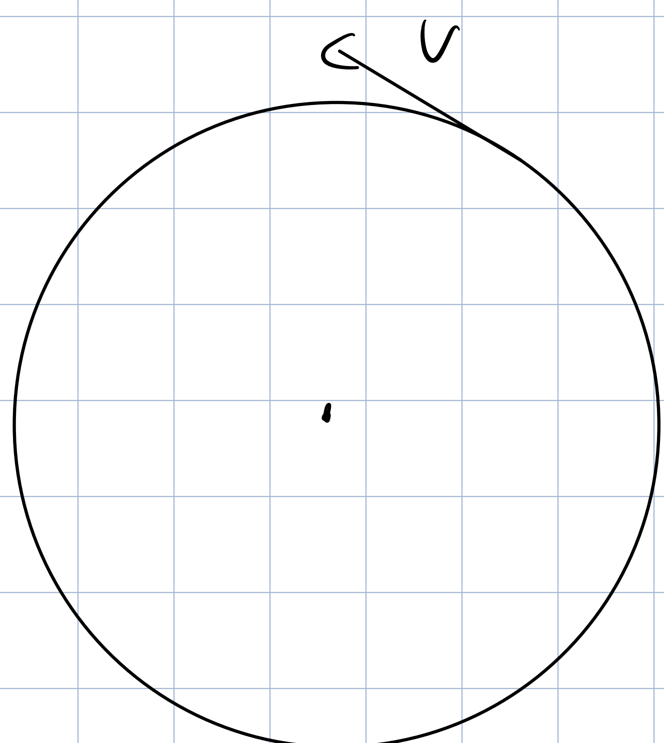
1

a) Non.

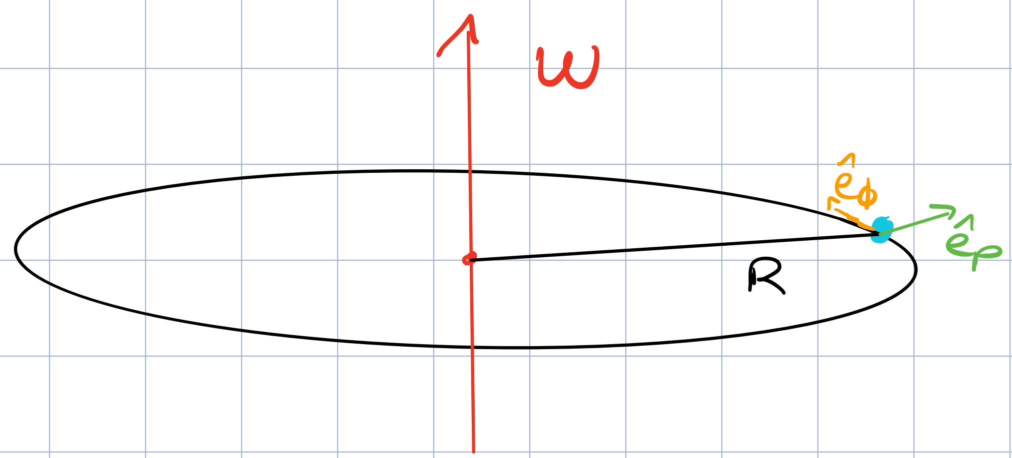
b) Oui, accélération radiale.
accélération tangentielle
$= \frac{dv}{dt} \vec{\tau}$

avec $\vec{\tau}$ tangent
au mur.

$$\frac{dv}{dt} = \frac{d\phi}{dt}$$



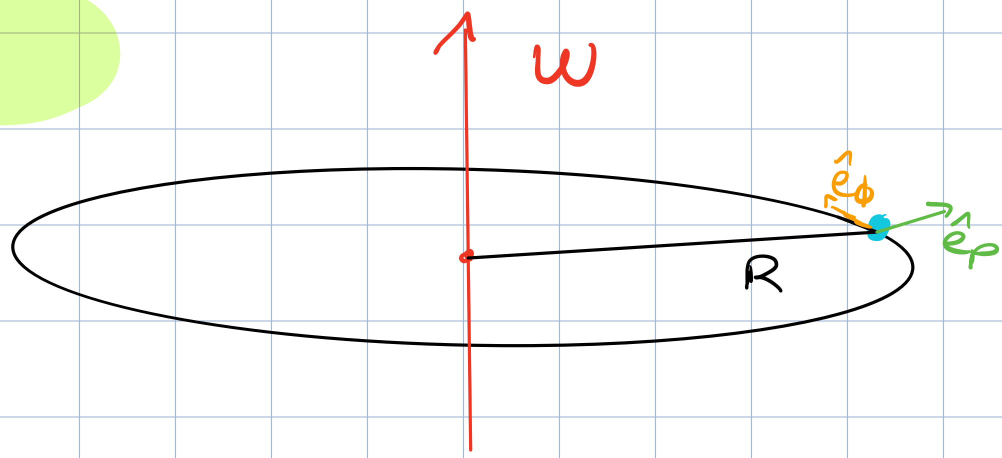
pas de variat° de rayon, ni de variat° de θ



$$\omega = \frac{d\phi}{dt}$$

$$v = \omega r = \frac{d\phi}{dt} r$$

$$\vec{a} = -\omega^2 \vec{r} \Rightarrow a = -\omega^2 R$$



$$\vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\,\hat{e}_\rho + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})\,\hat{e}_\phi + \cancel{\ddot{z}\,\vec{e}_z}$$

A $\quad \dot{\phi}$ cste $\Rightarrow \rho\ddot{\phi} = 0$
ρ cste $\Rightarrow 2\dot{\rho}\dot{\phi} = 0$ $\Big\}$ aucune accélératº

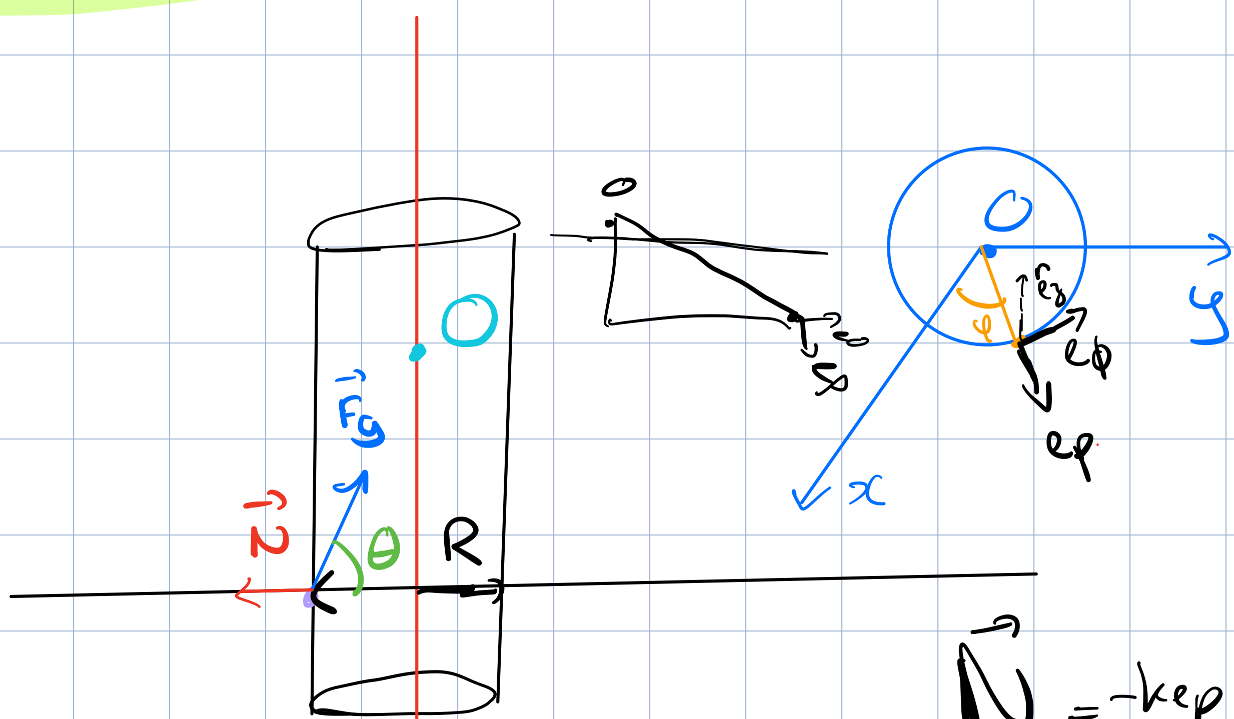
B $\quad \dot{\phi} \nearrow \Rightarrow \vec{a}_\phi = \rho\ddot{\phi} > 0$ accel tangentielle,
$\Rightarrow \vec{a}_\rho = -\rho\dot{\phi}^2 < 0$ accel radiale

$$\rho\ddot{\phi} = -\rho\dot{\phi}^2$$

$$\boxed{\Leftrightarrow \ddot{\phi} = -\dot{\phi}^2}$$

① Point sur un cylindre

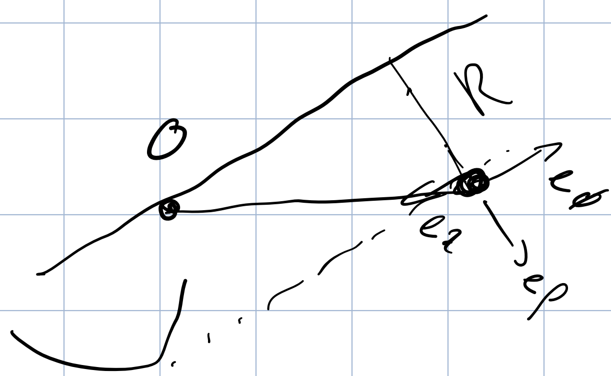
ⓐ $\vec{F_a} + \vec{N} = \sum \vec{F} = m\vec{a}$



$\vec{F_a} = F$

$\vec{N} = -k e_\rho$

$\vec{F_a} = k' \hat{e_z} + k'' \hat{e_\rho}$



$$\begin{cases} \vec{F}_a = -Rk\hat{e}_\rho - zk\hat{e}_z \\ \vec{N} = N\hat{e}_\rho \end{cases}$$

$$\sum \vec{F} = m\vec{a}$$

$$\begin{cases} m - R\dot{\phi}^2 = -Rk + N &) \hat{e}_\rho \\ m R\ddot{\phi} = 0 &) \hat{e}_\phi \\ m\ddot{z} = -kz &) \hat{e}_z \end{cases}$$

$\ddot{\phi} = 0 \Leftrightarrow \dot{\phi}$ cste.

On sait résoudre la dernière

$$\ddot{z} = \frac{-k}{m} z$$

$$z(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$\boxed{A\cos(\omega t + \varphi)}$$

$$\dot{z}(t) = -A\omega\sin(\omega t) + B\omega\cos(\omega t)$$

$=0$

- $z(0) = z_0$
- $\dot{z}(0) = 0$

- $B\omega = 0 \Leftrightarrow B = 0$

$$z(t) = A\cos(\omega t)$$

$$= z_0\cos(\omega t)$$

$$= z_0\cos\left(\sqrt{\frac{k}{m}}\, t + \theta_0\right)$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

pulsation naturelle sans force supplém.

$$N = -mR\dot{\phi}^2 + Rk$$

$$= -Rm\left(\dot{\phi}^2 + \frac{k}{m}\right)$$

?

$$= Rm\left(\omega_0^2 - \omega^2\right)$$

- Si $\omega > \omega_0$:

  $N_p < 0$, donc dirigée vers l'intérieur

- Si $\omega < \omega_0$:

  $N_p > 0$, dirigée vers l'extérieur

- Si $\omega = \omega_0$:

  $N_p = 0$

(b)

$$\begin{cases} d = \sqrt{R^2 + (\quad z)^2} \\ F_a = kd \end{cases} \Leftrightarrow d = \frac{F_a}{k}$$

$$\cos\theta = \frac{R}{d}$$

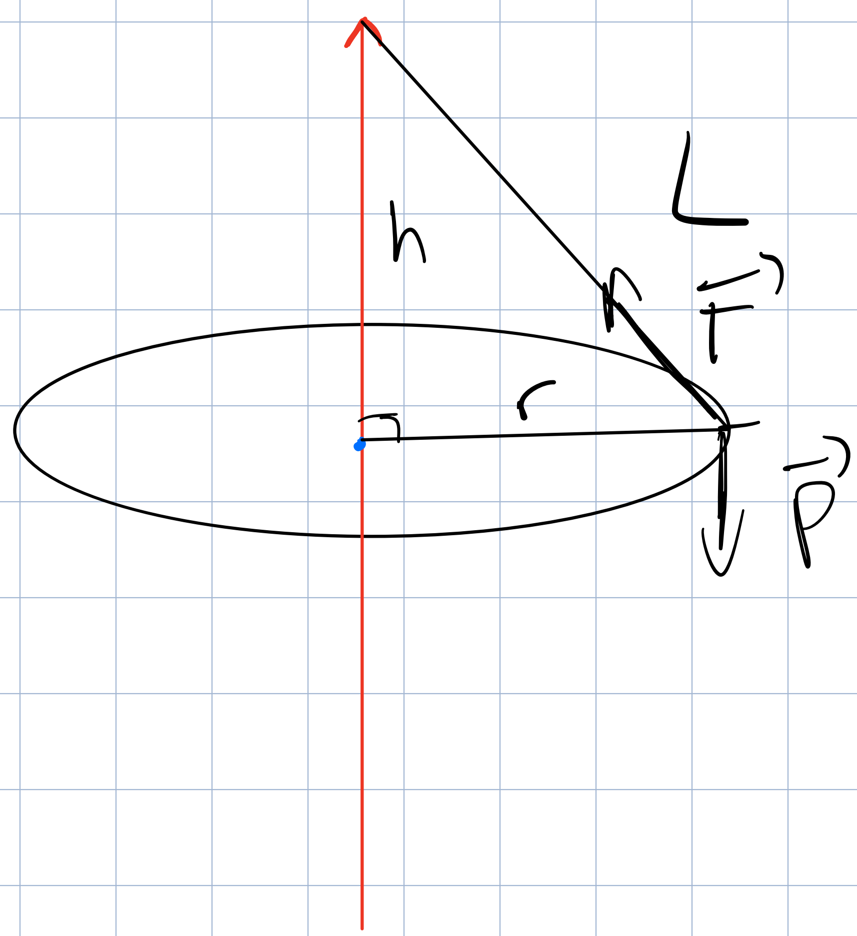
$$\vec{N} = -F_a \cdot \cos\theta \cdot \hat{e}_\rho$$

$$= -F_a \cdot \frac{R}{\frac{F_a}{k}} \cdot \hat{e}_\rho = -Rk\,\hat{e}_\rho$$

$$m\vec{a} = -Rk\,\hat{e}_\rho + kd\cos\theta\,\vec{e}_\rho + kd\sin\theta\,\hat{e}_z$$

$$\begin{cases} -R\dot{\phi}^2 = -Rk + kd\cos\theta \\ \ddot{z} = kd\sin\theta \end{cases}$$

② La chauve-souris physicienne



périmètre $= 2\pi r = d$

vitesse $= |\vec{v}|$

pulsation $\omega = \dfrac{|\vec{v}|}{d}$

rayon $r = \sqrt{L^2 + h^2}$

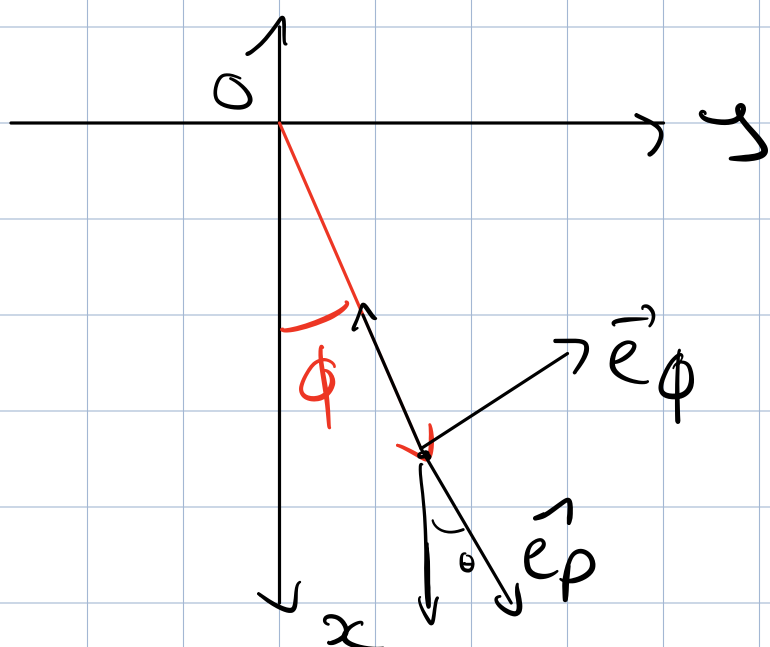
resoudre selon $\hat{e}_p$ $\hat{e}_\phi$

$$L^2 = r^2 + h^2$$

$$\Leftrightarrow r^2 = L^2 + h^2$$

$$r = \sqrt{L^2 + h^2}$$

$$\boxed{\omega = \frac{|\vec{v}|}{\sqrt{L^2 + h^2}}}$$



$$\begin{cases} \bullet\ \rho \text{ cste} = L,\ \dot{\rho} = 0,\ \ddot{\rho} = 0 \\ \bullet\ z = 0,\ \dot{z} = 0,\ \ddot{z} = 0 \end{cases}$$

$$\boxed{\begin{aligned} \vec{a} = & -L\dot{\phi}^2\,\hat{e}_\rho \\ & + L\ddot{\phi}\,\hat{e}_\phi \end{aligned}}$$

$$\sum \vec{F} = m\vec{a}$$

$$m\vec{a} = \vec{P} + \vec{T}$$

<u>On projette :</u>

$$-mL\dot{\phi}^2 = \cos\phi\, mg + T_\rho \quad)\ \vec{e}_\rho$$

$$mL\ddot{\phi} = -\sin\phi\, mg \quad)\ \vec{e}_\phi$$

$$mL\ddot{\phi} = -\sin\phi \, mg.$$

- $\sin\phi \simeq \phi$

$$mL\ddot{\phi} = -\phi \, mg$$

$$\Leftrightarrow L\ddot{\phi} = -\phi \, g$$

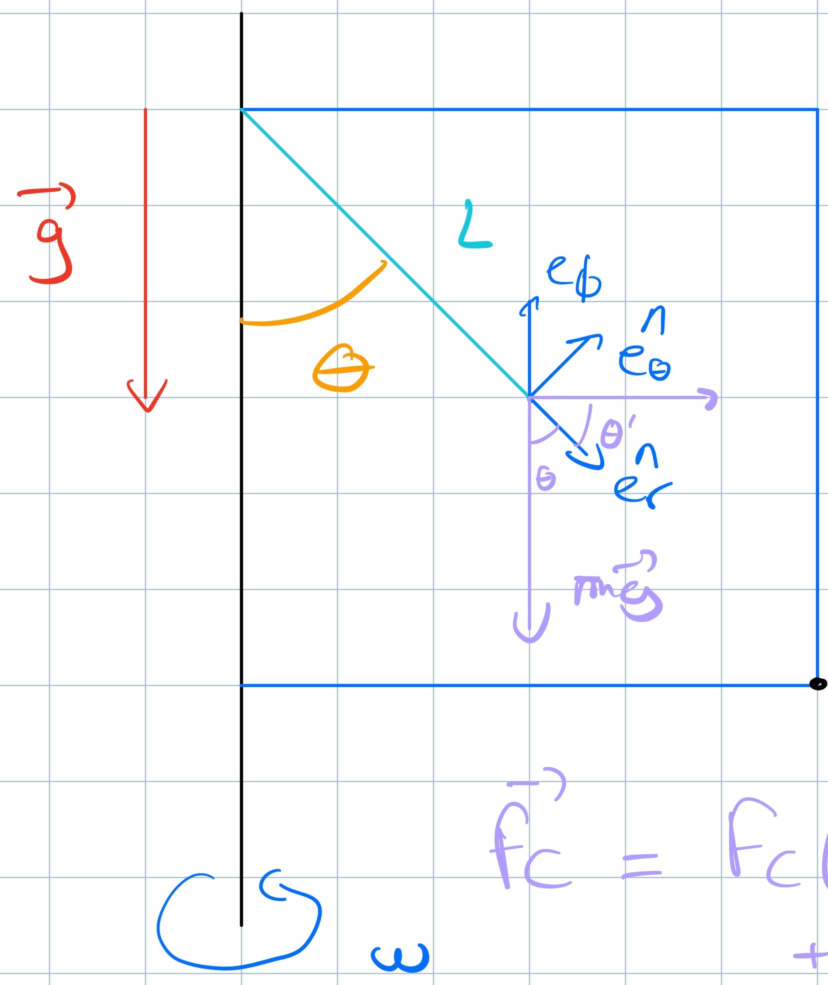
$$\Leftrightarrow \ddot{\phi} = \frac{-g}{L}\phi$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$\phi(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$\dot{\phi}(t) = -A\omega\sin(\omega t) + B\omega\cos(\omega t)$$

③



$$\begin{cases} \theta + \theta' = \frac{\pi}{2} \\ \theta' + \theta'' = \frac{\pi}{2} \end{cases}$$

$$\vec{f_c} = F_c\left(\sin\theta\, e_r + \cos\theta\, \hat{e}_\theta\right)$$

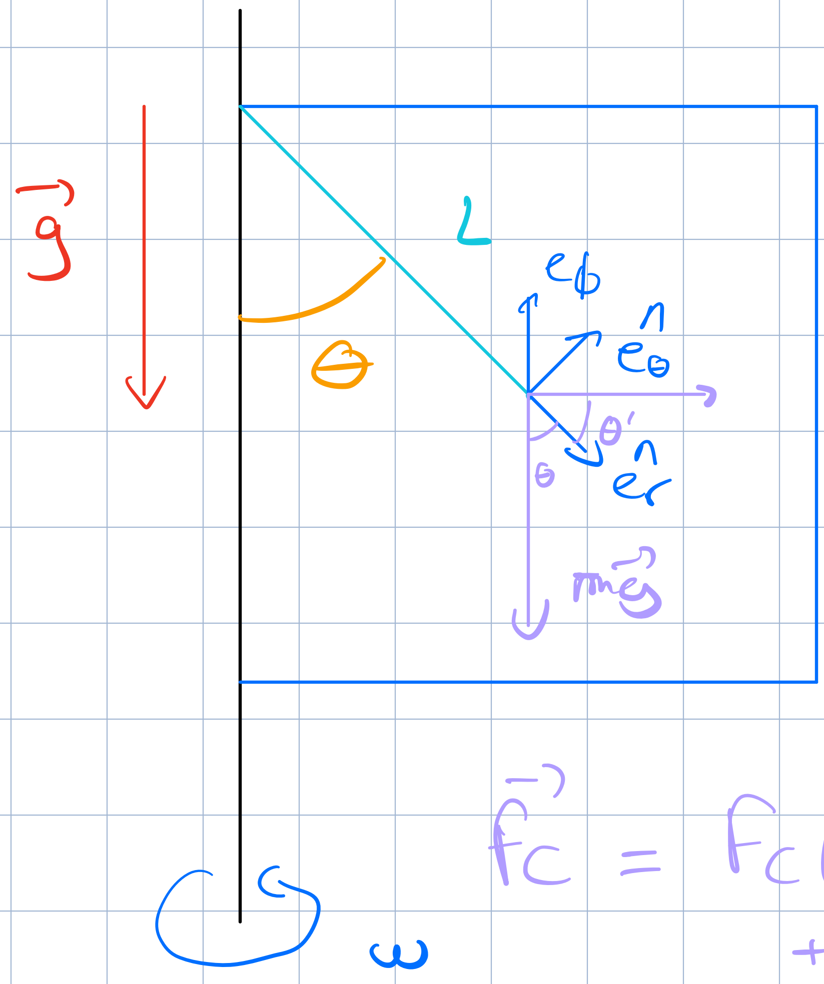
$$\sum \vec{F} = m\vec{a} = \vec{P} + \vec{T} + \vec{F_c}$$ (force inertie pendule)

$$m\left(-L\dot{\theta}^2 - L\dot{\phi}^2 \sin^2\theta\right) = \cos\theta\, mg - T_p \quad \Big|\ \hat{e}_r$$

$$m\left(L\ddot{\theta} - L\dot{\phi}^2 \sin\theta \cos\theta\right) = -\sin\theta\, mg \quad \Big|\ \hat{e}_\theta$$

$$m\left(2L\dot{\phi}\dot{\theta}\cos\theta\right) = m\omega^2$$

3



$$\begin{cases}\theta + \theta' = \frac{\pi}{2} \\ \theta' + \theta'' = \frac{\pi}{2}\end{cases}$$

$$\vec{F_c} = F_c(\sin\theta \, e_r + \cos\theta \, \hat{e}_\theta)$$

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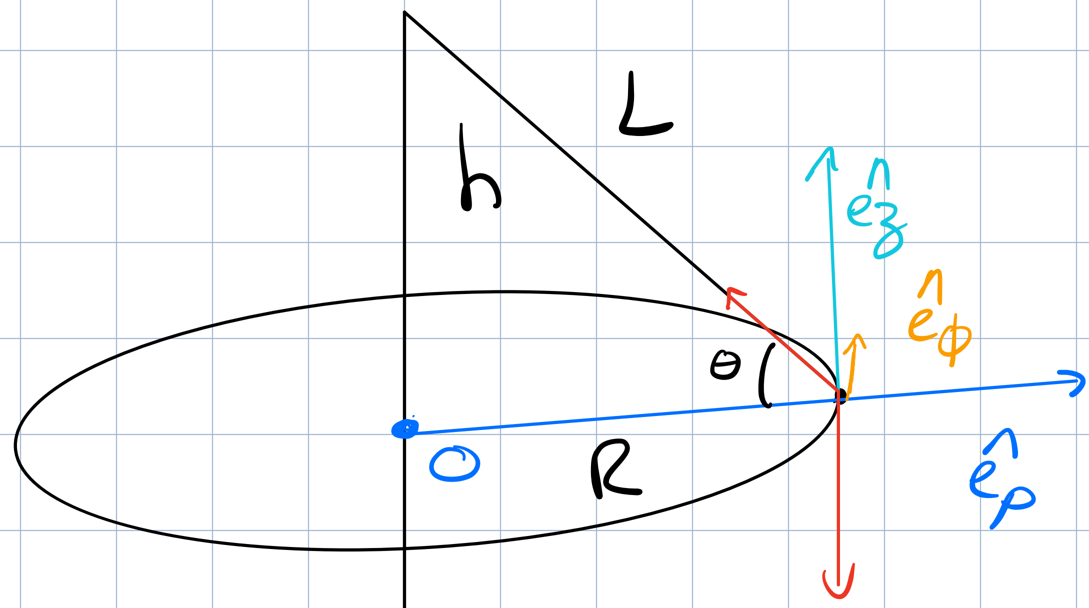
$$\begin{aligned} m(-L\dot{\theta}^2 - L\dot{\phi}^2 \sin^2\theta) &= \cos\theta \, mg - T_p \quad && \hat{e}_r \\ m(L\ddot{\theta} - L\dot{\phi}^2 \sin\theta\cos\theta) &= -\sin\theta \, mg && \hat{e}_\theta \\ m(2L\dot{\phi}\dot{\theta}\cos\theta) &= m\omega^2 && \end{aligned}$$

La chauve-souris physicienne



on choisit cylindrique car l'angle θ est cste.

$$\tan\theta = \frac{h}{R} \Leftrightarrow R = \frac{h}{\tan\theta}$$

$$\sum \vec{F} = m\vec{a}$$

$$= \vec{P} + \vec{T}$$

$$\begin{cases} m(\ddot{z}) = -mg + \sin\theta\, T & \vert\, \hat{e}_z \\ m(\cancel{\ddot{\rho}} - \rho\dot{\phi}^2) = \cos\theta\, T & \vert\, \hat{e}_\rho \\ m(\rho\ddot{\phi} + \cancel{2\dot{\rho}\dot{\phi}}) = 0 \end{cases}$$

On pose $\rho = R = \sqrt{L^2 - H^2}$

- $m R \ddot{\varphi} = 0 \Rightarrow \underline{\dot{\varphi} \text{ cste}} = \frac{\|\vec{v}\|}{R} = \omega$

- $z = z_i \Rightarrow \dot{z} = 0 \Rightarrow \ddot{z} = 0$

$$\Rightarrow \begin{cases} mg = \sin\theta T \\ -mR\omega^2 = \cos\theta T \end{cases}$$

$$\tan\theta = \frac{mg}{-mR\omega^2} = \frac{g}{R\omega^2}$$

$$\Rightarrow R\omega^2 = \frac{g}{\tan\theta}$$

$$\Leftrightarrow \frac{h}{\cancel{r\cos\theta}} \cdot \omega^2 = g \cdot \frac{1}{\cancel{r\cos\theta}}$$

$$\Leftrightarrow h \cdot \omega^2 = g$$

$$\Leftrightarrow \omega^2 = \frac{g}{h}$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{g}{h}}} \quad s^{-1}$$

Si elle avait oscillé, on sait que

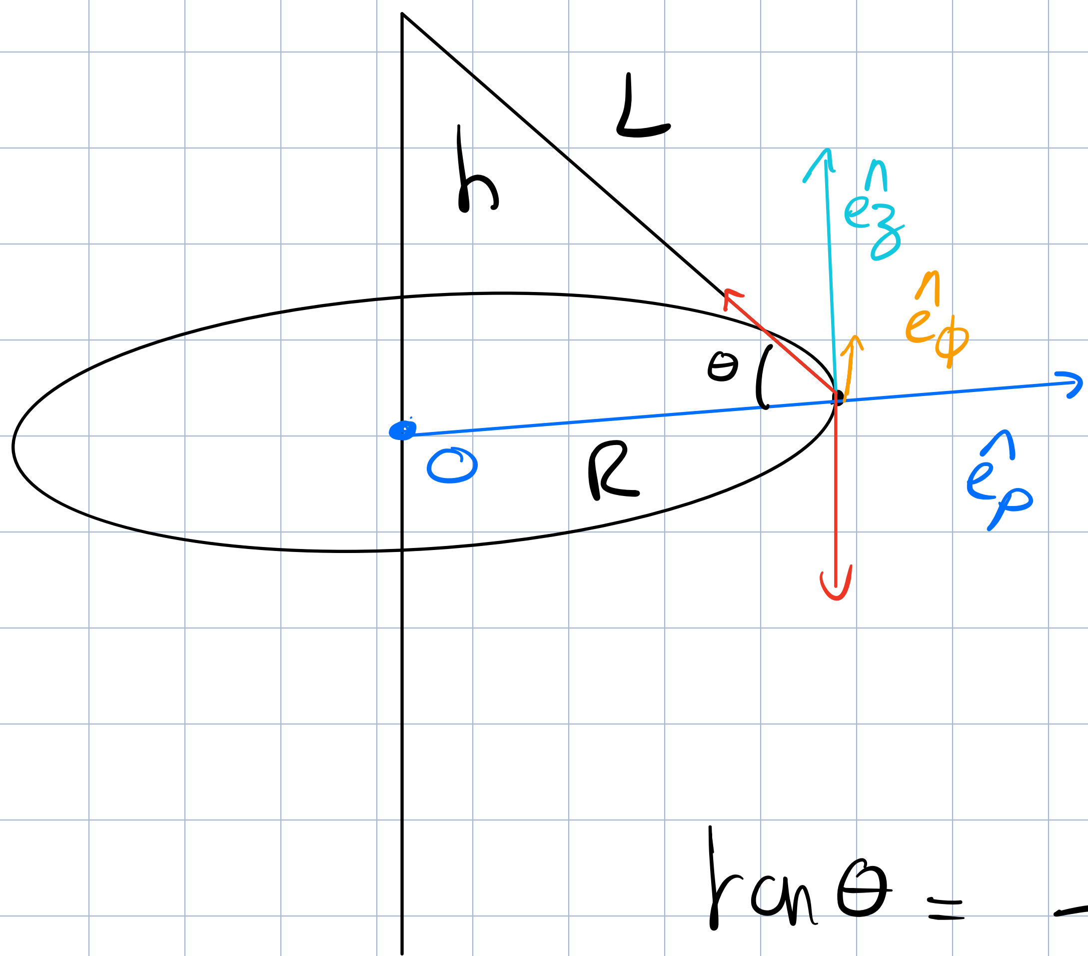
$$\omega = \sqrt{\frac{g}{L}} \text{ (pendule mathématique)}$$

$rad \cdot s^{-1}$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

$$T > 2\pi\sqrt{\frac{h}{g}}$$

pendule



$$\tan\theta = \frac{h}{R}$$

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$$= \vec{P} + \vec{T}$$

$$\begin{cases} m(\ddot{z}) = -mg + \sin\theta\, T & |\hat{e}_z \\ m(\cancel{\ddot{\rho}} - \rho\dot{\phi}^2) = \cos\theta\, T & |\hat{e}_\rho \\ m(\rho\ddot{\phi} + \cancel{2\dot{\rho}}\dot{\phi}) = 0 & \end{cases}$$

on pose ρ cste $= R$

$$= \sqrt{L^2 - H^2}$$

on pose z cste $\Rightarrow \dot{z} = 0$

on pose $\dot{\phi}$ cste $\Rightarrow \dfrac{\|\vec{v}\|}{R} = \omega_0$

$$\sin\theta T = mg$$
$$\cos\theta T = -m\rho\dot{\phi}^2$$

$$\tan\theta = \frac{mg}{-R\omega^2 m} = \frac{g}{-R\omega^2}$$

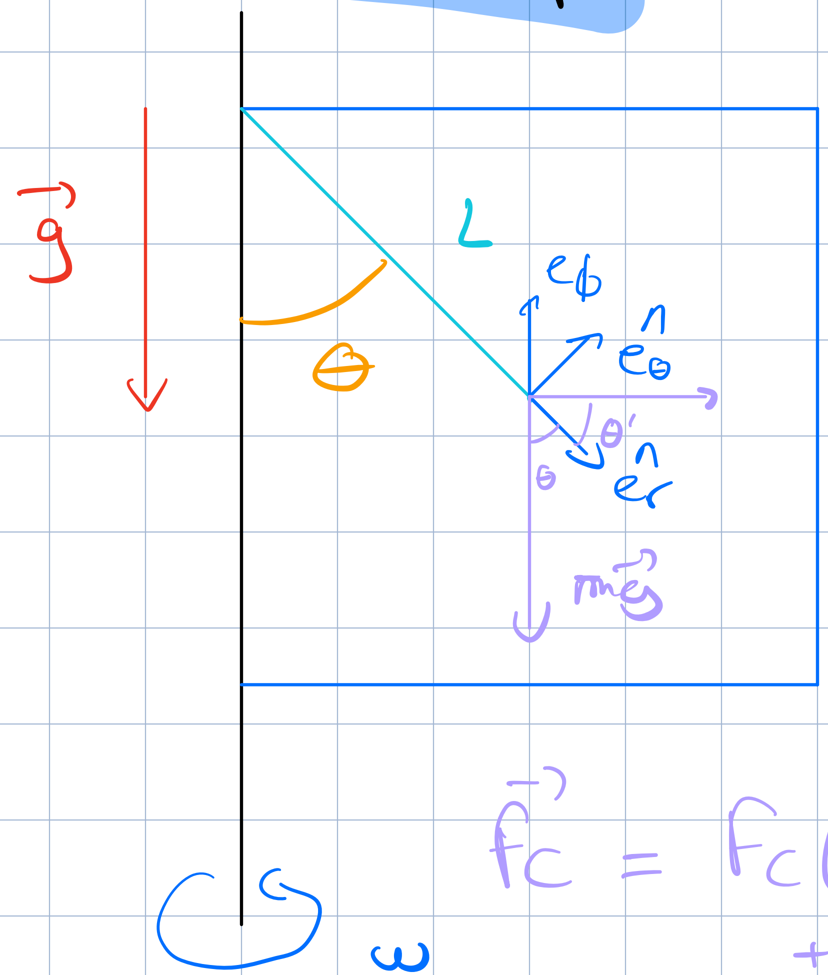
$$\frac{h}{R} = \frac{g}{-R\omega^2} \Leftrightarrow \frac{g}{-\omega^2} =$$

$$\Leftrightarrow \quad h = \frac{-w^2}{g}$$

$$\Leftrightarrow \quad -w^2 = h \cdot g$$

$$\Leftrightarrow \quad \omega = \sqrt{g \cdot}$$

③ Pendule sur une porte



$$\begin{cases} \theta + \theta' = \frac{\pi}{2} \\ \theta' + \theta'' = \frac{\pi}{2} \end{cases}$$

$$\vec{F_c} = F_c(\sin\theta \, e_r + \cos\theta \, \hat{e}_\theta)$$

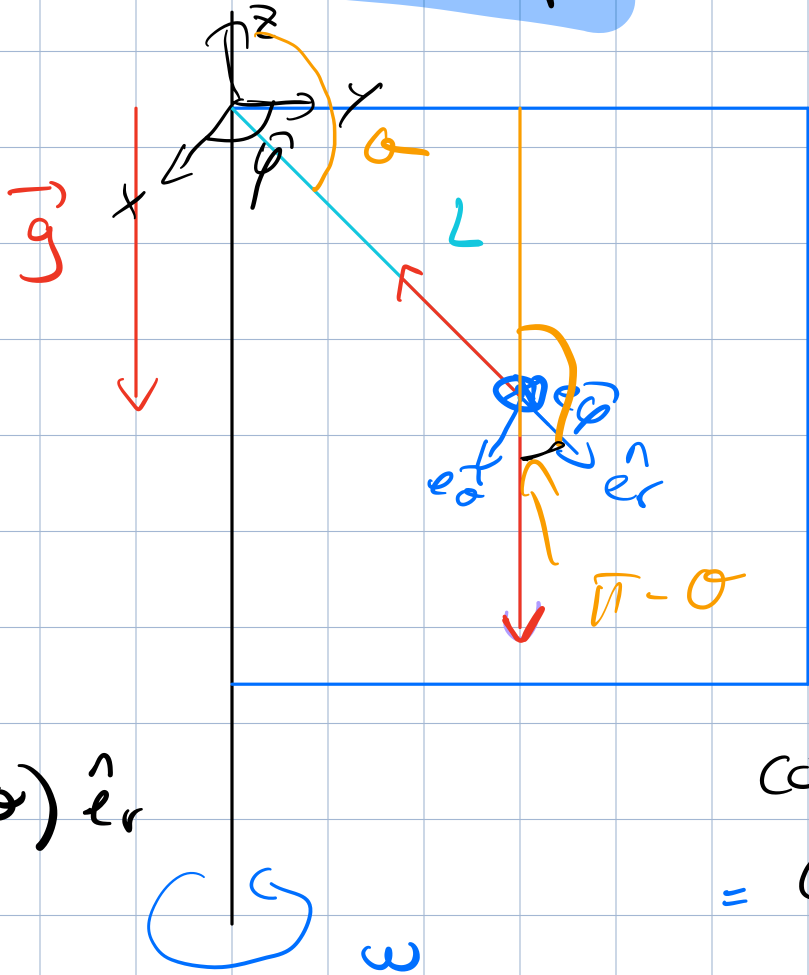
$$\sum \vec{F} = m\vec{a} = \vec{P} + \vec{T} + \vec{F_c}$$ (force inertie pendule)

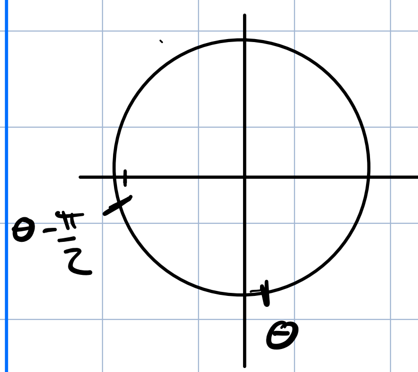
$$m(-L\dot{\theta}^2 - L\dot{\phi}^2 \sin^2\theta) = \cos\theta \, mg - T_p \qquad \hat{e}_r$$

$$m(L\ddot{\theta} - L\dot{\phi}^2 \sin\theta\cos\theta) = -\sin\theta \, mg \qquad \hat{e}_\theta$$

$$m(2L\dot{\phi}\dot{\theta}\cos\theta) = m\omega^2$$

③ Pendule sur une porte





$$\cos(\pi - \theta) = -\cos(\theta)\,\hat{e}_r$$

$$\cos\left(\frac{\pi}{2} - \pi + \theta\right) = \cos\left(\frac{-\pi}{2} + \theta\right) = \sin(\theta)\,\hat{e}_\theta$$

$$\sum \vec{F} = m\vec{a} = \vec{P} + \vec{T} + \vec{N}$$

$$\begin{array}{l} \hat{e}_r \\ \hat{e}_\theta \\ \hat{e}_\phi \end{array} \left\{ \begin{array}{l} \cancel{\ddot{r}} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta = -\cos\theta\, mg - T \\ r\ddot{\theta} + \cancel{2\dot{r}}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta = \sin\theta\, mg \\ \cancel{r\ddot{\phi}\sin\theta} + \cancel{2\dot{r}\dot{\phi}\sin\theta} + \underbrace{2r\dot{\phi}\dot{\theta}}_{\cos\theta} = \underbrace{N}_{\neq 0} \end{array} \right.$$

$\neq 0$

On pose $\dot{\phi}$ cste $= \omega$

$\Rightarrow \ddot{\phi} = 0$

r cste $\Rightarrow \dot{r} = 0 \Rightarrow \ddot{r} = 0$.

$$\Rightarrow \begin{cases} \hat{e}_r : & m(-r\dot{\theta}^2 - r\omega^2 \sin^2\theta) = -\cos\theta\, mg - T \\ \hat{e}_\theta : & m(r\ddot{\theta} - r\omega^2 \sin\theta \cos\theta) = \sin\theta\, mg \\ \hat{e}_\phi : & m\, 2r\omega\dot{\theta} = N \end{cases}$$

$$\Rightarrow \begin{cases} -mL(\dot{\theta}^2 + \omega^2 \sin^2\theta) = -T - \cos\theta\, mg \\ mL(\ddot{\theta} - \omega^2 \sin\theta\cos\theta) = \sin\theta\, mg \\ mL(2\omega\dot{\theta}) = N \end{cases}$$

$\omega \to 0$

$$\begin{cases} \hat{e}_\phi & N = 0 \\ \hat{e}_\theta & mR\ddot{\theta} = \sin\theta \, mg \\ \hat{e}_r & -mR\dot{\theta}^2 = -T - \cos\theta \, mg \end{cases}$$

Sans rotation de la perle
→ pendule simple

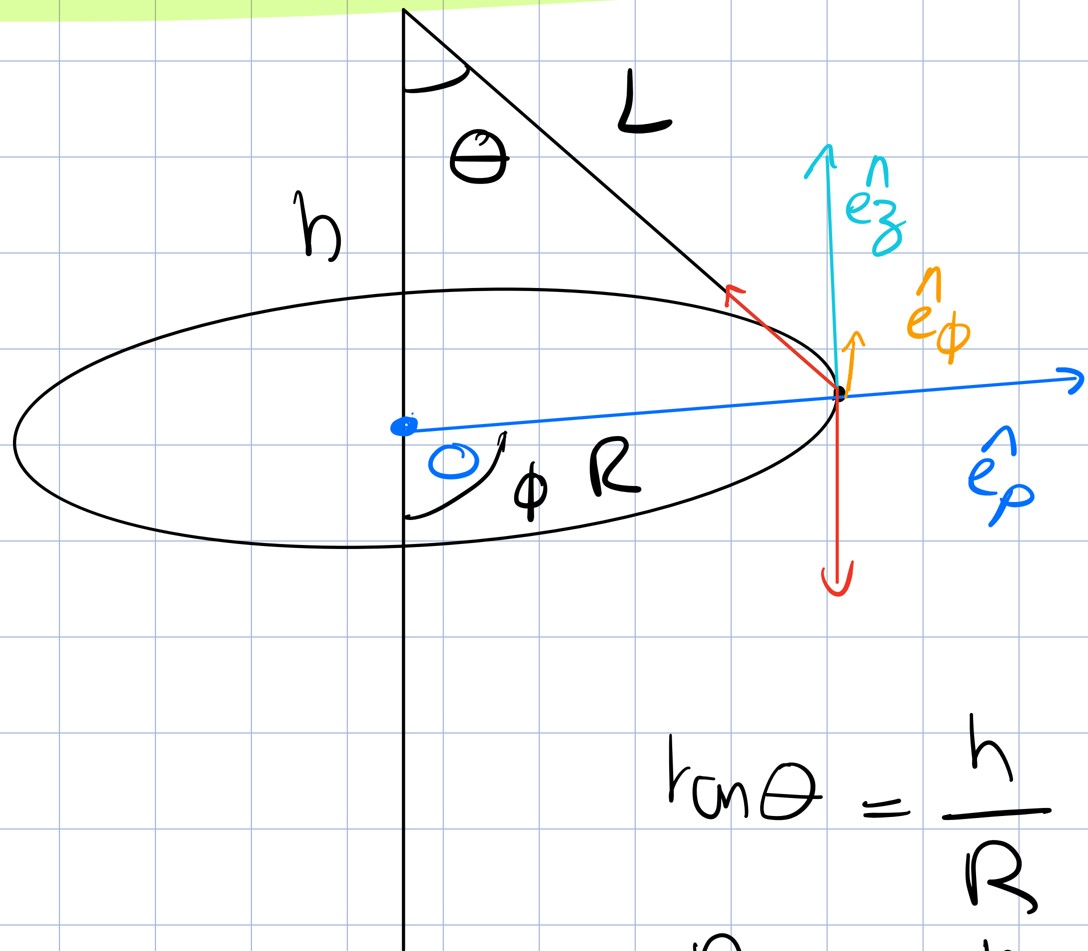
$L \to 0$

$$\begin{cases} -T - mg\cos\theta = \omega^2 \sin^2\theta \\ \end{cases}$$

d'où vient $\vec{N}$? car mvt pas ralenti ou accéléré

$\Rightarrow$ on considère que la porte est très lourde et que seule celle-ci a un impact sur le pendule mais pas l'inverse (on néglige)

Correct° chauve-souris physicienne



$$\tan\theta = \frac{h}{R}$$

$$\Leftrightarrow R = \frac{h}{\tan\theta}$$

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$$\begin{cases} m(\ddot{z}) = -mg + \sin\theta\, T & |\hat{e}_z \\ m(\cancel{\ddot{\rho}} - \rho\dot{\phi}^2) = \cos\theta\, T & |\hat{e}_\rho \\ m(\rho\ddot{\phi} + \cancel{2\dot{\rho}\dot{\phi}}) = 0 & \end{cases}$$

$\boxed{1 \times 3}$ $\boxed{1 \times 3}$

$A \quad T: \begin{pmatrix}1\\0\\0\end{pmatrix} \quad T\begin{pmatrix}0\\1\\0\end{pmatrix} \quad T\begin{pmatrix}0\\0\\1\end{pmatrix}$

$$T\begin{pmatrix}1\\0\\0\end{pmatrix} \to \begin{pmatrix}1\\3\\4\end{pmatrix}$$

$$T\begin{pmatrix}x\\y\\z\end{pmatrix} \Rightarrow T\begin{pmatrix}1\\0\\0\end{pmatrix} \quad T(x) = \begin{pmatrix}1\\3\\4\end{pmatrix}$$

$2 \times 2 = 2 \times 2$

$$T\begin{pmatrix}0\\1\\0\end{pmatrix} = T(y) \qquad \begin{pmatrix}1 & 4\\2 & 5\\3 & 6\end{pmatrix}^T = \begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix}$$

$3 \times 2 \to 2 \times 3$

$$A \times B = \begin{pmatrix} & \end{pmatrix}\begin{pmatrix} & \end{pmatrix}$$

$2 \times \underline{3} \neq \underline{2} \times 3$

$2 \times 3 \quad 3 \times 2$