Exercice 1

O

~~O~~ $k = 3k'$ $k = 3k' + 1$

$k = 3k' + 2$

O

O

Exercice 2

$x < y$ (il suffit d'inverser pour retrouver la prop)

Exercice 3

~~O~~

O

O

O

Exercice 4

1) Faux. Contre ex.

(2)

$$\neg\left(\forall x \exists y\, P(x,y)\right)$$

$$\equiv \exists x \forall y \neg P(x,y)$$

De Morgan Law

Exercice 5

① Faux

e.g.: $p = \text{false}$

$r = \text{false}$

$$\underbrace{(p \to q)}_{T} \to \underbrace{r}_{f} \qquad (T)$$

$$\underbrace{T \to f}_{F}$$

$$\underbrace{p}_{F} \to (q \to r)$$

$$\underbrace{F \to (q \to r)}_{T}$$

② $(\neg p \vee q) \wedge (\neg p \vee r)$ F $\Rightarrow$

$\rightarrow$ false

p^{T} et q F

a p^{T} et r F

$\neg p \vee q \vee r$

$\rightarrow$ True

F $\Rightarrow$ p (T)
q (F)
r (F)

p (T)
r (F)
q (T)

FAUX

③ $(\neg p \vee r) \wedge (\neg q \vee r)$

$\equiv (\neg p \wedge \neg q) \vee (\neg p \wedge r)$ $\neg p (\neg q \vee r)$

$\vee (r \wedge \neg q)$ $\vee ((r \wedge \neg q) \vee r$

$\vee (r \wedge r)$ $r (\neg p \wedge \neg q$

Ok

$\boxed{\neg p \vee q \vee r}$

proof!

r (F)

q (F)

$\neg p$ (F) $\Rightarrow$ p (T)

Exercice 6

s
g
h
r

- $s \rightarrow h \wedge r$
- $g \rightarrow h \wedge r$

① $(s \vee g) \rightarrow (h \wedge r)$ disjuncto ?

② $\neg r$

③ $s \rightarrow h \wedge r$ conjuncto

④ $s \rightarrow h$

⑤ $s \rightarrow r$

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⑥ $\neg s$ M.T (②,⑤)

7

1 $\forall x\,(P(x) \lor Q(x))$

2 $\forall x\,(\neg Q(x) \lor S(x))$

3 $\forall x\,(R(x) \to \neg S(x))$

4 $\exists x\,\neg P(x)$

5	$\neg P(b)$	E.I 4
6	$P(b) \lor Q(b)$	U.I 1
7	$\neg Q(b) \lor S(b)$	U.I 2
8	$R(b) \to \neg S(b)$	U.I 3
9	$Q(b)$	?? 5 7
10	$S(b)$	D. Syllogism 7 9

11 $\quad \neg R(b) \quad$ M.T 8 10

$\therefore \quad \exists x \, \neg R(x) \quad$ E.G 11

$\forall x\,((P(x) \vee Q(x)) \wedge (\neg Q(x) \vee S($

Exercice 7

$M(La)$ $\quad$ $S(La, Li)$

$\neg M(J)$ $\quad$ $S(Li, J)$

$\forall x \, \neg S(J, x)$

Lars can see Jeff

Exercice 8

① p : "x is irrational".

Let's assume $\neg p$.

Then $\sqrt{3} = \dfrac{a}{b}$, $\overbrace{a}^{\in \mathbb{Z}}, \overbrace{b}^{\in \mathbb{Z}^*}$; pgcd$(a,b) = 1$

$\Rightarrow 3 = \dfrac{a^2}{b^2}$

$\Rightarrow a^2 = 3b^2$ (1)

$\Rightarrow a = 3k$

(1) $\Rightarrow (3k)^2 = 3b^2$

$\Rightarrow 9k^2 = 3b^2$

$\Rightarrow b^2 = 3k^2$ (2)

$\Rightarrow b = 3k'$

Contradict° pgcd$(3k, 3k') \neq 1$. Then p.

(2) $\log_2(9) = 10$
$\iff 9 = 2^{10}$

Let's assume $\log_2(9)$ rational.

$$\log_2(9) = \frac{a}{b}, \quad \begin{cases} a \in \mathbb{Z},\ b \in \mathbb{Z}^* \\ \text{pgcd}(a,b) = 1. \end{cases}$$

$$\Rightarrow 9 = 2^{a/b}$$

$$\Rightarrow 9 = \sqrt[b]{2^a}$$

$$\Rightarrow \underbrace{9^b}_{\text{odd}} = \underbrace{2^a}_{\text{even}}$$

impossible, 9 et 2 premiers entre eux

(3)

$\exists x, y$

$\left(\text{is\_irr}(a) \land \text{is\_irr}(b) \land \text{is\_r}(x^y) \land (x = \sqrt{3} \lor y = \sqrt{3})\right)$

Non constructive ex.. proof. Let's assume

$\neg \exists x, y (\ldots)$

s'il n'existe pas de nb x, y

qui donne un rationnel x^y,

avec soit $x = \sqrt{3}$ soit

$y = \sqrt{3}$,

ce serait une contradict°.

- $\sqrt{3}^{\,y} \neq \frac{a}{b} \Rightarrow$ contradiction
- $x^{\sqrt{3}} \neq \frac{c}{d} \Rightarrow$ contradiction

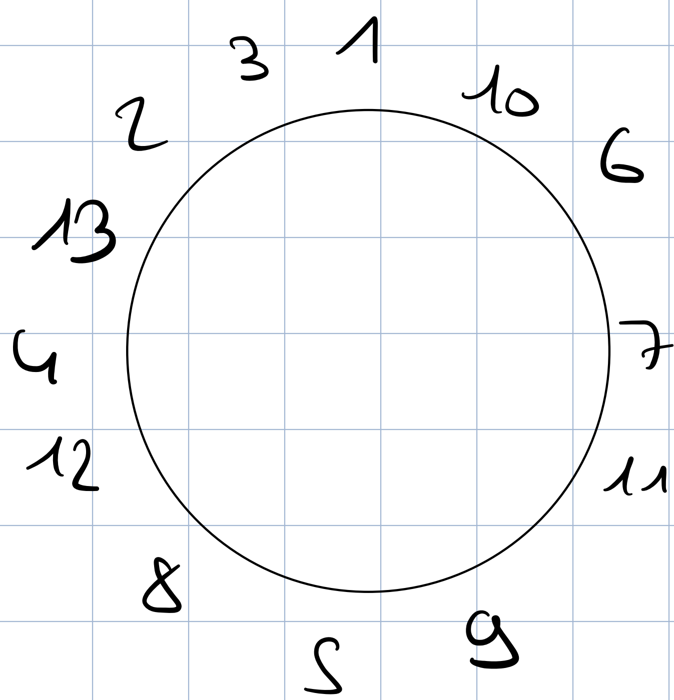
Let's suppose $\sqrt{3}^{\sqrt{3}}$ is irrational

then $\left(\sqrt{3}^{\sqrt{3}}\right)^{\sqrt{3}} = 3$ rational

4

$$\forall x \; \exists y \; (2x - y = -1)$$

Let's assume $\forall x \; \forall y \; (2x - y \neq -1)$



$$e_i + e_{i+1} + e_{i+2} + e_{i+3} + \text{reste} = 28$$

permutations $= 12!$

13

12 45.

11

10

$$\binom{13+5-1}{5}$$

$$= \binom{17}{5}$$

$$= \frac{17!}{5! \cdot \cancel{12!}}$$

$$= \frac{17 \times 16 \times 15 \times 14 \times 13}{}$$

$$6188$$