

Exercise 2

$$\textcircled{1} \quad \begin{cases} a_0 = 3 \\ a_{n+1} = a_n \end{cases}$$

$$\textcircled{2} \quad \begin{cases} a_0 = 0 \\ a_{n+1} = a_n + 2 \end{cases}$$

$$\textcircled{3} \quad \begin{cases} a_0 = 3 \\ a_{n+1} = a_n + 2 \end{cases}$$

$$\textcircled{4} \quad \begin{cases} a_0 = 1 \\ a_{n+1} = 5 \cdot a_n \end{cases}$$

⑤

$$\begin{cases} a_0 = 0 \\ a_{n+1} = (\sqrt{a_n} + 1)^2 \end{cases}$$

0	
1	
2.2	4.
3.3	9
4.4	16
5.5	25
	36

$$a_{n+1} = a_n + 2\sqrt{a_n} + 1$$

$$a_n + 2n + 1$$

$$n^2 + n$$

⑥

$$\begin{cases} a_0 = 0 \\ a_{n+1} = (\sqrt{a_n} + 1)^2 + \sqrt{a_n} \end{cases}$$

$$= a_n + 2n + 2$$

0	
$1^2 + 1$	2
$2^2 + 2$	6
$3^2 + 3$	12
$4^2 + 4$	20
$5^2 + 5$	30
$6^2 + 6$	42

7

$$\begin{cases} a_0 = 1 \\ a_n = a_{n-1} + 1 + (-1)^n \cdot 2 \end{cases}$$

$$1 + 1 - 2 = 0$$

$$0 + 1 + 2 = 3$$

$$3 + 1 - 2 = 2$$

$$2 + 1 + 2 = 5$$

$$5 + 1 - 2 = 4$$

	1	0
	0	1
	3	2
-1(2	3
+3	5	4
-1(4	5
+3	7	6
-1(6	7
	9	8
	8	9
		10

8

$$\begin{cases} a_0 = \underline{1} \\ a_n = a_{n-1} \cdot (n+1) \end{cases}$$

1	0
2	1
6	2
24	3

Exercise 3

① $a_n = 2 \cdot 3^n$

0 2

1 $(2) \cdot 3$

2 $(2) \cdot 3 \cdot 3$

② $a_n = a_{n-1} + 2$

$a_0 = 3$

0 3

1 $3 + 2$

2 $3 + 2 + 2$

$$a_n = 3 + 2 \cdot n$$

$$\textcircled{3} \quad a_n = a_{n-1} + n$$

$$a_0 = 1$$

$$0 \quad 1$$

$$1 \quad 1 + 1 = 2$$

$$2 \quad 1 + 1 + 2 = 4$$

$$3 \quad 1 + 1 + 2 + 3 = 7$$

$$4 \quad 1 + 1 + 2 + 3 + 4 = 11$$

$$5 \quad 1 + 1 + 2 + 3 + 4 + 5 = 16$$

$$6$$

$$a_n = 1 + \frac{(n-1)(n)}{2} + 1$$

$$a_1 = 1$$

$$a_2 = 3$$

$$a_3 = 6$$

$$a_4 = 10$$

$$a_5 = 15$$

$$\textcircled{4} \begin{cases} a_n = a_{n-1} + 2n + 3 \\ a_0 = 4 \end{cases}$$

$$0 \quad 4$$

$$1 \quad 4 + 2 \cdot 1 + 3 = 9$$

$$2 \quad 4 + 2 \cdot 1 + 3 + 2 \cdot 2 + 3 = 16$$

$$3 \quad 4 + 2 \cdot 1 + 3 + 2 \cdot 2 + 3 + 2 \cdot 3 + 3 =$$

$$4 + 2(1 + 2 + 3) + 3 \cdot (3) \quad 25$$

$$a_n = 4 + 2 \left(\frac{n(n+1)}{2} \right) + 3 \cdot n$$

$$\textcircled{5} \begin{cases} a_n = 2a_{n-1} - 1 \\ a_0 = 1 \end{cases}$$

$$0 \quad 1$$

$$1 \quad 2 \cdot 1 - 1 = 1$$

$$2 \quad 2(2 \cdot 1 - 1) - 1 = 1$$

$$3 \quad 2(2(2 \cdot 1 - 1) - 1) - 1 = 1$$

$$a_n = 1$$

⑥ $a_n = 3a_{n-1} + 1 \quad a_0 = 1$

0 1

1 $3 \cdot 1 + 1 = 4$

2 $3(3 \cdot 1 + 1) + 1 = 13$

3 $3(3(3 + 1) + 1) + 1 = 40$

$$= 3(3(3 + 1)) + 3 + 1$$

$$= 3(3(3)) + 3^2 + 3 + 1$$

4 $3^4 + 3^3 + 3^2 + 3 + 1$

$$a^n = \sum_{i=0}^n 3^i$$

$$= \frac{a(1-r^{n+1})}{1-r}$$

$$, \quad r=3$$

$$a = q_0 = 1$$

$$= 1 \left(\frac{1-3^{n+1}}{1-3} \right)$$

$$= \frac{1-3^{n+1}}{2}$$

⑦

$$\begin{cases} a_0 = 5 \\ a_n = n a_{n-1} \end{cases}$$

$$0 \quad 5$$

$$1 \quad 1(5) = 5$$

$$2 \quad 2(5) = 10 \quad 2(1(5))$$

$$3 \quad 3(10) = 30 \quad 3(2(1(5)))$$

$$4 \quad 4(30) = 120 \quad 4(3(2(1(5))))$$

$$5 \quad 5(120) = 100 + 500 = 600$$

$$a_n = \prod_{i=0}^n i \cdot 5 = n! \cdot 5$$

$$\textcircled{8} \begin{cases} a_n = 2^n a_{n-1} \\ a_0 = 1 \end{cases}$$

0

1

1

$2(1)(1)$

2

$2(2(2))$

3

$2(3)(2(2))$

4

$2(4)(2)(3)(2(2))$

5

$2(5)(2)(4)(\text{---})$

$$a_n = n! \cdot 2^n$$

Exercise 4

$$\textcircled{1} \quad \begin{cases} a_0 \\ a_{n+1} = 3 \cdot a_n \end{cases}$$

$$\textcircled{2} \quad \begin{cases} a_0 = 100 \\ a_n = 3^n \cdot a_0 \end{cases}$$

$$a_{10} = 3^{10} \cdot 100$$

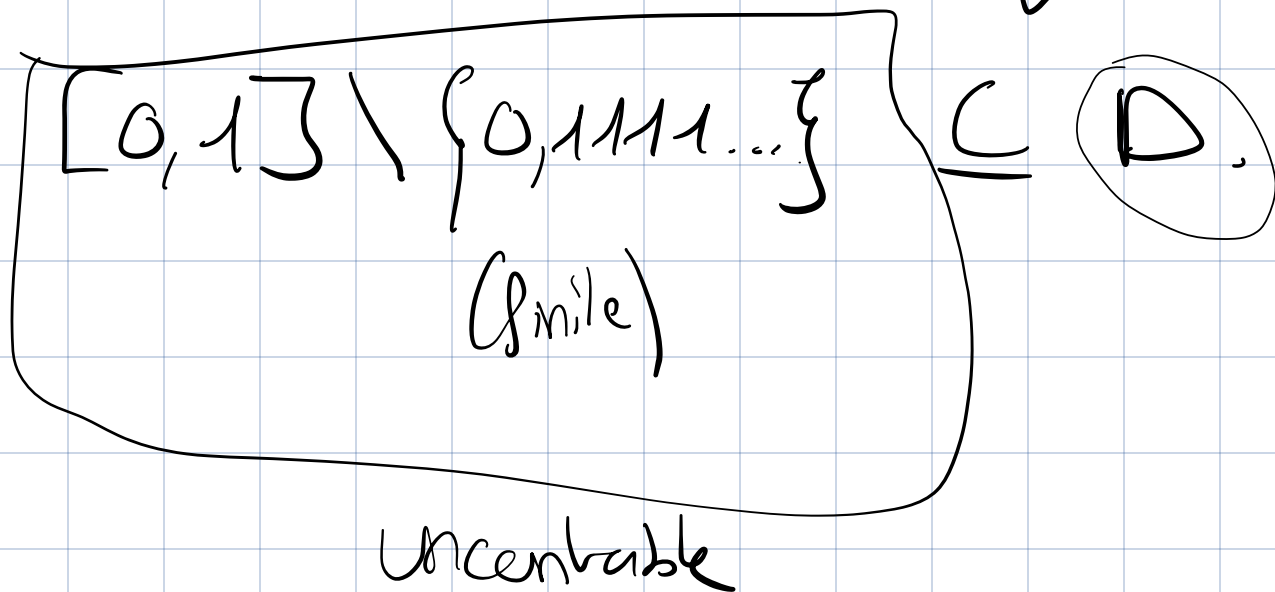
$$= 5904900.$$

Exercice 5

B is countable

D is uncountable

B ressemble à \mathbb{N}



Exercise 6

$$a_0 = 1$$

$$a_1 = 1/2$$

$$a_2 = 1/3$$

$$a_3 = 1/4$$

1	2/1	3/1	4/1	5/1
1/2	2/2	3/2	4/2	5/2
1/3	2/3	3/3	4/3	5/3
1/4	2/4	3/4	4/4	5/4
1/5	2/5			
1/6	2/6			

Handwritten notes on a grid background, showing a sequence of numbers and their combinations, with red circles and lines indicating relationships or groupings.

Row 1: 4 , 44 , 444 , 4444

Row 2: $4, 44$, $44, 44$, $444, 44$, $4444, 44$

Row 3: $4, 444$, $44, 444$, $444, 44$, $4444, 444$

Row 4: $4, 4444$, $44, 4444$, $444, 4444$, $4444, 4444$

Row 5: $4, 44444$, $4, 44444$

Red circles highlight the numbers 4 , 44 , 444 , 4444 , $4, 44$, $44, 44$, $444, 44$, $4444, 44$, $4, 444$, $44, 444$, $444, 44$, $4444, 444$, $4, 4444$, $44, 4444$, $444, 4444$, $4444, 4444$, $4, 44444$, and $4, 44444$.

Red lines connect the circled numbers, showing a branching structure from 4 to 44 to 444 to 4444 , and then to the combinations in the subsequent rows.

Exercice 1

①

On associe les nb

$$(a \cdot b)^k \text{ à } d$$

$$|A \times B| = |A| \times |B| \quad \begin{matrix} (a, \dots) \\ (a \cdot b)^{k+1} \text{ à } d \\ (b, \dots) \end{matrix}$$

$$\{a, b, c\} \times \{d, e\} \quad \begin{matrix} (a \cdot b)^k + (a \cdot b) \\ d \dots \end{matrix}$$

$$\begin{matrix} (a, d) & (b, d) & (c, d) \\ (a, e) & (b, e) & (c, e) \end{matrix}$$

\mathbb{R} is uncountable.

Ⓕ

but $\mathbb{N} \subseteq \mathbb{R}$,

and $|\mathbb{N}|$ is infinite

and \mathbb{N} is countable

\Rightarrow Therefore, false.

Ⓖ $\mathbb{N} \cup \{x \in \mathbb{R}, 0 < x < 1\}$

$\subseteq [0, 1]$

\uparrow

uncountable.

\Rightarrow uncountable

⑤

A set of real numbers

B set of real numbers with only
irrational and integers.

$A \cap B$

\Rightarrow set of integers

\mathbb{Z} .

Ex. 8

(A)

$$a = \frac{n}{2}$$

x	i	n
0	1	16
1	2	
2	4	
3	6	
4	8	
5	10	
6	12	
7	14	
8	16	

③

i	j	n
0	1	16
1	2	
2	4	
3	9	
4	16	

$$b = \sqrt{n}$$