(1)

$$\lim_{x \to 0} \frac{x^3}{3} + 3x^2$$

$$= 0$$

Oui, car $\lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$

(2)

$$\lim_{x \to 0} \frac{\sin(x)}{x} \times \frac{1}{\cos(x)}$$

$$= 1$$

continue ✓

(3) $\lim_{x \to 2^-} \frac{x-1}{x-2}$

$$= \frac{1^-}{0^-} = -\infty$$

$$\lim_{x \to 2^-} \frac{x-1}{x-2}$$

$$= \frac{1^+}{0^+} = +\infty$$

pero continue

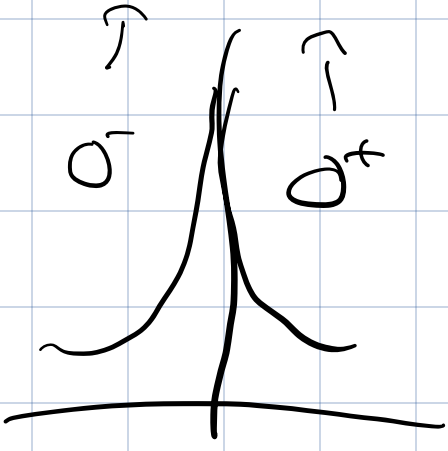
(4) $x \sin\left(\frac{1}{x}\right)$

$$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

$$-x \leq f(x) \leq x$$

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$$

continua



⑤ $h = x - 3$

$$j(x) = \frac{e^h - 1}{h}$$

$$\lim_{h \to 0} = \frac{e^h - e^0}{h - 0}$$

taux d'accroissement au point h.

$$\exp(0) = 1.$$

⑥

$$\lim_{x \to 0} -\sqrt{|x|} = 0$$

continue

Exercice 2

①

$$\lim \frac{x\left(1 + \frac{1}{x}\right)}{}$$

$$= \frac{\left(1+\frac{1}{2c}\right)x\sqrt{x^2\left(\frac{1}{2}+1+\frac{1}{x^2}\right)}}{x\left(1+\frac{1}{x}+\dots\right)}$$

$$= 1$$

(2) $$-1 \leq \sin(x) \leq 1$$

$$-x \leq x\sin(x) \leq x$$

$$\frac{-x}{x^2+2} \leq \text{“} \leq \frac{x}{x^2+2}$$

$$\frac{-1}{x+\frac{2}{x}} \leq \text{“} \leq \frac{1}{x+\frac{2}{x}}$$

$\to 0^-$, “ , $\to 0^+$

(3) $$\lim_{x \to +\infty} (3) = +\infty$$

$$x\left(\sin\left(\frac{1}{x}\right)+\cos\left(\frac{1}{x}\right)\right)$$

$\searrow$ $\searrow$ $\searrow$

$+\infty$ 0 1

Exercice 3

$$\lim_{x \to 0^-} \frac{1}{\ln(|x|)} = 0^-$$

$\ln(|x|) \to -\infty$

$$\lim_{x \to 0^+} \frac{1}{\ln(|x|)} = 0^-$$

Continue sur $\boxed{\mathbb{R}.}$

$$\lim_{x \to 0} \sin\left(x \cdot \frac{\pi}{2}\right)$$

$$= 0$$

$$\frac{\sin(x) - (ax^3 + bx)}{x^3}$$

$$-(ax^3 + bx) = x^3$$

$$a = (-1)$$

$$b \in \mathbb{R}$$

Exercice 4

① $l(x) = x^3 - 3x^2 + 3x - 1$ $[0;3]$

$l'(x) = 3x^2 - 6x + 3$

$x_1 = 1$

donc $l'(x) > 0$ donc l ↗ sur $[0,3]$

D'après le TVI :

$\text{Im}(l(x)) = [-1;8]$

$27 - 27 + 9 - 1$

② $m'(x) = \frac{-1}{x^2}$

↳ $m(-1) = 1$ $[-1;1]$

$m(1) = -1$

(3) $n(x) = \tan(x)$ sur $\left[\frac{-\pi}{4}; \frac{\pi}{3}\right]$

$$n'(x) = \frac{1}{\cos(x)^2}$$ ↗

$$n\left(\frac{-\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

$$n\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4}\right)$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \sqrt{3}$$

$$[1; \sqrt{3}]$$

Exercice 5

① $f(x) = x^3 - 2x - 1 - \sin(x)$

② $f'(x) = 3x^2 - 2 - \cos(x)$

$f(\pi) = \pi^3 - 2\pi - 1 - \sin(\pi) > 0$

$f(-\pi) = (-\pi)^3 + 2\pi - 1 - 0 < 0$

③ La fonction est