B-118 A-B998

① odd integer : $2k+1$

$2(2k+1) \rightarrow$ even.

③ $(2k+1)^2 = 4k^2 + 2 \cdot 2 \cdot k + 1$

$= 2(2k^2+2k)+1$

⑤ $m+n = 2k'+1$

$n+p = 2k+1$

$m = 2k' - n$

$p = 2k - n$

$m+p = 2k' + 2k - 2n$

$= 2(k'+k-n)$

direct proof

(7) $(k+1)^2$

$= k^2 + 2k + 1$

k^2

(9) Let's assume

$$\frac{a}{b} + r = \frac{c}{d}$$

$$\Leftrightarrow \quad r = \frac{c}{d} - \frac{a}{b} = \frac{c-a}{d-b}$$

r rational. $\Rightarrow \lightning \Leftarrow$.

(17) $\neg q \to \neg p$

$\equiv p \to q$

if $x < 1$ and $y < 1$, then

$x + y < 2$.

(19) $n^3 + 5 = 2k + 1$ (21) $(\Rightarrow n = 2k\ ?)$

(a) $\neg q \to \neg p$.

if $n = 2k' + 1$,

$$(2k'+1)^3 + 5$$
$$= 2^3 k'^3 + 3 \cdot 2^2 \cdot k'^2 + 3 \cdot 2 \cdot k' + 6$$
$$= 2(2^2 k^3 + 6k^2 + 3k + 3)$$

(b) Suppose n^3+5 odd
n odd

n^2 odd
n^3 odd

but the diff between two odd nb $\Rightarrow$ even

(21) $n > 1 \rightarrow n^2 > n$

$0 > 1 \Rightarrow 0 > 0$
F $\Rightarrow$ F

(23) $(a+b)^1 \geq a^1 + b^1$

$= a+b \geq a+b$

(25) 64 days ch. $\rightarrow$ sdw ≥ 10.

Let's assume sdw < 9

$\rightarrow 9 \times 7 = 63$

Contradiction

(11)

$$\frac{a_1 + a_2 + \dots + a_n}{2} \leq a_i$$

Let's assume

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq a_i$$

for some i between 1 and n

$$\frac{a_1 + a_2 + \dots + a_n}{n} = A$$

$$a_1 + a_2 + \dots + a_n < nA$$

(39)

① $p_1 \rightarrow p_4$

② $p_3 \rightarrow p_1$

③ $p_4 \rightarrow p_2$

④ $p_2 \rightarrow p_5$

⑤ $p_5 \rightarrow p_3$

⑥ $p_1 \rightarrow p_2$ H.S ①③

⑦ $p_1 \rightarrow p_5$ H.S ⑥④

⑧ $p_1 \rightarrow p_4$

$p_2 \rightarrow p_5$

$p_5 \rightarrow p_3$

$p_3 \rightarrow p_1$

$p_5 \rightarrow p_1$

$p_1 \rightarrow p_5$

$p_5 \rightarrow p_5$

$p_4 \rightarrow p_4$

(3) ① $1 \leq x \leq 4$

$4^3 < 100$

$5^3 > 100$

(5) if $x \geq y$:

$\max(x, y) = x$

$+ \quad \min(x, y) = y$

$= x + y.$

$x < y$:

$\max(x, y) = y$

$+ \quad \min(x, y) = x$

$= x + y$