Homework 13

Tanja Käser

Exercise 6. Derive the probability distribution of all the possible outcomes for the following random events:

1. The maximum of a roll of two regular dice.
2. The result of a roll of three indistinguishable dice is a triple, pair or all distinct
3. The result of a roll of five indistinguishable dice is a five of a kind, four of a kind, full house, tree of a kind, two pairs, one pair or all distinct

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	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

6 11/36
5 9/36
4 7/36
3 5/36
2 3/36
1 1/36

2 6·6·6 combinations

only 6 are triples $\quad p(\text{triple}) = \frac{1}{36}$

$\binom{6}{1}\binom{3}{1}\binom{5}{1}$ ← choose the other nb

choose the number

choose the position of the other nb *

*
CCS
SCC
CCS

$= 6 \cdot 3 \cdot 5 = 90$ pairs $\quad p(\text{pair}) = \frac{90}{6 \cdot 6 \cdot 6}$

$$p(\text{distinct}) = 1 - p(\text{triple}) - p(\text{pair})$$

$$= \frac{5}{9}$$

(k) Bust (none of the above)

Exercise 8. Consider five-card poker hands drawn from a regular deck where suits are irrelevant (i.e., the hand (2♡)(A♡)(2♠)(10♠)(Q♠) is the same as the hand (Q♡)(A♡)(2♢)(2♣)(10♣)).

1. What is the total of such poker hands?

- ~~6175~~
- 6188
- 5148
- 7462

2. What is the probability of the distinct poker hands that contain:

   (a) *One pair* (poker hand containing two cards of the same kind and three cards of three other, distinct kinds)

   (b) *Two pairs* (poker hand containing two cards of the same kind, two cards of another kind and one card of a third kind)

   (c) *Three of a kind* (poker hand containing three cards of the same kind and two cards of two other kinds)

   (d) *Straight* (poker hand containing five consecutive kinds, counting the aces both as the first and the last kind)

   (e) *Flush* (poker hand containing five cards of the same suits)

   (f) *Full house* (poker hand containing three cards of one kind and two cards of another kind)

   (g) *Four of a kind* (poker hand containing four cards of the same kind and one card of another kind)

   (h) *Straight flush* (poker hand containing five consecutive kinds of the same suit, counting the aces both as the first and the last kind)

   (i) *Royal flush* (poker hand containing the five highest kinds of the same suit; note that "royal" implies "straight")

   (j) *Five of a kind* (poker hand containing five cards of the same kind)

   (k) *Bust* (none of the above)

3. Compare your results with the previous exercise.

→ it's the same as asking "how to share out kinds in the hand? 4 aces, 1 king? 4 kings, 1 ace?"

stars and bars $\binom{13+5-1}{5} = 6188$

↑ not 5-1 because we need

minus 13 draws where we have more than 4 times the same kind (impossible)

a place to store the cards we leave in the draw pile

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a $\binom{13}{1} \cdot \binom{12}{3}$

Exercise 12. With 2.7 goals on average per soccer game and ten games per weekend, the probability of at least 45 goals per weekend

- ○ can be proved to be at most 50%.
- ○ can be proved to be 60%.
- ☒ can be proved to be at most 60%.
- ○ is not sufficiently specified by the data provided to make any type of prediction.

$\Rightarrow$ Markov

$$p(x(s) \geq a) \leq E(x)/a$$

$$p(x(s) \geq 45) \leq E(x)/45$$

$$= 27/45$$

$$= 0{,}6.$$

Exercise 13. A die is rolled three times resulting in an ordered triple (r_1, r_2, r_3) of independent random outcomes $r_1, r_2, r_3 \in \{1, 2, 3, 4, 5, 6\}$, and the random variable X is defined by $X((r_1, r_2, r_3)) = r_1 + 2r_2 + 3r_3 \in \mathbf{Z}$.

The expected value $\mathbb{E}(X)$ of X and the probability $p(X \leq r)$ that X takes at most the value r satisfy

- ○ $\mathbb{E}(X) = 21$ and $p(X \leq 8) = \frac{1}{72}$.
- ☒ $\mathbb{E}(X) = 21$ and $p(X \leq 8) = \frac{1}{54}$.
- ○ $\mathbb{E}(X) = 24.5$ and $p(X \leq 8) = \frac{1}{72}$.
- ○ $\mathbb{E}(X) = 24.5$ and $p(X \leq 8) = \frac{1}{54}$.

$$E(r_1 + 2r_2 + 3r_3)$$

$$= E(r_1) + 2E(r_2) + 3E(r_3)$$

$$= \frac{7}{2} + 7 + \frac{3 \cdot 7}{2} = 21$$

$p(x \leq 8)$. We want all the comb. st.

$$r_1 + 2r_2 + 3r_3 \leq 8$$

minimum is 5 (when everything is at 1)

1 1 1   2 1 1

1 2 1

3 1 1

$$\frac{4}{6^3} = \frac{1}{54}$$

cartesian product

Exercise 11. A die is rolled twice resulting in an ordered pair (r_1, r_2) of independent random outcomes $r_1, r_2 \in \{1, 2, 3, 4, 5, 6\}$, and the value $s = r_1 + 2r_2 - 4k \in \{1, 2, 3, 4\}$ is computed, where $k \in \mathbf{Z}$.

- ~~○~~ s is uniformly distributed over $\{1, 2, 3, 4\}$.
- ○ s is not uniformly distributed over $\{1, 2, 3, 4\}$, but it is if "$r_1 + 2r_2$" is replaced by "$r_1 + 3r_2$".
- ○ s is not uniformly distributed over $\{1, 2, 3, 4\}$, but it is if "$r_1 + 2r_2$" is replaced by "$r_2 + 2r_1$".
- ○ s is not uniformly distributed over $\{1, 2, 3, 4\}$, but it is if all outcomes with $r_1 + r_2 = 7$ are discarded.

$r_2 \backslash r_1$	1	2	3	4	5	6
1	3	4	1	2	3	4
2	1	2	3	4	1	2
3	3	4	1	2	3	4
4	1	2	3	4	1	2
5	3	4	1	2	3	4
6	1	2	3	4	1	2

$9 \cdot 3$

$9 \cdot 1$

$9 \cdot 4$

$9 \cdot 2$

Exercise 14. After summer, the winter tires of a car (with four wheels) are to be put back. However, the owner has forgotten which tire goes to which wheel, and the tires are installed "randomly", each of the $4! = 24$ permutations are equally likely.

1. What is the probability that tire 1 is installed in its original position?
2. What is the probability that all the tires are installed in their original position?
3. What is the expected number of tires that are installed in their original positions?
4. Redo the above for a vehicle with n wheels.
5. What is the probability that none of the tires are installed in their original positions?

1 equal distribution $\Rightarrow \frac{1}{4}$

2 $\frac{1}{24}$

3 $E(T_1 + T_2 + T_3 + T_4)$

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1.$

4 a equal distribution $\frac{1}{n}$. b $\frac{1}{n!}$.

c 1

5 number of derangements / $n!$