Homework 9

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Exercise 1. Use strong induction to show the following statement:
Any postage of at least 8 cents can be formed using just 3 cents and 5 cents stamps.

Basis Step:

$$8 = 5 + 3$$
$$9 = 3 \cdot 3$$
$$10 = 2 \cdot 5$$

Inductive Step:

Let's assume we can form any postage of k, $k-1$ and $k-2$ cents with only stamps of 3, 5.
Can we form $k+1$?

yes, take $k-2$ and add a stamp of 3.

Conclusion

it works.

Exercise 3. Find the mistake in the following proof:
Let $P(n)$ for $n \in \mathbf{Z}_{\geq 0}$ be the propositional function "all sets composed of n integers consist of only even integers," which is proved using strong induction:

Basis step $P(0)$ is true, since if S is an empty set of integers the statement "$\forall s\ s \in S \to s$ is even" is true.

Inductive step Let $k \geq 0$ and assume that $P(i)$ is true for $0 \leq i \leq k$. To prove that $P(k+1)$ is true we use the following steps:

1. Let T be an arbitrary set of integers with $|T| = k+1$.
2. Write T as the disjoint union of sets T_1 and T_2 such that $|T_1| = k$ and $|T_2| = 1$.
3. Because $|T_1| < |T|$ and $|T_2| < |T|$ the induction hypothesis applies to both T_1 and T_2, implying that all elements of both T_1 and T_2 are even.
4. Because $T = T_1 \cup T_2$ it follows that all elements of T are even as well.
5. Because T was arbitrarily chosen as a set of integers of cardinality $k+1$, it follows that $P(k+1)$ is true.

Because not all integers are even, the proof cannot be correct.

does not hold, as $|T_2|$ can be equal to 1

Exercise 4. Let $P(n)$ for $n \in \mathbf{Z}_{>0}$ be the propositional function "all sets composed of n integers consist of only odd integers," which is proved using strong induction:

all sets of 1 el. do not contain only 1 (can contain 3)

Basis Step $P(1)$ is true because 1 is odd.

Inductive step Let $k > 0$ and assume that $P(i)$ is true for $0 < i \leq k$. To prove that $P(k+1)$ is true we use the following steps:

1. Let S be an arbitrary set of integers with $|S| = k+1$.
2. Write S as the disjoint union of sets S_1 and S_2 such that $|S_1| = k$ and $|S_2| = 1$.
3. Because $|S_1| < |S|$ and $|S_2| < |S|$ the induction hypothesis applies to both S_1 and S_2,
4. implying that all elements of both S_1 and S_2 are odd.
5. Because $S = S_1 \cup S_2$ it follows that all elements of S are odd as well.
6. Because S is an arbitrarily chosen set of integers with $|S| = k+1$, it follows that $P(k+1)$ is true.

Exercise 5. Let n be a positive integer and x be an integer.

1. Give a recursive algorithm for computing nx (you can only use addition).
2. Prove that your recursive algorithm is correct.

1

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procedure compute (n, x) {
    if (n == 1) return x;
    else return x + compute(n-1, x)
}
```

2

Basis Step: compute$(1, x) = x$. $\boxed{\text{Ok}}$

Inductive Step:

Let's assume our algo works for compute(n, x).

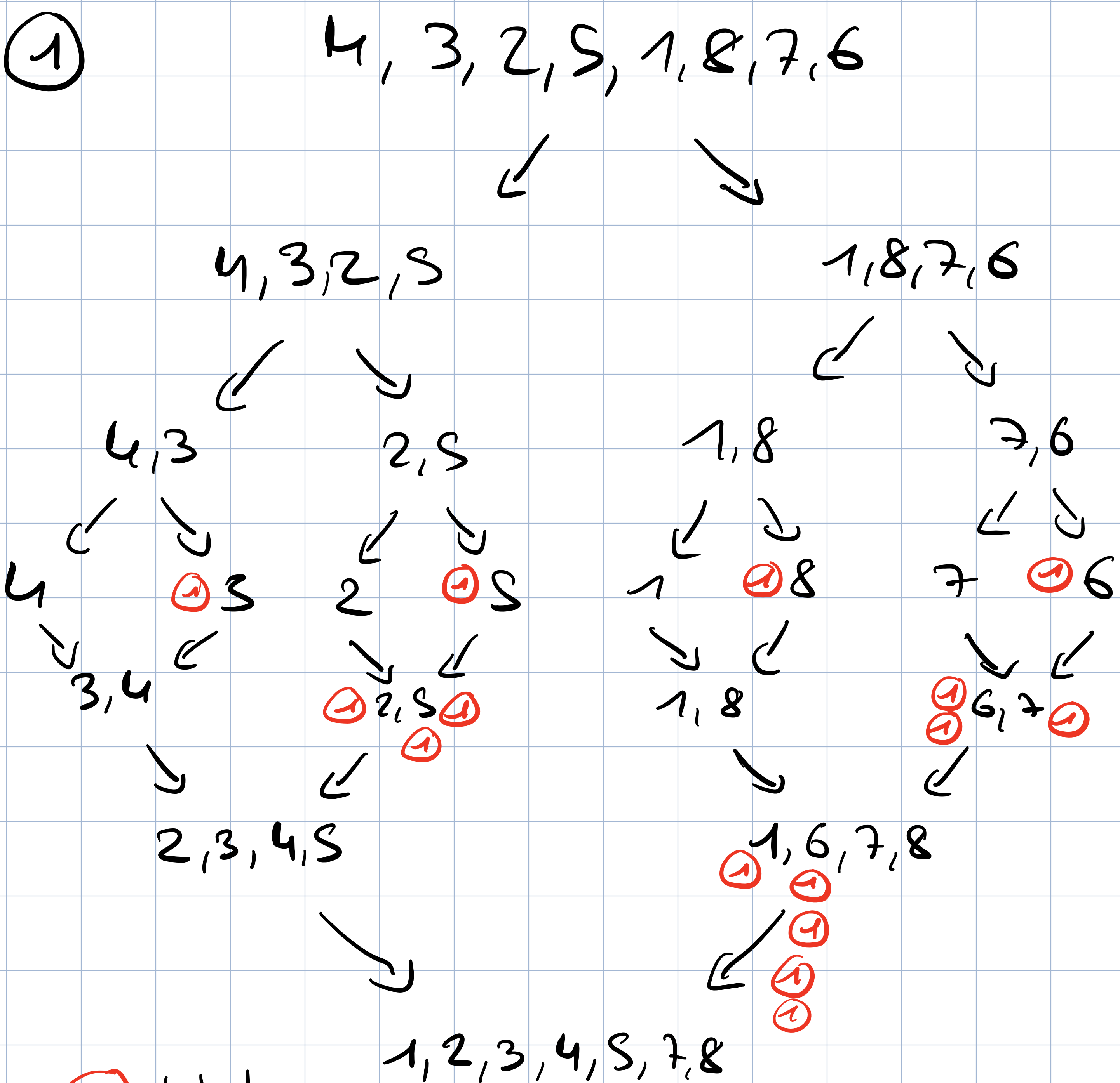
compute$(n+1, x) = x +$ compute(n, x).

Conclusion: it works.

Exercise 9. Let l be the following list of integers: 4, 3, 2, 5, 1, 8, 7, 6

1. Sort l using merge sort.
2. How many comparisons were performed ?

1



15 total