Matrice : transformation linéaire

fonction

vector $\mapsto$ vector

pas de courbes
origine ne bouge pas
lignes restent parallèles

e.g $\vec{v}\begin{pmatrix}5\\7\end{pmatrix}$ $\hat{x}\begin{pmatrix}1\\0\end{pmatrix}$ $\hat{y}\begin{pmatrix}0\\1\end{pmatrix}$

$\Downarrow$ after transformation

$\hat{x}\begin{pmatrix}3\\-2\end{pmatrix}$ $\hat{y}\begin{pmatrix}2\\1\end{pmatrix}$

$$\text{transformed } \vec{v} = 5(\text{transformed } \hat{x}) + 7(\text{transformed } \hat{y})$$

$$= \begin{pmatrix}3 & 2\\-2 & 1\end{pmatrix}\begin{pmatrix}5\\7\end{pmatrix}$$

matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with vector $\begin{pmatrix} a \\ c \end{pmatrix}$ colinéaire

$\begin{pmatrix} b \\ d \end{pmatrix}$

→ transformation leads to une droite

$$\underset{\text{2nd trans.}}{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}} \left(\underset{\text{1st trans.}}{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}} \begin{pmatrix} x \\ y \end{pmatrix} \right)$$

$\underbrace{\text{rotation} + \text{shear}}$ = composition of these transformations

$$\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$= \begin{pmatrix} ea + gb & fa + hb \\ ec + gd & fc + hd \end{pmatrix}$$

$$\underset{\text{rotate}}{M_1} \underset{\text{shear}}{M_2} \neq \underset{\text{shear}}{M_2} \underset{\text{rotate}}{M_1}$$

$$(AB)C = A(BC)$$

$\Rightarrow$ just ABC in the end

3D Transformation

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

product 2 3x3 matrix → applying 2 transformat^{c}

$$\begin{pmatrix} 0 & -2 & 2 \\ 5 & 1 & 5 \\ 1 & 4 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} -6+12 & 0-8+14 & 0-10+16 \\ 3+30 & 5+4+35 & 10+5+40 \\ 12-6 & 1+16-7 & 2+20-8 \end{pmatrix}$$

déterminant : facteur d'agrandissement
scale factor ds aires

$$\det\begin{pmatrix} 3 & 2 \\ 0 & 2 \end{pmatrix}$$

$$\det\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} = 0 \rightarrow \text{span is a line}$$

linearly dependent

$$\det(A) < 0$$

$\downarrow$

inverse l'orientation de l'espace

si y est à droite de x

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en 3D

$\hookrightarrow \det(A) \rightarrow$ facteur agrandissement du volume

(du cube de base

vers le nouveau
parallélépipède)

$$2x + 5y + 3z = -3$$

$$4x + 0y + 8z = 0$$

$$1x + 3y + 0z = 2$$

$$\rightarrow \underbrace{\begin{pmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{\vec{x}} = \underbrace{\begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix}}_{\vec{v}}$$

$$A\vec{x} = \vec{v}$$

we apply transfor. A to $\vec{x}$ and get $\vec{v}$

we want to reverse $\underline{\underline{A^{-1}}}$, apply to $\vec{v}$ to get $\vec{x}$

$$A^{-1}A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{(no change)}$$

$$\vec{x} = A^{-1}\vec{v}$$

if $\det(A) \neq 0$, A^{-1} exists

sinon on arrive dans une dimension ⊖ basse
on est pas
sûr que $\vec{x}$ existe

nb of dimensions in the column space
↓
rank 1 : line
rank 2 : plane
↳ normal pour une matrice 2x2
↳ pour une matrice 3x3, ① vect est une combinaison linéaire des 2 autres

full rank transformatⁿ

for a full-rank matrix, column space only contains one vector $\begin{pmatrix}0\\0\end{pmatrix}$ at the origin, but if it is not full rank, much more vectors
null space

Matrice 3x2

2D → 3D

$$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \rightarrow \begin{pmatrix}3 & 1\\1 & 4\\5 & 9\end{pmatrix}$$

$\hat{i}$ $\hat{j}$ → $\hat{i}$ $\hat{j}$

fullrank

tjrs un plan unique (car 2 vecteurs)
mais visualisé en 3D

2x3

3D → 2D

rank - 1

Matrices 1×2

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \rightarrow \text{to number line}$$

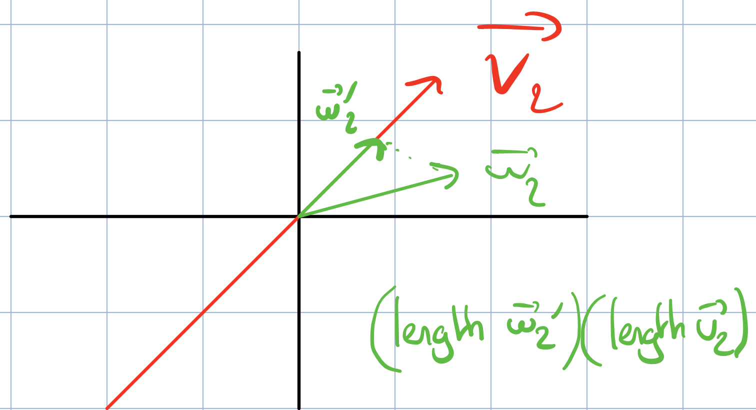
$\downarrow \quad \downarrow$

$\hat{\imath} \quad \hat{\jmath}$

Dot product

$$\begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 2 \\ 8 \end{pmatrix} = 2 \cdot 8 + 7 \cdot 2 + 1 \cdot 8$$

$\vec{v}$ $\quad$ $\vec{w}$



So when perp. $\Rightarrow 0$

pointing same dir > 0

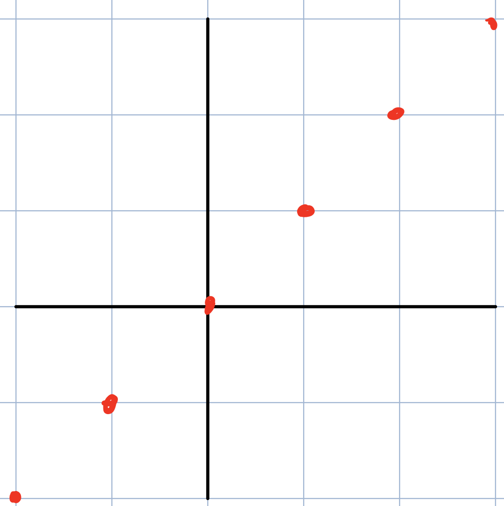
not " < 0

ORDER DOES NOT MATTER

DUALITY

$$\begin{bmatrix}2\\7\end{bmatrix} \to L(\vec{v}) \to [1.8]$$

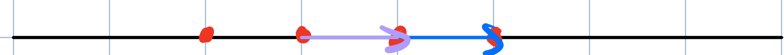
2d 1d



$\Downarrow$ transformat° linéaire vers 1D

e.g $(2\ 1)$

stays evenly spaced



for any vector $\vec{w}$, we can know where it landed with $[2\ 1]\begin{pmatrix}w_1\\w_2\end{pmatrix} = 2w_1 + 1w_2$

Same calculation as... dot product.

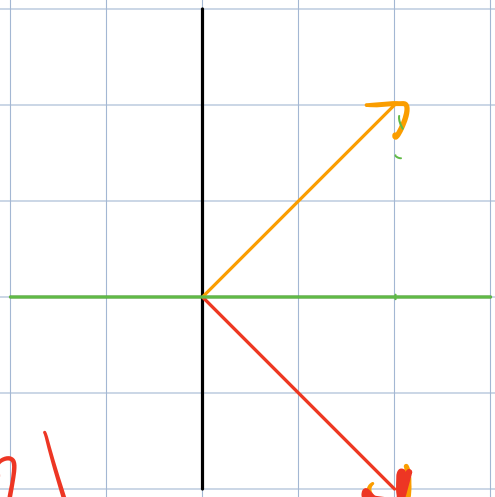
So we can make the association between

1x2 matrices $\Leftrightarrow$ 2d vectors

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \qquad \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \cdot \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$$

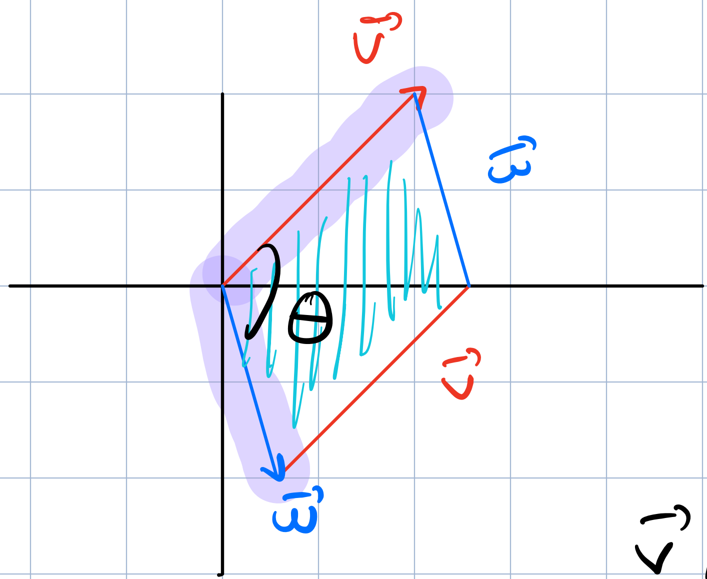
$$\text{"}\Leftrightarrow\text{"} \quad \underbrace{\begin{pmatrix} x_1 & y_1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}}_{\text{from 2D to 1D}}$$



$$\begin{pmatrix} 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = 0$$

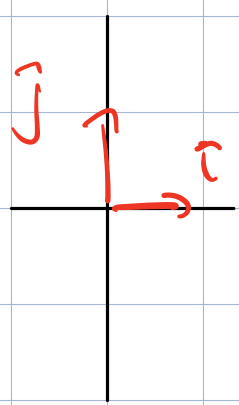
Cross Product



$\vec{v} \wedge \vec{w} =$ area of parallelogram

$\vec{w} \wedge \vec{v} =$ area of parallelogram

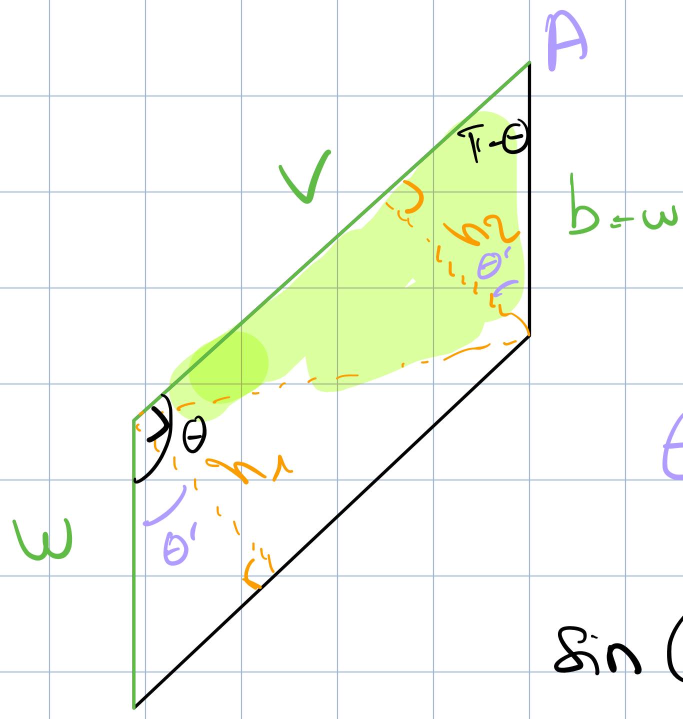
$\hat{\imath}$



$\hat{\imath} \times \hat{\jmath} = +$

$$|\vec{v} \wedge \vec{w}| = \det \begin{pmatrix} v_1 & w_1 \\ v_2 & w_2 \end{pmatrix}$$

↳ linear transf



Known values
v, w

$$\theta' = \theta - \frac{\pi}{2}$$

$$\sin(A) = \frac{opp}{hyp}$$

$$\pi = \frac{\pi}{2} + \theta' + A$$

$$= \frac{\pi}{2} + \left(\theta - \frac{\pi}{2}\right) + A$$

$$= \theta + A$$

$$= \frac{b \times h}{2} = \frac{v \times h_2}{2}$$

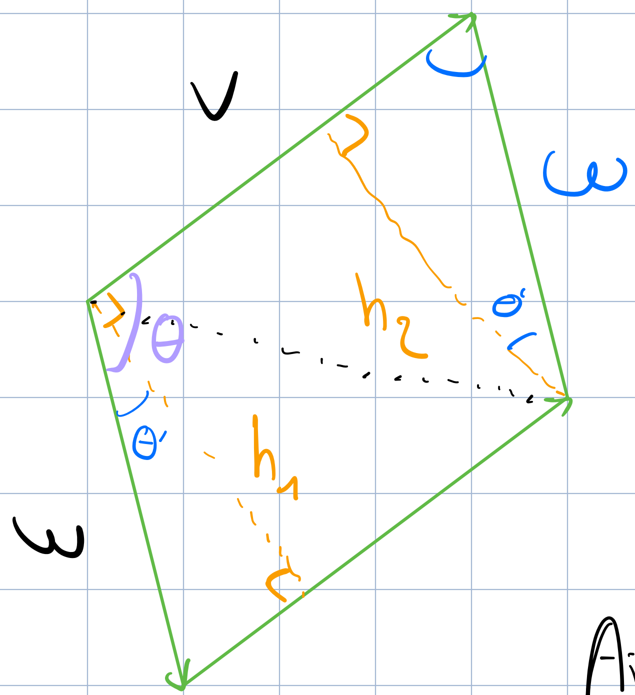
$$\begin{cases} \sin(A) = \frac{opp}{hyp} \\ \quad = \frac{h_2}{w} \\ \Leftrightarrow h_2 = \sin(A) \times w \end{cases}$$

$$= \frac{v \times \sin(A) \times w}{2}$$

$$\sin(\pi - \theta)$$

$$= -\sin\theta$$

(on fait ×2 pour avoir les deux)



$$\theta' = \theta - \frac{\pi}{2}$$

Aire$(v,w) =$ $v \times w \times \sin\theta$

$$\boxed{v \times h_2 .}$$

$$\pi = \frac{\pi}{2} + \theta' + A$$

$$\Leftrightarrow \pi = \frac{\pi}{2} + \theta - \frac{\pi}{2} + A$$

$$= \theta + A$$

$$\Leftrightarrow A = \pi - \theta$$

$$\sin(A) = \frac{opp}{hyp}$$

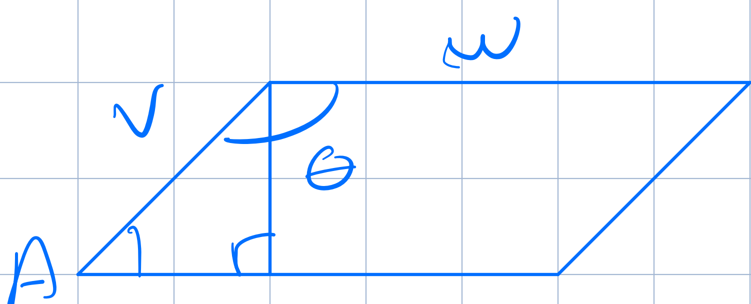
$$= \frac{h_2}{w}$$

$$\Leftrightarrow h_2 = \sin(A) \times w$$

$$v \times \sin(A) \times w$$

$$= v \times \sin(\pi - \theta) \times w$$

$$= v \times -\sin(\theta) \times w$$



$$A = \pi - \cancel{\frac{\pi}{2}} - \left(\theta - \cancel{\frac{\pi}{2}}\right)$$

$$= \pi - \theta$$

$$\sin A = \frac{opp}{hyp} = \frac{h}{v} \Leftrightarrow h = \sin A \, v$$

$\vec{v} \times \vec{w}$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \det\left(\begin{bmatrix} \hat{\imath} & v_1 & w_1 \\ \hat{\jmath} & v_2 & w_2 \\ \hat{k} & v_3 & w_3 \end{bmatrix}\right)$$

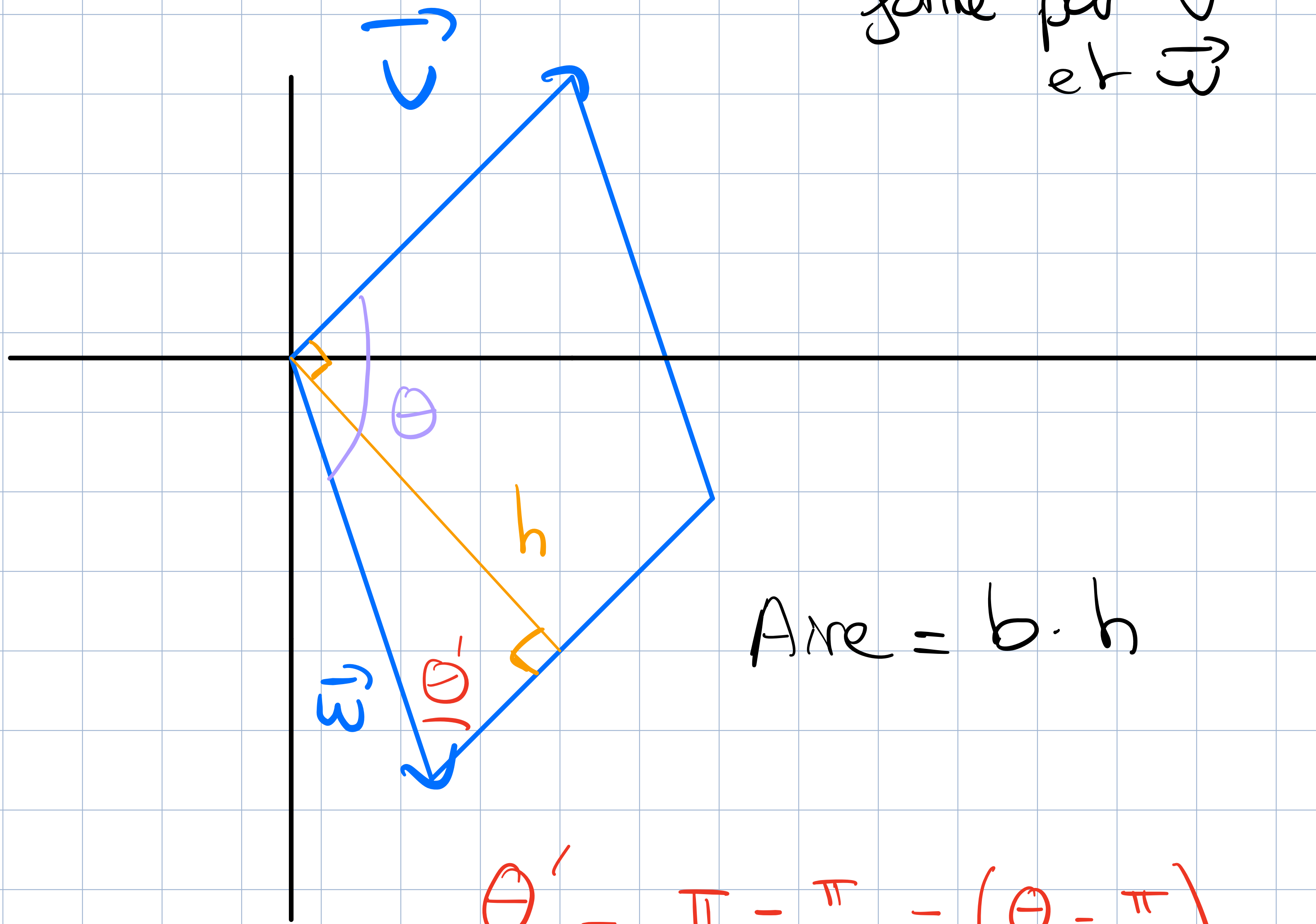
$$\left.\begin{aligned} = \ & \hat{\imath}(v_2 w_3 - w_2 v_3) \\ + \ & \hat{\jmath}(w_1 v_3 - v_1 w_3) \\ + \ & \hat{k}(v_1 w_2 - w_1 v_2) \end{aligned}\right\} \vec{p}$$

students are often told that it works

it gives:

- length = parallelogram's area $v\ w$
- and° = perpendicular to plan $v\ w$
- obeys right hand rule

- On cherche l'aire du parallélogramme formé par $\vec{v}$ et $\vec{w}$



$$\text{Aire} = b \cdot h$$

$$\theta' = \pi - \frac{\pi}{2} - \left(\theta - \frac{\pi}{2}\right)$$
$$= \pi - \theta$$

$$\sin(\theta') = \frac{\text{opp}}{\text{hyp}} = \frac{h}{w}$$

$$\Leftrightarrow h = \sin(\theta') \cdot w = \sin\theta \, w$$

$$\text{Aire} = v \cdot w \cdot (\sin\theta)$$