Exercises Rosen p. 491 Probability Theory

$$3\rho(T) = \rho(H)$$

$$3\rho(T) + \rho(T) = 1$$

3
$$p(z) + p(u) = 3(u \cdot p(0))$$

$$\rho(b) = 6\rho(0)$$

$$4 \cdot \rho(0) + 2 \cdot \rho(b) = 1$$

$$=$$
 $(4+12) p(0) = 1$

$$= \frac{1}{16} (4+12) \rho(0) = 1$$

$$= \frac{1}{16} \text{ and } \rho(b) = \frac{6}{16}$$

$$\int x_1 + x_2 = 7$$
 store and boxs

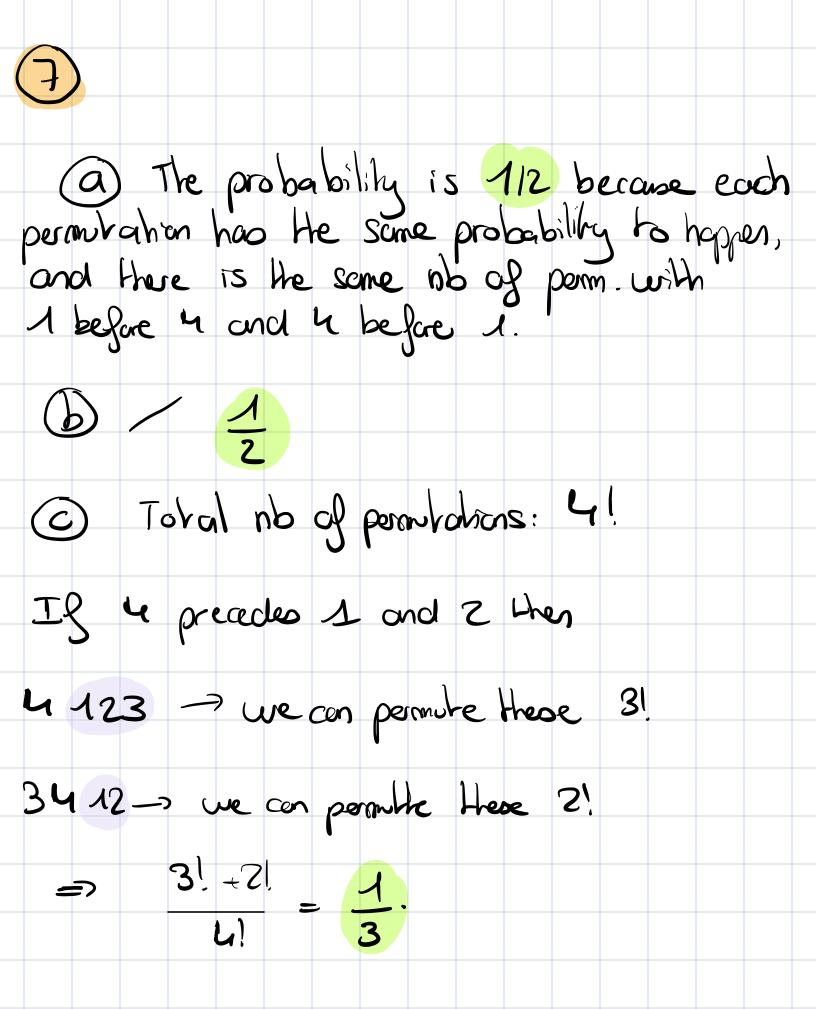
$$\begin{pmatrix} 7+2-1 \\ 2-1 \end{pmatrix} = 8 combinations$$

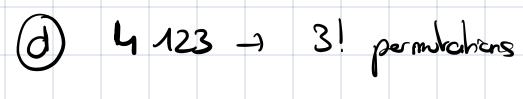
$$P(4 lher 3) = \frac{2}{7} \cdot \frac{2}{7} = \frac{4}{49}$$

· for the other conses:

$$\rho(\text{He others}) = \frac{1}{7} \cdot \frac{1}{7} \cdot (8-2-1) = \frac{5}{49}$$

$$=\frac{5}{48}+\frac{4}{45}-\frac{9}{48}$$





note that we can not pointe 4 and 2, Heave one already contred (2341.)



- (b) We gix z at the top. There are still 25! situations => 25! = p(E) 26!
- Symmetry: some nb of pernutations w/a before z and z before $a = p(E) = \frac{1}{z}$.
- (d) we can view a and z as a back and can't the nb of structure. $\frac{2S!}{z6!} = \frac{1}{26} \rho(E)$
- e) we can say that a, m, z are a vique dock, we have only 73 letters" $\frac{73!}{76!} = \frac{1}{15,600}$

11)
$$\rho(EUF) > \rho(E) = 0.7$$
.
 $\rho(EUF) = \rho(E) + \rho(F) - \rho(ENF)$
 $= 0.7 + 0.5 - \rho(ENF)$
 $= 1.2 - \rho(ENF) < 1$

Therefore p (ENF) is necessarily 20,2.

(13)
$$\rho(EUF) = \rho(E) + \rho(P) - \rho(ENF)$$

$$\Rightarrow \rho(ENP) = -\rho(EUP) + \rho(E) + \rho(F)$$

$$\Rightarrow$$
 -p(EUF) + p(E)+p(F) > -1+p(E)+p(F)

(15) We can prove this by induction: Boois Step: p(ENUEZ) = P(EN) + P(EZ) Jok Inductive Step: p (En U... U Er) & p (En) + ... + p (Ex) holds (*) = p(En U... UEL U Elex) = p(En U... U Elex) + p(Elex) - P((En U... UEL) NELEX) => there fore p (En U. UExxx) & p(En)+ +p(Exxx) Conclusion: it works.

$$\rho(E \cap F) = \rho(E) \cdot \rho(F)$$

$$= P(ENF) = P(EUF)$$

$$= P(ENF)$$

$$= P(ENF$$

$$P(\bar{E}) = 1 - \rho(\bar{E})$$

$$P(\bar{F}) = 1 - \rho(\bar{F})$$

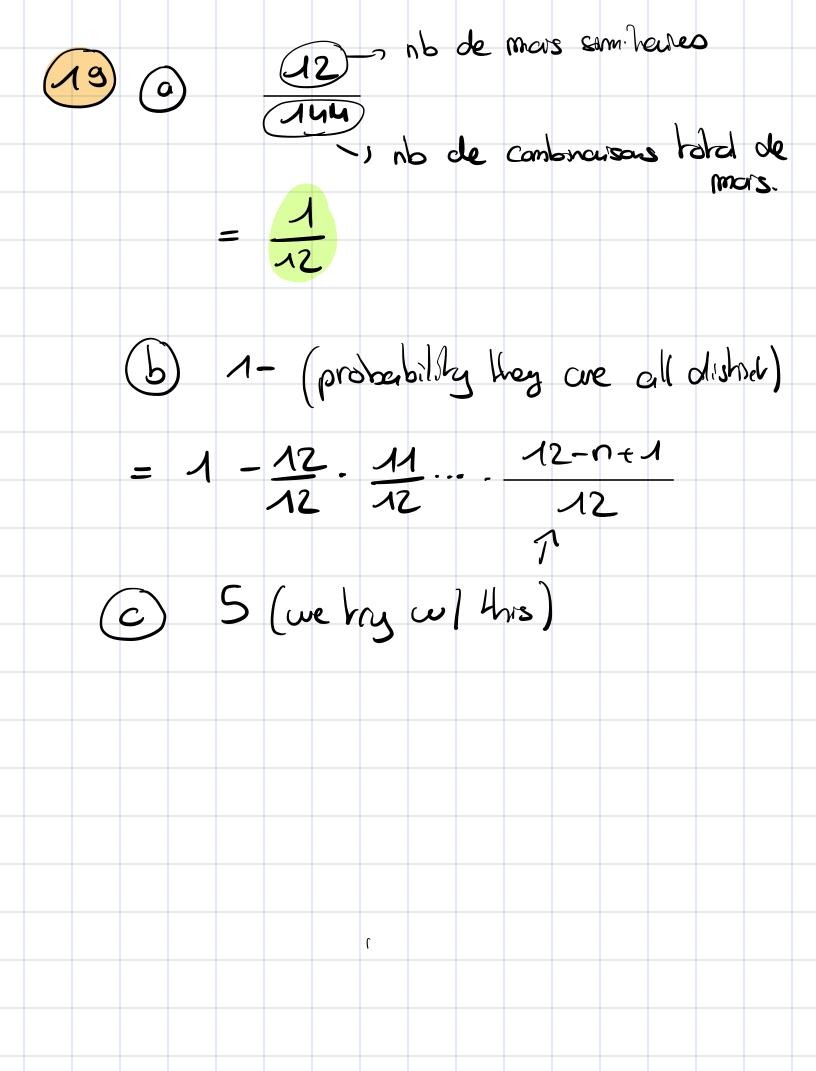
$$P(\bar{E}) = 1 - \rho(\bar{F})$$

$$P(\bar{E}) = 1 - \rho(\bar{F})$$

$$P(\bar{E}) = 1 - \rho(\bar{F})$$

$$= 1 - \rho(\bar{F}) - \rho(\bar{F}) + \rho(\bar{F}) + \rho(\bar{F})$$

Therefore Earl Findep => Earl Findep.



$$1 - \left(\binom{n}{0} \left(\frac{365}{366}\right)^n + \binom{n}{1} \left(\frac{365}{366}\right)^{n-1} \left(\frac{1}{366}\right)\right) \frac{1}{2}$$

$$=) - \left(\binom{0}{0} \left(\frac{365}{366}\right)^{1} + \binom{0}{1} \left(\frac{365}{366}\right)^{1-1} \left(\frac{1}{36}\right)\right) > \frac{-1}{2}$$

$$(\frac{365}{366})^{n} + n \left(\frac{365}{366}\right)^{n-1} \left(\frac{1}{366}\right) < \frac{1}{2}$$

23) F: "first flip come up heads"

E: "3 of He next 4 flips are heads" $\rho(E) = {n \choose 3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right) = {4 \choose 3} \left(\frac{1}{2}\right)^4$ $\rho\left(E/F\right) = \rho\left(E\cap F\right) = \frac{\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^{4} \cdot \frac{1}{2}}{1/2}$ $= \left(\frac{H}{3}\right) \cdot \left(\frac{1}{2}\right)^{4}$

= 1/4

We can have
$$00-1$$
, 100 or 000 .

Total number of combinations $2^3 = 8$.

Total number of combinations
$$2^3 - 8$$
.

$$P(E) = \frac{3}{8}$$

$$n=2$$
 $A = \int GG, GB, BG, BB$

$$\rho(E) = \frac{1}{2} \quad \rho(P) = \frac{3}{4}$$

$$\rho(E \cap P) = \frac{1}{2} \quad \rho(E) \cdot \rho(P) = \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

$$n = u$$
 $A = \int 6666, 666B, 66B6, ...$
 $A = \begin{cases} 16. \end{cases}$

$$P(E) = \frac{14}{16}$$
 $P(P) = \frac{1}{16}$
 $P(P) = \frac{$

$$p(EnP) = \frac{4!}{3!} \cdot \frac{1}{16}$$
 $= \frac{1}{4}$

$$p(E) \cdot p(P) \neq p(EDP)$$

$$\frac{29}{1} \left(\frac{6}{2} \right) \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{6}{6} \right) \left(\frac{1}{2} \right) \left(\frac{1}{2} \right)$$

$$=\frac{3}{16}$$

$$\begin{array}{c|c}
\hline
B & S & 49 \\
S & 100
\end{array}$$

$$\begin{array}{c}
2,8\% \\
100
\end{array}$$

$$\bigcirc 1 - \bigcirc S - \frac{1}{100}$$

$$= 1-0,51+\frac{1}{100}$$

$$\rho(E) = \left(\frac{SO}{100}\right)\left(\frac{S1}{100}\right)\left(\frac{S2}{100}\right)\left(\frac{S3}{100}\right)\left(\frac{S4}{100}\right)$$

$$= \rho(E) + \rho(P) - \rho(E \cap P)$$

$$= \frac{1}{2} + \frac{2^3}{2^5} - \frac{2^2}{2^5} = \frac{5}{8}$$

$$\bigcirc P^{n} + \binom{n}{2} \binom{n}{p}^{n-1} (1-p)$$

$$= \rho^{n} + n(\rho^{n-1})(1-\rho)$$