

$$E_{\text{kin}} = \sum_{\alpha} \frac{1}{2} m_{\alpha} \vec{v}_{\alpha}^2$$

$$= \frac{1}{2} \sum_{\alpha} m_{\alpha} (\vec{v}_A + \vec{\omega} \wedge \vec{AP}_{\alpha})^2$$

car schale modell

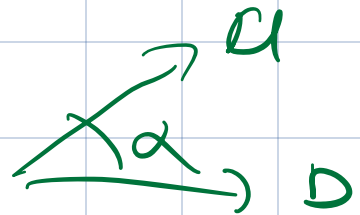
$$= \frac{1}{2} M (\vec{v}_A)^2 + \sum_{\alpha} m_{\alpha} (\vec{v}_A \cdot (\vec{\omega} \wedge \vec{AP}_{\alpha})) + \frac{1}{2} \sum_{\alpha} m_{\alpha} (\vec{\omega} \wedge \vec{AP}_{\alpha})^2$$

$$= M \vec{v}_A \cdot (\vec{\omega} \wedge \vec{AG})$$

$$(\vec{a} \wedge \vec{b}) = a^2 b^2 \sin^2 \alpha$$

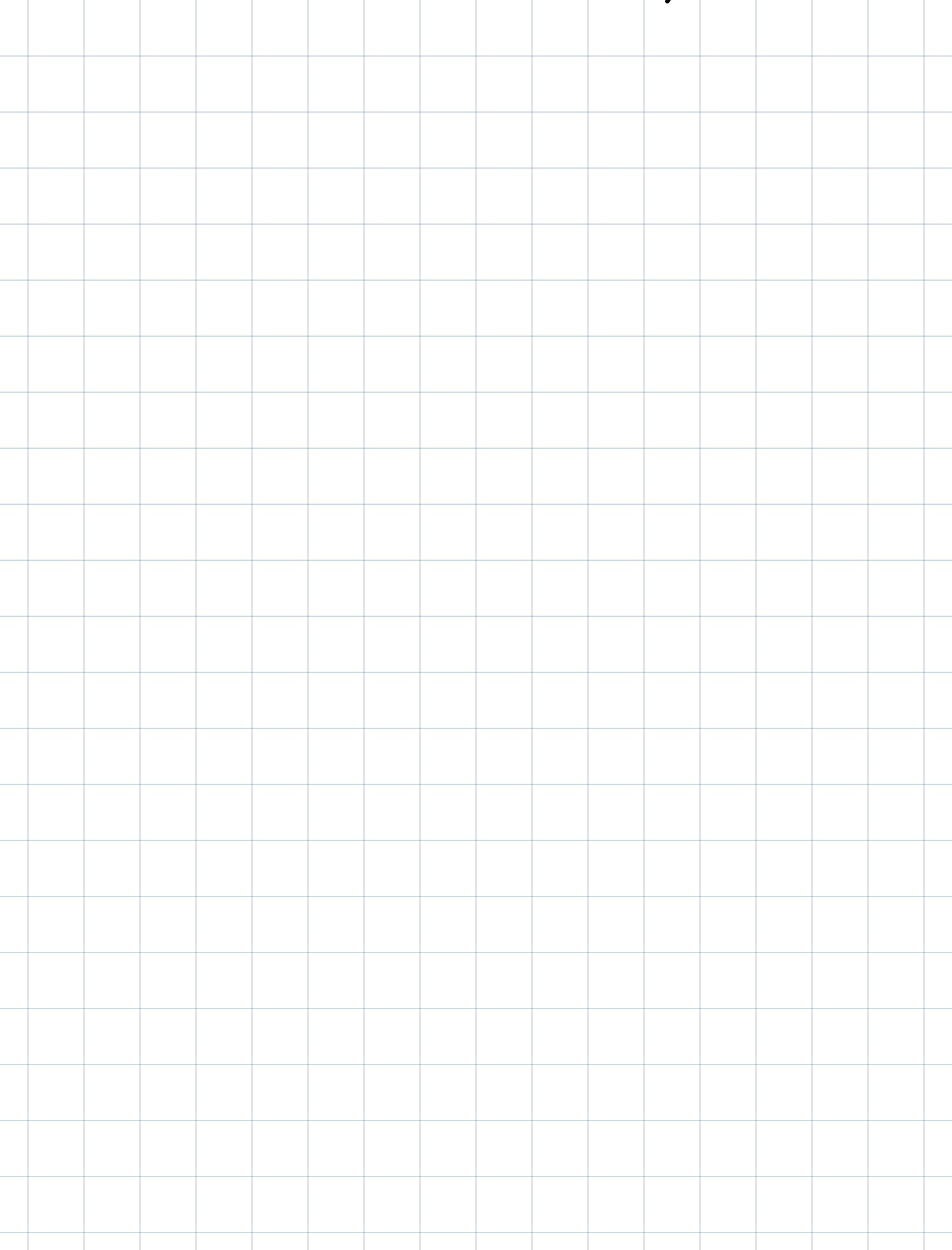
$$(\vec{a} \cdot \vec{b}) = a^2 b^2 \cos^2 \alpha$$

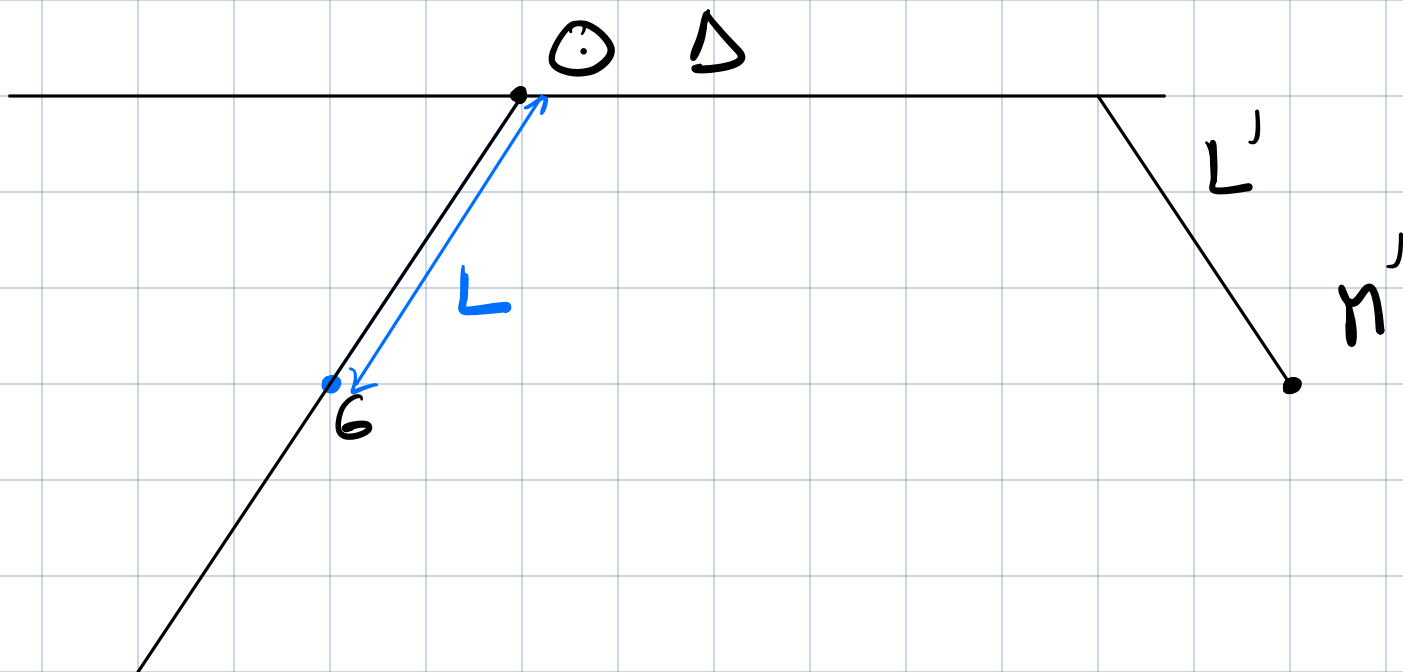
$$\Rightarrow (\vec{a} \wedge \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = a^2 b^2$$



$$\begin{aligned}
\text{or } & \frac{1}{2} \sum_{\alpha} m_{\alpha} (\vec{\omega} \wedge \vec{AP}_{\alpha})^2 \\
&= \frac{1}{2} \sum_{\alpha} m_{\alpha} \left( \vec{\omega}^2 (\vec{AP}_{\alpha})^2 - (\vec{\omega} \cdot \vec{AP}_{\alpha})^2 \right) \\
&\quad \text{(on axis } \omega_x^2 + \omega_y^2 + \omega_z^2) \\
&= \frac{1}{2} \sum_{\alpha} m_{\alpha} \left( \sum_{i,j} \omega_i \omega_j \delta_{ij} (\vec{AP}_{\alpha})^2 \right. \\
&\quad \left. + \sum_{i,j} \omega_i (\vec{AP}_{\alpha})_i \omega_j (\vec{AP}_{\alpha})_j \right) \\
&= \frac{1}{2} \sum_{i,j} \omega_i \omega_j \sum_{\alpha} m_{\alpha} \left( (\vec{AP}_{\alpha})^2 \delta_{ij} \right. \\
&\quad \left. - (\vec{AP}_{\alpha})_i (\vec{AP}_{\alpha})_j \right) \\
&= \frac{1}{2} \sum_{i,j} \omega_i \omega_j (\tilde{I}_A)_{ij} \\
&= \frac{1}{2} \vec{\omega} \cdot (\tilde{I}_A \vec{\omega})
\end{aligned}$$

$$\begin{aligned}
\Rightarrow E_{\text{cm}} &= \frac{1}{2} M \vec{V}_A^2 + M \vec{V}_A \cdot (\vec{\omega}_A \wedge \vec{AG}) \\
&\quad + \frac{1}{2} \vec{\omega} \cdot (\tilde{I}_A \vec{\omega})
\end{aligned}$$





$$I_{G,\Delta} = \frac{1}{12} m (2L)^2$$

que doit valoir  $L'$  pour que les périodes  
soient égales.

$$T = T'$$

$$\Leftrightarrow \omega = \omega'$$

$$\Leftrightarrow \omega^2 = (\omega')^2$$

$$\left. \begin{aligned} \omega^2 &= \frac{L M g}{I_D} \\ (\omega')^2 &= \frac{g}{L'} \end{aligned} \right\} \Rightarrow \frac{L M g}{I_{G,D} + M L^2} = \frac{g}{L'}$$

$$\Rightarrow \frac{L M g}{\underbrace{\frac{1}{12} M 4 L^2 + M L^2}_{\frac{4}{3} M L^2}} = \frac{g}{L'}$$

$$\Rightarrow \frac{L M}{\frac{4}{3} M L^2} = \frac{1}{L'}$$

$$\Rightarrow L' = \frac{4}{3} L$$