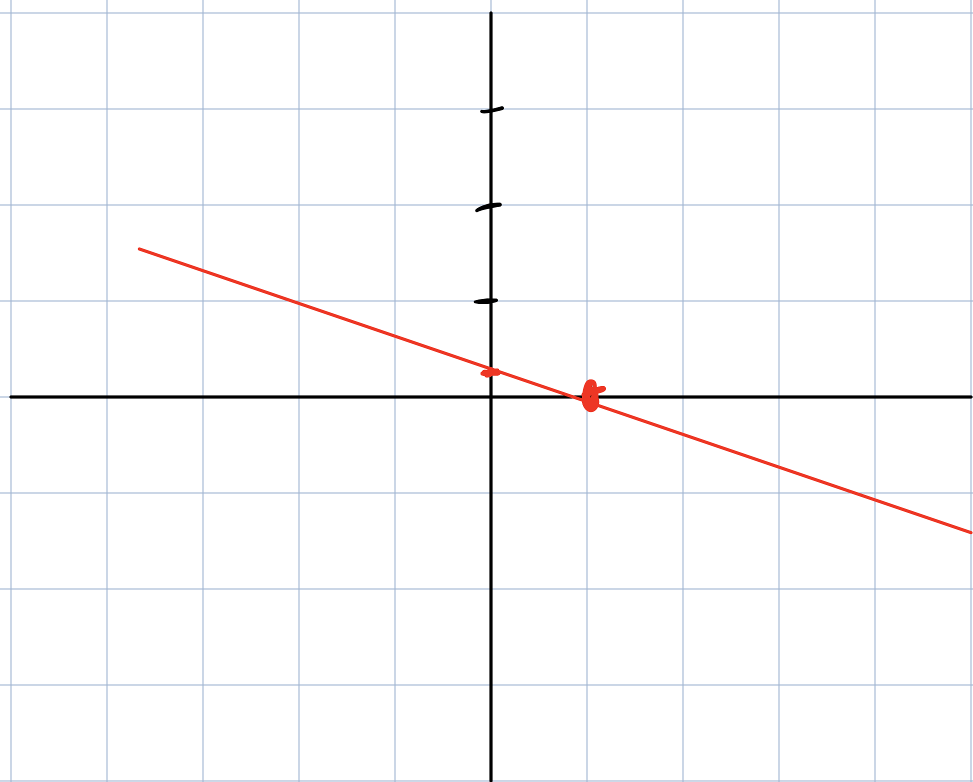
Ex 1

1. N
2. Y
3. Y
4. N
5. Y

Ex 2

a)



$(1;0)$
$(0;\frac{1}{3})$

(b) $\alpha x + \beta y = 1 \quad (1)$

- $\left(0 ; \frac{1}{\beta}\right)$ $\quad \vec{v}_1 \begin{pmatrix} 1/\alpha \\ -1/\beta \end{pmatrix}$
- $\left(\frac{1}{\alpha} ; 0\right)$

$-x + y = -1$

- $(0 ; -1)$ $\quad$ • $\vec{v}_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
- $(1 ; 0)$

$\vec{v}_1 = k \vec{v}_2$

$1 = k \times \frac{1}{\alpha} \iff \alpha = k$

$$1 = k \times \frac{-1}{\beta} \quad \text{et} \quad \beta = -k$$

$$\text{et} \quad \alpha = -\beta$$

c

$(0; -1)$

$$\begin{cases} -x_1 + x_2 = -1 \\ \alpha x_1 + \beta x_2 = 1 \\ \cancel{(\alpha - 1) x_1 + (\beta + 1) x_2 = 0} \end{cases}$$

$0 \qquad \beta - 1$

1 $\begin{cases} \alpha = 1 \\ \beta = -1 \end{cases}$

comme ça $(0; -1)$ appartient aux 2 droites et $\alpha = -\beta$

2 $\begin{cases} \alpha = 0 \\ \beta = 0 \end{cases}$

③

$$\begin{cases} -x_1 + x_2 = -1 \\ \alpha x_1 - \alpha x_2 = 1 \end{cases}$$

droites sécantes
$$\left| \begin{array}{l} -x_1 + x_2 = -1 \quad (0, -1) \\ \alpha x_1 + \beta x_2 = 1 \end{array} \right.$$

$$\alpha\, 0 + \beta - 1 = 1$$

$$\alpha = 6 \quad \overset{=}{-1}$$

Exercice 3

(i) (a)

$$\left(\begin{array}{cc|c} 1 & -2 & -1 \\ -1 & 3 & 3 \end{array}\right)$$

(b)

$$\left(\begin{array}{ccc|c} 3 & 2 & -1 & 12 \\ 2 & -4 & 1 & -1 \\ -4 & 1 & 2 & -8 \end{array}\right)$$

(c)

$$\left(\begin{array}{ccc|c} 6 & -3 & 2 & 11 \\ -3 & 2 & -1 & -4 \\ 5 & -3 & 2 & 9 \end{array}\right)$$

d)

$$\left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{array}\right)$$

Ex 2 (c)

$$\left(\begin{array}{cc|c} -1 & 1 & -1 \\ \alpha & \beta & 1 \\ (\alpha-1) & (\beta+1) & 0 \end{array}\right)$$

$$\Leftrightarrow \left(\begin{array}{cc|c} -1 & 1 & -1 \\ \alpha & \beta & 1 \\ -1 & 1 & -1 \end{array}\right) \mathrm{II} + \alpha(\mathrm{III})$$

$$\Leftrightarrow \left(\begin{array}{cc|c} -1 & 1 & -1 \\ 0 & \beta+\alpha & 1-\alpha \\ 0 & 0 & 0 \end{array}\right)$$

H1 $\beta + \alpha = 0$

donc $1 - \alpha = 0$

et $-x_1 + x_2 = -1$

H2 $\beta + \alpha \neq 0$

donc ça donne 1 sol unique

H3 $\beta + \alpha = 0$

et $1 - \alpha = 0$

donc imp. $S = \emptyset$

$$\begin{cases} \beta + \alpha = 0 \\ 1 - \alpha = 0 \end{cases}$$

$$\Leftrightarrow \beta = -\alpha$$

$$\text{et } \alpha = -1$$

$$\Leftrightarrow \beta = 1$$

a)

$$\left(\begin{array}{cc|c} 1 & -2 & -1 \\ -1 & 3 & 3 \end{array}\right)$$

$II \leftarrow II + I$

$$= \left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 1 & 2 \end{array}\right)$$

$I \leftarrow I + 2(II)$

$$= \left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right)$$

$S = \{(3;2)\}$

(b)

$$\left(\begin{array}{ccc|c} 3 & 2 & -1 & 12 \\ 2 & -4 & 1 & -1 \\ -4 & 1 & 2 & -8 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 3 & 2 & -1 & 12 \\ 2 & -4 & 1 & -1 \\ 0 & -7 & 4 & -10 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 6 & 4 & -2 & 24 \\ 6 & -12 & 3 & -3 \\ 0 & -7 & 4 & -10 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 6 & 4 & -2 & 24 \\ 0 & -16 & 5 & -27 \\ 0 & -7 & 4 & -10 \end{array}\right)$$

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$+ \frac{1}{4}$ (II)

$$= \left(\begin{array}{ccc|c} 6 & 0 & -2+\frac{5}{4} & 24+\frac{1}{4}(-27) \\ 0 & -16 & 5 & -27 \\ 0 & -7 & 4 & -10 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 6 & 0 & -3/4 & \overset{\frac{69}{4}}{\cancel{\frac{96}{4}+\frac{1}{4}(-27)}} \\ 0 & -16 & 5 & -27 \\ 0 & -7 & 4 & -10 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 24 & 0 & -3 & 69 \\ 0 & -16 & 5 & -27 \\ 0 & -7 & 4 & -10 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 24 & 0 & -3 & 69 \\ 0 & -64 & 20 & -108 \\ 0 & -35 & 20 & -50 \end{array}\right)$$

-27×4

$= 120 - 12$

$= 108$

$$= \left(\begin{array}{ccc|c} 24 & 0 & -3 & 69 \\ 0 & -29 & 0 & -58 \\ 0 & -35 & 20 & -50 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & \underline{1} \end{array}\right)$$

$$-35(2) + 20x_3 = -50$$

$$\Leftrightarrow \quad x_3 = \frac{-50 + 35(2)}{20}$$

$$= 1$$

$$24x_1 - 3 = 69$$

$$\Leftrightarrow \quad x_1 = \frac{69+3}{24}$$

$$= 3$$

c

$$\left(\begin{array}{ccc|c} 6 & -3 & 2 & 11 \\ -3 & 2 & -1 & -4 \\ 5 & -3 & 2 & 9 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ -15 & 10 & -5 & -20 \\ 15 & -9 & 6 & 27 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 7 \\ 15 & -9 & 6 & 27 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 15 & -9 & 6 & 27 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 15 & -9 & 0 & 3 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 15 & 0 & 0 & 30 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 2 \end{array}\right)$$

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d) $\left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{array}\right)$

$$= \left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 1 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 0 & 7 & 7 \\ 0 & 1 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \end{array}\right)$$

Exercice 4

a $\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$ T.

$$= \begin{pmatrix} 0 & 0 \\ 1 & 2 \end{pmatrix}$$

(b) F. multiplier par $k \neq 0$

(c) T.

Exercice 5

$$\begin{cases} a_{1,1}x_1, \dots, a_{1,n}x_n = b_1 \\ a_{m,1}x_1, \dots, a_{m,n}x_n = b_m \end{cases}$$

Exercice 6

(a)

$$\left(\begin{array}{cccc|c} 3 & 1 & -2 & 4 & 3 \\ 1 & 2 & -1 & 7 & 4 \\ 2 & 2 & 1 & 3 & 4 \\ 1 & -1 & -1 & 1 & 1 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 3 & 1 & -2 & 4 & 3 \\ 1 & 2 & -1 & 7 & 4 \\ 0 & 4 & 3 & 1 & 2 \\ 1 & -1 & -1 & 1 & 1 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 3 & 1 & -2 & 4 & 3 \\ 0 & 3 & 0 & 6 & 3 \\ 0 & 4 & 3 & 1 & 2 \\ 1 & -1 & -1 & 1 & 1 \end{array}\right)$$

$$\frac{6 \times 4}{3}$$

$$= \left(\begin{array}{cccc|c} 3 & 1 & -2 & 4 & 3 \\ 0 & 4 & 0 & 8 & 4 \\ 0 & 4 & 3 & 1 & 2 \\ 1 & -1 & -1 & 1 & 1 \end{array}\right) \qquad = 2 \qquad \frac{3 \times 4}{3}$$

$$= \left(\begin{array}{cccc|c} 3 & 1 & -2 & 4 & 3 \\ 0 & 0 & -3 & 7 & 2 \\ 0 & 4 & 3 & 1 & 2 \\ 1 & -1 & -1 & 1 & 1 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & 4 & 1 & 1 & 0 \\ 0 & 0 & -3 & 7 & 2 \\ 0 & 4 & 3 & 1 & 2 \\ 1 & -1 & -1 & 1 & 1 \\ -3 & 3 & 3 & -3 & -3 \end{array}\right)$$

$$=$$

$$\left(\begin{array}{cccc|c} 0 & 4 & 1 & 1 & 0 \\ 0 & 0 & -3 & 7 & 2 \\ 0 & 0 & 2 & 0 & 2 \\ 1 & -1 & -1 & 1 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 0 & 4 & 1 & 1 & 0 \\ 0 & 0 & 0 & 7 & 5 \\ 0 & 0 & 1_3 & 0 & 1_3 \\ 1 & -1 & -1 & 1 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 0 & 4 & 0 & 1 & -1 \\ 0 & 0 & 0 & 7 & 5 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & -1 & 1 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 0 & 4 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 5/7 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & 2 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 0 & 4 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 5/7 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 9/7 \end{array}\right) \quad \begin{array}{l} 2-5/7 \\ = 9/7 \end{array}$$

$$\left(\begin{array}{cccc|c} 0 & 4 & 0 & 0 & -1 - 5/7 = -\frac{12}{7} \\ 0 & 0 & 0 & 1 & 5/7 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 9/7 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 0 & 1 & 0 & 0 & -\frac{3}{7} \\ 0 & 0 & 0 & 1 & 5/7 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 9/7 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 0 & 1 & 0 & 0 & -\frac{3}{7} \\ 0 & 0 & 0 & 1 & 5/7 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & \frac{6}{7} \end{array}\right)$$

Exercice 8

$$\left(\begin{array}{cccc|c} 2 & 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 & 3 \\ 1 & 3 & 2 & 1 & 5 \\ 3 & 4 & 3 & 3 & 7 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 2 & 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 & 3 \\ 0 & 5 & 3 & 2 & 2 \\ 3 & 4 & 3 & 3 & 7 \end{array}\right)$$

$$\begin{array}{ccccc} & 5 & 5/3 & 10/3 & 10/3 \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 0 & 3\left(\times \frac{5}{3}\right) & 1 & 2 & 2 \\ 1 & 2 & 1 & 1 & 3 \end{array}\right.$$

$$\left(\begin{array}{cccc|c} 0 & 5 & 3 & 2 & 2 \\ 3 & 4 & 3 & 3 & 7 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 0 & 15 & 5 & 10 & 10 \\ 1 & 2 & 1 & 1 & 3 \\ 0 & 0 & -1 & 1 & 1 \\ 3 & 4 & 3 & 3 & 7 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & 15 & 5 & 10 & 10 \\ 1 & 2 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 3 & 4 & 3 & 3 & 7 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & 15 & 5 & 10 & 10 \\ 1 & 2 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & 2 & 2/3 & 20/15 & 20/15 \\ 1 & 2 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & -1/2 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & 2 & 2/3 & 20/15 & 20/15 \\ 1 & 0 & 1/3 & -5/15 & 28/15 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & -1/2 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & 0 & 2/3 & 20/15 & 35/15 \\ 1 & 0 & 1/3 & -5/15 & 25/15 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & -1/2 \end{array}\right.$$

Exercice 2

$$\left(\begin{array}{cc|c} -1 & 1 & -1 \\ \alpha & \beta & 1 \\ (\alpha-1) & (\beta+1) & 0 \end{array}\right)$$

$$= \left(\begin{array}{cc|c} -1 & 1 & -1 \\ \alpha & \beta & 1 \\ & & \end{array}\right)$$

$$= \left(\begin{array}{cc|c} -1 & 1 & -1 \\ 0 & \beta+\alpha & 1-\alpha \\ & & \end{array}\right)$$

$\hookrightarrow$ $\beta + \alpha = 0$ et $1-\alpha \neq 0$
pas de sol^{o}

$$\left(00 \mid c\right)$$

$\hookrightarrow$ $\beta + \alpha = 0$ et $1-\alpha = 0$
infinité sol^{o}

$\hookrightarrow$ $\beta + \alpha = k$ et $1-\alpha = k'$
une sol^{o}

7) e)

$$\left(\begin{array}{cccc|c} 3 & 1 & -2 & 4 & 3 \\ 1 & 2 & -1 & 7 & 4 \\ 2 & 2 & 1 & 3 & 4 \\ 1 & -1 & -1 & 1 & 1 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 3 & 1 & -2 & 4 & 3 \\ 1 & 2 & -1 & 7 & 4 \\ 0 & -2 & 3 & -11 & -4 \\ 1 & -1 & -1 & 1 & 1 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 3 & 1 & -2 & 4 & 3 \\ 1 & 2 & -1 & 7 & 4 \\ 0 & -2 & 3 & -11 & -4 \\ 0 & -3 & 2 & -6 & -3 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & -5 & 1 & -17 & -9 \\ 1 & 2 & -1 & 7 & 4 \\ 0 & -2 & 3 & -11 & -4 \\ 0 & -3 & 2 & -6 & -3 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & -10 & 2 & -34 & -18 \\ 1 & 2 & -1 & 7 & 4 \\ 0 & -10 & 15 & -55 & -20 \\ 0 & -3 & 2 & -6 & -3 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & -10 & 2 & -34 & -18 \\ 1 & 2 & -1 & 7 & 4 \\ 0 & 0 & 17 & -89 & -38 \\ 0 & -3 & 2 & -6 & -3 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & 3 & -6/10 & \frac{102}{10} & 54 \\ 1 & 2 & -1 & 7 & 4 \\ 0 & 0 & 17 & -89 & -38 \\ 0 & -3 & 2 & -6 & -3 \end{array}\right)$$

$$\frac{\times \quad \times 3}{10}$$

$$\frac{-30}{10} - \frac{6}{10}$$

$$= \left(\begin{array}{cccc|c} 0 & 3 & -6 & 102 & 540 \\ 1 & 2 & -1 & 7 & 4 \\ 0 & 0 & 17 & -89 & -38 \\ 0 & 0 & 14 & -66 & -36 \end{array}\right)$$

$$=$$

$$17x - 89y = -38$$

$$14x - 66y = -36$$

b

$$\left(\begin{array}{cccc|c} 4 & 4 & -3 & 3 & 0 \\ 3 & 2 & 1 & 2 & 0 \\ 2 & 0 & 5 & 1 & 0 \\ 1 & 2 & -4 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 4 & 4 & -3 & 3 & 0 \\ 3 & 2 & 1 & 2 & 0 \\ 2 & 0 & 5 & 1 & 0 \\ 2 & 4 & -8 & 2 & 0 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 4 & 4 & -3 & 3 & 0 \\ 3 & 2 & 1 & 2 & 0 \\ 2 & 0 & 5 & 1 & 0 \\ 0 & 4 & -13 & -1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & 4 & -13 & 1 & 0 \\ 3 & 2 & 1 & 2 & 0 \\ 2 & 0 & 5 & 1 & 0 \\ 0 & 4 & -13 & 1 & 0 \end{array}\right) -2(\mathrm{III})$$

$$= \left(\begin{array}{cccc|c} 0 & 4 & -13 & 1 & 0 \\ 6 & 4 & 2 & 4 & 0 \\ 6 & 0 & 15 & 3 & 0 \\ 0 & 4 & -13 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & 4 & -13 & 1 & 0 \\ 0 & 4 & -13 & 1 & 0 \\ 6 & 0 & 15 & 3 & 0 \\ 0 & 4 & -13 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & -13 & 1 & 0 \\ 6 & 0 & 15 & 3 & 0 \\ 0 & 4 & -13 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 0 & 4 & -13 & 1 & 0 \\ 6 & 0 & 15 & 3 & 0 \end{array}\right)$$

x_3, x_4 free

$$4x_2 - 13x_3 + x_4 = 0$$

$$6x_1 + 15x_3 + x_4 = 0$$

$$x_1 = \frac{5}{2}t - \frac{1}{2}u$$

$$x_2 = \frac{13x_3 - x_4}{a_4}$$

Ex 8

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & -2 & -1 & 3 \\ 1 & 3 & 2 & 5 \\ 3 & 4 & 3 & 7 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & -2 & -1 & 3 \\ 0 & 5 & 3 & 2 \\ 3 & 4 & 3 & 7 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 0 & 5 & 3 & -5 \\ 1 & -2 & -1 & 3 \\ 0 & 5 & 3 & 2 \\ 3 & 4 & 3 & 7 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 0 & 0 & 0 & -7 \\ & & & \\ & & & \\ & & & \end{array}\right)$$