$$\boxed{5.8}$$

5.1 $$f'(x) = \frac{(1+x^4) - x(4x^3)}{(x^4)^2}$$

$$= \frac{1-3x^3}{x^8}$$

5.2 $$f'(x) = \frac{2x(1+x^2+x^4) - x^2(2x+4x^3)}{(1+x^2+x^4)^2}$$

$$= \frac{2x + \cancel{2x^3} + 2x^5 - \cancel{2x^3} - 4x^5}{(1+x^2+x^4)^2}$$

$$= \frac{2x - 2x^5}{(1+x^2+x^4)^2}$$

S.3 $(g \circ f)' = g'(f(x)) \cdot f'(x)$

$$-\sin\left(\sqrt{1+x^2}\right) \cdot \frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}}$$

$$= \frac{-x}{\sqrt{1+x^2}} \sin\left(\sqrt{1+x^2}\right)$$

S.4

$$f'(x) = \cos\left(\sqrt{1+x^2+x^4}\right) \cdot \frac{2x+3x^4}{2\sqrt{1+x^2+x^4}}$$

S.5 $f'(x) = \dfrac{-\sin(x)}{\cos(x)} = -\tan(x)$

S.6

$$g'(x) = \frac{\cos(x)\cos(x) - (1+\sin(x))(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\sin^2(x) + \sin(x) + \cos^2(x)}{\cos^2(x)}$$

$$= \frac{1+\sin(x)}{\cos^2(x)}$$

$$f'(x) = \frac{\dfrac{1+\sin(x)}{\cos^2(x)}}{\dfrac{1+\sin(x)}{\cos(x)}}$$

$$= \frac{(1+\sin(x))(\cos(x))}{\cos^2(x)(1+\sin(x))}$$

$$= \frac{\cancel{(1+\sin(x))}\cancel{(\cos(x))}}{\cos^{2}(x)\cancel{(1+\sin(x))}}$$

$$= \boxed{\frac{1}{\cos(x)}}$$

5.7

$$f(x) = \begin{cases} x^2 x & \text{si } x \in \mathbb{Z}^* \\ x^2 [x] & \text{si } x \notin \mathbb{Z}^* \end{cases}$$

$$f'(x) = \begin{cases} x^2 [x] & \text{si } x \notin \mathbb{Z}^* \\ \text{pas dérivée} & \end{cases}$$

S.8

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{si } x \neq 0 \\ 0 & \text{si } x = 0 \end{cases}$$

$$f'(x) = \begin{cases} 2x \sin\left(\frac{1}{x}\right) + x^2\left(\frac{-1}{x^2}\right)\cos\left(\frac{1}{x}\right) & \text{si } x \neq 0 \\ 0 & \text{si } x = 0 \end{cases}$$

$(*)$

$(*)$ $2x\sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$

S.9 $$f'(x) = \begin{cases} -(-2x^{-3})e^{-\frac{1}{x^2}} & \text{si } x \neq 0 \\ 0 & \text{si } x = 0 \end{cases}$$

S.10 $$\lim_{x \to 0} \frac{e^x - 1}{x} \qquad \frac{0}{0}$$

$$= \Big| \lim_{x \to 0} \frac{e^x}{1} = 1.$$

BH %

S.11 $$\lim_{x\to 0} \frac{e^{x^2}-1}{x}$$

$$= \Big|_{BH\,\%} \lim_{x\to 0} \frac{2xe^{x^2}}{1} = 0$$

S.12 $$\lim_{x\to \pi/2} \frac{\cos(x)}{x-\pi/2}$$

$$= \Big|_{BH\,\%} \lim_{x\to \pi/2} \frac{-\sin(x)}{1} = -1$$

S.13 $$\lim_{x\to \sqrt{2}} \frac{x^x - \sqrt{2^{\sqrt{2}}}}{x-\sqrt{2}}$$

$$\left[\left((2)^{1/2}\right)^{(2)^{1/2}} - \left(2^{(2)^{(1/2)}}\right)^{(1/2)} \right.$$

$$= 2^{1/2} - 2^{1/2} \qquad 0$$

$$= \lim_{x \to \sqrt{2}} \frac{\left(e^{x\ln(x)}\right)'}{1}$$

BH $0/0$

$$= \lim_{x \to \sqrt{2}} \left(\ln(x) + x\frac{1}{x}\right)\left(e^{x\ln(x)}\right)$$

$$= \left(\frac{1}{2}\ln(2) + 1\right)\left(e^{\sqrt{2}\ln(\sqrt{2})}\right)$$

$$= \left(\underline{\qquad\qquad}\right)\left(\sqrt{2}^{\sqrt{2}}\right)$$

$$= \left(\underline{\qquad\qquad}\right)\left(\left(\left(2\right)^{(1/2)}\right)^{\left(2^{(1/2)}\right)}\right)$$

$$= \left(\underline{\qquad\qquad}\right)\left(2^{1/2 \cdot 2 \cdot 1/2}\right)$$

$$= \left(\underline{\qquad\qquad}\right)\left(\sqrt{2}\right)$$

S.14

$$\lim_{x \to \pi} \frac{\sin x}{x - \pi} \quad \frac{0}{0}$$

$$\underset{\text{BH \%}}{=} \lim_{x \to \pi} \frac{\cos x}{1} = -1.$$

S.16

$$\lim_{x \to 0} \frac{x - \sin(x)}{1 - \cos(x)} \quad \frac{0}{0}$$

$$\underset{\text{BH \%}}{=} \lim_{x \to 0} \frac{1 - \cos(x)}{\sin(x)} \quad \frac{0}{0}$$

$$\underset{\text{BH \%}}{=} \lim_{x \to 0} \frac{\sin(x)}{\cos(x)} = 0$$

S. 17

$$\tan'(x) = \left(\frac{\sin(x)}{\cos(x)}\right)'$$

$$= \frac{1}{\cos^2(x)} = 1 + \tan^2(x)$$

$$\left(\frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}\right)$$

$$\lim_{x \to 0} \frac{\arctan(x)}{x} \qquad \frac{0}{0}$$

$$\left(f^{-1}\right)' = \frac{1}{f'(f^{-1}(x))}$$

$$\arctan'(x) = \frac{1}{1 + \tan^2(\arctan(x))}$$

$$= \frac{1}{1 + x^2}$$

$$= \Big| \quad \left(\frac{1}{1+x^2}\right) \cdot \frac{1}{1}$$

BH %

$$= \underline{1}$$

S. 18

$$\lim_{x \to 1} \frac{\arctan\left(\frac{1-x}{1+x}\right)}{x-1} \quad \frac{0}{0}$$

$$= \Big|_{\text{BH \%}} \lim_{x \to 1} \frac{1}{1+\left(\frac{1-x}{1+x}\right)} \cdot \frac{1}{1}$$

$$= \frac{1}{\quad} = \frac{2}{3}.$$

$$\lim_{x \to 0^+} \frac{\ln(x)}{\frac{1}{x^\alpha}} \quad \frac{-\infty}{+\infty}$$

$$\underset{BH}{=} \frac{-\infty}{+\infty} \quad \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-\alpha x^{\alpha-1}}{x^{2\alpha}}}$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{x} x^{2\alpha}}{-\alpha x^{\alpha-1}}$$

$$= \lim_{x \to 0^+} \frac{x^\alpha}{-\alpha x^{\alpha-1}} = 0$$

$$\lim_{x \to \infty} \frac{\ln(x)}{x^{\alpha}} \quad \frac{+\infty}{+\infty}$$

$$\overset{\text{BH}}{=} \frac{\frac{1}{x}}{\frac{-\alpha x^{\alpha-1}}{x^{2\alpha}}} = \frac{x^{2\alpha}}{-\alpha x^{\alpha}}$$

$\boxed{S.21}$

$$\left(1+\frac{1}{u}\right)^{u \cdot \alpha} = \left(\left(1+\frac{1}{u}\right)^{u}\right)^{\alpha}$$

$$= e^{\alpha}$$

$$u = \frac{x}{\alpha}$$

$$\lim_{x \to \infty} \frac{e^x}{x^n} \quad \frac{+\infty}{+\infty}$$

$$= \Big|_{BH\ +\infty/+\infty} \quad \lim_{x \to \infty} \frac{e^x}{nx^{n-1}}$$

$$= \lim_{x \to \infty} \frac{e^x x}{nx^n}$$

$$\ln\left(\frac{e^x}{x^n}\right)$$

$$= \ln(e^x) - \ln(x^n)$$

$$= x - n\ln(x)$$

$$\lim_{x \to \infty} x\left(1 - n\frac{\ln(x)}{x}\right)$$

$\downarrow 0$

$= +\infty$

$e^{+\infty} = +\infty$

$\boxed{S.23}$

$$f(x) = \sum_{k}^{n} \frac{f^{k}(a)}{k!} (x-a)^{k} + o\left((x-a)^{n}\right)$$

DL en a de degré n.

$$\lim_{x \to 0} \frac{o(x^{n})}{x^{n}} = 0$$

$$\begin{cases} \sin \\ \cos \\ \ln(1+x) = \\ e^{x} \end{cases}$$

$$= x + \frac{x^2}{2} + \frac{x^4}{4}$$

$$-1 \leqslant \sin(x) \leqslant 1$$

$$-x \leqslant \sin(x)x \leqslant x$$

$|\sin x| \nearrow$

$$\begin{cases} \lim\limits_{x \to 0^+} \sin(x) = 0. \\ \lim\limits_{x \to 0^+} x = 0. \end{cases}$$

$$\begin{cases} |\sin(x)| \nearrow \text{ sur }]0; \frac{\pi}{2}] \\ |x| \nearrow \text{ sur }]0; \frac{\pi}{2}] \end{cases}$$

$$\sin\left(\frac{\pi}{2}\right) = 1 < \frac{\pi}{2}.$$

$$\frac{|\sin x|}{|x|} < \frac{|x|}{|x|}$$

$$\Leftrightarrow \left|\frac{\sin x}{x}\right| < 1$$

$$f'(x) = \cos(x)$$

$$f'(x) = x$$

$\cancel{|x| \geq 1}$

$\cancel{|\sin x| \leq 1 \leq |x|}$

$$f'(x) \geq m > 0$$

- dérivée positive
- $\nearrow$ strictement

$$\forall x_1, x_2,\ x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$$

---

e^x | $\dfrac{e^x + \boxed{mx}}{\boxed{e^x + m}}$

Si $f'(x) \geq m > 0$

alors $\lim\limits_{x \to -\infty} f(x) < 0$

et $\lim\limits_{x \to \infty} f(x) =$

(les autres termes deviennent
négligeables)