$$\left|\begin{array}{ccc|cc} (2-\lambda) & 4 & 3 & (2-\lambda) & 4 \\ -4 & (-6-\lambda) & -3 & -4 & (-6-\lambda) \\ 3 & 3 & (1-\lambda) & 3 & 3 \end{array}\right.$$

$$= (2-\lambda)(-6-\lambda)(1-\lambda)$$

$$+ (4)(-3)(3)$$

$$+ (3)(-4)(3)$$

$$- (3)(-6-\lambda)(3)$$

$$- (2-\lambda)(-3)(3)$$

$$- (4)(-4)(1-\lambda)$$

$$= (-12 - 2\lambda + 6\lambda + \lambda^2)(1-\lambda)$$

$$-36-36-9(-6-\lambda)$$

$$+9(2-\lambda)$$

$$+16(1-\lambda)$$

$$=-\cancel{1}2-2\lambda+6\lambda\cancel{+\lambda^2}$$

$$+\cancel{12\lambda}\cancel{+2\lambda^2}\cancel{-6\lambda^2}\cancel{-\lambda^3}$$

$$-36-36+54+\cancel{9\lambda}+18\cancel{-9\lambda}$$

$$+\cancel{1}6-16\lambda$$

$$=4+4\lambda-3\lambda^2$$

$$-\lambda^3+16\lambda-72+54$$

$$+18$$

$$\begin{vmatrix} 2-\lambda & 4 & 3 \\ -4 & -6-\lambda & -3 \\ 3 & 3 & 1-\lambda \end{vmatrix}$$

$$= 4 - 3\lambda^2 - \lambda^3 = (1-\lambda)(\lambda^2+4\lambda+4)$$
$$= (1-\lambda)(\lambda-2)^2$$

$$\begin{array}{l|l} -\lambda^3 - 3\lambda^2 + 4 & 1-\lambda \\ \hline -(-\lambda^3+\lambda^2) & \lambda^2+4\lambda+4 \\ \hline 0 \; -4\lambda^2 + 4 & \\ -(-4\lambda^2+4\lambda) & \end{array}$$

$$\begin{array}{l} \hline 0 \; -4\lambda + 4 \\ -(-4\lambda+4) \\ \hline \end{array}$$

$$0$$

Pour $\lambda = 1$ :

$$\ker \left( \begin{pmatrix} 1 & 4 & 3 \\ -4 & -7 & -3 \\ 3 & 3 & 0 \end{pmatrix} \right)$$

$$\begin{pmatrix} 1 & 4 & 3 \\ 0 & -23 & -15 \\ 3 & 3 & 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 4 & 3 \\ 0 & -23 & -15 \\ 0 & -9 & -9 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 4 & 3 \\ 0 & -23 & 23 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\rightarrow \left(\begin{array}{ccc} 1 & 1 & 0 \\ 0 & -23 & 15 \\ 0 & 1 & 1 \end{array}\right|$$

$$\rightarrow \left( \phantom{\begin{array}{c} 0 \\ 0 \\ 0 \end{array}} \right.$$

# Exercice 2

A diagonalisable

$\Leftrightarrow \exists P$ inversible sh $A = PDP^{-1}$

$\Rightarrow A^2 = PDP^{-1}PDP^{-1}$

$= PD^2P^{-1}$

et $A^k = PD^kP^{-1}$

---

$A^k = PD^kP^{-1}$

$A^{k-1} = PD^kP^{-1}A^{-1}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} a^2 + cb & ba + db \\ ac + cd & bc + d^2 \end{pmatrix}$$

$$\begin{cases} a^2 + cb = 0 \\ ba + db = 0 \\ ac + cd = 0 \\ bc + d^2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} a^2 - d^2 = 0 \Rightarrow a = d \\ \\ \end{cases}$$

$$\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-1 & 1-1 \\ -1+1 & -1+1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\det \begin{pmatrix} 1-\lambda & 1 \\ -1 & -1-\lambda \end{pmatrix}$$

$$= (1-\lambda)(-1-\lambda) + 1$$

$$= \cancel{-1} - \cancel{\lambda} + \cancel{\lambda} + \lambda^2 + \cancel{1}$$

$$= \lambda^2$$

$$\Delta = (-2)^2 - 4 \cdot 0$$

$$= 4$$

$$\sqrt{\Delta} = 2$$

$$\lambda = \frac{2 \pm 2}{2} \quad \begin{cases} + \rightarrow 2 \\ - \rightarrow 0 \end{cases}$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\det \begin{pmatrix} 0-\lambda & 0 \\ 1 & 0-\lambda \end{pmatrix}$$

$$= (0-\lambda)^2$$

$$\ker \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$= \text{Vect} \left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$

# Exercice 5

(a) $\dfrac{1}{z} = \dfrac{1}{z}\dfrac{\bar{z}}{z}$

$\bar{i} = -i$

$\overline{i^2} = \overline{-1} = -1$

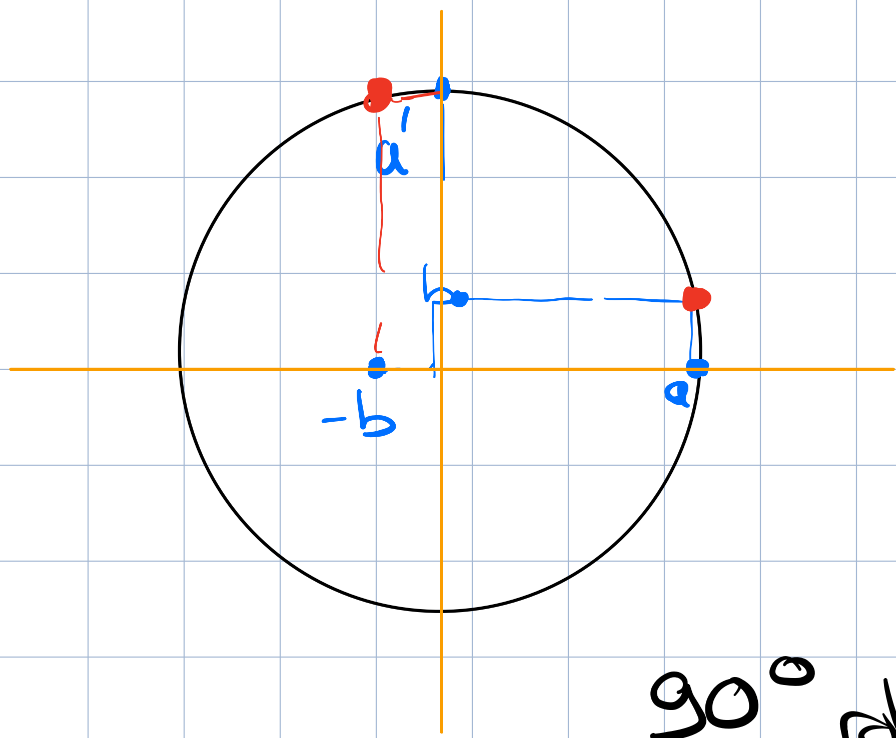
$(\bar{i})^2 = (-i)^2 = 1$

$\dfrac{1}{i} = \dfrac{1}{i}\dfrac{-i}{i} = \dfrac{-i}{i^2} = i$

(b) $iz = i(\overbrace{a+ib}^{z})$

$= ai + i^2 b$

$= ai - b$




90° rotation

# Exercice 6

ⓐ $A = PBP^{-1}$

$\Leftrightarrow \quad AP = PB$

$\Leftrightarrow \quad P^{-1}AP = B$

$\Leftrightarrow \quad QAQ^{-1} = B$

TRUE ? ~~new~~ $PA = PB$

i know specific case

$A = PQP^{-1}$

same eigenvalues

(b) FALSE

multiplicité géométrique 2

(c) FALSE $\quad A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

"

(d) TRUE

$$A = PBP^{-1}$$

$$B = QCQ^{-1}$$

$$A = PQCQ^{-1}P^{-1}$$

$$= X \; C \; X^{-1}$$