

# Homework 6

Tanja Käser

**Exercise 6.** Let  $F$  be the set of real numbers with decimal representation consisting of all fours (and possibly a single decimal point). Examples of numbers contained in  $F$  are 4, 44, 4444444, 44.4, 4.444444, 444.44444, ... etc.

Let  $G$  be the set of real numbers with decimal representation consisting of all fours or sixes (and possibly a single decimal point). Examples of numbers contained in  $G$  are 4, 6, 44, 66, 46, 64, 4464464, 46.46, 6.644464, 646.64646464, 446.6666666, . . . etc.

☒ The set  $F$  is countable and the set  $G$  is not countable.

☐ The sets  $F$  and  $G$  are both countable.

☐ The set  $G$  is countable and the set  $F$  is not countable.

☐ The sets  $F$  and  $G$  are both not countable.

① We can count them. There is a bijection between this set and the set of rational nb.

$p \Rightarrow$  position of the dot  
 $q \Rightarrow$  number of 4

② We can use Cantor. Alphabet of more than 1 character of infinite length.

**Exercise 7.** Which of the following statements is incorrect?

- ☐ The Cartesian product of finitely many countable sets is countable.
- ☒ Any subset of infinite cardinality of an uncountable set is uncountable.
- ☐  $N \cup \{x \in \mathbf{R}, 0 < x < 1\}$  is uncountable.
- ☐ The intersection of two uncountable sets can be countably infinite.

$\mathbb{Q} \subseteq \mathbb{R}.$

**Exercise 8.**

```
function f1() {  
  x=0  
  i=1  
  while (i ≤ n) {  
    x=x+1  
    i=x+x  
  }  
  a=x  
}
```

```
function f2() {  
  y=0  
  j=1  
  while (j ≤ n) {  
    y=y+1  
    j=y*y  
  }  
  b=y  
}
```

After execution of the two program fragments f1 and f2, it is the case that

- ☒  $a \approx \frac{n}{2}, b \approx \sqrt{n}.$
- ☐  $a \approx n, b \approx \log_2(n).$
- ☐  $a \approx \frac{n}{2}, b \approx \log_2(n).$
- ☐  $a \approx n, b \approx \sqrt{n}.$

$$i = 2x$$

$$j = y^2$$