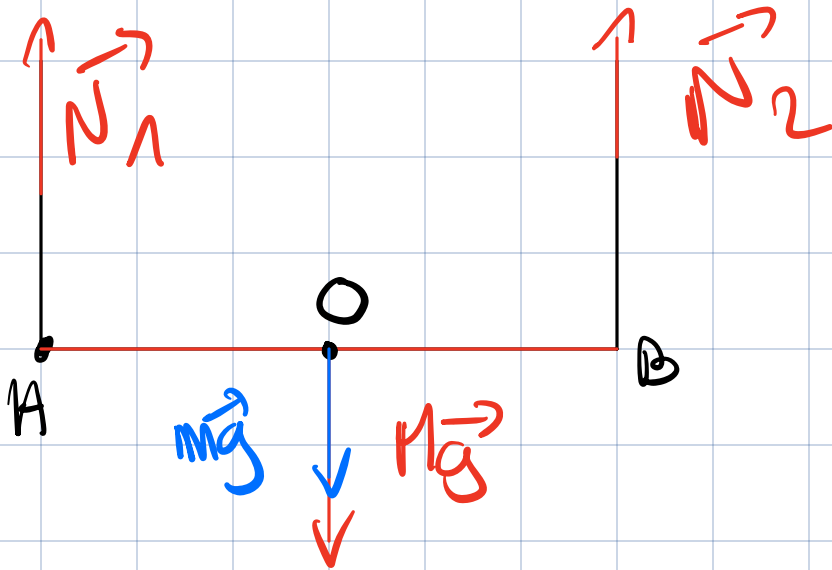


600 cm/s

1200 eV



① Se torções

$$\vec{L}_O = \vec{r} \wedge m\vec{v} = \vec{0}$$

$$\Rightarrow \vec{\Pi}_O = \vec{0}$$

② Se torções

$$\sum \vec{F} = \vec{0} = m\vec{a}$$

$$\vec{\Pi}_O(m\vec{g}) = \vec{OO} \wedge m\vec{g} = \vec{0}$$

$$\vec{\Pi}_O(M\vec{g}) = \vec{OO} \wedge M\vec{g} = \vec{0}$$

$$\vec{M}_0(\vec{N}_1) = \vec{OA} \wedge \vec{N}_1 = -OA \cdot N_1 \cdot \vec{e}_3$$

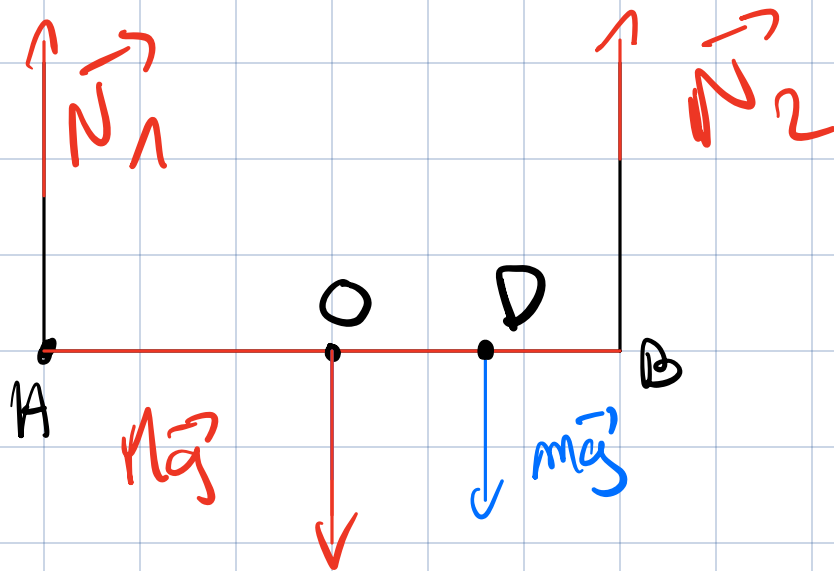
$$\vec{M}_0(\vec{N}_2) = \vec{OB} \wedge \vec{N}_2 = OB \cdot N_2 \cdot \vec{e}_3$$

$$\vec{OA} \wedge \vec{N}_1 + \vec{OB} \wedge \vec{N}_2 = \vec{0}$$

$$\vec{OB} = -\vec{OA}$$

$$\vec{OA} \wedge \vec{N}_1 - \vec{OA} \wedge \vec{N}_2 = \vec{0}$$

$$\cancel{\vec{OA}} \wedge \vec{N}_1 = \cancel{\vec{OA}} \wedge \vec{N}_2 \quad \underline{N_1 = N_2}$$

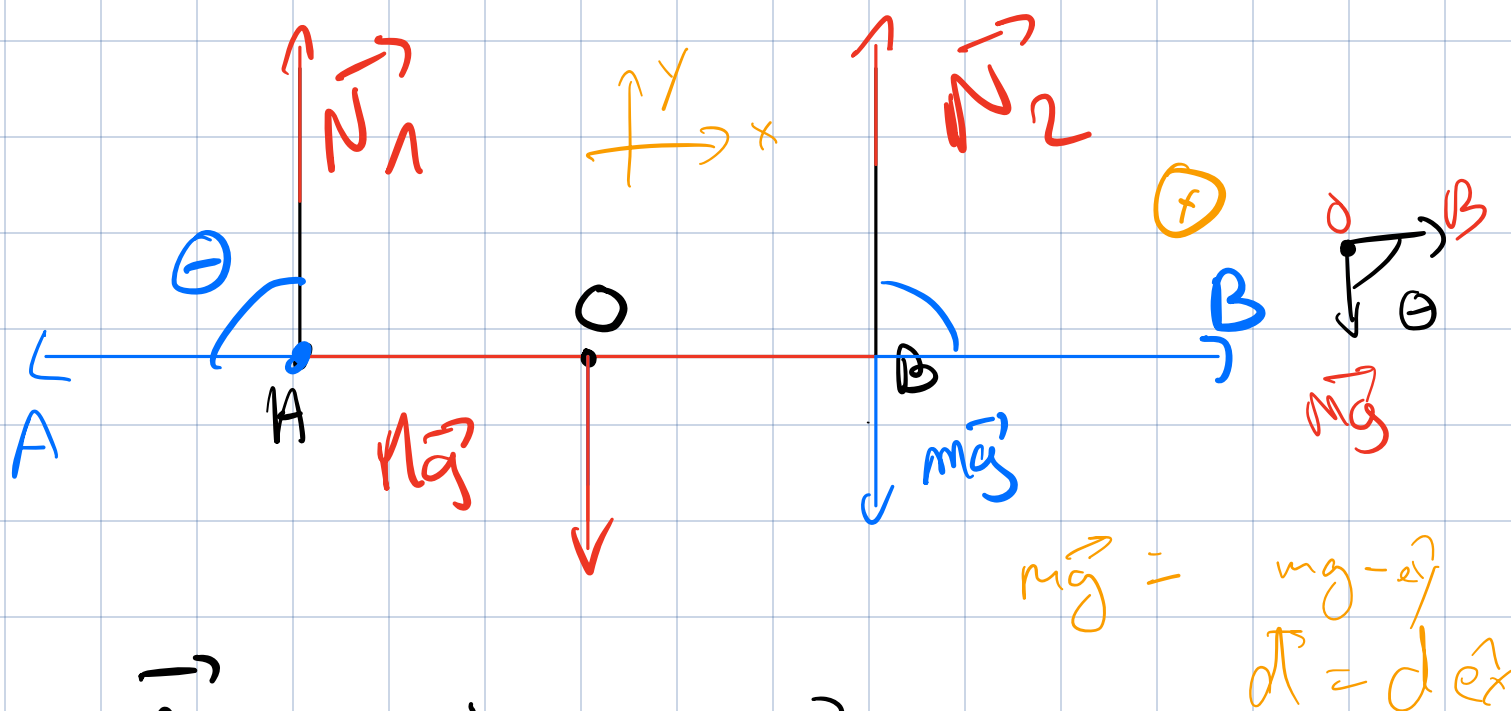


$$M_O(Mg) = 0$$

$$M_O(mg) = OD \wedge mg$$

$$= OD \cdot mg$$

$$N_1 + N_2 = Mg + mg = 1200N$$



$$\vec{M}_O(m\vec{g}) = \vec{OB} \wedge m\vec{g}$$

$$= OB \cdot mg \cdot \hat{e}_z$$

$$\vec{M}_O(\vec{N}_1) = \vec{OA} \wedge \vec{N}_1$$

$$= OA \cdot N_1 \cdot \hat{e}_z$$

$$\vec{M}_O(\vec{N}_2) = \vec{OB} \wedge \vec{N}_2$$

$$= OB \cdot N_2 \cdot \hat{e}_z$$

$$OB \cdot mg + OB \cdot N_2 - OA \cdot N_1 = 0$$

$$\Leftrightarrow (OA)(mg + N_2 - N_1) = 0.$$

$$\Leftrightarrow \underline{mg = N_1 - N_2.}$$

① Voyage vers Mars

$$a) \quad k = \frac{T_M^2}{(2R_M)^3} = \frac{T_T^2}{(2R_T)^3}$$

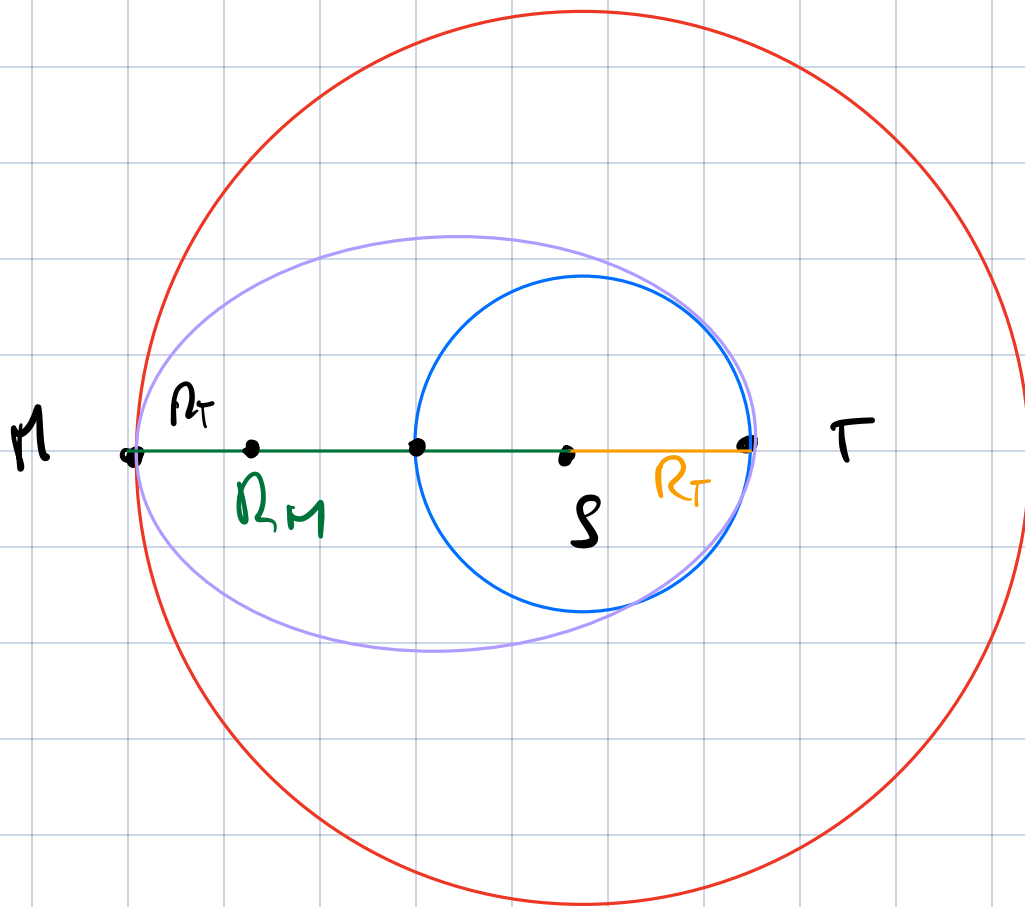
$$\Leftrightarrow T_M = \sqrt{\frac{(2R_M)^3}{(2R_T)^3} \cdot T_T^2}$$
$$\approx 684 \text{ jours}$$

$$b) \quad S_M = 2\pi \cdot R_M$$

$$V_M = \frac{2\pi \cdot R_M}{T_M}$$

$$c) \quad \vec{M}_{SM} = \vec{r}_M \wedge \vec{F} = \vec{0}.$$

$$\vec{L}_{SM} = \vec{0}.$$



$$\frac{R_T + R_M}{2} = a.$$

$$\frac{T_V^2}{(2a)^3} = \frac{T_M^2}{(2R_M)^3}$$

$$\Leftrightarrow T_V^2 = \sqrt{\frac{T_M^2}{(2R_M)^3} \cdot (2a)^3}$$

$$\textcircled{d} \quad E_m = K + V(r)$$

$$\sum \vec{F} = \vec{F}_G = -G \cdot \frac{M_s \cdot m}{R^2} \cdot \frac{\vec{r}}{r}$$

Comme \vec{F}_G est conservatrice $\Rightarrow E_m$ conservée.

$$E_{m0} = \frac{1}{2} m (V_{\text{départ}})^2 - G \frac{M_s \cdot m}{(R_T)}$$

$$E_{m1} = \frac{1}{2} m (V_{\text{arrivée}})^2 - G \frac{M_s \cdot m}{(R_L)}$$

$$\frac{1}{2} m (V_{\text{départ}})^2 - 2G \frac{M_s \cdot m}{R_T} = \frac{1}{2} m (V_{\text{arrivée}})^2 - 2G \frac{M_s \cdot m}{R_L}$$

$$V_{\text{départ}}^2 - 2G \frac{M_s}{R_T} = V_{\text{arrivée}}^2 - 2G \frac{M_s}{R_L}$$

$L_0 = \cancel{cst}$ car force
centrale

$$\vec{L}_0 = R_T \frac{\vec{r}}{r} \wedge m \vec{v}_{\text{depart}} = R_T \cdot m \cdot v_{\text{depart}}$$

$$\vec{L}_g = R_M \frac{\vec{r}}{r} \wedge m \vec{v}_{\text{arrivée}} = R_M \cdot m \cdot v_{\text{arrivée}}$$

$$\frac{T^2}{a^3} =$$

$$R_T \cdot m \cdot v_{\text{depart}} = R_M \cdot \cancel{m} \cdot v_{\text{arrivée}}$$

$$\Leftrightarrow v_{\text{depart}} = \frac{R_M}{R_T} \cdot v_{\text{arrivée}}$$

$$\left(\frac{R_M}{R_T} \cdot v_A\right)^2 - 2G \frac{M_S}{R_T} = v_A^2 - 2G \frac{M_S}{R_M}$$

$$\Leftrightarrow \left(\frac{R_M}{R_T}\right)^2 \cdot v_A^2 - v_A^2 = -2G \frac{M_S}{R_M} + 2G \frac{M_S}{R_T}$$

$$\Leftrightarrow \left(\left(\frac{R_M}{R_T}\right)^2 - 1\right) v_A^2 = 2G \left(\frac{M_S}{R_T} - \frac{M_S}{R_M}\right)$$

$$\Leftrightarrow \left(\frac{R_M^2 - R_T^2}{R_T^2}\right) v_A^2 = 2G \left(\frac{M_S R_M - M_S R_T}{R_T R_M}\right)$$

$$\Leftrightarrow \quad // \quad = 2G M_S \left(\frac{R_M - R_T}{R_T R_M}\right)$$

$$v_A^2 \left(\frac{R_M^2 - R_T^2}{R_T^2}\right) = 2G M_S \left(\frac{R_M - R_T}{R_T R_M}\right)$$

$$R_M = x \quad R_T = y$$

$$\frac{x^2 - y^2}{y^2} = k \left(\frac{x - y}{xy} \right)$$

$$\Leftrightarrow (x - y)(x + y)(xy) = k(x - y) \cdot y$$

$$\Leftrightarrow (x + y)(xy) = ky^2$$

$$\Leftrightarrow (x + y)x = ky$$

$$k = \frac{2GM_S}{v_A^2}$$

$$(R_M + R_T) R_M = \frac{2GM_S}{v_A^2} R_T$$

$$\frac{2R_T}{\dots}$$

$$\sqrt{\dots}$$

$$\Rightarrow \frac{(R_M + R_T) R_M \cdot V_A^2}{2 R_T} = \boxed{G M_s}$$

Cette équation est valable pour le vaisseau mais aussi pour la Terre de vivane

$$\frac{(R_M + R_T) R_M \cdot V_A^2}{2 R_T} = R_T \cdot (V_T)^2$$

$$V_A^2 = (V_T)^2 \cdot 2 (R_T)^2 \cdot \frac{1}{(R_M + R_T) R_M}$$

$$\Leftrightarrow V_A^2 = 2 (V_T)^2 \cdot \frac{(R_T)^2}{R_M^2 + R_T R_M}$$

=

$$V_A^2 = 2 V_T^2 \frac{R_T^2}{R_T \left(\frac{1}{R_M^2} + R_M \right)}$$

$$\Rightarrow (v_A)^2 = 2(v_T)^2 \cdot \frac{(R_T)^2}{R_M^2 + R_T R_M}$$

② L'échelle

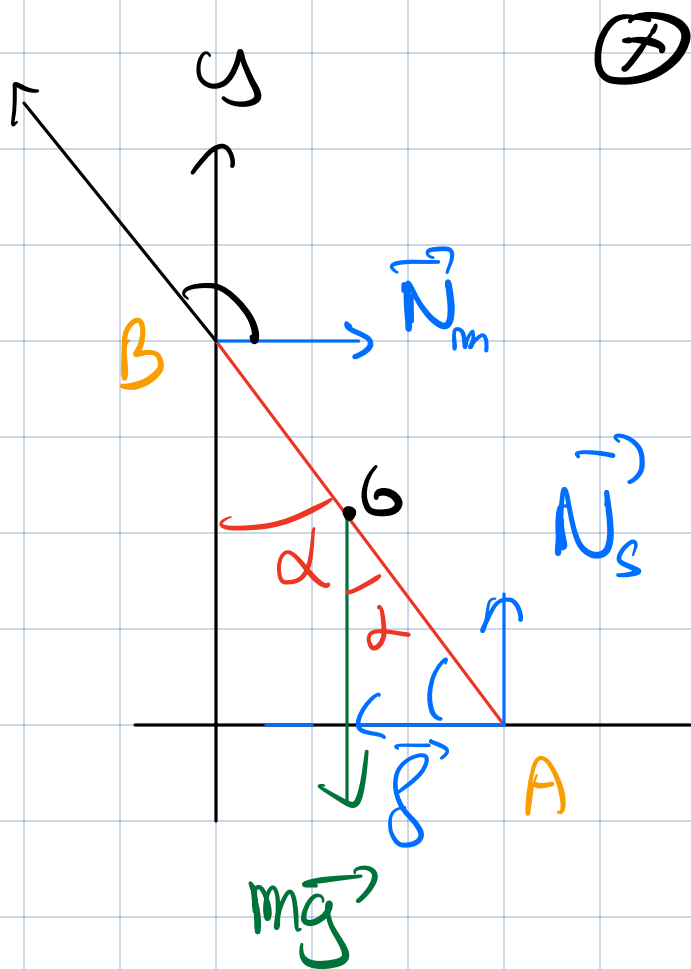


Diagram illustrating a differential vector element $d\vec{N}$ in a 2D coordinate system. The vector $d\vec{N}$ is shown in orange, pointing along the positive x-axis. The components dx and dy are shown in purple, representing the differential elements along the x and y axes respectively.

$$d\vec{N} = (dy \hat{e}_y + dx \hat{e}_x) \times (N \hat{e}_x) = N dy (\hat{e}_y \times \hat{e}_x + \hat{e}_x \times \hat{e}_x)$$

on cherche les conditions pour rester
en équilibre (moment nul)

On choisit comme origine **A**

$$\vec{AA} \wedge \vec{g} = 0$$

$$+ \vec{AA} \wedge \vec{N}_s = 0$$

$$+ \vec{AG} \wedge \vec{mg}$$

$$+ \vec{AB} \wedge \vec{N}_m$$

$$= \vec{0}$$

$$\Rightarrow \left(\frac{l}{2} \cdot mg \cdot \sin \alpha \right.$$

$$\left. - l \cdot \sin \left(\alpha + \frac{\pi}{2} \right) \cdot N_m \right) \hat{e}_z = \vec{0}$$

$$N_m = -\frac{l}{2} \cdot mg \cdot \sin \alpha - \frac{1}{\cos \alpha} \cdot \frac{-1}{l}$$

$$= \frac{1}{2} \cdot mg \cdot \tan \alpha$$

$$\sum \vec{F} = N_m + \vec{f} + \vec{N}_S + m\vec{g}$$

$$\begin{cases} (\hat{e}_y) \quad m\vec{a} = \vec{N}_S + m\vec{g} = \vec{0} \\ (\hat{e}_x) \quad m\vec{a} = \vec{f} + \vec{N}_m = \vec{0} \end{cases}$$

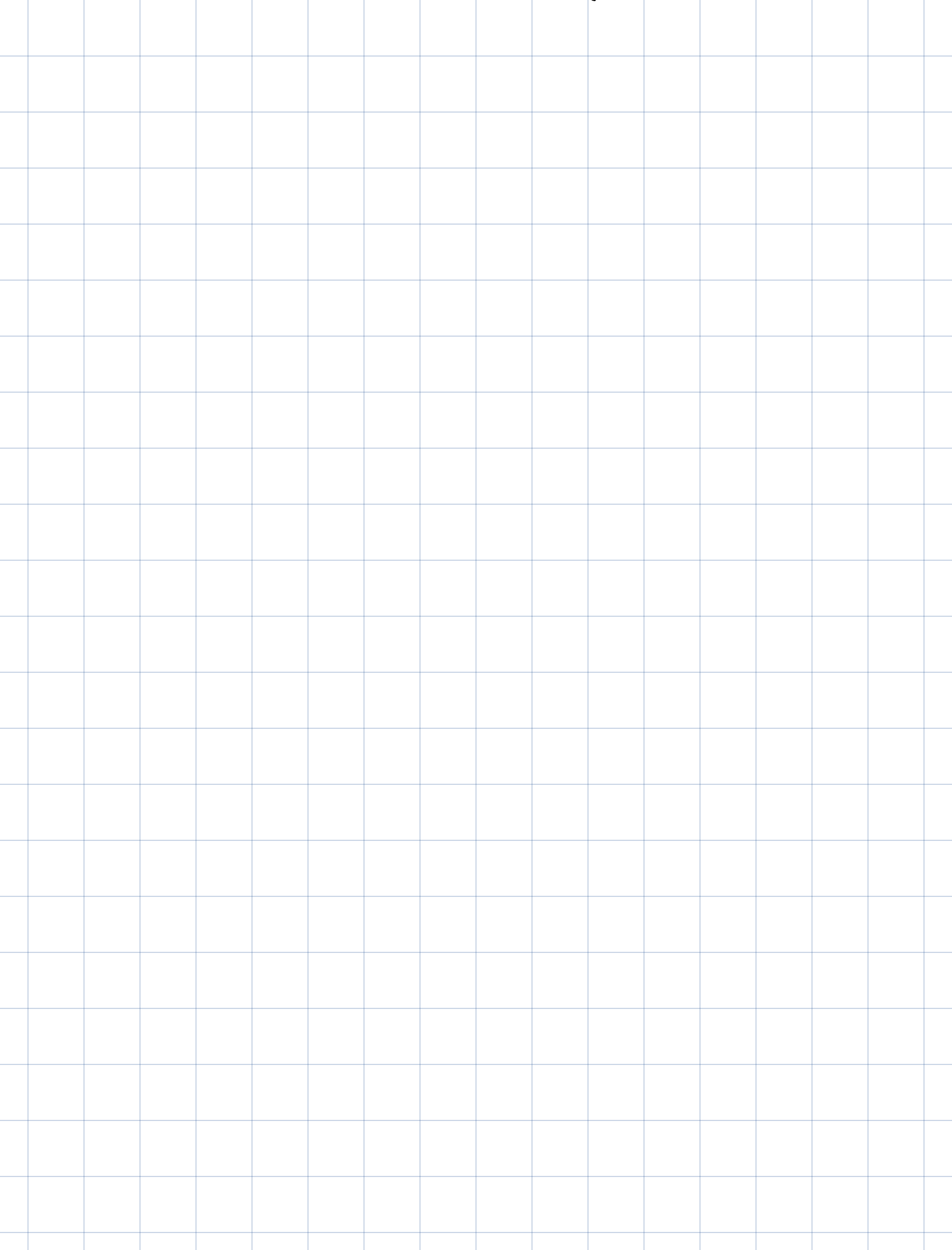
$$F_{\text{frict. st. max}} + N_m = 0$$

$$\Leftrightarrow -\mu_{\text{max}} N_S + \frac{1}{2} \cdot mg \cdot \tan \alpha = 0$$

$$\Leftrightarrow -\mu_S (mg) + \frac{1}{2} mg \cdot \tan \alpha = 0$$

$$\Leftrightarrow \frac{1}{2} mg \tan \alpha = \mu mg$$

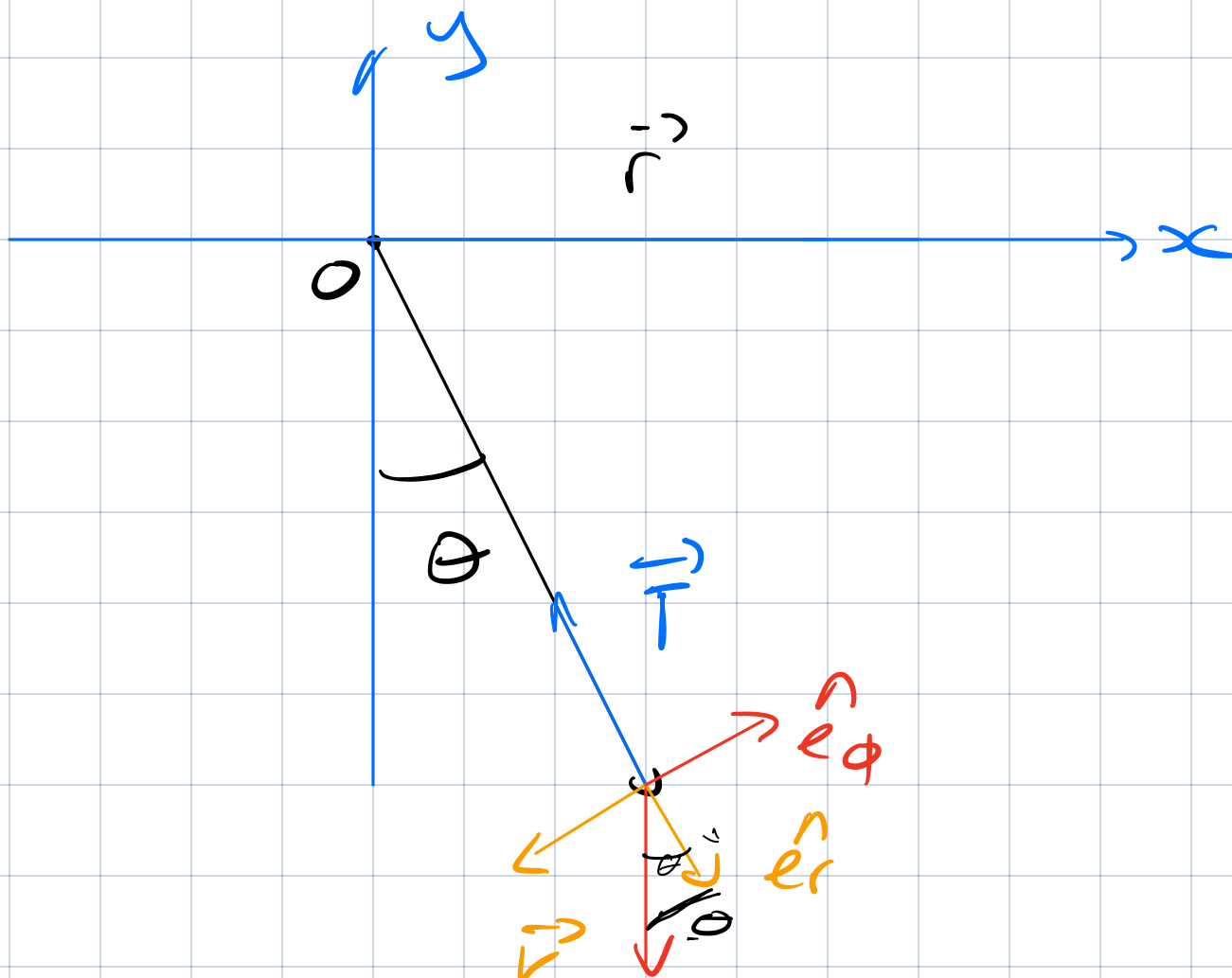
$$\Leftrightarrow \tan \alpha = 2\mu$$



③

$$\frac{d\vec{L}_O}{dt} = \vec{\tau}_O$$

④



$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\phi$$

$$\vec{L}_0 = \vec{r} \wedge m\vec{v}$$

$$\Leftrightarrow \vec{L}_0 = l \vec{e}_\rho \wedge m l \dot{\phi} \vec{e}_\phi$$

(coord. cylind.)

$$\Leftrightarrow \vec{L}_0 = (l \cdot m l \dot{\phi}) \vec{e}_z$$

$$= l^2 m \dot{\phi} \vec{e}_z$$

$$\Rightarrow \frac{d\vec{L}_0}{dt} = l^2 m \ddot{\phi} \vec{e}_z$$

$$= \vec{M}_0$$

$$\vec{M}_0 = \vec{r} \wedge \vec{F}$$

$$= l \vec{e}_r \wedge ($$

$$\cancel{mg \cdot \cos \Theta \vec{e}_r} - mg \cdot \sin \Theta \vec{e}_\phi)$$

$$= l \hat{e}_r \wedge (-mg \sin \Theta \hat{e}_\phi)$$

$$= -l mg \sin \Theta \hat{e}_3$$

$$= \vec{\Pi}_0$$

$$-l mg \sin \Theta = Q^2 g \ddot{\phi}$$

$$\Leftrightarrow \frac{-g}{r} \sin \Theta = \ddot{\phi}$$