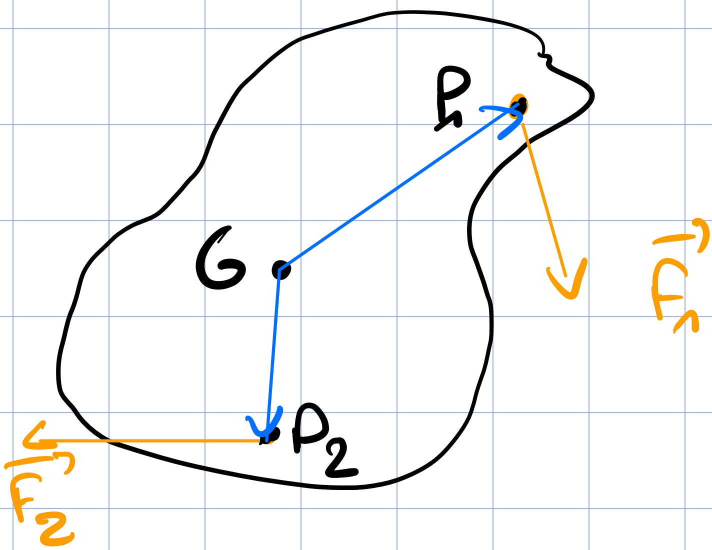
$$\frac{d\vec{L_Q}}{dt} = \vec{M_Q}^{ext} - \vec{v_Q} \wedge M\vec{v_G}$$

si $\vec{v_Q} = 0 \Rightarrow \dfrac{d\vec{L_O}}{dt} = \vec{M_O}^{ext}$

si $Q = G \Rightarrow \dfrac{d\vec{L_G}}{dt} = \vec{M_G}^{ext}$



RAPPEL

$$\begin{cases} \vec{M_G}^{(P_1)} = \vec{GP_1} \wedge \vec{F_1} \\ \vec{M_G}^{(P_2)} = \vec{GP_2} \wedge \vec{F_2} \end{cases}$$

Par rapport à un point $A \in$ Solide :

- $$\vec{L}_A = \sum_\alpha \overrightarrow{AP_\alpha} \wedge m_\alpha \vec{V}_\alpha$$

car solide

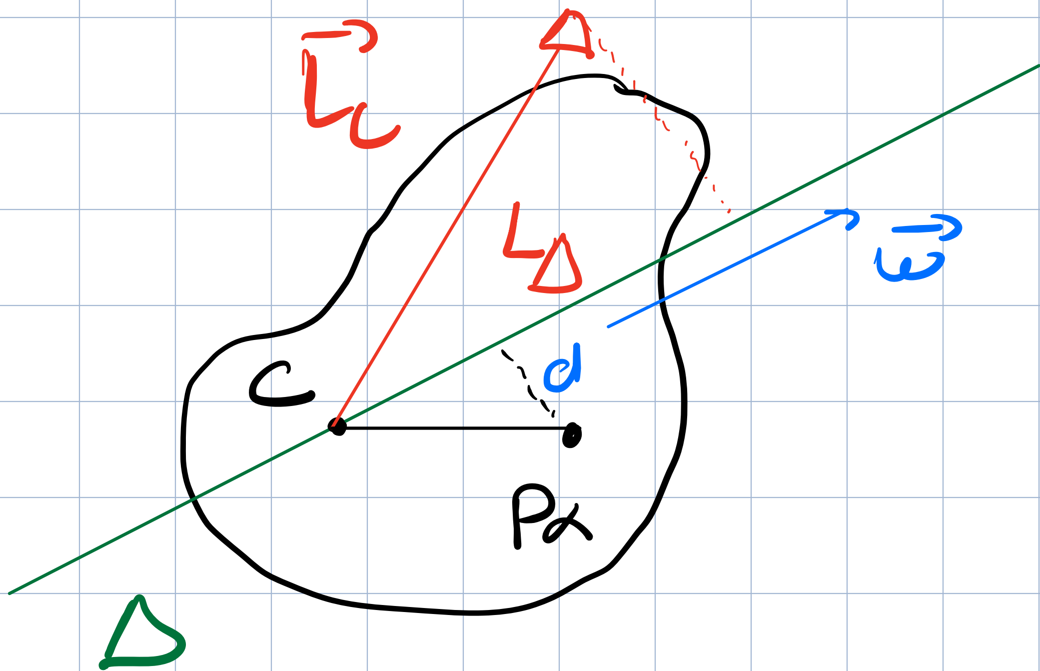
$$\stackrel{\downarrow}{=} \sum_\alpha \overrightarrow{AP_\alpha} \wedge m_\alpha \left(\vec{V}_A + \vec{\omega} \wedge \overrightarrow{AP_\alpha}\right)$$

$$= \underbrace{\sum_\alpha m_\alpha \overrightarrow{AP_\alpha}}_{M\overrightarrow{AG}} \wedge \vec{V}_A + \sum_\alpha \underbrace{\left(\overrightarrow{AP_\alpha} \wedge \left(\vec{\omega} \wedge \overrightarrow{AP_\alpha}\right)\right)}$$

$$\vec{a} \wedge (\vec{b} \wedge \vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$$

$$= \overrightarrow{AG} \wedge M\vec{V}_A + \underbrace{\sum_\alpha m_\alpha \left(\left(\overrightarrow{AP_\alpha}\right)^2 \vec{\omega} - \left(\overrightarrow{AP_\alpha} \cdot \vec{\omega}\right) \overrightarrow{AP_\alpha}\right)}_{*}$$

Rotation autour d'un axe fixe :



$\vec{L}_C =$ (*) car vitesse nulle donc 1er terme nul

$$= \sum_\alpha m_\alpha \left((\vec{CP}_\alpha)^2 \vec{\omega} - (\vec{CP}_\alpha \cdot \vec{\omega}) \vec{CP}_\alpha \right)$$

On cherche composante de $\vec{L}_C$ selon Δ

$$L_\Delta = \vec{L}_C \cdot \hat{\omega} = \vec{L}_C \cdot \frac{\vec{\omega}}{\omega}$$

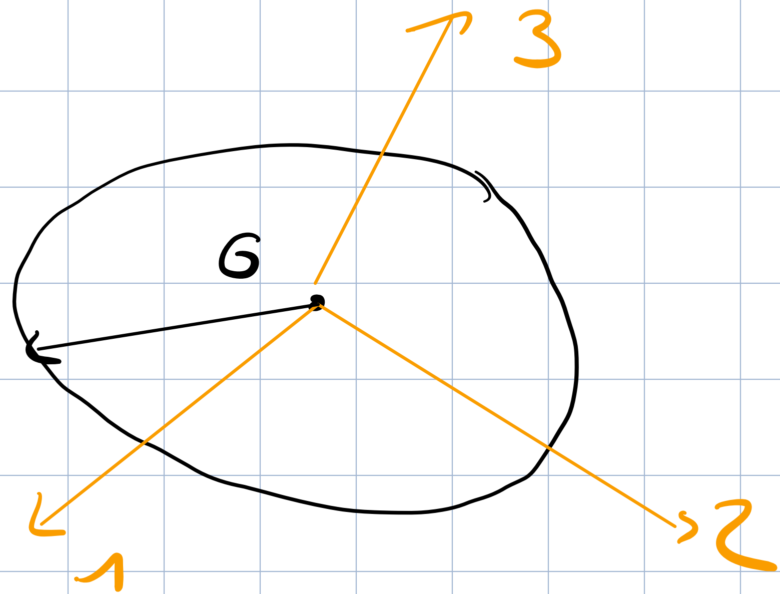
$$= \frac{1}{\omega} \cdot \sum_\alpha m_\alpha \left((\vec{CP}_\alpha)^2 \underbrace{\vec{\omega} \cdot \vec{\omega}}_{\omega^2} - \underbrace{(\vec{CP}_\alpha \cdot \vec{\omega})}_{\omega(\vec{CP}_\alpha \cdot \hat{\omega})} \cdot \underbrace{\vec{CP}_\alpha \cdot \vec{\omega}}_{\omega(\vec{CP}_\alpha \cdot \hat{\omega})} \right)$$

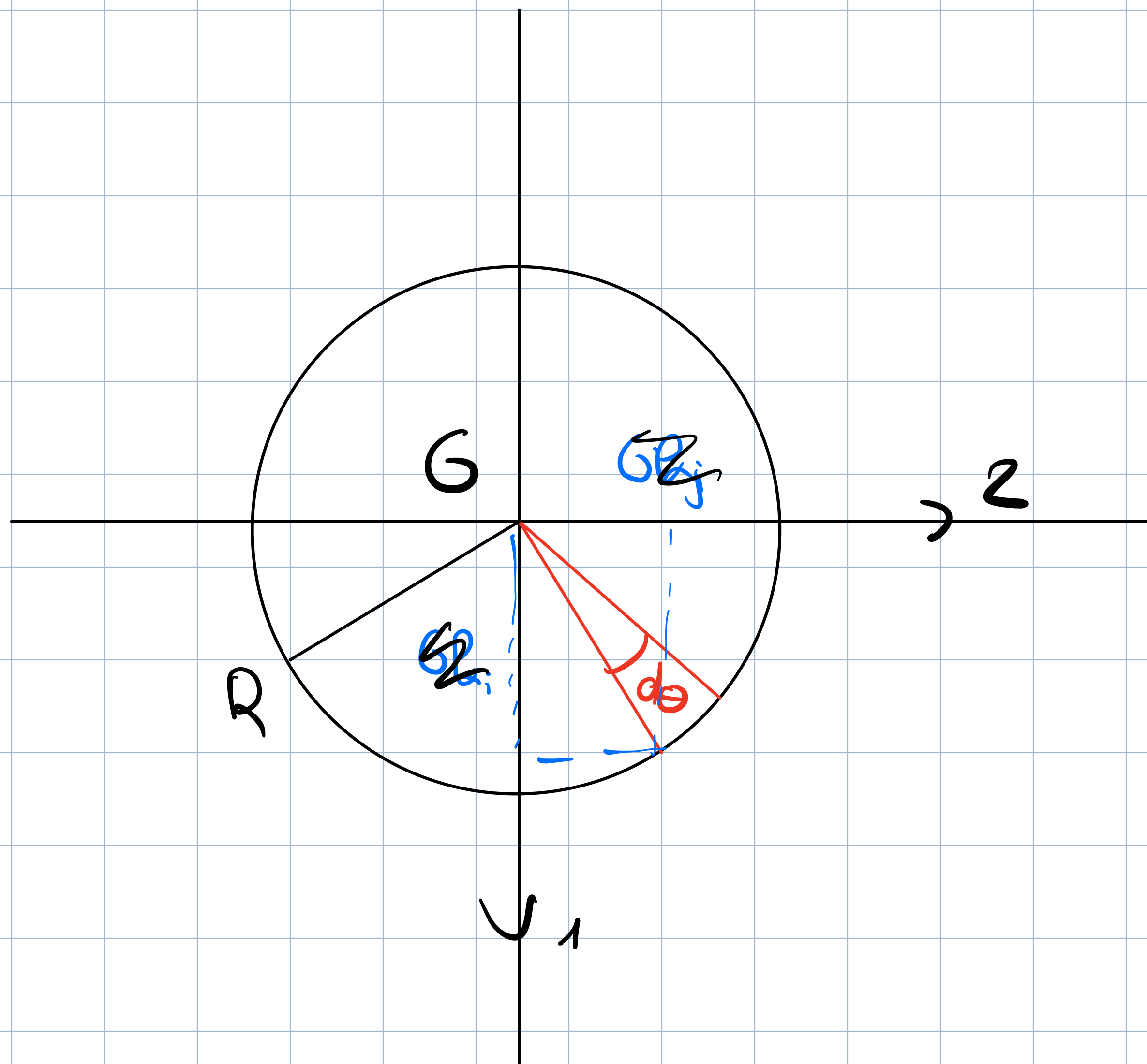
$$= \omega \sum_\alpha m_\alpha \Big(\underbrace{(\vec{CP_\alpha})^2 - (\vec{CP_\alpha} \cdot \hat{u})^2}_{d^2} \Big)$$

$$= \omega \underbrace{\sum_\alpha m_\alpha (d_\alpha)^2}_{I_\Delta}$$

il dépend que de la géométrie du solide (et de l'axe...)

moment d'inertie





$$\left.\begin{array}{l} M \longleftrightarrow 2\pi \\ \Leftrightarrow dm \longleftrightarrow d\theta \end{array}\right\} \frac{dm}{d\theta} = \frac{M}{2\pi}$$

$$(\tilde{I}_G)_{ij} \overset{\text{parcelles}}{=} \sum_\alpha m_\alpha \left[(\vec{GP_\alpha})^2 \delta_{ij} - (\vec{GP_\alpha})_i (\vec{GP_\alpha})_j \right]$$

car système est continu

$$= \int_{\text{roue}} dm \left(R^2 \cdot \delta_{ij} - x_i x_j \right)$$

$$\boxed{x_1 = R \cdot \cos\theta \qquad x_3 = 0. \qquad x_2 = R \cdot \sin\theta}$$

$$= \frac{M}{2\pi} \int_0^{2\pi} d\theta \left[R^2 \delta_{ij} - x_i x_j \right]$$

$$\left.\begin{array}{l} \text{Si } i = 3, \ j \neq i \\ j = , \ j \neq i \end{array}\right\}$$

$$\Rightarrow (\tilde{I})_{ij} = \frac{-M}{2\pi} \int^{2\pi} d\theta \, x_{1,2} \, x_3 = 0$$

$$\tilde{I}_G = \begin{pmatrix} \frac{1}{2}MR^2 & 0 & 0 \\ 0 & \frac{1}{2}MR^2 & 0 \\ 0 & 0 & MR^2 \end{pmatrix}$$

$$i = j = 3 \Rightarrow (\tilde{I})_{33}$$

$$= \left[\frac{MR^2}{2\pi}\int_0^{2\pi} d\theta\right] - 0 = MR^2$$

$$i = 1,\ j = 2 \Rightarrow (\tilde{I})_{12} = (\tilde{I})_{21}$$

$x_i x_j$

$$= -\frac{MR^2}{2\pi}\int_0^{2\pi} d\theta \cdot \sin\theta\cos\theta$$

$$= \frac{MR^2}{2\pi}\,\frac{1}{2}\sin^2\theta\,\Big|_0^{2\pi} = 0$$

$$i = j = 1 \Rightarrow (\tilde{I})_{11}$$

$$= \frac{MR^2}{\cancel{2\pi}} \cdot \cancel{2\pi} - \frac{MR^2}{2\pi} \underbrace{\int_0^{2\pi} d\theta \cos^2\theta}_{}$$

ou $\sin^2\theta$

$$= \frac{1}{2}\left[\theta + \sin\theta\cos\theta\right]$$

$$= MR^2 - \frac{MR^2}{2\pi} \frac{1}{2}\left[\theta \pm \sin\theta \cos\theta\right]_0^{2\pi}$$

$$= \frac{1}{2} MR^2$$