Homework 7

Tanja käser

Week 7

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Exercise 1. The following three algorithms sort the input sequence $a_1, \ldots, a_n$ of real numbers in ascending order.

Algorithm 1 Bubble Sort

```
for i = 1 to n − 1 do
    for j = 1 to n − i do
        if a_j > a_{j+1} then
            swap a_j and a_{j+1}
```

Algorithm 2 Selection Sort

```
for i = 1 to n − 1 do
    min ← i + 1
    for j = i + 1 to n do
        if a_min > a_j then
            min ← j
    if a_i > a_min then
        swap a_i and a_min
```

Algorithm 3 Insertion Sort

```
for j = 2 to n do
    i ← 1
    while a_j > a_i do
        i ← i + 1
    m ← a_j
    for k = 0 to j − i − 1 do
        a_{j−k} ← a_{j−k−1}
    a_i ← m
```

Use Bubble Sort, Selection Sort and Insertion Sort to sort the following sequence:

`9, 12,-43, 20, -2, 3, 7, 28, 19.`

(1)

9, 12, -43, 20, -2, 3, 7, 28, 19

9, -43, 12, -2, 20, 3, 7, 19, 28

-43, 9, -2, 12, 3, 7, 19, 20, 28

-43, -2, 3, 7, 9, 12, 19, 20, 28

(2)

9, 12, -43, 20, -2, 3, 7, 28, 19
-43, 12, 9, 20, -2, 3, 7, 28, 19
-43, -2, 9, 20, 12, 3, 7, 28, 19
-43, -2, 3, 20, 12, 9, 7, 28, 19
-43, -2, 3, 7, 12, 9, 20, 28, 19
-43, -2, 3, 7, 9, 12, 20, 28, 19
-43, -2, 3, 7, 9, 12, 19, 28, 20
-43, -2, 3, 7, 9, 12, 19, 20, 28.

(3)

9, 12, -43, 20, -2, 3, 7, 28, 19
-43, 9, 12, 20, -2, 3, 7, 28, 19
-43, -2, 9, 12, 20, 3, 7, 28, 19
-43, -2, 3, 9, 12, 20, 7, 28, 19
-43, -2, 3, 7, 9, 12, 20, 28, 19
-43, -2, 3, 7, 9, 12, 19, 20, 28

Exercise 2. Adapt the bubble sort algorithm so that it stops when no interchanges are required. Express this more efficient version of the algorithm in pseudocode.

```
procedure bubble_sort(a_1, ..., a_n : list) {
    for i = 1 to n-1 do {
        has_changed = false
        for j = 1 to n-i {
            if (a_j > a_{j+1}) {
                swap a_j and a_{j+1}
                has changed = true
            }
        }
    }
    if (!has_changed) { exit(1) }
}
```

Exercise 5.

$$L_{x_1} = (y_3, y_1, y_2) \qquad L_{y_1} = (x_2, x_1, x_3)$$
$$L_{x_2} = (y_2, y_3, y_1) \qquad L_{y_2} = (x_1, x_3, x_2)$$
$$L_{x_3} = (y_1, y_2, y_3) \qquad L_{y_3} = (x_3, x_2, x_1)$$

Let L_x for $x \in X = \{x_1, x_2, x_3\}$ be the preference list of x as given above
and let L_y for $y \in Y = \{y_1, y_2, y_3\}$ be the preference list of y as given above.
We say that a matching is X-optimal (resp. Y-optimal) if all elements of X (resp. Y) are matched with their highest preference.
The matching $\{(x_1, y_1), (x_2, y_3), (x_3, y_2)\}$ is

- ○ unstable.
- ○ stable and Y-optimal.
- ○ stable and X-optimal.
- ☒ stable but not a stable matching that is X- or Y-optimal.

Let's see if X-optimal

(x_1, y_3)
(x_2, y_2)
(x_3, y_1)

↳ no

Y-optimal?

(y_1, x_2)
(y_2, x_1)
(y_3, x_3)

↳ no

Stable:

- from x_1: yes. $x_1 \to y_3$ but $y_3 \heartsuit x_2$.
- from y_1: yes. $y_1 \to x_2$ but $x_2 \heartsuit y_3$

- from x_2: yes. $x_2 \to y_2$ but $y_2 \not\heartsuit x_3$
- from y_3: yes. $y_3 \to x_3$ but $x_3 \not\heartsuit y_2$

- from x_3: yes. $x_3 \to y_1$ but $y_1 \not\heartsuit x_1$
- from y_2: yes. $y_2 \to x_1$ but $x_1 \not\heartsuit y_1$

Exercise 7. Let $\{A, B, C, D\}$ be a set of men, and $\{a, b, c, d\}$ a set of women. We want to match up men and women using the Gale-Shapley algorithm in two different ways. The preferences of men and women are given in the following lists, going from most preferable on the left to least preferable on the right.

Men	1st	2nd	3rd	4th
A	c	d	b	a
B	d	c	a	b
C	a	c	b	d
D	b	d	a	c

Women	1st	2nd	3rd	4th
a	D	A	B	C
b	C	B	A	D
c	C	B	A	D
d	D	A	B	C

1. If the men propose, and women accept/reject, what is the matching after the algorithm terminates?
2. If the women propose, and men accept/reject, what is the matching after the algorithm terminates?
3. Who is the best possible (stable) valid partner for "a"?

1. (A, c)
(B, d)
(C, a)
(D, b)

2. ~~(a, D)~~
~~(b, c)~~
(c, C)
(d, D)
(a, A)
(b, B)

3. Women-optimal matching ⇒ Therefore a prefers A.