Question 1

$$[x^{-1}]x^{23} = [1]_{23}$$

$$x^{23}$$

$$[x^{-1}]x^{23} = [1]_{23}$$

$$x^{22} \equiv [x^{-1}]_{23}$$

$$22 \equiv -1\ [23]$$

$$x^{25} \equiv [x^{2}]_{23}$$

$$x \equiv [x]_{23}$$
$$x^2 \equiv [x^2]_{23}$$

$$x^{23} x^{-1} = x^{22}$$

$x^{22} = 1 \quad x^{23} \equiv 1$

$x^{23} x^{-1} \equiv x^{-1}$

$x^{23} = 1 \quad x^{22} \equiv x^{-1}$

$x^n \cdot x^{-n}$

$x^{22} = x$

$x^{23} - x^{22}$

$x^{23}\left(x^{22} - x^{21} + x^{-3} + x^{-4} + \ldots\right)$

$x^{22} - x^{21} + x^{20} + \ldots$

$= x^{-1} - x^{-2} + x^{-3}$

$x + x^{-1}$

$x^{23}(x^{-1} - x^{-2}$

$-x + x^{-1}$

$x^{13} = 2$

$x^{22} = 1$

$x^{21} = 12$

$x^{20} = 6$

$x^{19} = 3$

$x^{18} = 13$

$x^{17} = 18$

$x^{16} = 9$

$x^{15} = 16$

$x^{14} = 8$

$x^{13} = 4$

$x^{12} = 2$

$x^{11} = 1$

$x^{10} = 12$

$x^{9} = 6$

$x^{8} = 3$

$x^{7} = 13$

$x^{6} = 18$ ∧

$x^{5} = 9$ ∪

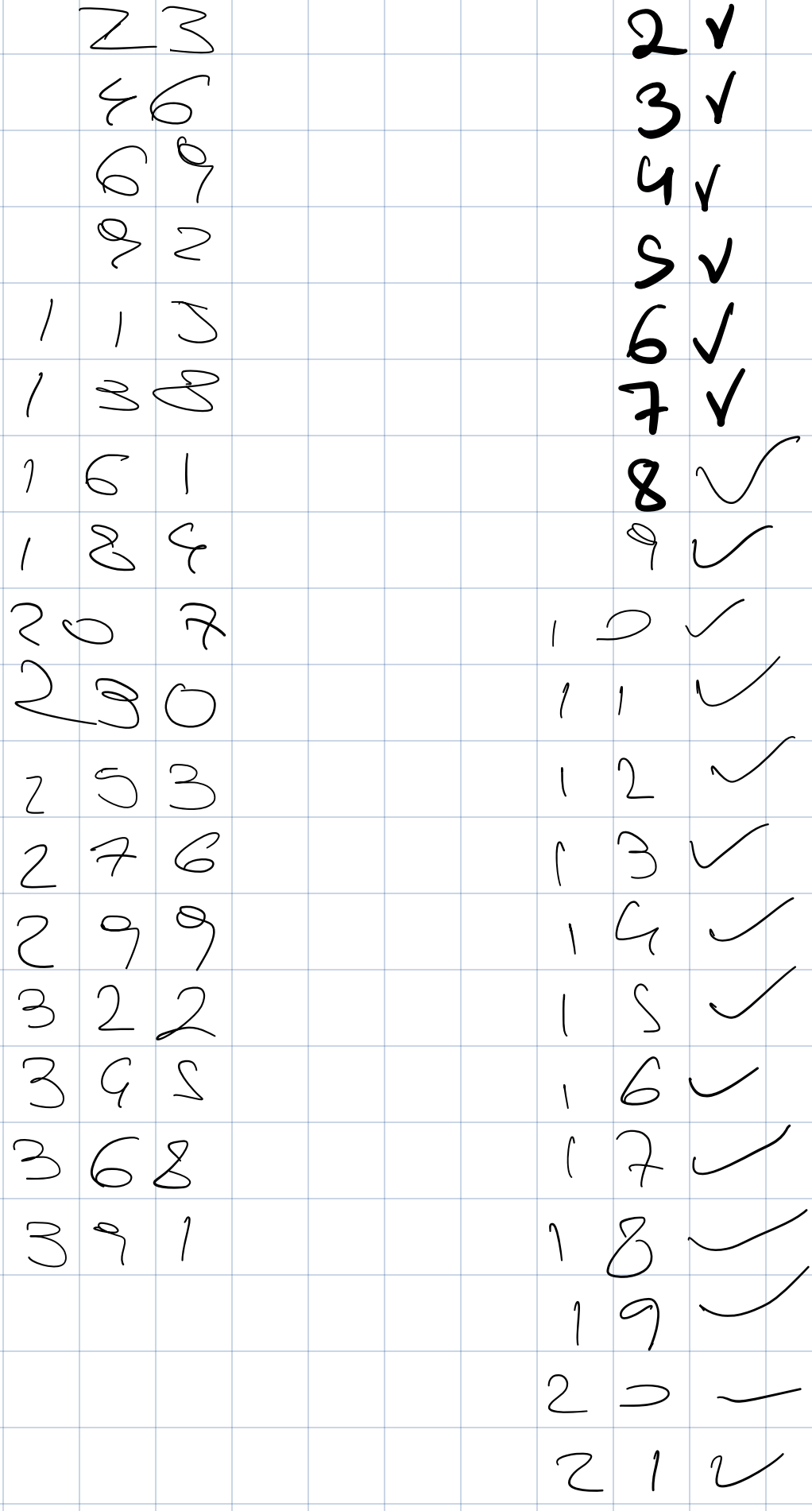
$x^{4} = 16$ ∧

$x^{3} = 8$ ∪

$x^{2} = 4$ ∧

$x = 2$ ∪

$\phi(23) = 22$



22X

441 −
230
211

484 −
230
254

93

$$\begin{matrix} \times 2 \\ \curvearrowright \end{matrix}\left(\begin{array}{cccc|c} 1 & 6 & 3 & 4 & 0 \\ 3 & 6 & 1 & 3 & 0 \\ 5 & 2 & 1 & 3 & 0 \end{array}\right)$$

$$\begin{matrix} \times 2 + \curvearrowright \\ \\ \end{matrix}\left(\begin{array}{cccc|c} 1 & 6 & 3 & 4 & 0 \\ 3 & 6 & 1 & 3 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 6 & 3 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$x_1 = -3x_3$$
$$x_2 = 0$$
$$x_3 = 0$$

$$\dim \operatorname{Im} A = \dim \operatorname{Im} A^T$$

(Q4)

0 0 1

0 0 1 1

0 1 1 0 ✓

1 1 1