Exercice 2

① $\begin{cases} a_0 = 3 \\ a_{n+1} = a_n \end{cases}$

② $\begin{cases} a_0 = 0 \\ a_{n+1} = a_n + 2 \end{cases}$

③ $\begin{cases} a_0 = 3 \\ a_{n+1} = a_n + 2 \end{cases}$

④ $\begin{cases} a_0 = 1 \\ a_{n+1} = 5 \cdot a_n \end{cases}$

⑤ $\begin{cases} a_0 = 0 \\ a_{n+1} = \left(\sqrt{a_n} + 1\right)^2 \end{cases}$

0	
1	
2·2	4
3·3	9
4·4	16
5·5	25
	36

$a_{n+1} = a_n + 2\sqrt{a_n} + 1$

$a_n + 2n + 1$

$n^2 + n$

⑥ $\begin{cases} a_0 = 0 \\ a_{n+1} = \left(\sqrt{a_n} + 1\right)^2 + \sqrt{a_n} \end{cases}$

$\left(= a_n + 2n + 2 \right)$

0		
1^2+1	2	
2^2+2	6	6
3^2+3	12	
4^2+4	20	8
5^2+5	30	10
6^2+6	42	12

7

$$\begin{cases} a_0 = 1 \\ a_n = a_{n-1} + 1 + (-1)^n \cdot 2 \end{cases}$$

$1+1-2=0$

$0+1+2=3$

$3+1-2=2$

$2+1+2=5$

$5+1-2=4$

	1	0
	0	1
	3	2
-1	2	3
$+3$	5	4
-1	4	5
$+3$	7	6
-1	6	7
	9	8
	8	9
		10

8

$$\begin{cases} a_0 = 1 \\ a_n = a_{n-1} \cdot (n+1) \end{cases}$$

1	0
2	1
6	2
24	3

Exercice 3

(1) $a_n = 2 \cdot 3^n$

0	2
1	$(2) \cdot 3$
2	$(2) \cdot 3 \cdot 3$

(2) $a_n = a_{n-1} + 2$

$a_0 = 3$

0	3
1	$3+2$
2	$3+2+2$

$$a_n = 3 + 2 \cdot n$$

(3) $a_n = a_{n-1} + n$

$a_0 = 1$

0	1
1	1+1 = 2
2	1+1+2 = 4
3	1+1+2+3 = 7
4	1+1+2+3+4 = 11
5	1+1+2+3+4+5 = 16
6	

$$a_n = \frac{(n-1)(n)}{2} + 1$$

$a_1 = 1$

$a_2 = 3$

$a_3 = 6$

$a_4 = 10$

$a_5 = 15$

(4) $\begin{cases} a_n = a_{n+1} + 2n + 3 \\ a_0 = 4 \end{cases}$

0	4
1	$4 + 2\cdot 1 + 3 = 9$
2	$4 + 2\cdot 1 + 3 + 2\cdot 2 + 3 = 16$
3	$4 + 2\cdot 1 + 3 + 2\cdot 2 + 3 + 2\cdot 3 + 3 =$
	$4 + 2(1 + 2 + 3) + 3\cdot(3)$ 25 .

$$a_n = 4 + 2\left(\frac{n(n+1)}{2}\right) + 3\cdot n$$

(5) $\begin{cases} a_n = 2a_{n-1} - 1 \\ a_0 = 1 \end{cases}$

0	1
1	$2 \cdot 1 - 1 = 1$
2	$2(2 \cdot 1 - 1) - 1 = 1$
3	$2(2(2 \cdot 1 - 1) - 1) - 1 = 1$

$a_n = 1$

(6) $a_n = 3a_{n-1} + 1 \quad a_0 = 1$

$0 \quad 1$

$1 \quad 3 \cdot 1 + \underline{1} = 4$

$2 \quad 3(3 \cdot 1 + 1) + \underline{1} = 13$

$3 \quad 3(3(3+1)+1)+1 = 40$

$= 3(3(3+1)) + 3 + \underline{1}$

$= 3(3(3))) + 3^2 + 3 + \underline{1}$

$4 \quad 3^4 + 3^3 + 3^2 + 3 + \underline{1}$

$$a^n = \sum_{i=0}^{n} 3^i$$

$$= \frac{a(1-r^{n+1})}{1-r} \quad , \quad r = 3$$

$$a = a_0 = 1$$

$$= 1\left(\frac{1-3^{n+1}}{1-3}\right)$$

$$= \frac{1-3^{n+1}}{2}$$

7 $\begin{cases} a_0 = 5 \\ a_n = n a_{n-1} \end{cases}$

0	5		
1	$1(5) = 5$		
2	$2(5) = 10$	$2(1(5))$	
3	$3(10) = 30$	$3(2(1(5)))$	
4	$4(30) = 120$	$4(3(2(1(5))))$	
5	$5(120) = 100 + 500 = 600$		

$$a_n = \prod_{i=0}^{n} i \cdot 5 = n! \cdot 5$$

8 $\begin{cases} a_n = 2n a_{n-1} \\ a_0 = 1 \end{cases}$

0	1
1	2(1)(1)
2	2(2(2))
3	2(3)(2(2))
4	2(4)(2)(3)(2(2))
5	2(5)(2)(4)( )

$$\boxed{a_n = n! \cdot 2^n}$$

Exercise 4

(1)
$$\begin{cases} a_0 \\ a_{n+1} = 3 \cdot a_n \end{cases}$$

(2)
$$\begin{cases} a_0 = 100 \\ a_n = 3^n \cdot a_0 \end{cases}$$

$$a_{10} = 3^{10} \cdot 100$$

$$= 5\,904\,900.$$

Exersise 5

B is contable D is uncontable

B ressemble à $\mathbb{N}$

$[0,1] \setminus \{0,1111\ldots\} \subseteq D$.

(finite)

uncontable

Exercise 6

$a_0 = 1$
$a_1 = 1/2$
$a_2 = 1/3$
$a_3 = 1/4$

1	2/1	3/1	4/1	5/1
1/2	2/2	3/2	4/2	5/2
1/3	2/3	3/3	4/3	5/3
1/4	2/4	3/4	4/4	5/4
1/5	2/5			
1/6	2/6			

4	44	444	4444
4,44	44,44	444,44	4444,44
4,444	44,444	444,444	4444,444
4,4444	44,4444	444,4444	4444,444
4,44444	4,44444		

Exercice 7

v) On associe les nb

$(a \cdot b)k$ à $(a, \ldots)$

$$|A \times B| = |A| \, (a \cdot b)k + 1 \text{ à } (b, \ldots)$$

$$|A| \times |B|$$

$$\{a, b, c\} \times \{d, e\} \quad (a \cdot b)k + (a \cdot b) \text{ à } (\ldots, \ldots)$$

(a, d) (b, d) (c, d)

(a, e) (b, e) (c, e)

$\mathbb{R}$ is uncountable. (F)

but $\mathbb{N} \subseteq \mathbb{R}$,
and $|\mathbb{N}|$ is infinite
and $\mathbb{N}$ is countable

$\Rightarrow$ therefore, false.

(V) $\mathbb{N} \cup \{x \in \mathbb{R},\ 0 < x < 1\}$

$\subseteq\ ]0,1[$

$\uparrow$

uncountable.

$\Rightarrow$ uncountable

(v)

A set of real numbers

B set of real numbers with only irrational and integers.

$A \cap B$

$\Rightarrow$ set of integers

$\mathbb{N}$.

Ex. 8

A

x	i	n
0	1	16
1	2	
2	4	
3	6	
4	8	
5	10	
6	12	
7	14	
8	16	

$$a = \frac{n}{2}$$

B

y	j	n
		16
0	1	
1	2	
2	4	
3	9	
4	16	

$$b = \sqrt{n}$$