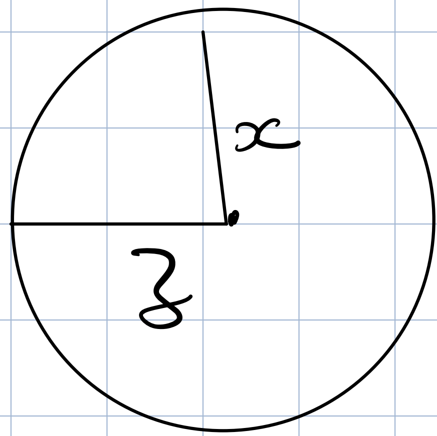
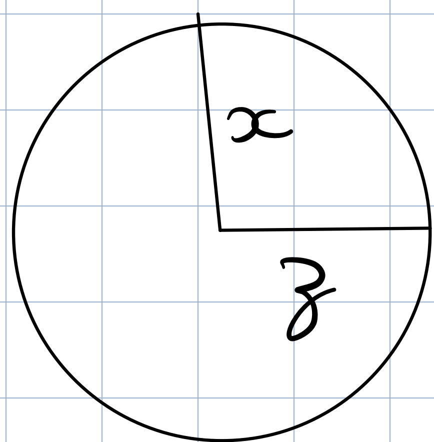
Concepts

(a)



y vers le mur

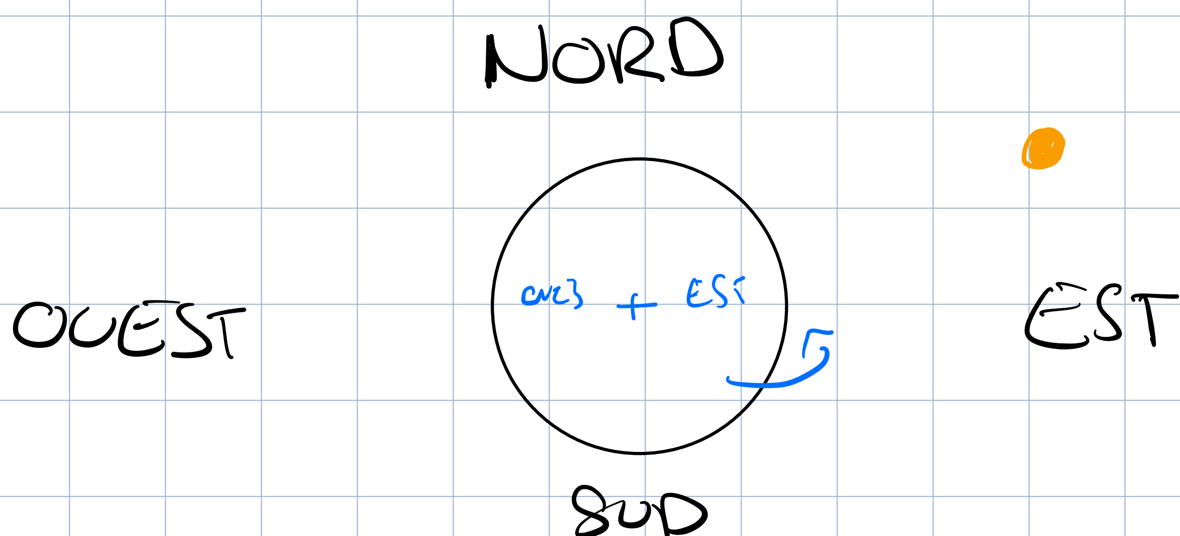
⊗



y vers la personne qui lit l'heure

⊙

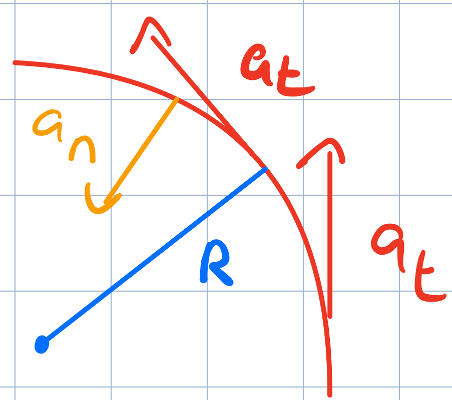
(b)



du sud vers le nord (sens de $\vec{\omega}$ est le sens de rotatio)

(1) Accélération dans un virage

(a)



- $$\boxed{\vec{a}(t) = \frac{dv}{dt}(t) \cdot \hat{\tau} + \frac{v(t)^2}{R} \cdot \hat{n}}$$

$\Rightarrow \quad v(t) = a_t \cdot t$

$$\cdot\ v(t) = \omega(t) \cdot R$$

$$\begin{cases} \omega(t) = \dfrac{a_t \cdot t}{R} \\ \omega(t) = \dfrac{d\theta}{dt}(t) \end{cases}$$

slide du cours

$$v(t) = a_t \cdot t$$

$$s(t) = \frac{1}{2} a_t t^2$$

$$\Rightarrow \theta(t) = \frac{1}{2R} a_t \cdot t^2$$

$$\theta(T) = \frac{\pi}{2}$$

$$\frac{1}{2R} a_t \cdot T^2 = \frac{\pi}{2}$$

$$\Leftrightarrow \boxed{a_t = \frac{\pi}{2} \cdot 2R \cdot \frac{1}{T_2} = \frac{R\pi}{T^2}}$$

(b) $v(T) = \omega(T) \cdot R$

$$= \frac{a_t \cdot T}{R} \cdot R$$

$$= \frac{R\pi}{T^{\cancel{2}}} \cdot \cancel{T} \cdot \cancel{\frac{1}{R}} \cdot R$$

$$\boxed{v(T) = \frac{R\pi}{T}}$$

(c) $\vec{a_n}(T) = \frac{v(T)^2}{R} \cdot \hat{n}$

$$= \left(\frac{R\pi}{T}\right)^2 \cdot \frac{1}{R} \cdot \hat{n}$$

$$= \frac{R^2\pi^2}{RT^2} \cdot \hat{n}$$

$$\boxed{\vec{a_n}(T) = \frac{R\pi^2}{T^2}}$$

(d) $a_n(t_{eq}) = a_t$

$$a_n(t_{eq}) = \frac{v(t_{eq})^2}{R}$$

$v(t) = a_t \cdot t$

$=$

$$= \left(\frac{a_t \cdot t_{eq}}{R} \cdot R\right)^2 \cdot \frac{1}{R}$$

$$= \frac{(a_t)^2 \cdot t_{eq}^2}{R}$$

$$(a_t)^2 \cdot t_{eq}^2 \cdot \frac{1}{R} = a_t$$

$\boxed{\text{Sol 1}: t = 0}$

$$\Leftrightarrow \quad a_t \cdot t_{eq}^2 \cdot \frac{1}{R} = 1$$

$$\Leftrightarrow \quad t_{eq}^2 = \frac{R}{a_t}$$

a 0.

$$\Leftrightarrow \quad t_{eq} = -\sqrt{\frac{R}{a_t}}, \sqrt{\frac{R}{a_t}}$$

$$\frac{R}{\frac{R\pi}{T^2}}$$

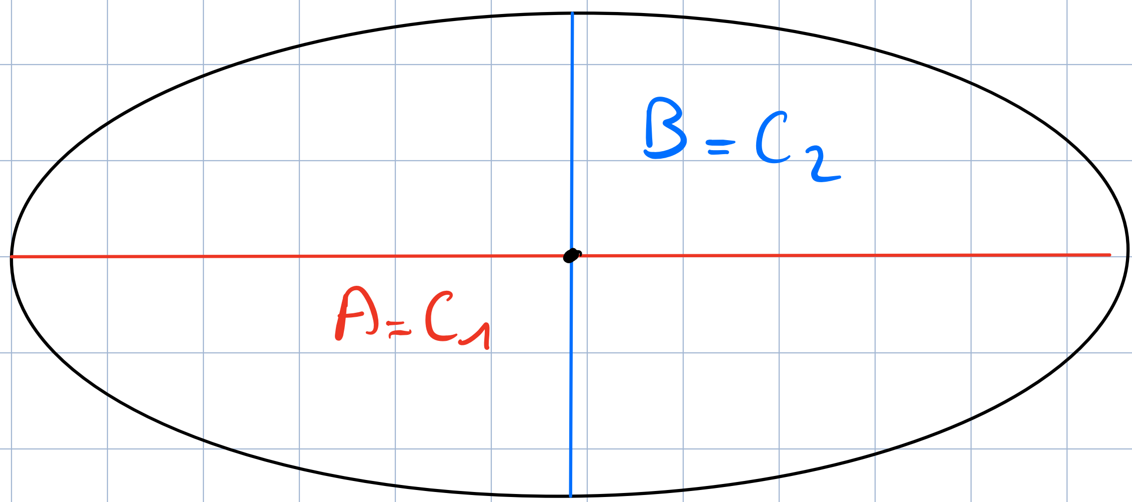
$$= \frac{\cancel{R}T^2}{\cancel{R}\pi}$$

$$= \sqrt{\frac{T^2}{\pi}}$$

$$= \pm\frac{T}{\sqrt{\pi}}$$

Ex 2

1



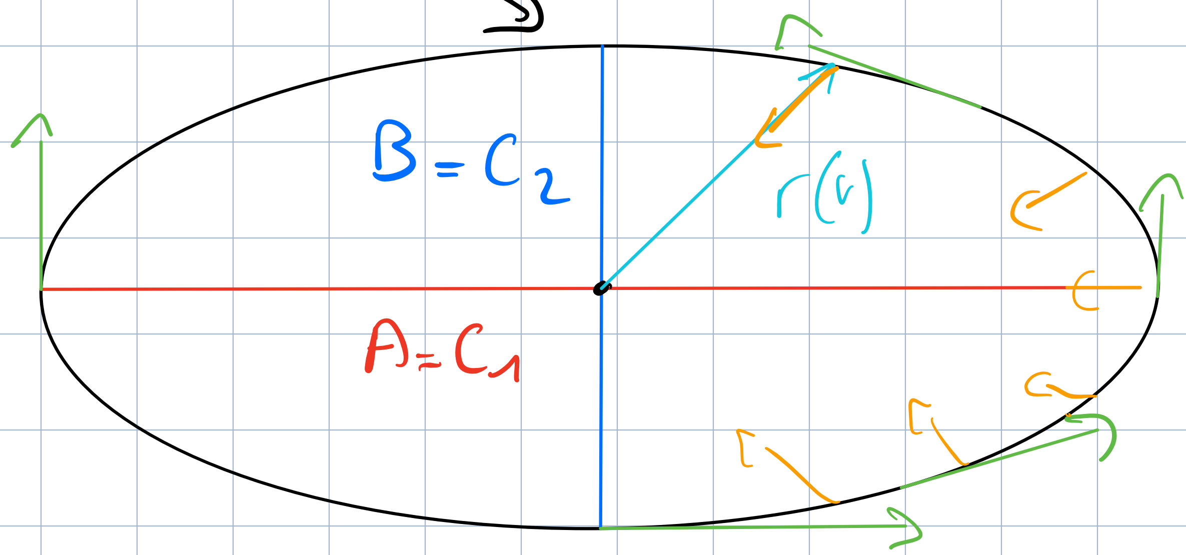
$$\begin{cases} x = C_1 \cos(\omega t) \\ y = C_2 \sin(\omega t) \end{cases}$$

$$\frac{C_1^2 \cos^2(\omega t)}{C_1^2} + \frac{C_2^2 \sin^2(\omega t)}{C_2^2}$$

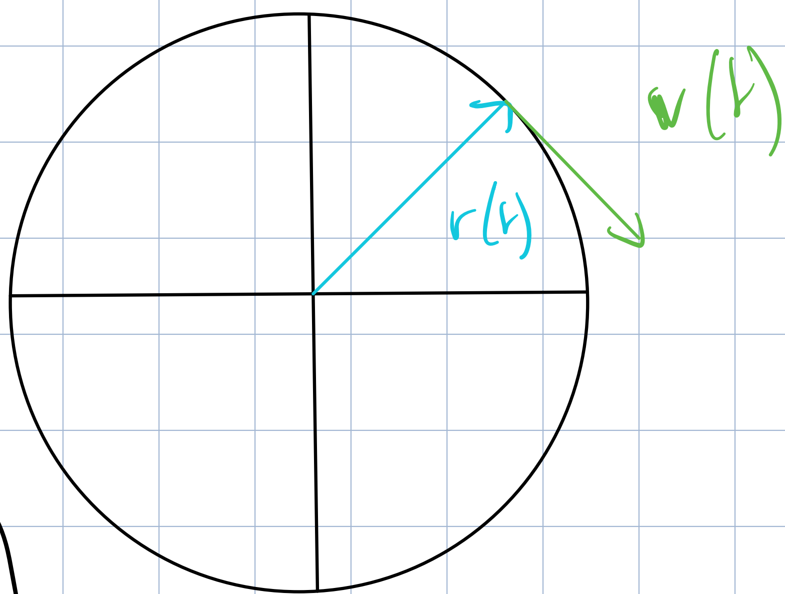
$$= \cos^2(\omega t) + \sin^2(\omega t) = \underline{1}$$

donc le point matériel parcourt une ellipse.

C_1 et C_2 sont les demi axes



$C_1 = C_2$



Maher ?

ils peuvent être orthogonaux par ex ici

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$= -C_1 \omega \sin(\omega t)\,\hat{\imath}$$

$$+ C_2 \omega \cos(\omega t)\,\hat{\jmath}$$

$$\begin{pmatrix} -C_1 \omega \sin(\omega t) \\ C_2 \omega \cos(\omega t) \end{pmatrix} \cdot \begin{pmatrix} C_1 \cos(\omega t) \\ C_2 \sin(\omega t) \end{pmatrix}$$

$$-C_1^2 \omega \sin(\omega t)\cos(\omega t) + C_2^2 \omega \cos(\omega t)\sin(\omega t) \quad (*)$$

Si $C_1 = C_2$, $(*) = 0$.
Si $t = 0$
etc. mais pas cas general

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= \left(-C_1 \omega^2 \cos(\omega t)\,\hat{\imath} - C_2 \omega^2 \sin(\omega t)\,\hat{\jmath} \right)$$

or $\sum \vec{F} = m\vec{a}$

$$\Leftrightarrow \vec{a} = \frac{\vec{F}}{m}$$

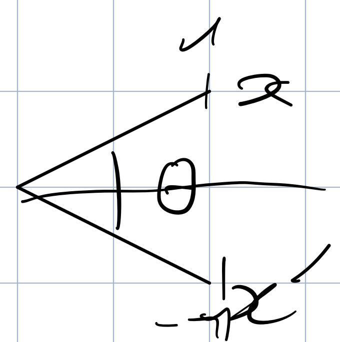
$$\Leftrightarrow -C_1 \omega^2 \cos(\omega t)\,\hat{\imath} - C_2 \omega^2 \sin(\omega t)\,\hat{\jmath} = \frac{\vec{F}}{m}$$

$$\Leftrightarrow \vec{F} = m\omega^2 \overbrace{\left(-C_1 \cos(\omega t)\,\hat{\imath} - C_2 \sin(\omega t)\,\hat{\jmath} \right)}^{-\vec{r}}$$

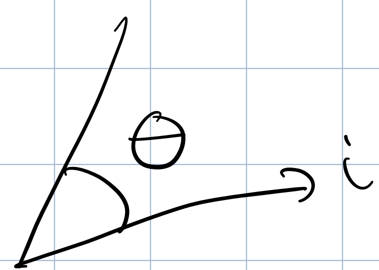
c force non constante, dépend de
la position de l'objet

Ex3

(a) $\vec{V} \Big(\; V_1$ $\qquad$ cos



matrice rotation



$$\begin{bmatrix} \sin\theta & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = A$$

$$A \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Si $\theta = \frac{\pi}{2}$

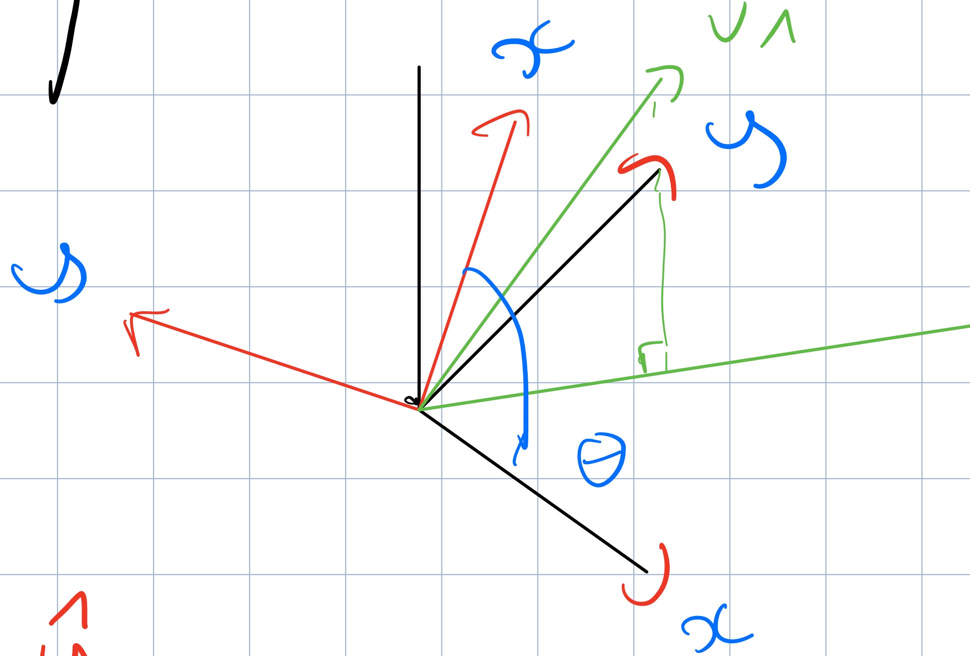
$$x \quad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad y$$

$$y \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad -x$$

$$\underbrace{\begin{bmatrix} \sin\theta & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{décrit la rotation}} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \sin\theta & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

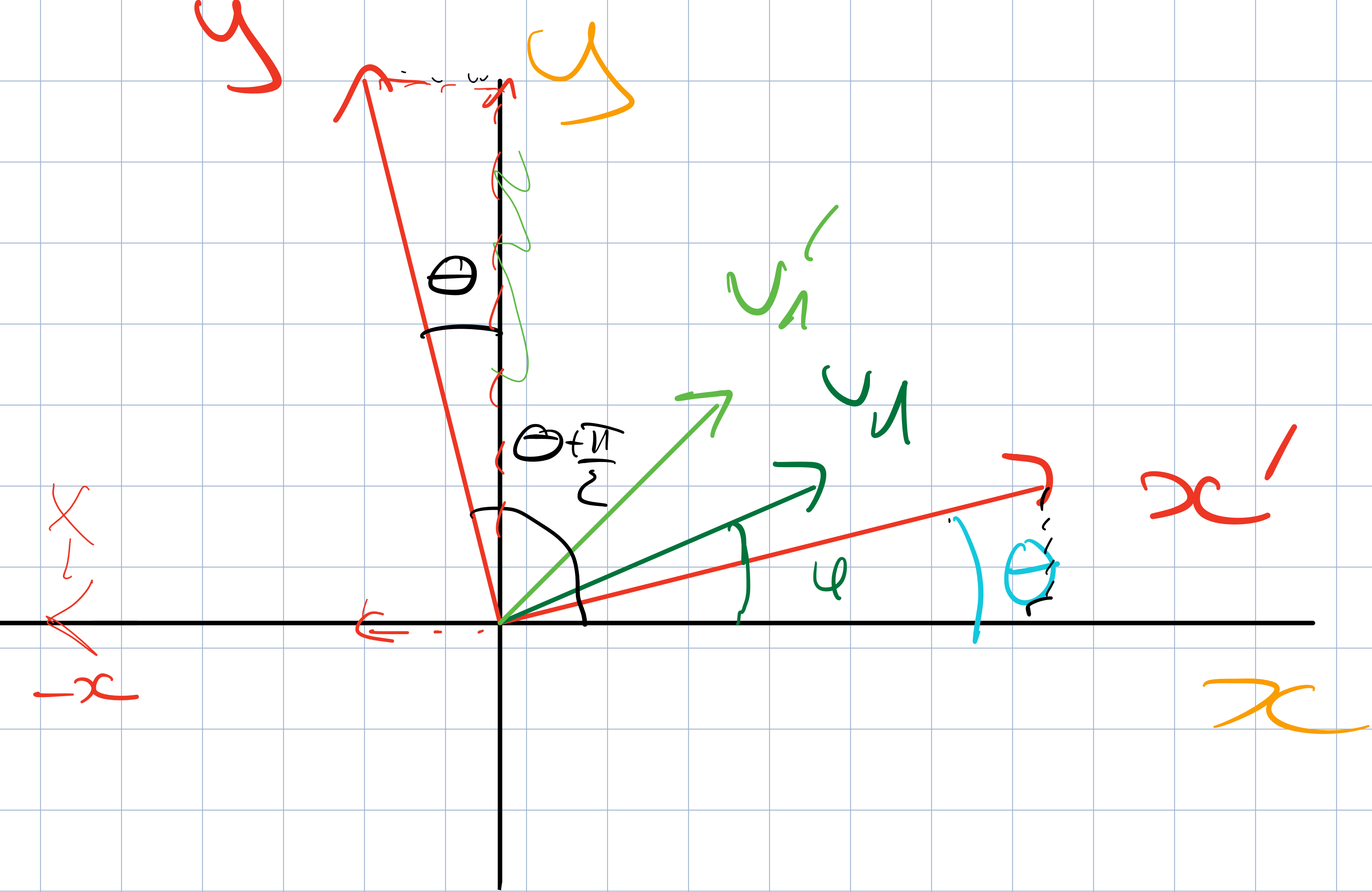
$$= \begin{pmatrix} v_1 \sin\theta \\ v_2 \cos\theta \\ v_3 \end{pmatrix}$$



$$\hat{u}' = \sin\left(\theta + \frac{\pi}{2}\right)\hat{y}$$

$$- \cos\left(\frac{\pi}{2} - \theta\right)\hat{x}$$

$$\hat{y}' = \cos(\theta)\,\hat{y} - \sin\theta\,\hat{x}$$



$$V_1 = V_1 \cos\varphi + V_1 \sin\varphi$$

$$V_2 = V_1 \cos(\varphi + \theta) + V_1 \sin(\varphi + \theta)$$

$$\hat{x}' = \cos\theta\, \hat{x} + \sin\theta\, \hat{y} + 0$$

$$\hat{y}' = -\sin\theta\, \hat{x} + \cos\theta\, \hat{y} + 0$$

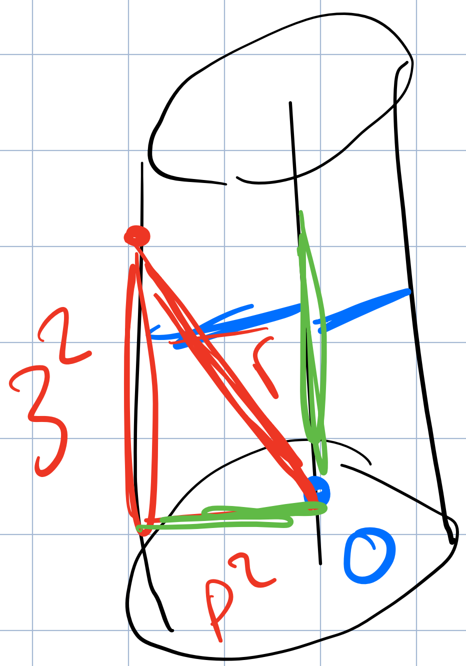
$$\hat{z}' = 0\, \hat{y} + 0\, \hat{y} + \hat{z}$$

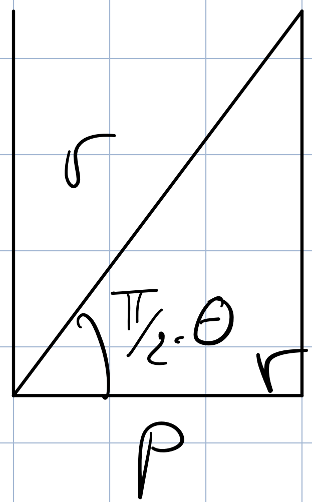
$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{pmatrix} x\cos\theta + y\sin\theta \\ x(-\sin\theta) + y\cos\theta \\ z \end{pmatrix}$$

(b) $\vec{OP} =$

$\rho\cos(\phi)\hat{x} + \rho\sin(\phi)\hat{y} + z\hat{z}$





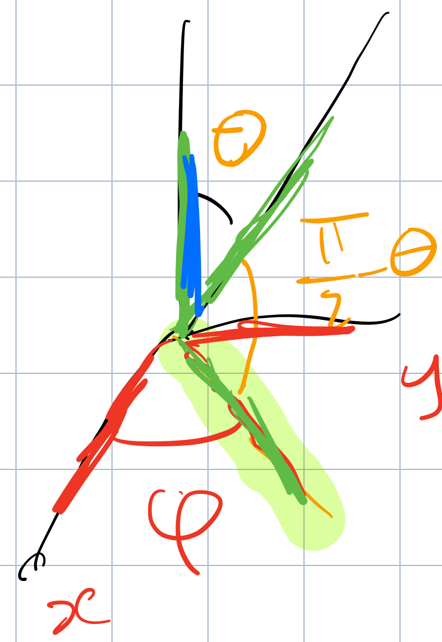
$$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{\rho}{r}$$

$$\Leftrightarrow \rho = r\cos\left(\frac{\pi}{2} - \theta\right)$$

$\vec{OP} =$

$$r\cos(\theta)\hat{z} + r\cos\left(\frac{\pi}{2} - \theta\right)\cos(\phi)\hat{x} + r\cos\left(\frac{\pi}{2} - \theta\right)\sin(\phi)\hat{y}$$

$$\vec{OP} = r\cos(\theta)\,\hat{z}$$
$$+\ r\sin(\theta)\cos(\phi)\,\hat{x}$$
$$+\ r\sin(\theta)\sin(\phi)\,\hat{y}$$



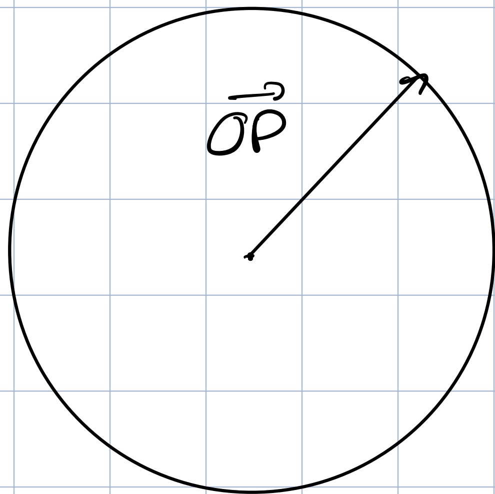
OP sur plan xy $\quad \cos\left(\frac{\pi}{2}-\theta\right) r = \sin\theta\, r$

$\cos\varphi\ \sin\theta\, r \qquad r\,\hat{x}$

$\sin\varphi\ \sin\theta\, r \qquad \hat{y}$

$r\cos\theta\ \hat{z}$

(c) $\overrightarrow{OP}$.



$$\text{Norme } \overrightarrow{OP} = \sqrt{(x)^2 + (y)^2 + (z)^2}$$

Convertir avec f[illegible]

Sphère

$$\boxed{(x)^2+(y)^2+(z)^2 = R^2}$$

$$\boxed{(\rho)^2+(z)^2 = R^2}$$

$\leq$ si plein

$$\boxed{r = R}$$

Cylindrique

$$\boxed{\begin{array}{l} x^2+y^2 = R^2 \\ -\frac{L}{2} \leq z \leq \frac{L}{2} \end{array}}$$

$$\boxed{\begin{array}{l} \rho \leq R \\ -\frac{L}{2} \leq z \leq \frac{L}{2} \end{array}}$$

$$\boxed{\begin{aligned} \cos\left(\frac{\pi}{2}-\theta\right) r &= R \\ -\frac{L}{2} \leq \cos(\theta)\, r &\leq \frac{L}{2} \end{aligned}}$$

(b) $\vec{OP} =$

$$\rho\cos(\phi)\,\hat{x} + \rho\sin(\phi)\,\hat{y} + z\,\hat{z}$$

$$(x)^2 + (y)^2 + (z)^2 = R^2$$

$$\Leftrightarrow (\rho \cos\phi \, \hat{x})^2 + (\rho \sin\phi \, \hat{y})^2 + (z\hat{z})^2 = R^2$$

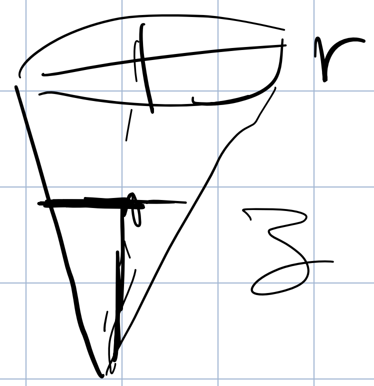
$$\Leftrightarrow \rho^2 \cos^2\phi + \rho^2 \sin^2\phi + z^2 = R^2$$

$$\Leftrightarrow \rho^2 + z^2 = R^2$$

Cône de révolution

$$0 \leq z \leq h$$

$$\tan \varphi = \frac{r}{h} \Leftrightarrow r = \tan \varphi \, h$$



$$\Leftrightarrow r = h \tan \varphi$$

$$x^2 + y^2 = z^2 \tan^2 \varphi$$

$$= z^2 \frac{R^2}{H^2}$$

$$0 \leq z \leq h$$

$$\frac{z}{h} = \sqrt{\frac{x^2+h^2}{R^2}}$$

$$0 \leq z \leq h$$

ϕ libre

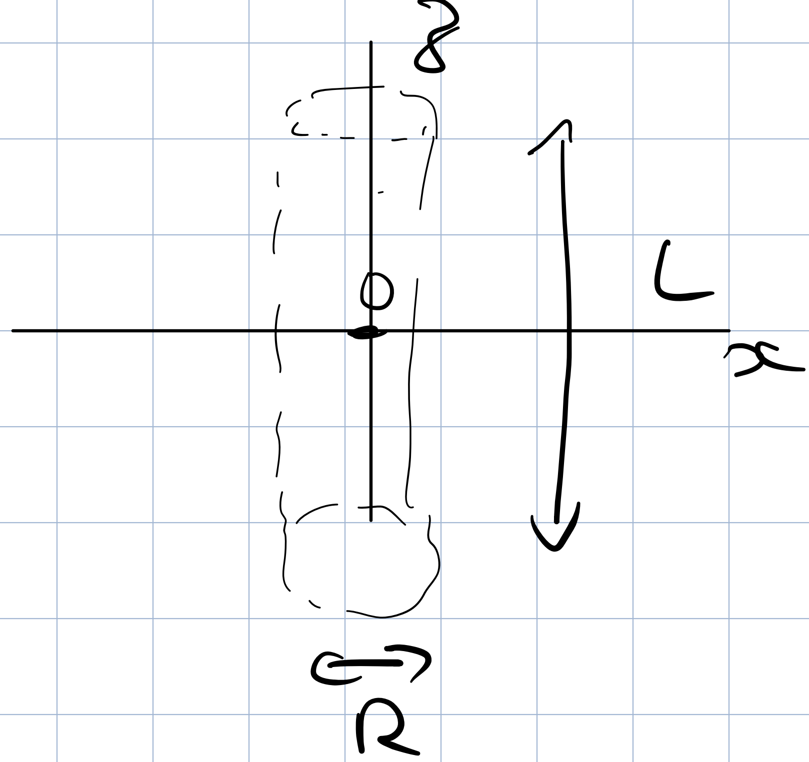
$$\rho = z \cdot \tan\varphi = z \cdot \frac{R}{H}$$

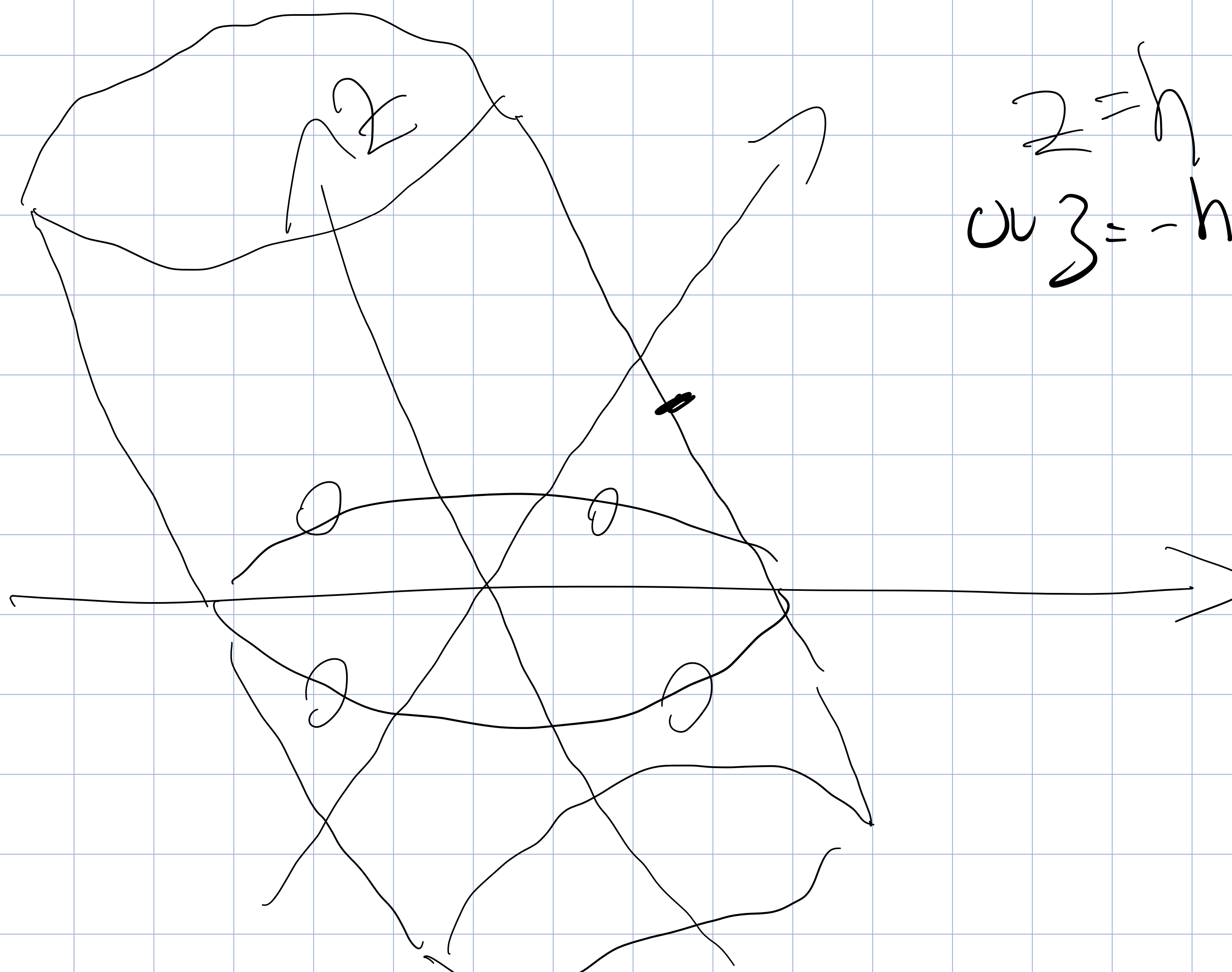
- $0 \leq \cos\theta\, r \leq h$

  $$\sin\theta\, r = \cos\theta\, r \cdot \frac{R}{H}$$

- $\tan\theta = \frac{R}{H}$

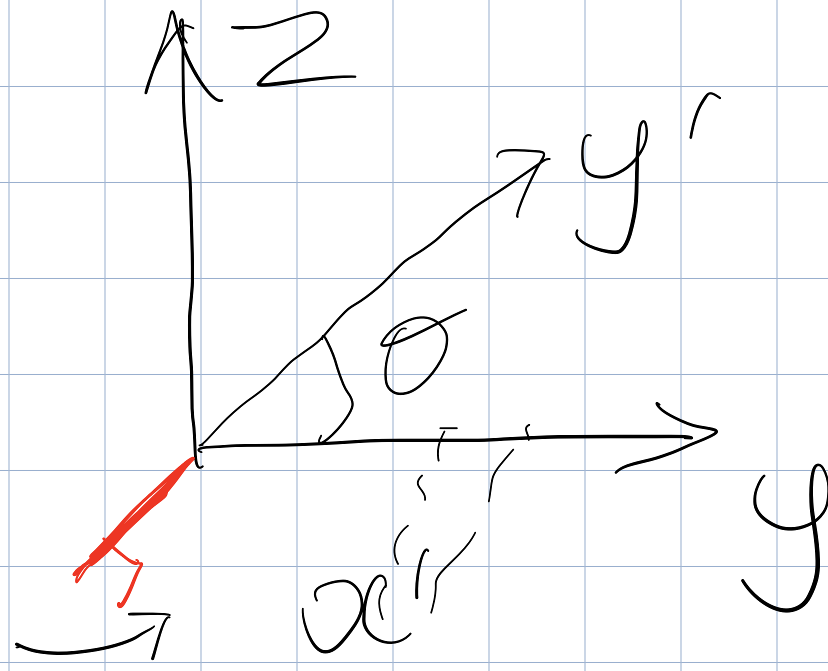
$$\begin{cases} -\dfrac{h}{2} \leq z \leq \dfrac{h}{2} \\ -\dfrac{R}{2} \leq x \leq \dfrac{R}{2} \\ -\dfrac{R}{2} \leq y \leq \dfrac{R}{2} \end{cases}$$

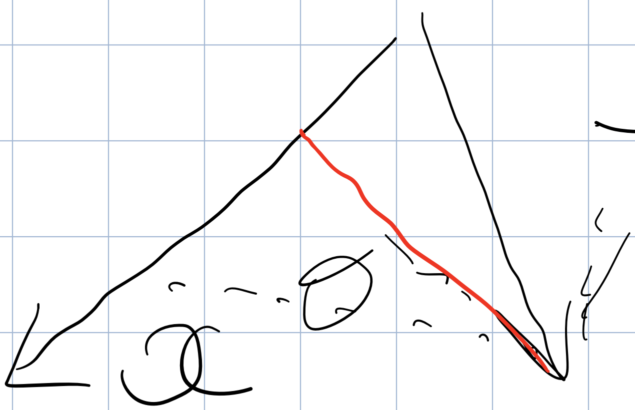




$z = h$

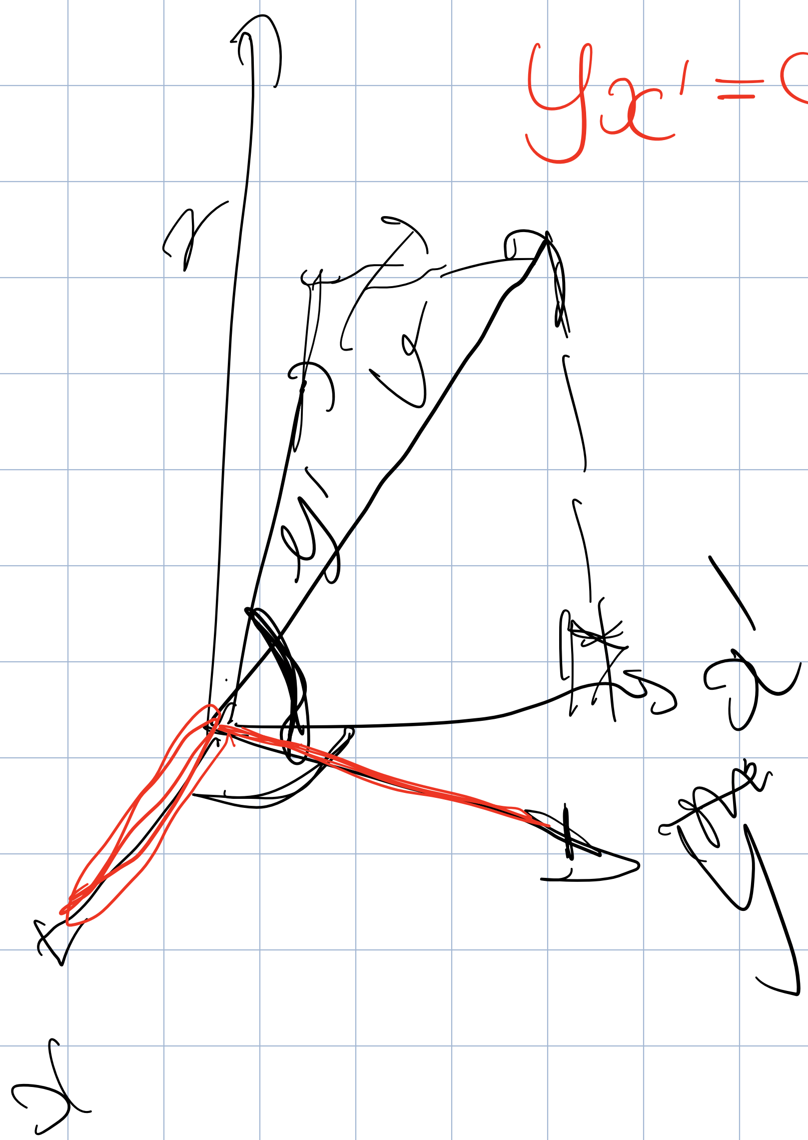
ou $z = -h$

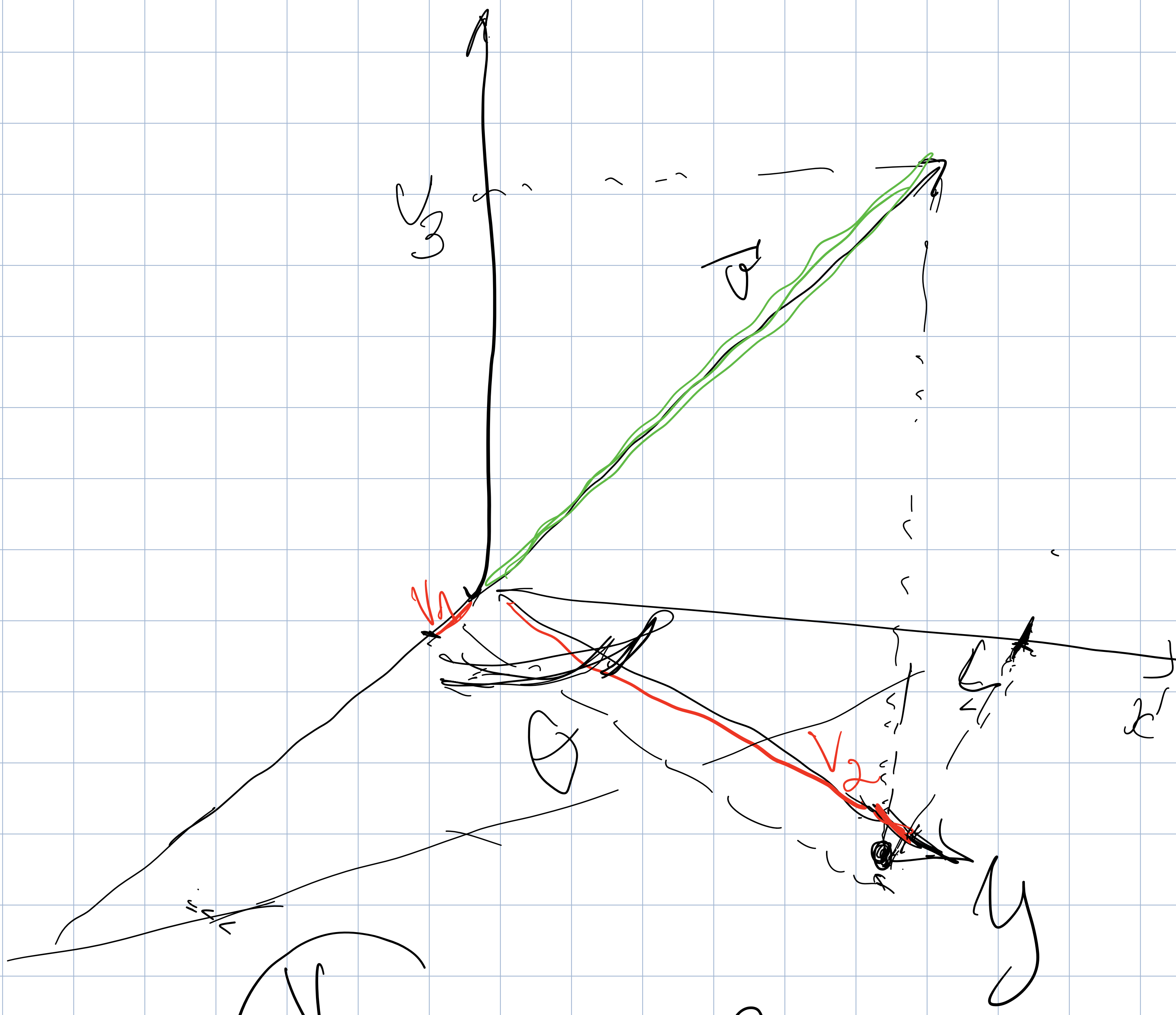




$$x_{x'} = \sin\theta$$

$$y_{x'} = \cos\theta$$

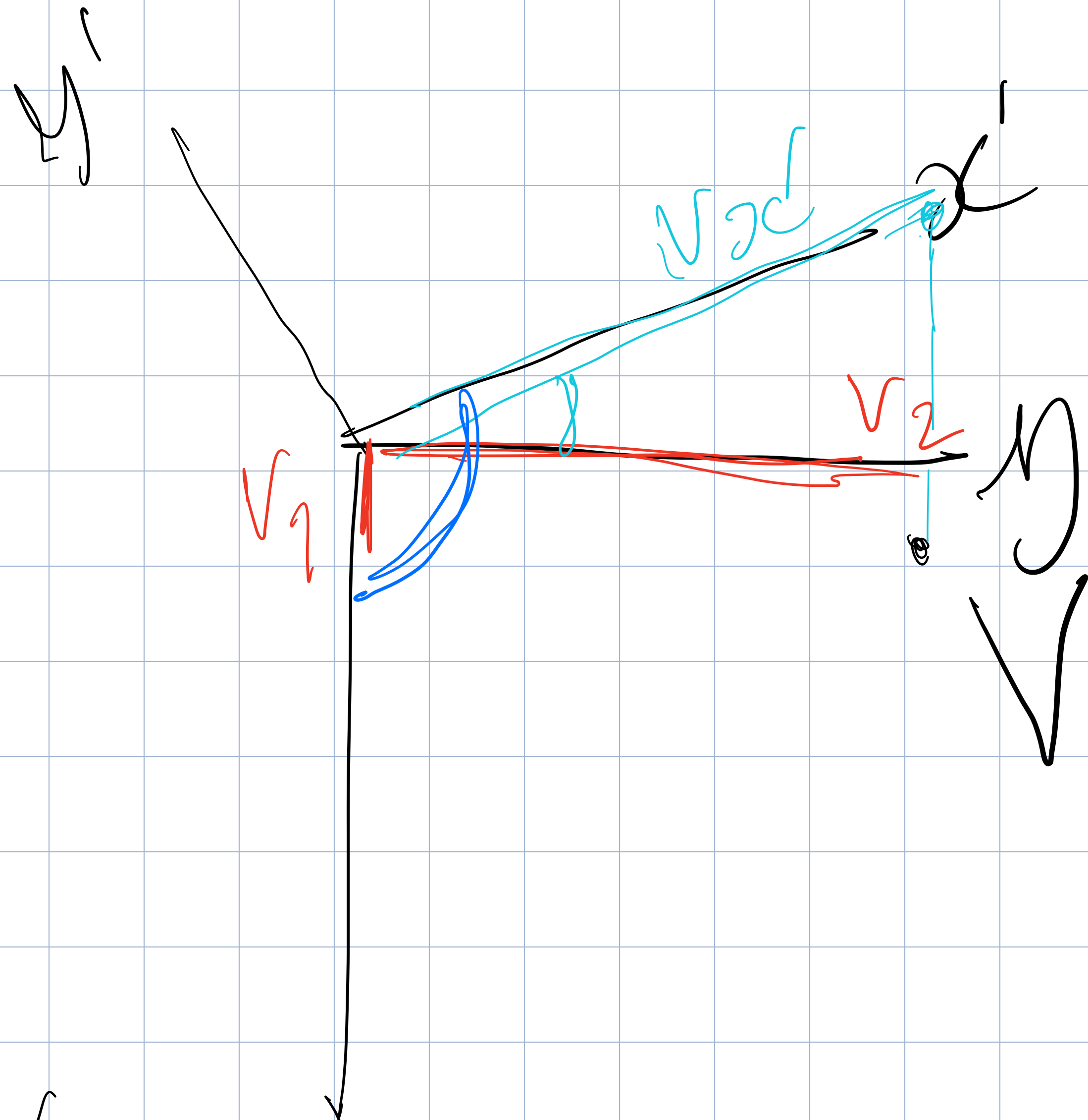




$V_x = 90 - \theta$

$V_x = \cos(90-\theta)\, V_2$

$\cos(90-\theta) = \frac{V_x}{V_2}$



$$\underbrace{\cos(\theta - 90)}_{}{}^{x} = \frac{V_2}{V_{x'}}$$

$$V_{x'} = \frac{U_2}{\cos\left(\theta - \frac{\pi}{2}\right)}$$

$$= \left(\frac{V_2}{\sin\theta}\right)$$

$$|d\tilde{\tau}| = \theta$$