Exercise 1

(a)

(i) 2^{8} → bits

(ii) $2^{10} \cdot 2$ → 11 bits

(iii) $3581 - 3325 = 256$ → 8 bits

(iv) $657 + 879 = 1536$

$2^{10} < 1536 < 2^{11}$

→ 11 bits

(b) $\lceil \log_2 (b-a) \rceil$

(c) 2^{n}

(d)

Decimal	Binary	Octal	Hexadecimal
139	10001011	213	8B
185	10111001	271	B9
58	00111010	072	3A
201	011001001	311	C9
56	111000	70	38
211	11010011	323	D3
27	00011011	33	1B
108	01101100	154	6C

Table 1: Conversion between decimal, binary and hexadecimal formats.

$$
\begin{aligned}
139 &= 69 \cdot 2 + 1 \\
69 &= 34 \cdot 2 + 1 \\
34 &= 17 \cdot 2 + 0 \\
17 &= 8 \cdot 2 + 1 \\
8 &= 4 \cdot 2 + 0 \\
4 &= 2 \cdot 2 + 0 \\
2 &= 1 \cdot 2 + 0 \\
1 &= 2 \cdot 0 + 1
\end{aligned}
$$

$$
\begin{aligned}
185 &= 92 \cdot 2 + 1 \\
92 &= 46 \cdot 2 + 0 \\
46 &= 23 \cdot 2 + 0 \\
23 &= 11 \cdot 2 + 1 \\
11 &= 5 \cdot 2 + 1 \\
5 &= 2 \cdot 2 + 1 \\
2 &= 1 \cdot 2 + 0 \\
1 &= 0 \cdot 2 + 1
\end{aligned}
$$

$$2 + 8 + 16 + 32 = 58$$

$$1 + 8 + 2^6 + 2^7$$

$$= 1 + 8 + 64 + 128 = 201$$

$$2^3 + 2^4 + 2^5 = 8 + 16 + 32 = 56$$

$1 + 2 + 2^4 + 2^6 + 2^7$

$= 1 + 2 + 16 + 64 + 128$

$= 131 + 16 + 64 = 80 + 131 = 211$

$1 + 2 + 8 + 16 = 27$

$4 + 8 + 32 + 64 = 108$

Exercise 2

ⓐ

ⓘ 2 3 6

$\downarrow$ $\downarrow$ $\downarrow$

0010 0011 0110

6 1

$\downarrow$ $\downarrow$

0110 0001

1 5 8

$\downarrow$ $\downarrow$ $\downarrow$

0001 0101 1000

ⓘⓘ 0011 1001

$\downarrow$ $\downarrow$

3 9

~~0110 1110~~ → 10 not valid

~~1110 0011~~ → 13 not valid

(iii) giga grand (inefficace)

(b) (i) no

(ii)
$$236 = 118 \cdot 2 + 0$$
$$118 = 59 \cdot 2 + 0$$
$$59 = 29 \cdot 2 + 1$$
$$29 = 14 \cdot 2 + 1$$
$$14 = 7 \cdot 2 + 0$$
$$7 = 3 \cdot 2 + 1$$
$$3 = 1 \cdot 2 + 1$$
$$1 = 0 \cdot 2 + 1$$

$$(236)_{10} = (1110\ 1100)_2$$

xor

$$(10011010)_{gray}$$

$$61 = 30 \cdot 2 + 1$$
$$30 = 15 \cdot 2 + 0$$
$$15 = 7 \cdot 2 + 1$$
$$7 = 3 \cdot 2 + 1$$

$$3 = 1 \cdot 2 + 1$$
$$1 = 0 \cdot 2 + 1$$

$$(0)(00111101)_2 = (61)_2$$

xor
↓

$$(00100011)_{gray}$$

$$158 = 79 \cdot 2 + 0$$
$$79 = 39 \cdot 2 + 1$$
$$39 = 19 \cdot 2 + 1$$
$$19 = 9 \cdot 2 + 1$$
$$9 = 4 \cdot 2 + 1$$
$$4 = 2 \cdot 2 + 0$$
$$2 = 1 \cdot 2 + 0$$
$$1 = 0 \cdot 2 + 1$$

$$(10011110)_2 = (158)_{10}$$
$$(11010001)_{gray}$$

Exercice 3

$[0; 2^n]$

ⓐ $[0; \cancel{2^n} - 1]$ unsigned

$[-2^{n-1}; 2^{n-1}-1]$ signed

→ when we only need > 0 integers.

ⓑ

Decimal	Sign-magnitude	Two's complement
−32	10100000	11100000
73	01001001	01001001
−98	11100010	10011110
47	0 0101111	0 0101111
−39	10100111	11011001
86	01010110	01010110

Table 2: Conversion of decimal numbers to signed binary format

$-98 = -128 + 30$

$73 = 36 \cdot 2 + 1$

$36 = 18 \cdot 2 + 0$

$18 = 9 \cdot 2 + 0$

$30 = 15 \cdot 2 + 0$

$15 = 7 \cdot 2 + 1$

$7 = 3 \cdot 2 + 1$

$9 = 4 \cdot 2 + 1$

$4 = 2 \cdot 2 + 0$

$2 = 1 \cdot 2 + 0$

$1 = 0 \cdot 2 + 1$

$3 = 1 \cdot 2 + 1$

$1 = 0 \cdot 2 + 1$

1	0	1	1	1	0	
2	1	1	0	1	1	
4	0	1	0	1	1	
8	0	1	1	0	0	
16	0	0	1	0	1	1
32	1	1	0	1	0	1
64	1	0	1	0	1	1

$-39 = -128 + 28 + 61$

$= -128 + 89$

d) Table 3 contains 4-bit binary numbers. Extend them to 8 bits and convert the 8-bit binary numbers to the different listed formats.

$-128 + 124 = -4$

4-bit Sign-magnitude	8-bit Sign-magnitude	Hexadecimal	Decimal	8-bit Two's complement
0110	0000 0110	6	6	0000 0110
1100	10000100	?	−4	1111, 1~~011~~ + 1 100
1111	10000111	?	−7	11111001
0001	00000001	1	1	0000 0001
4-bit Two's complement	**8-bit Two's complement**	**Hexadecimal**	**Decimal**	**8-bit Sign-magnitude**
1010	1111 1010	?	−8+2=−6	1000 0110
0101	0000 0101	5	5	0000 0101
1100	1111 1100	?	−4	1000 0100
1000	1111 1000	?	−8	1000 1000

Table 3: Sign extension and conversion between different formats.

e) i) not redundant

ii) we can use the same algo

we need separate algo

iii) larger numerical range -> not redundant

$$\begin{array}{rr} & 1111 \\ + & 0100 \\ \hline & 1011 \end{array} \qquad \begin{array}{r} -7 \\ +4 \\ -3 \end{array}$$

$$\begin{array}{rr} & 101 \\ + & 001 \\ \hline & 110 \end{array} \qquad \begin{array}{r} -1 \\ 1 \end{array}$$

Exercise 5

(a)

A	HW(A)	B	HW(B)	HD(A, B)
01001010	3	11011110	6	3
00010100	2	01000000	1	3
01000010	2	01010101	4	4
11010110	5	10010000	2	3
10011111	6	01010010	3	5

Table 6: Table for hamming distance and hamming weight

(b) (i) There are 7 0 and 3 8.

(ii) $\binom{10}{7} = 120.$

(c) 1 1 0 1 0 1 0 1 0 1

0 0 1 0 1 0 1 0 1 0

max : 10

min : 2

1 1 1 1 1 1 0 0 0 0

1 1 1 1 0 0 0 0 0 0

(d) The hamming distance is 1

(e) max 3   001, 110

min 1   010, 110

(f) (i) $\binom{n}{1} \cdot \binom{2}{1} \cdot 2^{n-1}$

choose place for diff

choose diff

$n \cdot 2^n$

(i) $2^n \cdot 2^n = 2^{2n}$

2^n similar strings

$2^{2n} - 2^n$

(ii) $1 \dots n \Rightarrow n$?

Exercise 4

Unsigned					
A	Bin(A)	A>>1	A>>3	A<<1	A<<3
39	00100111	00010011	00000100	01001110	00111000
192	11000000	01100000	00011000	10000000	00000000
75	01001011	00100101	00001001	10010110	01011000
Signed (two's complement)					
96	01100000	00110000	00001100	11000000	00000000
−38	11011010	11101101	11111011	10110100	11010000
49	00110001	00011000	00000110	01100010	10001000
−106	10010110	11001011	11110010	00101100	10110000

Table 4: Binary Shift operations

1		1	0	0	1	0
2		1	0	1	0	1
4		0	0	0	0	1
8		1	0	1	0	0
16		0	0	1	1	1
32		0	1	0	1	0
64	1	1	1	1	0	0
128	1	0	0	1	0	1

$-64 + 26 = -38$

$4 + 20 + 2$

$-128 + 28 - 6 = -106$

$= -128 + 22 = -106$

(b) (i) 8 + 1 bits for the first sum

$16/2 = 8$ nb of 9 bits
$8/2 = 4$ nb of 10 bits
$4/2 = 2$ nb of 11 bits
1 nb of 12 bits

only 1s

It's the same as multiplying by 16.
(2^4)

$\cdot\ 2^8 \leftarrow$ bits

$= 12$ bits

(ii) we take only 1s (worst case)

worst case $a \times b = 16$.

$16/2 \rightarrow 8$ nb of 16 bits
$8/2 \rightarrow 4$ nb of 32 bits
$4/2 \rightarrow 2$ nb of 64 bits
$2/2 \rightarrow 1$ nb of 128 bits

(c)

A	B	A+B	Bin(A)	Bin(B)	Bin(A)+Bin(B)
6	7	13	110	111	1101
3	-8	-5	0011	1000	1011
-2	-8	-10	1110	1000	OVERFLOW
5	-3	2	0101	1101	0010
1	5	6	0001	0101	0110

Table 5: Binary addition

```
   1
 1 111
+  110
------
  1101
```

```
  0011
+ 1000
------
  1011
```

```
 1 1110
+  1000
-------
 1 0110
```

```
  1 1
  0101
+ 1101
------
  0010
```

```
     1
  0001
+ 0101
------
  0110
```

(d) $(12)_{10} = (1100)_2$

$(6)_{10} = (0110)_2$

$$
\begin{array}{rr}
 & 1100 \\
\cdot & 0110 \\
\hline
 & 0000 \\
+ & 0000 \\
\hline
 & 00000 \\
+ & 11000 \\
\hline
 & {}^{1}0111000 \\
+ & 110000 \\
\hline
 & 1001000 \\
+ & 0000000 \\
\hline
 & 1001000
\end{array}
$$

-3

(e) Two's complement:

- compute the first product
- add the partial sum
- add an extra sign extension (1 or 0)

↳ etc.

Sign magnitude :

- same as normal operation

But add 1 if signs are different