Exercice 1

ⓐ $\vec{v_1} = 1\vec{v_2} + 2\vec{v_3}$

$W = \text{Span}\{\vec{v_2}, \vec{v_3}\}$

2 dimensions (2 vecteurs forment une base)

ⓑ $\{\vec{v_2}, \vec{v_3}\}$ (car indep)

ⓒ $\vec{v_1} + \vec{v_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

ex : $\vec{v_3} = \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\}$. $(\vec{v_1} + \vec{v_2}, \vec{v_3}$ indep

Exercice 2

$$\vec{x} = a\begin{pmatrix}1\\5\\0\\0\end{pmatrix} + b\begin{pmatrix}-3\\0\\1\\0\end{pmatrix} + c\begin{pmatrix}6\\0\\-2\\0\end{pmatrix} + d\begin{pmatrix}0\\4\\-1\\5\end{pmatrix}$$

$$\begin{pmatrix}1 & -3 & 6 & 0\\5 & 0 & 0 & 4\\0 & 1 & -2 & -1\\0 & 0 & 0 & 5\end{pmatrix}$$

$$\rightarrow \begin{pmatrix}1 & -3 & 6 & 0\\5 & 0 & 0 & 0\\0 & 1 & -2 & 0\\0 & 0 & 0 & 1\end{pmatrix}$$

$$\rightarrow \begin{pmatrix}1 & 0 & 0 & 0\\1 & 0 & 0 & 0\\0 & 1 & -2 & 0\\0 & 0 & 0 & 1\end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

<u>dimension 2</u>

Exercice 3

$$[x]_B = 3\begin{pmatrix}1\\-4\\3\end{pmatrix} + 0\begin{pmatrix} \\ \\ \end{pmatrix} - \begin{pmatrix}4\\-7\\0\end{pmatrix}$$

$$= \begin{pmatrix}3 - 4\\-12 + 7\\9\end{pmatrix}$$

$$= \begin{pmatrix}-1\\-5\\9\end{pmatrix}$$

on cherche a, b, c t.q

$$a\vec{b_1} + b\vec{b_2} + c\vec{b_3} = \vec{y}.$$

$$\left(\begin{array}{ccc|c} 1 & 5 & 4 & 10 \\ -4 & 2 & -7 & -9 \\ 3 & -2 & 0 & 1 \end{array}\right)$$

à la fin ça donne $\vec{y} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ d'après Pedro.

Exercice 4

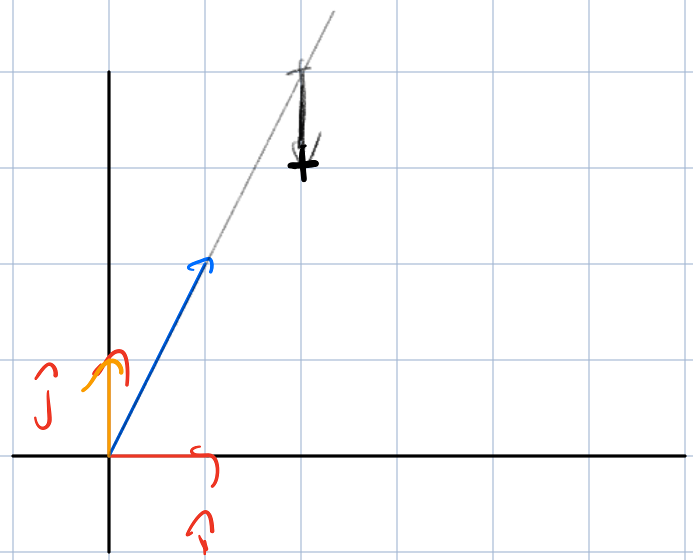
ⓐ $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = P_B \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2+0 \\ 4+3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 7 \end{pmatrix}$

ⓑ $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 2+3+0 \\ 0+3+0 \\ 2+0+1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix}$



$$P_B \left[\vec{x}\right]_B$$

$= \vec{x}$ en base cano.

$$P_B = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$$

$$P_B^{-1} = \frac{1}{\det P_B} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

$$P_B^{-1} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2+0 \\ -4+3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right)$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & -3 & +0 \\ 0 & +3 & +0 \\ -2 & +3 & +1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$

Exercice 5

ⓐ

$$c_1(1-3t^2)$$
$$+ c_2(2+t-5t^2)$$
$$+ c_3(1+2t) = 0$$

$$\Leftrightarrow \quad c_1 + 2c_2 + c_3$$
$$+ t(c_2 + 2c_3)$$
$$+ t^2(-3c_1 - 5c_2) = 0$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -3 & -5 & 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ -3 & -5 & 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & -5 & -9 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Ok. Indep.

(b)

$$\left\{ \begin{array}{l} t \mapsto 1 \\ t \mapsto t \\ t \mapsto t^2 \end{array} \right\}$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -3 & -5 & 0 \end{pmatrix}$$

(c) $a\begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix} + b\begin{pmatrix} 2 \\ 1 \\ -5 \end{pmatrix} + c\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ -3 & -5 & 0 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ -3 & -5 & 0 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -5 & -9 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$-3\left(\vec{b_1}\right)+2\left(\vec{b_2}\right)-1\left(\vec{b_3}\right)$$

Faire

(3, -2 ankip)

Exercice 6

a

$$c_1(1)$$
$$+c_2(2t)$$
$$+c_3(-2+4t^2)$$
$$+c_4(-12t+8t^3)$$
$$=0$$

$$\Leftrightarrow (c_1-2c_3)$$
$$+t(2c_2-12c_4)$$
$$+t^2(4c_3)$$
$$+t^3(8c_4)=0$$

$$\Leftrightarrow \begin{cases} c_1 - 2c_3 = 0 \\ 2c_2 - 12c_4 = 0 \\ 4c_3 = 0 \\ 8c_4 = 0 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 2 & 0 & -12 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \end{array}\right)$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \boxed{\text{Ok}}$$

ⓑ $\begin{cases} c_1 - 2c_3 = 7 \\ 2c_2 - 12c_4 = -12 \\ 4c_3 = -8 \\ 8c_4 = 12 \end{cases}$

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 7 \\ 0 & 2 & 0 & -12 & -12 \\ 0 & 0 & 4 & 0 & -8 \\ 0 & 0 & 0 & 8 & 12 \end{array}\right)$$

$$\to \left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 7 \\ 0 & 2 & 0 & -12 & -12 \\ 0 & 0 & 2 & 0 & -4 \\ 0 & 0 & 0 & 8 & 12 \end{array}\right)$$

$$\to \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & -12 & -12 \\ 0 & 0 & 2 & 0 & -4 \\ 0 & 0 & 0 & 8 & 12 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & -12 & -12 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 8 & 12 \end{array}\right) \qquad \frac{8\cdot}{8} \qquad \frac{12\cdot 12}{8}$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & 0 & -12+\left(\frac{12\cdot 12}{8}\right) \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 12/8 \end{array}\right) \qquad \cdot 12$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & 0 & -12+18 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 12/8 \end{array}\right) \qquad \begin{array}{l} 12\cdot 12 \\ 12\cdot 6\cdot 2 \end{array}$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 6/2 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 12/8 \end{array}\right) \qquad \begin{array}{l} (60+12)\cdot 2 \\ 72\cdot 2 \\ = 144 \end{array}$$

$$\begin{pmatrix} 3 \\ 3 \\ -2 \\ 3/2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6/2 \\ -2 \\ 12/8 \end{pmatrix} \qquad 12/8 = 6/4 = 3/2$$

Exercice 7

$$\begin{pmatrix} c_1 - c_2 + 2c_4 & 2c_1 + 2c_3 + c_4 + 3c_5 \\ 2c_1 + 2c_3 + c_4 + 3c_5 & c_1 + c_2 + 2c_2 \end{pmatrix}$$

$$\begin{pmatrix} c_1 - c_2 & 2c_1 + 2c_3 \\ 2c_1 - 2c_3 & c_1 + c_2 + c_3 \end{pmatrix}$$

- $c_1 = c_2$

$$= \begin{pmatrix} 0 & 2c_1 + 2c_3 \\ 2c_2 - 2c_3 & 2c_2 + c_3 \end{pmatrix}$$

- $c_3 = 2c_2$

$$= \begin{pmatrix} 0 & 2c_2 + 2c_3 \\ 2c_3 - 2c_3 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & c_3 \\ 0 & 0 \end{pmatrix}$$

On a 3 matrices indépendantes

(+) on sait que la base est composée de 3 mat.

Exercice 8

$$\alpha(\underbrace{a + bt + ct^2 + dt^3}_{u})$$

$$= \alpha a + \alpha bt + \alpha ct^2 + \alpha dt^3$$

$$T(\alpha u)$$

$$= (\alpha a + \alpha b + \alpha c + \alpha d)$$

$$+ (\alpha a + \alpha b)t$$

$$+ (\alpha c + \alpha d)t^2$$

$$= \alpha(\text{———}) + \alpha(\text{—})t + \alpha(\text{—})t^2$$

$$= \alpha T(u)$$

$$T(u+v)$$

$$= (a+a'+b+b'+c+c'+d+d')$$

$$+ (a+a'+b+b')t$$

$$+ (c+c'+d+d')t^2$$

$$= (a+b+c+d) \text{ etc.}$$

$$= T(u) + T(v).$$

(b)

$$\begin{pmatrix} a+b+c+d \\ a+b \\ c+d \end{pmatrix} \begin{matrix} 1 \\ t \\ t^2 \end{matrix}$$

$$= a\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + d\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\to \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$B = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

Dimension 2

c

$$\left(\begin{array}{cc|c} 1 & 1 & 7 \\ 1 & 0 & 5 \\ 0 & 1 & 2 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 1 & 0 & 5 \\ 0 & 1 & 2 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 0 & 5 \\ 0 & 1 & 2 \end{array}\right)$$

$$5\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \\ 2 \end{pmatrix}$$

(d)

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right)$$

$T(\vec{x}) = 0$

$a + b + c + d = 0$

$a + b = 0$

$c + d = 0$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} a = -b \\ c = -d \end{cases}$$

$\alpha = a$

$\alpha = -b$

$\beta = c$

$\beta = -\alpha$

$$\text{base}(\ker T) = \left\{ \alpha \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} \right\}$$

<u>Dimension 2</u>

$$\left(\begin{array}{cc|c} 1 & 0 & 2 \\ -1 & 0 & -2 \\ 0 & 1 & -5 \\ 0 & -1 & 5 \end{array}\right)$$

$$\to \left(\begin{array}{cc|c} 0 & 0 & 0 \\ -1 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & -1 & 5 \end{array}\right)$$

coordonnées :

$$\begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

Ex. 9

① ☐

☐

☒ $\det(A) \neq 0$

☐

② $[M]_E$

☐ ☐

☒ ☐

$$\begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix}$$

③

$$\begin{pmatrix} c_1 + c_3 + c_4 & c_2 + c_3 \\ c_1 + c_2 & c_1 + c_2 \end{pmatrix}$$

- $c_1 = -c_2$

- $c_3 = -c_2$

$$\begin{pmatrix} -c_2 - c_2 + c_4 & 0 \\ 0 & 0 \end{pmatrix}$$

- $c_4 = 2c_2$

Test: $c_2 = 1$

$$-\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\boxed{1}$ ~~impossible~~

$$\begin{pmatrix} c_1 + c_3 & c_2 \\ c_1 & c_2 + c_4 \end{pmatrix}$$

$$c_1 = 0$$
$$c_2 = 0$$
$$c_3 = -c_1 = 0$$
$$c_2 = -c_4 = 0.$$

$\boxtimes$

$$\begin{pmatrix} c_1 + c_2 + c_3 & c_2 + c_3 + c_4 \\ c_1 + c_2 + c_4 & c_1 + c_2 + c_3 \end{pmatrix}$$

$$c_1 = -c_2 - c_3$$

$-c_2 - c_3 = c_2 - c_4$

$2c_2$

$-$

$$\begin{pmatrix} c_1 + c_2 & c_2 + c_3 \\ c_1 + c_2 + c_3 + c_4 & c_2 + c_3 \end{pmatrix}$$

- $c_2 = -c_1$
- $c_3 = -c_2$

$$\begin{pmatrix} 0 & 0 \\ c_3 + c_4 & 0 \end{pmatrix}$$

- $c_3 = -c_4$ Seulement si linéaire

- $\boxed{c_3 = -c_4 = -c_2 = c_1 = 0}$

Exercice 10

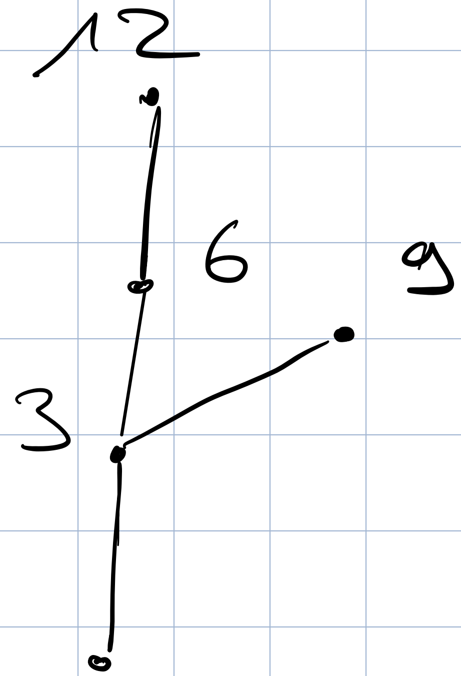
(a) Faux (vect nul

(b) $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

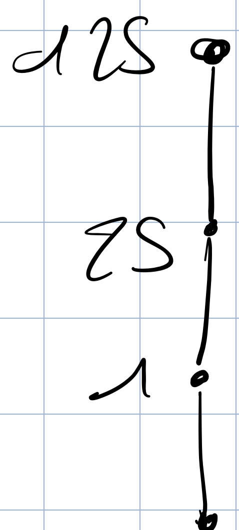
(c) V.

(d) $\underset{m \times n}{A} \underset{n \times 1}{\vec{x}} = 0$ V.

Exercice 7



NON



OUI

$$S = \{a, b\}$$

$$P(S) = \{\{a\}, \{b\}, \phi\}$$

$$\begin{tikzcd}[column sep=small]
 & \{a,b\} \arrow[dl, no head] \arrow[dr, no head] & \\
\{a\} \arrow[d, no head] & & \{b\} \arrow[dll, no head] \\
\phi & &
\end{tikzcd}$$

S^* uncountable

A uncountable

$$(-2)^{-2} = \frac{-1}{4}$$

$$+ \ (1-2)^{1-2} = -\underline{1}$$

$$+ \ 1 = 1$$

$$\boxed{\begin{array}{l} 1 \to 1 \\ (1-2)^{1-2} = -1 \end{array}}$$

$$-1 \to 3$$

$$(-1-2)^{-1-2} = (-3)^{-3} = \cancel{-\frac{1}{27}}$$

$$(-2)^{-2} = + \cancel{\frac{1}{4}}$$

$$= \cancel{-1}$$

$= \cancel{1}$

$$(3-2)^{3-2} = \boxed{+\ 1}$$

$$(-2)^{-2} = \boxed{\frac{1}{4}}$$

$$\cancel{-1}$$

$$\cancel{1}$$

$$A\left(\frac{1}{i}\right) + B\left(\frac{1}{i+2}\right)$$

$$\frac{A(i+2)}{i(i+2)} + \frac{Bi}{i(i+4)}$$

$$= \frac{Ai + 2A + Bi}{i(i+2)}$$

$$i(A+B) + 2A = 1$$

$$A = \frac{1}{2}$$

$$B = -A = -\frac{1}{2}$$