Logic :

Proposition is a declarative sentence that is T or F

$x + 5 = 12$ is not a proposition because x undef

Atomic propositº : cannot be expressed in terms of simpler prop.

Compound propositº : other prop + logical connectives

* **Negatº** $\neg p$ "not p" opposite truth value

$a :=$ "earth is round" (T)
$\neg a$ (F)

Truth table : list all possible truth values

* Conjunctᵒ $p \wedge q$ "p and q"

* Disjunctᵒ $p \vee q$ "p or q"

⚠ or in natural language
- inclusive or "you need a Master or a PhD"
- exclusive or "soup or salad"

* Exclusive or $p \oplus q$ "xor"

* Implication $p \rightarrow q$ "conditional statement"

premise conclusion "if p then q"

(F) if p is (T) and q is (F)

no need for connectᵒ between the two

e.g "if I'm elected, then I will lower the taxes"

⚠ if q is always (T) then $(p \rightarrow q)$ is (T)
if p is always (F) then $(p \rightarrow q)$ is (F)

⚠ $p \rightarrow q \not\Leftrightarrow q \rightarrow p$

In natural language...

"if p then q"

"q when p"

In mathematical language...

"a necessary conditio for p is q"

"a sufficient conditio for q is p"

* Biconditional $p \Leftrightarrow q$ "bi-implications"

(T) if p and q have the same truth values.

$$\neg(p \Leftrightarrow q) \Leftrightarrow p \oplus q$$

E.g: "p if and only if q"

Cours 20/09

* Precedence of logical connectives

$\neg$

$\wedge \ \vee$

$\rightarrow \ \leftrightarrow$

$$(p \vee (\neg q)) \rightarrow (p \wedge q)$$

... $\oplus$? no convention

$$\begin{cases} p := \text{"it is below freezing"} \\ q := \text{"it is snowing"} \\ p \Leftrightarrow q \end{cases}$$

"l'un implique l'autre"

* Claroff

- Tautology

  always true. e.g $p \vee \neg p$

- Contradiction

  always false. e.g. $p \wedge \neg p$

- Contingency

  sometimes true, false e.g. p

E.g. tautology: $(p \wedge q) \to (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$\to$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

great... but a bit long... reasoning?

① if we have $a \to b$, it is false if and only if a (T) and b (F).

② a is (T) iff p (T) and q (T)

③ p is (T), q is (T), then $(p \vee q)$ is (T)

④ Therefore, impossible to make it false

* Satisfiability

is there a solutionº in truth table?

→ either a tautology or contingency

* Logical equivalences

$p \leftrightarrow q$ is a tautology

⇒ then p and q are logically equivalent

⇒ $p \equiv q$

e.g. $\neg p \vee q \equiv p \rightarrow q$

⚠ Cheat sheet with all the laws

e.g.: identity, domination, idempotent, double-negationº, commutative, De Morgan's, associative, ...

De Morgan's

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$p \rightarrow q \equiv \neg p \vee q$$

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

* Contrapositive

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

* Converse | Inverse

$$q \rightarrow p \equiv \neg p \rightarrow \neg q$$

* Equivalence Proofs

e.g. Show that $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

- $\neg p \wedge \neg(\neg p \wedge q) \equiv \neg p \wedge \neg q$ De Morgan
- $\neg p \wedge (p \vee \neg q) \equiv \neg p \wedge \neg q$ Distrib.
- $\neg p \wedge \neg q \equiv \neg p \wedge \neg q$ ■ Neg. Law (T)

eg. show that $(p \vee q) \rightarrow (p \vee q)$

- $\neg(p \wedge q) \vee (p \vee q)$ Law on condit.
- $(\neg p \vee \neg q) \vee (p \vee q)$ De M.
- $(\neg p \vee p) \vee (\neg q \vee q)$ Associative law
- $T \vee T$ ■ (T)

⟹ Method 1: truth table

⟹ Method 2: perform an equivalence proof

⟹ Method 3: find a counter-example

* Functionally complete logical operators set

you can do everything with only $\wedge$, $\vee$, $\neg$

* Normal forms (e.g. DNF or CNF)

they use only $\wedge$, $\vee$, $\neg$.
⟹ easy to compare

* DNF → disjunctive normal form

- has to be disjunct° of conjunctions

each conjunction has to be a minterm
(each variable or its negat° appears once in it)

e.g. $(p \wedge q) \vee (p \wedge \neg q)$

e.g. $(p \wedge \neg q) \vee p$ / q not appearing

$\neg(p \vee q)$ doesn't work

$p \vee (q \wedge (r \vee s))$ not a minterm

* CNF → conjunctive normal form

• has to be conjuct° of disjunct°

each disjunction has to be a clause (each variable or its neg appears once)

Construct DNF	Construct CNF
- build truth table	- build truth table
- search for (T) rows to construct minterms	- search for (F) rows to construct clauses
- for each term:	- for each term:
- if (T), take it directly	- if (T), negate it
- if (F), negate it	- if (F), use it
- join with OR $\vee$	- join with $\wedge$