Exercice 1

$3\times 2 \quad 2\times 1$

3×1

ⓐ $A = \begin{pmatrix} 4 & 3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$ injective

ⓑ $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$ surjective

ⓒ $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ bijective

ⓓ $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ rien

ⓔ $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ bijective

8 (imp.)

Exercice 2

$3\times4 \quad 4\times1 \quad : \quad 3\times1$

$$\begin{pmatrix} 1 & 3 & 5 & 7 \\ -1 & 3 & 0 & 0 \\ 1 & 2 & 3 & 7 \end{pmatrix}$$

est-ce qu'on peut vraiment regarder les coeffs?

$$\left(\begin{array}{cccc|c} 1 & 3 & 5 & 7 & 0 \\ -1 & 3 & 0 & 0 & 0 \\ 1 & 2 & 3 & 7 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 0 & 6 & 5 & 7 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 1 & 2 & 3 & 7 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 3 & 7 & 0 \\ 0 & 0 & 5 & 7 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 5 & 7 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 7/5 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x = 2z \\ y = 0 \\ u = 7/5 z \\ z \text{ libre} \end{cases}$$

$$A\vec{x} = \vec{0}$$

$$\vec{x} = t\begin{pmatrix} 2 \\ 0 \\ 7/5 \\ 1 \end{pmatrix}$$ donc pas injective

Les colonnes de A engendrent $\mathbb{R}^3$
(un pivot par ligne)
$\Rightarrow$ surjective

$2 \times 1 \to 2 \times 1$

Exercice 3

ⓐ

$3 \times 3 \quad 1 \times 3 \to 1 \times 3$

AB

$$\begin{pmatrix} 1 & 3 & -2 \\ 3 & 4 & 2 \\ 4 & 7 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Exercice 4

(a)

$$\begin{pmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6+2+1 & 2+2+4 \\ 0+2+2 & 0+2+8 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 8 \\ 4 & 10 \end{pmatrix} \quad AB$$

(b)

$$\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 4 & 5 \\ 4 & 4 & 6 \\ 2 & 3 & 9 \end{pmatrix} \quad BA$$

(c) $2 \times 3 \;-\; 2 \times 2$

(d)

$$\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1+3 & 1+6 \\ 4 & 2+3 & 2+6 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 4 & 7 \\ 4 & 5 & 8 \end{pmatrix} \quad CA$$

e $\begin{pmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix}$

$$= \begin{pmatrix} 3+2 & 9+3 \\ 2+4 & 6+6 \\ 1+8 & 3+12 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 12 \\ 6 & 12 \\ 9 & 15 \end{pmatrix} \quad BC$$

f) $2\times 2 \cdot 3\times 3$ CB

g) $2\times 2 \cdot 3\times 1$ CD

h) $$\begin{pmatrix} 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 15 \end{pmatrix}$$ EC

i) $2\times 3 \cdot 1\times 2$ EA

b

a $A \cdot A$

Exercice 5

ⓐ Soit $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$$AB = \begin{pmatrix} 1+0 & 1+0 \\ 0+1 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1+0 & 1+0 \\ 1+0 & 1+0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

(b) $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$AB = \begin{pmatrix} 1+0 & 1+0 \\ 0-1 & 0-1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$

$BA = \begin{pmatrix} 1+0 & 0-1 \\ 1+0 & 0-1 \end{pmatrix}$

$= \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$

Exercice 6

ⓐ $\begin{pmatrix} A\vec{b_1} & A\vec{b_2} \end{pmatrix} = 0$

+ intelligent que ça ?

$$\begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} a+2c & b+2d \\ 3a+6c & 3b+6d \end{pmatrix}$$

$$\begin{cases} a+2c = 0 \iff a = -2c \\ b+2d = 0 \iff b = -2d \end{cases} \quad c, d \in \mathbb{R}$$

Méthode 2

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad B = \begin{pmatrix} x_1 & x_3 \\ x_2 & x_4 \end{pmatrix}$$

$$AB = \begin{pmatrix} x_1 a + x_2 b & x_3 a + x_4 b \\ x_1 c + x_2 d & x_3 c + x_4 d \end{pmatrix}$$

$$= \begin{pmatrix} A\vec{b_1} & A\vec{b_2} \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 a + x_2 b \\ x_1 c + x_2 d \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} a + b & b + d \\ 3a + 2b & 3b + 2d \end{pmatrix}$$

$$a + b = 0$$

$$\Leftrightarrow \quad a = -b.$$

$$-3b + 2b = 0$$
$$\Rightarrow \quad b = 0$$
$$\Rightarrow \quad a = 0$$

or $\quad b + d = 0$
$$\Leftrightarrow \quad d = -b$$
$$\Leftrightarrow \quad d = 0.$$

$$3b + 2d = 0$$

Non. pas de sol

(c)

$$\begin{pmatrix} 3 & -4 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} 7 & 4 \\ 5 & k \end{pmatrix} \quad AB$$

$$= \begin{pmatrix} 21-20 & 12-4k \\ -35+5 & -20+k \end{pmatrix}$$

$$=$$

BA

$$\begin{pmatrix} 7 & 4 \\ 5 & k \end{pmatrix}\begin{pmatrix} 3 & -4 \\ -5 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 21-20 & -28+4 \\ 15-5k & -20+k \end{pmatrix}$$

$$\begin{cases} 15-5k = -30 \\ -24 = 12-4k \end{cases}$$

$$\Leftrightarrow \begin{cases} k = \dfrac{-45}{-5} = 9. \\ -36 = -4k \end{cases}$$

$$\Leftrightarrow \begin{cases} k = 9. \\ k = 9. \end{cases}$$

d

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} a^2 + cb & ba + db \\ ac + cd & bc + d^2 \end{pmatrix}$$

$$\begin{cases} a^2 + cb = 0 \\ ba + db = 0 \\ ac + cd = 0 \\ bc + d^2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} b = \dfrac{-a^2}{c} \cup c = 0 \text{ et } a^2 = 0 \\ a = -d \cup b = 0 \\ a = -d \cup c = 0 \\ b = \dfrac{-d^2}{c} \cup c = 0 \text{ et } d^2 = 0 \end{cases}$$

On peut poser $c = 0$

$\Rightarrow a^2 = 0 \Rightarrow a = 0$

$\Rightarrow d^2 = 0$

$\Rightarrow b = k$

Exercice 7

(a)

$$T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix}$$

$$T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$A_1 = \begin{pmatrix}1 & 0\\0 & 1\\1 & 0\end{pmatrix}$$

(b) $T\begin{pmatrix}1\\0\\0\end{pmatrix} = 1$ $\quad T\begin{pmatrix}0\\0\\1\end{pmatrix} = 1$

$$T\begin{pmatrix}0\\1\\0\end{pmatrix} = 1$$

$$A_2 = \begin{pmatrix} 1, 1, 1 \end{pmatrix}$$

$T_2 \circ T_1$

$$A_2 \; A_1 \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1, 1, 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \end{pmatrix}$$

(b) $\mathbb{R}^2$ dép. $\mathbb{R}$ arrée

Prouver $(AB)^T = B^T A^T$ (Ex 9)

A de taille $m \times n$
B de taille $n \times p$

AB compatible de taille $m \times p$.

$$\begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$A^T_{ij} = A_{ji} \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$B^T_{ij} = B_{ji} \qquad = \begin{pmatrix} ea+gb & \text{--} \\ ec+gd & \text{--} \end{pmatrix}$$

$$AB_{ij} = \sum_{k=1}^{n} A_{ik} \cdot B_{kj}$$

$$AB^T_{ij} = AB_{ji} = \sum_{k=0}^{n} A_{jk} \cdot B_{ki}$$

$$= \sum_{k=0}^{n} A^T_{kj} \cdot B^T_{ik} \qquad = \sum_{k=0}^{n} B^T_{ik} \cdot A^T_{kj}$$

$$= \underline{B^T \cdot A^T}$$

Exercice 8

(a) $$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\det(A) = 8-4 = 4.$$

$$A^{-1} = \frac{1}{4}\begin{pmatrix} 4 & -2 \\ -2 & 2 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 2 & 2 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cc|cc} 1 & 1 & 0.5 & 0 \\ 1 & 2 & 0 & 0.5 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cc|cc} 1 & 1 & 0.5 & 0 \\ 0 & 1 & -0.5 & 0.5 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cc|cc} 1 & 0 & 1 & -0.5 \\ 0 & 1 & -0.5 & 0.5 \end{array}\right)$$

(b) $A \to I_2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right)$$

Exercice 10

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & 3 & 4 \\ 2 & 4 & 5 & 1 \\ 7 & 14 & -1 & -3 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 3 & -3 \\ 7 & 14 & -1 & -3 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & -8 & -17 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & -15 \\ 0 & 0 & -8 & -17 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ pas d'inverse

$$\begin{pmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & -2 & -1 \\ -2 & -6 & 3 & 2 \\ 3 & 5 & 8 & -3 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 3 & 0 \\ 3 & 5 & 8 & -3 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 3 & 5 & 0 & -3 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -4 & 0 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ $\exists$ an inverse

c)

$$\begin{pmatrix} 1 & -4 & -7 & 3 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 18 & 3 \\ 0 & 0 & 0 & 17 \end{pmatrix}$$

pivot dans chaque ligne $\rightarrow$ $\exists$ inverse

D

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\rightarrow$ $\exists$ inverse

(E) $\rightarrow$ pas d'inverse (pas pivot col 1)

Exercice 11

(a) $B, C \in \mathbb{R}^{n \times m}$

$B + C = D \Rightarrow D \in \mathbb{R}^{n \times m}$

$D_{ij} = B_{ij} + C_{ij}$

$$(AD)_{ij} = \sum_{k=1}^{n} A_{ik} \cdot D_{kj}$$

$$(AB)_{ij} = \sum_{k=1}^{n} A_{ik} \cdot B_{kj}$$

$$(AC)_{ij} = \sum_{k=1}^{n} A_{ik} \cdot C_{kj}$$

Or $(AB + AC)_{ij}$

$$= \sum_{k=1}^{n} A_{ik} \cdot B_{kj}$$

$$+ \sum_{k=1}^{n} A_{ik} \cdot C_{kj}$$

$$= \sum_{k=1}^{n} A_{ik} \cdot \left(\sum_{k=1}^{n} B_{kj} + C_{kj} \right)$$

$$= \sum_{k=1}^{n} A_{ik} \cdot \sum_{k=1}^{n} D_{kj}$$

$$= \sum_{k=1}^{n} A_{ik} \cdot D_{kj}$$

$$= AD_{ij}$$

(b) $$AB_{ij} = \sum_{k=1}^{n} A_{ik} \cdot B_{kj}$$

$$r(AB_{ij}) = r\left(\text{———} \right)$$

$$= \sum_{k=1}^{n} r \cdot A_{ik} \cdot B_{kj}$$

$$= (rA)B$$

$$= A(rB)$$

$$B \in \mathbb{R}^{n \times p}$$

(c) $$I_{n_{ij}} = \begin{cases} 0 & \text{si } i \neq j \\ 1 & \text{si } i = j \end{cases}$$

$$(AI_n)_{ij} = \sum_{k=1}^{n} A_{ik} \cdot I_{kj}$$

on veut que $= A_{ij}$

qd $k=j$, seul cas où $I_{kj} = 1$.

$$= A_{ij} \cdot I_{jj} + 0 + 0 + 0 \ldots$$

$$= A_{ij}$$

Par définition :

$$I_{m_{ij}} = \begin{cases} 0 \text{ si } i \neq j \\ 1 \text{ si } i = j \end{cases}$$

et $(I_m A)_{ij} = \sum_{k=1}^{n} A_{ik} \cdot I_{kj}$

or $I_{kj} \neq 0$ ssi $k = j$

$$\Rightarrow (I_m A)_{ij} = A_{ij} \cdot I_{jj}$$

$$= A_{ij}$$

Exercice 12

Si $A \in \mathbb{R}^{n \times n}$ est inversible,

$\forall \vec{b} \in \mathbb{R}^n$,

$\exists ! \, \vec{x} \in \mathbb{R}^n$ t.q

$A\vec{x} = \vec{b}$.

$\Rightarrow$ bijectivité $\Rightarrow$ un pivot par ligne

$\Rightarrow$ indépendance linéaire

$\Rightarrow$ engendre $\mathbb{R}^n$.

Exercice 13

1

$$C = \begin{pmatrix} 0 & 3 & 4 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 3 & 4 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 3 & 4 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 15 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad C^2$$

$$\begin{pmatrix} 0 & 0 & 15 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 3 & 4 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad C^3$$

2

$$(I_3 + C + C^2)(I_3 - C)$$

$$= I_3 - \cancel{I_3 C} + \cancel{C I_3} - \cancel{C^2} + \cancel{C^2} - C^3$$

$$= I_3 - C^3$$

$$= I_3$$

3 $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 3 & 4 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 1 & -3 & -4 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{pmatrix} = I - C.$$

$$\left(\begin{array}{ccc|ccc} 1 & -3 & -4 & 1 & 0 & 0 \\ 0 & 1 & -5 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & -3 & 0 & 1 & 0 & 4 \\ 0 & 1 & 0 & 0 & 1 & 5 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 19 \\ 0 & 1 & 0 & 0 & 1 & 5 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

(4) $C\vec{x} + \vec{b} = \vec{x}$

$$\left(\begin{array}{ccc|c} 1 & -3 & -4 & 1 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & -3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$I - C \rightarrow \vec{b}$$

$$(I - C)\begin{pmatrix} 7 \\ 2 \\ 0 \end{pmatrix} = \vec{b}$$

$$\vec{x} - C\vec{x} = \vec{b}$$

$$(I - C)\vec{x} = \vec{b}.$$

$$\Rightarrow (I - C)\begin{pmatrix} 7 \\ 2 \\ 0 \end{pmatrix} = \vec{b}$$

$$\vec{x} = \begin{pmatrix} 1 & 3 & 19 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1+6 \\ 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ 2 \\ 0 \end{pmatrix}$$

Exercice 14

$3\times 1 \quad 1\times 3$

(a)

$$\begin{pmatrix}1\\2\\3\end{pmatrix}\begin{pmatrix}1 & 2 & 3\end{pmatrix}$$

$$= \begin{pmatrix}1 & 2 & 3\\2 & 4 & 6\\3 & 6 & 9\end{pmatrix}$$

(b)

$$\begin{pmatrix}1\\0\\1\end{pmatrix}\begin{pmatrix}1 & 2\end{pmatrix}$$

$3\times 1 \quad 1\times 2$

$$= \begin{pmatrix}1 & 2\\0 & 0\\1 & 2\end{pmatrix}$$

c) $\begin{pmatrix} 1 \\ 4 \end{pmatrix} \begin{pmatrix} 1 & 3 & 5 \end{pmatrix}$

$= \begin{pmatrix} 1 & 3 & 5 \\ 4 & 12 & 20 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1+2 \\ 2+1 \end{pmatrix}$

$$(AB)_{ij} = \sum_{k=1}^{n} A_{ik} \cdot B_{ik}$$