Q conceptuelles

(a) force de frottement, le sol.

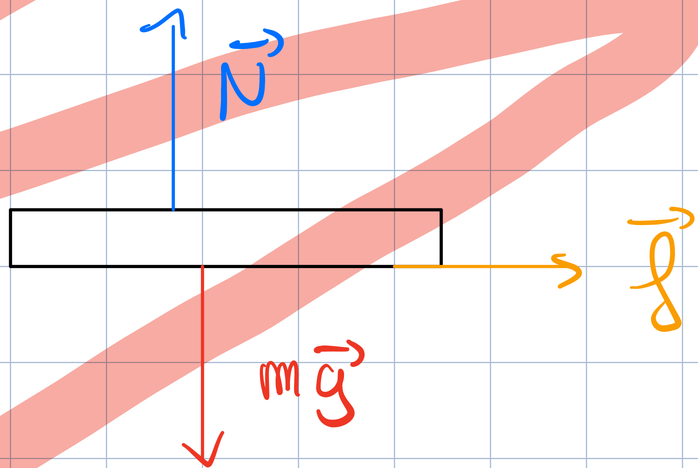
(b) force de frottement de l'air dépend de la masse volumique.

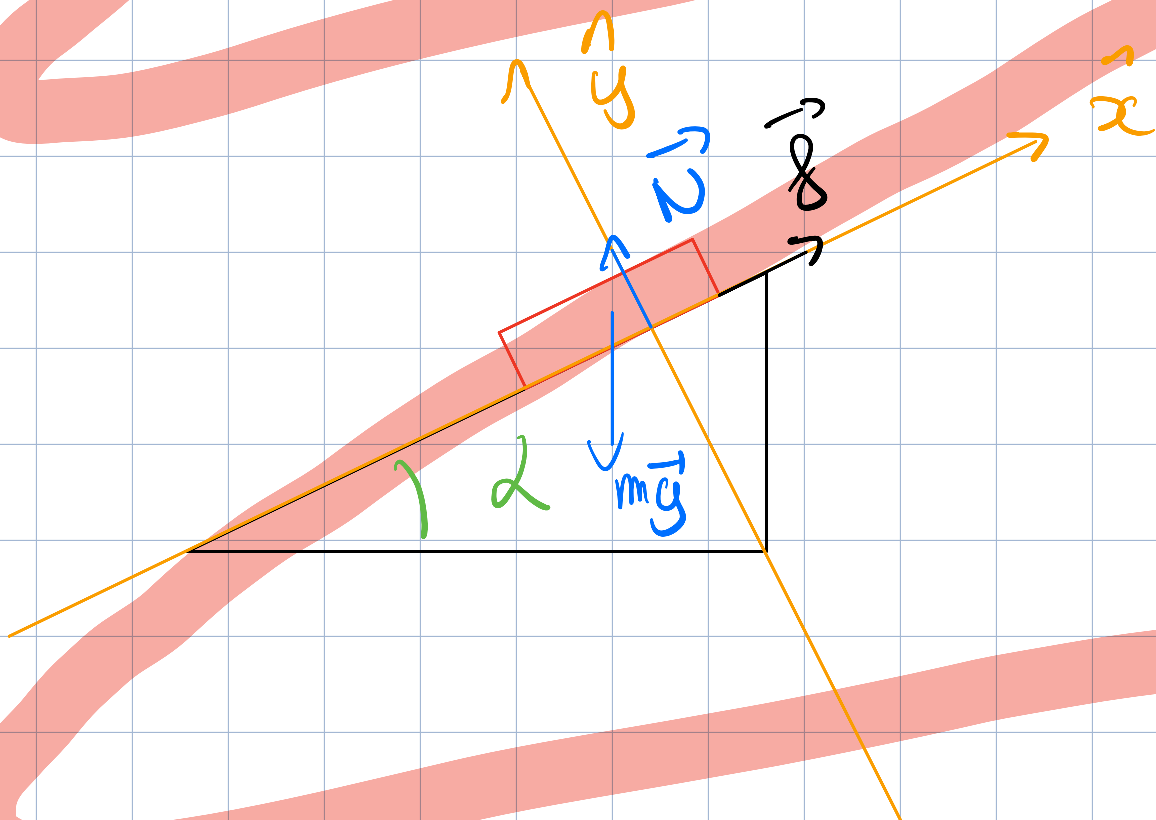
Si masse vol égale, alors chutent même temps.

(c) Oui (force normale)

Exercice 1

a





$$\sum \vec{F} = m\vec{a}$$

$$= m\vec{g} + \vec{N} + \vec{f}$$

On projette :

$$\begin{cases} m\ddot{y} = N - mg\cos\alpha \\ m\ddot{x} = f - mg\sin\alpha \end{cases}$$

On pose $f = f_{max} = N\mu_s$.

on a $\ddot{x} = 0$ et $\ddot{y} = 0$.

$$\begin{cases} N = mg\cos\alpha \\ N\mu_s = mg\sin\alpha \end{cases}$$

$$\mu_s = \tan\alpha .$$

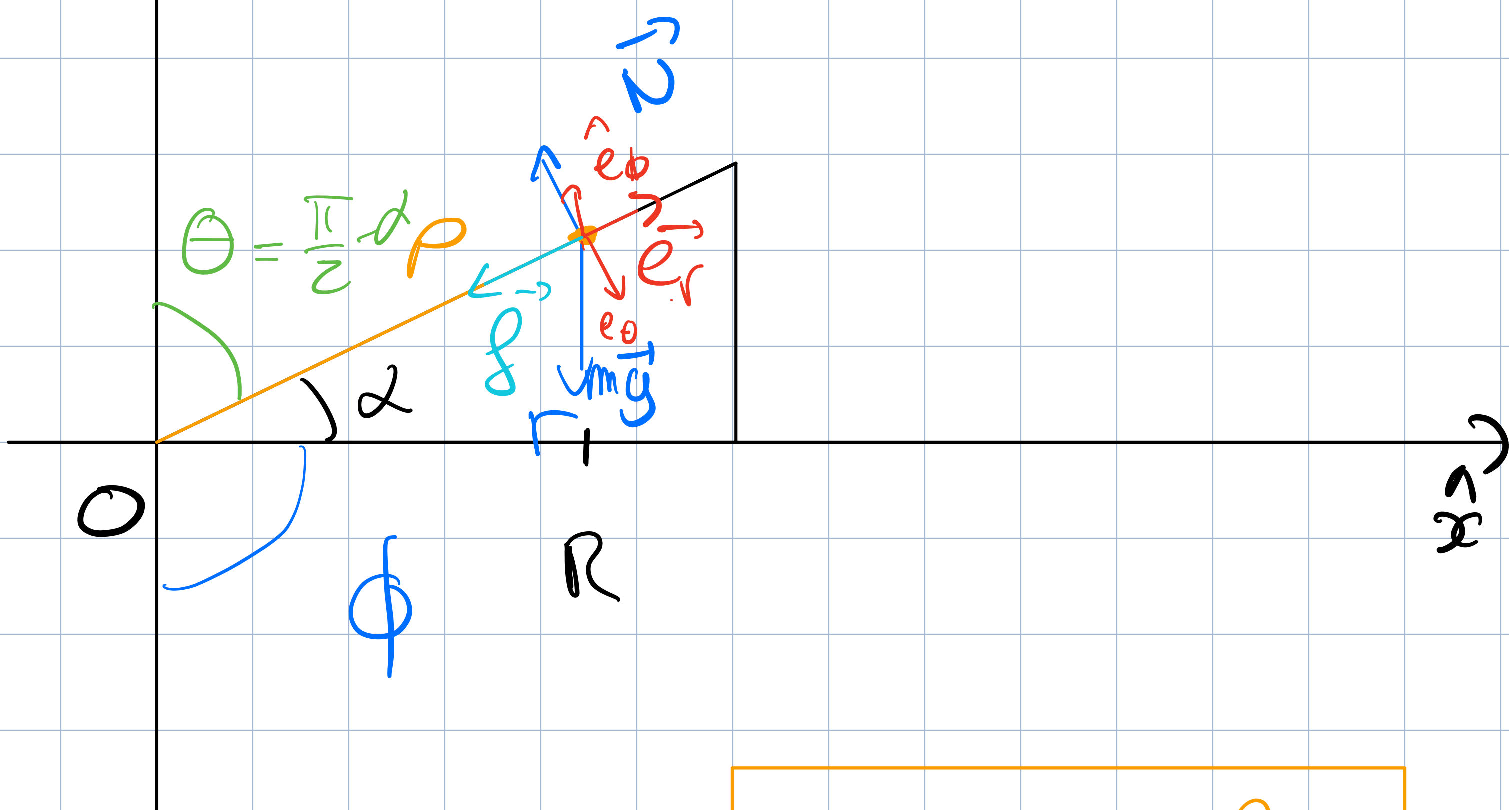
Exercice 1

$$\boxed{a_N = \frac{v^2}{R}} \qquad \vec{v}_N$$

$m\vec{g}_N + f_N$

$$\frac{v^2}{R}$$

$= mg \sin \alpha$



$$\boxed{\sin(\theta) = \frac{R}{\rho}}$$

$$\sum \vec{F} = m\vec{a}$$

$$= \vec{N} + m\vec{g} + \vec{f}$$

On projette

$$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos(\theta)$$

$$\begin{cases} m a_\rho = f - \sin(\alpha) mg \\ m a_\theta = \cos(\alpha) mg + N \\ m a_\phi = 0 \end{cases}$$

$$\cos\left(\frac{\pi}{2}-\alpha\right) = \sin(\theta)$$

- On pose ρ constant $= \dfrac{R}{\sin\theta}$
- $\dot{\phi}$ constant $= \dfrac{V}{R}$
- θ constant $= \dfrac{\pi}{2} - \alpha$.
- $\mu_c N = f$

$$\begin{cases} -m \dfrac{\cancel{R}}{\cancel{\sin\theta}} \dfrac{v^2}{R^{\cancel{2}}} \sin^{\cancel{2}}\theta = f - \sin\alpha \, mg & (\hat{e_r}) \\ -m \dfrac{\cancel{R}}{\cancel{\sin\theta}} \dfrac{v^2}{R^{\cancel{2}}} \cancel{\sin\theta} \cos\theta \, \vec{e_\theta} = \cos\alpha \, mg - N & (\hat{e_\theta}) \\ 0 = 0 & (\hat{e_\phi}) \end{cases}$$

$$\Leftrightarrow \begin{cases} -\dfrac{mv^2}{R} \cos\alpha = -f - \sin\alpha \, mg \\ -\dfrac{mv^2}{R} \sin\alpha = \cos\alpha \, mg - N \end{cases}$$

$$\Leftrightarrow \begin{cases} f = -\sin\alpha \, mg + \dfrac{mv^2}{R} \cos\alpha \\ N = -\cos\alpha \, mg - \dfrac{mv^2}{R} \sin\alpha \end{cases}$$

$$\Leftrightarrow \begin{cases} f = mg\cos\alpha\left(\dfrac{-\sin\alpha \cancel{mg}}{\cancel{mg}\cos\alpha} + \dfrac{v^2}{Rg}\right) \\ N = \cos\alpha\, mg\left(1 + \dfrac{v^2}{Rg}\tan\alpha\right) \end{cases}$$

$$\Leftrightarrow \begin{cases} f = mg\cos\alpha\left(-\tan\alpha + \dfrac{v^2}{Rg}\right) \\ N = mg\cos\alpha\left(1 + \dfrac{v^2}{Rg}\tan\alpha\right) \end{cases}$$

On veut $f \leqslant \mu_s N$ (la force max qu'on peut exercer avant que la voiture dérape et qu'on passe en frottements cinétiques)

$$f \leqslant \mu_s N$$

$$\Leftrightarrow (\cancel{mg\cos\alpha})\left(\frac{v^2}{Rg} - \tan\alpha\right) \leq \mu_s (\cancel{mg\cos\alpha})\left(1 + \frac{v^2}{Rg}\tan\alpha\right)$$

$$\Leftrightarrow \frac{v^2}{Rg} - \tan\alpha \leq \mu_s\left(1 + \frac{v^2}{Rg}\tan\alpha\right)$$

$$\Leftrightarrow \frac{v^2}{Rg} - \tan\alpha - \mu_s - \frac{v^2}{Rg}\tan\alpha\,\mu_s \leq 0.$$

$$\Leftrightarrow \frac{v^2}{Rg}(1 - \tan\alpha\,\mu_s) \leq \mu_s + \tan\alpha.$$

<u>Si $1 - \tan\alpha\,\mu_s \leq 0$:</u>

pas de dérapage

$$-\tan\alpha\,\mu_s \leq -1$$

$$\tan\alpha\,\mu_s \geqslant 1$$

$$\Leftrightarrow \boxed{\mu_s \geqslant \frac{1}{\tan\alpha}}$$

Si $1 - \tan\alpha\,\mu_s > 0$:

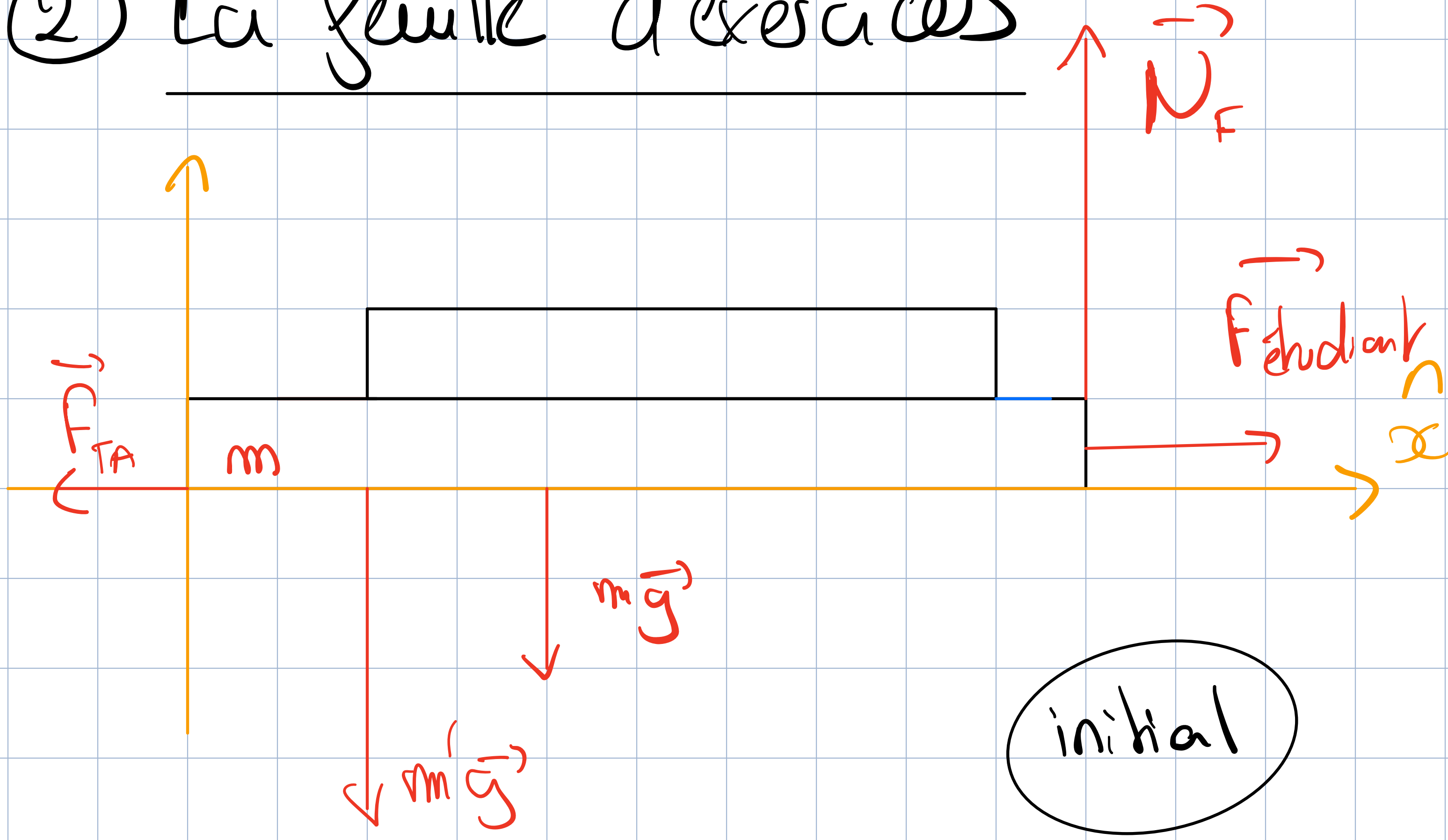
$$\frac{v^2}{Rg} \leqslant \frac{\mu_s + \tan\alpha}{1 - \tan\alpha\,\mu_s}$$

$$\Leftrightarrow v \leq \sqrt{Rg\left(\frac{\mu_s + \tan\alpha}{1 - \tan\alpha\,\mu_s}\right)}$$

Application numérique :

$v \leq 72 \text{ m} \cdot \text{s}^{-1}$.

② La feuille d'exercices



Système de la feuille + téléphone

$$\sum \vec{F} = (m + m')\vec{a}$$

$$= m'\vec{g} + m\vec{g} + \vec{N}_f + \vec{F}_e + \vec{F}_{TA}$$

On projette :

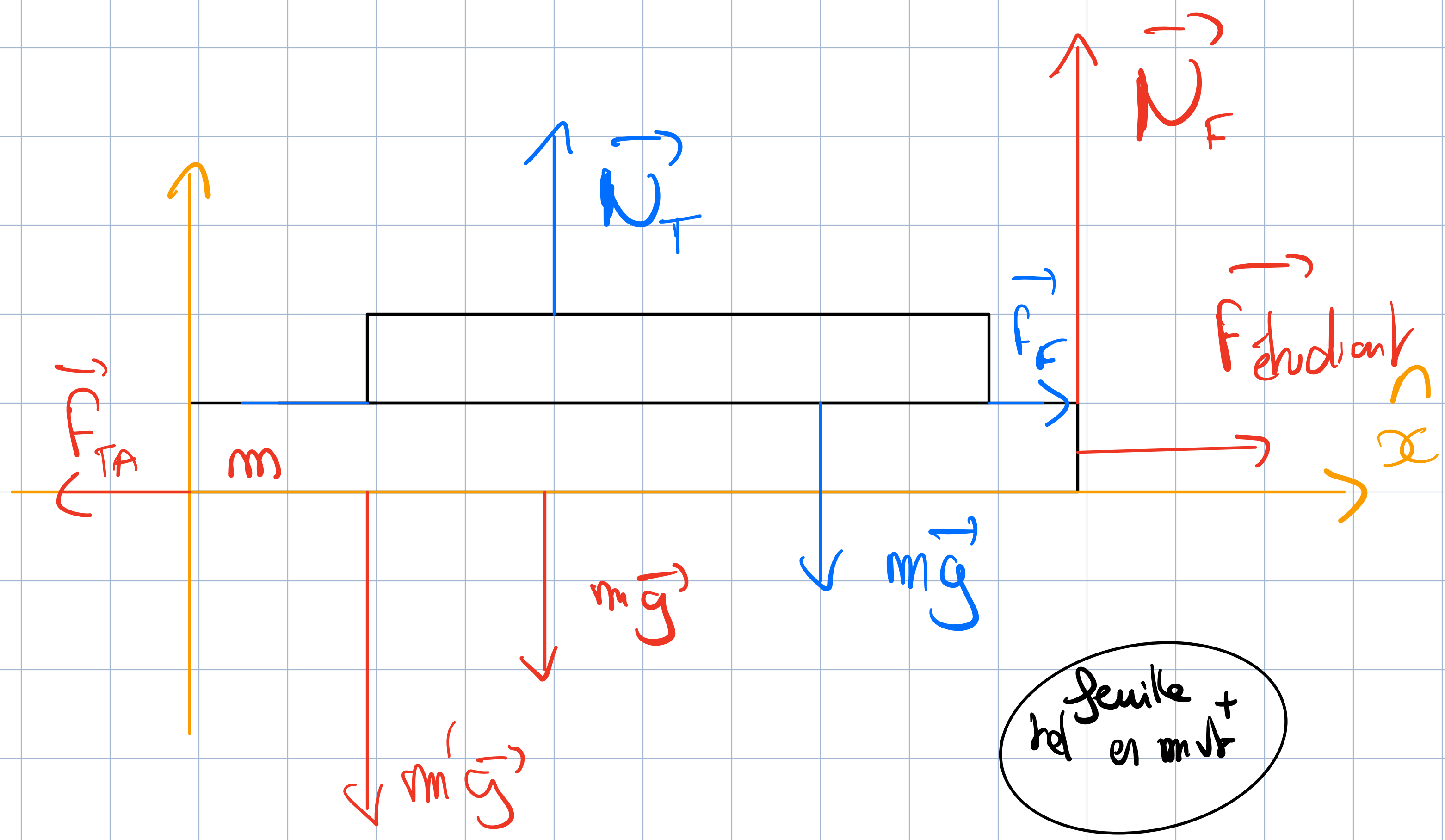
$$\begin{cases} (m+m')\ddot{y} = (m'+m)g - N \\ (m+m')\ddot{x} = F_e - F_{TA} \end{cases}$$

On pose $F_{TA} = \mu_s \cdot N$ et $\ddot{x} = 0$.

$$\begin{cases} N = (m'+m)g \\ 0 = F_e - \mu_s \cdot N - \mu_s' \cdot N \end{cases}$$

$$\boxed{F_e \geq \mu_s \cdot (m'+m)g}$$

Maintenant, on sépare les deux systèmes



Par le système tel + feuille :

$$\begin{cases} N = (m' + m)g \\ (m+m')\ddot{x} = F_e - \mu_c \cdot (m+m')g. \end{cases}$$

Pour le système téléphone ;

$$\sum \vec{F} = m\vec{a}$$

$$= m\vec{g} + \vec{N}_T + \vec{F}_F$$

<u>On projette</u>

$$\begin{cases} m\ddot{y} = mg - N \\ m\ddot{x} = F_F \end{cases}$$

On pose $F_F = \mu_s \cdot N$

or, avant que le tel bouge par rapport à la feuille $\ddot{x} = \ddot{x}$.

$\Leftrightarrow$ tant que $F_e < F_F^{max}$.

On a $\ddot{x} = \frac{F_F}{m'}$

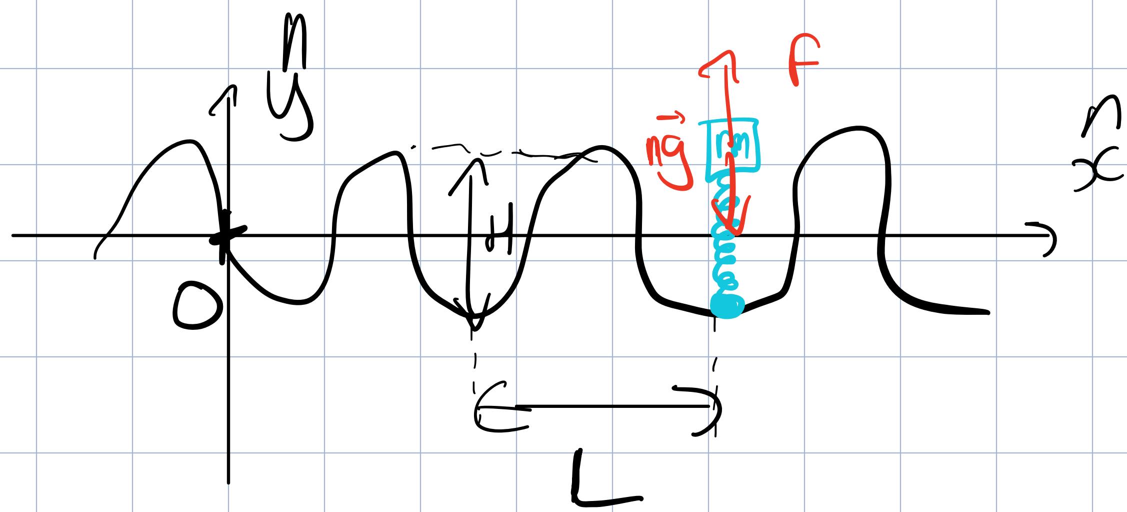
$$F_e = (m+m')\ddot{x} + \mu_c \cdot (m+m')g$$
$$= (m+m') \cdot \frac{F_F}{m} + \mu_c (m+m')g$$
$$= (m+m') \cdot \frac{\mu_s' \cdot mg}{m} + \mu_c (m+m')g$$
$$= (\mu_s' + \mu_c) \cdot (m+m') \cdot g$$

mais peut-être que $\mu_c > \mu_s$ (la force minimale pour faire bouger la feuille une fois serait trop faible pour la faire glisser).

$$\boxed{F_e \geq \left(\max\left(\mu_s \,;\, (\mu_s' + \mu_c)\right)\right) \cdot (m + m') \cdot g .}$$

③ Champ de bosses

$$\dot{x} = v_x \Rightarrow \ddot{x} = 0 .$$



$$h(t) = \frac{H}{2}\cos(\omega t)$$

$$T = \frac{2\pi}{\omega} \Leftrightarrow \boxed{\omega = \frac{2\pi}{T}}$$

$$T = \frac{L}{V_x} \text{ donc } \omega = \frac{2\pi}{L} \cdot V_x$$

$$\boxed{h(t) = \frac{H}{2} \cos\left(\frac{2\pi}{L} \cdot v_x t\right)}$$

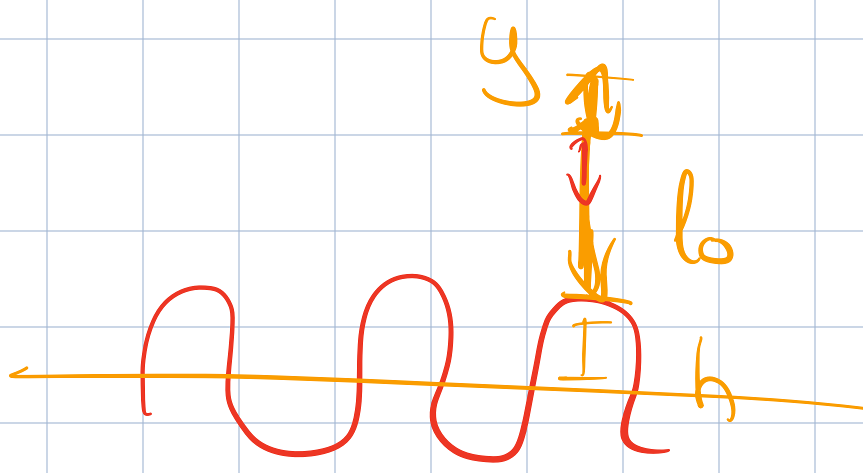
$$\sum \vec{F} = m\vec{a}$$

$$= \vec{P} + \vec{F}$$

$$\begin{cases} m\ddot{y} = mg - k\Delta z \\ m\ddot{x} = 0 \end{cases}$$

$$\Delta y = y(t) - h(t) - l_0$$

$$\begin{cases} m\ddot{y} = mg - k(y - h - l_0) \\ 0 = 0 \end{cases}$$



$$m\ddot{y} = mg - ky + kh + kl_0$$

$$m\ddot{y} = -mg - ky + k\left(\frac{H}{2}\cos\left(\frac{2\pi}{L}\cdot V_x t\right)\right) + kl_0$$

$$\ddot{y} = -g - \frac{k}{m}y + \frac{k}{m}\frac{H}{2}\cos\left(\frac{2\pi}{L}\cdot V_x t\right) + \frac{k}{m}l_0$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\ddot{y} = -g - \omega_0^2 y + \omega_0^2\frac{H}{2}\sin\left(\left(\frac{\pi}{2} - \frac{2\pi}{L}\cdot V_x\right)t\right) + \omega_0^2 l_0$$

$$\ddot{y} + g + \omega_0^2 y - \omega_0^2 l_0 = \omega_0^2\frac{H}{2}(-)$$

$$\Leftrightarrow \ddot{y} + \omega_0^2\left(\frac{g}{\omega_0^2} + y - l_0\right) = \omega_0^2\frac{H}{2}(-)$$

$$u = \frac{g}{\omega_0^2} - l_0 + y \quad \Big| \quad \alpha_0 = \omega_0^2\frac{H}{2}$$

$$\Rightarrow \ddot{u} + \omega_0^2 u = \alpha_0 \sin(\omega t)$$

$$\omega = \frac{\pi}{2} - \frac{2\pi}{L} \cdot V_x$$

d) $$\begin{cases} u(t) = p\sin(\omega t - \phi) \quad \phi = 0 \\ \dot{u}(t) = p\omega\cos(\omega t) \\ \ddot{u}(t) = -p\omega^2 \sin(\omega t) \end{cases}$$

$$-p\omega^2 \sin(\omega t) + \omega_0^2 (p\sin(\omega t)) = \alpha_0 \sin(\omega t)$$

$$\Leftrightarrow \sin(\omega t)(-p\omega^2 + p\omega_0^2) = \alpha_0 \sin(\omega t)$$

$$\Leftrightarrow -p\omega^2 + p\omega_0^2 = \alpha_0$$

$$\Leftrightarrow p(-\omega^2 + \omega_0^2) = \alpha_0$$

$$\Leftrightarrow p = \frac{\alpha_0}{-\omega^2 + \omega_0^2}$$

$$P = \frac{M}{2} \cdot \omega_0^2 \cdot \frac{1}{-\omega^2 + \omega_0^2}$$

$$= \frac{M}{2} \cdot \frac{\cancel{\omega_0^2}}{\cancel{\omega_0^2}\left(1 + \frac{-\omega^2}{\omega_0^2}\right)}$$

$$= \frac{M}{2} \left(\frac{1}{1 - \frac{\omega^2}{\omega_0^2}} \right)$$

$\omega = \omega_0$, résonance $\times$

$$\sqrt{\frac{k}{m}} = \frac{\pi}{2} - \frac{2\pi}{L} V_x \qquad \frac{2\pi}{L} \cdot k = \frac{\pi}{2}$$

$$\sqrt{\frac{k}{m}} - \frac{\pi}{2} = \frac{-2\pi}{L} V_x \qquad k = \frac{L\cancel{\pi}}{4\cancel{\pi}}$$

$$\Leftrightarrow \frac{-L}{2\pi}\left(\sqrt{\frac{k}{m}} - \frac{\pi}{2}\right) = V_x$$

$$\Leftrightarrow \quad V_x = \frac{L}{2\pi}\left(-\sqrt{\frac{k}{m}} + \frac{\pi}{2}\right)$$

$\omega \to +\infty$
alors $V_x \to -\infty$
et $p \to 0$

$\omega \to -\infty$
alors $V_x \to +\infty$
$p \to 0$

$\omega \to 0$
alors $V_x \to \frac{L}{4}$
et $p \to 0$