$$V(X) + V(Y) = V(X+Y)$$

$$V(Z) = \sum_{s \in S} (Z(s) - E(Z))^2 p(s)$$

Soit $Z = X + Y$

Si X et Y indep $Z(s) = X(s) + Y(s)$

$E(Z) = E(X) + E(Y)$

$$V(X+Y) = \sum_{s \in S} (X(s) + Y(s) - E(X) - E(Y))^2$$

$$= (X(s) - E(X))^2 - 2(-) + (Y(s) - E(Y))^2$$

$$= V(X) + V(Y)$$

$$V(Z) = E(Z^2) - E(Z)^2$$

$$\left(-\left(E(X)^2 + 2E(X)E(Y) + E(Y)^2\right)\right)$$

$$= E\left((X+Y)^2\right) - E(X)^2 - 2E(X)E(Y) + E(Y)^2$$

$$= E(X^2) + 2E(X)E(Y) + E(Y^2)$$

$$- E(X)^2 - 2E(X)(Y) - E(Y)^2$$

$$= V(X) + V(Y)$$

$$g(x^2) - g^2(x)$$

Exercise 2

① $E(X) = 0$

$E(Y) = 20$

$E(X+20) = E(X) + 20$
$= E(Y)$

② $V(X) = E(X^2) - \underbrace{E(X)}_{\to 0}{}^2$

$$\sum_{k=\{100,81,\dots\}} p(X=k)\cdot k = \frac{2}{21}\sum_{k=1}^{10} k^2$$

$$\frac{2}{21}\cdot 100 + \frac{2}{21}\cdot 81 + 0\cdot \frac{1}{21}$$

$$\frac{10(10+1)(\cancel{21})}{6} \cdot \frac{2}{\cancel{21}}$$

$$= 2\cdot\left(\frac{100+10}{6}\right) = \frac{110}{3}$$

$$V(X) = \frac{110}{3}$$

$$V(Y) = V(X+20)$$

$$= E((X+20)^2) - E(X+20)^2$$

$$= E(X^2 + 40\cdot X + 400)$$

$$- (E(X) + E(20))^2$$

$$= E(X^2) + 40E(X) + \cancel{400}$$

$$- E(X)^2 - 40E(X) - \cancel{400}$$

$= V(x)$

$V(z) = V(y)$

Exercise 3

$\underline{n=4}$

$$E(X) = \sum_{s \in S} p(X=s)\, p(s)$$

$$= \frac{1}{n}\left(\frac{n(n+1)}{2}\right)$$

$$= \boxed{\frac{(n+1)}{2}}$$ for $n = 4, 6, 8, 12, 20$

$E(XYZAB)$

$= E(X) \cdot E(Y) \ldots \cdot E(B)$

$= \underline{\text{for all } n}$

$$X_n = X_T + X_H$$

$$E(X_n) = E(X_T) + E(X_H)$$

$$= 0 \quad \text{(même proba)}$$

2

Exercise 7

$, L(x) = R(x)$

Basis Step

- $L(\lambda) = R(\lambda)$

Inductive Step

Let's assume $L(a) = L(a)$
and $L(c) = L(c)$

Then, $L([a]) = R([a])$
and $L(ac) = L(ac)$

Ex 10

- $\frac{b}{a} = k \Leftrightarrow b = ka$
- $a = k'b$

$$\{ (a,b) \mid (a \neq b) \land ((a,b) \in R_1 \lor (a,b) \in R_2) \}$$

$$(x+y)^n$$

$$= \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \, y^{k}$$

$$\left(x^2 - \frac{1}{x}\right)^9$$

$$\sum_{k=0}^{n} \binom{n}{k} x^{2(n-k)} \quad x^{-k}$$

$k = 4 \Rightarrow 1$ terme apparaît

coef $\binom{9}{4}$

$$\frac{9!}{4!\,5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$$

$$
\begin{aligned}
&= 3 \cdot 2 \cdot 7 \cdot 3 \\
&= 9 \cdot 14 \\
&= 90 + 4 \cdot 9 \\
&= \underline{126}
\end{aligned}
$$

$$x_1 + x_2 + x_3 = 37$$

$8 \quad 3 \quad 4$

boxes.

$$\binom{\quad +3-1}{3}$$