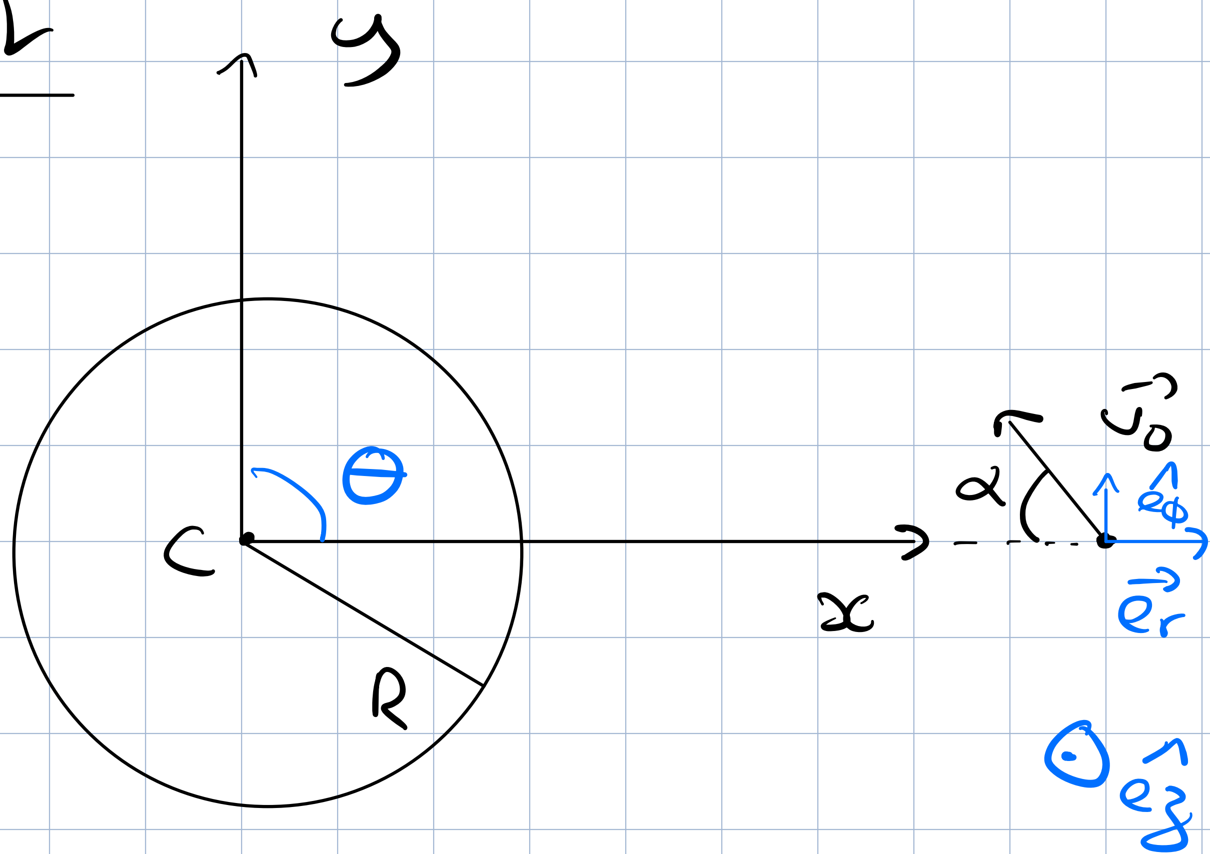
Minitest



ⓐ Quantités conservées :

- E_m (force conservative)
- L_O (force centrale).

$$E_m = K + V$$

$$= \frac{1}{2} m v^2 - \frac{GMm}{r}$$

$$= \frac{1}{2} m \left(\dot{r} \vec{e_r} + r\dot{\theta} \hat{e}_\theta \right)^2 - \frac{GMm}{r}$$

car $\vec{e_r} \cdot \vec{e_\theta} = 0$

$$= \frac{1}{2} m \left(\dot{r}^2 + (r\dot{\theta})^2\right) - \frac{GMm}{r}$$

$$= \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 - \frac{GMm}{r}$$

donc $E_{m_0} = \frac{1}{2} m (v_0)^2 - \frac{GMm}{D}$

$$\boxed{\vec{L_c} = \vec{r} \wedge m\vec{v}}$$

$$= r\vec{e_r} \wedge m\left(\dot{r}\hat{e_r} + r\dot{\theta}\hat{e_\theta}\right)$$

$$= r \cdot m \cdot r\dot{\theta}\,\hat{e_z} = r^2 m \hat{e_z}$$

donc $L_{c_0} = m D v_0 \sin\alpha$

(b) $L_c^2 = m^2 r^4 \dot{\theta}^2$

$$\dot{\theta}^2 = \frac{L_c^2}{m^2 r^4}$$

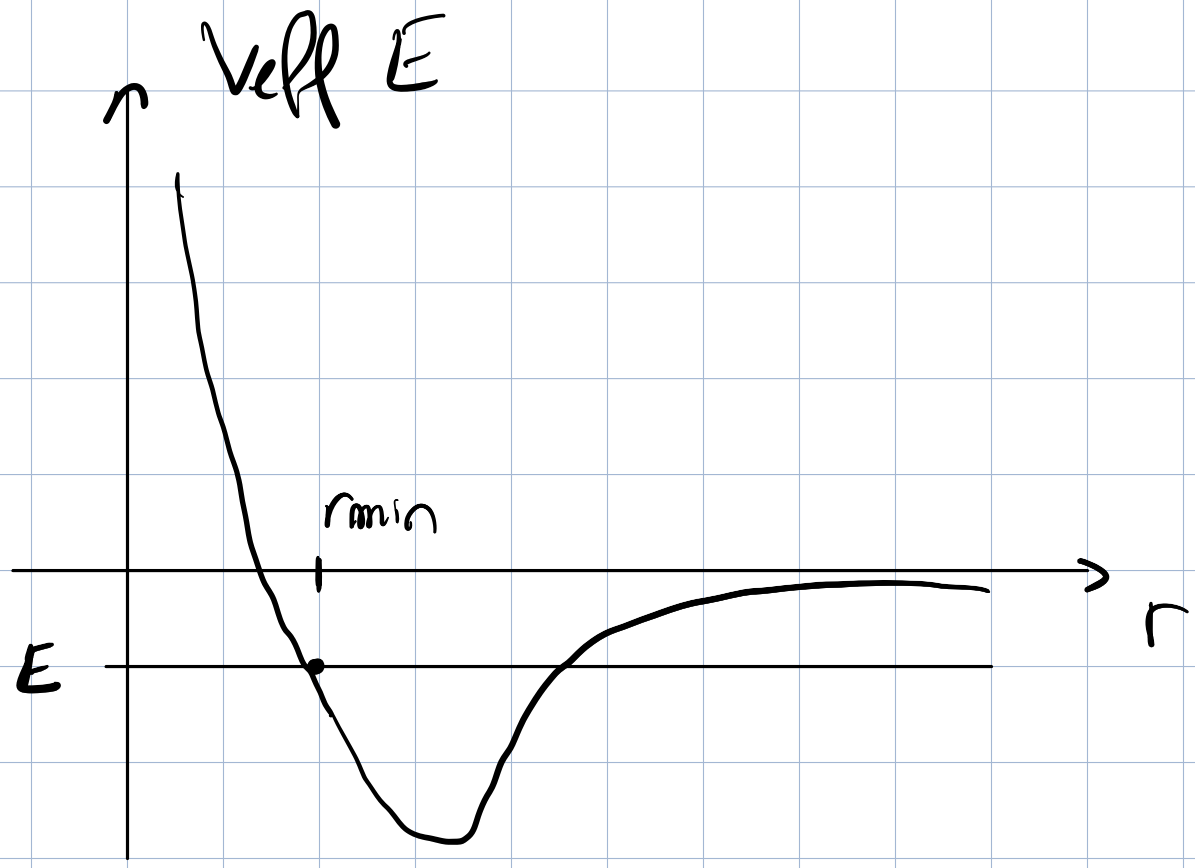
donc

$$E_m = \frac{1}{2} m\dot{r}^2 + \frac{1}{2} \cancel{mr^2} \frac{L_c^2}{m^{\cancel{2}} r^{\cancel{4}\,2}} - \frac{GMm}{r}$$

$$= \frac{1}{2} m\dot{r}^2 + \underbrace{\frac{L_c^2}{2mr^2} - \frac{GMm}{r}}_{V_{eff}}$$

Rayon minimum vérifie $\dot{r}_{min} = 0$ donc

On aura $E_{r_{min}} = V_{eff}(r_{min})$


Veff E
rmin
E
r

ⓒ Si $E = 0$,

$$0 = \frac{L_c^2}{2 m r_{min}^2} - \frac{GMm}{r_{min}}$$

$$\Leftrightarrow \frac{L_c^2}{2m} - GMm\, r_{min} = 0$$

$$\Leftrightarrow r_{min} = \frac{L_c^2}{2m^2 GM}$$

Si $E \neq 0$:

$$\frac{L_c^2}{2 m r_{min}^2} - \frac{GMm}{r_{min}} = E$$

$$\Leftrightarrow \frac{L_c^2}{2m} - GMm\, r_{min} = E\, r_{min}^2.$$

$$\Leftrightarrow \frac{L_c^2}{2mE} - \frac{GMm\,r_{min}}{E} - r_{min}^2 = 0$$

$$\Delta = \left(\frac{-GMm}{E}\right)^2 - 4\cdot(-1)\cdot\left(\frac{L_c^2}{2mE}\right)$$

$$= \left(\frac{GMm}{E}\right)^2 + \frac{2L_c^2}{mE}$$

$$r_{min} = \frac{-1}{2}\left(\frac{GMm}{E} \pm \sqrt{\Delta}\right)$$

$$r_{min} = \frac{-1}{2}\frac{GMm}{E}\left(1 \pm \phantom{\sqrt{\Delta}}\right)$$

$$= \frac{-1}{2}\frac{GMm}{E}\left(1 \pm \frac{\sqrt{\left(\frac{GMm}{E}\right)^2\left(1 + \frac{2L_c^2E}{G^2M^2m^3}\right)}}{\frac{GMm}{E}}\right)$$

$$r_{min,\pm} = \frac{-1}{2}\frac{GMm}{E}\left(1 \pm \sqrt{1 + \frac{2L_c^2E}{G^2M^2m^3}}\right.$$

on veut r_{min} positif.

donc. si $E > 0$,

on choisit seulement r_+.

si $E < 0$,

on peut choisir les deux car

$$\sqrt{1 + \frac{2L_e^2 E}{G^2 m^3 M^2}} < 1,$$ donc dans

tous les cas on aura un produit neg.(neg)

positif.

d) Quand le rayon est infini, l'énergie potentielle est nulle.

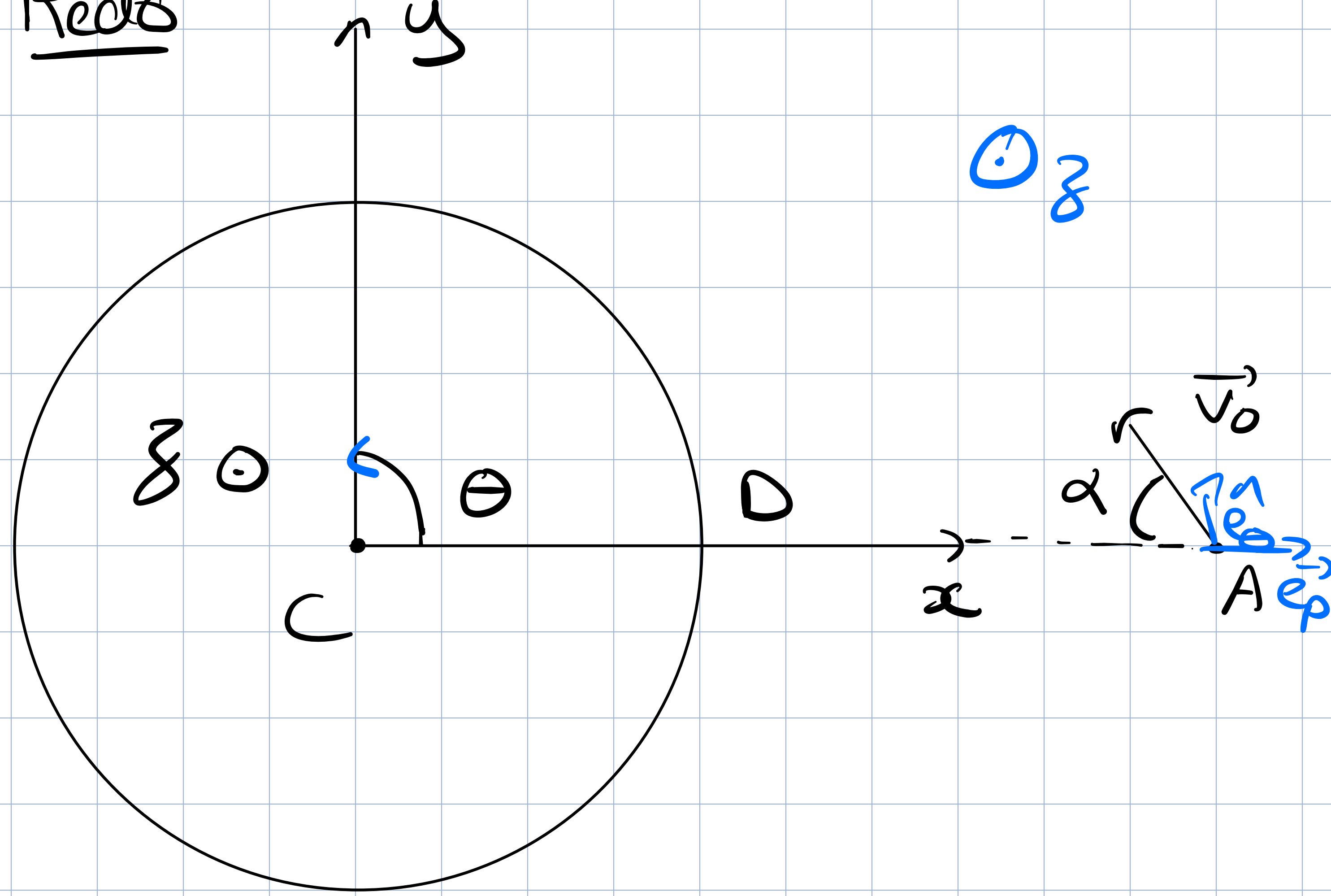
$$E_m = \frac{1}{2} m (v_\infty)^2$$

$$\frac{1}{2} m(v_\infty)^2 = \frac{1}{2} m (v_0)^2 - \frac{GMm}{D} \geq 0.$$

$$\Leftrightarrow \quad v_0 \geq \sqrt{\frac{2GM}{D}}$$

pas d'autre conditiᵒ.

Redo



ⓐ
- Energie mécanique (force conservative)
- Moment cinétique (force centrale)

$$E_m = \frac{1}{2} m v^2 - \frac{GMm}{r}$$

$$= \frac{1}{2} m \left(\dot{r}\hat{e_r} + r\dot{\theta}\hat{e_\theta}\right)^2 - \frac{GMm}{r}$$

$$= \frac{1}{2} m \left(\dot{r}^2 + r^2\dot{\theta}^2\right) - \frac{GMm}{r}$$

$$E_{m_0} = \frac{1}{2} m v_0 + \frac{GMm}{D}$$

$$\vec{L}_C = \vec{r} \wedge m\vec{v}$$

$$= r\hat{e}_r \wedge m(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta)$$

$$= mr^2\dot{\theta}\,\hat{e}_z$$

$$L_0 = D m v_0 \sin\alpha .$$

ⓑ $L_c^2 = m^2 r^4 \dot{\theta}^2$

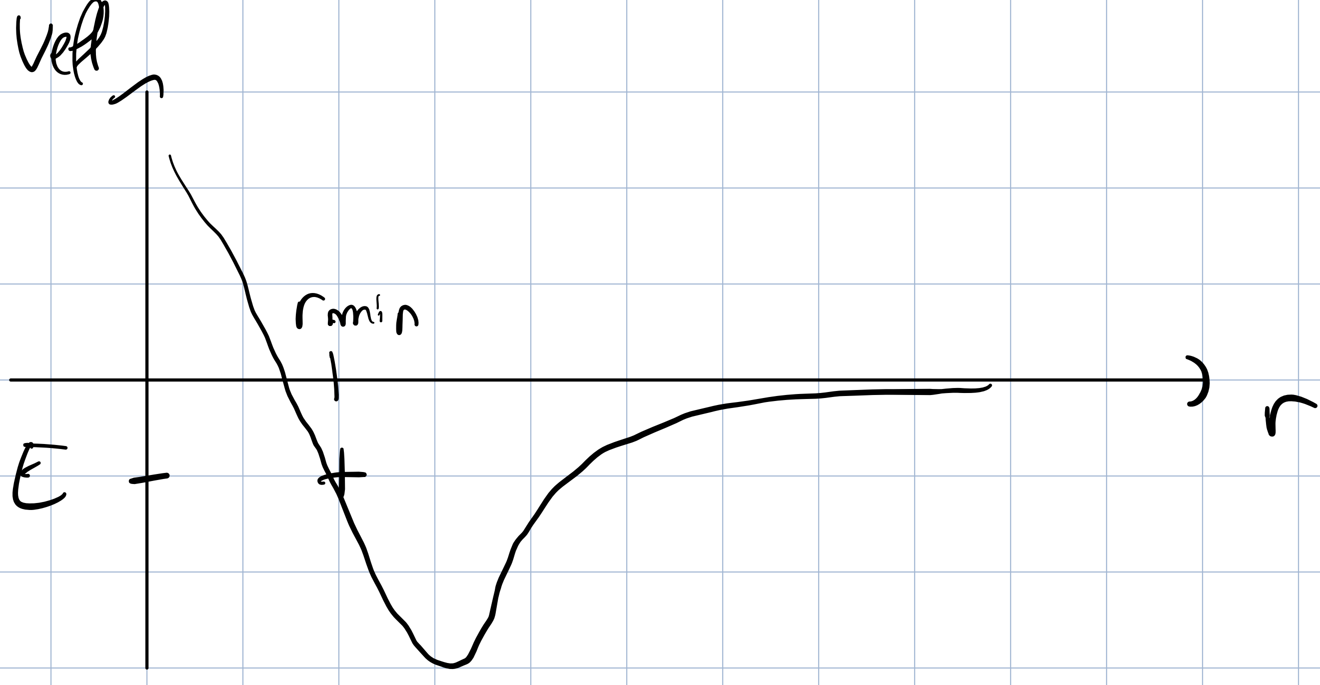
$$\dot{\theta}^2 = \frac{L_c^2}{m^2 r^4}$$

et $E = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \left(\frac{L_c^2}{m^2 r^4} \right) - \frac{GMm}{r}$

$$= \frac{1}{2} m \dot{r}^2 + \underbrace{\frac{1}{2} \frac{L_c^2}{m r^2} - \frac{GMm}{r}}_{V_{eff}}$$

Quand $r = r_{min}$, $\dot{r} = 0$

donc $E = V_{eff}$



c) Si $E = 0$

on a donc

$$\frac{1}{2}\frac{L_c^2}{m r_{min}^2} - \frac{GMm}{r_{min}} = 0$$

$$\Leftrightarrow \frac{L_c^2}{m} - GMm\, r_{min} = 0$$

$$\Leftrightarrow r_{min} = \frac{L_c^2}{GMm^2}$$

Si $E \neq 0$

$$\frac{1}{2}\frac{L_c^2}{m r_{min}^2} - \frac{GMm}{r_{min}} = E$$

$$\Leftrightarrow \frac{L_c^2}{2mE} - \frac{GMm}{E} r_{min} = r_{min}^2$$

$$\Leftrightarrow \frac{L_c^2}{2mE} - \frac{GMm}{E} r_{min} - r_{min}^2 = 0$$

$$\Delta = \left(\frac{GMm}{E}\right)^2 - 4 \cdot \frac{L_c^2}{2mE} \cdot (-1)$$

$$= \left(\frac{GMm}{E}\right)^2 + \frac{2L_c^2}{mE}$$

$$r_{min\pm} = \frac{-1}{2}\left(\frac{GMm}{E} \pm \sqrt{\Delta}\right)$$

$$= \frac{-1}{2}\frac{GMm}{E}\left(1 \pm \frac{\sqrt{\left(\frac{GMm}{E}\right)^2\left(1 + \frac{2L_c^2E^2}{mEG^2M^2m^2}\right)}}{\frac{GMm}{E}}\right)$$

$$= \frac{-1}{2}\frac{GMm}{E}\left(1 \pm \sqrt{1 + \frac{2L_c^2E}{m^3G^2M^2}}\right)$$

On veut r_{min} positif.

Si $E > 0$, on prend la version r_{min+}

Si $E < 0$, on peut prendre les deux.
car
$$\sqrt{1 + \frac{\quad}{\quad}} < 1.$$
donc ça fait $\ominus \cdot \ominus$

(c) Par conservat° de l'énergie,

$$\frac{1}{2} m v_\infty = \frac{1}{2} m (v_0)^2 - \frac{GMm}{D} \geq 0$$

$$\boxed{v_0 \geq \sqrt{\frac{2GM}{D}}}$$

pas d'autres conditi°