Problem 3.1

1. $p_{S_1|S_0}(H|F) = 1/2 = p_{S_2|S_0}(H|F)$
$p_{S_1|S_0}(T|F) = 1/2 = \text{"}$
$p_{S_1|S_0}(H|R) = 3/4 = \text{"}$
$p_{S_1|S_0}(T|R) = 1/4 = \text{"}$

$$p_{S|S_0}((H,H)|F) = \frac{1}{4}$$
$$p_{S|S_0}((H,T)|F) = \frac{1}{4}$$
$$p_{S|S_0}((T,H)|F) = \frac{1}{4}$$
$$p_{S|S_0}((T,T)|F) = \frac{1}{4}$$

$$p_{S|S_0}((H,H)|R) = \frac{9}{16}$$
$$p_{S|S_0}((H,T)|R) = \frac{3}{16}$$
$$p_{S|S_0}((T,H)|R) = \frac{3}{16}$$
$$p_{S|S_0}((T,T)|R) = \frac{1}{16}$$

because S_1, S_2 indep. if we know the coin.

(2) $$H(S_0) = \sum_{s \in S_0} - p(s) \log_2(p(s))$$

$$= \frac{2}{3} \log_2\left(\frac{2}{3}\right) + \frac{1}{3} \log_2\left(\frac{1}{3}\right)$$

$$\simeq 0.92.$$

There are two possible outcomes for S_1:

$$p_{S_1}(H) = \frac{1}{3} \cdot \frac{3}{4} + \frac{2}{3} \cdot \frac{1}{2} = \frac{7}{12}$$
$$p_{S_1}(T) = \frac{1}{3} \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{1}{2} = \frac{5}{12}$$

$$H(S_1) = -\frac{7}{12} \log_2\left(\frac{7}{12}\right) - \frac{5}{12} \log_2\left(\frac{5}{12}\right)$$

$$= \underline{0.97} = H(S_2)$$

$$p_S((H,H)) = \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{9}{16} = \frac{17}{48}$$

$$p_S((H,T)) = \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{3}{16} = 11/48$$

$$p_S((T,H)) = \frac{2}{3}\cdot\frac{1}{4} + \frac{1}{3}\cdot\frac{3}{16} = 11/48$$

$$p_S((T,T)) = \frac{2}{3}\cdot\frac{1}{4} + \frac{1}{3}\cdot\frac{1}{16} = 9/48$$

$H(S) \approx 1.95$

$$-\frac{17}{48}\log_2\left(\frac{17}{48}\right)-\frac{2\times 11}{48}\log_2\left(\frac{11}{48}\right)-\frac{9}{48}\cdots$$

1.957385648

no $H(S_1,S_2) < H(S_1) + H(S_2)$

$= H(S)$

③ $H(S_1 | S_0 = y)$

$$= -\sum_{s \in A} p_{S_1|S_0}(s|y) \log_b p_{S_1|S_0}(s|y)$$

$$H(S_1|S_0) = H(S_1 | S_0 = F) \cdot p(S_0 = F) + H(S_1 | S_0 = R) \cdot p(S_0 = R)$$

$$= \frac{2}{3}\left(-2\log\left(\frac{1}{2}\right)\cdot\frac{1}{2}\right) + \frac{1}{3}\left(-\frac{3}{4}\log\left(\frac{3}{4}\right) - \frac{1}{4}\log\left(\frac{1}{4}\right)\right)$$

$$= 0.93 = H(S_2|S_0)$$

$$H(S|S_0) = H(S|S_0 = F)\cdot p(S_0 = F) + H(S|S_0 = R)\cdot p(S_0 = R)$$

$$= \frac{1}{3}\left(-\frac{9}{16}\log_2\left(\frac{9}{16}\right) - 2\cdot\frac{3}{16}\log_2\left(\frac{3}{16}\right)\right.$$

$$- \frac{1}{16}\log_2\left(\frac{1}{16}\right)\Big)\Big)$$

$$+ \frac{2}{3}\left(-4 \cdot \log\left(\frac{1}{4}\right) \cdot \frac{1}{4}\right)$$

$$H(S|S_0) = 1.87$$

As we know S_0, entropy is $\downarrow$.

(4) Yes, we can do:

$$H(S_1|S_0) + H(S_2|S_0) = H(S/S_0)$$

(5) $H(S_0, S_1, S_2)$

$$= H(S_0) + H(S_1|S_0) + H(S_2|S_0, S_1)$$

$$= 0.92 + 0.93 + 0.93$$

$$= \underline{2.7}$$

$$H(S_2 / S_0, S_1)$$

$$= H(S_2 \mid S_0 \cancel{S_1} = \{F\cancel{H}\}) \cdot p(S_0 S_1 = \{FH\})$$
$$+ H(S_2 \mid S_0 \cancel{S_1} = \{F\cancel{T}\}) \cdot p(S_0 S_1 = \{FT\})$$
$$+ H(S_2 \mid S_0 \cancel{S_1} = \{R\cancel{H}\}) \cdot p(S_0 S_1 = \{RH\})$$
$$+ H(S_2 \mid S_0 \cancel{S_1} = \{R\cancel{T}\}) \cdot p(S_0 S_1 = \{RT\})$$

$$= H(S_2 / S_0)$$

$$H(S_0 / S)$$

$$= H(S_0, S_1, S_2) - H(S)$$

$$= 2.7 - 1.95$$

Problem 3.2

① $H(X)$

$$= 6 \cdot \frac{1}{12} \log(12) + \frac{1}{2} \log(2)$$

$\simeq \underline{2.2924}$

③ $H(Y) = H(X)$

when we know X we know Y

④ $H(Y/X) = 0$

⑤ $H(X/Y) = 0$

⑥ $H(X,Y)$

ⓐ $H(X,Y)$

$$= H(X) + H(Y/X)$$

$$= H(X)$$

(b) $H(X, Y)$

$$= -\sum_{x,y} p_{XY}(x,y) \log\left(p_{XY}(x,y)\right)$$

$$= -\sum_{x} p_X(x)$$

$$p_{XY}(x,y) = p_X(x) \underbrace{p_{Y/X}(y/x)}_{\substack{=1 \text{ si } y=f(x) \\ 0 \text{ si } y \neq f(x)}}$$

$$p_{XY}(x,y) = p_X(x)\, p_Y(y/x)$$

$$\forall y \neq f(x) \quad p_{YX}(x,y) = 0$$

$$y = f(x) \Rightarrow p_{xy}(x,y) = p_x(x)$$

$$-\sum_{xy} p_{xy}(x,y) \log p_{xy}(x,y) =$$

ces termes seront 0 sauf 0

$$= -\sum_{x} p_x \log p_x$$

$$= H(x)$$

7

$$Z = \{\underset{1/2}{0}, \underset{1/6}{1}, \underset{1/6}{4}, \underset{1/6}{9}\}$$

$$H(z) = \frac{1}{6} \cdot (3) \cdot \log(6) + \frac{1}{2} \cdot \log(2) = \underline{1.79}$$

⑧ $H(Z/X) = 0$

$$= \sum_{x \in X} p_X(x) \left(- \sum_{z \in Z} p_{Z/X}(z,x) \log p_{X,Y}(z,x) \right)$$

$$= 0$$

$H(X/Z) =$

$$\sum_{z \in Z} p_Z(z) \left(- \sum_{x \in X} p_{X/Z}(x/z) \log p_{X/Z}(x,z) \right)$$

$$\overset{0}{=} \frac{1}{2}\left(0 \text{ (car proba = 1)} \right)$$

$+ 3 \cdot \frac{1}{6}\left(\frac{1}{2} \cdot \log(2) + \frac{1}{2} \cdot \log(2)\right)$

$+ \; - -$

$+ \; - -$

$= \frac{1}{2}.$

10 $H(X,Z) = H(X) + H(Z|X)$

a $= H(Z) + H(X/Z)$

$= 1.79 + 0.5$

$= \underline{2.29} = H(X)$

b $H(X,Y)$

$$= -\sum_{x,y} p_{XY}(x,y) \log\left(p_{XY}(x,y)\right)$$

we think about the cases $p_{XY}(x,y)$ will not be 0:

$$x = -1, 1 \quad y = 1$$
$$-2, 2 \quad y = 4$$
$$-3, 3 \quad y = 9$$
$$0 \quad y = 0$$

$$= 6 \cdot \frac{1}{12} \cdot \frac{1}{6} \cdot \log(12 \cdot 6)$$
$$+ \frac{1}{2} \cdot \frac{1}{2} \log(4)$$

$$\sum_{x}\sum_{z} p_{XZ}(x,z) \log p_{XZ}(x,z)$$

$$p_{XZ}(x,z) = \underline{p(x)}\underbrace{p(z|x)}_{1 \text{ or } 0}$$

$$= -\sum_x p_x(x) \log(p_x(x))$$

$$= H(x)$$

$$\sum_{x,z} p_{XZ}(x,z) =$$

$$= \sum_x \sum_z p_{ZX}(x z)$$

not a bijection

Problem 3.3

1

$$H(S) = -\sum_{S} p_s \log p_s$$

$$L(S, r_{SF})$$

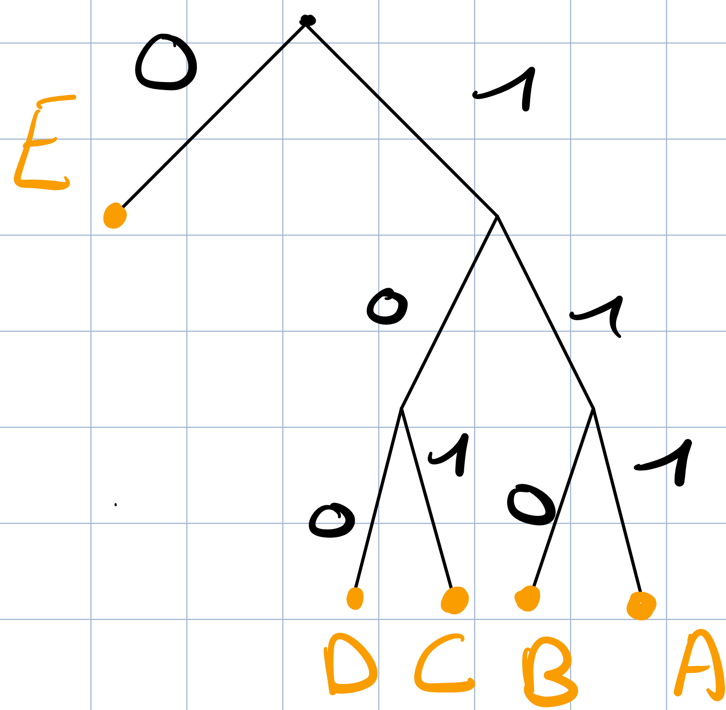
$$= \sum_{s \in S} p(s) \cdot \lceil -\log_2 (p_S(s)) \rceil$$

$$= \frac{10}{52} \cdot 3 + \frac{9}{52} \cdot 3 + \frac{8}{52} \cdot 3$$

$$+ \frac{5}{52} \cdot 4 + \frac{20}{52} \cdot 2$$

$$= \frac{141}{52} \simeq \boxed{2{,}7.}$$

②



$$L(S, r_H) = \frac{29}{13} \simeq 2.23$$

③ $p_{X_1}(\cdot)$

$1 \swarrow \quad \searrow 0$

$\frac{32}{52} \qquad \frac{20}{52}$

$p_{X_2|X_1}(\cdot|1)$

$1 \swarrow \quad \searrow 0$

$\frac{19}{52} \qquad \frac{13}{52}$

$P_{X_3|X_1,X_2}(\cdot \mid 1,0)$

$1 \swarrow \qquad \searrow 0$

$\frac{8}{52} \qquad\qquad \frac{5}{52}$

$P_{X_3|X_1,X_2}(\cdot \mid 1,1)$

$1 \swarrow \qquad \searrow$

$\frac{10}{52} \qquad\qquad \frac{2}{52}$

Adam ? calculs

4

$2 + 20 = 22$

$\frac{20}{52}$, $\frac{16}{52}$ / $\frac{9}{52}$, $\frac{8}{52}$, $\frac{5}{25}$

e 00

a 01

b 10

c 110

d 111

$L(S, r_F) = \underline{2.56}$.

1

$$H(X_1, X_2, X_3)$$

$$= H(X_1) + H(X_2) + H(X_3)$$

$$= \frac{1}{3}\log(3)\cdot 3 \;\begin{smallmatrix}a\\b\\c\end{smallmatrix} + \frac{1}{3}\log(3)\cdot 3 + \frac{1}{3}\log(3)\cdot 3$$

$$= 3\log(3)$$

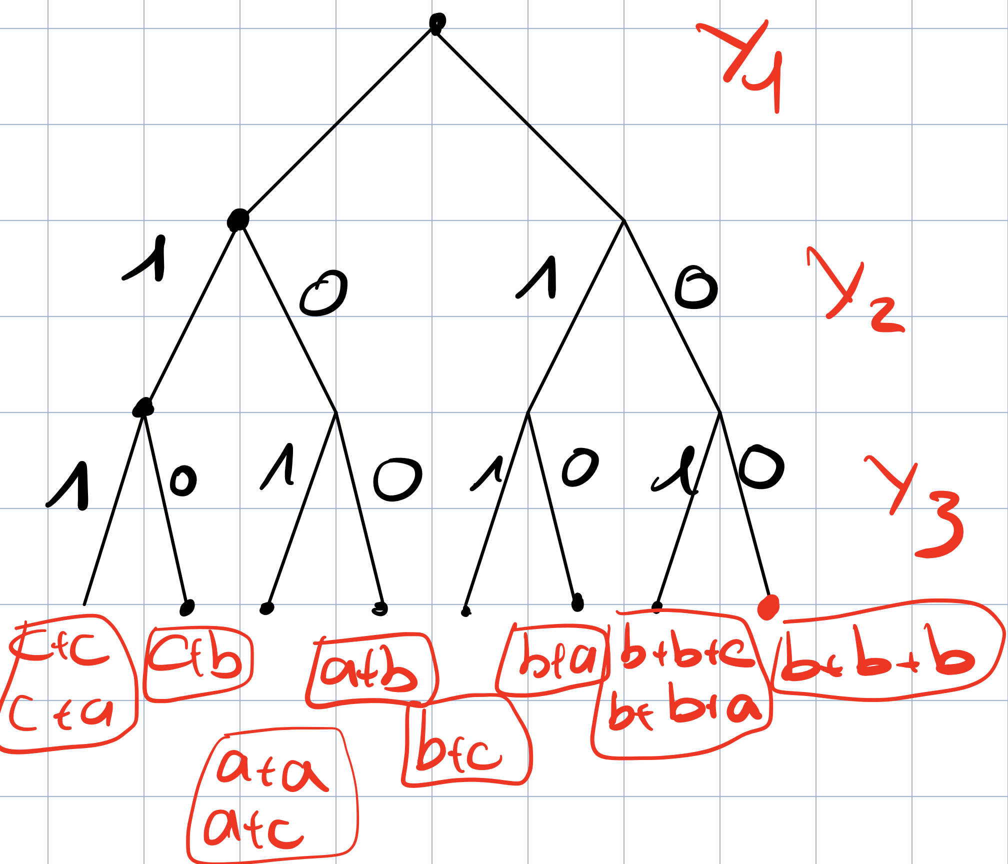
$$L(X, r)$$

$$= \frac{1}{3}\cdot 2 + \frac{1}{3} + \frac{1}{3}\cdot 2 = \frac{5}{3}$$

② $p_{Y_1 Y_2 Y_3}(0,0,1)$ 0

$(1,0,0)$

$(0,1,1)$



$p(0,0,0) = \frac{1}{27}$

$p(0,0,1) = \frac{2}{27}$

$p(0,1,0) = \frac{3}{27}$

$p(0,1,1) = \frac{3}{27}$

$$p(1,0,0) = \frac{3}{27} \qquad p(1,0,1) = \frac{6}{27}$$

$$p(1,1,0) = \frac{3}{27} \qquad p(1,1,1) = \frac{6}{27}$$

$$H(Y_1, Y_2, Y_3)$$

$$= \frac{3}{27} \cdot \log_2\left(\frac{27}{3}\right) \cdot 4 + \frac{6}{27} \cdot \log_2\left(\frac{27}{6}\right) \cdot 2$$

$$+ \frac{1}{27} \cdot \log_2\left(27\right) + \frac{2}{27} \cdot \log_2\left(\frac{27}{2}\right)$$

$$\approx 2{,}83 \text{ bits}$$

$$H(Y_1) = \frac{\overset{3+3+6+6}{18}}{27} \log_2\left(\frac{27}{18}\right) + \frac{9}{27} \log\left(\frac{27}{9}\right)$$

$$\approx 0.91 \text{ bits}$$

$$H(Y_2) = \frac{15}{27}\log_2\left(\frac{27}{15}\right) + \frac{12}{27}\log_2\left(\frac{27}{15}\right)$$

$$\simeq 0.99 \text{ bits}$$

$$H(Y_2|Y_1)$$

$$= \sum_{y_1 \in Y_1} \left[\sum_{y_2 \in Y_2} p(y_2/y_1) \cdot \log_2(p(y_2/y_1)) \right]$$

$$P(Y_2=0 \mid Y_1=0)$$

$$= \frac{P(Y_2=0 \cap Y_1=0)}{P(Y_1=0)}$$

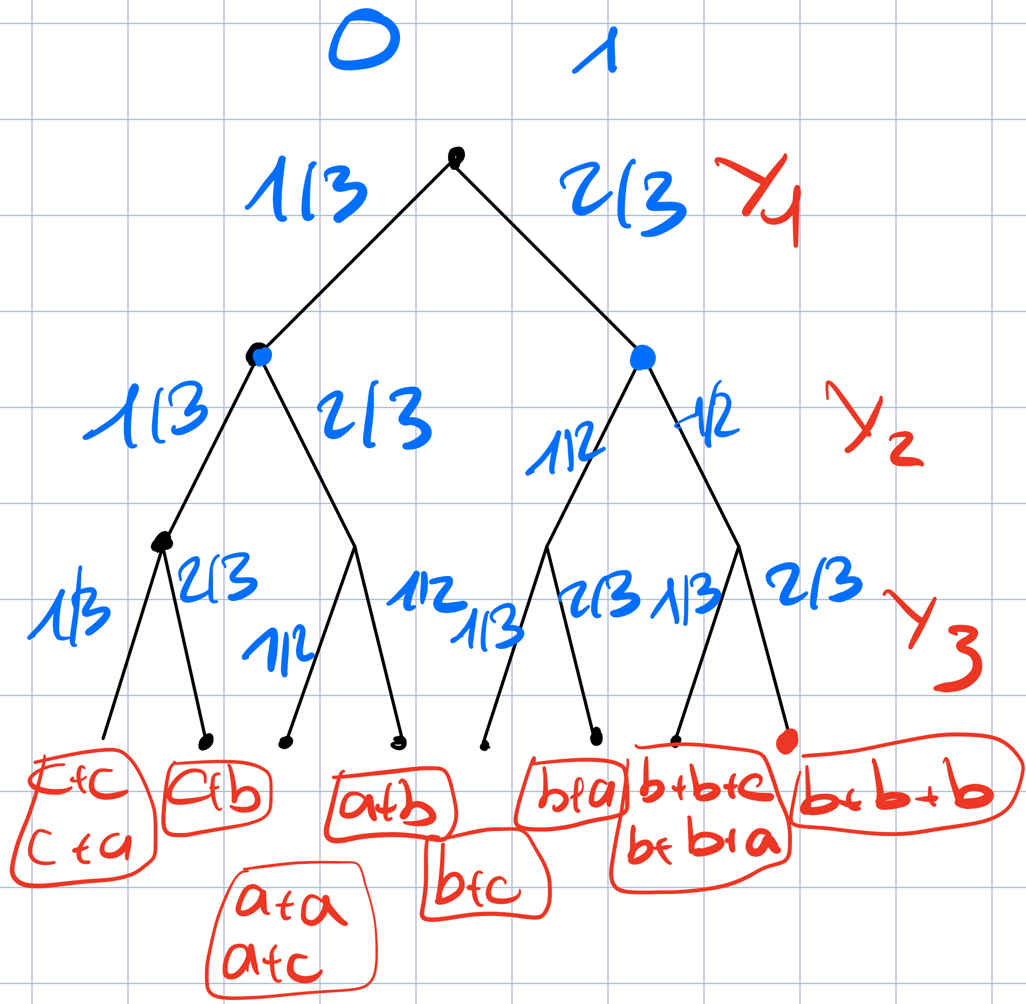
$$= \frac{3/27}{1/3} = \frac{1}{3}$$

$$P(Y_2=1 \mid Y_1=0) = \frac{2}{3}$$

$$P(Y_2=0 \mid Y_1=1)$$

$$= \frac{9}{27} \cdot \frac{27}{18} = \frac{1}{2}$$

$$P(Y_2=1 \mid Y_1=1) = \frac{1}{2}$$



$$H(Y_2|Y_1)$$

$$= \frac{1}{3}\left(\frac{1}{3}\log(3) + \frac{2}{3}\log\left(\frac{3}{2}\right)\right)$$

$$+ \frac{2}{3}\left(2 \cdot \log(2) \cdot \frac{1}{2}\right)$$

$$H(Y_3/Y_2)$$

$$= \frac{1}{9} \cdot \left(\frac{1}{3}\log(3) + \frac{2}{3}\log\left(\frac{3}{2}\right)\right)$$

$$+\frac{2}{9}\cdot\left[\frac{1}{2}\cdot 2\cdot \log(2)\right]$$

$$+\frac{2}{6}\left(\frac{1}{3}\log(3)+\frac{2}{3}\log\left(\frac{3}{2}\right)\right)\Big]$$

$$+\frac{2}{6}\left[\frac{1}{3}\log(3)+\frac{2}{3}\log\left(\frac{3}{2}\right)\right]\Big]$$

4

$$H(Y_1,\dots,Y_n)=\log 3+\frac{1}{3}H(Y_1,\dots,Y_{n-1})+\frac{2}{3}H(Y_1,\dots,Y_{n-2}). \tag{1}$$

Justify every step of the following proof. ← no extra info

$$\begin{aligned}
H(Y_1,\dots,Y_n) &= H(Y_1,\dots,Y_n,X_1) && (2)\\
&= H(X_1)+H(Y_1,\dots,Y_n|X_1) \quad \text{chain r.} && (3)\\
&= \log 3+\frac{1}{3}H(Y_1,\dots,Y_n|X_1=\mathrm{a})\\
&\quad +\frac{1}{3}H(Y_1,\dots,Y_n|X_1=\mathrm{b})\\
&\quad +\frac{1}{3}H(Y_1,\dots,Y_n|X_1=\mathrm{c}) \quad \boxed{2} && (4)\\
&= \log 3+\frac{1}{3}H(Y_1,\dots,Y_{n-2}) \leftarrow \text{we now know 2 bits (10)}\\
&\quad +\frac{1}{3}H(Y_1\dots,Y_{n-1}) \quad \text{1 bit (0)}\\
&\quad +\frac{1}{3}H(Y_1,\dots,Y_{n-2}) \quad \text{2 bits (11)} && (5)\\
&= \log 3+\frac{1}{3}H(Y_1\dots,Y_{n-1})+\frac{2}{3}H(Y_1,\dots,Y_{n-2}). && (6)
\end{aligned}$$

$\boxed{2}$ $H(X/Y)$

$$= p(Y=y_1)\cdot H(X/Y=y_1)$$
$$+ p(Y=y_2)\cdot H(X/Y=y_2)$$

5

$$H(Y_n/Y_1\dots Y_{n-1}) + H(Y_1\dots Y_{n-1}) = \log 3 + \frac{1}{3}H(Y_1\dots Y_{n-1}) + \frac{2}{3}H(Y_1\dots Y_{n-2})$$

$$\Leftrightarrow \frac{2}{3}H(Y_1\dots Y_{n-1}) + H(Y_n/Y_1\dots Y_{n-1}) = \log 3 + \frac{2}{3}H(Y_1\dots Y_{n-2})$$

$$\Leftrightarrow \frac{2}{3}\left[H(Y_{n-1}/Y_1\dots Y_{n-2}) + \cancel{H(Y_1\dots Y_{n-2})}\right] + H(Y_n/Y_1\dots Y_{n-1}) = \log 3 + \cancel{\frac{2}{3}H(Y_1\dots Y_{n-2})}$$

$$\Leftrightarrow \frac{2}{3}H(Y_{n-1}/Y_1\dots Y_{n-2}) + H(Y_n/Y_1\dots Y_{n-1}) = \log 3$$

(6)

$$S_n + \frac{2}{3} S_{n-1} = \log_3$$

$$x + \frac{2}{3}x = \log_3$$

$$\Rightarrow \frac{5}{3}x = \log_3$$

$$\Rightarrow x = \frac{3\log_3}{5}$$

Problem 3.1

① $ps1/so\,(H/F) = \frac{1}{2} = ps2/so\,(\sim)$

$ps1/so\,(H/R) = \frac{3}{4} = $ ——

$ps1/so\,(T/F) = \frac{1}{2} = $ ——

$ps1/so\,(T/R) = \frac{1}{4} - $ ——

$ps/so\,((H,H)/F) = \frac{1}{4}$

$ps/so\,((H,T)/F) = \frac{1}{4}$

$ps/so(\,(H,T)/R) = \frac{3}{16}$

$(H,T)/R = \frac{3}{16}$

$(T,T)/R) = \frac{1}{16}$

$(H,H)/R) \quad \frac{9}{16}$

(2) $H(S_0) = \frac{1}{3}\log(3) + \frac{2}{3}\log\left(\frac{3}{2}\right)$

$= \log(3) - \frac{2}{3}$

$H(S_1) = \frac{1}{3}\left[\frac{3}{4}\log\left(\frac{4}{3}\right) + \frac{1}{4}\log(4)\right]$

$+ \frac{2}{3}\left[\frac{1}{2}\log(2)\cdot 2\right]$

$H(S_2) = H(S_2)$

$H(S) = \frac{2}{3}\left[\frac{1}{2}\cdot\frac{1}{2}\cdot\log(\text{—})\cdot 4 \right]$ FHH FHT FTH FTT

$+ \frac{1}{3}\left[\frac{3}{4}\cdot\frac{1}{4}\log(\text{—})\right.$ RHT RTH

$+ \frac{1}{4}\cdot\frac{1}{4}\cdot\text{—}$

$\left.+ \frac{3}{4}\cdot\frac{3}{4}\cdot\text{—}\right]$

$\frac{}{4} \quad \frac{}{4}$

No because $H(S) < H(S_1) + H(S_2)$

③ $H(S_1 / S_0)$

$$= p(S_0 = F) \cdot H(S_1 / S_0 = F)$$
$$+ p(S_0 = R) \cdot H(S_1 / S_0 = R)$$

$$= \frac{2}{3} \left(\frac{3}{4} \log\left(\frac{4}{3}\right) + \frac{1}{4} \log(4) \right)$$
$$+ \frac{1}{3} \left(\frac{1}{2} \log(2) - 2 \right)$$

$$H(S_2 / S_0) = H(S_1 / S_0)$$

$H(S/S_0)$

$= \frac{2}{3}\left(4 \cdot \log(4) \cdot \frac{1}{4}\right)$

$+ \frac{1}{3}\left(\frac{3}{16} \log(\quad \text{—} \quad)\right)$

$H(S_0, S_1 S_2)$

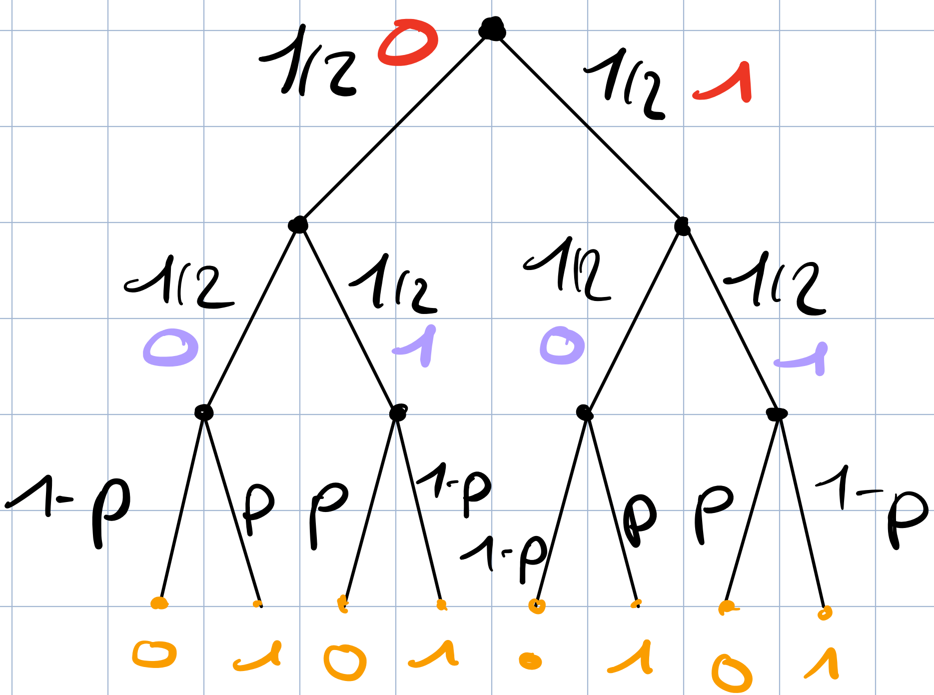
$= H(S_1, S_2 \mid S_0) + H(S_0)$

$= H(S_1 \mid S_0) + H(S_2 \mid S_0) + H(S_0)$

$$H(S_0/S) + H(S)$$
$$= H(S_0, S)$$

4.3

(1) $p_{Si}(0) =$



$$
\begin{aligned}
p_{S_2}(0) &= \frac{1}{2}\cdot\frac{1}{2}\cdot(1-p) \\
&+ \frac{1}{2}\cdot\frac{1}{2}\cdot p \\
&+ \frac{1}{2}\cdot\frac{1}{2}\cdot(1-p) \\
&+ \frac{1}{2}\cdot\frac{1}{2}\cdot p = \frac{1}{2}p + \frac{1}{2}(1-p) = 1/2
\end{aligned}
$$

Basis Step

$$p_{s_1}(0) = \frac{1}{2}$$

Inductive Step

Let's assume $P_{s_{i-1}}(0) = \frac{1}{2}$.

Then:

$$p_{s_i}(0)$$

$$= p_{s_{i-2}}(0) \cdot p_{s_{i-1}}(0) \cdot (1-p)$$

$$+ p_{s_{i-2}}(0) \cdot p_{s_{i-1}}(1) \cdot p$$

$$+ p_{s_{i-2}}(1) \cdot p_{s_{i-1}}(0) \cdot (1-p)$$

$$+ p_{s_{i-2}}(1) \cdot p_{s_{i-1}}(1) \cdot p$$

$$= 1/2$$

2. $H(S) =$ infinite bits ?

3.

Problem 3.4 (redo)

(1) IID Source:

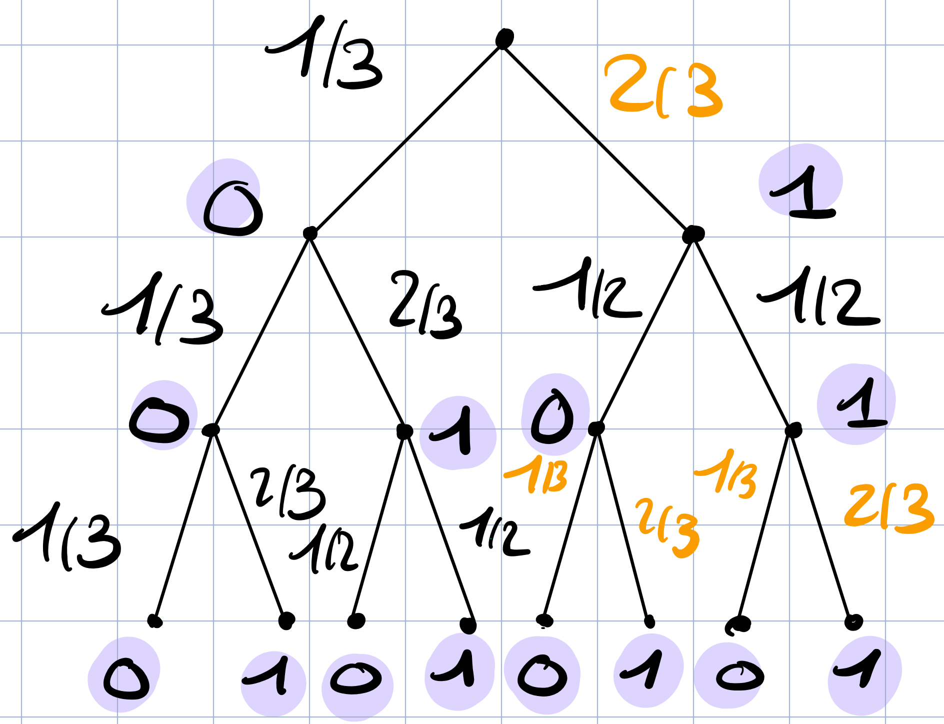
$$H(X_1, X_2, X_3) = H(X_1) + H(X_2) + H(X_3)$$
$$= 3H(X_1)$$
$$= 3 \cdot \left(3 \cdot \frac{1}{3}\log(3)\right)$$
$$= \underline{3\log(3)}$$

$$L(x, r) = \frac{1}{3} \cdot 2 \cdot 2 + \frac{1}{3} \cdot 1$$
$$= \frac{5}{3}.$$

②

001 011 101 111

010 100 110 000



1/3 - 2/3 become b or a or c at the beginning

1/2 because we know it's a or c

③ $H(Y_1 Y_2 Y_3)$

$$= \frac{1}{27}\log(27) + \frac{2}{27}\log\left(\frac{27}{2}\right) + \frac{2}{18}\log\left(\frac{18}{2}\right)\cdot 4 + \frac{4}{18}\cdot\log\left(\frac{18}{4}\right)\cdot 2$$

$\simeq 2.83$ bits

$$H(Y_1) = \frac{2}{3}\log\left(\frac{3}{2}\right) + \frac{1}{3}\log(3)$$

$$H(Y_2) = \frac{8}{18}\log\left(\frac{18}{8}\right) + \frac{10}{18}\log\left(\frac{18}{10}\right)$$

$$p(Y_2=0) = \frac{1}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{1}{2} = \frac{2}{18} + \frac{6}{18}$$

$$H(Y_2|Y_1) = \frac{1}{3}\left[\frac{1}{3}\log 3 + \frac{2}{3}\log\frac{3}{2}\right] + \frac{2}{3}\left[\frac{1}{2}\log(2) + \frac{1}{2}\log(2)\right]$$

moyenne pondérée

④

$$H(Y_1,\ldots,Y_n) = \log 3 + \frac{1}{3}H(Y_1,\ldots,Y_{n-1}) + \frac{2}{3}H(Y_1,\ldots,Y_{n-2}). \quad (1)$$

Justify every step of the following proof.

$$\begin{aligned}
H(Y_1,\ldots,Y_n) &= H(Y_1,\ldots,Y_n,X_1) && (2)\\
&= H(X_1) + H(Y_1,\ldots,Y_n|X_1) && (3)\\
&= \log 3 + \frac{1}{3}H(Y_1,\ldots,Y_n|X_1=\mathrm{a})\\
&\quad + \frac{1}{3}H(Y_1,\ldots,Y_n|X_1=\mathrm{b})\\
&\quad + \frac{1}{3}H(Y_1,\ldots,Y_n|X_1=\mathrm{c}) && (4)\\
&= \log 3 + \frac{1}{3}H(Y_1,\ldots,Y_{n-2})\\
&\quad + \frac{1}{3}H(Y_1\ldots,Y_{n-1})\\
&\quad + \frac{1}{3}H(Y_1,\ldots,Y_{n-2}) && (5)\\
&= \log 3 + \frac{1}{3}H(Y_1\ldots,Y_{n-1}) + \frac{2}{3}H(Y_1,\ldots,Y_{n-2}). && (6)
\end{aligned}$$

conditional entropy
- chain rule

← on connaît 2 caract.

(5)

$$H(Y_1 \dots Y_n) = \log 3 + \frac{1}{3} H(Y_1 \dots Y_{n-1}) + \frac{2}{3} H(Y_1 \dots Y_{n-2})$$

$$\Leftrightarrow H(Y_n | Y_1 \dots Y_{n-1}) + H(Y_1 \dots Y_{n-1}) = \log 3 + \dots$$

$$\Leftrightarrow H(Y_n | Y_1 \dots Y_{n-1}) = \log 3 - \frac{2}{3} H(Y_1 \dots Y_{n-1}) + \frac{2}{3} H(Y_1 \dots Y_{n-2})$$

$$= \log 3 - \frac{2}{3} H(Y_{n-1} | Y_1 \dots Y_{n-2}) .$$

(6)

$$u_n + \frac{2}{3} u_{n-1} = \log 3$$

$$\ell + \frac{2}{3} \ell = \log 3$$

$$\Rightarrow \frac{5}{3} \ell = \log 3 \Rightarrow \frac{3}{5} \log 3 = \ell .$$