Exercise 1

(1) not three consecutive 0.

A_{n-3} + 100
A_{n-2} + 10
A_{n-1} + 1

$A_3 = 2^3$
$A_2 = 2^2$
$A_1 = 2$

(2) A_{n-k}
$A_{n-(k+1)}$
$\vdots$
A_{n-1}

Exercise 2

\# 3 cons. 0 $\Rightarrow A$
\# 4 cons. 1 $\Rightarrow B$

Si commence avec :

$1 \Rightarrow |A_{n-1}|$

$$0 \Rightarrow \begin{cases} 01 \Rightarrow |A_{n-2}| \\ 00 \Rightarrow |A_{n-3}| \\ 000 \Rightarrow 2^{n-3} \text{ possibilités} \end{cases}$$

$$\Rightarrow |A_n| = |A_{n-1}| + |A_{n-2}| + |A_{n-3}| + 2^{n-3}$$

$A_1 = 0 \quad A_2 = 0 \quad A_3 = 1$

$A_4 = 1 + 2 = 3$

$A_5 = 3 + 1 + 0 + 2^2 = 8$

$A_6 = 8 + 3 + 1 + 2^3 = 20$

$A_7 = 20 + 8 + 3 + 2^4 = 47$

$A_8 = 47 + 20 + 8 + 2^5 = 102$

si on commence avec :

$0 \Rightarrow |B_{n-1}|$

$1 \Rightarrow \begin{cases} 10 \Rightarrow |B_{n-2}| \\ 110 \Rightarrow |B_{n-3}| \\ 1110 \Rightarrow |B_{n-4}| \\ 1111 \Rightarrow 2^{n-4} \text{ poss} \end{cases}$

$\Rightarrow |B_n| = |B_{n-1}| + |B_{n-2}| + |B_{n-3}| + 2^{n-4}$

$B_0 = 0 \quad B_1 = 0 \quad B_4 = 1$

$B_2 = 0 \quad B_3 = 0 \quad B_5 = 1 + 2 = 3$

$B_6 = 3 + 1 + 2^2 = 8 \quad B_7 = 28$

$B_8 = 28 + 8 + 3 + 2^5$

000 A 1111	2
A000 1111	1
1111 A000	2
1111000 A	1
A1111000	1
0001111 A	1
	= 8

$$|X| = |A| + |B| - |A \cup B|$$

Exercise 3

1

$$r^n = 3r^{n-1} - 3r^{n-2} + r^{n-3}$$

$$r^3 - 3r^2 + 3r - 1 = 0$$

$$(r-1)(r^2 - 2r + 1)$$

$$(r-1)(r-1)(r-1)$$

$$\boxed{\begin{aligned} &= r^3 - 2r^2 + r - r^2 + 2r - 1 \\ &= r^3 - 3r^2 + 3r - 1 \text{ ok} \end{aligned}}$$

$$\Rightarrow a_n = (\alpha_{1,0} + \alpha_{1,1} n + \alpha_{1,2} n^2) r_1^n$$

$r_1 = 1$

$a_0 = 1 \iff 1 = \alpha_{1,0}$

$a_1 = 3 \iff 3 = \alpha_{1,1} + \alpha_{1,2}$

$a_2 = 7 \iff 7 = 2\alpha_{1,1} + 4\alpha_{1,2}$

$\iff \alpha_{1,0} + 2\alpha_{1,1} + 4\alpha_{1,2} = 7$

$\alpha_{1,0} = 1$ et $\alpha_{1,1} = 1$ et $\alpha_{1,2} = 1$

Ainsi $a_n = 1 + n + n^2$

② $r^3 + r^2 - r - 1 = 0$

$(r-1)(r^2+2r+1)$

$= (r-1)(r+1)(r+1)$

$a_n = (\alpha_{1,1}) + (\alpha_{2,1} + \alpha_{2,2} n)(-1)^n$

$a_0 = 3 = \alpha_{1,1} + \alpha_{2,1} = 3$

$a_1 = -4 = \alpha_{1,1} - \alpha_{2,1} - \alpha_{2,2}$

$a_2 = 9 = \alpha_{1,1} + \alpha_{2,1} + 2\alpha_{2,2}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 1 & -1 & -1 & -4 \\ 1 & 1 & 2 & 9 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 1 & -1 & -1 & -4 \\ 0 & 0 & 2 & 6 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & -2 & -1 & -7 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & -1/2 & 7/2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$1/2$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 7/2 - 6/2 \\ 0 & 1 & 1/2 & 7/2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 1/2 \\ 0 & 1 & 0 & 7/2 - 3/2 \\ 0 & 0 & 1 & 3 \end{array}\right) \begin{array}{c} \\ 2 \\ \\ \end{array}$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1/2 + 3/2 \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \end{array}\right) \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

$$a_n = 1 + (2 + 3n)(-1)^n$$

Exercise 4

derangement

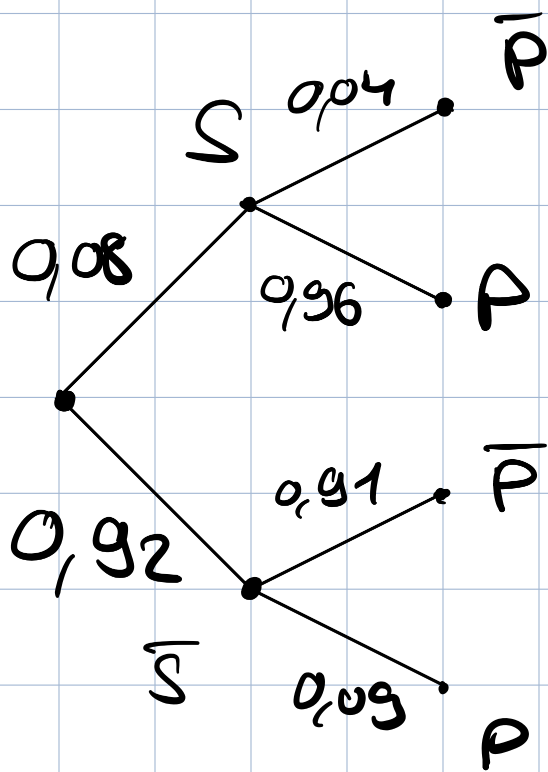
$$D_6 = 6!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right]$$

$$= 265.$$

Exercise 5

$P(S) = 0{,}08 \qquad P_{\overline{S}}(P) = 0{,}09$

$P_S(P) = 0{,}96$



$$P_P(S) = \frac{P(S \cap P)}{P(P)}$$

$$P(P) =$$
$$0{,}08 \cdot 0{,}96 + 0{,}92 \cdot 0{,}09 = 0{,}1596.$$

$$P(S \cap P) = 0{,}08 \cdot 0{,}96 = 0{,}0768.$$

$$P_p(S) = \frac{0{,}0768}{0{,}1596} \simeq 0{,}48.$$

Exercise 6

- There are 250 000 integers divisible by 4.
- There are 166 666 integers divisible by 6.

83 333 entiers

$$250\,000 + 166\,666 - 83\,333$$

$$= \underline{333\,333}.$$

Exercise 7

$$\sum_{k=1}^{5} |A_k|$$

$$- \sum_{k=1}^{5-1} |A_k \cap A_{k+1}|$$

$$+ \sum_{k=1}^{5-2} |A_k \cap A_{k+1} \cap A_{k+2}|$$

$$- \sum_{k=1}^{5-3} |A_k \cap A_{k+1} \cap A_{k+2} \cap A_{k+3}|$$

$$+ \sum_{k=1}^{5-4} |A_k \cap A_{k+1} \cap A_{k+2} \cap A_{k+3} \cap A_{k+4}|$$

$$- |A_k \cap A_{k+1} \cap A_{k+2} \cap A_{k+3} \cap A_{k+4} \cap A_{k+5}|$$

Exercise 8

$$P(n) : p\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} p(A_i)$$

Basis Step

pour $n = 1$

$$p(A_1) = p(A_1) \quad \boxed{\text{Ok}}$$

Inductive Step

Let's assume $p\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} p(A_i)$

Let's prove it implies

$$p\left(\bigcup_{i=1}^{n+1} A_i\right) \leq \sum_{i=1}^{n+1} p(A_i)$$

$$\Leftrightarrow p\left(\bigcup_{i=1}^{n} A_i \cup A_{n+1}\right) \leq \sum_{i=1}^{n} p(A_i) + p(A_{n+1})$$

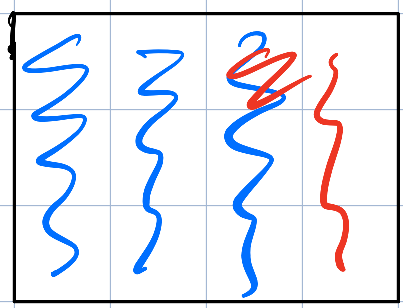
$$\Leftrightarrow p\left(\bigcup_{i=1}^{n} A_i\right) + p(A_{n+1}) - p(A_n \cap A_{n+1})$$
$$\leq \sum_{i=1}^{n} p(A_i) + p(A_{n+1})$$

$$\Leftrightarrow -p(A_n \cap A_{n+1}) \leq 0 . \quad \boxed{\text{Ok}} .$$

Exercise 9

① $p(A \cap B)$ can be at max $\frac{1}{3}$

it can be at min $\frac{1}{12}$.



un dé avec 12 faces

$\frac{3}{4} \to$ avoir entre 1 et 8

$\{1,2,3,4,5,6,7,8\}$

$\frac{1}{3} \to$ avoir $\{9,10,11,12\}$

2 $p(A \cup B)$ max 1.

min $\frac{1}{3}$

Exercise 10

$$P(S/X)\,P(X)$$

$$\boxed{\begin{aligned} P(S/A)\cdot P(A) \\ P(S/B)\cdot P(B) \\ P(S/C)\cdot P(C) \end{aligned}} \begin{aligned} &= 0{,}4 \cdot 0{,}01 = \boxed{0{,}4\%} \\ &= 0{,}35 \cdot 0{,}02 = \boxed{0{,}7\%} \\ &= 0{,}25 \cdot 0{,}03 = \boxed{0{,}75\%} \end{aligned}$$

$$P(\text{mange } X \mid \text{sick})$$

$$= \frac{P(\text{sick} \mid \text{mange } X)\cdot p(\text{mange } X)}{\boxed{P(\text{sick})}}$$

$$P(\text{sick}) = P(\text{sick} / \text{mange } A)$$

sauf qu'on sait ça este donc on compare juste les numérateurs

Exercice 11

$\Leftrightarrow$ " a-t-on autant de chance d'avoir
$S = 1, 2, 3, 4$ ".

$$r_1 + 2r_2\ [4] \rightarrow \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix}$$ même proba?

$$|\{1,6\} \times \{1,6\}| = 36$$

$$\underbrace{(r_1 - 1) + (r_2 - 1)\,6}_{} \in [0 \dots 35]$$

va de 0 à 5

$\frac{36 \text{ valeurs}}{4} = 9.$

donc • est équiprobable

$$ab \bmod 4 \equiv \underbrace{(a[4] \cdot (b[4]))[4]}_{2}$$

$$3 \cdot 2$$

$$(r_1 - 1) + (r_2 - 1)\,2$$

$$\Downarrow$$

$$r_1 + 2r_2 - 3 \equiv r_1 + 2r_2 + 1$$

Exercice 12

$i \geq 1$
$j \leq 3$

$$p(E_1 \cap E_3 \cap E_2 \cap E_3) \overset{?}{=} p(E_1 \cap E_3) \cdot p(E_2 \cap E_3)$$

Exercice 13

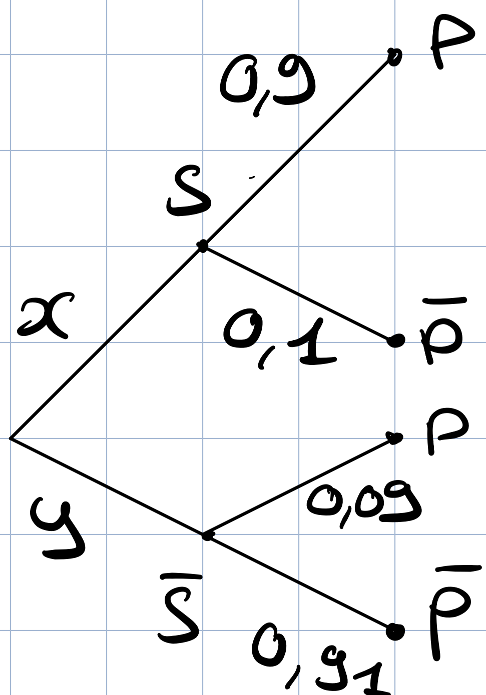
R : successful

S : success predicted

$P_S(R) = 0{,}9$ $\quad$ $P($

$P_S(\bar{R}) = 0{,}09$

$P_S(R)$



$x + y = 1$

$\Leftrightarrow y = 1 - x$

$$P(P) = x \cdot 0{,}9 + y \cdot 0{,}09$$

$$P_P(S) = \frac{x \cdot 0{,}9}{x \cdot 0{,}9 + y \cdot 0{,}09}$$

$$= \frac{x \cdot 0{,}9}{x \cdot 0{,}9 + (1-x)0{,}09}$$

$$= \frac{x \cdot 0{,}9}{x(0{,}9 - 0{,}09) + 0{,}09}$$

$$= \frac{0{,}9}{0{,}81 + \frac{0{,}09}{1{,}3}}$$

$$= \boxed{\frac{5}{6}}$$

$$P(S \mid P) = \frac{\overset{0{,}9 \cdot x}{P(P \mid S) \times P(S)}}{P(P)}$$

Exercise 14

35 W 10 B	35 W b B

$$P(\text{first box} \mid \text{black ball}) = \frac{1}{3}$$

$$= \frac{P(\text{black ball} \mid \text{first box}) \cdot P(\text{first box})}{p(\text{black ball} \mid \text{first box})\, p(\text{first box}) + p(\text{black ball} / \text{second box})\, p(\text{second box})}$$

$$= \frac{1/2\left(\frac{10}{45}\right)}{\frac{1}{2}\left(\frac{10}{45}\right) + \frac{1}{2}\left(\frac{b}{35+b}\right)} = \frac{1}{3}$$

$$\Leftrightarrow \quad \frac{10}{45} + \frac{b}{35+b} = 3 \cdot \left(\frac{10}{45}\right)$$

$$\Leftrightarrow \quad \frac{b}{35+b} = \frac{20}{45}$$

$$\Leftrightarrow \quad \frac{45b}{35+b} = 20$$

$$\Leftrightarrow \quad 45b = 20(35+b)$$

$$= 700 + 20b$$

$$\Rightarrow \quad b = \frac{700}{25} = 7 \cdot \frac{100}{25} = \underline{28.}$$

$$p(E_1 \cap E_3 \cap E_2) \overset{?}{=} p(E_1 \cap E_3)\, p(E_2 \cap E_3)$$

E_1 = 3 sort $\quad p(E_1) = \frac{1}{6}$

E_3 =

E_2 = j'arrive à finir ici $\quad p(E_2) = \frac{1}{10}$

$$p(E_1)\, p(E_2) = \frac{1}{60}$$

$$6 \times 10$$

$$60$$