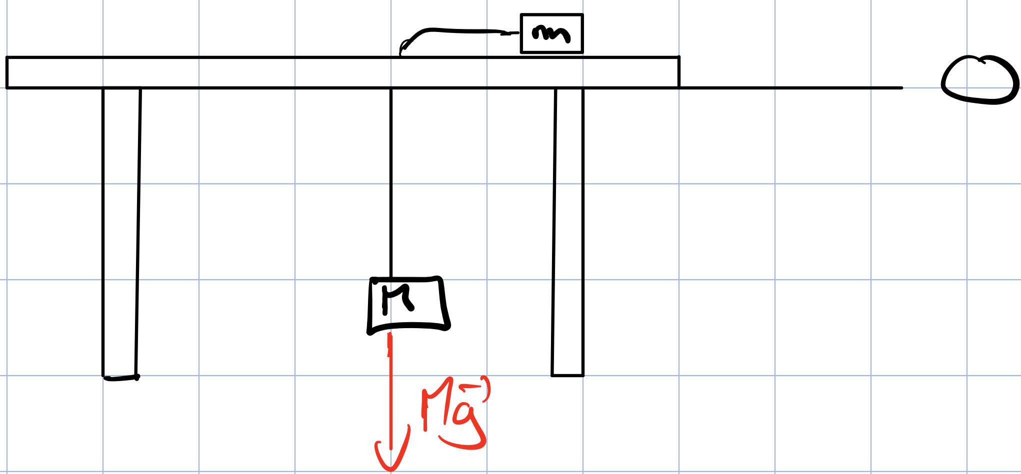
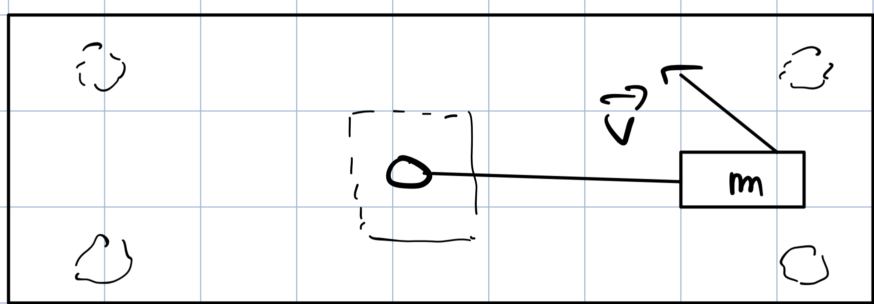
pas vitesse init ← elle tombe

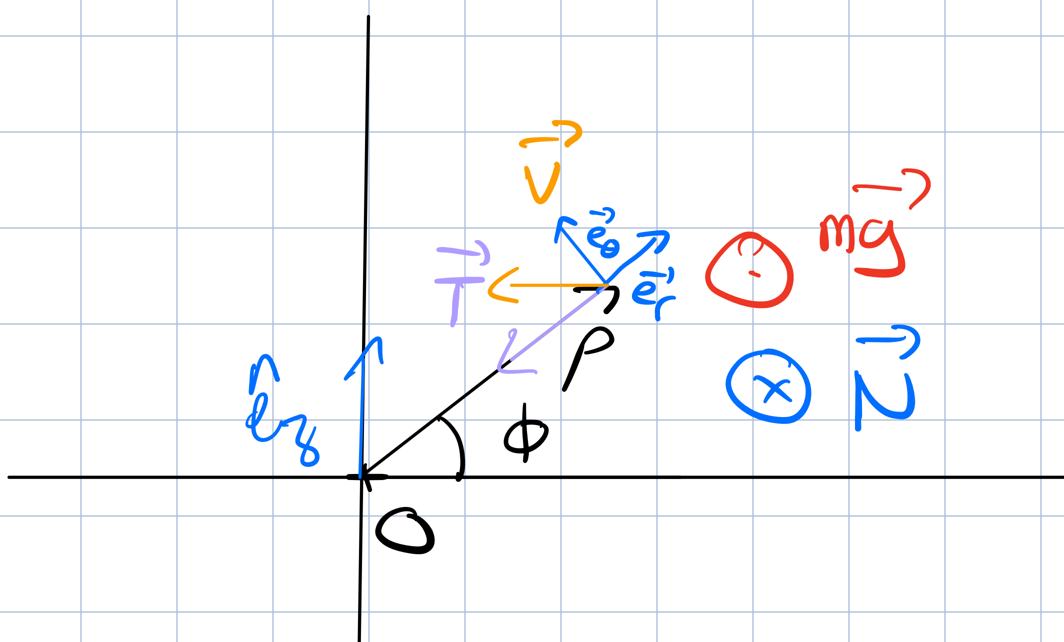
moment cinétique constant

moment cinétique nul pr tomber

1 La table à trou







(a) $E_m = \frac{1}{2} m v_m^2 + \frac{1}{2} M v_M^2 + Mgh$

⚠ avant de pouvoir écrire ça, on doit montrer que la tension travaille mais que T et T' s'annulent

Soit $\vec{T}$ la tension du fil sur la masse m.
et $\vec{T'}$ la tension du fil sur la masse M.

$$dW_T = \vec{T} \cdot d\vec{s} = \vec{T} \cdot (dr\,\hat{e_r} + d\theta\,\hat{e_\theta})$$
$$= \cos(\pi) \cdot \|\vec{T}\| \cdot dr$$
$$= -T\,dr$$

$$dW_{T'} = \vec{T'} \cdot d\vec{z} = \vec{T'} \cdot (dz\,\hat{e_z})$$
$$= \cos(0) \cdot \|\vec{T'}\| \cdot dz$$

$$= T' dz$$

or $T' = T$, donc $dW_{T'} = -dW_T$

et $dr = dz$ (le fil a une longueur cste).

donc on a bien :

$$E_m = \frac{1}{2} m v_m^2 + \frac{1}{2} M v_M^2 + V_M(z)$$

$$= \frac{1}{2} m v_m^2 + \frac{1}{2} M v_M^2 + Mgz$$

$V_M(x) - V_M(0)$

$$= -\int_0^z M\vec{g} \cdot d\vec{z} \quad \left(= -\int_0^z -Mg\vec{e_z} \cdot dz\,\vec{e_z} \right)$$

$$= Mgz + V(z_0)$$

(on l'a redémontré mais on peut utiliser les slides du cours)

donc on a que des forces conser-
vatrices et l'énergie mécanique est constante.

(b) Force centrale

$\Rightarrow$ E mécanique conservée
$\Rightarrow$ moment cinétique ?

$$\vec{L} = \vec{r} \wedge m\vec{v}$$

$$\frac{d}{dt}\left(\vec{r} \wedge m\vec{v}\right)$$

$$= \vec{r} \wedge m\vec{a} + \cancel{\vec{v} \wedge m\vec{v}} = \vec{0}$$

$$= m\left(\vec{r} \wedge \ddot{\vec{r}}\right)$$

$$\left.\begin{array}{l} m\ddot{\vec{r}} = -Mg\hat{e}_r \\ \text{et } \vec{r} = r\hat{e}_r \end{array}\right) \text{ donc } m\left(r\hat{e}_r \wedge (-Mg)\hat{e}_r\right) = \vec{0}.$$

donc moment cinétique constant.

ⓒ

$$\begin{cases} m(\ddot{\rho} - \rho\dot{\phi}^2) = -T \\ m(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}) = 0 \\ M\ddot{z} = T' - Mg = M\ddot{\rho} \end{cases}$$

$$\Rightarrow T' = M\ddot{\rho} + Mg$$

donc $m(\ddot{\rho} - \rho\dot{\phi}^2) + T' = -T + T'$

$$\Leftrightarrow m(\ddot{\rho} - \rho\dot{\phi}^2) + T' = 0$$

$$\Leftrightarrow m(\ddot{\rho} - \rho\dot{\phi}^2) + M\ddot{\rho} + Mg = 0$$

OU

$$\frac{dL_z}{dt} = m\left(2\rho\dot\rho\dot\phi + \rho^2\ddot\phi\right)$$

$$= 0 \quad \left(\text{nur ein } \atop \text{cshe}\right)$$

$$0 = \frac{dE}{dt}$$

$$= (M+m)\dot\rho\ddot\rho$$

$$+ m\rho\dot\rho\dot\phi^2$$

$$+ m\rho^2\dot\phi\ddot\phi$$

$$+ Mg\dot\rho$$

$$= (M+m)\dot\rho\ddot\rho$$

$$+ m\rho\dot\phi(\dot\rho\dot\phi + \rho\ddot\phi) + Mg\dot\rho$$

$$0 = (M+m)\ddot{\rho} - m\rho\dot{\phi}^2 + Mg$$

$$0 = (M+m)\ddot{\rho} - \frac{L_z^2}{m\rho^3} + Mg.$$

② Pendule asymétrique

ⓐ

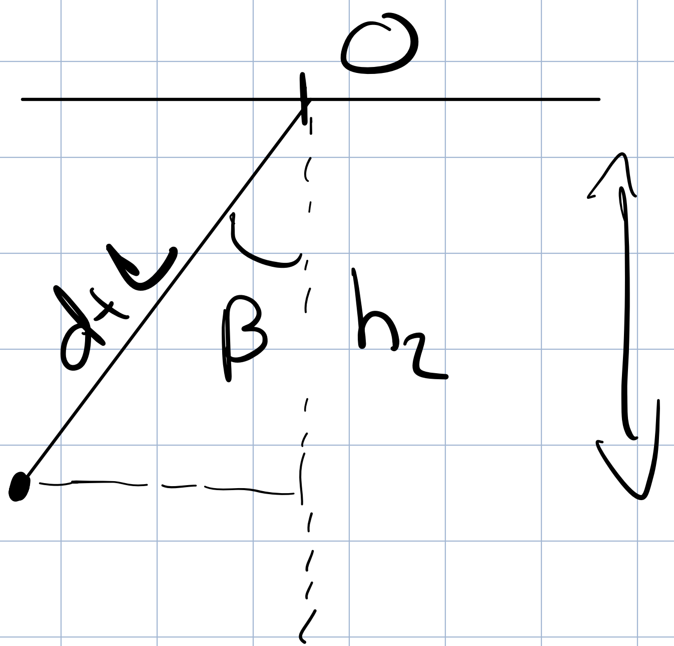
$$\vec{L} = \vec{r} \wedge m\vec{v}$$

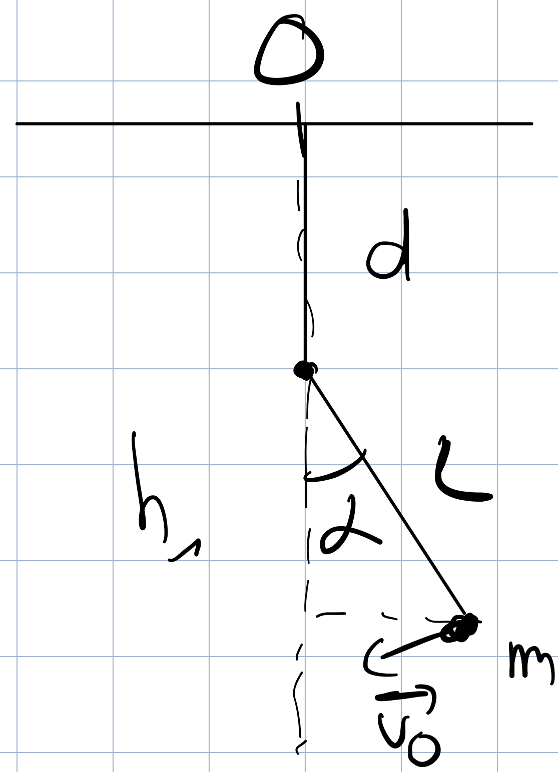
$$= |\vec{r}| \cdot |\vec{v}| \cdot \sin$$

$$E_{m_2} = \frac{1}{2} m v^2 + mgh$$

$$= mg(\cos\beta)(d+L)$$

$$E_{m_1} = \frac{1}{2} m v_0^2 + mg((\cos\alpha)(L)+d)$$





$$\cos \beta = \frac{adj}{hyp} = \frac{h_2}{d+L}$$

$$\Leftrightarrow h_2 = \cos \beta \cdot (d+L)$$

$$\cos \alpha = \frac{adj}{hyp} = \frac{h_1}{L}$$

$$\Leftrightarrow h_1 = \cos \alpha \cdot L + d$$

$$\frac{1}{2} m v_0^2 - mg\left((\cos\alpha)(L)+d\right)$$

$$= -mg(\cos\beta)(d+L)$$

On pose $a = \cos\alpha = \cos\beta$.

$$\frac{1}{2} m v_0^2 - mg(aL+d) = -mga(d+L)$$

$$\Leftrightarrow \frac{1}{2} m v_0^2 = mg(-ad - aL + aL + d)$$

$$\Leftrightarrow \frac{1}{2} m v_0^2 = mg(-ad + \cancel{aL} - \cancel{aL} + d)$$

$$\Leftrightarrow v_0^2 = 2g(ad+d)$$

$$\Leftrightarrow v_0 = \sqrt{2gd(-\cos\alpha + 1)}$$

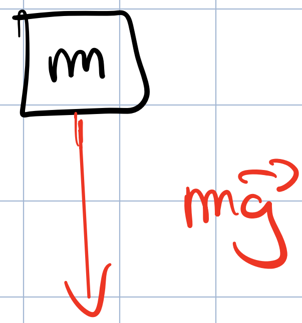
$$\frac{1}{2} m v_{max}^2 - mg(d+L) = - mg\cos\beta(d+L)$$

$$\Leftrightarrow \frac{1}{2} m v_{max}^2 = mg\left(-\cos\beta(d+L) + d + L\right)$$

$$\Leftrightarrow m v_{max}^2 = 2mg(d+L)(1-\cos\beta)$$

$$\Leftrightarrow v_{max} = \sqrt{2g(d+L)(1-\cos\beta)}$$

③



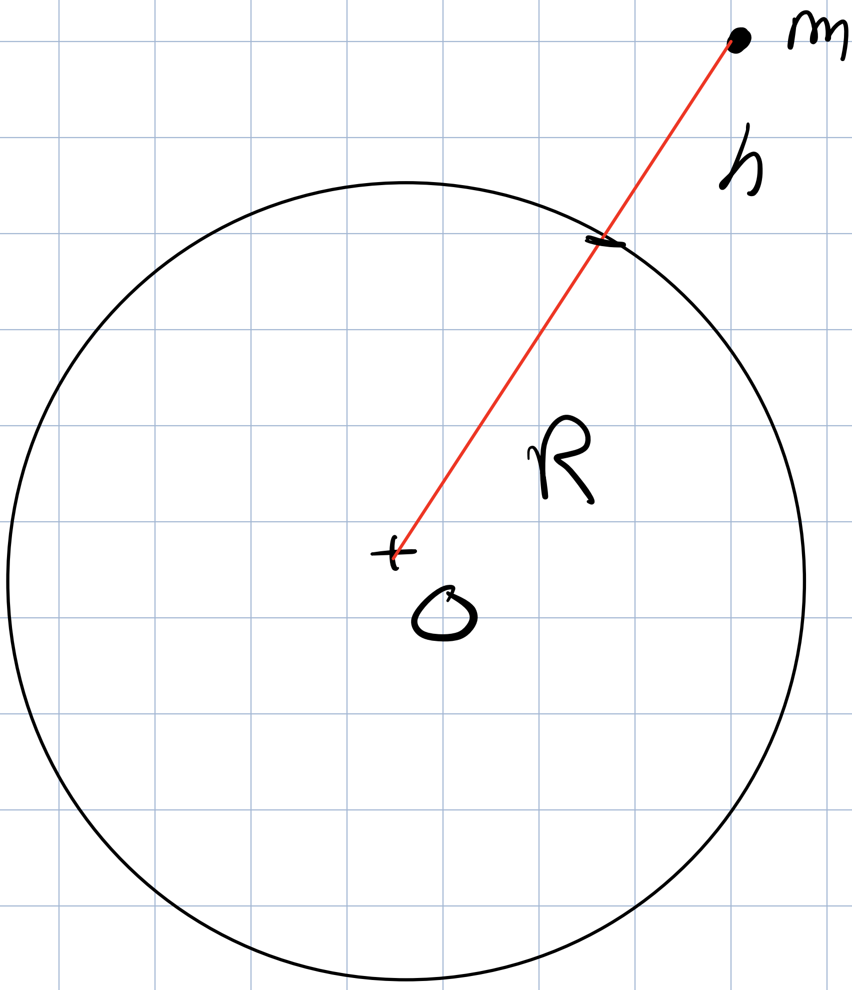
$$\sum \vec{F} = m\vec{a} .$$

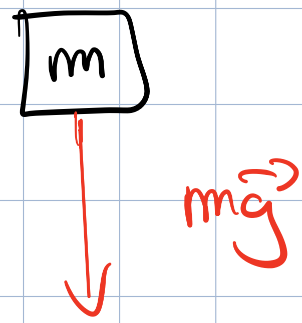
$$= -G \frac{Mm}{(R_T+z)^2}\hat{z}$$

$$\begin{cases} m\ddot{z} = -G \dfrac{M_T\, m}{(R_T+z)^2} \end{cases}$$

$$r(t) = R_T + z(t)$$

Exercice 3





$$\sum \vec{F} = m\vec{a}.$$

$$= -G \frac{Mm}{(R_T + z)^2}\hat{z}$$

$$m\ddot{z} = -G \frac{M_T m}{(R_T + z)^2}$$

$$\Leftrightarrow \ddot{z} = -G \frac{M_T}{(R_T + z)^2}$$

(2) On multiplie par $\dot{z}$ des 2 côtés

$$m\ddot{z}\dot{z} = -\frac{GmM_T}{(R_T+z)^2}\dot{z}$$

$$\dot{z} = \frac{dz}{dt}$$

$$\int m\ddot{z}\dot{z}\,dt = \int -GmM_T\left(\frac{1}{(R_T+z)^2}\right)\dot{z}\,dt$$

$$\Leftrightarrow \frac{1}{2}m\dot{z}^2 = -GmM_T\int\frac{1}{(R_T+z)^2}\,dz$$

$$\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$$

$$\frac{1}{2}m\dot{z}^2 = \frac{GmM_T}{(R_T+z)} + C$$

À une hauteur H du sol, v est nulle.

$$\frac{1}{2}mv^2 = 0 \Rightarrow C = \frac{-GmM_T}{(R_T+H)}$$

donc :

$$E(z, \dot{z}) = \frac{1}{2} m \dot{z}^2 - \frac{G m M_T}{R_T + z} = - \frac{G m M_T}{R_T + H}$$

(c)

On pose $z = 0$

$$E = -\frac{GmM_T}{R_T + H} = \frac{1}{2} m {V_{max}}^2 - \frac{GmM_T}{R_T}$$

$$\Leftrightarrow \quad {V_{max}}^2 = \left(\frac{-GmM_T}{R_T + H} + \frac{GmM_T}{R_T} \right) \frac{2}{m}$$

$$= \frac{-GmM_T R_T + GmM_T (R_T + H)}{(R_T + H)(R_T)} \frac{2}{m}$$

$$= \frac{2GM_T (\cancel{-R_T} \cancel{+R_T} + H)}{R_T (R_T + H)}$$

$$= \frac{2GM_T H}{R_T (R_T + H)}$$

$$V_{max} = \sqrt{\frac{2GM_T H}{R_T (R_T + H)}}$$

d) $$E = \frac{1}{2} m v^2 + m \frac{G M_T}{R_T^2} z$$

$$= m \frac{G M_T}{R_T^2} H$$

$$\Rightarrow \frac{1}{2} m v_{max}^2 = m \frac{G M_T}{R_T^2} H$$

$$\Leftrightarrow v_{max}^2 = 2 \frac{G M_T}{R_T^2} H$$

$$\Leftrightarrow v_{max} = \sqrt{2 \frac{G M_T}{R_T^2} H}$$

$$\frac{v_{max_1} - v_{max_2}}{v_{max_1}}$$

$$=$$

$$\frac{1-\sqrt{\frac{2GMH}{R^2}}}{\sqrt{\frac{2GMH}{R(R+H)}}} =$$

$$= 1-\sqrt{1/R^2} \Big/ \sqrt{1/(R+H)R}$$

$$1-\sqrt{1/R} \Big/ \sqrt{1/R+H}$$

$$1-\sqrt{\frac{1/R}{1/R+H}}$$

$$1-\sqrt{\frac{1}{1/1+H/R}}$$

$$1-\sqrt{1+H/R}$$