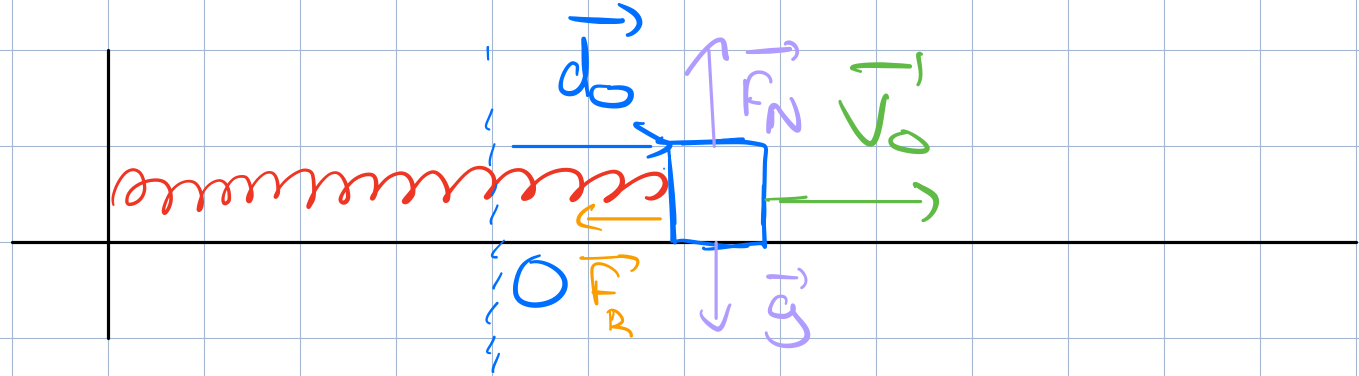
Ex 1



① $\sum \vec{F} = m\vec{a}$

$= F_n \hat{e}_y - g\hat{e}_y - \underbrace{F_R \hat{e}_x}_{= -k \cdot \vec{d}}$

② On projette les forces :

$$\begin{cases} m\ddot{y} = F_n - g \\ m\ddot{x} = -k \cdot x \end{cases}$$

On pose les contraintes :

$$y = 0 \Rightarrow \dot{y} = 0 \Rightarrow \ddot{y} = 0.$$

Donc

$$\begin{cases} F_n = -g \\ m\ddot{x} = -k \cdot x \end{cases}$$

$$x(t_0) = x_0$$
$$v(t_0) = v_0.$$

(H) $$x(t) = x_0 \cos(\omega_0 (t - t_0)) + \frac{v_0}{\omega_0} \sin(\omega_0 (t - t_0))$$

$$\dot{x}(t) = -x_0 \omega_0 \sin(\omega_0 (t - t_0)) + \frac{v_0}{\cancel{\omega_0}} \cancel{\omega_0} \cos(\omega_0 (t - t_0))$$

$$\ddot{x}(t) = -x_0 \omega_0^2 \cos(\omega_0(t-t_0)) - v_0 \omega_0 \sin(\omega_0(t-t_0))$$

$$m\left(-x_0 \omega_0^2 \cos(\omega_0(t-t_0)) - v_0 \omega_0 \sin(\omega_0(t-t_0))\right)$$

$$= -k\left(x_0 \cos(\omega_0(t-t_0)) + \frac{v_0}{\omega_0} \sin(\omega_0(t-t_0))\right).$$

$$\Leftrightarrow -m \cdot \omega_0^2 \left(x(t)\right) = -(\omega_0^2 \cdot m)(x(t))$$

$$\Leftrightarrow T.$$

$$k = \omega_0^2 \cdot m$$

$$f(t+T) = f(t) \quad \forall t$$

$$T = \frac{2\pi}{\omega_0}$$

$$\cos\left(\omega_0\left(t + \frac{2\pi}{\omega_0} - t_0\right)\right)$$

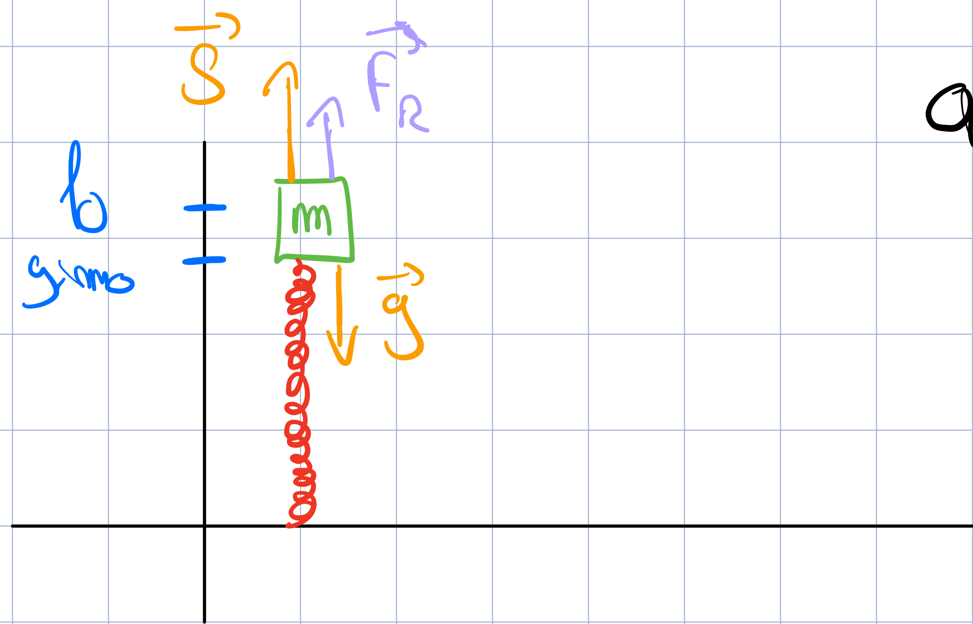
$$= \cos\left(\omega_0(t - t_0) + 2\pi\right)$$

$$= \cos\left(\omega_0(t - t_0)\right)$$

$$\sin\left(\omega_0\left(t + \frac{2\pi}{\omega_0} - t_0\right)\right)$$

$$= \sin\left(\omega_0(t - t_0)\right) \qquad \boxed{\underline{ok}}$$

Exercice 2



On étudie la masse m

$$\sum \vec{F} = m\vec{a}$$

On pose les contraintes :

$$y = 0 \Rightarrow \dot{y} = 0 \Rightarrow \ddot{y} = 0$$

$$\begin{cases} m\ddot{y} = -mg + k \cdot d \\ \quad\;\; = -mg + k \cdot (l_0 - y) \end{cases}$$

$$\vec{d} = l_0 - y$$

$$mg = k \cdot l_0 - k \cdot y$$

$$\Leftrightarrow \quad mg - k \cdot l_0 = -k \cdot y$$

$$\Leftrightarrow \quad y = -\frac{mg}{k} + l_0$$