Exercice 1

(i) $$\int_0^2 \left(x^3 - y^{1/3}\right) dx$$

$$= \left[\frac{1}{4}x^4 - xy^{1/3}\right]_0^2$$

$$\int_0^1 4 - 2y^{1/3} \, dy$$

$$= \left[4y - 2\cdot\frac{3}{4}y^{4/3}\right]_0^1$$

$$= 4 - \frac{3}{2} = \frac{5}{2}.$$

$$\int_0^2 \underbrace{\int_0^1 \left(x^3 - y^{1/3}\right) dy}_{x^3 - \frac{3}{4}} \, dx$$

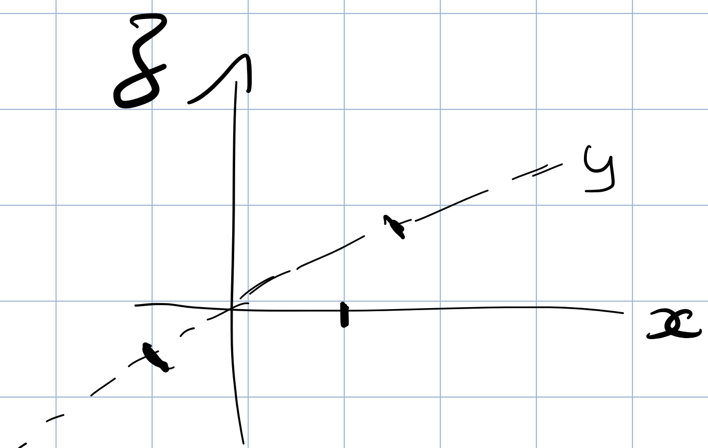
$$= \int_0^2 x^3 - \frac{3}{4} \, dx$$

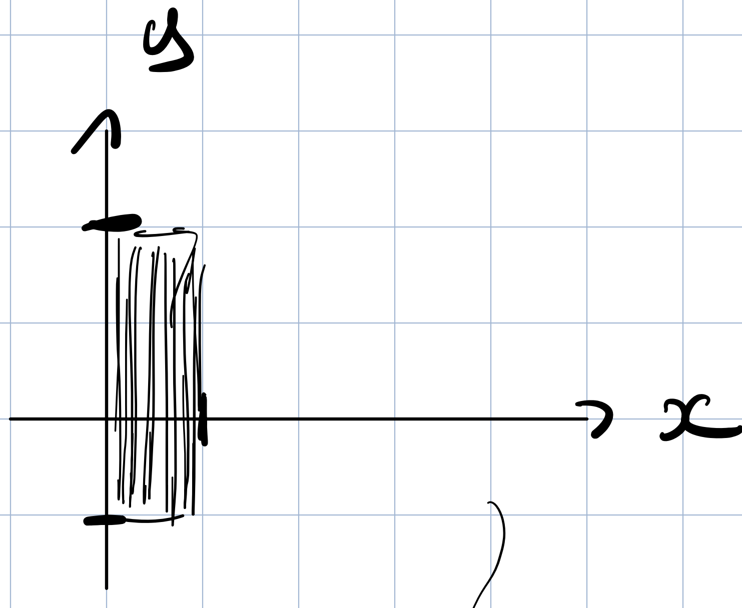
$$= \left[\frac{1}{4}x^4 - \frac{3}{4}x\right] dx$$

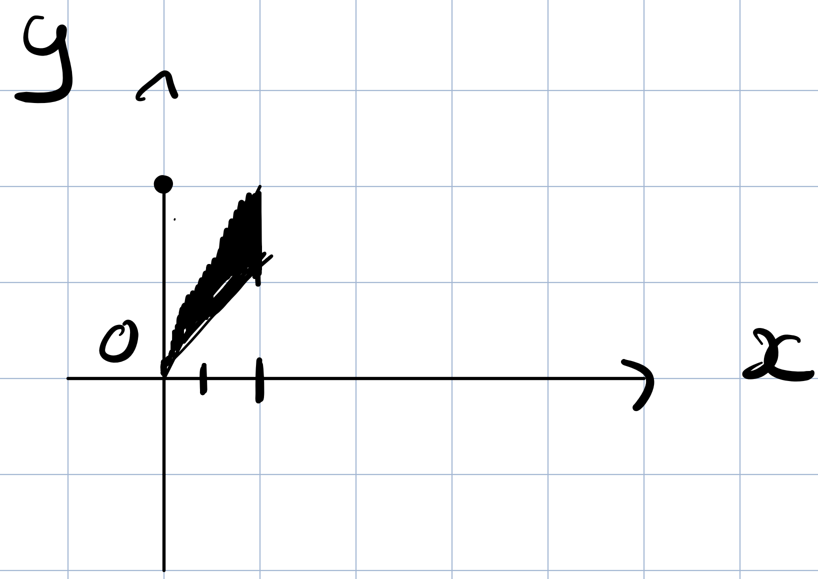
$$= 4 - \frac{3}{2} = \frac{8}{2} - \frac{3}{2} = \frac{5}{2}.$$

Exercice 2

(i) $$\int_{-1}^{2}\int_{0}^{1} \cos(x+y)\,dx\,dy$$







$$\int_0^1 \cos(x+y)\,dx$$

$$= \left[\sin(x+y)\right]_0^1$$

$$= \sin(1+y) - \sin(y)$$

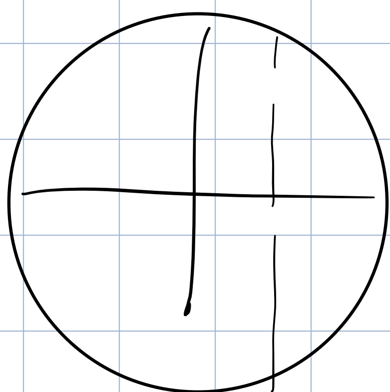
$$\int_{-1}^{2} \sin(1+y) - \sin(y)\,dy$$

$$= \left[-\cos(1+y) + \cos(y)\right]_{-1}^{2}$$

$$= -\cos(3) + \cos(2)$$

$$- (\underbrace{-\cos(0)}_{1} + \underbrace{\cos(-1)}))$$

$$= -\cos(3) + \cos(2) - \cos(-1) + 1$$



$$\cos(a) = \cos(-a)$$

$$\Rightarrow -\cos(3) + \cos(2) - \cos(1) + 1$$

ii) $$\int_0^1 \underbrace{\int_x^{2x} e^{x+y}\, dy}\, dx$$

$$\hookrightarrow \left[e^{x+y}\right]_x^{2x}$$

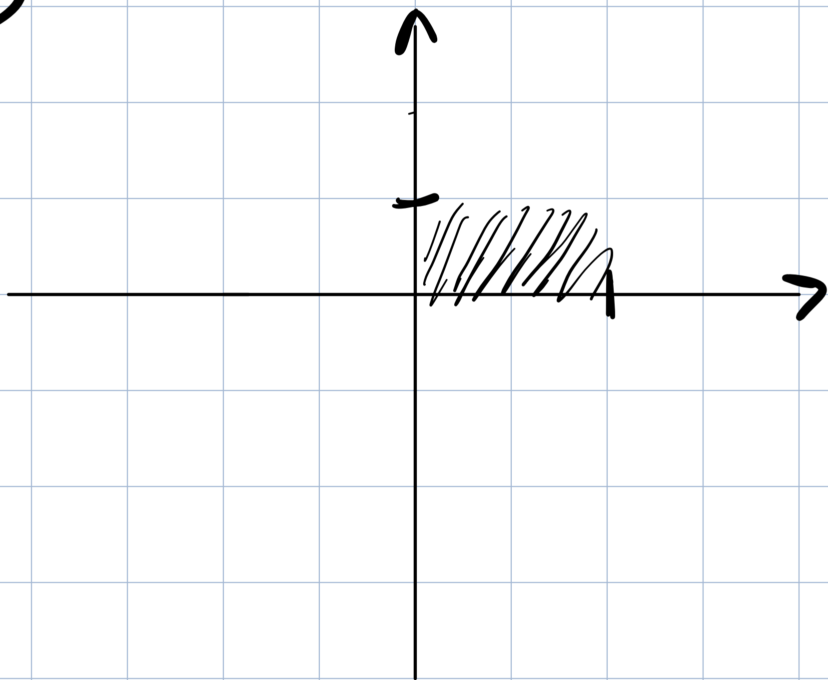
$$= \int_0^1 e^{3x} - e^{2x}\, dx$$

$$= \left[\frac{1}{3} e^{3x} - \frac{1}{2} e^{2x} \right]_0^1$$

$$= \frac{1}{3} e^3 - \frac{1}{2} e^2 - \frac{1}{3} + \frac{1}{2}$$

Exercice 3

(i)



$$\int_0^1 \underbrace{\int_0^2 (x+y)^{1/2}\,dx\,dy}$$

$$\int_0^1 \left[\frac{2}{3}(x+y)^{3/2}\right]_0^2 dy$$

$$= \int_0^1 \left(\frac{2}{3} \left[(2+y)^{3/2} - y^{3/2} \right] \right) dy$$

$$= \frac{2}{3} \left[\frac{2}{5} (2+y)^{5/2} - \frac{2}{5} y^{5/2} \right]_0^1$$

$$= \frac{4}{15} (3)^{5/2} - \frac{4}{15} - \frac{4}{15} (2)^{5/2}$$

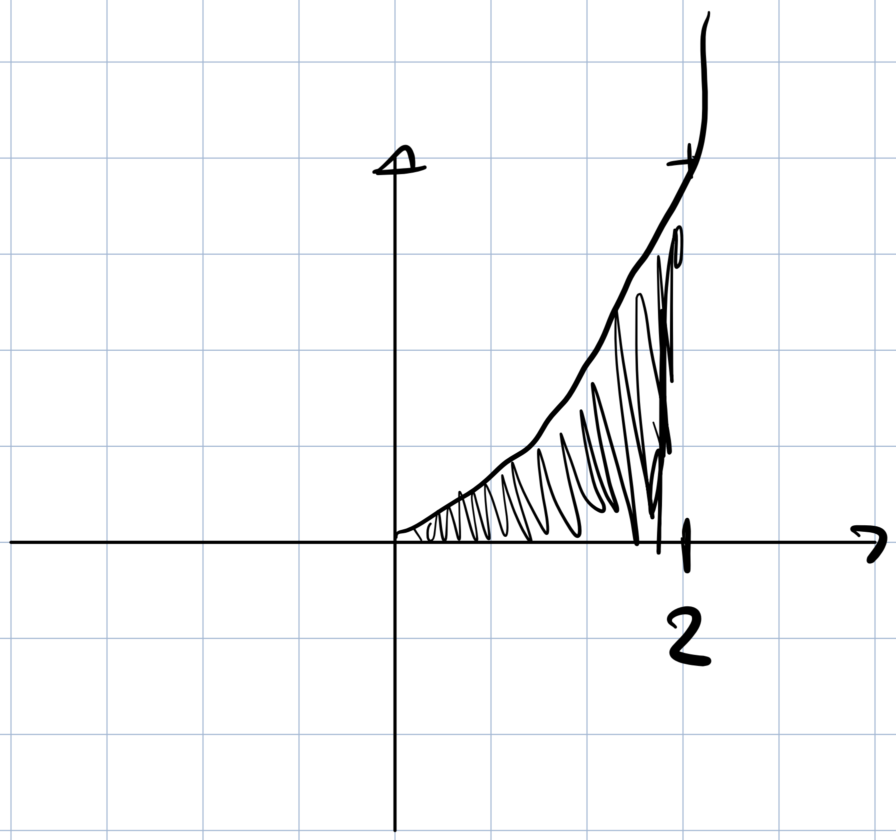
$$= \frac{4}{15} \left((\sqrt{3})^5 - 1 - (\sqrt{2})^5 \right)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$3 \cdot 3 \cdot \sqrt{3} \qquad 2 \cdot 2 \cdot \sqrt{2}$$

$$= \frac{4}{15} \left(9\sqrt{3} - 4\sqrt{2} - 1 \right)$$

(ii)



$$\int_0^2 \left(\int_0^{x^2} x^2 y \, dy \right) dx$$

$\Downarrow$

$$\left[\frac{1}{2} x^2 y^2 \right]_0^{x^2}$$

$$= \frac{1}{2} x^6$$

$$\int_0^2 \frac{1}{2} x^6 \, dx$$

$$= \left[\frac{1}{2} \cdot \frac{1}{7} x^7 \right]_0^2$$

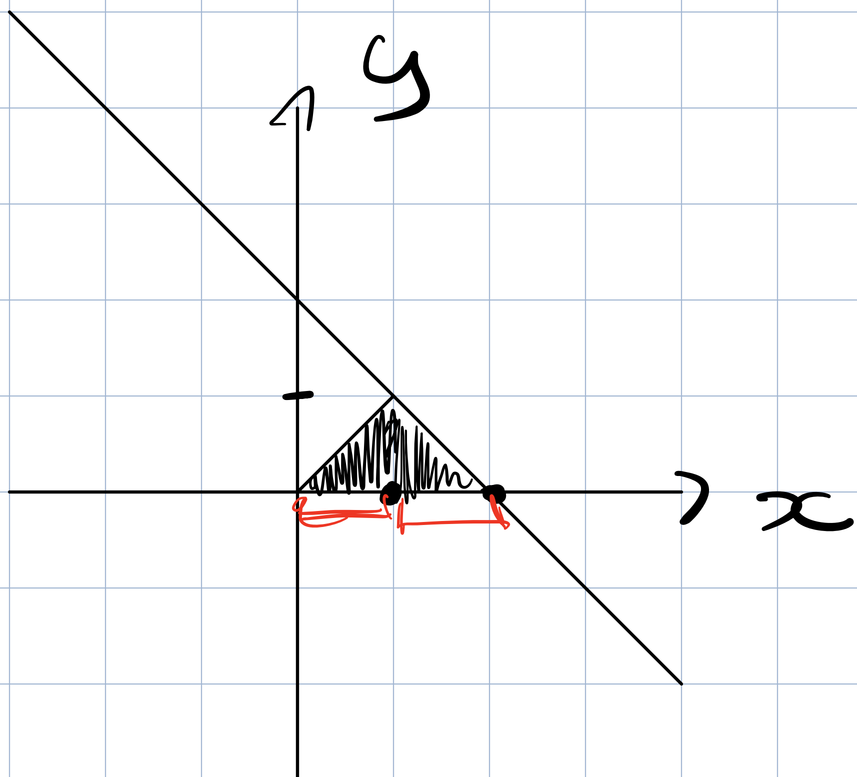
$$= \frac{2^6}{7} \cdot$$

$(2^3)^2$

$8^2 = 64$

$$= \frac{64}{7}$$

(iii)



$x+y-2 \leq 0 \iff y \leq -x+2$

$0 \leq y \leq x$

$y \leq -x+2$

$\forall x \leq 1$, condiz° importante è $0 \leq y \leq x$

$\forall x \geq 1$, condiz° imp $y \leq -x+2$

$$\int_0^1 \int_0^x f(x,y)\,dx\,dy$$

$$+ \int_1^2 \int_0^{-x+2} f(x,y)\,dx\,dy$$

$x-y \geq 0$

et $x+y-2 \leq 0$

$\Rightarrow$ on peut enlever la valeur

absolue et mettre un $\ominus$

$$\int_0^1 \int_0^x (x-y)(x+y-2)\,dy\,dx$$

$$= \int_0^1 \int_0^x x^2 + \cancel{xy} - 2x - \cancel{xy} - y^2 + 2y \,dy\,dx$$

$$= \int_0^1 \int_0^x x^2 - y^2 + 2y - 2x \,dy\,dx$$

$$= \int_0^1 \left[x^2 y - \frac{1}{3} y^3 + y^2 - 2xy \right]_0^x dx$$

$$= \int_0^1 \Big(\underbrace{x^3 - \frac{1}{3}x^3}_{\frac{2}{3}x^3} + \underbrace{x^2 - 2x^2}_{-x^2}\Big)\,dx$$

$$= \left[\frac{2}{3}\cdot\frac{1}{4}x^4 - \frac{1}{3}x^3\right]_0^1$$

$$= \frac{2}{12} - \frac{1}{3} = \frac{1}{6} - \frac{2}{6} = \frac{-1}{6}$$

$$\int_1^2 \int_0^{-x+2} x^2 - y^2 + 2y - 2x \; dy\, dx$$

$$= \int_1^2 \left[x^2y - \frac{1}{3}y^3 + y^2 - 2xy\right]_0^{-x+2} dy$$

$$x^2(-x+2) - \frac{1}{3}(-x+2)^3 + (-x+2)^2 - 2x(-x+2)$$

$$= -x^3 + 2x^2 - \frac{1}{3}(-x+2)^3 + (-x+2)^2 + 2x^2 - 4x$$

$$\left[-\frac{1}{4}x^4 + \frac{1}{3}\cdot\frac{1}{4}\cancel{(-x+2)^4} - \frac{1}{3}\cancel{(-x+2)^3} + 4\cdot\frac{1}{3}x^3 - 2x^2\right]_1^2$$

$$= -\frac{1}{4}(2^4) + \frac{4}{3}2^3 - 2(2)^2 + \frac{1}{4} - \frac{1}{12} + \frac{1}{3} - \frac{4}{3} + 2$$

$$= -\frac{1}{6}$$

$$-\left(-\frac{1}{6}\right)-\left(-\frac{1}{6}\right)=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$$

le - de la valeur absolue

Exercice 4

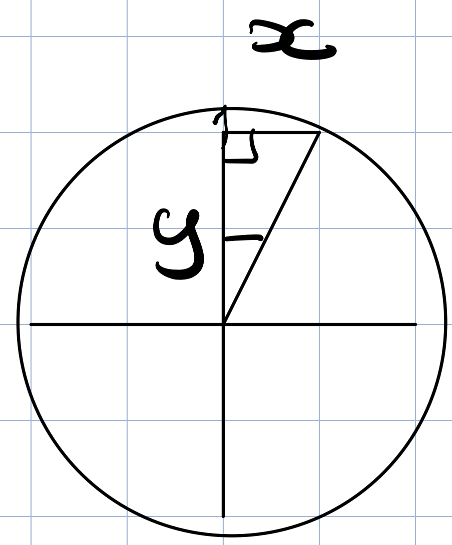
(i) $$G(\rho, \theta) = \begin{pmatrix} \rho\cos\theta \\ \rho\sin\theta \end{pmatrix}$$

$$J_G = \begin{pmatrix} \cos\theta & -\rho\sin\theta \\ \sin\theta & \rho\cos\theta \end{pmatrix}$$

$$\det(J_G) = \rho$$

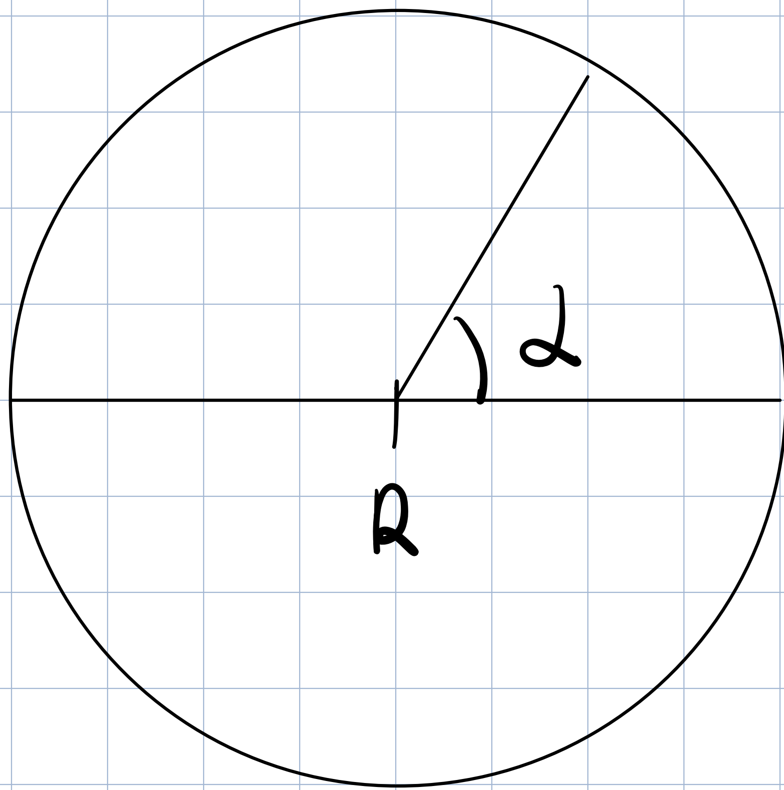
$$\tan\alpha = \frac{x}{y}$$

$$\Rightarrow \alpha = \arctan\left(\frac{x}{y}\right)$$



$$H(x, y) = \begin{pmatrix} \sqrt{x^2 + y^2} \\ \arctan(x/y) \end{pmatrix}$$

$$\det(H) = \frac{1}{\det G} = \frac{1}{\rho} = \frac{1}{\sqrt{x^2+y^2}}$$



$$\iint 1 \, dx\,dy$$

det

$$= \int_0^{\alpha} \int_0^{R} 1 \, |\rho| \, d\rho \, d\phi$$

$$= \int_0^{\alpha} \left[\frac{1}{2} \rho^2 \right]_0^R d\phi$$

$$= \int_0^{\alpha} \frac{1}{2} R^2 \, d\phi$$

$$= \frac{1}{2} R^2 \alpha$$

$$\int_0^R \int_0^\alpha 1 \, \rho \, d\phi \, d\rho$$

$$= \int_0^R \left[\rho \phi \right]_0^\alpha d\rho$$

$$= \int_0^R \rho \alpha \, d\rho$$

$$= \frac{1}{2} R^2 \alpha \qquad \checkmark$$

$\Rightarrow$ qu'on integre ds un sens ou ds l'autre la technique du det marche iii

$$y \leq |\sqrt{3}x|$$

$$(x-1)^2 + (\sqrt{3}x)^2 < 1$$

$$\iff x^2 - 2x + 1 + 3x^2 < 1$$

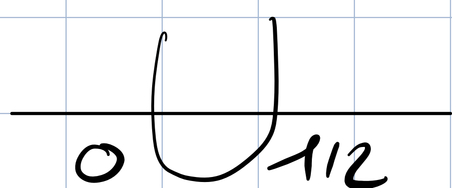
$$\iff 4x^2 - 2x \cancel{+1} < \cancel{1}$$

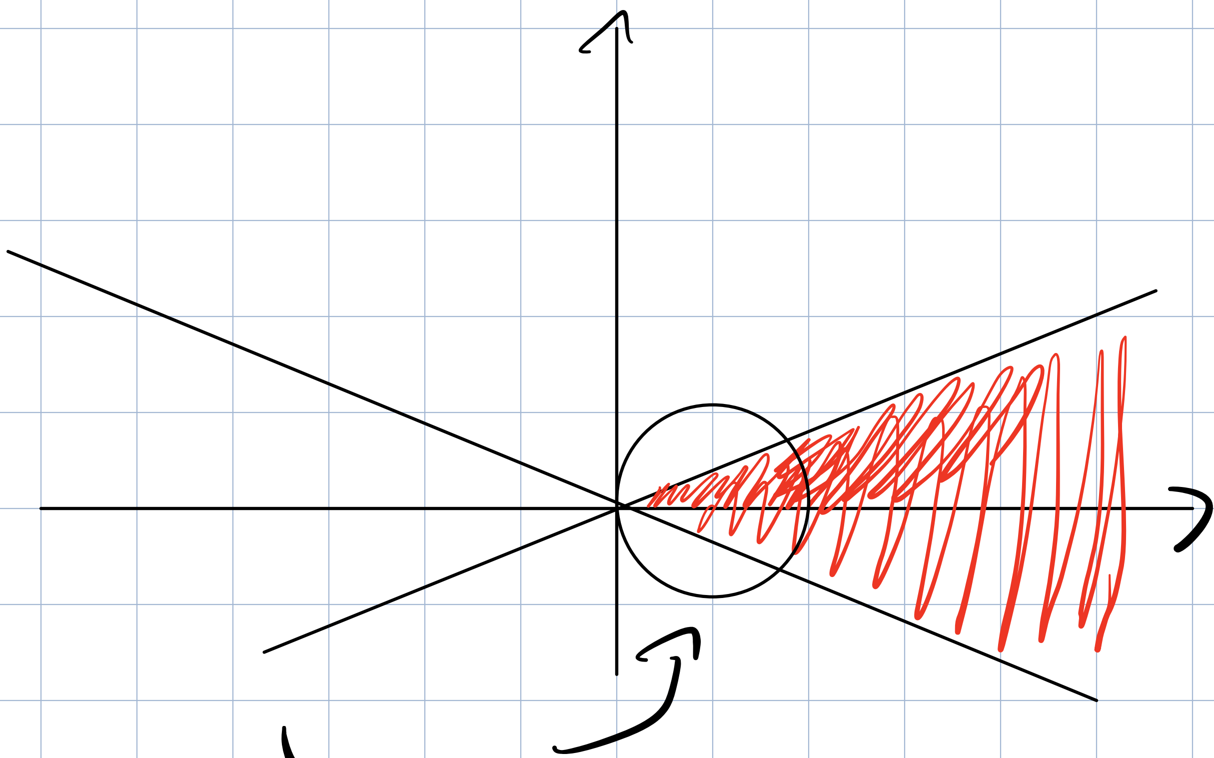
$$\iff 4x^2 - 2x < 0$$

$$\Delta = (-2)^2 = 4$$

$4/8 = \frac{1}{2}$

$$x = \frac{2 \pm 2}{8} \to 0$$





cercle
décalé de 1

$$x - 1 = \beta$$

$$\beta^2 + y^2 < 1$$

$$y = \sqrt{3}x$$

$$\frac{x}{y} = \sqrt{3}$$

$\arctan(\sqrt{3})$

Symétrique

$$2 \cdot \int_0^1 \int_0^{\pi/3} \rho \, d\phi \, d\rho$$

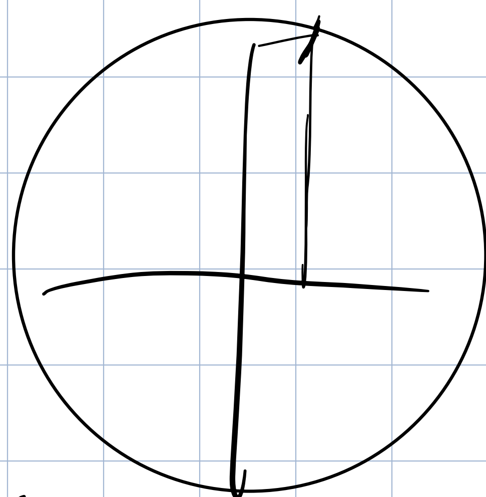
$$\cancel{= 2\int_0^1 \frac{\pi}{3}\rho\, d\rho}$$

$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \to \pi/3$$

$$\cancel{= 2\left[\frac{\pi}{3}\cdot\frac{1}{2}\rho^2\right]_0^1}$$

$\pi/3$

$$\cancel{= 2\,\frac{\pi}{3}\cdot\frac{1}{2}}$$



$$= \pi/3$$

faux car le rayon n'est pas 1
Hr le temps

$$(x-1)^2 + y^2 < 1$$

$$\Rightarrow x^2 - 2x + 1 + y^2 < 1$$

$$\Rightarrow \rho < 2x$$

$$\Rightarrow \rho^2 < 2\rho\cos\phi$$

$$\Rightarrow \rho < 2\cos\phi$$

$$2\int_0^{\pi/3}\int_0^{2\cos\phi} \rho\, d\rho\, d\phi$$

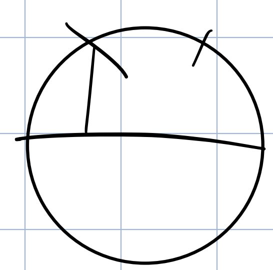
$$= 2\int_0^{\pi/3} \left[\frac{1}{2}\rho^2\right]_0^{2\cos\phi} d\phi$$

$$= 2\int_0^{\pi/3} \frac{1}{2}(2\cos\phi)^2\, d\phi$$

$$= 2\int_0^{\pi/3} 2\cos^2\phi\, d\phi$$

$$= 2\int_0^{\pi/3} 2\left(\frac{1}{2}\left(1+\cos(2\phi)\right)\right) d\phi$$

$$= 2\left(\left[\phi + \sin(2\phi)\right]_0^{\pi/3}\right)$$



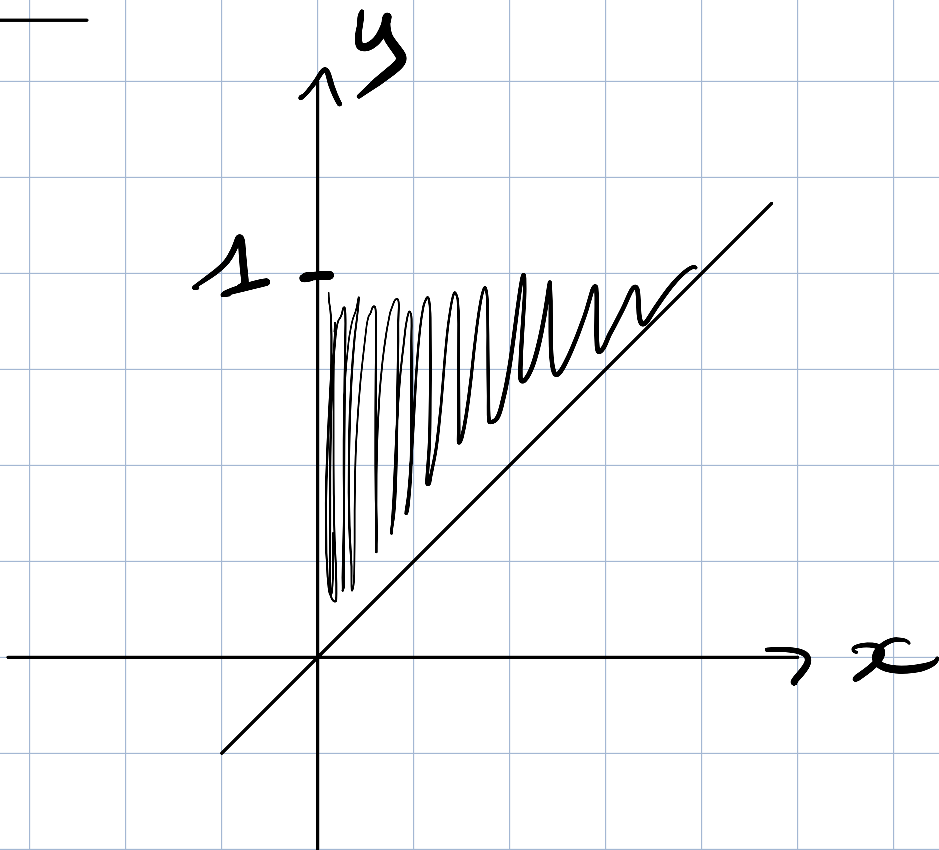
$$= \boxed{2\pi/3 + \frac{\sqrt{3}}{2}}$$

$$(\cos)^2 = \left(\frac{e^{i\phi} + e^{-i\phi}}{2}\right)^2$$

$$= \frac{e^{2i\phi} + 2 + e^{-2i\phi}}{4}$$

$$= \frac{1}{2} + \frac{1}{2}\cos(2\phi)$$

Exercice 4



$$\int_0^1 \int_y^1 e^{(x^2)} \, dx \, dy$$

$$= \int_0^1 \Big[$$

$$e^{x^2}\, dx = \frac{e^u}{2\sqrt{u}}\, du$$

$u = x^2$

$du = 2x\, dx$

$$\Rightarrow dx = \frac{du}{2x} = \frac{du}{2\sqrt{u}}$$

$\Rightarrow$ imp. on inverse les bornes

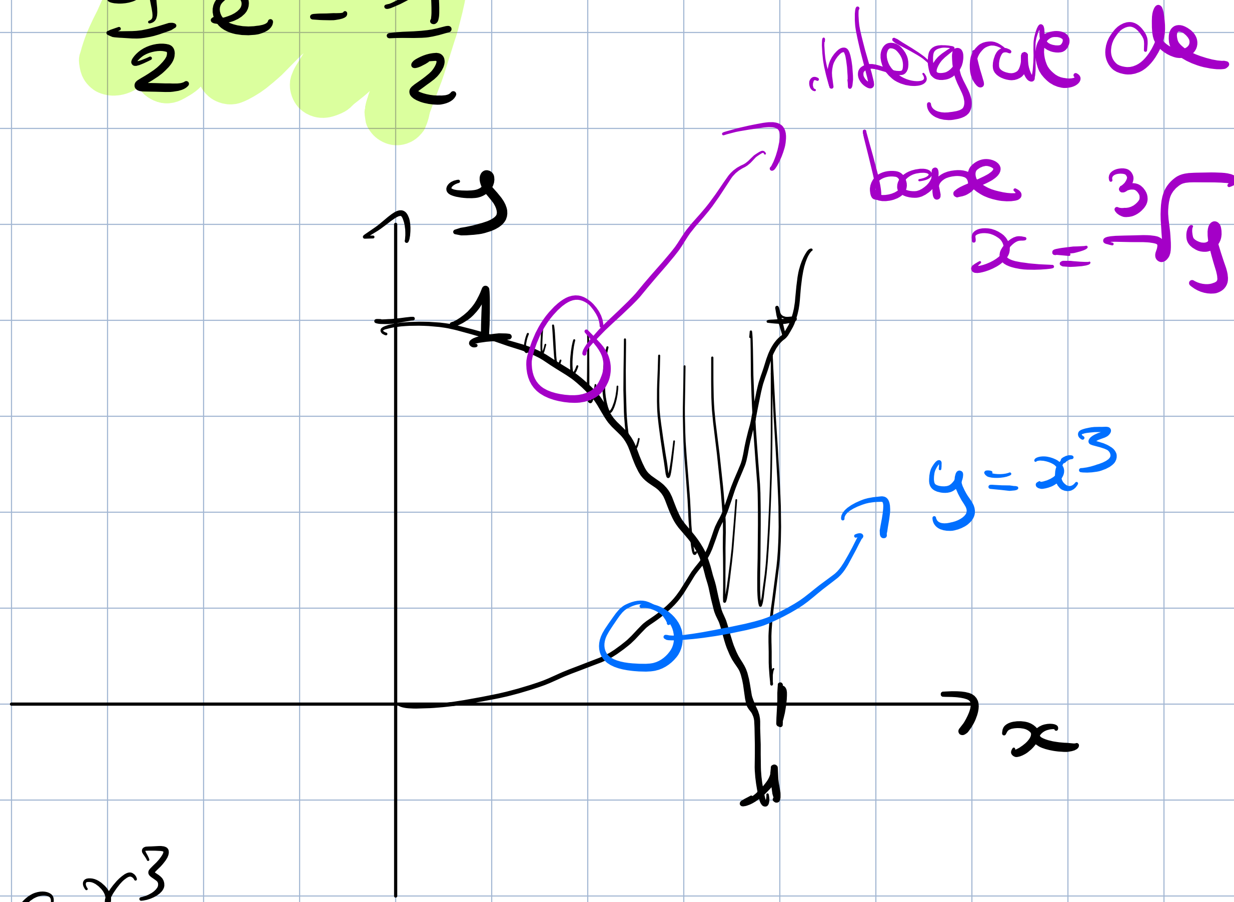
$$\int_0^1 \int_0^x e^{(x^2)}\, dy\, dx$$

$$= \int_0^1 \left[y e^{x^2} \right]_0^x dx$$

$$= \int_0^1 x e^{x^2}\, dx$$

$$= \left[\frac{1}{2} e^{x^2} \right]_0^1$$

$$= \frac{1}{2}e - \frac{1}{2}$$



$$\int_0^1 \int_0^{x^3}$$

$$x = \sqrt[3]{y}$$

$$y = x^3$$

$$\int_0^1 \int_{\sqrt[3]{y}}^1 \sqrt{1 + x^4}\, dx\, dy$$

$$\int_0^1 \int_0^{x^3} \sqrt{1+x^4}\, dy\, dx$$

$$(1+u)^{1/2} \frac{1}{4} u'$$

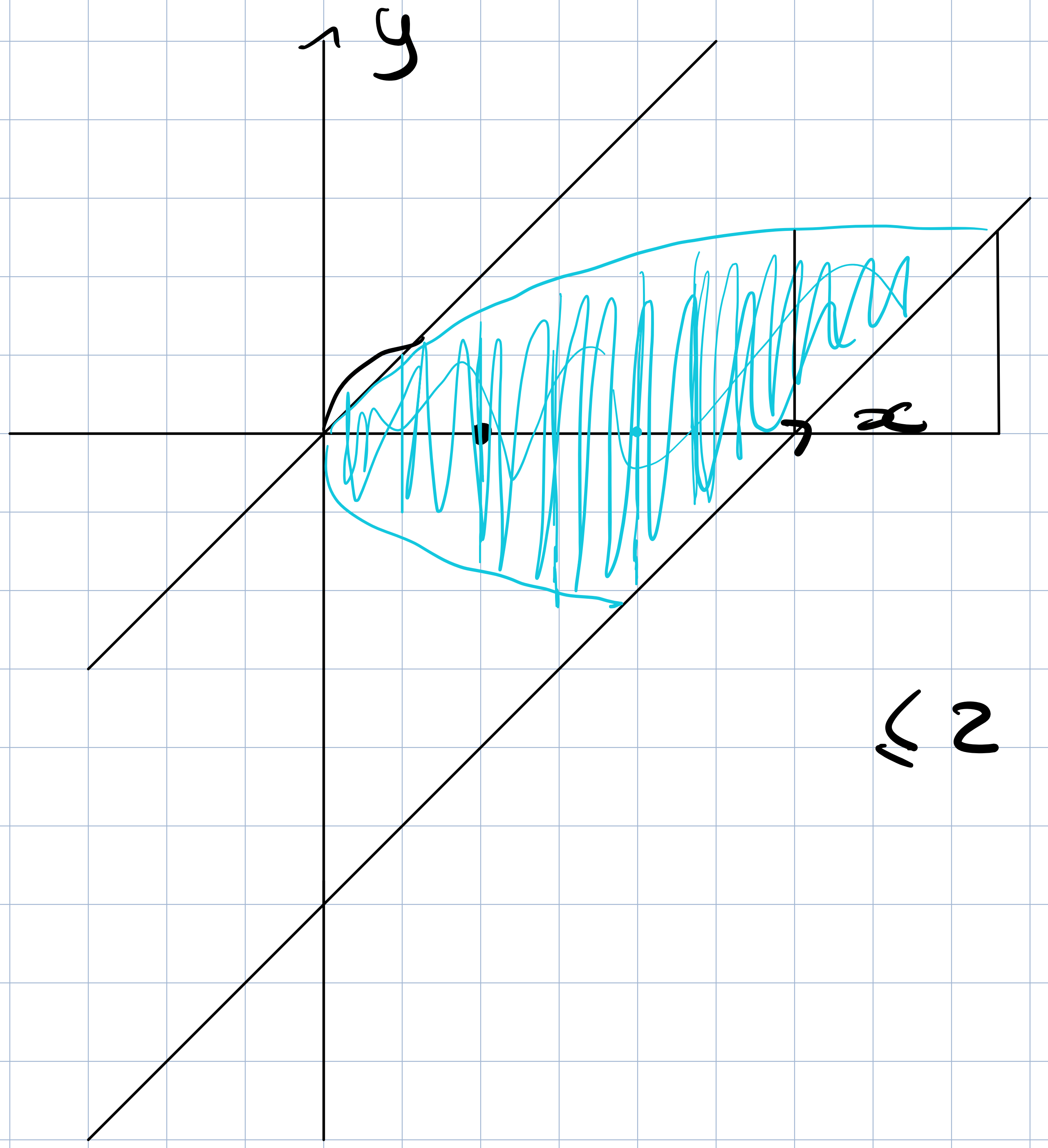
$$= \int_0^1 (1+x^4)^{1/2} x^3 \, dx$$

$$= \left[\frac{2}{3} \cdot \frac{1}{4} (1+x^4)^{3/2} \right]_0^1$$

$$= \frac{2}{12} (2)^{3/2} - \frac{2}{12}$$

$$= \frac{1}{6} \left(2\sqrt{2} - 1 \right)$$

Exercice 5



$$\sqrt{x} = x-6$$

$$\Rightarrow \quad x = x^2 - 12x + 36$$

$$\Leftrightarrow \quad x^2 - 13x + 36 = 0$$

$$(13)^2 = 144 + 12 + 13 = 164 + 5 = 169$$

$$12 \cdot 12 + 12 \quad + 13$$

$$13 \cdot 12$$

$$\Delta = 169 - 4 \cdot 36$$
$$= 169 - 144$$
$$= 25$$

$$\begin{array}{r} {}^{2} \\ 36 \\ \cdot \quad 4 \\ \hline 144 \end{array}$$

$$\sqrt{\Delta} = \frac{13 \pm 5}{2} \begin{array}{l} \nearrow 9 \\ \searrow 4 \end{array}$$

$$\cancel{\int_0^9 \int_0^{\sqrt{x}} 1 \, dy \, dx} - \frac{3 \cdot 3}{2}$$

$$\int_0^4 \int_0^{\sqrt{x}} 1\,dy\,dx + 2$$

$$\Rightarrow \int_0^9 \left[y\right]_0^{\sqrt{x}} dx - 6$$

$(x)^{1/2}$

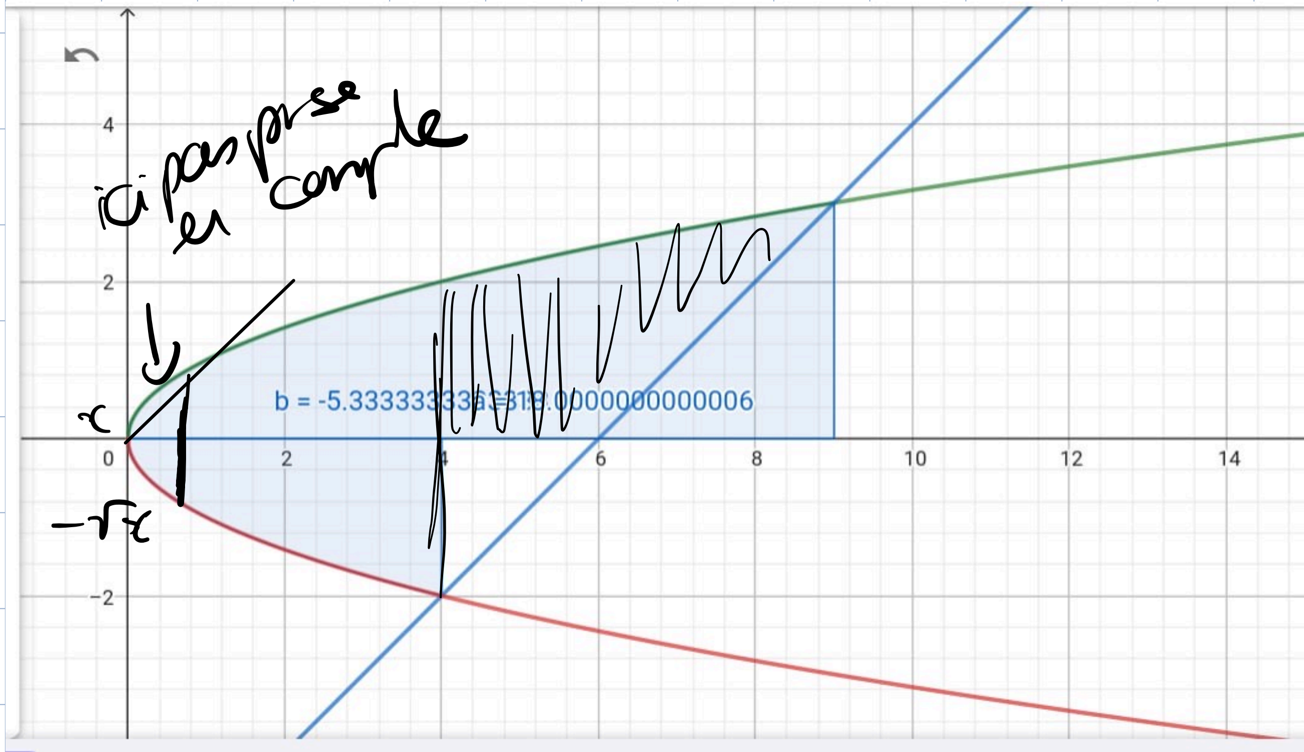
$2(x)^{3/2}$

$$\Rightarrow \int_0^9 \sqrt{x}\,dx - 6$$

$$\Rightarrow \left[\frac{2}{3}(x)^{3/2}\right]_0^9 - 6$$

$$= \frac{2}{3}(9)^{3/2} - 6$$

$$\frac{2}{3}(9)^{3/2} - 6 + \frac{2}{3}(4)^{3/2} - \frac{1}{2}$$



$$\int_0^1 \int_{-\sqrt{x}}^{x} 1\,dy\,dx$$

$$+ \int_1^4 \int_{-\sqrt{x}}^{\sqrt{x}} 1\,dy\,dx$$

$$+ \int_4^9 \int_{x-6}^{\sqrt{x}} 1\,dy\,dx$$

$$= \int_0^1 [y]_{-\sqrt{x}}^{x} \, dx$$

$$+ \int_1^4 [y]_{-\sqrt{x}}^{\sqrt{x}} \, dx$$

$$+ \int_4^9 [y]_{x-6}^{\sqrt{x}} \, dx$$

$$= \int_0^1 x + \sqrt{x} \, dx$$

$$+ \int_1^4 \sqrt{x} + \sqrt{x} \, dx$$

$$+ \int_4^9 \sqrt{x} - x + 6 \, dx$$

$$= \left[\frac{1}{2}x^2\right]_0^1 + \left[\frac{2}{3}(x)^{3/2}\right]_0^1$$

$$+ \left[2 \, \frac{2}{3} \, (x)^{3/2}\right]_1^4$$

$$+\left[\frac{2}{3}(x)^{3/2}\right]_4^9 - \left[\frac{1}{2}x^2\right]_4^9 + \left[6x\right]_4^9$$

$$= \frac{1}{2} + \frac{2}{3} + \frac{4}{3}(8) - \frac{4}{3}$$

$$+ \frac{2}{3}(27) - \frac{2}{3}(8) - \frac{1}{2}81 + \frac{16}{2}$$

$$+ 54 - 24$$

$$= \underline{62/3}$$

Exercice 7

$(6,5)$

$(2,3)$

$$\frac{5-3}{6-2} = \frac{2}{4} = \frac{1}{2}$$

$$2 \leq y \leq \frac{1}{2}x$$

$$\int_1^6 \int$$

$2(2)+c=3$

$\Leftrightarrow c=-1$

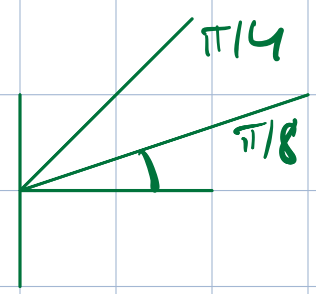
$\frac{1}{2}6+c=5$

$3+c=5$

$\Leftrightarrow 2=c$

$$\int_1^6 \int_{1/2x+1/2}^{1/2x+2} 1\,dy\,dx$$

$-$



$$\frac{1}{2} - \frac{4}{2} = \frac{-3}{2}$$

$$\Leftrightarrow \frac{1}{2}$$

$$(1+2^2) = H^2$$

$$\Leftrightarrow 5 = H^2$$

$$\Leftrightarrow H = \sqrt{5}$$

l'aire du parallélogramme représenté ci-dessous d'abord sans et ensuite avec change-
variables. Un changement de variables vous semble-t-il utile dans ce cas? 1

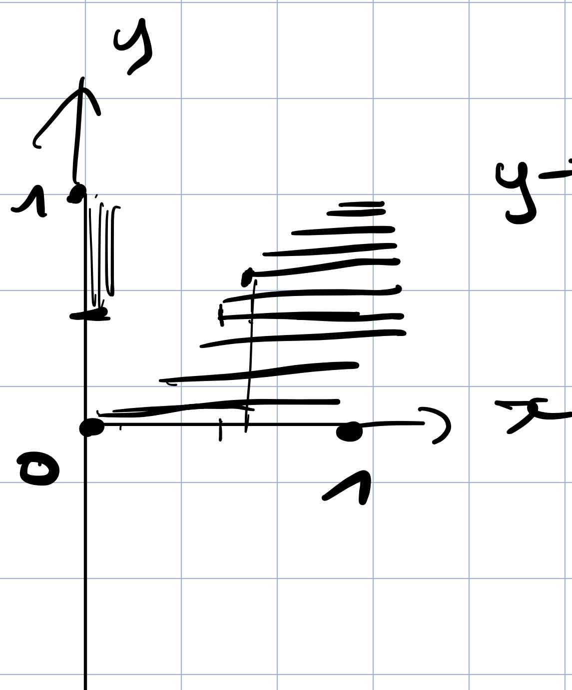
2 1 ≤ y ≤ 2

1→6

dy dx

$$\int_0^1 \int_y^1$$

$$\int_0^1 \int_0^x$$



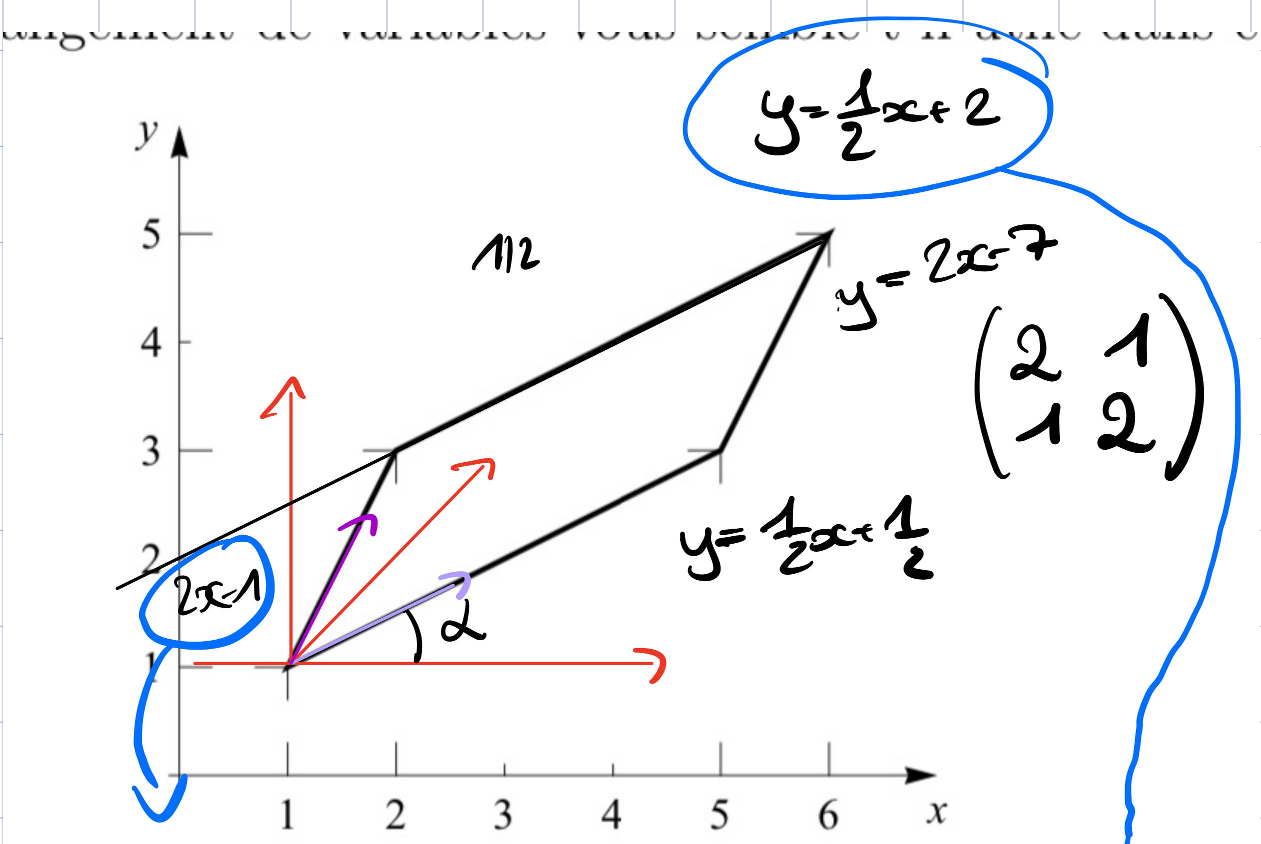
y→1

$\begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} 1 \\ a \end{pmatrix} \begin{pmatrix} 1 \\ b \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \qquad H(x,y) = \begin{pmatrix} 2x+y \\ x+2y \end{pmatrix}$$

$$J_H = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \qquad \int_0^2$$



$y = 2x - 1 \iff \underline{2x - y = 1}$

$y = \frac{1}{2}x + 2$

$\iff 2y - x = 4$

$\iff \underline{x - 2y = -4}$

$$\begin{cases} u = 2x - y = H_1 \\ v = x - 2y = H_2 \end{cases}$$

$$J_H = \begin{array}{c} \partial x \\ \partial x \end{array} \begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}$$

$\det(J_H) = -4 + 1 = -3$

$\det(J_G) = -1/3.$

$$\iint_D dx\,dy = \int_1^7 \int_{-4}^{-1} -\frac{1}{3}\, du\,dv = 6$$

Exercice 8

(i) 8 $\sqrt{4\cdot 2} = 2\sqrt{2}$

Si n n'est pas premier alors

$n = p_1^{k_1} \cdot p_2^{k_2} \cdot \ldots \cdot p_t^{k_t}$

supposons que $n = p_1 \cdot q_1$ (le cas minimal)

avec p_1 et $q_1 > \sqrt{n}$, impossible car $n < \sqrt{n}$

(ii)

- ~~règle de divisibilité par 2 ⇒ ok car n pairs~~

- ~~——— 3 ⇒ somme des chiffres est un multiple de 3 ⇒ ok car $n = 222$~~

- ~~——— 4 ⇒ les 2 derniers chiffres sont divisibles par 4 p. ex $n = \dots 20$~~

- ~~——— 5 ⇒ n finit par 0~~

- ~~——— 6 ⇒ n est divisible par 2 (finit par 0 p. ex) et par 3 ($\Sigma = 3k$)~~

- ——— 7

on sait que si on prend $n+1$ nombres dans m, on a deux nombres dans m minimum qui sont congrus à la même chose k mod n.
si on les prend et on les soustrait on aura un autre nombre dans m qui est congru à 0 mod n.

(ici on pose m = l'ensemble des nb de $\mathbb{N}$ qui ne sont composées que de 2)

(iii) $P(n)$: ...

Initialisation

$f_0 = 0$ ✓

$f_1 = \frac{\sqrt{5}}{\sqrt{5}} = 1$ ✓

$f_0 + f_1 = 1 \quad f_2 = 1.$

Hérédité

$$f_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

$$f_{n+1} = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right)$$

$$f_{n+2} = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+2}\right)$$

$$f_n + f_{n+1} = \frac{1}{\sqrt{5}}\Bigg($$

$$\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n} - \left(\frac{1-\sqrt{5}}{2}\right)^{n} + \left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$$

$$\Bigg)$$

car $\left(\frac{1+\sqrt{5}}{2}\right)^2 = \left(\frac{3+\sqrt{5}}{2}\right)$

$$\left(\frac{1+\sqrt{5}}{2}\right)^{n}\left(\overset{\frac{2}{2}}{1} + \frac{1+\sqrt{5}}{2}\right) -$$

$$\left(\frac{1-\sqrt{5}}{2}\right)^{n}\left(\frac{3-\sqrt{5}}{2}\right)$$

✓ Concls ok

(iv) Inițializare

$f_0 f_0 = 0$

$P(0,0)$

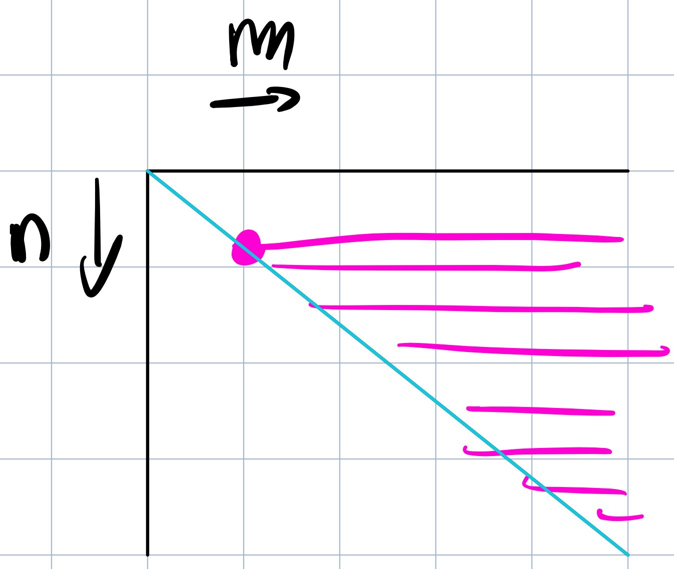
$P(m,0)$

$P(m+1,n) \rightarrow$

$$f_m f_0 = 0 = \frac{1}{5}\left(l_m - (-1)^0 l_m\right)$$

$$= 0$$

$\boxed{\text{ereditare}}$



$P(m,n)$

$\rightarrow P(m+1,n)$

$P(m,n)$

$\rightarrow P(m+1,n+1)$