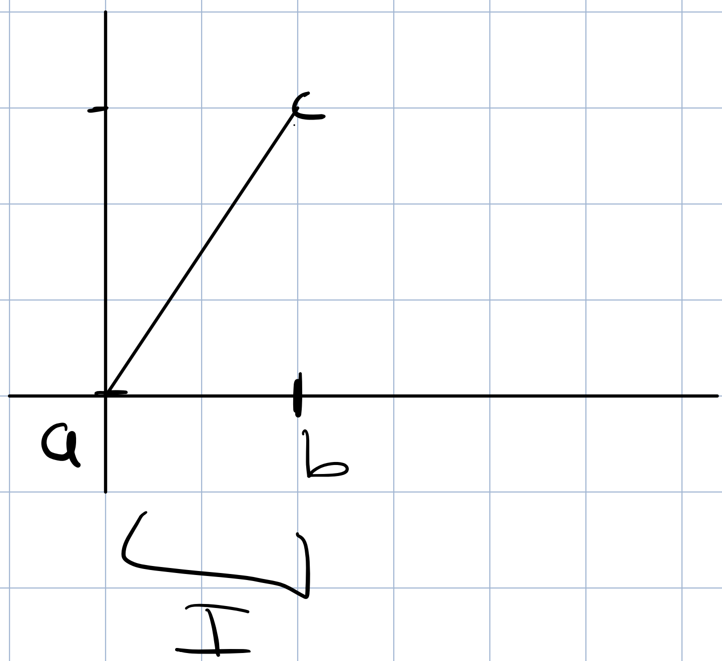
$\boxed{6}$

$]0,2[$



$\boxed{7}$

ⓐ $$e^{x-1} - x - 1 = 0$$

$$f(x) := e^{x-1} - x - 1.$$

on cherche $f(x) = 0$.

- f continue sur $\mathbb{R}$.
- $x = 1$, $e^0 - 2 < 0$
- $x = -1$, $e^{-2} + 1 - 1 > 0$.

par le TVI, f continue sur $[-1, 1]$
(un intervalle fermé)

$$f(-1) < 0, \quad f(1) > 0$$

$\Rightarrow$ une sol existe.

(b) $x^2 - \frac{1}{x} - 1 = 0$

$f(x) := x^2 - \frac{1}{x} - 1$

on cherche $f(x) = 0$.

- f est continue sur $\mathbb{R}$
- posons $a = f(1) < 0$
- posons $b = f(10) > 0$.

et f continue sur $[1, 10]$, intervalle fermé

et par le TVI, f prend toutes les valeurs

entre $f(1)$ et $f(10)$, donc $\exists x_0$ t.q

$$f(x_0) = 0.$$

Exercice 8

$$x^3 + x - 1 = 0$$

on pose $f(x) = x^3 + x - 1$.

on pose $a = 1$, $f(1) = 1$

on pose $b = -1$, $f(-1) = -1$

$$u = \frac{1-1}{2} = 0$$

$f(u) = -1$.

$b = u$.

$$u = \frac{1}{2}.$$

$$f(u) = \frac{1}{8} + \frac{4}{8} - \frac{8}{8} = \frac{-3}{8}.$$

$$b = \frac{-3}{8}$$

$$u = \frac{5}{8} \cdot \frac{1}{2} = \frac{5}{16}.$$

$$f(u) < 0.$$

$$b = \frac{5}{16}$$

$$u = \frac{5/16 + 1}{2} = \frac{21}{32}$$

$$f(u) < 0.$$

$$u = \frac{53}{64}$$

$$f(u) > 0$$

$$a = \frac{53}{64}$$

$$u = \frac{95}{128}$$

$$f(u) > 0$$

$$a = \frac{95}{128}$$

la sol° est entre $\left[\frac{95}{128} ; \frac{21}{32}\right]$.

Exercice 9

$$\boxed{f'(x) := \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Si f est paire :

$$f(x) = f(-x)$$

Soit $g(x) := -x$

$$f(g(x))' = f'(g(x)) \cdot g'(x)$$

$$= f'(-x) \cdot -1$$

$$= -f'(-x)$$

$\Leftrightarrow f'(x) = -f'(-x).$

<u>Si f est impaire :</u>

$$f(x) = -f(-x)$$

$$\Leftrightarrow f'(x) = -(-f'(-x))$$

$$\Leftrightarrow f'(x) = f'(-x)$$

<u>Si f périodique :</u>

$$f(x) = f(x+T)$$

$$\Leftrightarrow f'(x) = f'(x+T)$$

Exercice 10

$$f'(x) = \lim_{h \to 0} \frac{2\sin(2x+2h) - \sin(2x)}{2 \cdot h}$$

$h' = 2h$

$$= 2 \lim_{h' \to 0} \frac{\sin(2x+h') - \sin(2x)}{h'}$$

$h' = h$

$$= 2 \lim_{h \to 0} \frac{\sin(2x)\cos(h) + \cos(2x)\sin(h) - \sin(2x)}{h}$$

$$= 2\sin(2x) \lim_{h \to 0} \frac{(\cos(h) - 1)}{h}$$

$$+ 2\cos(2x) \lim_{h \to 0} \frac{\sin(h)}{h}$$

$$= 2\cos(2x).$$

Exercice 11

$\boxed{1}$ Faux

$$f(x) = \begin{cases} x^2, & x \in \mathbb{Q} \\ 0, & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$

$f'(0) = 0$

$\lim \dfrac{f(x) - 0}{x - 0} = 0$ (à montrer)

$\boxed{2}$ Faux. ex. $|x|$

$\boxed{3}$ Faux. $\sqrt{x^2}$ en prendre la valeur abs

$\boxed{4}$ $f(x) = x^2 - 2x$

$$f'(x) = 2x-2$$

$$f'(1) = 2-2 = 0$$

$$(f\circ f)'(1) = f'(f(1)) \cdot f'(1) = 0$$

Exercice 1

(a)

$\alpha = 1$

$\beta = \dfrac{1}{2}$

$$\lim_{x \to 3^-} \frac{1}{2}x - 4 = \frac{3}{2} - 4$$

$$= \frac{3}{2} - \frac{8}{2}$$

$$= \frac{-5}{2}$$

(b) $\alpha = 1$

$\beta = \frac{5}{3}$

$$\lim_{x \to 3^-} \frac{5}{3}x - 4 = 5 - 4 = 1^-$$

$$\lim_{x \to 3^+} \frac{3x^2 - 10x + 3}{x^2 - 2x - 3}$$

$$= \lim_{x \to 3^+} \frac{\cancel{(x-3)}(3x-1)}{\cancel{(x-3)}(x+1)}$$

$$= \lim_{x \to 3^+} \frac{8}{4} = 2.$$

FAUX

(c) $\alpha = 2$

$$\lim_{x \to 3^-} \frac{5}{3} \cdot x - 4 = 1$$

(d)

(e) $\alpha = 2$

$$\lim_{x \to 3^-} 2 \cdot x - 4 = 2$$

$\boxed{\text{Ok}}$

Exercice 2

$$f(x) = \begin{cases} x^2 - x + 3 , & x \leq 1 \\ \alpha x + \beta , & x > 1 \end{cases}$$

$$\lim_{x \to 1^+} x^2 - x + 3 = 3.$$

$$\lim_{x \to 1^-} \alpha x + \beta$$

$$= \alpha + \beta$$

$$\alpha + \beta = 3$$

$$\Leftrightarrow \alpha = 3 - \beta , \quad \beta \in \mathbb{R}.$$

$$f'(x) = \begin{cases} 2x-1 & , x \leq 1 \\ \alpha & , x > 1 \end{cases}$$

$$f'(1^-) = f'(1^+) = 1$$

$$\Leftrightarrow \underline{\alpha = 1}$$

$$\beta = 3 - 1 = 2.$$

Exercice 3

ⓐ f'

$$= \frac{u'v - uv'}{v^2}$$

$$f'(x) = \frac{5(3x^2-1) - (5x+2)(3\cdot 2x)}{(3x^2-1)^2}$$

$$= \frac{15x^2 - 5 - (30x^2 + 12x)}{(3x^2-1)^2}$$

$$= \frac{-15x^2 - 12x - 5}{(3x^2-1)^2}$$

(b) $f' = \dfrac{u'v - uv'}{v^2}$ $\qquad \boxed{(\sqrt{u})'} = \dfrac{u'}{2\sqrt{u}}$

$$v'(x) = \frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}$$

$$f'(x) = \frac{(2x)(\sqrt{1-x^2}) - (x^2)\left(\frac{-x}{\sqrt{1-x^2}}\right)}{(1-x^2)}$$

$$= \frac{(2x)(\sqrt{1-x^2}) + x^3(1-x^2)^{-1/2}}{(1-x^2)}$$

$$= \frac{2x(1-x^2)^{1/2} + x^3(1-x^2)^{-1/2}}{(1-x^2)^1}$$

$$= 2x(1-x^2)^{-1/2} + x^3(1-x^2)^{-3/2}$$

(c) $f' = u'v + uv'$

$u(x) = \sin(x)^2$

$u'(x) = 2\sin(x) \cdot (\sin(x)')$

$= 2\sin(x)\cos(x)$

(d) $f'(x) = \left(\dfrac{\sin(x)}{\cos(x)}\right)'$

$= \dfrac{\cos(x)\cos(x) + \sin(x)\sin(x)}{(\cos(x))^2}$

$= \dfrac{1}{\cos^2(x)}$

(e) $f(x) = \sin\left(\sqrt{\sin(x)}\right)^{1/2}$

$= \sin\left(\sin(x)^{1/2}\right)^{1/2}$

$$\frac{\left(\sin\left(\sqrt{\sin(x)}\right)\right)'}{2\sqrt{\sin\left(\sqrt{\sin(x)}\right)}}$$

$$= \frac{\cos\left(\sqrt{\sin(x)}\right) \cdot \left(\sqrt{\sin(x)}\right)'}{2\sqrt{\sin\left(\sqrt{\sin(x)}\right)}}$$

$$= \frac{\cos\left(\sqrt{\sin(x)}\right)}{2\sqrt{\sin\left(\sqrt{\sin(x)}\right)}} \cdot \frac{(\sin(x))'}{2\sqrt{\sin(x)}}$$

$$= \frac{\cos\left(\sqrt{\sin(x)}\right)}{2\sqrt{\sin\left(\sqrt{\sin(x)}\right)}} \cdot \frac{\cos(x)}{2\sqrt{\sin(x)}}$$

$$= \frac{\cos\left(\sqrt{\sin(x)}\right) \cdot \cos(x)}{2 - \sqrt{\sin\left(\sqrt{\sin(x)}\right) \cdot \sin(x)}}$$

f) $\left(\left(2x^4 + e^{-(4x+3)}\right)^3\right)^{1/5}$

$$= \left(2x^4 + e^{-4x-3}\right)^{3/5}$$

$f'(x)$

$$= \frac{3}{5}\left(2x^4 + e^{-4x-3}\right)^{-2/5}$$

$$= \frac{3}{5}\left(2x^4 + e^{-4x-3}\right)^{-2/5} \cdot \left(2x^4 + e^{-4x-3}\right)'$$

$$= \frac{3}{5}\left(2x^4 + e^{-4x-3}\right)^{-2/5} \cdot \left(8x^3 + -4e^{-4x-3}\right)$$

(g) $\log_a(x)' = \dfrac{1}{x\ln(a)}$

$$\cosh(x) = \frac{e^{x} - e^{-x}}{2}$$

$$\cosh'(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right) = \sinh(x)$$

$$f'(x) = \frac{1}{x\ln(3)} \cdot \cosh(x)'$$

$$= \frac{1}{x\ln(3)} \cdot \sinh(x)$$

4

$$\ln\left(4^{\sin(x)}\right)$$

$$= \ln\left(e^{\sin(x)\ln(4)}\right)$$

$$\ln\left(4^{\sin(x)}\right)$$

$$= \sin(x)\ln(4)$$

$$f'(x) = \ln(4)\cdot\cos(x)\, e^{\cos(4x)}$$

$$+ \sin(x)\ln(4)\left(-4\sin(4x)\right) e^{\cos(4x)}$$

$$= e^{\cos(4x)}\ln(4)\left(\cos(x) + \sin(x)\left(-4\sin(4x)\right)\right)$$

Exercice 4

ⓐ $f^{(n)} = m \cdot (m-1) \cdots (m-n+1) x^{m-n}$

Si $m = 0$, $f^{(n)} = 0$

ⓑ $f'(x)$

$= 2\cos(2x) - 2\sin(x)$

$$\boxed{\begin{array}{l} 4k = 2^n \sin(2x) + 2\cos(x) \\ 4k+1 = 2^n \cos(2x) - 2\sin(x) \\ 4k+2 = -2^n \sin(2x) - 2\cos(x) \\ 4k+3 = -2^n \cos(2x) + 2\sin(x) \end{array}}$$

(c) $f'(x) = \frac{1}{x}$

$$f''(x) = \frac{-1}{x^2}$$

$$f'''(x) = \left(\frac{-1}{x} \cdot \frac{-1}{x}\right)'$$

$$= \frac{1}{x^2} \cdot \frac{-1}{x} + \frac{-1}{x} \cdot \frac{1}{x^2}$$

$$= 2\left(\frac{-1}{x^3}\right)$$

$$f''''(x) = -6x^{-4}$$

$$(n-2)(n-1) \cdot (-1)^{n+1} \cdot \left(\frac{1}{x^n}\right)$$

$$f^{(1)} = \frac{1}{x}$$

$$f'(1) = -\frac{1}{x^2}$$

Exercice 5

(a)

$$f(x) = 2x + 3 + (e^x - 1)\sin(x)^7 \cos(x)^4$$

$$g(x) = \ln(x)^3 \Rightarrow g'(x) = \frac{3\ln^2(x)}{x}$$

$$z(x) = x^3$$

$$(z \circ g)' = z'(g(x)) \cdot g'(x)$$

$$= 3(\ln(x)^2) \cdot \frac{1}{x}$$

$$= \frac{3\ln^2(x)}{x}$$

$$(g \circ f)'(0) = g'(f(0)) \cdot f'(0)$$

$$f'(0) = 3$$

$$g'(3) = \frac{\cancel{3}\ln^2(3)}{\cancel{3}} = \ln^2(3)$$

$$g'(f(0)) \cdot f'(0) = \ln^2(3) \cdot 2$$

$$= 2\ln^2(3).$$

$$(g \circ f)'(0) = g'(f(0)) \cdot f'(0)$$

$$g'(x) = 4(x-1)^3$$

$$f(0) = 0$$

$$g'(0) = 4(-1)^3 = \underline{-4}.$$

$$f'(0)$$

$$= \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$= \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \to 0} \frac{f(x)}{x}$$

$$= \lim_{x \to 0} \frac{\cancel{x}^2 \sin\left(\frac{1}{x}\right) + 2x}{\cancel{x}}$$

$$= \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) + 2$$

$$= \underline{2}.$$