Exercice 1

$$\det(A) \cdot \det(A) \cdot \det(A) = 0.$$

$$(I_n + A + A^2)(I_n - A)$$

$$= I_n - A + A - A^2 + A^2 - A^3$$

$$= \underline{I_n}.$$

Exercice 2

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & \lambda \end{vmatrix} \begin{matrix} + \\ - \\ + \end{matrix}$$

$$\lambda \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$\boxed{\lambda \neq 0}$$

$$= \underline{-\lambda}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 2 & \lambda & 0 & 0 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 2 & \lambda & 0 & 0 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 1 & 2 & \lambda & 0 & 0 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 2 & \lambda & 0 & -1 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & \lambda & -2 & 1 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -2/\lambda & 1/\lambda & 1/\lambda \end{array}\right)$$

Exercice 3

$\det(A)$

$$= c_1^2 \cdot \begin{vmatrix} 1 & 1 \\ c_2 & c_3 \end{vmatrix}$$

$$- c_2^2 \cdot \begin{vmatrix} 1 & 1 \\ c_1 & c_3 \end{vmatrix}$$

$$+ c_3^2 \cdot \begin{vmatrix} 1 & 1 \\ c_1 & c_2 \end{vmatrix}$$

$$= c_1^2 \cdot (c_3 - c_2)$$

$$- c_2^2 \cdot (c_3 - c_1)$$

$$+ c_3^2 \cdot (c_2 - c_1)$$

$$= c_1^2 \cdot c_3 - c_1^2 \cdot c_2$$
$$- c_2^2 \cdot c_3 + c_2^2 \cdot c_1$$
$$+ c_3^2 \cdot c_2 - c_3^2 \cdot c_1$$

$$(c_2 - c_1)(c_3 - c_1)(c_3 - c_2)$$

$$= (c_2 \cdot c_3 - c_1 \cdot c_2 - c_1 \cdot c_3$$
$$+ c_1^2)(c_3 - c_2)$$

$$= c_2 \cdot c_3 (c_3 - c_2) - c_1 \cdot c_2 ((c_3 - c_2)$$
$$- c_1 \cdot c_3 ((c_3 - c_2) + c_1^2 (c_3 - c_2)$$

$$= c_2 \cdot c_3^2 - c_2^2 \cdot c_3 - \cancel{c_1 \cdot c_2} \cdot c_3$$
$$+ c_1 \cdot c_2^2 - c_1 \cdot c_3^2 + \cancel{c_1 \cdot c_3} \cdot c_2$$
$$+ c_1^2 \cdot c_3 - c_1^2 \cdot c_2$$

$c_2 - c_1 \neq 0 \iff c_2 \neq c_1$

$c_3 \neq c_1$

$c_3 \neq c_2$

$c_1 \neq c_2 \neq c_3$

Exercice 4

(a)

$$\det \begin{pmatrix} a_{1x} & a_{2x} \\ a_{1y} & a_{2y} \end{pmatrix}$$

$$= \det \begin{pmatrix} a_{2x} & a_{2y} \\ a_{1x} & a_{1y} \end{pmatrix} \quad \det(A^T) = \det(A)$$

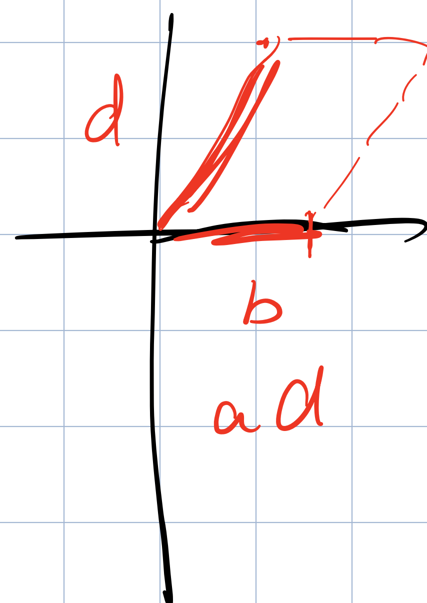
$$= \det \begin{pmatrix} a_{2x} + c a_{1x} & a_{2y} + c a_{1y} \\ a_{1x} & a_{1y} \end{pmatrix}$$

ajouter mult
ligne keep det

(b)

Aire parallélogramme

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



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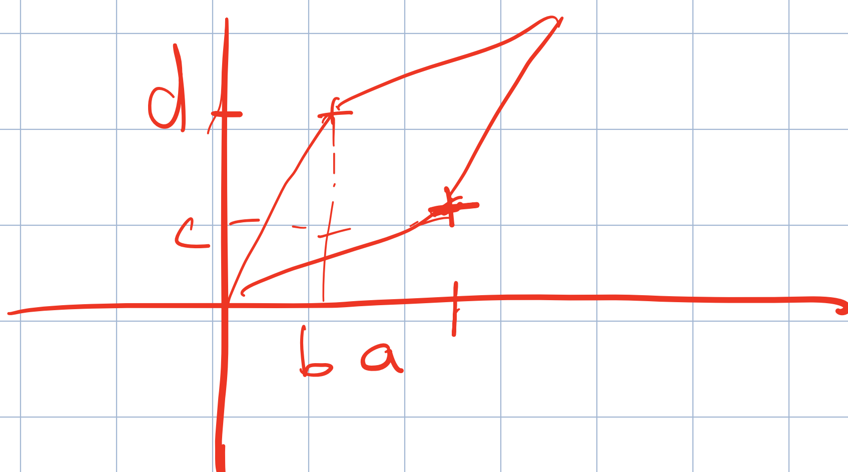

Exercice 5

$$\left(\begin{array}{ccc|ccc} 0 & 2 & 3 & 1 & 0 & 0 \\ 3 & 5 & 7 & 0 & 1 & 0 \\ 4 & 6 & 9 & 0 & 0 & 1 \end{array}\right)$$

x -3

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

$$\det(E_1) = 1$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 0 & 2 & 3 & 1 & 0 & 0 \\ 3 & 5 & 7 & 0 & 1 & 0 \\ 4 & 0 & 0 & -3 & 0 & 1 \end{array}\right)$$

$$E_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3/4 & 0 & 1/4 \\ 3 & 5 & 7 & 0 & 1 & 0 \\ 0 & 2 & 3 & 1 & 0 & 0 \end{array}\right)$$

$$\det(E_2) = -1$$

$$E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

x -3

$$\det(E_3) = 1/4$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3/4 & 0 & 1/4 \\ 0 & 5 & 7 & 9/4 & 1 & -3/4 \\ 0 & 2 & 3 & 1 & 0 & 0 \end{array}\right)$$

$$E_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2/5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\det(E_4) = \frac{2}{5}$$

$\times 2/5$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 0 & 1 \\ 0 & 2 & 14/5 & 18/20 & 2/5 & -6/20 \\ 0 & 2 & 3 & 1 & 0 & 0 \end{array}\right) \cdot 5/5$$

-1

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & 2 & 14/5 & 18/5 & 2/5 & 0 \\ 0 & 0 & 1/5 & -13/5 & -2/5 & 6/20 \end{array}\right)$$

$$\det(A^{-1}) = 1 \cdot 2 \cdot 1/5 = 2/5.$$

$$\det(A) = \frac{1}{2/5} = \frac{5}{2}$$

$$\begin{pmatrix} -2 & 0 & 1 \\ 18/5 & 2/5 & 0 \\ -13/5 & -2/5 & 6/20 \end{pmatrix}$$

$$\begin{vmatrix} -2 & 0 & 1 \\ 18/5 & 2/5 & 0 \\ -13/5 & -2/5 & 6/20 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{vmatrix}$$