

Exercise 1

a) i)

$$\log(256) = \text{8 bits}$$

ii)

$$[-2^{n-1} - 1; 2^{n-1}]$$

$$2^{n-1} = 1024 \Rightarrow n-1 = \log(1024) \\ \Rightarrow n-1 = 10 \\ \Rightarrow n = 11 \text{ bits}$$

iii)

3881

$$\begin{array}{r} - 3325 \\ \hline 0256 \end{array}$$

$\Rightarrow Q1$

iv)

$$\begin{array}{r} 11 \\ 879 \\ + 657 \\ \hline 1536 \end{array}$$

$$1024 \cdot 2 > 1536$$

\Rightarrow

11 bits

⑥ $\lceil \log(b-a) \rceil$

⑦ 2^n

⑧

Decimal	Binary	Octal	Hex
139	10001011	213	8B
185	10111001	271	B9
58	00111010	72	3A
201	11001001	311	C9
56	00111000	70	38
211	11010011	323	D3
27	00011011	33	1B
108	01101100	154	6C

16
32
64
80
96
112
128
144
160
176
192
208

$$139 = 16 \cdot 8 + 11$$

$$185 = 16 \cdot 11 + 9$$

Exercise 2

i $(236)_{10}$

$$= (00100110110)_{BCD}$$

$$(61)_{10}$$

$$= (01100001)_{BCD}$$

$$(158)_{10}$$

$$= (000101011000)_{BCD}$$

ii

$$\begin{array}{r} 0011 \quad 1001 \\ \hline 3 \qquad \qquad 9 \end{array}$$



$$\begin{array}{r} 0110 \quad 1010 \\ \hline 6 \qquad \qquad 10 \end{array}$$



$$\begin{array}{r} 1110 \quad 0011 \\ \hline 14 \end{array} \quad \times$$

iii too many bits

b

$$\begin{array}{r} 0110 \\ 1110 \\ 0101 \end{array} \begin{array}{l} \swarrow \\ \searrow \\ \times \end{array}$$

c

$$\begin{aligned} 236 &= 118 \cdot 2 + 0 \\ 118 &= 59 \cdot 2 + 0 \\ 59 &= 29 \cdot 2 + 1 \\ 29 &= 14 \cdot 2 + 1 \\ 14 &= 7 \cdot 2 + 0 \\ 7 &= 3 \cdot 2 + 1 \\ 3 &= 1 \cdot 2 + 1 \\ 1 &= 0 \cdot 2 + 1 \end{aligned}$$

$$(11101100)_2$$

11101100
 $(10011010)_{\text{Gray}}$

$$61 = 16 \cdot 3 + 13$$

$(00111101)_2$

$(00100011)_{\text{Gray}}$

$$158 = 16 \cdot 9 + 14$$

$(10011110)_2$

$= (11010001)_{\text{Gray}}$

Exercise 3

a) Signed $[-2^{n-1} - 1, 2^{n-1}]$

Unsigned $[0; 2^n - 1]$

Signed is better if app does not need neg. integers.

b) Signed: 0 → max positive → max negative → 0

Unsigned: 0 → max positive → 0

c)	Decimal	Sign and Flags.	1's compl.
	-32	101000000	11100000
	73	01001001	01001001
	-98	11100010	10011110
	47	00101111	00101111
	-39	10100111	11011001
	86	01010110	01010110

(d)

4-bit SN	8-bit SM	Hex	Dec	8-bit 2's Comp.
0110	0000 0110	6	6	0000 0110
1100	1000 0100	84	-4	1111 1100
1111	1000 0111	87	-7	1111 1001
0001	0000 0001	1	1	0000 0001

4-bit 2's	8-bit 2's	Hex	Dec	8-bit SM
1010	1111 1010	FA	-6	1000 0110
0101	0000 0101	S	S	0000 0101
1100	1111 1100	FC	-4	1000 0100
1000	1111 1000	F8	-8	1000 1000

e) i) 2's complement : non-redundant

ii) 2's complement : addition is the same
as subtraction

iii) 2's complement : better numerical range
+ different overflows

Exercise 4

a)

Unsigned

A	Bn(A)	A >> 1	A >> 3	A << 1	A << 3
39	00100111	00010011000001000100110	001110000000000000000000	001110000000000000000000	001110000000000000000000
192	11000000	01100000000110001000000	000000000000000000000000	000000000000000000000000	000000000000000000000000
75	01001011	0001001010000100110010100	000000000000000000000000	000000000000000000000000	000000000000000000000000

Signed

A	Bn(A)	A >> 1	A >> 3	A << 1	A << 3
39	00100111	00010011000001000100110	000000000000000000000000	010011000000000000000000	001110000000000000000000
-38	11011010	111010111111011	101101001101000	101101001101000	110100000000000
49	00110001	00010000000001000100010	000000000000000000000000	011000101000100010001000	100010000000000000000000
-106	10010110	110001111110010	001011001010100010100000	001011001010100010100000	101100000000000000000000

$$-38 = -64 + 26 \quad | -106 = -128 + 22$$

⑥ 16 nb de 8 bits
8 nb de 9 bits
4 nb de 10 bits
2 nb de 11 bits
1 nb de 12 bits

16 nb de 8 bits
8 nb de 16 bits
4 nb de 32 bits
2 nb de 64 bits
1 nb de 128 bits

⑦ $(6+7)_{10} = (0110 + 0111)_2$

$$\begin{array}{r} 1 \\ 0 \quad 1 \quad 1 \quad 0 \\ + \quad 0 \quad 1 \quad 1 \quad 1 \\ \hline 1 \quad 1 \quad 0 \quad 1 \end{array} = (-3)_{10}$$

⚠

$$(3-8)_{10} = (0011 + 1000)_2$$

$$= (1011)_2 - 5$$

$$(-2-10)_{10} = (1110 + 1000)_2$$

$$= (0110)_2 \quad 6 \quad \Delta$$

$$(5-3)_{10} = (0101 + 1101)_2$$

$$= (0010)_2 \quad 2$$

$$(1+5)_{10} = (0001 + 0101)_2$$

$$= (0110)_2 \quad 6$$

d) $(12)_{10} = (1100)_2$
 $(6)_{10} = (0110)_2$

$$\begin{array}{r}
 1100 \\
 \cdot 0110 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 0000 \\
 + 111000 \\
 + 110000 \\
 \hline
 \end{array}$$

$$1001000$$

e i remove the sign, multiply the magnitude, assign the sign with a $\text{XOR}(s_1, s_2)$

ii Convert to SFN \rightarrow multiply
 \rightarrow convert back to 2's compl.

Exercise S

a) Count the number of 1s and the number of diff. positions.

b) i) $\begin{array}{|c|} \hline 7 \\ \hline \end{array} \times (1s)$
 $3 \times (0s)$

ii) $\binom{10}{2}$

c) minimum distance: 2
max distance: 10

d) 1

e) $\begin{array}{c} 110 \\ 001 \end{array} \Rightarrow HD 3$ $\begin{array}{c} 001 \\ 101 \end{array} \Rightarrow HD 1$

⑧ i 2^n different strings

$(2^n)^2$ couples str. (u, v)

we want $u \neq v$ then

$$(2^{2n}) - (2^n)$$

nb of (u, v)

ii from 0 to n, $(n+1)$ distinct distances.

Exercise 6

a) $B_{\text{bias}} = 2^{m-1} - 1 = 2^3 - 1 = 7$

$$E = (0100)_2 = (4)_{10} = (-3)_{10}$$

$$\left(1 + \frac{1}{2}\right) \cdot 2^{-3}$$

- b) $(-1) \cdot \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{16}\right) \cdot 2^5$
- c) $(1) \cdot \left(1 + \frac{1}{2} + \frac{1}{4}\right) \cdot 2^8$
- d) $(1) \cdot \left(1 + \frac{1}{16}\right) \cdot 2^6$

⑥

i

$$(2.5)_{10} = (10.1)_2$$

$$= (.01)_2 \cdot 2^1$$

$$1 = E - B$$

$$\Leftrightarrow E = 1 + B$$

$$= 128 = 1000 \text{ 0000}$$

$$M = 0100 \dots 000$$

ii

$$(3.75)_{10} = (11.110)_2$$

$$(.1110) \cdot 2$$

$$E = 128$$

$$M = 11100 \dots$$

iii $(-4.25)_{10} = (-100.01)_2$

$$(0001)_2$$

$$E = 128 \quad S=1$$
$$\Pi = 0001$$

iv $(-5)_{10} = (-101.00)_2$

$$(0100)_2$$

$$E = 128 \quad S=1$$
$$\Pi = 0100$$

v $(6.25)_{10} = (110.01)_2$

$$E = 128 \quad S=0$$
$$\Pi = 1001$$

c) $(0.75)_{10} = (0.11)_2$

$$\begin{aligned}E_{\text{single}} &= -1 + B_{\text{single}} \\&= -1 + 127 \\&= 126\end{aligned}$$

$$\begin{aligned}E_{\text{double}} &= -1 + B_{\text{double}} \\&= -1 + (2^{10} - 1) \\&= 1022\end{aligned}$$

Exercise 7

a) i) $(1010.1101)_2$

$$\begin{aligned}&= 10 + 0.75 + 0.0625 \\&= 10.8125\end{aligned}$$

ii) $(10001010.1101)_2$

$$= -128 + 10.8125$$

$$= -118 + 0.8125$$

$$= -117.1875$$

iii

$$(10101001.0001)_2$$

$$= -128 + 32 + 8 + 1 + 0.0625$$

$$= -86.9375$$

iv

$$(0101001.1001)_2$$

$$= 4 + 1 + 16 + 64 + 0.5 + 0.0625$$

$$= 85.5625$$

v

$$(01011010.1111)_2$$

$$= 2 + 8 + 16 + 64 + 0.5 + 0.25 + 0.125$$

$$= 90.875 + 0.0625 \quad + 0.0625$$

$$= 90.9375$$

b

i

most positive number:

$$2^{n_1} + (1 - 2^{-n_2})$$

ii

$$2^{-n_2}$$

iii

$$-2^{n_1-1}$$

iv

$$-2^{-n_2}$$

c

$$(1.5)_{10} = (0001.1000)_2$$

$$(5.25)_{10} = (1001.0100)_2$$

$$(-0.125)_{10} = (-111.1110)_2$$

$$(-3.0)_{10} = (-1101.0000)_2$$

$$(-6.75)_{10} = (-1001.0100)_2$$

① $(01101.0010)_2$

$$E = 3 + 127 = 130$$

$$\Pi = 1010010$$

$$S = 0$$

ii $(11010.1101)_2$

$$E = 130$$

$$S = 1$$

$$\Pi = 0101101$$

iii $(01111.1111)_2$

$$E = 130$$

$$S = 0$$

$$\Pi = 1111\dots$$

iv $(11001.1001)_2 \Rightarrow E = 130$

$$S = 1$$

$$\Pi = 0011001\dots$$

Exercise 8

a) i) max value:

$$\begin{aligned} E_{\max} &= (2^8 - 1) - 3 \\ &= 2^8 - 2^7 \\ &= 2^7 \end{aligned}$$

Max menkasse: 111....

$$\begin{aligned} &= \left(1 + \frac{1}{2} + \frac{1}{4} + \dots\right) \\ &= 1 \cdot \frac{1 - \left(\frac{1}{2}\right)^{24}}{1 - \frac{1}{2}} \\ &= 2 \left(1 - 2^{-24}\right) \end{aligned}$$

$$\text{Value} = 2^{128} (2 - 2^{-23})$$

Range 2 · Value (symmetric because of ST)

II Some calculation with S3 instead of 24.

ii resolution: diff min non-zero value repres. and zero

$$E_{\min} = 0 - 127$$

$$\Pi_{\min} = (1 + 2^{-23})$$

$$\Pi_{\text{zero}} = 1$$

$$V_{\min} = (1 + 2^{-23}) \cdot 2^{-127} - 2^{-122}$$

$$= 2^{-23} \cdot 2^{-127} = \underline{\underline{2^{-150}}}$$

$$\begin{aligned} & 0.5 \\ & + 0.250 \\ & + 0.125 \\ & + 0.0625 \\ & + 0.03125 \end{aligned}$$

b $(2.7)_{10} = (10.1011)_2$

$$\begin{aligned} E &= 1 + (2^{2-1} - 1) \\ &= 2 \end{aligned}$$

$$(3.9)_{10} = (11.11100)_2$$

$$\begin{aligned} E &= 1+1 \\ &= 2 \end{aligned} \quad 1111$$

$$(-0.52) = (0.10000)_2$$

$$\begin{aligned} D &= 0000... \\ E &= -1+1=0 \end{aligned}$$

$$(-0.67) = (0.10101)_2$$

$$\begin{aligned} D &= 0110 \\ E &= 0 \end{aligned}$$

$$\begin{array}{r} + 125 \\ + 125 \\ + 000 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ + 250 \\ + 125 \\ + 0325 \\ \hline 4075 \end{array}$$

$$\textcircled{c} \quad (7.2)_{10} = (0111.0011)_2$$

$$(-6.42)_{10} = -(0110.01101)_2$$

$$\begin{aligned} &= -(0110.0111)_2 \\ &= (1001.1001)_2 \end{aligned}$$

$$(-3.67)_{10} = -(0011.10101)_2$$

$$\begin{aligned} &= -(0011.1011)_2 \\ &= (1100.0101)_2 \end{aligned}$$

$$(5.33)_{10} = (0101.01010)_2$$

$$= (0101.0101)_2$$

$$\begin{array}{r} 280 \\ + 0625 \\ \hline 3125 \end{array}$$

$$\textcircled{d} \quad \text{Resolution} = 2^{-23}$$

$$\text{Range} = (2^8 - 1) + (1 - 2^{-23})$$

312S

+ 0.5
0.0Exercises

a) $(14.51)_{10} = (1110.1000)_2$

$$B = 2^4 - 1 = 15$$

0.625

0.3125

0.15625

$$E = 3 + 1S = 18$$

$$\begin{aligned} I &= 10100001 \\ S &= 0 \end{aligned}$$

$$(-21.75)_{10} = (10101.11000)_10$$

$$E = 4 + 1S = 19$$

$$S = 1$$

$$I = 010111000$$

$$\begin{array}{r} 7 \\ 250 \\ + 125 \\ \hline 375 \end{array}$$

$$(7.45) = (111.01110...)_2$$

$$\pi = 101110011$$

$$E = 2eV$$
$$S = 0$$

$$(-2.725) = (10.101100)_2$$

$$\begin{array}{r}
 & 0.5 \\
 + & 0.125 \\
 + & 0.0625 \\
 \hline
 & 6875 \\
 - & 3125 \\
 \hline
 & 7\overline{1}875 \\
 & + 1562 \\
 \hline
 & 3437
 \end{array}$$

$$E = 16$$

$$S = 1$$

$$P = 0101110$$

+ many more is

(so it gives 111
when you add 1 to the
last bit)

We take the largest exponent

1011

$$14.51 \cap = 0.110\ldots$$

- b) i) takes less space, better range
better resolution
 - ii) takes less space

Exercise 10

$$(0.125)_{10} = (0.001)_2$$

$$\begin{array}{l} E = -3 \\ n = 0000 \dots \end{array}$$

$$0 \quad \underbrace{0111111}_{E=0} \quad 0000\dots$$

$$\Rightarrow (0.125)_{10} = (0 \quad 01111100 \quad 000\dots)$$

$$(0.25)_{10} = (0.0100)_2$$

$$\Rightarrow (0 \quad 01111101 \quad 000\dots)$$

We take the largest exponent:

$$\begin{array}{r} 0.25 \\ 0.125 \\ \hline 0.125 \end{array} \quad \begin{array}{l} 0 \quad 01111101 \quad 1.00000 \\ 0 \quad 01111101 \quad 0.10000 \\ \hline 0 \quad 01111101 \quad 1.10000 \end{array}$$

$\frac{28}{725}$

$$(-0.375)_{10} = (0.011)_2$$

$$(0.5)_{10} = (0.1)_2$$

0.5	0 0111110	1.0000...
-0.375	1 0111110	0.1100...
Sum	0 0111110	1.1100...
\Rightarrow	0 011110	0.01000...
\Rightarrow	0 0111100	1.0000...

$$(0.625)_{10} = (0.101)_2$$

$$(0.75)_{10} = (0.110)_2$$

0.625	0 0111110	1.0100000
0.75	0 0111110	1.1000000
Sum	0 0111110	1.0100000
	0 0111111	01000

$$\begin{array}{r}
 250 \\
 +125 \\
 \hline
 875
 \end{array}$$

$$\begin{aligned}
 (0.875)_{10} &= (0.111)_2 \\
 (1.0)_{10} &= (1.0)_2
 \end{aligned}$$

$$\begin{array}{r}
 1 | & 1 & 01111111 & 1.00000 \\
 0.875 | & 1 & 01111111 & 0.11100 \\
 \text{Sum} & 1 & 01111111 & 1.11100...
 \end{array}$$

$$\begin{aligned}
 (1.125) &= (1.001)_2 \\
 (-1.25) &= (1.010)_2
 \end{aligned}$$

$$\begin{array}{r}
 1.125 | & 0 & 01111111 & 1.0010000 \\
 -1.25 | & 1 & 01111111 & 1.0100000 \\
 \hline
 & & 01111111 & 1.111 \\
 \Rightarrow & 1 & 01111111 & 0.0010000 \\
 \Rightarrow & 1 & 01111100 & 00000
 \end{array}$$

b

$$(1.125)_{10} = (1.001)_2$$

$$(-3.25)_{10} = (-11.010)_2$$

$$\begin{array}{r} 1 \\ 11.010 \\ + 1.001 \\ \hline 100.011 \end{array}$$

$$\begin{array}{r} 250 \\ - 125 \\ \hline 375 \end{array}$$

$$(4.375)_{10} = (10100.010)_2$$

$$(-4.375)_{10} = (-1011.1010)_2$$

$$(6.5)_{10} = (110.100)_2$$

$$\begin{array}{r} 11011.1010 \\ + 0110.1000 \\ \hline 110010.0010 \end{array}$$

$$(2.625)_{10} = (10.101)_2$$

$$\begin{aligned} (-7.75)_{10} &= -(0111.1100)_2 \\ &= (1000.0100)_2 \end{aligned}$$

$$\begin{aligned} (-0.875)_{10} &= -(0.1110)_2 \\ &= (1111.0010)_2 \end{aligned}$$

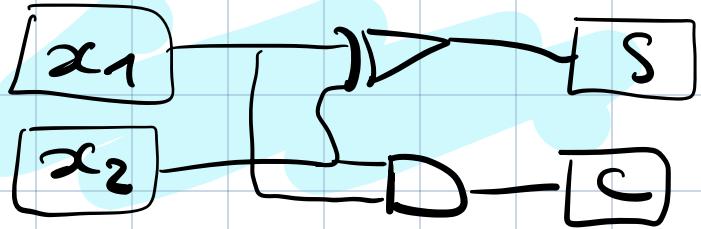
$$\begin{aligned} (-1.125)_{10} &= -(1.0010)_2 \\ &= (1110.1110)_2 \end{aligned}$$

$$(6.25)_{10} = (110.0100)_2$$

$$\begin{aligned} (-3.875)_{10} &= -(11.1110)_2 \\ &= (100.0010)_2 \end{aligned}$$

(half-adder)

x_1	x_2	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



(full-adder)

x_1	x_2	c_i	s_0	c_0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

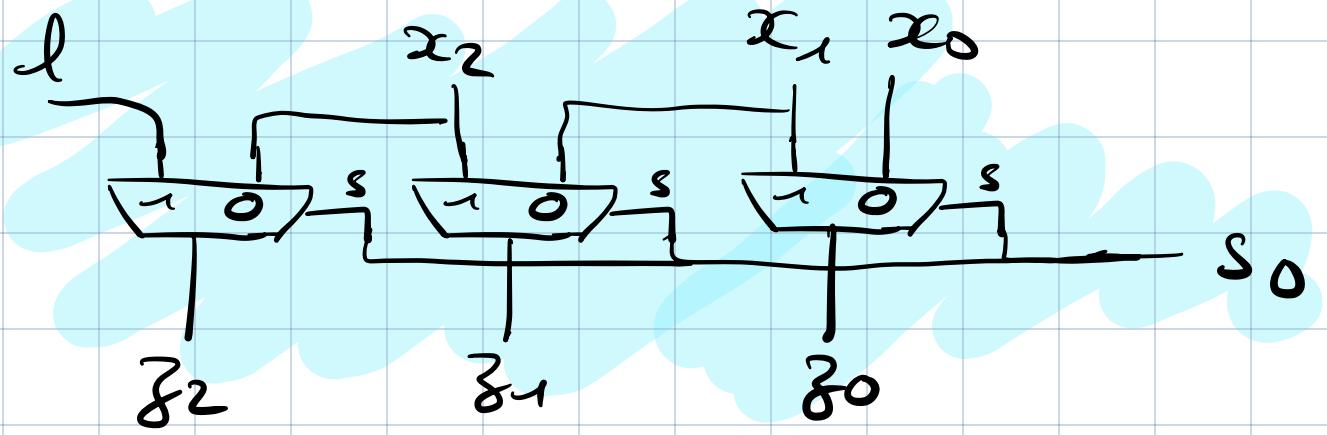
$$(s_0, c_1) = HA(x_1, x_2)$$

$$(s_0, c_2) = HA(s_0, c_i)$$

$$c_0 = c_1 \text{ or } c_2$$

S	OP
0	$x+y$
1	$x+\bar{y}+1$

$\Rightarrow x + (y \oplus S) + S$



Exercise 1

a

$$2^n$$

b

$$Y_{\omega}$$

c

$$\overline{x_1 x_2} \equiv \overline{x_1} + \overline{x_2}$$

d

$$p_2 = x_1 x_2$$

$$p_1 = (\overline{x_1 x_2})(x_1 + x_2)$$

$$p_3 = p_1 x_3$$

$$p_4 = p_1 + x_3$$

$$f = p_2 + p_3$$

$$g = \overline{p_3} p_4$$

Exercise 2

(a) $\boxed{i} \subset (\bar{a} + b)$

$\boxed{ii} a + (\bar{b}c)$

$\boxed{iii} a(\bar{b} + c) + (b)$

(b)

$$\begin{aligned} & \cancel{abc} = abc \\ & \cancel{ac} + b \\ & \cancel{a} + \cancel{bc} \end{aligned}$$

(c)

$$\begin{aligned} a + bc &= a + bc \quad \checkmark \\ (a+b)(a+\bar{b}) &= a \quad \checkmark \end{aligned}$$

Exercise 3

a) $A = x_3(x_1 + \bar{x}_2) = x_3x_1 + x_3\bar{x}_2$
 $B = x_1(\bar{x}_2 + \bar{x}_3) = x_1\bar{x}_2 + x_1\bar{x}_3$
 $C = x_2(x_1 + \bar{x}_3) = x_2x_1 + x_2\bar{x}_3$

$$AB + BC + AC$$

$$(x_3x_1 + x_3\bar{x}_2)(x_1\bar{x}_2 + x_1\bar{x}_3)$$

$$= x_3x_1\bar{x}_2 = AB$$

$$BC = (x_1\bar{x}_2 + x_1\bar{x}_3)(x_2x_1 + x_2\bar{x}_3)$$

$$= x_1x_2\bar{x}_3$$

$$AC = (x_3x_1 + x_3\bar{x}_2)(x_2x_1 + x_2\bar{x}_3)$$

$$= x_3x_2x_1$$

$$x_1 \overbrace{x_2}^{x_1 x_2} x_3 + x_1 x_2 \overbrace{x_3}^{x_1 x_2 x_3} + x_1 x_2 x_3$$

$$\equiv x_1 (\bar{x}_2 x_3 + x_2 \bar{x}_3 + x_2 x_3)$$

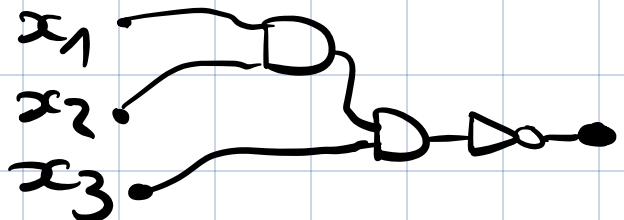
$$\equiv x_1 (x_2 + x_3)$$

b

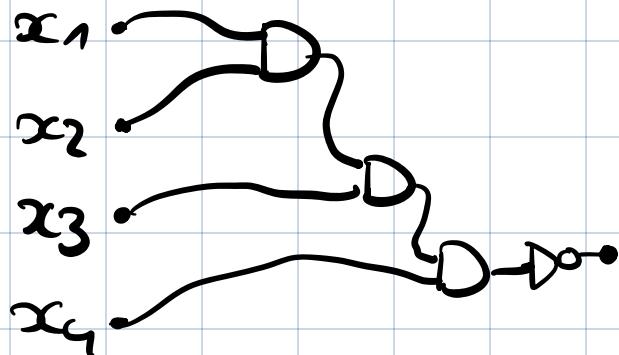
i

x_1	x_2	x_3	s
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

ii

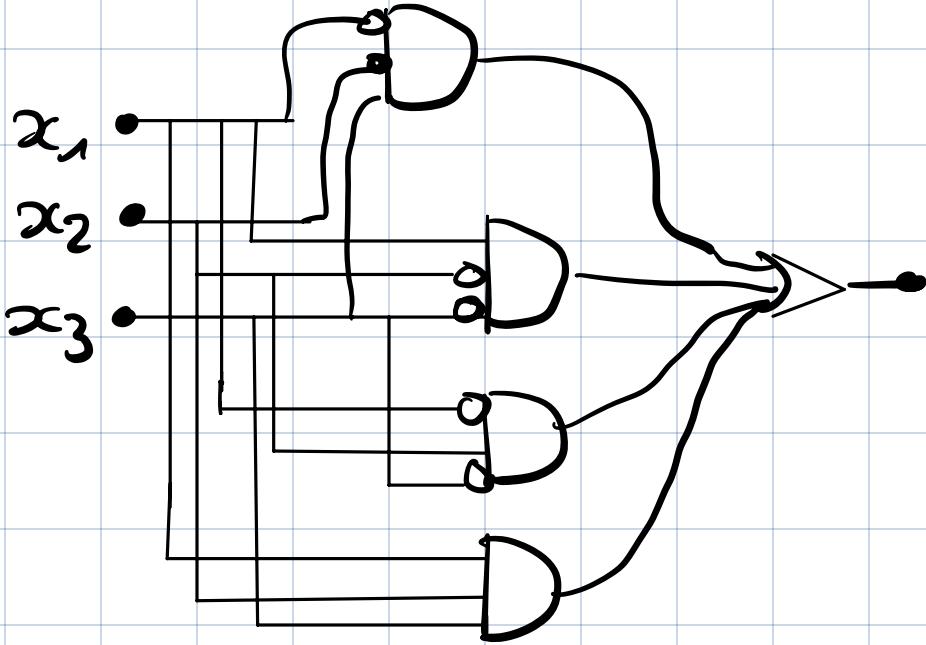


iii



c

ji



Exercise 4

a

a

b

c

p₁

p₂

p₃

p₄

f

b

a

b

c

p₁

p₂

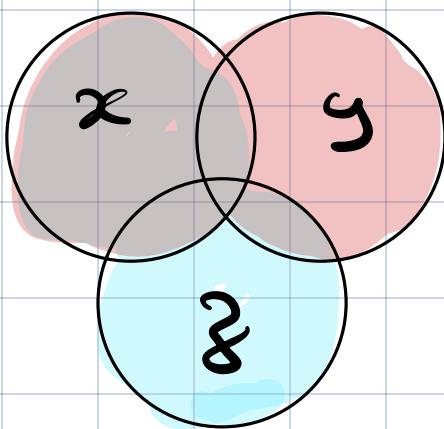
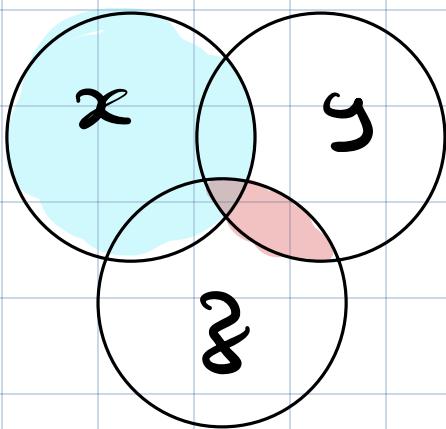
p₃

p₄

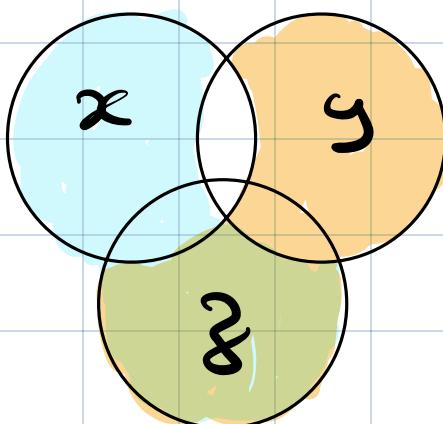
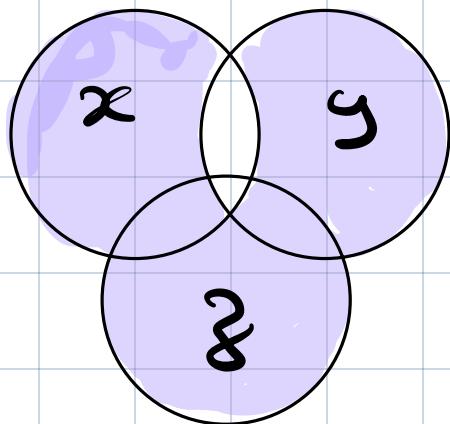
g

Exercise 5

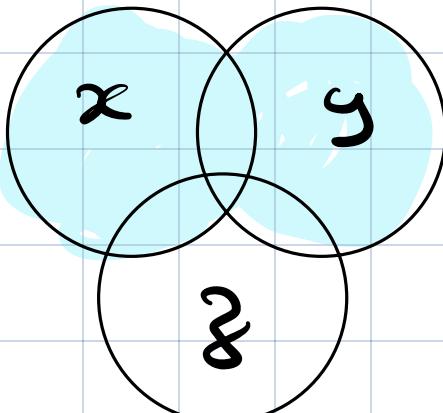
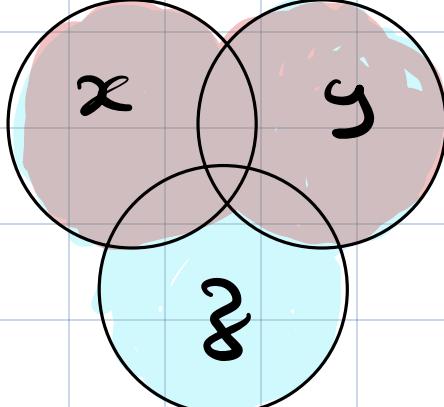
a



b



c



Exercise 7

a /
b /

c time for C_1N_1 combination:

$$x_1 + x_2 (\text{Se}) : 3\text{ns}$$

$$x_1 + x_2 (\text{Ce}) : 2\text{ns}$$

$$8 + \text{C}_1\text{N}_0 (\text{S}_0) : 3\text{ns}$$

$$8 + \text{C}_1\text{N}_0 (\text{C}_0) : 2\text{ns}$$

$$\text{C}_0\text{C}_1 : 2\text{ns}$$

$$3\text{ns} + 2\text{ns} + 2\text{ns} : \boxed{7\text{ns}}$$

time for S_1 combination:

$$x_1 + x_2 (\text{St}) : 3\text{ns} \quad \checkmark$$

$$\text{time } \text{C}_1\text{N}_0 \rightarrow \text{C}_1\text{N}_1 : 2\text{ns} + 2\text{ns} = \boxed{4\text{ns}}$$

$$\tau + 4 \cdot (\text{ripple_cnt} - 1)$$

$$= 7 \cdot 4 \cdot 3 = 12 \cdot 7 = 19 \text{ ns}$$

$$\tau + 4(\tau) = 28 + 7 = \underline{\underline{35 \text{ ns}}}$$

Exercise 8

a) $\bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}\bar{c}$
+ $\bar{a}\bar{b}\bar{c} = f$

$$(a+b+\bar{c})(a+\bar{b}+\bar{c})(\bar{a}+b+c)$$
$$(\bar{a}+\bar{b}+\bar{c}) = f$$

i) $\bar{a}\bar{b}c + ab\bar{c} + abc = f$

$$(a+b+c)(a+\bar{b}+c)(a+\bar{b}+\bar{c})$$
$$(\bar{a}+b+c)(\bar{a}+b+\bar{c}) = f$$

b) $f = \bar{a}\bar{b}c + \bar{a}bc + \bar{a}\bar{b}\bar{c} + abc$

$m_1 \quad m_3 \quad m_2 \quad m_7$

$$f = \prod_O + \prod_U + \prod_S + \prod_G$$

$$= (\bar{a}+b+c)(\bar{a}+\bar{b}+c)(\bar{a}+b+\bar{c})(\bar{a}+\bar{b}+\bar{c})$$

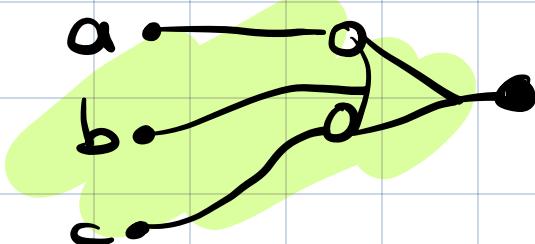
ii

$$\begin{aligned} f = & \underset{m_3}{\bar{a}\bar{b}c} + \underset{m_2}{\bar{a}\bar{b}\bar{c}} + \underset{m_1}{\bar{a}\bar{b}c} + \underset{m_0}{\bar{a}\bar{b}\bar{c}} \\ & + \underset{m_7}{ab\bar{c}} + \underset{m_6}{a\bar{b}\bar{c}} + \underset{m_4}{\bar{a}\bar{b}\bar{c}} \end{aligned}$$

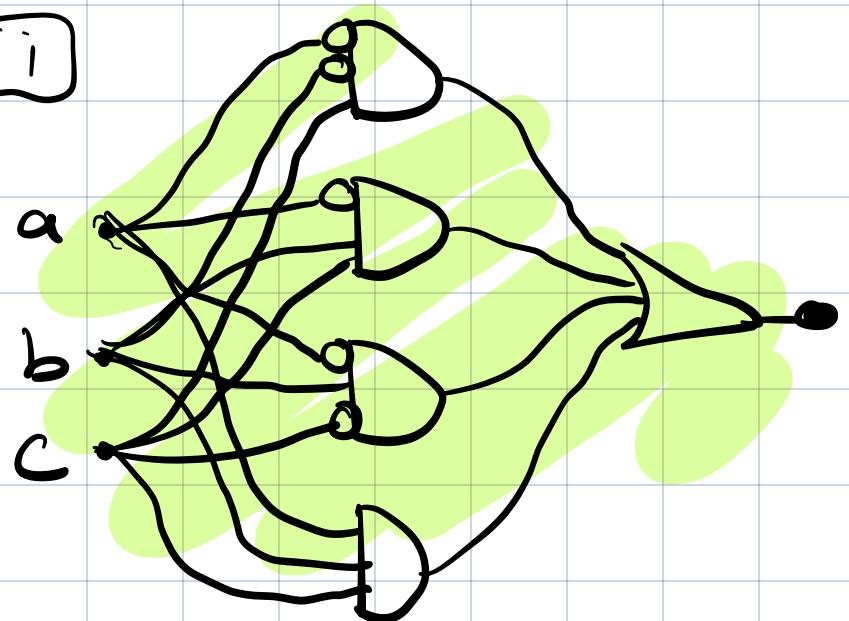
$$f = (\bar{a}+b+\bar{c})$$

c b

ii



i



Exercise 9

a) $\bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}\bar{c}$ $m_2 \quad m_0$
 $+ a\bar{b}c + \bar{a}bc$ $m_7 \quad m_3$
 $+ a\bar{b}c + a\bar{b}\bar{c}$ $m_5 \quad m_4$



$\bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c$ $m_2 \quad m_3$
 $+ ab\bar{c} + a\bar{b}c$ $m_7 \quad m_5$
 $+ a\bar{b}\bar{c} + \bar{a}\bar{b}\bar{c}$ $m_4 \quad m_0$

b) $\bar{a}bc + \bar{a}\bar{b}c$ $m_3 \quad m_1$
 $+ ab\bar{c}$ m_6
 $+ \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c$ $m_2 \quad m_3$
 $+ a\bar{b}c + a\bar{b}\bar{c}$ $m_5 \quad m_4$



$\bar{a}\bar{b}c + \bar{a}\bar{b}\bar{c}$ $m_5 \quad m_1$
 $+ ab\bar{c} + a\bar{b}\bar{c}$ $m_6 \quad m_4$
 $+ ab\bar{c} + \bar{a}\bar{b}\bar{c}$ $m_6 \quad m_2$
 $+ \bar{a}bc$ m_3

(c) $a\bar{b}\bar{c} + \bar{a}\bar{b}\bar{c}$ m_6 m_4
 $+ a\bar{b}c + \bar{a}bc$ m_7 m_3
 $+ a\bar{b}\bar{c} + \bar{a}\bar{b}\bar{c}$ m_4 m_0

$$\Pi_2 + \Pi_1 + \Pi_S$$



(d) $(a+c)(\bar{a}+\bar{b}+c)(\bar{a}+b)$

$$= (a+\bar{b}+c)(a+b+c)(\bar{a}+\bar{b}+\bar{c})(\bar{a}+b+c)$$

$$(\bar{a}+b+c)$$

$$\Pi_2 \quad \Pi_0 \quad \Pi_7 \quad \Pi_4 \quad \Pi_S$$

$$(a+b)(b+c)(\bar{a}+\bar{c})$$



$$= (a+b+c)(a+b+\bar{c})(a+\bar{b}+c)(\bar{a}+\bar{b}+c)$$

$$(\bar{a}+\bar{b}+\bar{c})(\bar{a}+\bar{b}+c)$$

$$\Pi_0 \quad \Pi_1 \quad \Pi_0 \quad \Pi_4 \quad \Pi_S \quad \Pi_7$$

Exercise 10

a) i)

$$f = \bar{a}\bar{b}c + \bar{a}b\bar{c} + a\bar{b}\bar{c} + abc$$

$$f = \bar{a}\bar{b}c + \bar{a}b\bar{c} + a\bar{b}\bar{c} + abc$$

$$= (\bar{a}\bar{b}c)(\bar{a}b\bar{c})(a\bar{b}\bar{c})(abc)$$

ii) $f = (ab + cd) + ((a+b)(\bar{c}+d))$

$$= ab + cd + a\bar{c} + ad + \bar{b}\bar{c} + \bar{b}d$$

$$= (\bar{a}b)(\bar{c}d)(a\bar{c})(ad)(\bar{b}\bar{c})(\bar{b}d)$$

iii) $f = (\bar{a}\bar{b}c)(\bar{a}b\bar{c})(a\bar{b}\bar{c})(abc)$

$$\textcircled{b} \quad f = (a+b+c)(\bar{a}+\bar{b}+\bar{c})(\bar{a}+b+\bar{c}) \\ (\bar{a}+\bar{b}+c)$$

$$= (\underline{a+b+c}) + (\underline{a+\bar{b}+\bar{c}}) + (\bar{a}+b+\bar{c}) + (\bar{a}+\bar{b}+c)$$

$$\textcircled{ii} \quad f = (\bar{\bar{a}}\bar{b} + \bar{c}\bar{d}) + ((a+\bar{b})(\bar{c}+d))$$

$$= (\bar{a}+\bar{b}) + (\bar{c}+\bar{d}) + (\underline{a+\bar{b}}) + (\bar{c}+d)$$

$$= (\bar{a}+\bar{b}) + (\bar{c}+\bar{d}) + (\bar{a}+\bar{b}) + (\bar{c}+d)$$

$$\textcircled{iii} \quad f = \underset{m_2}{\bar{a}\bar{b}c} + \underset{m_4}{\bar{a}\bar{b}\bar{c}} + \underset{m_6}{a\bar{b}\bar{c}} + \underset{m_7}{ab\bar{c}}$$

$$= \underset{N_0}{(a+b+c)} \underset{N_1}{(a+b+\bar{c})} \underset{N_3}{(a+\bar{b}+\bar{c})} \underset{N_5}{(\bar{a}+\bar{b}+c)}$$

$$= (\bar{a}+\bar{b}+c) + (\bar{a}+\bar{b}+\bar{c})(a+\bar{b}+\bar{c})\bar{a}+\bar{b}+\bar{c})$$

Exercise 12

a

	w	x	y	z	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	0	1	1	0	1	0	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

$$a = (w + x + y + \bar{z})(w + \bar{x} + y + z)$$

$$= w + y + \bar{z}\bar{x} + xz$$

$$b = (w + \bar{x} + y + \bar{z})(w + \bar{x} + \bar{y} + z)$$

$$= W + \bar{X} + YZ + \bar{Z}\bar{Y}$$

$\bar{Y}Z$	$\bar{W}X$	00	11	01	00
10	00	11			
11	00	11	11		
01	10	10	10		
00	10	01			

$$W\bar{X}\bar{Y} + \bar{W}Y + \bar{W}XZ + \bar{W}\bar{X}\bar{Z}$$

Exercise 13

① $a \square f = \bar{ac} + cb$

ii

a

b

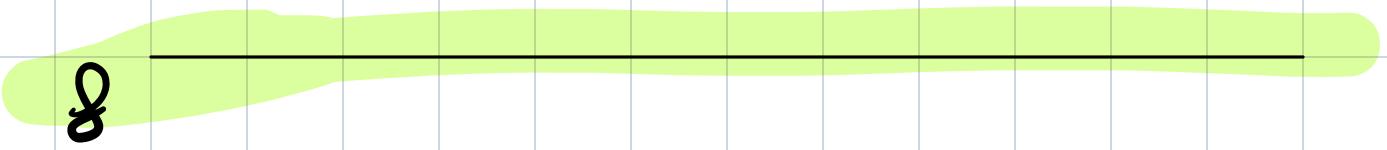
c

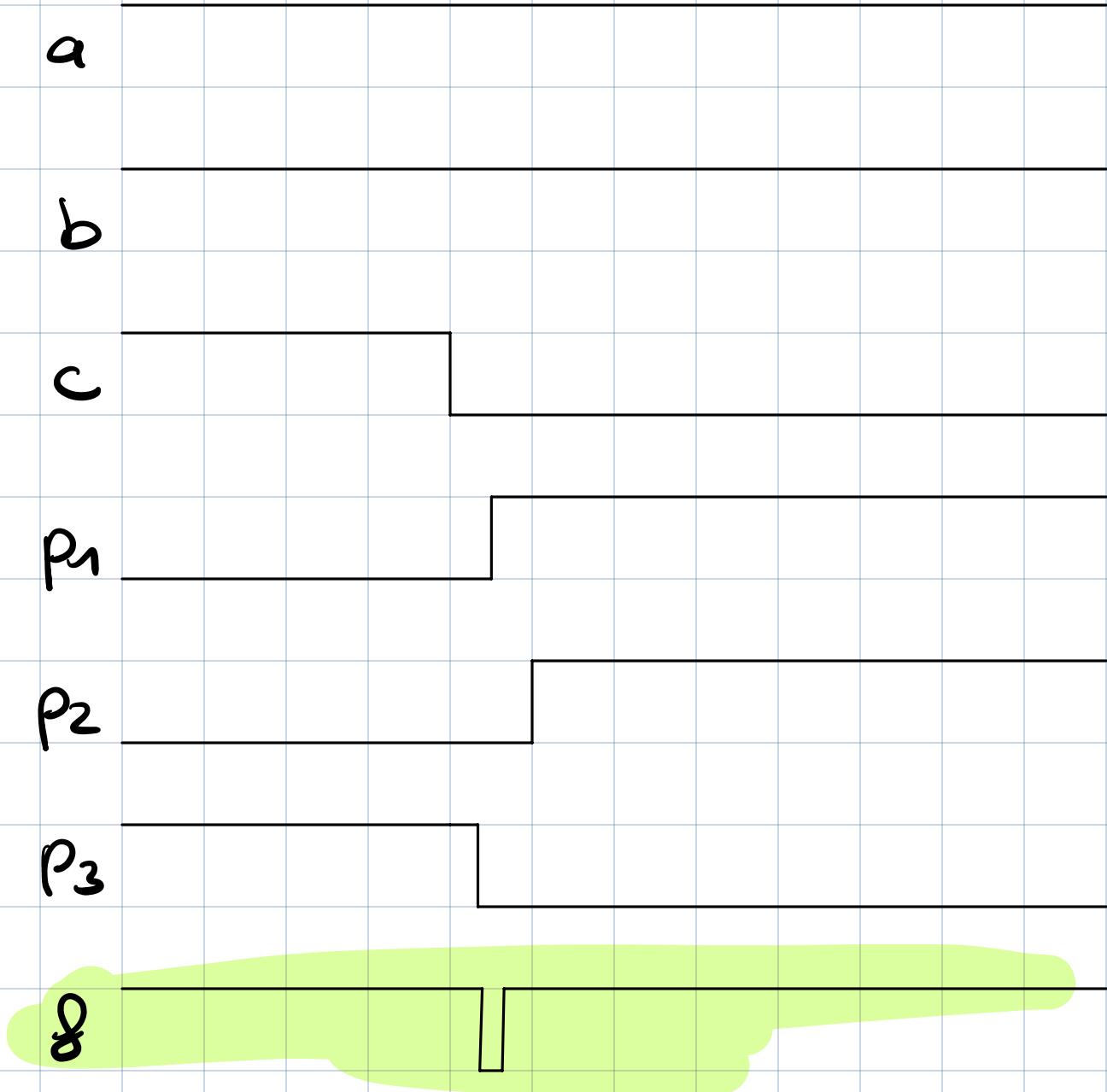
p₁

p₂

p₃

f





6

i

$$f = \bar{ac} + bc + ab$$

a

b

c

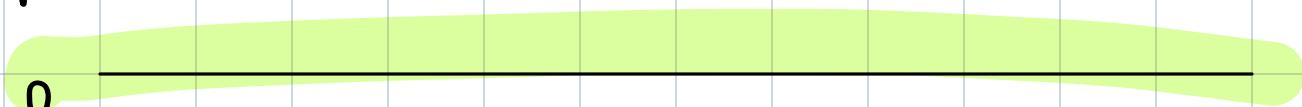
p₁

p₂

p₃

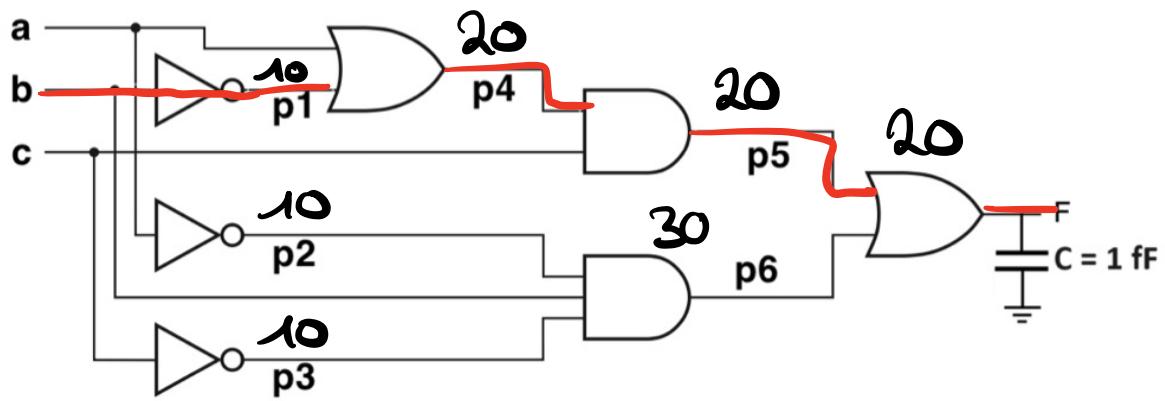
p₄

f



Exercise 14

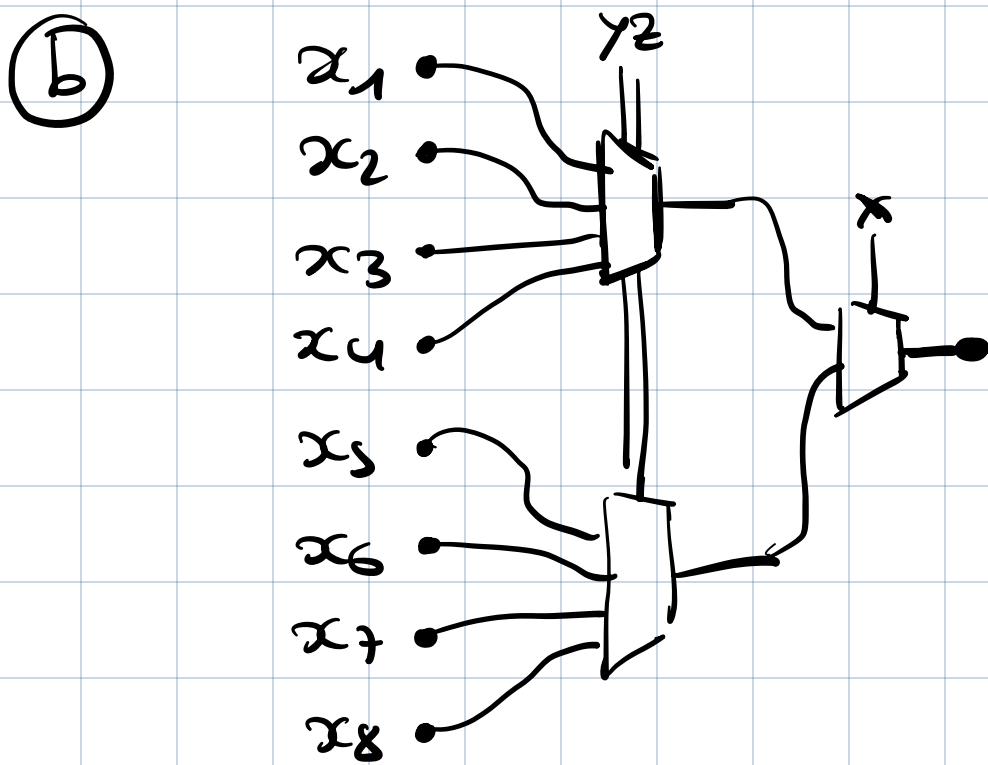
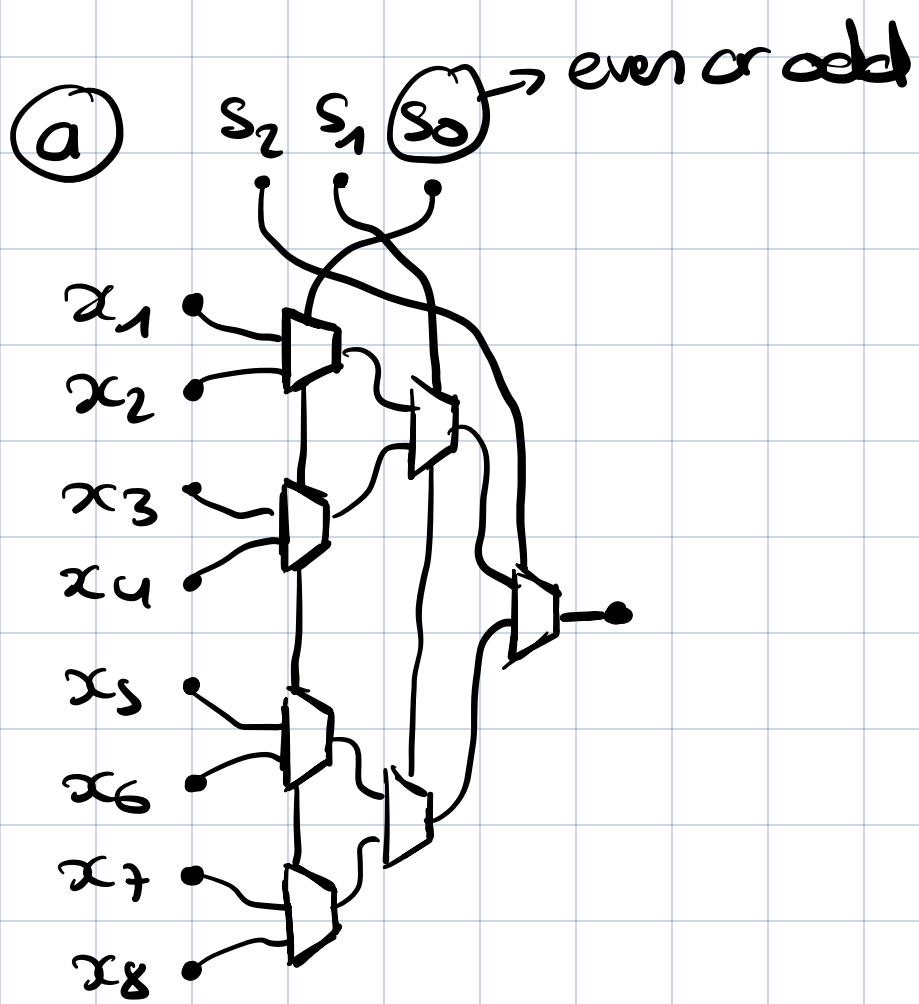
a



b

$$10 + 20 + 20 + 20 = \underline{70 \text{ ns}}$$

Exercise 1S



Exercise 16

$$fC(V_{DD})^2 = P_{Diss}$$

a) $\begin{cases} f = 320 \cdot 10^6 \text{ Hz} \\ V_{DD} = 1.05 \text{ V} \\ C = 100 \text{ fF} = 10^{-13} \text{ F} \end{cases}$

$$P_1 = 320 \cdot 10^6 \cdot 100 \cdot 10^{-13} \cdot (1.05)^2$$

b) 

c) 

d) 

Exercise 17

a) delay p_1 : 60ns + 0ns

delay p_2 : 20ns + 0ns

delay p_3 : 40ns + 0ns

delay p_4 : 30ns + $\overbrace{\max(p_1, p_2)}$ 60ns

delay p_5 : 60ns + 0ns

delay p_6 : 60ns + $\max(p_1, p_3)$

delay p_7 : 30ns + $\max(p_2)$

delay p_8 : 20ns + $\max(p_1, p_3, p_6)$

delay p_9 : 20ns + $\max(p_5, p_6, p_7)$

worst-case path: a $\rightarrow p_1 \rightarrow p_6 \rightarrow f$

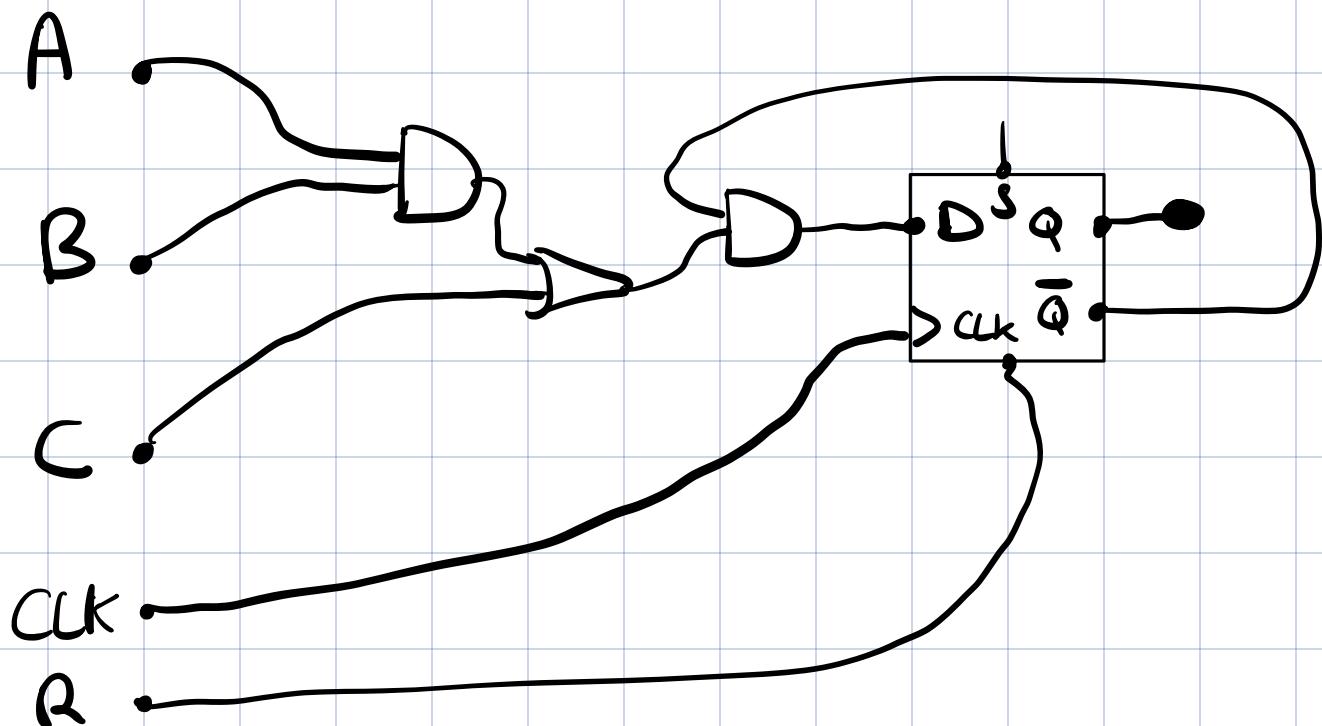
Exercise 18, 19, 20, 21, 22, 23, 24

// Verilog

Exercise 25

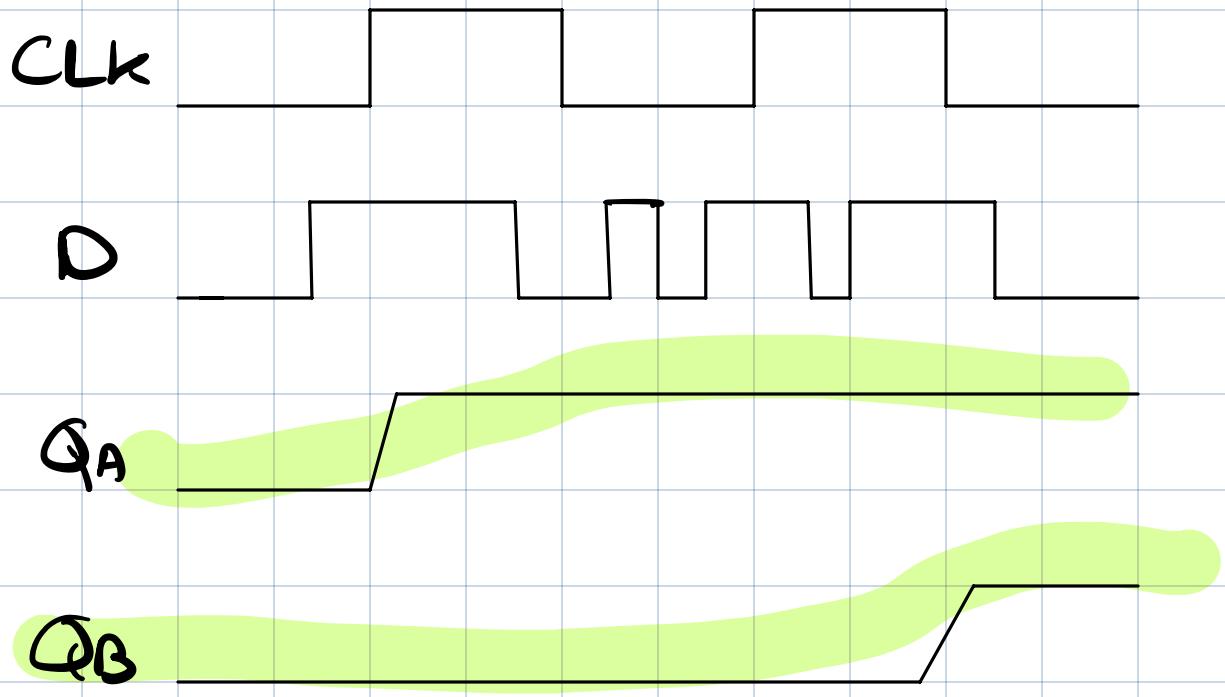
a

$$y(t) = \bar{y}(AB + C)$$





Exercise 27



Exercise 28

① $X_0 X_1 X_2 X_3 = \begin{pmatrix} 1000 \\ 0100 \\ 1100 \\ 0010 \end{pmatrix}$

⋮

$Y_0 Y_1 Y_2 Y_3 = \begin{pmatrix} 1000 \\ 0100 \\ \vdots \end{pmatrix}$

② a) 10 possible states: $\{0, 1, 2, 3, \dots, 9\}$

b) $10 \cdot 10$ possible states: $\{0, \dots, 99\}$

③ No can't from 0 to 99.

④ \neq Verilog

Exercise 27

a



$2^3 - 1$

	0	0	1
	0	1	0
1	0	0	
	0	1	1
	1	1	0
	1	1	1
	1	0	1

$$Q_1 = \overline{Q}_0 \text{ XOR } \overline{Q}_2$$

$$Q_0 = \overline{Q}_2$$

$$Q_2 = \overline{Q}_1$$

b

multiplexers

Exercise 30

a

module up-down-center (

input reset, load, clk,
input [0:4] load_val,
input φ,
output reg [0:4] out

);

always @ (posedge clk) begin

if (reset) out = 0;
else if (load) out = load_val;
else out = up ? out++ : out--;

?

endmodule

module up-down-counter-hb;

reg load = 1;

reg reset = 0;

reg clk = 0;

reg [3:0] load_val = 4'b0000;

wire [3:0] ar;

up-down-counter udc (

- . load(load),
- . reset(reset),
- . clk(clk),
- . load_val(load_val),
- . ar(ar)

);

always begin

#S:

clk' = ~clk

end

initial begin

#20; wait for 4 clock cycles

load = 0;

#10; wait for a clock cycle

if(out != 0001) begin

\$error("something went wrong");

end else \$display("ok! [%b]", out);

#10;

if(out != 0010) begin

\$error("something went wrong");

else

(

:

)

/

Exercise 31

a) $\log(4096) = 12$

b) ✓ Verilog

Exercise 32

a) $\lceil \log(k) \rceil$ bits

b) // Verilog

Exercise 33

$$\begin{aligned}Q_0^* &= b \\Q_1^* &= Q_0 \\Q_2^* &= Q_1 \\x^* &= (\bar{Q}_2)Q_1\end{aligned}$$

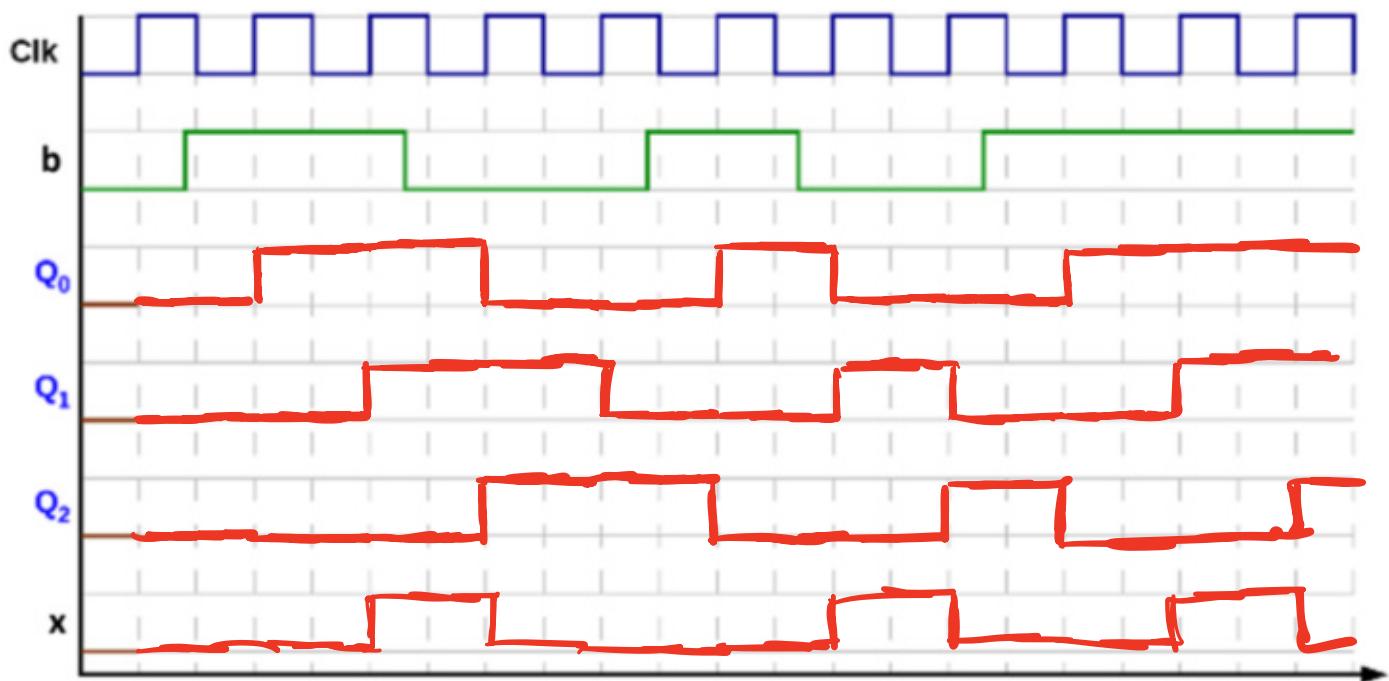


Figure 70: Waveform Template

- We add several flip-flops to avoid metastability issues
- it is a rising edge detector

Exercise 34

① $Y_1 = (\bar{y}_1 w) + (y_2 w) = w(\bar{y}_1 + y_2)$

$$Y_2 = (y_1 w) + (\bar{y}_2 w) = w(y_1 + \bar{y}_2)$$

$$Z = Y_1 Y_2$$

②

$$w=0 \quad w=1$$

$y_1 y_2$	$\bar{y}_1 \bar{y}_2$	$\bar{y}_1 y_2$	z
0 0	00	10	0
0 1	00	11	0
1 0	00	01	0
1 1	00	11	1

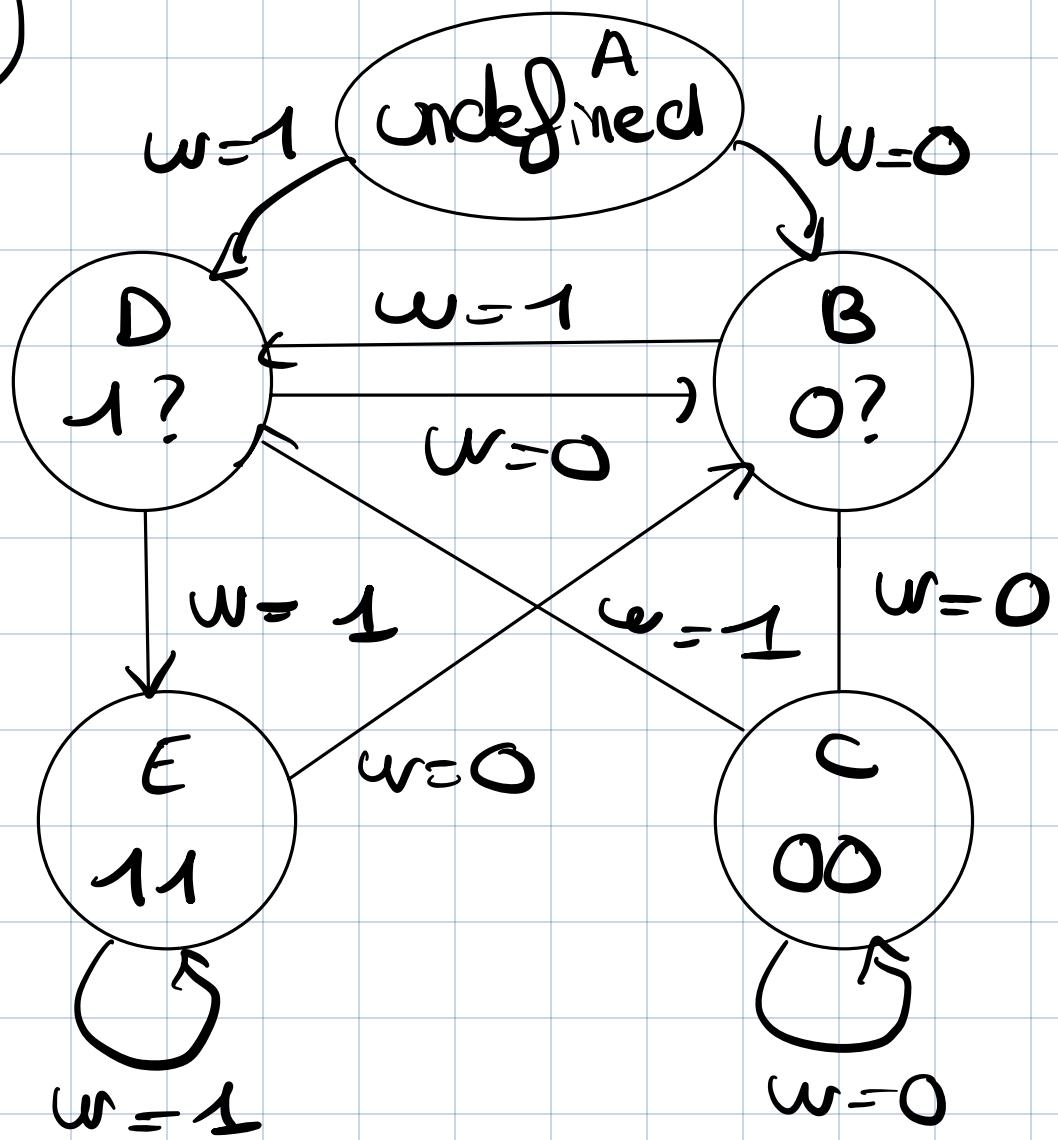
③ $00 \rightarrow 10 \rightarrow 01 \rightarrow 11^L$

Sequence detector

Exercise 3S

1 S

2



(3)

$\gamma_2 \gamma_1 \gamma_0$	Next state		output	
	$w=0$	$w=1$		Z
000 A	001 B	011 D		0
001 B	010 C	011 D		0
010 C	010 C	011 D		1
011 D	001 B	100 E		0
100 E	001 B	100 E		1

$$\gamma_2^* = (\bar{\gamma}_2 \gamma_1 \gamma_0 + \gamma_2 \bar{\gamma}_1 \bar{\gamma}_0) w$$

$$\gamma_1^* = \bar{\gamma}_2 (\bar{\gamma}_0 + \bar{\gamma}_1 \gamma_0) w$$

$$+ \bar{\gamma}_2 (\gamma_1 \text{ XOR } \gamma_0) \bar{w}$$

$$\gamma_0^* = w (\gamma_1 \text{ XOR } \gamma_0 + \bar{\gamma}_2 \bar{\gamma}_1 \bar{\gamma}_0)$$

$$+ \bar{w} (\gamma_1 \text{ XOR } \gamma_0)$$

$$Z = \bar{Y}_2 Y_1 \bar{Y}_0 + Y_2 \bar{Y}_1 \bar{Y}_0$$

$\bar{Y}_3 Y_2 Y_1 Y_0$	Next state		Output
	$w=0$	$w=1$	Z
00001 A	00010 B	01000 D	0
00010 B	00100 C	01000 D	0
00100 C	00100 C	01000 D	1
01000 D	00010 B	10000 E	0
10000 E	00000 B	10000 E	1

① —

$$\begin{aligned} ② Y_4^* &= w(Y_4 + Y_3) \\ Y_3^* &= \bar{w}(Y_1 + Y_2 + Y_3) \\ Y_2^* &= \bar{w}(Y_3 + Y_4) \\ Y_1^* &= \bar{w}(Y_3 + Y_4) \\ Y_0^* &= 0 \end{aligned}$$

③ a) ↑ more flip-flops

b) ↓ less

Exercise 36

$$\textcircled{1} \quad y_2 = w(\bar{y}_2 \bar{y}_1)$$

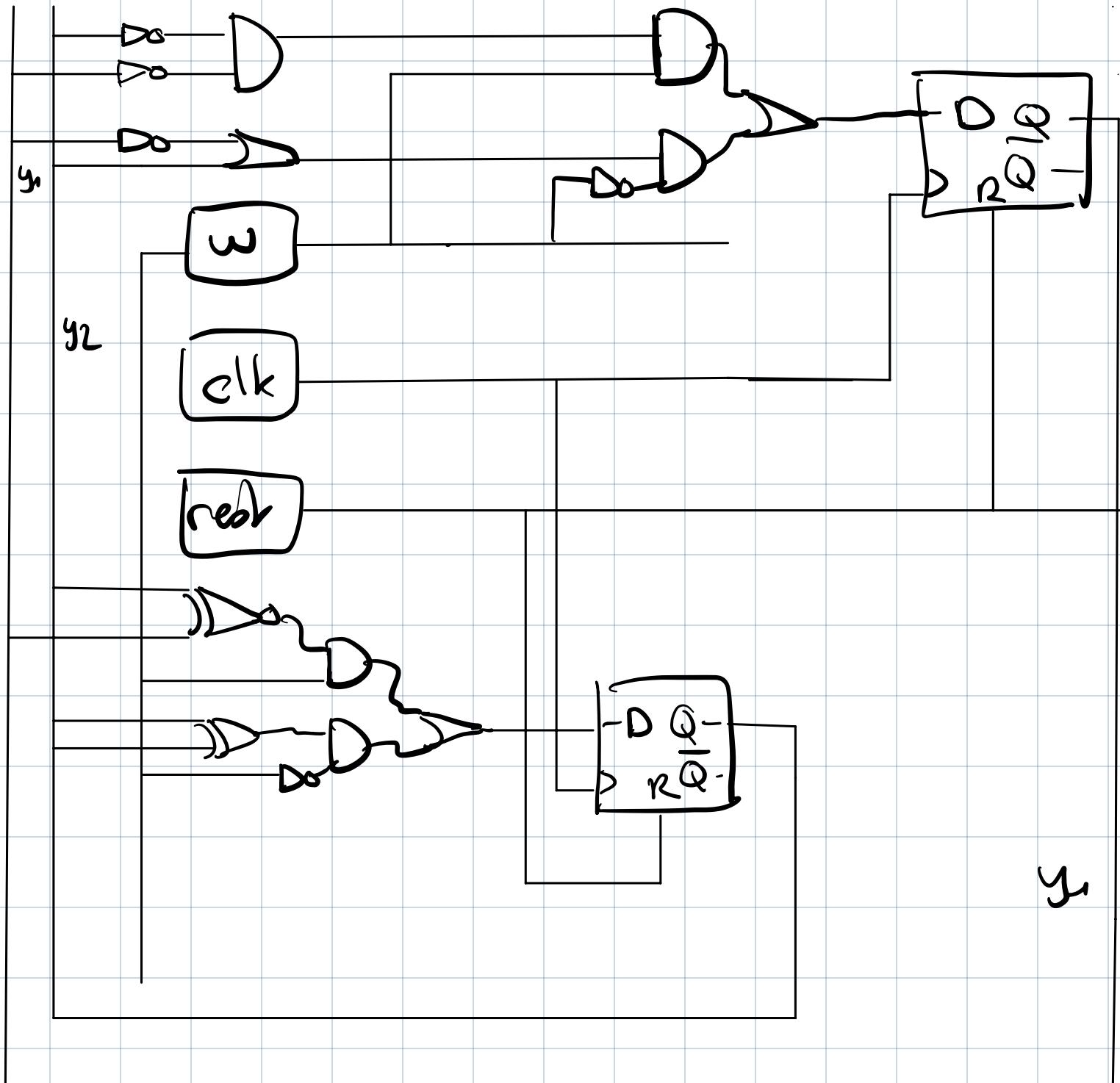
$$+ \bar{w}(y_2 + \bar{y}_1)$$

$$y_1 = w(\bar{y}_1 \oplus y_2)$$

$$+ \bar{w}(y_1 \oplus y_2)$$

$$z = y_2 y_1$$

\textcircled{2} Moore \rightarrow only depends on the previous state



Exercise 37

①

inputs: $\{1, 2, 5\}$

outputs: $\{0, 1\}$

internal states: $\{0, 1, 2, 3, 4, 5, 6, 7\}$

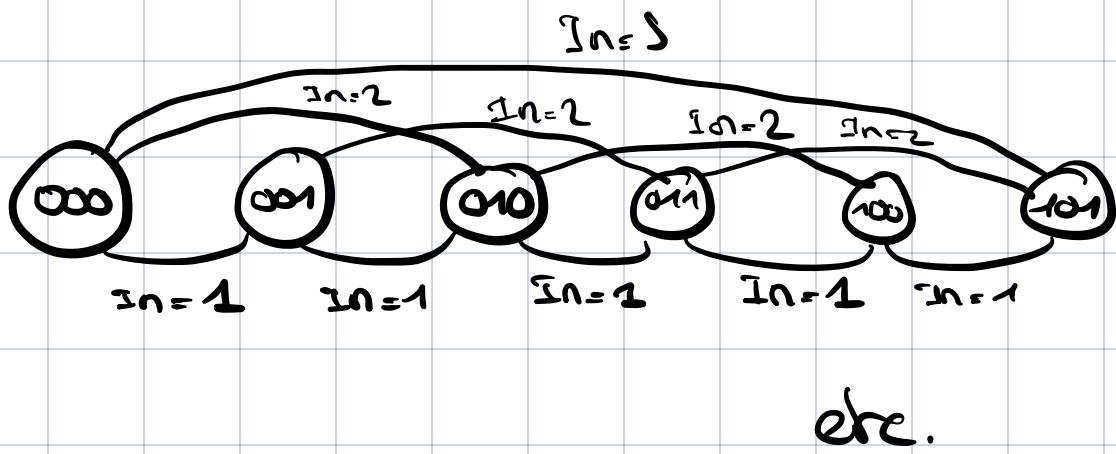
②

inputs: $\{1, 2, 5\}$

outputs: $\{0, 1\}$

internal states: $\{000, 001, \dots, 111\}$

3



4

States	Inputs			Output		
	1	2	5	1	2	5
0	1	4	8	0	0	0
1	2	3	6	0	0	0
2	3	4	7	0	0	0
3	3	5	0	0	0	1
5	5	6	1	0	0	1
6	6	0	2	0	1	1
7	7	0	3	0	1	1
	0	1	4	1	1	1

Inputs

States	1	2	S	Output
0	1	4	5	0
1	2	3	6	0
2	3	4	7	0
3	3	5	0	0
4	5	6	1	0
5	6	0	2	0
6	7	0	3	0
7	8	9	12	0
8				1
9				1
10			7	1
11	12	S		1
12				1

Exercise 38

a) $t_{\text{setup}} + t_{\text{comb,max}} + t_{\text{q,rmax}}$ has to fit within one clock cycle.

$$0.6 \text{ ns} + (1 + 0.2) \cdot 2 \text{ ns} + 1 \text{ ns}$$

$$= 0.6 + 2.4 + 1 = 4 \text{ ns} \quad (\text{Q}_1 \text{ to } \text{D}_2)$$

$$0.6 \text{ ns} + (1 + 0.1) \text{ ns} + 1 \text{ ns}$$

$$= 2.7 \text{ ns} \quad (\text{Q}_2 \text{ to } \text{D}_3)$$

$$0.6 \text{ ns} + (0) \text{ ns} + 1 \text{ ns} = 1.6 \text{ ns} \quad (\text{Q}_3 \text{ to } \text{D}_1)$$

$$0.6 \text{ ns} + (1.2) \text{ ns} + 1 \text{ ns} = 2.8 \text{ ns} \quad (\text{Q}_2 \text{ to } \text{D}_2)$$

$$0.6ns + (2.4)ns + 1ns = 4ns$$

(Q₃ to D₂)

$$\delta_{\max} = \frac{1}{c_{ns}}$$

$$t_{\text{hold}} \leq t_{\text{comb}_{\min}} + t_{\text{cq}, \min}$$

Q1/D ₂	\Rightarrow	3.2ns	✓	-t _{setup}
Q2/D ₃	\Rightarrow	1.9ns	✓	$-\Delta(t_{\text{comb}_{\max}}, t_{\text{comb}_{\min}})$
Q3/D ₁	\Rightarrow	0.8ns	✓	
Q2/D ₂	\Rightarrow	2ns	✓	
Q3/D ₂	\Rightarrow	3.2ns	✓	

b

Q₁ to D₂

$$0.6\text{ns} + 2.4\text{ns} + 1 - 0.7\text{ns} = 3.3\text{ns}$$

Q₂ to D₃

$$0.6\text{ns} + 1.1\text{ns} + 1 + 0.7\text{ns} = 3.4\text{ns}$$

Q₃ to D₂

$$0.6\text{ns} + 2.4\text{ns} + 1 - 0.7\text{ns} = 3.3\text{ns}$$

Q₂ to D₂

$$0.6\text{ns} + 1.1\text{ns} + 1 = 2.7\text{ns}$$

Q₃ to D₁

$$0.6\text{ns} + 1\text{ns} = 1.6\text{ns}$$

$$f_{\text{max}} = \frac{1}{3.4 \text{ ns}}$$

$t_{\text{comb,min}} + t_{Q,\min} > t_{\text{hold}} + \text{skew}$
 (it has to stay longer)

$$Q_1/D_2 : 3.2 \text{ ns} - 0.7 \text{ ns} = 2.5 \text{ ns}$$

$$Q_2/D_3 : 1.9 \text{ ns} + 0.7 \text{ ns} = 2.6 \text{ ns}$$

$$Q_3/D_1 : 0.8 \text{ ns}$$

$$Q_2/D_2 : 2 \text{ ns}$$

$$Q_3/D_2 : 3.2 \text{ ns} - 0.7 \text{ ns} = 2.5 \text{ ns}$$

(c)

$$Q_1/D_2 : 3.2 \text{ ns}$$

$$Q_2/D_3 : 1.9 \text{ ns} - 0.5 \text{ ns} = 1.4 \text{ ns}$$

$$Q_3/D_1 : 0.8 \text{ ns} - (1 - 0.5 \text{ ns}) = 0.3 \text{ ns}$$

$$Q_2/D_2 : 2 \text{ ns}$$

$$Q_3/D_2 : 3.2 \text{ ns} + 0.5 \text{ ns} = 3.7 \text{ ns}$$

f_{max} :

a

$$Q_1 / D_2 = C_{ns} + 1_{ns} - S_{ns}$$

$$f_{\text{max}} = \boxed{\frac{1}{S_{ns}}}$$

Exercise 3g

a

Shake	In	out
	0 1	
IDLE	IDLE	A0 0
A0	IDLE	A1 0
A1	IDLE	A2 0
A2	IDLE	A3 0
A3	IDLE	A4 0
A4	IDLE	A5 0
A5	IDLE	A6 0
A6	IDLE	A7 0
A7	IDLE	A8 0
A8	IDLE	A9 0
A9	IDLE	B 0
B	IDLE	B 1

State	Counter	In	NextS	NextC	out
IDLE	X	1	A	0	0
A	0	1	A	1	0
A	1	1	A	2	0
A	2	1	A	3	0
A	3	1	A	3	0
A	4	1	A	3	0
A	5	1	A	3	0
A	6	1	A	3	0
A	7	1	A	3	0
A	8	1	A	3	0
A	X	0	IDLE	X	1
B	9	X	B	9	0
IDLE	X	0	IDLE	9	0
			IDLE	X	0

module ten_counter(

 input clk, reset, in,
);

 reg IDLE_STATE = 2'b00;
 reg A_STATE = 2'b01;
 reg B_STATE = 2'b10;
 reg current_state = 2'b00;
 reg next_state = 2'b00;

 reg counter
 = 0;

 always @* begin

 if (in) begin

 if (current_state == IDLE_STATE) begin
 next_state <= A_STATE;
 counter <= 0;

 end else if (current_state == B_STATE) begin
 next_state <= IDLE_STATE;

 end else if (current_state == A_STATE)

reset = B

end else begin
if (current-state == A-STATE) begin
 next-state <= IDLE-STATE;
end else if (current-state == B-STATE) begin
 next-state <= IDLE-STATE
end
end

always @ (posedge clock) begin

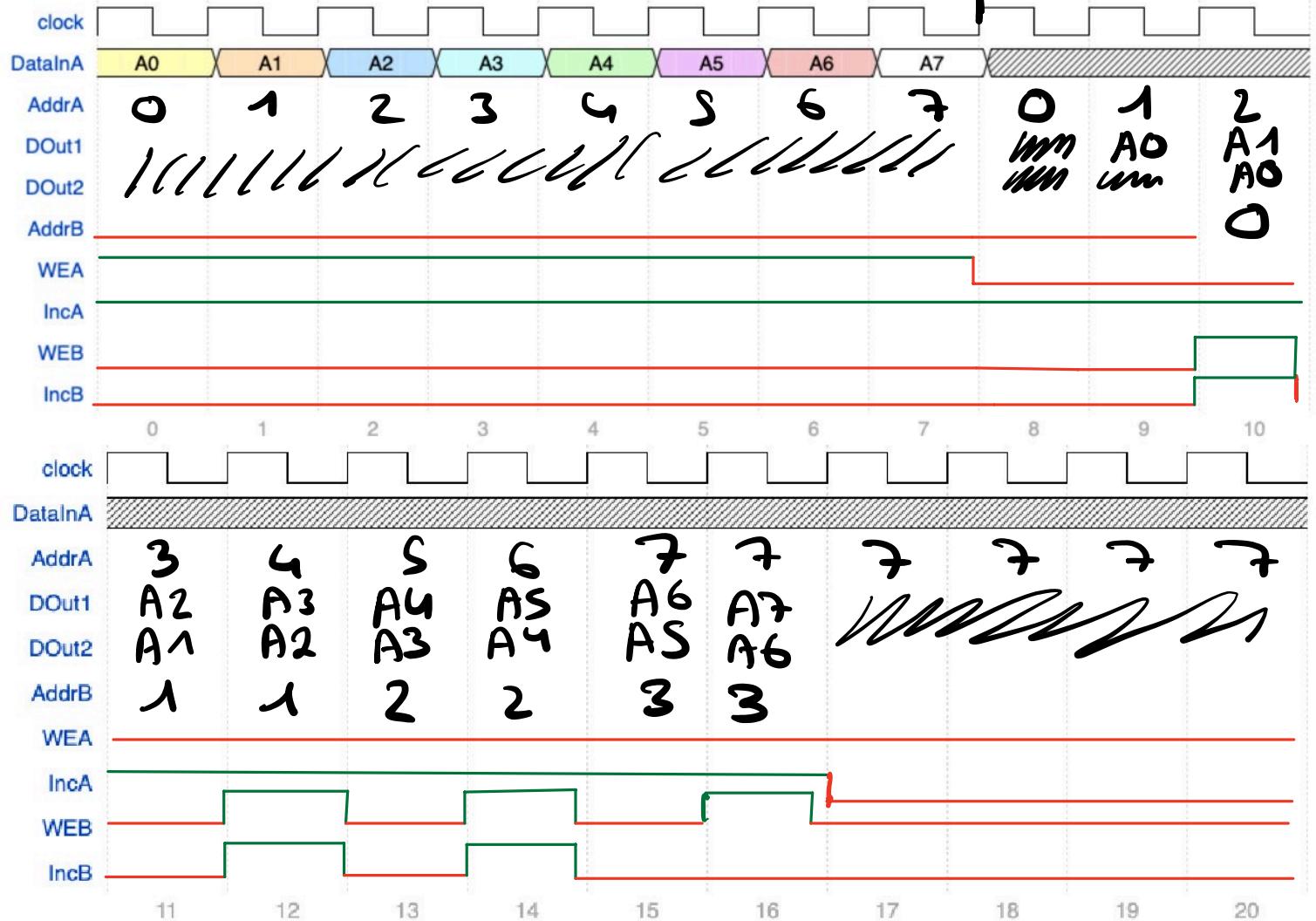
if (reset) begin
 current-state <= IDLE-STATE;
 counter <= 0;
end else begin
 current-state <= next-state;
 if (A-STATE) counter <= counter+1;
end
end

always @* begin
 at = current-state == B-STATE

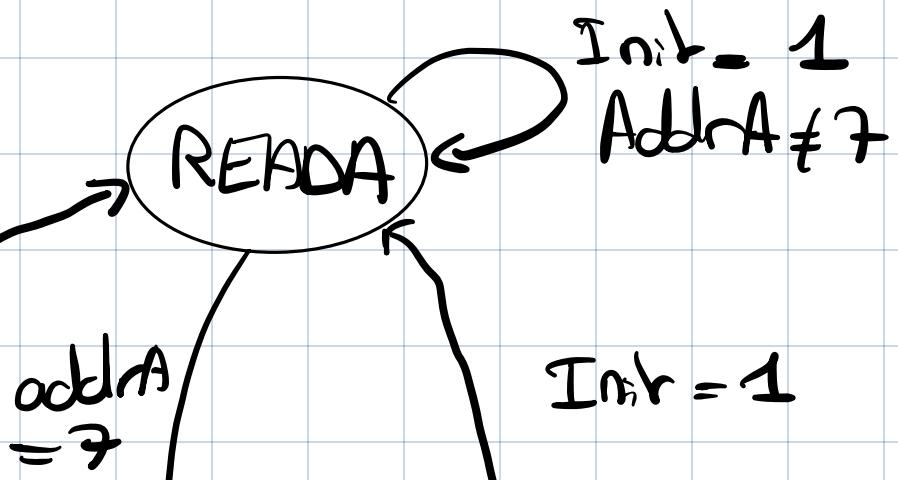
end

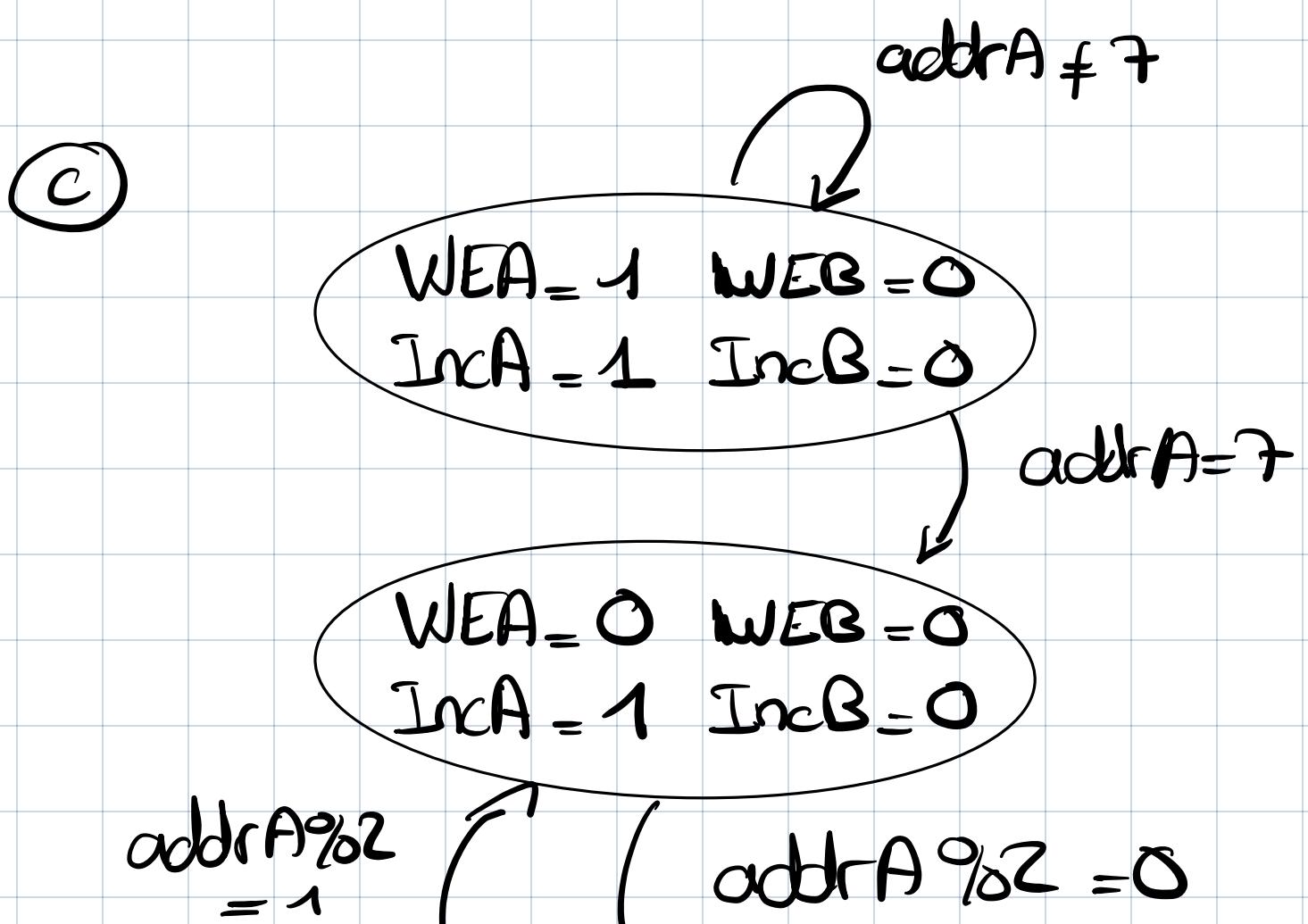
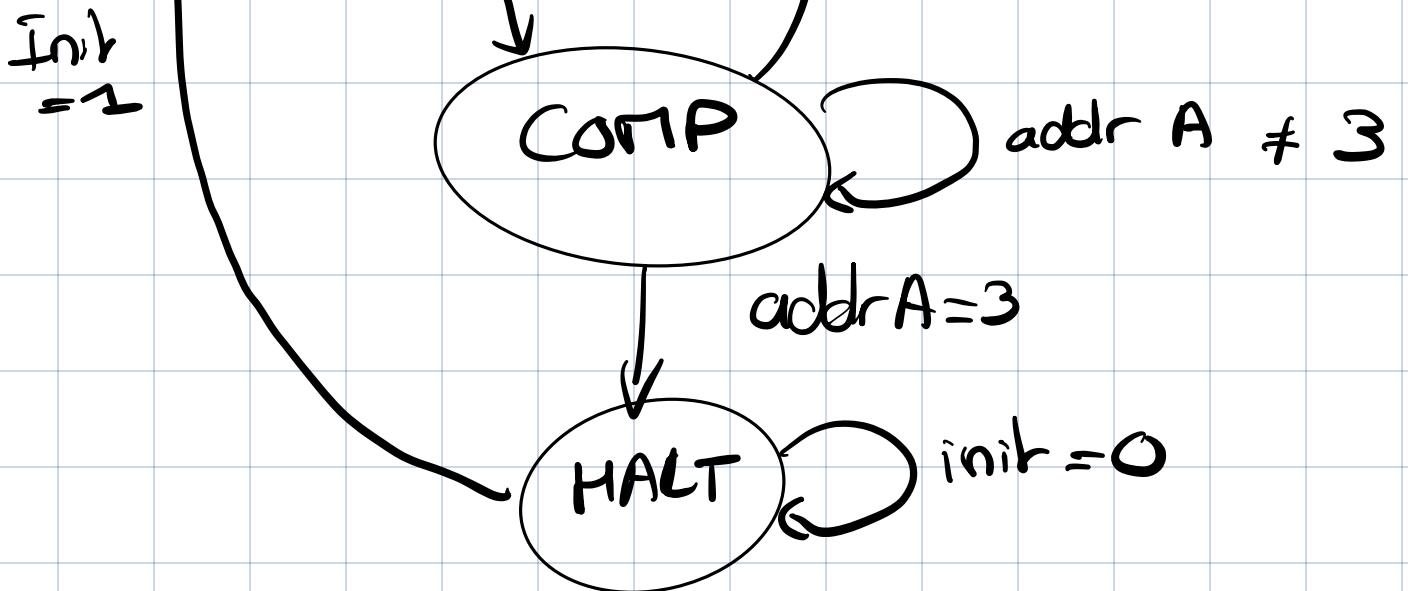
Exercise 40

end of
initialization



b





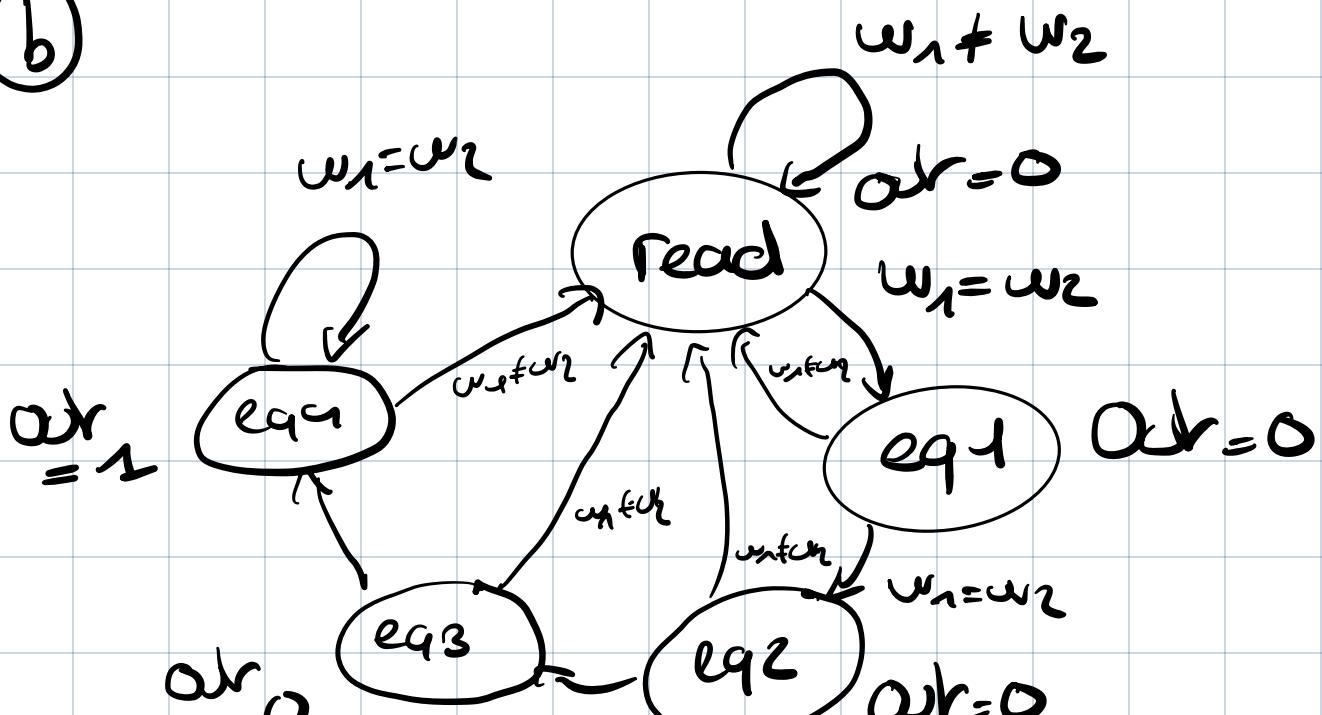
$WEA = 0$ $WEB = 1$
 $IncA = 1$ $IncB = -1$

Sr	Current			Next			Ours			
	CnA	CnB	Int	8t	CnA	CnB	IntA	IntB	WEA	WeB
R	0	0	0	R	1	0	1	0	1	0
R	1	0	0	R	2	0	1	0	1	0
R	2	0	0	R	3	0	1	0	1	0
R	3	0	0	R	4	0	1	0	1	0
R	4	0	0	R	S	0	1	0	1	0
R	5	0	0	R	6	0	1	0	1	0
R	6	0	0	R	7	0	1	0	1	0
R	7	0	0	C	0	0	1	0	1	0
C	0	0	0	C	1	0	1	0	0	0
C	1	0	0	C	2	0	1	0	0	0
C	2	0	0	C	3	0	1	1	0	1
C	3	1	0	C	4	1	1	0	0	0
C	4	1	0	C	5	1	1	1	0	1
C	5	2	0	C	6	2	1	0	0	0
C	6	2	0	C	7	2	1	1	0	1
C	7	3	0	C	0	3	1	0	0	0
C	0	3	0	H	0	0	0	1	0	1
H	0	0	0	H	0	0	0	0	0	0

Exercise 4.1

a) S

b)



Next State

c)

State	$w_1=w_2$	$w_1 \neq w_2$	Output
R	E_n	R	0
E_1	E_2	R	0
E_2	E_3	R	0
E_3	E_4	R	0
E_4	E_4	R	1

d module two-input-seq-detector (
input clk, reset, w1, w2,
output out
);

reg [2:0] next-state;
reg [2:0] current-state;
parameter [2:0] R = 3'b000,
parameter [2:0] E1 = 3'b001,
parameter [2:0] E2 = 3'b010,
parameter [2:0] E3 = 3'b011,
parameter [2:0] E4 = 3'b100;

always @* begin
next-state = current-state;
if (w1 == w2) begin

case (current-state)

R: next-state = E1;

E1: next-state = E2;

E2: next-state = E3;

$E_3: \text{next_state} = E_4;$

$E_4: \text{next_state} = E_4.$

default: $\text{next_state} = E_4;$

endcase

end else begin

$\text{next_state} = R;$

end

end

always @ (posedge clk or posedge reset) begin

if (reset) current_state $\in Q;$

else current_state $\leftarrow \text{next_state};$

end

always @* begin

out = current_state == $E_4 ? 1 : 0;$

end

endmodule

Exercise 42

(a)

```
module bit_counter(  
    input clk,  
    input reset,  
    input [7:0] load_data,  
    input load,  
    input enable,  
    output reg [3:0] count,  
    output reg done  
);
```

```
reg currently-reading-pos;  
reg [7:0] data;  
reg [1:0] next-state;  
reg [1:0] count-state;
```

parameters S1 = 2'b00, S2 = 2'b01,
S3 = 2'b11,

always @* begin

```
if (current-state == S1) begin
    if (load) begin
        data = load-data;
    end
    if (enable) begin
        next-state = S2;
    end
end else if (current-state == S2) begin
    if (currently-reading-pos == 7) begin
        next-state = S3
    end
end
end
```

always @ (posedge clock or posedge Reset)

begin

if (Reset) begin

currently_reading_pos <= 0;

cnt <= 0;

end else if (current_state == S2) begin

curr <= data[current_reading_pos] == 1 ? curr + 1 : curr;

currently_reading_pos <= currently_reading_pos + 1;

end

end

always @ (posedge clock or
posedge Reset) begin

if (Reset) begin

curr_state <= S1;

end else begin

curr_state <= next_state;

end

end

always @ * begin

done = current_state == S3 ? 1 : 0;
end

Exercise 43

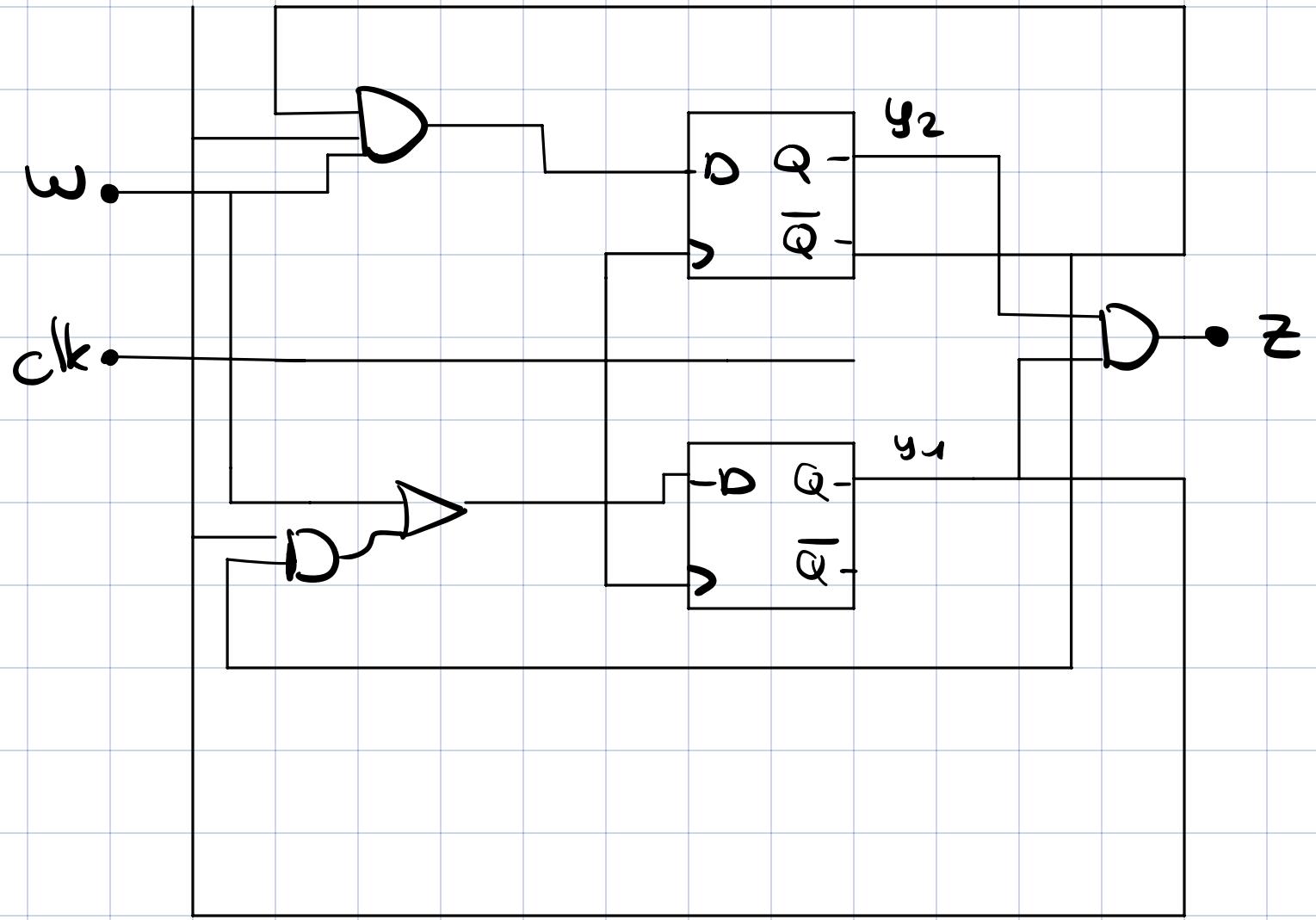
a

State $y_2 y_1$	Input		Output
	$w=0$	$w=1$	
00 A	00 A	01 B	0
01 B	01 B	1C	0
11 C	00 A	01 B	1

$$y_2^* = w(\bar{y}_2 y_1)$$

$$y_1^* = w + \bar{y}_2 y_1$$

$$z = y_2 y_1$$



$Q_2 \rightarrow D_2$

$t_{\text{setup}} + t_{CQ,\max} + t_{\text{comb},\max} > t_{\text{cycle}}$

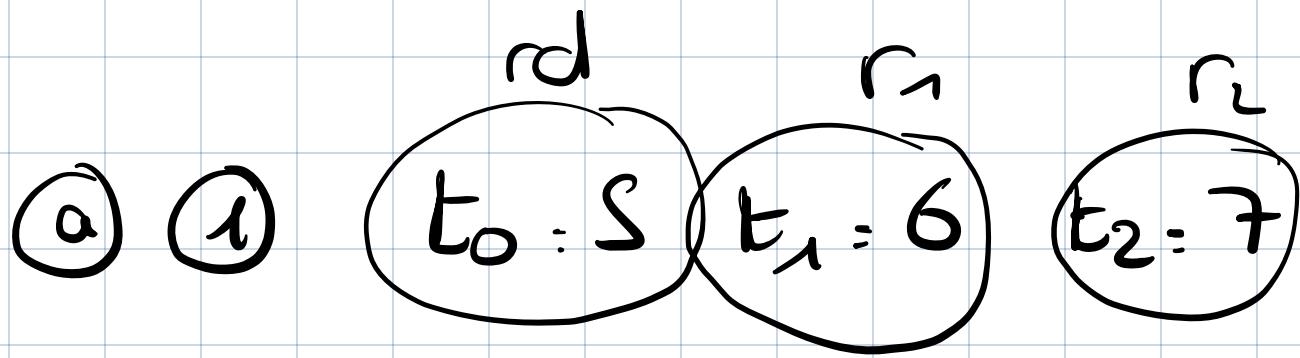
$$100 + 200 + 400 = 700 \text{ ps}$$

$Q_1 \rightarrow D_1$

$$100 + 200 + 300 + 400 = 1000 \text{ ps}$$

$$f_{\max} = \frac{1}{1000}$$

Exercise 2



R-type

| 0_x00 | 0111 | 0110 | 0_x0 | 0101 | 0_x33

→ | 0000000 | 00111 | 00110 | 000 | 00101 |
 0110011

$$t_0 = 1S = F$$

② $t_4 = 1$

$$b_4 = x_{29} \quad s_0 = x_8 \quad s_3 = x_{19}$$

0000000 | 10011 | 01000 | 010 | 11101 | 0110011

③ $t_2 = x_7 \quad t_5 = x_{30}$

$t_2 = 0$

0000000 | 11110 | 00111 | 011 | 00111 | 0110011

④ $t_0 = x_5 \quad t_1 = x_6$

00000010000 | 00110 | 000 | 00101 |
010011

$t_0 = 0x00$

⑤ $s_1 = x_11 \quad s_2 = x_{18}$

00000000011 | 10010 | 001 | 10010 |

010011

~~$s_2 = 100001111010000$~~

⑥ $t_6 = x_{31}$

1hex

0x00012300

4·3 = 12
3hex

0x12300000

0x00012300 | 11111 | 0110111

Exercise 3

101
101

a)

00010010001101000

$$t_0' = t_0 \oplus t_2$$

$$t_2' = (t_0 \oplus t_2) \oplus t_2 = t_0$$

$$\begin{aligned} t_0'' &= t_0 \oplus t_2 \oplus t_0 \\ &= \underline{t_2} \end{aligned}$$

bne r₁, r₁, 13 ≡

→ bne r₁, r₂, 12

bne r_1, r_2, imm
= u_2

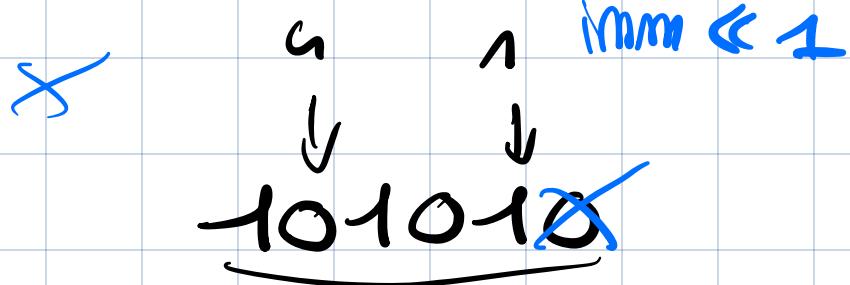
Exercise S

① $t_0 = x_3 \quad t_1 = x_6$

$$\begin{array}{r} 32 \\ \times 8 \\ \hline 2 \end{array}$$

u_0

u_2



B | 00000001 | 00110 | 00101 | 000 | 0010 |

10,0011

② bltu

$s_1 : x_9$

$s_2 : x_{18}$

- $u_1 : 11111111110\cancel{0}$

B | 1111111 | 10010 | 01001 | 110 | 110 | -

10,0011

③ lw $t_2 = x_7$
 $s_3 = x_{19}$

B | 00000000|100(100-11|010)00111|000011

④ t₀ : x_5 sb
s₆ : x_{22}
imm : 4 100

S | 00000000|00101 | 10110|000|00100|
0-100011

⑤ t₀ : x_5 0x 12345

0001, 0010 0011, 0100, 0101

5|0,0110100010,0,0010010|00101|
01100011

⑥ jdr $t_1, 16(t_2)$

$t_1 = \text{rd} : x_6$
 $t_2 = \text{rs}_1 : x_7$

000000010000|00111 (000)00110|
1100111



Exercise 10



Hello, yes, you should try to understand what the program does so you can do the computation by yourselves without executing assembly on paper.

Here is how I did it (maybe not the best way but it worked for the few exercises we have)

first, let's start with the `start` part:

- hum, t0, t1, r3 get initialized with the value 0, they are probably some counters (we don't know what they will count yet)
- then t5 takes the value s0... from the statement... all right, it is the starting address for the matrix. it will probably be a pointer towards the current memory address that we will increment when we want to read a new address

ok, now the `loop` part.

- `lub` says we want to load a unsigned byte (8 bits) to a register... why would we want to do so? ok the statement says we are storing a bunch of 8 bits unsigned number in our memory, so that's probably the first element of our matrix!
- then... $t2 = t2 + s3$. ok, we don't know why they do this yet
- then they are comparing $t2$ and $t3$... and jumping to skip if... $t2 < t3$ is 1 which means that $t2$ is indeed less than $t3$ (signed comparison!). same thing, no idea why they would want to this yet but anyway let's keep reading

I personnally then went to the `skip` part

- so if $t2 < t3$ then we store at address `t5` (the current address) to the value of `t2` but wait what is `t2` now? it is $t2 + s3$ right and what is `s3` again? ok, the unsigned number they said to be chosen arbitrarily. So basically... if the current element + the arbitrary number is less than the counter `t3` (we don't know what it is yet)... we set current element to itself + `s3`. Right.
- then, we increase `t5` which was the pointer to the address (so basically we set the address to the next address)
- then we increase `t0` by 1... right
- then we are comparing `t0` with `s1..` which is the number of rows! ahah, we found it, now we know that `t0` is the current index of the row, and `t1` is probably the current index of column!
- that's confirmed by the rest of the `skip` part, we see that if we arrive at the end of the rows, we are resetting the counter for the rows and increasing the counter for the column
- and then loop again to read the next element and process

now we know what is `t0` (idx of row), `t1` (idx for col), `t2` (current element + arbitrary number), `t4` (the result of current element + arbitrary number $< t3$) (even though we don't know what is `t3` yet)

and we also know that when (the result of current element + arbitrary number $< t3$), we are saving as the current element itself + arbitrary number.

So what is `t3`? well, let's see what's after bne

`s4 = t0 + 0 ... s5 = t1 + 0 ...` oh! so `s4` and `s5` are row and column indices!

and `t3 = t2 + 0` oh so if `t2 > t3` then... `t3 = t2`? that really seems like an algorithm to find an a max!

now we know what are all the registers used in our program

i

$$S_L = \# \text{ rows}$$

$$S_S = \# \text{ col}$$

$$12 + 78 + 35 = 125$$

(reg)

$$S_L = 3$$

$$S_S = 1$$

ii

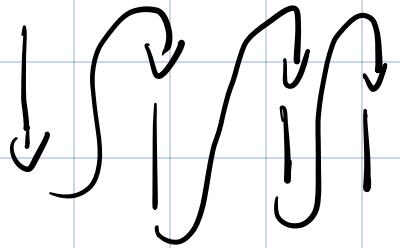
12

iii all elements get $+P$ (because we will not have any negative value)

iv whether we want the absolute max or just max

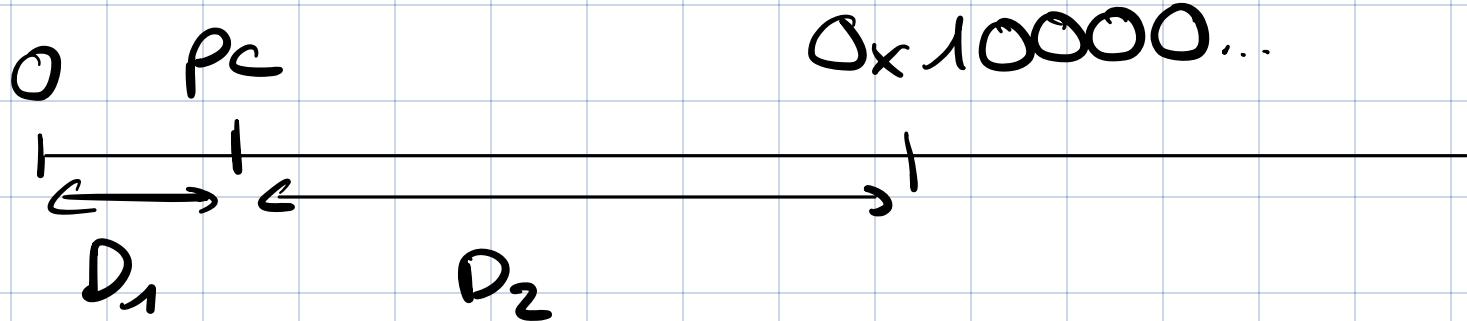
b See ED.

c $0 \times 1000 \quad 0000 \Rightarrow$
12
78
35



le programme
est calc
par ligne
comme ça

11
34
:
:
:



au ipc x_8 $0x10000...$

$$\Rightarrow x_8 = D_1 + 0x10000...$$

add x_8 , 300, -D1

$$\Rightarrow x_8 = 0x10000...$$

e

i) 8b+8b can lead to 9 bits

ii) k save \Rightarrow 32 bits to 8 bits.

srl t₆, t₂, 8

beq t₆, zero, Continue

addi t₂, zero, 0xFF

Continue

=

Exercise 11

will be my current pointer

add t₀, zero, s₀

will be my counter of A[i] > B[i]

add s₁, zero, zero

loop:

lw t₂, 0(t₀)

beg t₂, zero, finish

add t₃, zero, t₂

srai t₃, t₃, 16) clean up B

srai t₃, t₃, 16) A

srl t₄, t₃, t₂

add s₁, s₁, t₄

addi t₀, t₀, 4

j loop