Homework 8

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Exercise 1. The Selection Sort algorithm sorts the input sequence $a_1, ..., a_n$ of real numbers in ascending order

Algorithm 1 Selection Sort

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for i = 1 to n − 1 do
    min ← i + 1
    for j = i + 1 to n do
        if a_min > a_j then
            min ← j
        end if
    end for
    if a_i > a_min then
        swap a_i and a_min
    end if
end for
```

Which sequence will result in the most operations when using Selection Sort (swaps and comparisons)?

- ○ 9, 3, 8, 6, 5, 1, 7
- ⊗ 4, 7, 2, 9, 5, 6, 3
- ○ 0, 1, 2, 3, 4, 5, 6 → really fast
- ○ 7, 6, 5, 4, 3, 2, 1 → really fast

Exercise 2. Which function below grows fastest when n goes to infinity?

- ○ $(\log_3(33))^{n-3}$
- ○ 3^n
- ○ $n^{3\log_3(n)}$
- ○ $n^3 \log_3(n)$

a) $3^{\log_3\left(\log_3(33)^{n-3}\right)}$

$= 3^{(n-3)\underbrace{\log_3(\log_3(33))}_{>1}}$

(b) 3^n

(c) $3^{\log(n^3)\cdot\log(n)} = 3^{3\log(n)^2}$

(d) $3^{\log(n^3\cdot\log(n))} = 3^{3\log(n\cdot\log(n))}$

Exercise 7. Consider the two statements below, where k and ℓ are constants with $k > \ell \geq 2$ and $m \to \infty$:

$$\log_m(k) \text{ is } \Theta(\log_m(\ell)) \qquad k^{\log_\ell(m)} \text{ is } O(\ell^{\log_k(m)}).$$

- ○ They are both false.
- ☒ Only the first is true.
- ○ Only the second is true.
- ○ They are both true.

$$\left.\begin{aligned}\log_m(k) &= \frac{\log_e(k)}{\log_e(m)}\\ \log_m(\ell) &= \frac{\log_e(\ell)}{\log_e(m)}\end{aligned}\right)$$

Same order

Let $c = \log_e(k)$.

$$k^{\log_e(m)} = e^{\log_e(k) \cdot \log_e(m)} = e^{\log_e(m) \cdot c}$$

$$e^{\log_k(m)} = e^{\frac{\log_e(m)}{\log_e(k)}} = e^{\log_e(m) \cdot (-c)}$$

as $c > 1$, when $m \to \infty$, m^c is not $o(m^{1/k})$

Exercise 9. *Given the two statements below, where $d > 0$ is an integer constant and a_i for all $i \in \mathbf{Z}$ are positive integers with $\max_{i \in \mathbf{Z}}(a_i) = D$ for a constant $D > 0$,*

$$\sum_{i=0}^{n} a_i i^d \text{ is } \Theta(n^{d+1}) \qquad \sum_{i=0}^{d} a_i n^i \text{ is } \Theta(n^d)$$

- ○ *They are both true.*
- ○ *Only the first is true.*
- ○ *Only the second is true.*
- ○ *They are both false.*

$$\sum_{i=0}^{n} a_i i^d \text{ is } \Theta(n^{d+1})$$

$$= a_1 1^d + a_2 2^d + \dots + a_n n^d$$

$$\geq (n \cdot D) n^d = Dn^{d+1}$$

$$\sum_{i=0}^{n} a_i i^d \text{ is } O(n^{d+1})$$

$$\sum_{i=0}^{n} a_i i^d \leq c\, n^{d+1} \quad ?$$

$$\sum_{i=0}^{n} a_i i^d$$

$$= \sum_{i=0}^{n/2-1} a_i i^d + \sum_{i=n/2}^{n} a_i i^d$$

$$\geq \sum_{i=n/2}^{n} a_i i^d$$

try with $n=4$
$\frac{n}{2}=2$, we will do $2,3,4$
which is $\frac{n}{2}+1$.

$$\geq \sum_{i=n/2}^{n} a_i \left(n/2\right)^d$$

$$\geq m \sum_{i=n/2}^{n} \left(n/2\right)^d = m\left(\frac{n}{2}+1\right)\left(\frac{n}{2}\right)^d$$

$$\sum_{i=0}^{n} a_i i^d \text{ is } \Omega(n^{d+1}) \qquad = \frac{m}{2^{d+1}} n^{d+1}$$

$$\sum_{i=0}^{d} a_i n^i$$

$$= a_1 n + a_2 n^2 + a_3 n^3 + \dots + a_4 n^4$$ ✓