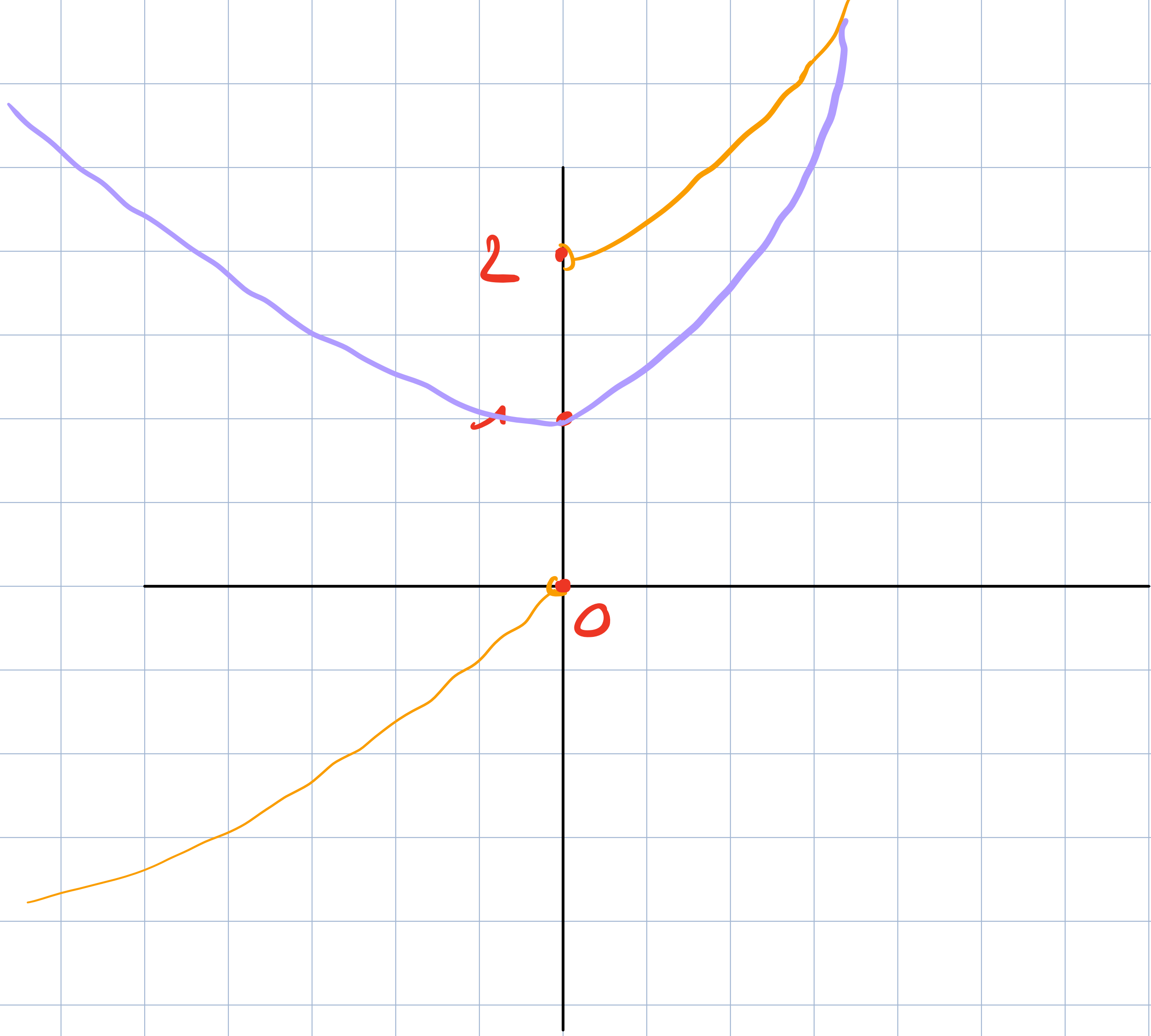
$$f'(x) = \begin{cases} -1 + e^{x} & si\ x < 0 \\ 1 + e^{x} & si\ x > 0 \end{cases}$$

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \to 0} \frac{|x| + e^{-x} - e^{0}}{x}$$

$$=$$


2
1
0

Exercice 2

$$\boxed{(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))}}$$

$$f'(x) = 1 + e^{x}$$

$$(f^{-1})'(1) = \frac{1}{1 + e^{(f^{-1}(1))}}$$

$$f(x) = 1 \iff x + e^{x} = 1$$

$$\iff x = 0$$

$$(f^{-1})'(1) = \frac{1}{1+e^{0}}$$

$$= \frac{1}{2} \neq 1 + \frac{1}{e}.$$

Exercice 3

$$\lim_{x \to 0} \frac{x \sin(x) \sin\left(\frac{1}{x}\right)}{x}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \cdot x \sin\left(\frac{1}{x}\right)$$

$$= \lim_{x \to 0} 1 \cdot 0$$

$$= 0$$

$$f'(x) =$$

$$\left(\sin(x) + x\cos(x)\right) \sin\left(\frac{1}{x}\right) = 0$$

$$\underbrace{+ \underbrace{\cancel{x}\sin(x)\frac{-1}{x^{\cancel{2}}}}_{=1}\cos\left(\frac{1}{x}\right)}_{\text{lim en 0 n'existe pas}}$$

$\lim\limits_{x \to 0} f'(x)$ n'existe pas

$\Rightarrow$ <u>non continue</u>.

Exercice 4

ⓐ $\left(f^{-1}\right)'(y)$

$$= \frac{1}{f'(f^{-1}(y))}$$

$$= \frac{1}{\cos(\arcsin(y))}$$

$$= \frac{1}{\sqrt{1-\sin^2(\arcsin(y))}}$$

$$= \frac{1}{\sqrt{1-y^2}}$$

$$\cos^2(x) + \sin^2(x) = 1$$

$$\Leftarrow \quad \cos(x) = -\sqrt{1 - \sin^2(x)}$$

$$(f^{-1})(y) = \arcsin(y)$$

(b) $(f^{-1})'(y) = \dfrac{1}{f'(f^{-1}(y))}$

$f'(x) = -\sin(x)$

$f^{-1}(y) = \arccos(y)$

$(f^{-1})'(y) = \dfrac{1}{-\sin(\arccos(y))}$

$\sin(x) = -\sqrt{1-\cos^2(x)}$

$= \dfrac{1}{-\sqrt{1-y^2}}$

(c) $f(x) = \dfrac{\sin(x)}{\cos(x)}$

$$f'(x) = \frac{\cos^2(x) + \sin^2(x)}{(\cos^2(x))}$$

$$= \frac{1}{\cos^2(x)}$$

$$f^{-1}(x) = \arctan(x)$$

$$(f^{-1})'(y) = \frac{1}{\frac{1}{\cos^2(\arctan(y))}}$$

$$= \cos^2(\arctan(y))$$

$$\tan^2(x) = \frac{\sin^2(x)}{\cos^2(x)}$$

$$e) \quad \cos^2(x) = \frac{\sin^2(x)}{\tan^2(x)}$$

$$= \frac{1}{1+\tan^2(x)}$$

$$= \frac{1}{1+\tan^2(\arctan(y))}$$

$$= \frac{1}{1+y^2}$$

(d) $f(x) = e^{x}$

$$f^{-1}(y) = \ln(y)$$

$$\left(f^{-1}\right)'(y) = \frac{1}{e^{\ln(y)}} = \frac{1}{y}.$$

(e) $f(x) = e^{-x}$

$$f^{-1}(y) = -\ln(y)$$

$$\left(f^{-1}\right)'(y) = \frac{1}{e^{-\ln(y)}} = \frac{1}{y \cdot e^{-1}}$$

$$= \frac{1}{e^{\ln(\frac{1}{y})}}$$

$$= \frac{1}{\frac{1}{y}} = y$$

(8) $f'(x) = \left(e^{\ln(a^x)}\right)'$

$$= \left(e^{x\ln(a)}\right)'$$

$$= \ln(a)\, e^{x\ln(a)}$$

$$= \ln(a) \cdot a^x$$

$$y = a^x$$

$$\Leftrightarrow \ln(y) = x\ln(a)$$

$$\Leftrightarrow \frac{\ln(y)}{\ln(a)} = x$$

$$\Leftrightarrow \log_a(y) = x$$

$$f^{-1}(y) = \log_a(y)$$

$$\left(f^{-1}(y)\right)' = \frac{1}{f'(f^{-1}(y))}$$

$$= \frac{1}{\ln(a) \cdot a^{\log_a(y)}}$$

$$= \frac{1}{\ln(a) \cdot y}$$

(9) $f(x) = \sinh(x) = \dfrac{e^{x} - e^{-x}}{2}$

$$f'(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right) = \cosh$$

$$\left(f^{-1}\right)(y) = \operatorname{arcsinh}(y)$$

$$\left(f^{-1}\right)'(y) = \frac{1}{\cosh(\operatorname{arcsinh}(y))}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\left(f^{-1}\right)'(y) = \frac{1}{\sqrt{1 + \sinh^2(\operatorname{arcsinh}(y))}}$$

$$= \frac{1}{\sqrt{1+y^2}}$$

(h) $f(x) = \cosh(x) = \dfrac{e^{x}+e^{-x}}{2}$

$$f'(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right) = \sinh(x)$$

$$\left(f^{-1}\right)'(x) = \frac{1}{\sinh(\operatorname{arccosh}(y))}$$

$$= \frac{1}{\sqrt{y^2 - 1}}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\Leftrightarrow \sinh^2(x) = -1 + \cosh^2(x)$$

$$\Leftrightarrow \sinh(x) = \sqrt{\cosh^2(x) - 1}$$

j) $f(x) = \dfrac{e^{x} + e^{-x}}{e^{x} - e^{-x}} = \dfrac{\cosh(x)}{\sinh(x)}$

$$f'(x) = \frac{\sinh(x)\sinh(x) - \cosh(x)\cosh(x)}{\sinh^2(x)}$$

$$= \frac{1}{\sinh^2(x)}$$

$$(f^{-1})(y) = \operatorname{arccoth}(y)$$

$$(f^{-1})'(y) = \frac{1}{\dfrac{1}{\sinh^2(\operatorname{arccoth}(y))}}$$

$$= \sinh^2(\operatorname{arccoth}(y)) \quad (*)$$

$$\coth^2(x) = \frac{\cosh^2(x)}{\sinh^2(x)} = \frac{1+\sinh^2(x)}{\sinh^2(x)}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\Leftrightarrow -1+\coth^2(x) = \frac{-\sinh^2(x)}{\sinh^2(x)} + \frac{1+\sinh^2(x)}{\sinh^2(x)}$$

$$\Leftrightarrow \coth^2(x) - 1 = \frac{1}{\sinh^2(x)}$$

$$\Leftrightarrow \sinh^2(x) = \frac{1}{\coth^2(x) - 1}$$

$$\circledast \quad = \frac{1}{\coth^2(\operatorname{arcoth}(y)) - 1}$$

$$= \frac{1}{y^2 - 1}$$

1 $f^{-1}(y) = \operatorname{arctanh}(y)$.

$$f'(x) = \left(\frac{\sinh(x)}{\cosh(x)}\right)'$$

$$= \frac{\cosh(x)^2 - \sinh(x)^2}{\cosh(x)^2}$$

$$= \frac{1}{\cosh^2(x)}$$

$$(f^{-1})'(y) = \frac{1}{\frac{1}{\cosh^2(\operatorname{arctan}(y))}}$$

$$= \cosh^2(\operatorname{arctanh}(x))$$

$$= \frac{1}{1 - \tanh(\operatorname{arctanh}(y))^2} = \frac{1}{1-y^2}$$

II

$$(g \circ f)' = g'(f(x)) \cdot f'(x)$$

$$\begin{cases} f(x_0) = y_0 \\ g'(y_0) = 3 \\ f'(x_0) = -4 \end{cases}$$

III

$$(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}$$

$$f'(x_0) = -4$$

$$f(x_0) = y_0$$

$$\Leftrightarrow f^{-1}(y_0) = x_0$$

Exercice 5

$\boxed{1}$

$$\left[\frac{\pi}{6} ; \frac{5\pi}{24}\right]$$

$$\frac{5\pi}{24} - \frac{\pi}{6} = \frac{5\pi}{24} - \frac{4\pi}{24} = \frac{\pi}{24}$$

- fon continue
- fon dérivable

$\Rightarrow$ TAF

$$\boxed{f(x+h) = f(x) + f'(x+uh)h, \quad u \in [0,1].}$$

$$f\left(\frac{\pi}{6}+\frac{\pi}{24}\right) = f\left(\frac{\pi}{6}\right) + \left(\frac{1}{\cos^2\left(\frac{\pi}{6}+\lambda\frac{\pi}{24}\right)}\right)\frac{\pi}{24}$$

$$= \frac{\sqrt{3}}{3} + \frac{1}{\cos^2\left(\frac{\pi}{6}+\lambda\frac{\pi}{24}\right)}\,\frac{\pi}{24}$$

$$1 \geq \cos^2\left(\frac{\pi}{6}+\lambda\frac{\pi}{24}\right) \geq 0$$

$$\frac{\pi}{24} \geq \frac{1}{\cos^2(\text{—})} \geq 0$$

entre

- $\frac{\sqrt{3}}{3} + \frac{\pi}{24}$
- $\frac{\sqrt{3}}{3}$

On sait que $\dfrac{1}{\cos^2(x)}$ ↗ sur $\left]\dfrac{\pi}{6} ; \dfrac{\pi}{4}\right[$

$$\Rightarrow \quad \frac{4}{3} = \frac{1}{\cos^2\left(\frac{\pi}{6}\right)} \leq \frac{1}{\cos^2\left(\frac{\pi}{6}+\frac{\pi}{24}\right)}$$

$$\leq \frac{1}{\cos^2\left(\frac{\pi}{4}\right)} = 2.$$

On obtient

$$\frac{\sqrt{3}}{3} + \frac{\pi}{18} \leq \tan\left(\frac{5\pi}{24}\right) \leq \frac{\sqrt{3}}{3} + \frac{\pi}{12}$$

$\boxed{2}$

$$f(x) - f(x+h) > 0 \Rightarrow \searrow$$

$$\sin(x) \cancel{- x} + \frac{1}{6}x^3 - \sin(x+h)$$
$$+ \cancel{x} + h - \frac{1}{6}(x+h)^3$$

$$= \sin(x) - \sin(x+h) + \frac{1}{6}x^3$$
$$- \frac{1}{6}\left(x^3 + 3x^2h + 3xh^2 + h^3\right)$$

on remarque que f paire $\Rightarrow$ on s'intéresse juste à $x \geq 0$.

f continue et dérivable $\Rightarrow$ TAF (corollaire 3)

$$f(x) = \cos(x) - 1 + \frac{1}{2}x^2$$

$\Rightarrow$ si dérivée de $f \geq 0$ et $f(a) \geq 0$ sur $[a,b]$, $f \geq 0$ sur $[a,b]$.

$$f(0) = \cos(0) - 1 + 0 = 0$$

$$f'(x) = -\sin(x) + x$$

On applique le cor 3 sur f' de la même façon.

$f'(0) = 0$ et $f''(x) = -\cos(x) + 1 \geqslant 0.$

$\Rightarrow f'(x) \geqslant 0.$ $+$ $f(0) \geqslant 0$

$\Rightarrow f(x) \geqslant 0.$