4.1)

$$\dot{x} = f(x) + g(x)\,u$$
$$y = h(x)$$

$$\dot{x} = \begin{bmatrix} x_3 - x_2^3 \\ -x_2 \\ x_1^2 - x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = x_1$$

• Relativer Grad r)

≙ Anzahl der Differentiationen (zeitl.) auf Ausgang damit u explizit erscheint

$$L_g L_f^{r-1} h(x) \neq 0$$
$$L_g L_f^{k} h(x) = 0 \qquad k = 0, \dots, r-2$$

$$L_g h(x) = \underbrace{\frac{\partial h(x)}{\partial x}}_{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}} \cdot \underbrace{g(x)}_{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} = 0$$

$$L_g L_f h(x) = \left[\frac{\partial}{\partial x}\left(\underbrace{\frac{\partial h(x)}{\partial x} f(x)}\right)\right] g(x) = 1 \neq 0$$

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot f(x) = x_3 - x_2^3$$

$$\frac{\partial}{\partial x}(x_3 - x_2^3) = \begin{bmatrix} 0 & -3x_2^2 & 1 \end{bmatrix}$$

$$\Rightarrow \underline{\underline{r = 2}}$$

• Byrnes - Isidori - Normalform

(1)

$$z = \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \\ \phi_3(x) \end{bmatrix} = \Phi(x)$$

$$\phi_1(x) = h(x) = x_1 = z_1$$

$$\phi_2(x) = L_f h(x) = x_3 - x_2^3 = z_2$$

$$L_f h(x) = \frac{\partial h(x)}{\partial x} f(x) = x_3 - x_2^3$$

$$[1 \;\; 0 \;\; 0] \cdot f(x)$$

$\phi_3(x)$ sodass $\Phi(x)$ ein lokaler Diffeomorphismus

$$L_g \phi_3(x) = \frac{\partial}{\partial x} \phi_3(x) \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

(Versuch 1)

$$\frac{\partial \phi_3(x)}{\partial x_3} = 0 \rightarrow \phi_3(x) = \phi_3(x_1, x_2) = x_1 + x_2$$

$$z = \Phi(x) = \begin{bmatrix} x_1 \\ x_3 - x_2^3 \\ x_1 + x_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$x_1 = z_1$ ✓ $\qquad x_2 = z_3 - z_1$ ✓

$x_3 - x_2^3 = z_2$ $\qquad x_3 = z_2 + (z_3 - z_1)^3 = z_2 + z_3^3 - 3z_3^2 z_1 + 3z_3 z_1^2 - z_1^3$ ✓

$x_1 + x_2 = z_3$

→ invertierbar ✓

→ $\Phi(x)$ & $\Phi^{-1}(z)$ glatte Abbildungen ✓

(3)

$$L_f^2 h(x) = L_f(x_3 - x_2^3)$$

$$[0 \;\; -3x_2^2 \;\; 1] \cdot f(x)$$

$$= +3x_2^3 + x_1^2 - x_3$$

$$L_g L_f^1 h(x) = 1$$

$$L_f \phi_3(x) = -x_2$$

$$\dot{z} = \begin{bmatrix} z_2 \\ 2 \cdot z_3^3 + z_1^2 - z_2 + u \\ -z_3 \end{bmatrix}, \; y = z_1$$

• $\Phi(x)$ invertierbar

• $\Phi(x)$ & $\Phi^{-1}(z)$ glatte Abbildungen

(2) (2. Versuch) Einfacher

$\phi_3(x) = x_2$ $\qquad x_1 = z_1$ ✓

$x_2 = z_3$ ✓

$x_3 = z_2 + z_3^3$ ✓

$$\Phi(x) = \begin{bmatrix} x_1 \\ x_3 - x_2^3 \\ x_2 \end{bmatrix}$$

$$\Phi^{-1}(z) = \begin{bmatrix} z_1 \\ z_3 \\ z_2 + z_3^3 \end{bmatrix}$$

- Zustandsregelgesetz

$$u = \underbrace{\frac{1}{L_g L_f h(x)}}_{=1} \left(-L_f^2 h(x) + v\right)$$

Satz 6.1 (6.17)

für $r=2$

$$L_f^2 h(x) = L_f \Big(\underbrace{\frac{\partial h(x)}{\partial x} f(x)}_{x_3 - x_2^3}\Big) = \underbrace{\frac{\partial (x_3 - x_2^3)}{\partial x}}_{\begin{bmatrix} 0 & -3x_2^2 & 1 \end{bmatrix}} f(x)$$

$$= + 3x_2^3 + x_1^2 - x_3$$

$$u = -3x_2^3 - x_1^2 + x_3 + v$$

$$\left[\begin{array}{l} \dot{x} = \begin{bmatrix} x_3 - x_2^3 \\ -x_2 \\ 3x_2^3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v \\ y = x_1 \end{array}\right]$$

lineares Eingangs-Ausgangsverhalten ?

BRUNNEN

• Nulldynamik

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = L_f^2 h(x) + (L_g L_f h(x))\,u = z_1^2 - z_2 - 2 \cdot z_3^3 + u$$

$$\left[\begin{array}{lcl} L_f^2 h(x) = +3x_2^3 + x_1^2 - x_3 & \xrightarrow{z} & +3 \cdot z_3^3 + z_1^2 - z_2 - z_3^3 \\ L_g L_f h(x) = 1 & & = z_1^2 - z_2 + 2 z_3^3 \end{array}\right]$$

$$\dot{z}_3 = L_f \phi_3(x) = \frac{\partial(x_2)}{\partial x} f(x) = -x_2 = -z_3$$

$$[0 \quad 1 \quad 0]$$

① $y(t) = h(x) = z_1 = 0$

$$\dot{y} = L_f h(x) = z_2 = 0$$

$$\xi = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad \eta = [z_3]$$

$$\dot{\eta} = q(\xi, \eta)$$

Nulldynamik: $\dot{\eta} = q(0, \eta_0) = -\eta$

$\eta = \eta_0 \cdot e^{-t} \rightarrow$ stabil ✓