicular to the top and the bottom of the waveguide zero at the sides if half of $\sin k_x$ fits in the width of the guide. Which is the case when k_x equals $n\pi$, where n is an integer. Now the electric field must satisfy Maxwell's equations. So:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0 \tag{125}$$

This equation becomes:

$$k_x^2 E_y + k_z^2 E_y - \frac{\omega^2}{c^2} E_y = 0 \tag{126}$$

If one assumes E_y is not zero everywhere, the equation holds true for:

$$k_x^2 + k_z^2 - \frac{\omega^2}{c^2} = 0 \tag{127}$$

k_x is fixed, so that k_z is:

$$k_z = \pm\sqrt{(\frac{\omega^2}{c^2}) - (\frac{\pi^2}{a^2})} \tag{128}$$

Where a is the width of the waveguide (and the spacing between the wires). In the case where k_z is negative, the waves will travel in the negative z direction, while for the positive k_z the waves will travel in the positive z direction. In a waveguide, one would like the waves to travel in both directions, however, in the case of a shielding, it should be used to reverse the direction of the waves. The cutoff frequency is the frequency at which point the waves cannot pass trough the waveguide anymore, exactly wat is required for the corner reflector. For higher frequencies k_z increases, until k is equal to $\frac{\omega}{c}$ If the frequency decreases, the square root becomes negative. This is the cutoff frequency. This cutoff frequency is equal to:

$$\omega_c = \frac{\pi c}{a} \tag{129}$$

For this, an imaginary k_z is required:

$$k_z = \pm ik' \tag{130}$$

$$k = \sqrt{(\frac{\pi^2}{a^2}) - (\frac{\omega^2}{c^2})} \tag{131}$$

Equation 124 can now be rewritten as:

$$E_y = E_0 e^{\pm k' z} e^{i\omega t} \sin k_x x \tag{132}$$

One can see that the electric field oscillates with $e^{i\omega t}$ and travels along z with $e^{\pm k' z}$.The accompanying plot is given in figure 16:

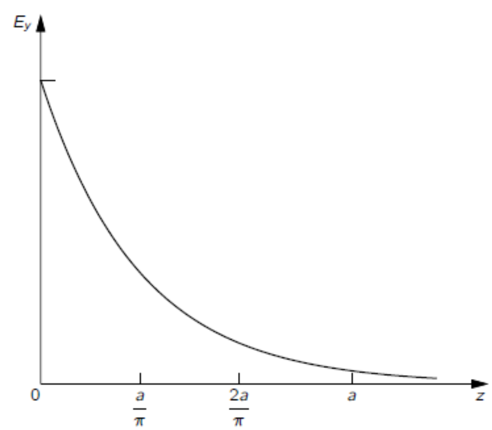


Figure 16: The electric field for a frequency below the cutoff frequency. [27]

D.3 Separation distance

Image theory can be used to construct 3 mirrored images of the source. Now it is posible to draw 4 paths from our source to a receiver in front of the corner reflector. Now required to make sure that these 4 signals are in phase such that we can get the maximal increase in signal strength. To calculate the phase change as a result of distance traveled we use the formula $shift(rad) = fracslamda$ with $lamda = w = wavelength$.

Now the addition distance each path takes compared to the source needs to be calculated. It is assumed that our receiver will be an infinite distance away. This allows to say that all paths will be leaving parallel to each other. It also makes it possible to say that once the signal is parallel to the source signal it will travel the same distance. These assumption are made to decrease the difficulty of the calculations.

The perfectly conducting plates of the corner reflector are at an angle of 90° and as we can see from Figure (ref), $l1$ and $l2$ are parallel. Therefore, we can find that $\alpha = 45°$. This gives us that $\beta 2 = 45°$ which is equal to $\beta 2$ because the angle at which the signal comes in is equal to the angle at which it will be reflected. From this we can see that $L3 = S\ \tan(45°) = S$. due to symmetry $L3 = L4$ thus we can sum these 2 signals because they will be equal. The additional distance needed to travel from $m3$ can directly be seen and is $2s$.

Because a signal that is reflected has a phase shift of 180° we have to take into account that the signal coming from $m1$ and $m2$ is shifted 180°. The signal coming from $m3$ can be constructed bt reflecting the signal 2 times near the back corner giving a total shift of $360° = 0$.

To construct the signal we will receive we will use superposition to sum the 4 signals. Where the properties of every signal will be put the sin function $sin(\omega t + \phi)$ where $w = freq$, $t = time$, $\phi = phaseshift(rad)$. This gives us the following expressions

$$source = \sin(\omega t) \tag{133}$$

$$m1 = m2 = \sin(\omega\ t + (1 + \frac{2s}{\lambda})\pi) \tag{134}$$

$$m3 = \sin(\omega(t + \frac{4s}{\lambda})\pi) \tag{135}$$

combining in the final expression:

$$signal = \sin(\omega t) + 2\sin(\omega t + (1 + \frac{2s}{\lambda})\pi) + \sin(\omega t + (\frac{4s}{\lambda})\pi) \tag{136}$$

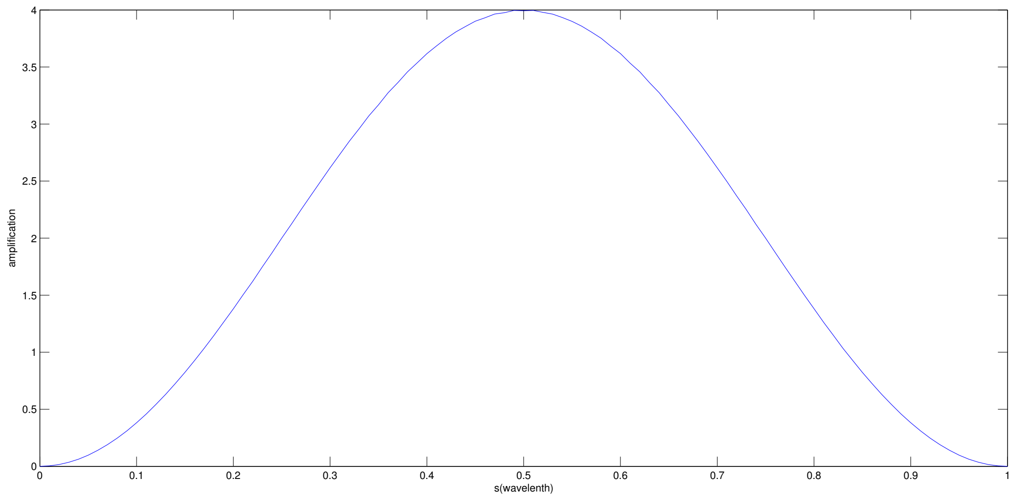


Figure 17: separation distance vs amount of sources received

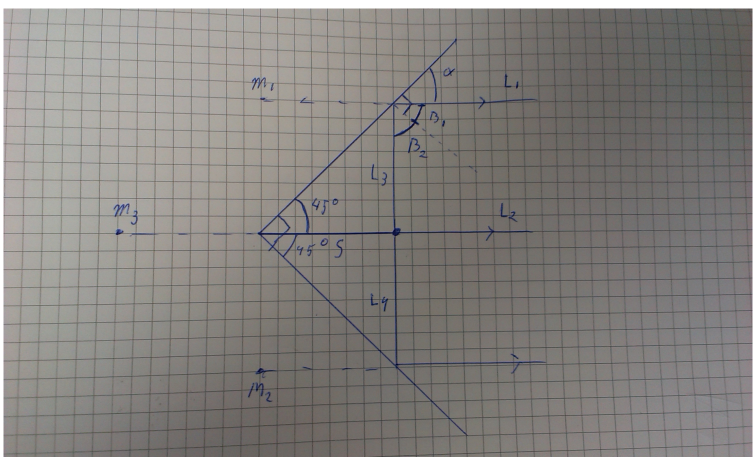


Figure 18: geometry

The mathlab code used to plot S vs amplification:

```
clc
clear all
close all
w=433000000;
a=1*pi;
number=1;
c=299792458;
lamda=c/w;
s=0;
numbers=1;
for s=0:0.01*lamda:lamda
    number=1;
    for t=0:0.0000000001*pi:(2*pi)/w
        awnser1(number)=sin(w*t)+2*sin(w*t+(1+(s/(1/2*lamda)))*pi)+sin(w*t+(2*s/(1/2*lamda))*pi);
        number=number+1;
    end
    power(numbers)=max(awnser1);
    numbers=numbers+1;
end
x=[0:100]./100;
plot(x,power)
```

Figure 19: Mathlab script

D.4 Chicken wire

An inexpensive wire sheet widely available for commerce is chicken wire. This makes it a suitable candidate to be used as a corner reflector. One could approximate the dimensions of chicken wire as follows: the thickness of the wire is about 0.5mm thick, while the spacing between the wires is at most 5 centimeters. As mentioned previously, the cutoff frequency of a waveguide is given by $\omega_c = \frac{\pi c}{a}$. For a distance a=0.05m, this would result in a cutoff frequency of about 19GHz. One where 433MHz is far below. As discussed in section D.2, the electric field travels along z according to $e^{\pm k' z}$, where k is equal to $\sqrt{(\frac{\pi^2}{a^2}) - (\frac{\omega^2}{c^2})}$. Since the thickness of the wires is about 0.5mm, this value can be used for z Filling in these values results in an magnitude of E_y of $2.6 \times 10^{-30} E_0$. So one could conclude that chicken wire would provide an sufficient shielding to function as a corner reflector in stead of a solid conducting plate. One should note, however, that it is assumed here that the chicken wire is made of a perfect conductor, which is obviously not the case in reality. Also, the values approximated here, are worst case scenarios, the finally used chicken wire might have more favorable dimensions.

E Simulations

E.1 4NEC2

The model for this report is of a standard folded dipole antenna analyzed using 4NEC2 software. It consisted of two long element sections which resemble the wires of equal diameter as NEC models tend to fail with unequal diameters. In addition to this it occurred that NEC models would be inconclusive if the transformation of the feedpoint impedance was done by ratios other than 4:1. To set up the antenna a source was placed at the center of one of the long element sections and the wires were divided into segments. The figure below showcases the general layout.

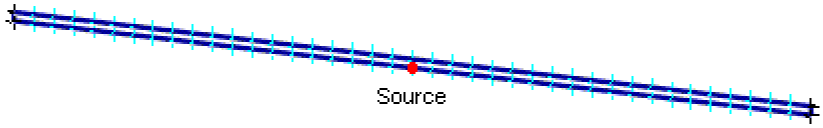


Figure 20: Model representation of a Folded Dipole Antenna

Two models were created, one in which all wires have a diameter of 2.5 cm and are 5 cm apart and one in which only the source wire has a diameter of 2.5 cm and all other wires 0.25 cm. In both cases the wires are of lossless material. We expect the folded dipole to show a source impedance close to 280 Ω.

Table 2: Folded Dipole Simulations Part 1

Diameter (cm)	Wire Length (cm)	Gain (dB)	Source Impedance	Corrected Gain (dB)
Source/Other			R +/- jX Ω	
2.5 / 2.5	243.6	2.13	$281.5 - j0.96$	2.13
2.5 / 0.25	248.0	2.96	$164.5 + j0.60$	2.14

The corrected Gain was found using the Average Gain Test (AGT) which is a function within the NEC software. The wire length refers to half the long element length. The equal diameter model results are very promising. Furthermore some experimentation with the alignment of the segments was pursued. Three different ratios lead to the following table of results.

Table 3: Folded Dipole Simulations Part 2

Segments	Wire Length (cm)	Gain (dB)	Source Impedance	Corrected Gain (dB)
Source/Other			R +/- jX Ω	
61 / 31	243.6	1.71	$329.1 - j2.33$	2.12
31 / 61	243.6	2.00	$273.1 + j1.26$	2.12
41 / 41	243.6	2.13	$281.5 - j1.77$	2.13

It is obvious from this that NEC does not handle misalignment very well and that good segment alignment remains an important modeling practice in order to obtain proper source impedances. Furthermore it must be stated that none of the models used so far fully incorporate the impact of the wires connecting the two long elements. The figure below shows the 3-dimensional and E-plane patterns for the equal-diameter model in free space. These patterns would be identical for the nonequal-diameter model.

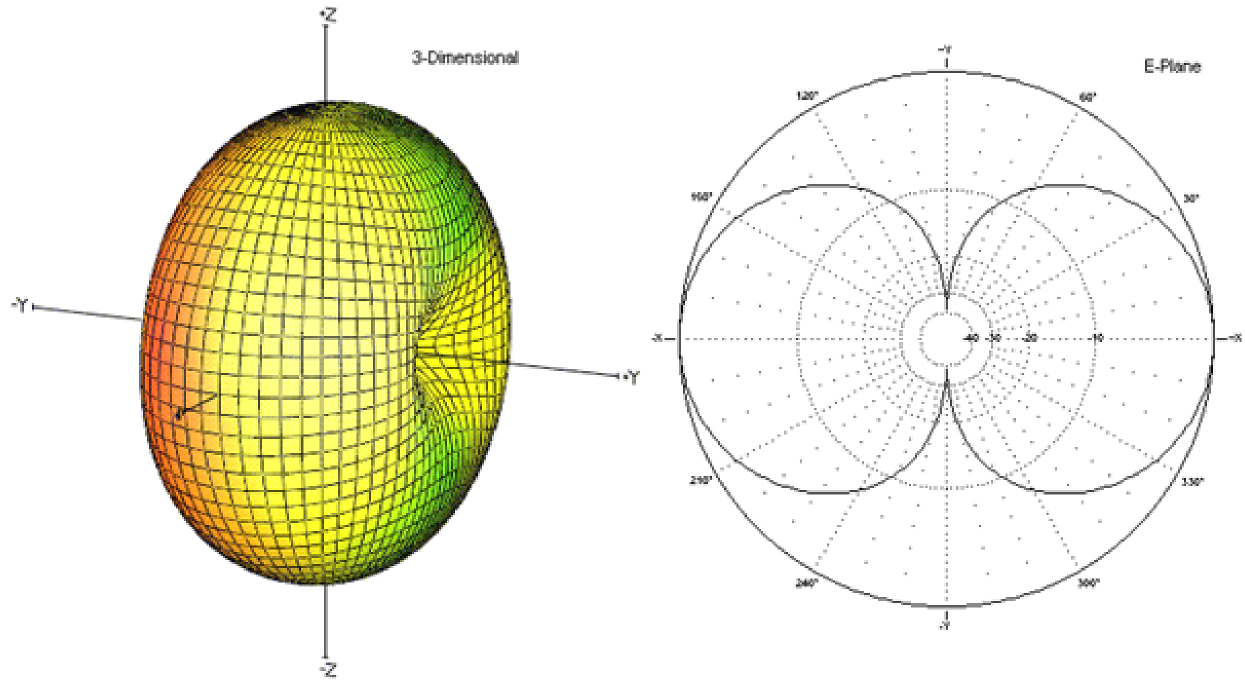


Figure 21: Folded Dipole Antenna Patterns

E.2 Comsol

To commence Comsol simulations a standard dipole antenna was modeled to gain further understanding of the software. After everything had been set up a parametric sweep was initialized to find to optimum values for the geometry. The input Impedance was found to be $120.9 + j28\Omega$ which is far lower than that of a folded dipole. Comsol also created several useful graphs such as the ones below. Figure 22 shows both the near and far field radiation patterns of a standard dipole antenna while Figure 23 gives us an understanding that the directivity of a dipole antenna is equal in all directions if no corner reflector is used. Figure 24 is very similar in shape to the radiation pattern found earlier in 4NEC2 of a folded dipole antenna.

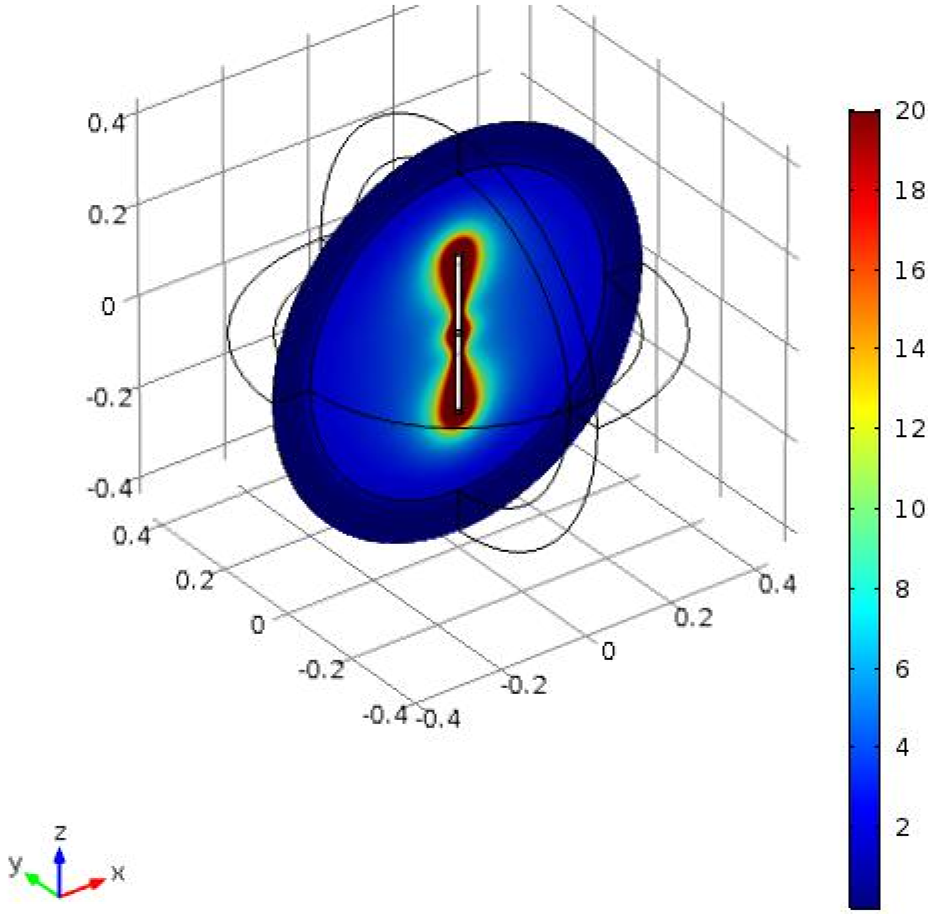


Figure 22: Near and Far Field Radiation Patterns

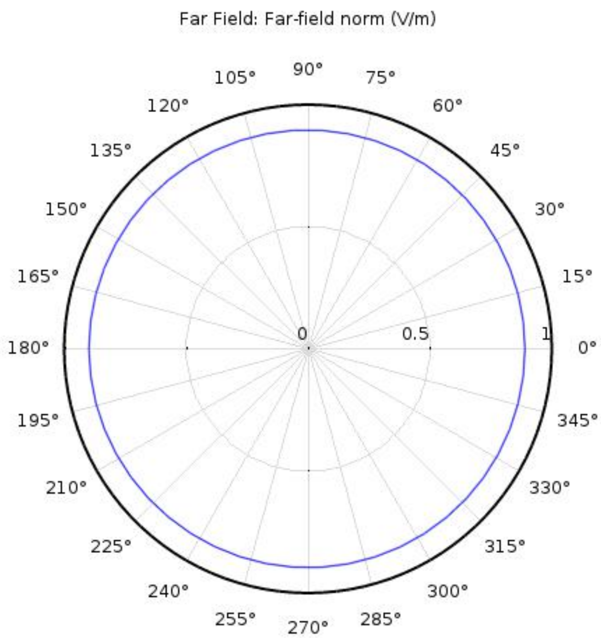


Figure 23: 2D Far Field Radiation Pattern

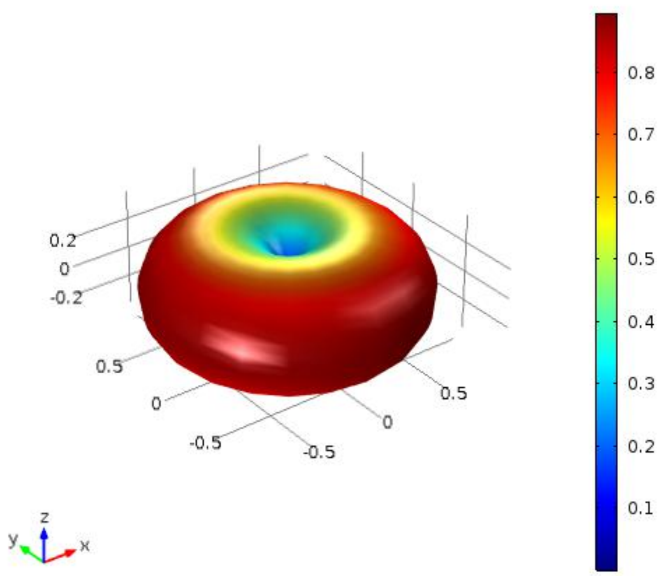


Figure 24: 3D Far Field Radiation Pattern

F Individual Contributions

- Simo Gabriel

  Section 4

  Appendix E

- Jasper Jonker

  Section 2.1

  Appendix A excluding A.9

  Appendix B.2

- Nehal Mathur

  Section 3.3

  Section 3.4

  Appendix A.9

- Duc Tan Nguyen

  Section 5.2

- Jules van Liefland

  Introduction

  Conclusion

- Samuel Oyediran

  Section 3.1

  Section 3.2

  Appendix B.1

- Andrea Veciana

  Section 2.3

  Section 3.5

  Appendix C

- Jasper Ringoot

  Section 5.1

- Jeroen Schlief

  Section 2.2

  Appendix D

- Jasper van der Schaaf