Exercice 1

ⓐ On veut calculer le moment d'inertie par rapport à l'axe $A\hat{\varepsilon}_1$.

$$I_{\hat{\varepsilon}_1} = \sum_{\alpha} m_\alpha (d_\alpha)^2$$

$$= mb^2 + m(-b)^2 + mh^2$$

$$= 2mb^2 + mh^2$$

$$I_{\hat{\varepsilon}_2} = mb^2 + m(b^2+h^2)$$

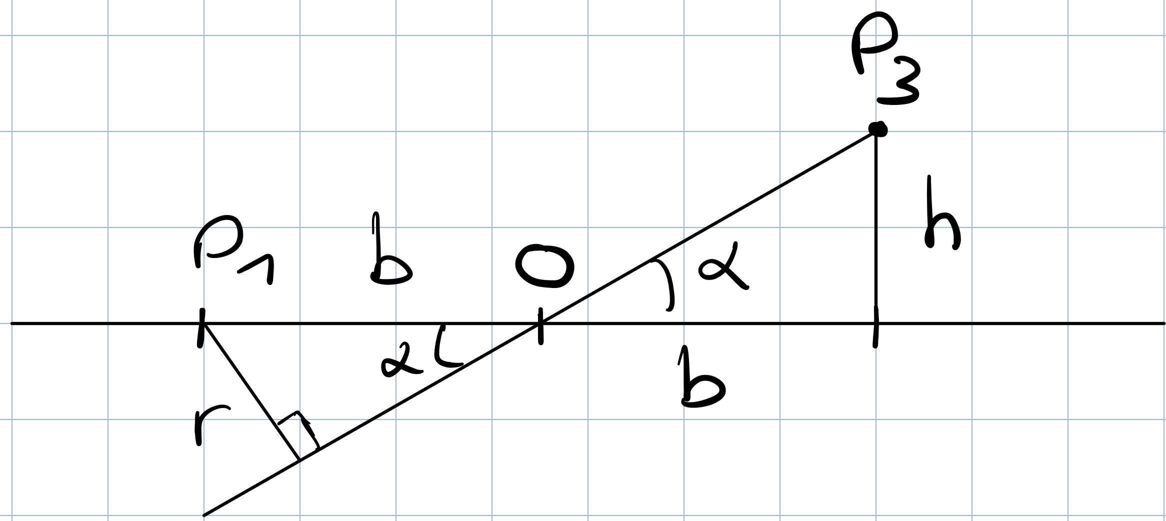
$$= 2mb^2 + mh^2$$

$$I_{\hat{\varepsilon}_3} = mb^2 + mb^2 + mb^2 + mb^2$$

$$= 4mb^2$$

$$I_O = mb^2 + mb^2 + mh^2$$

$$= \cancel{2mb^2 + mh^2}$$



- $\sin \alpha = h \cdot \dfrac{1}{\sqrt{b^2+h^2}}$
- $\sin \alpha = \dfrac{r}{b}$

$$r = \sin \alpha \cdot b$$

$$= \frac{hb}{\sqrt{b^2+h^2}}$$

$$I_\Delta = mb^2 + mb^2 + m\frac{h^2b^2}{b^2+h^2}$$

$$= mb^2\left(2 + \frac{h^2}{b^2+h^2}\right)$$

$$= \frac{mb^2}{b^2+h^2}\left(2(b^2+h^2) + h^2\right)$$

$$= \frac{mb^2}{b^2+h^2}\left(2b^2+3h^2\right)$$

(e) tenseur par rapport au point A

$$(\tilde{I}_A)_{ij} = \sum_\alpha m_\alpha\left[(\vec{AP_\alpha})^2\delta_{ij} - (AP_\alpha)_i(AP_\alpha)_j\right]$$

$$\begin{pmatrix} m(2b^2+h^2) & 0 & mhb \\ 0 & m(2b^2+h^2) & 0 \\ mhb & 0 & 4mb^2 \end{pmatrix}$$

COEFF DIAGONAUX

$$(I_A)_{11} = \sum_\alpha m_\alpha \left((\vec{AP_\alpha})^2 - ((AP_\alpha)_1)^2 \right)$$

$$= m(b^2 - b^2) \quad P_1$$

$$+ m(b^2 - 0^2) \quad P_2$$

$$+ m((-b)^2 - 0^2) \quad P_3$$

$$+ m(h^2 + b^2 - b^2) \quad P_4$$

$$= I_{A,1}$$

$$(I_A)_{22}$$

$$= m(b^2 - 0^2) + m((-b)^2 - b^2)$$
$$+ m(b^2 - b^2) + m(h^2 + b^2 - 0^2)$$

$$= mb^2 + m(h^2 + b^2) = 2mb^2 + mh^2$$

$$(I_A)_{3|3}$$

$$= m(b^2 - 0^2) + m(b^2 - 0^2)$$

$$+ m((-b)^2 - 0^2) + m(h^2 + b^2 - h^2)$$

$$= 4mb^2$$

COEFF NON DIAGONAUX

$$(I_A)_{1,2} = \sum_\alpha m_\alpha \left(-(AP_\alpha)_1 (AP_\alpha)_2\right)$$

$$= m(-b \cdot 0) + m(-b \cdot b)$$

$$+ m(-b \cdot (-b)) + m(-b \cdot 0)$$

$$= 0$$

$(I_A)_{1,3}$

$$= m(-b \cdot 0) + m(-b \cdot 0)$$

$$+ m(0 \cdot 0) + m(b \cdot h)$$

$$= mbh$$

$(I_A)_{2,1}$

$$= m(0 \cdot b)$$

$$+ m(b \cdot 0)$$

$$+ m(0 \cdot b)$$

$$+ m(-b \cdot 0) \quad = 0$$

$(I_A)_{2,3}$

$= m(0 \cdot 0)$

$+ m(b \cdot 0)$

$+ m(0 \cdot h)$

$+ m(-b \cdot 0)$ $= 0$

$(I_A)_{3,1}$

$= m(0 \cdot b)$

$+ m(0 \cdot 0)$

$+ m(h \cdot b)$

$+ m(0 \cdot 0)$ $= mhb$

$$(I_A)_{3,2}$$

$$= m(0 \cdot 0)$$
$$+ m\left(0 \cdot b\right)$$
$$+ m(h \cdot 0)$$
$$+ m(0 \cdot (-b))$$
$$= 0$$

Exercice 2

$$E_{cin} = \frac{1}{2} M \vec{v_A}$$
$$+ M \vec{v_A} \cdot (\vec{\omega} \wedge \vec{AG})$$
$$+ \frac{1}{2} \vec{\omega} \cdot (\tilde{I}_A \cdot \vec{\omega})$$

On pose A = G (centre de masse du ballon)

$$E_{cin} = \frac{1}{2} M \vec{v_A} + \frac{1}{2} \vec{\omega} \cdot (\tilde{I}_A \cdot \vec{\omega})$$
$$= \frac{1}{2} M \vec{v_A} + \frac{1}{2} I_\Delta \, \omega^2$$

$$\begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix} \begin{pmatrix} \omega \\ 0 \\ 0 \end{pmatrix}$$

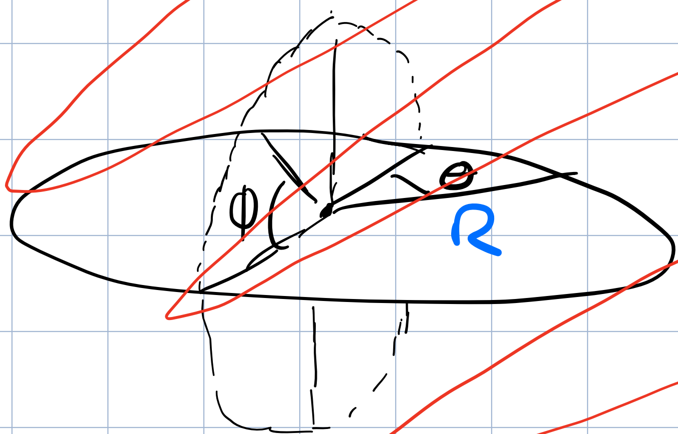
A ω

$$I_\Delta = \int_{\text{bolken}} m R^2$$

$$= \int_0^{2\pi} \int_0^{\pi} R\sin\theta \, d\theta$$

coordonées polaire

$r = R\sin\theta$



$$\sin\theta = \frac{opp}{hyp}$$

$$\Leftrightarrow \sin\theta = \frac{d}{R}$$

$\Leftrightarrow$

$$I_\Delta = \int_{\text{bolken}} R^2 m$$

surface sphere : $4\pi R^2$

$$I_\Delta = 4\pi R^2 \cdot R^2 \cdot m = 4R^4\pi \ .$$

$$\cancel{E_{cin} = \frac{1}{2} M \vec{V}_G + \frac{1}{2} 4R^4\pi}$$

$$\cancel{= \frac{1}{2}}$$

On ne connaît pas $\vec{V}_G$ mais on sait que $\vec{v}_C$ (le point de contact) a une vitesse nulle

$$\vec{V}_G = \vec{V}_C + \underbrace{\vec{\omega} \wedge \vec{CG}} = \vec{\omega} \wedge \vec{CG}$$

$$\Leftrightarrow \quad V_G = R\omega$$

$$I_\Delta \text{ (boule creuse)} = \frac{2}{3} MR^2$$

$$\Rightarrow E_{cin} = \frac{1}{2} M \vec{V}_G^{\,2} + \frac{1}{2} \cdot \frac{2}{3} MR^2 \omega^2$$

$$= \frac{1}{2} M \vec{v}_G^{\,2} + \frac{1}{3} M \vec{v}_G^{\,2}$$

$$= \frac{5}{6} M \vec{v}_G^{\,2}$$

(b)