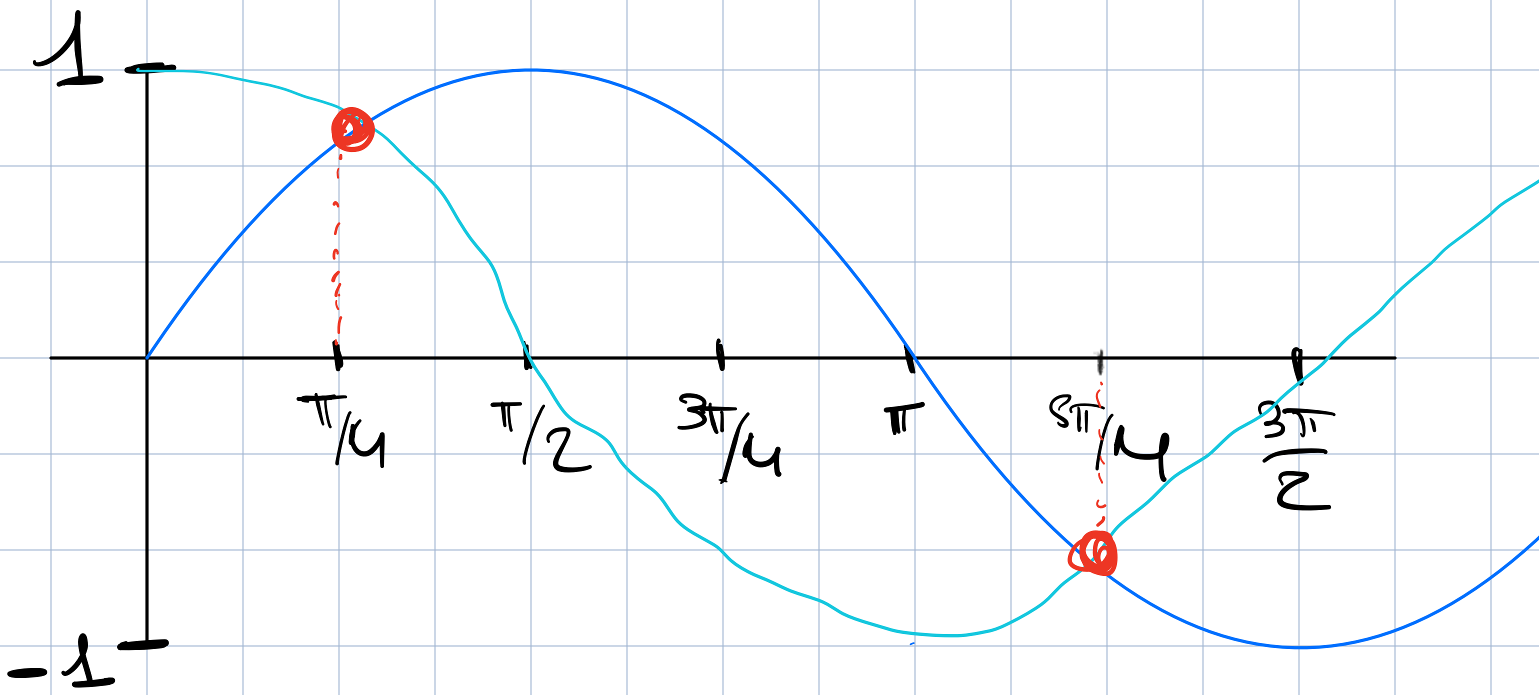
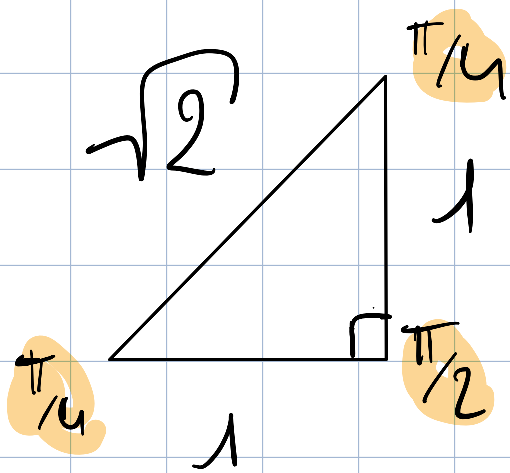
Fonct° élémentaires



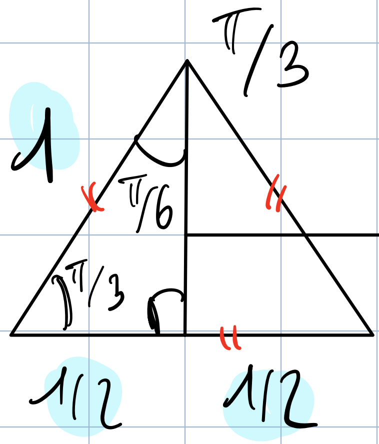
$$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$$



$$\left(\sqrt{2} = \sqrt{1^2 + 1^2}\right)$$

Pythagore

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$



par Pythagore $\frac{\sqrt{3}}{2}$

$$\sin\left(\frac{\pi}{6}\right) = \frac{1/2}{1} = \frac{1}{2}$$

$$\cos\left(\pi/6\right) = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$\sin\left(\pi/3\right) = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$$

$$\cos\left(\pi/3\right) = \frac{1/2}{1} = \frac{1}{2}$$

i

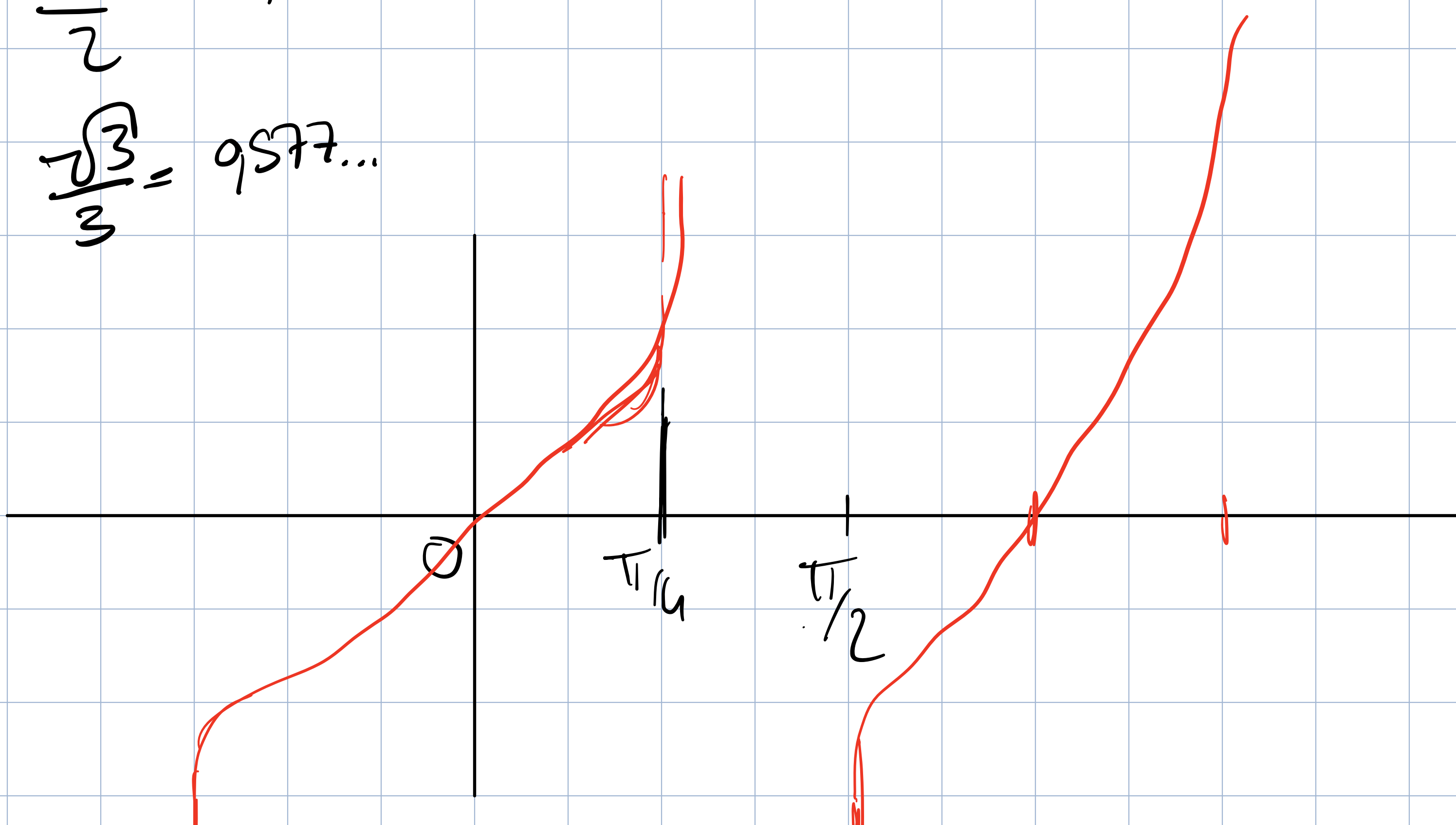
$\sqrt{2} = 1{,}414\ldots$

$\frac{\sqrt{2}}{2} = 0{,}707\ldots$

$\sqrt{3} = 1{,}732\ldots$

$\frac{\sqrt{3}}{2} = 0{,}866\ldots$

$\frac{\sqrt{3}}{3} = 0{,}577\ldots$



$$\mathrm{tg}(x) \equiv \tan(x)$$

$$\tan\left(\frac{\pi}{4}\right) = \frac{\sin \pi/4}{\cos \pi/4} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

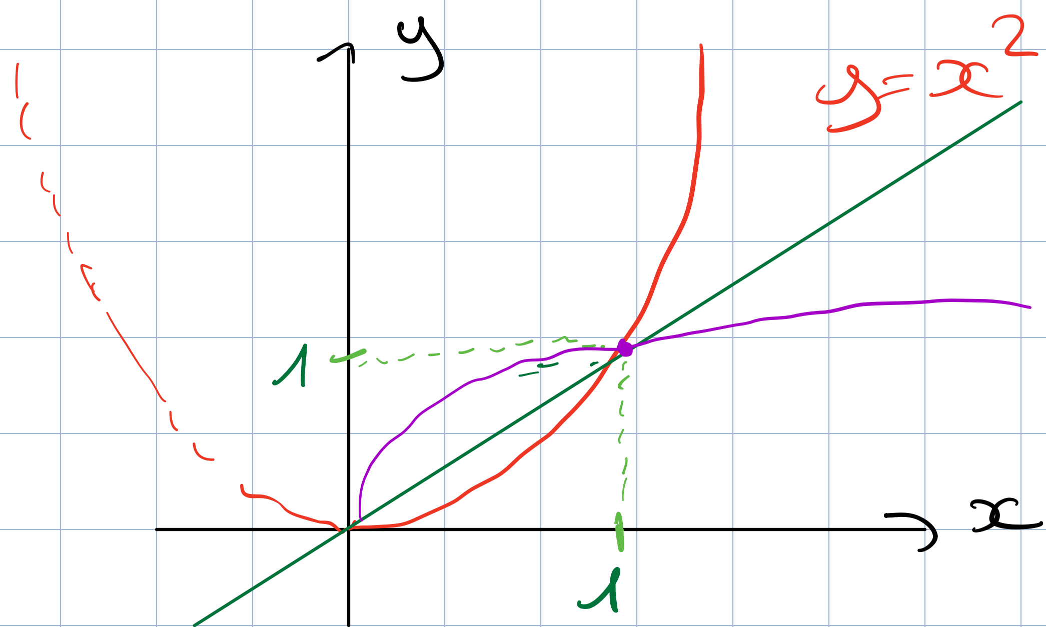
$$\tan\left(\pi/6\right) = \frac{\sin \pi/6}{\cos \pi/6} = \frac{1/2}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{3}$$

$$\tan\left(\pi/3\right) = \frac{\sin \pi/3}{\cos \pi/3} = \frac{\frac{\sqrt{3}}{2}}{1/2} = \sqrt{3}$$

Paires de fonct° réciproques

e.g. $x^2, \sqrt{x}$



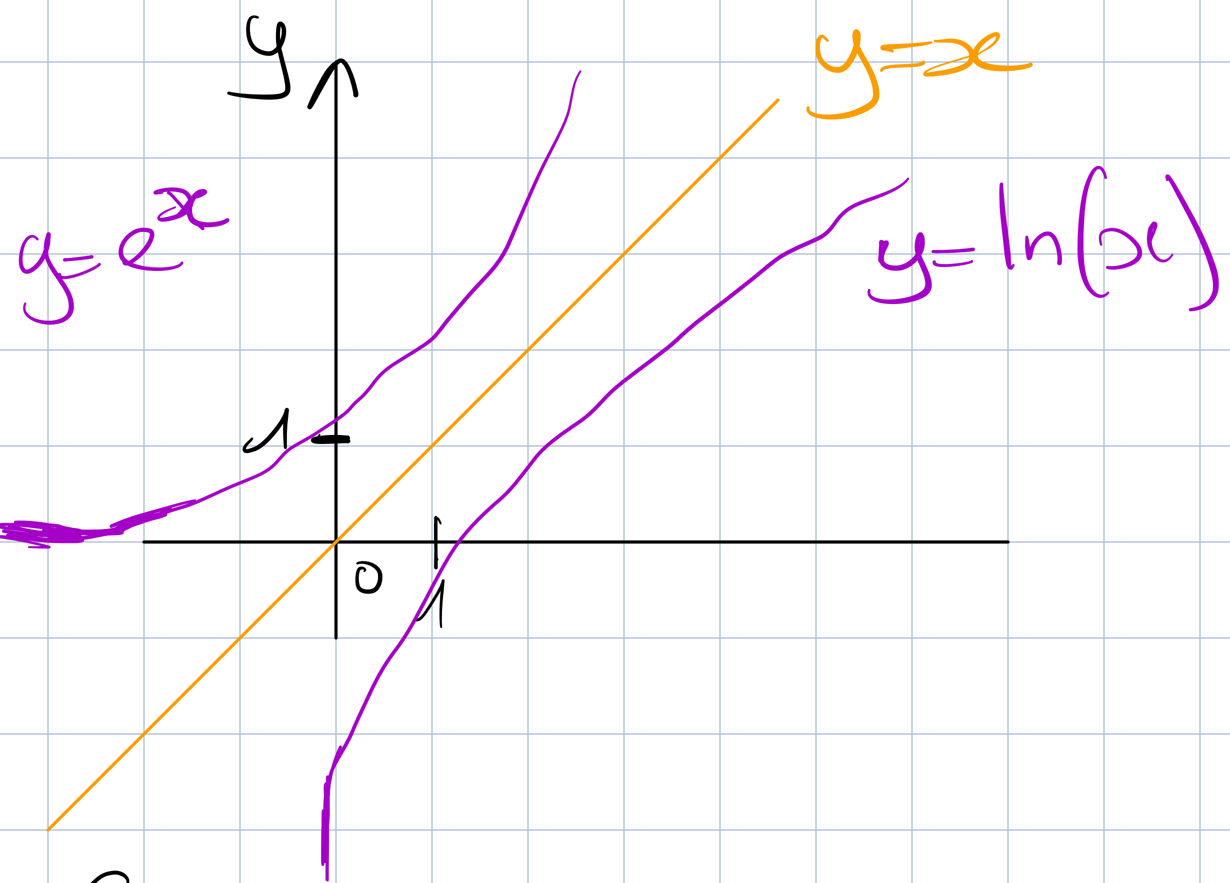
x

$\sqrt{x^2} = x$

$f : D_1 \to T_1 \quad g : D_2 \to T_2$

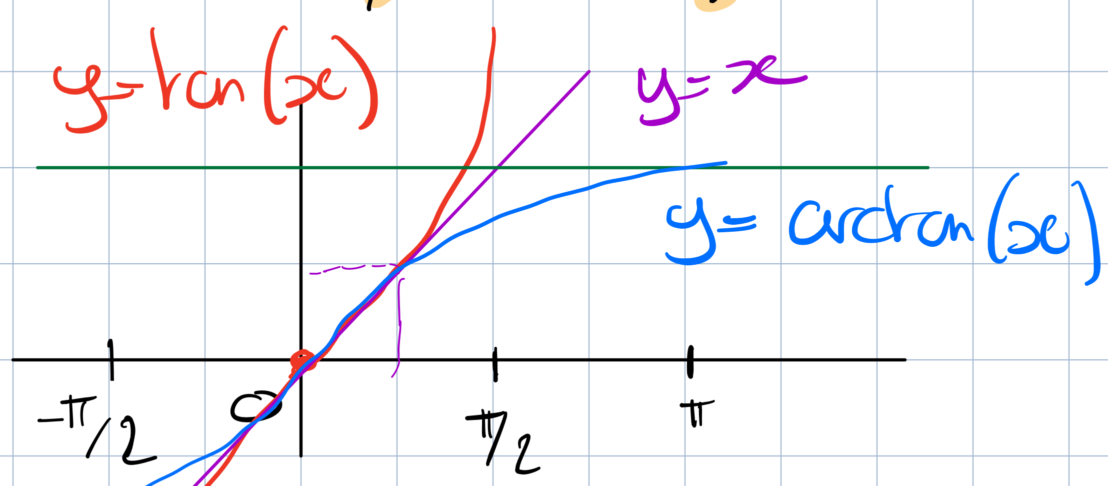
(f et g sont réciproques ssi , $f(g(x)) = x$
$\forall x \in D_1$ et $x \in T_2$ et $g(f(x)) = x$) ?

$\exp(x) = e^x, \ln(x)$ (logarithme népérien)



$$\begin{cases} e^{\ln(x)} = x \\ \ln(e^x) = x \end{cases}, \quad x \in \mathbb{R}$$

tan(x), arctan(x)



$$\begin{cases} \tan(\arctan(x)) = x \\ \arctan(\tan(\alpha)) = \alpha \end{cases}, \quad \alpha \in \left]-\frac{\pi}{2} ; \frac{\pi}{2}\right[\text{ et } x \in \mathbb{R}$$

Puissances, racines, etc

$\forall a, b \in \mathbb{R}_+^*$ et $m, n \in \mathbb{R}^+$: $\mathbb{R}$? ou $\mathbb{N}$?

$\boxed{\text{on a } \quad a^0 = 1 \quad (\text{convention})}$

$$a^n \cdot a^m = a^{(n+m)}$$

$$(a^n)^m = a^{n \cdot m} = (a^m)^n$$

$$(a \cdot b)^m = a^m \cdot b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} = a^m \cdot b^{-m}$$

$$= \frac{b^{-m}}{a^{-m}} = \left(\frac{b}{a}\right)^{-m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

On a les membres règles de calculs pour a^x, $x \in \mathbb{R}$

$$\boxed{\sqrt[n]{a} = a^{1/n} \quad (\text{où } a > 0 \text{ !!})}$$

si $\left(a^{1/n}\right)^n = a$ c-a-d ?

pour $a = 2n+1$, $n \in \mathbb{N}^+$, alors :

$$\sqrt[n]{a} = -\sqrt[n]{|a|}$$

réciproque de a^x, $a > 0$, $a \neq 1$ est :

$\log_a(x)$

$$\begin{cases} a^{\log_a(x)} = x \;, & x \in \mathbb{R}^+ \\ \log_a\left(a^{x'}\right) = x' \;, & x' \in \mathbb{R}. \end{cases}$$

identités qui en découlent

$$\boxed{\log_a(1) = 0 \qquad \log_a(a) = 1}$$

$$\log_a(x \cdot y) = \log_a(x) + \log_a(y)$$

$$\log_a\left(\frac{1}{x}\right) = -\log_a(x)$$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$

$x, y \in \mathbb{R}^+$

$$\log_a(x^r) = r \cdot \log_a(x)$$

$x \in \mathbb{R}^+$

$r \in \mathbb{R}$

Remarque $\ln(x) \equiv \log_e(x)$

$e \simeq 2{,}71828182\,8\ldots$

$\log_a(\sqrt{5}) = \log_a(5^{1/2}) = \frac{1}{2}\log_a(5)$

$\neq (\log_a(5))^{1/2}$

Challenge

$\frac{1}{2}\log_a(5) = (\log_a(5))^{1/2}$

$\Leftrightarrow \frac{1}{4}(\log_a(5))^2 = \log_a(5)$

$\Leftrightarrow \frac{1}{4} = \frac{1}{\log_a(5)}$ $\qquad \log_a(a) = 1$

$\Leftrightarrow 4 = \log_a(5)$ $\qquad a^4 = 5$

$\Leftrightarrow a = +\sqrt[4]{5} \simeq 1{,}6$

$\log_a(b) = c$

$$a^c = b$$