

Exercise 1

FALSE

$$(p \vee \neg q) \wedge (\neg p \vee q)$$

$$\cancel{p \wedge \neg p} \vee (p \wedge q) \vee (\neg q \wedge \neg p) \vee \cancel{q \wedge q}$$

$$\equiv p \leftrightarrow q$$

③

$$(p \vee q) \wedge (\neg p \vee \neg q)$$

$$\equiv (p \vee \neg q) \wedge (\neg p \vee q) \quad ?$$

$$\cancel{(p \wedge \neg p)} \vee \underline{(p \wedge q)} \vee (q \wedge \neg p) \vee \cancel{(q \wedge \neg q)}$$

$$(\cancel{p \wedge \neg p}) \vee (p \wedge q) \vee (\neg q \wedge \neg p) \vee (\cancel{\neg q \wedge q})$$

Common & compare?

Exercise 2

$$\textcircled{1} \quad p \oplus (q \wedge r)$$

$$\equiv (p \wedge \neg(q \wedge r)) \vee (\neg p \wedge (q \wedge r))$$

$$\equiv (p \wedge (\neg q \vee \neg r)) \vee (\neg p \wedge q \wedge r)$$

$$\equiv (p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \wedge q \wedge r)$$

$$\textcircled{2} \quad [(p \rightarrow q) \vee (r \rightarrow s)] \wedge u$$

$$\equiv [\neg p \vee q \vee \neg r \vee s] \wedge u$$

Exercise 3

- ☐ The 17-year old student drinking soda. ①
- ☒ The 17-year old student drinking vodka. ②
- ☐ The 18-year old student drinking water. ③
- ☐ The 18-year old student drinking tequila. ④

	$A(x)$	\rightarrow	$18(x)$	
①	F	\rightarrow	F	\Rightarrow T
②	T	\rightarrow	F	\Rightarrow F <u>answer</u>
③	T	\rightarrow	T	\Rightarrow T
④	T	\rightarrow	T	\Rightarrow T

- ① ☐ The 17-year old student drinking soda.
- ② ☐ The 17-year old student drinking vodka.
- ③ ☐ The 18-year old student drinking water.
- ④ ☐ The 18-year old student drinking tequila.

$A(x) \quad \wedge \quad 18(x)$

①	F	\wedge	F	\Rightarrow	F
②	T	\wedge	F	\Rightarrow	F
③	T	\wedge	F	\Rightarrow	F
④	T	\wedge	T	\Rightarrow	T

Exercice 4

- ① Tous les animaux, s'ils sont des lapins, sautent.
- ② Il existe un animal qui, s'il est un lapin, saute.
- ③ Tous les animaux sont des lapins qui sautent.
- ④ Il existe un animal qui est un lapin et qui saute

Exercice 5

$$\exists x \forall y \neg P(x, y)$$

\neg (pour tout x , il y a un y qui vérifie)

\Rightarrow il existe un x avec aucun y qui vérifie

\Rightarrow il existe un x tel que pour tous les y ça vérifie pas.

Exercise 6

$$① \forall x L(x, \text{"Sharon"})$$

$$② \forall x \exists y L(x, y)$$

$$③ \exists y \forall x L(x, y)$$

$$④ \neg(\exists x \forall y L(x, y))$$

$$\equiv \forall x \exists y \neg L(x, y)$$

$$⑤ \exists y \neg L(\text{"Daisy"}, y)$$

$$⑥ \exists y \forall x \neg L(x, y)$$

$$⑦ \exists! y \forall x L(x, y) \equiv \exists y \forall x (L(x, y) \vee y = x)$$

$$⑧ \exists! y \exists! z (y \neq z \wedge L(\text{"Marsus"}, y) \wedge L(\text{"Marsus"}, z))$$

$$(9) \quad \forall x \, L(x, x)$$

$$(10) \quad \exists x \, \forall y \, (L(x, x) \wedge \neg L(x, y))$$

est-ce ces deux brues sont équiv?

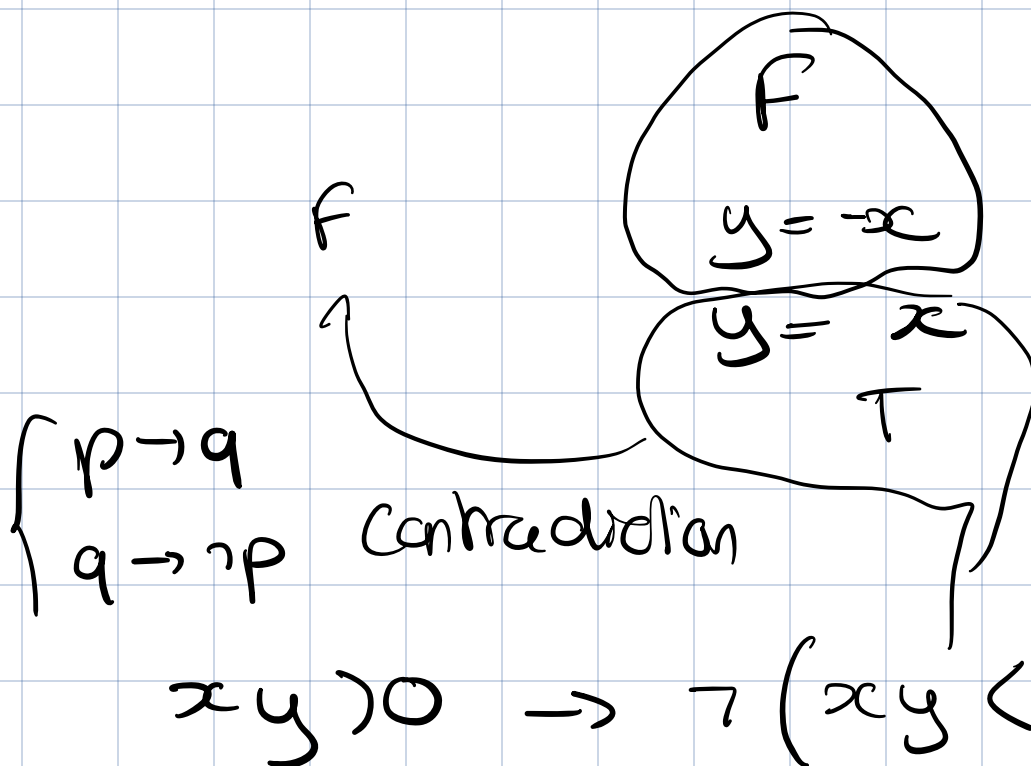
$$\exists x \, \forall y \, ((x \neq y) \rightarrow \neg L(x, y))$$

$$\exists x \, \forall y \, ((x \neq y) \rightarrow \neg L(x, y) \wedge (x = x) \rightarrow L(x, x))$$

Exercice 1

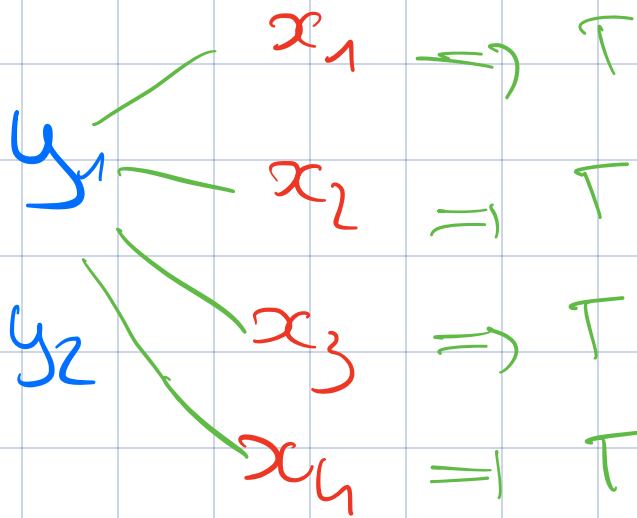
$$(1) \quad \text{faux} \quad (\forall x \, \exists y \, (x \neq 0 \rightarrow xy =$$

(2)



Exercise 8

① $\forall a_i$



②

$\forall y, z \quad y \neq z \Rightarrow \text{FALSE}$
counterexample
 $y=2 \quad z=2$

$\exists x \quad x^x = x! \Rightarrow \text{FALSE}$
counterexample
 $x=1$

$F \rightarrow F \Rightarrow \textcircled{T}$

Exercise 9

x
 y
⋮

$x \neq y$

$$\forall x \in E, \forall y \in E, \exists p_1 \in P, \\ \exists p_2 \in P (C(p_1, x, y) \wedge C(p_2, x, y)) \\ p_2 \neq p_1$$

②

$$\neg (\exists x, y \in E (x \neq y \wedge \forall p, q (p = q \\ \vee \neg C(p, x, y) \vee \neg C(q, x, y))))$$

$$\forall x, y \in E \rightarrow (\text{---})$$

$$\forall x, y \in E (x = y \vee \exists p, q \in P \\ \neg (\text{---}))$$

$$\forall x, y \in E (x = y \vee \exists p, q \in P \\ (p \neq q \wedge C(p, x, y) \wedge \dots))$$

REPONSE 3

Exercise 10

$$\exists! x T(x)$$

Inverse du \odot

$$\exists x \neg (\text{---})$$

$$\equiv \exists x \neg (T(x) \vee \exists y \neq x T(y))$$

$$\equiv \exists x (\neg T(x) \wedge \forall y \neq x \neg T(y))$$

$$\exists x (T(x) \wedge \forall y [\neg T(y) \vee y=x])$$

$$\neg (\forall x (T(x) \rightarrow \exists y \neq x (T(y)))$$

$$\equiv \exists x \neg (T(x) \rightarrow \exists y \neq x T(y))$$

$$\equiv \exists x (\neg T(x) \vee \exists y \neq x T(y))$$

$$\equiv \exists x (T(x) \wedge \forall y \neq x \neg T(y))$$

Exercise 11

$$\left(\forall x (G(x) \rightarrow A(x)) \right) \\ \wedge \left(\exists! y (F(x) \wedge \neg A(x)) \right),$$

$$\neg((\exists y(G(y) \wedge \neg A(y))) \wedge \neg(\forall x(F(x) \rightarrow A(x)))$$

$$\forall y \neg(G(y) \wedge \neg A(y))$$

$$\wedge \exists x \neg(F(x) \rightarrow A(x))$$

$$\forall y(\neg G(y) \vee A(y)) \wedge$$

$$\exists x \neg(\neg F(x) \vee A(x))$$

$$\equiv \forall y(G(y) \rightarrow A(y)) \wedge \exists x(F(x) \wedge \neg A(x))$$

RESPONSE 1