Problem 2.1

$$p_{XYZ}(x,y,z) = p_X(x)\,p_Y(y)\,p_Z(z)$$

$$= p(\{X=x\} \cap \{Y=y\} \cap \{Z=z\})$$

$$= p_X(x) \cdot p_Y(y)$$

$$p_{XY}(x,y) = p(\{X=x\} \cap \{Y=y\})$$

sum over all possible values of z →

$$= \sum_z p_{XYZ}(x,y,z)$$

$$= \sum_z p_X(x)\,p_Y(y)\,p_Z(z)$$

$$= p_X(x)\,p_Y(y)$$

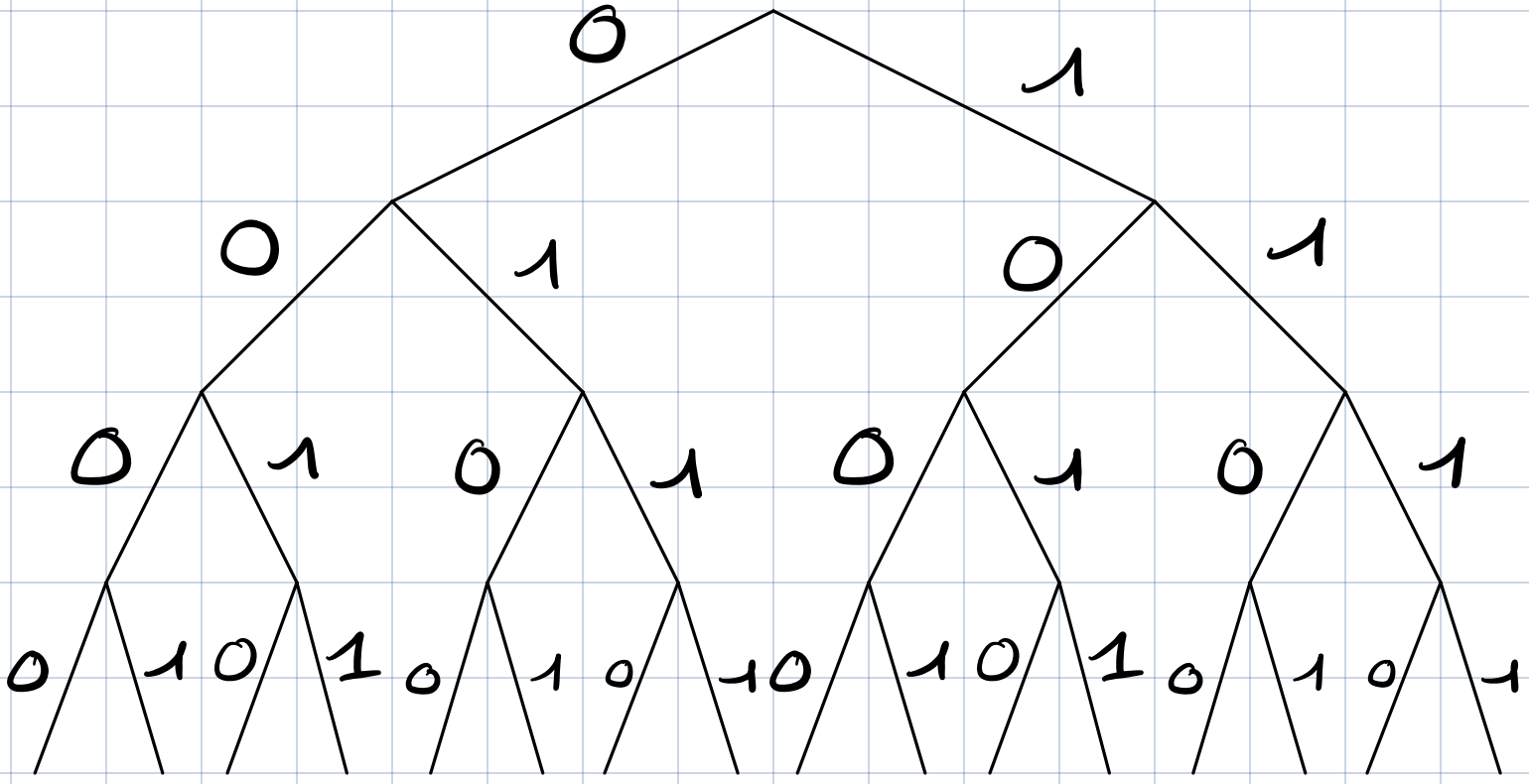
Problem 2.2

① Source $\{a, b, c, \dots\}$

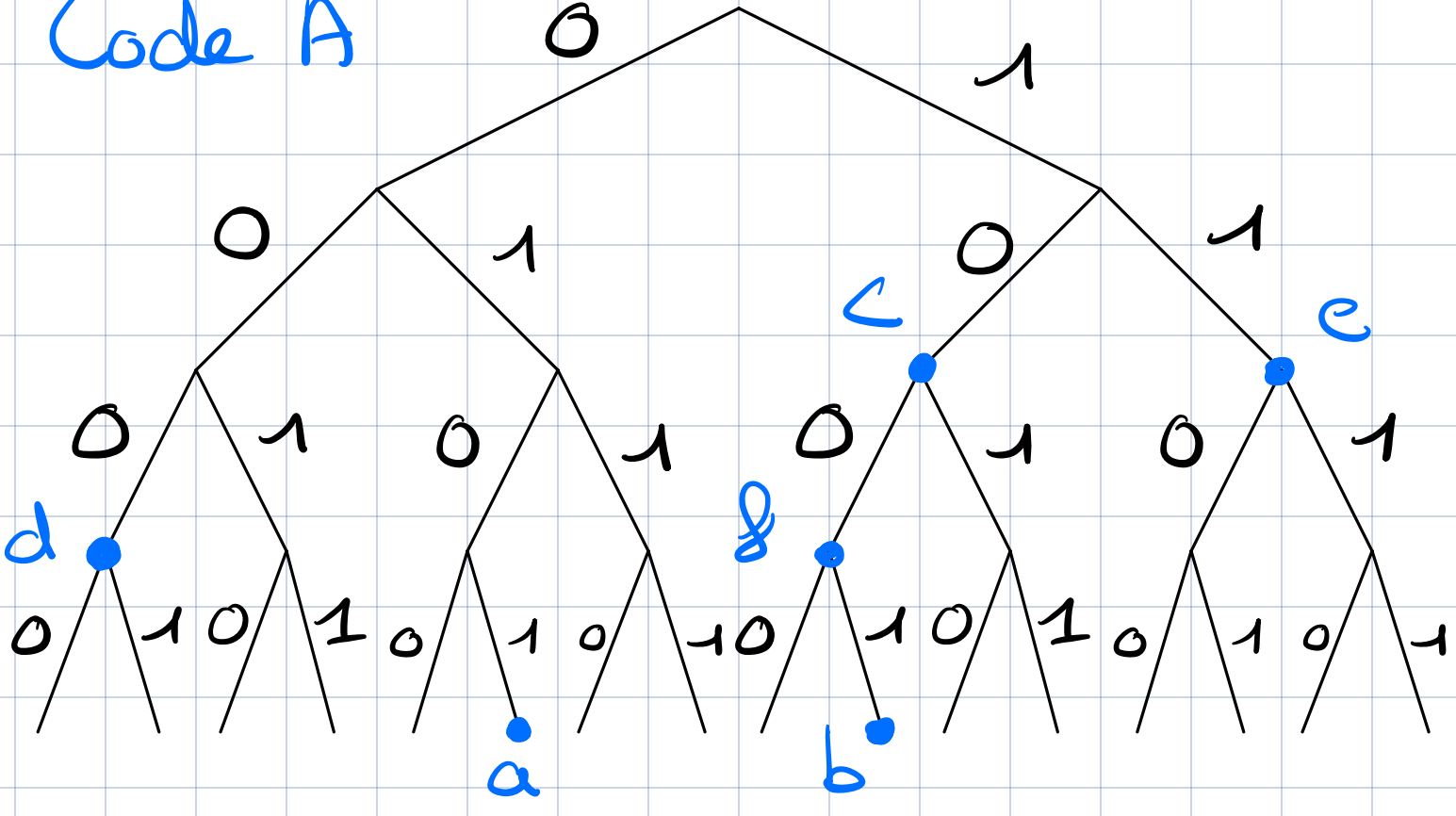
Code $\{0, 1\}$

(2)

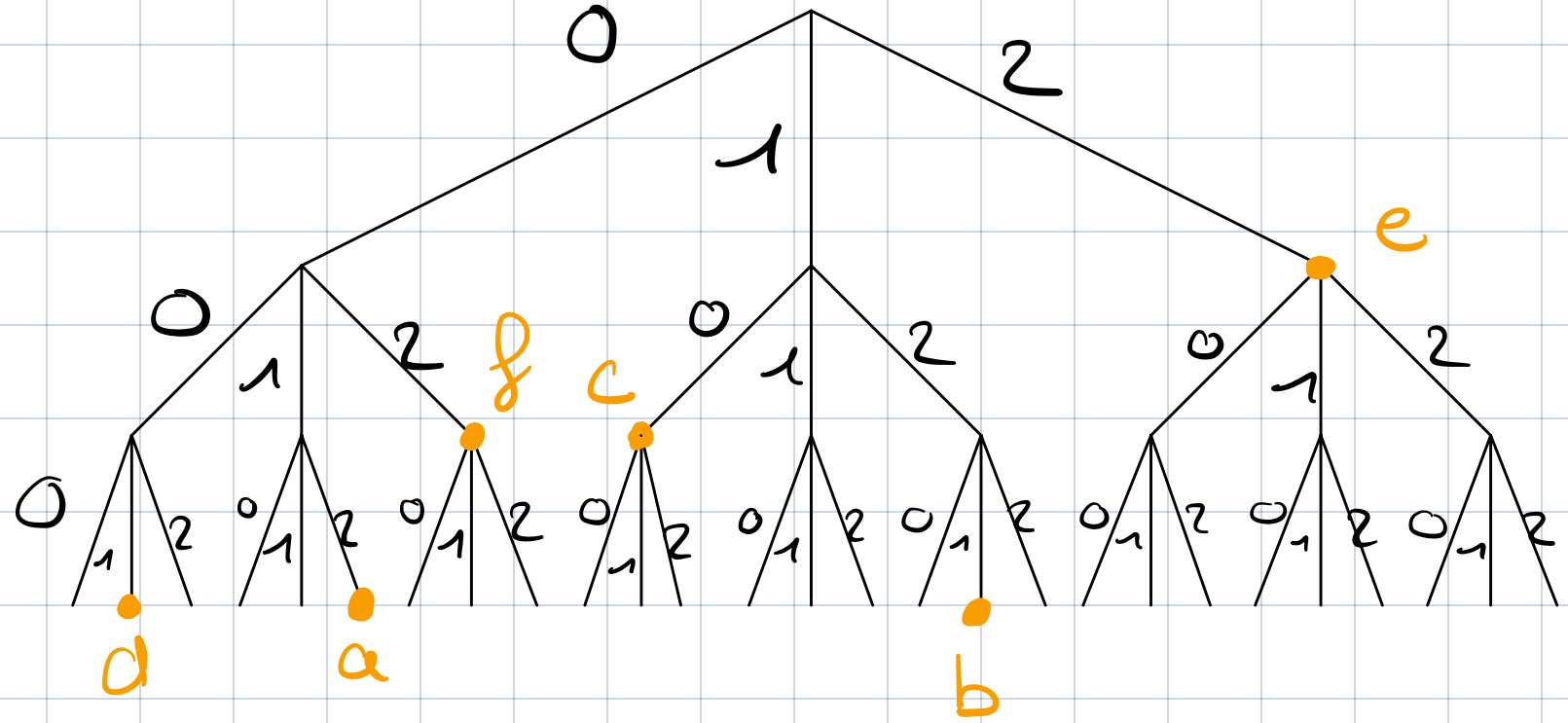
to copy and paste



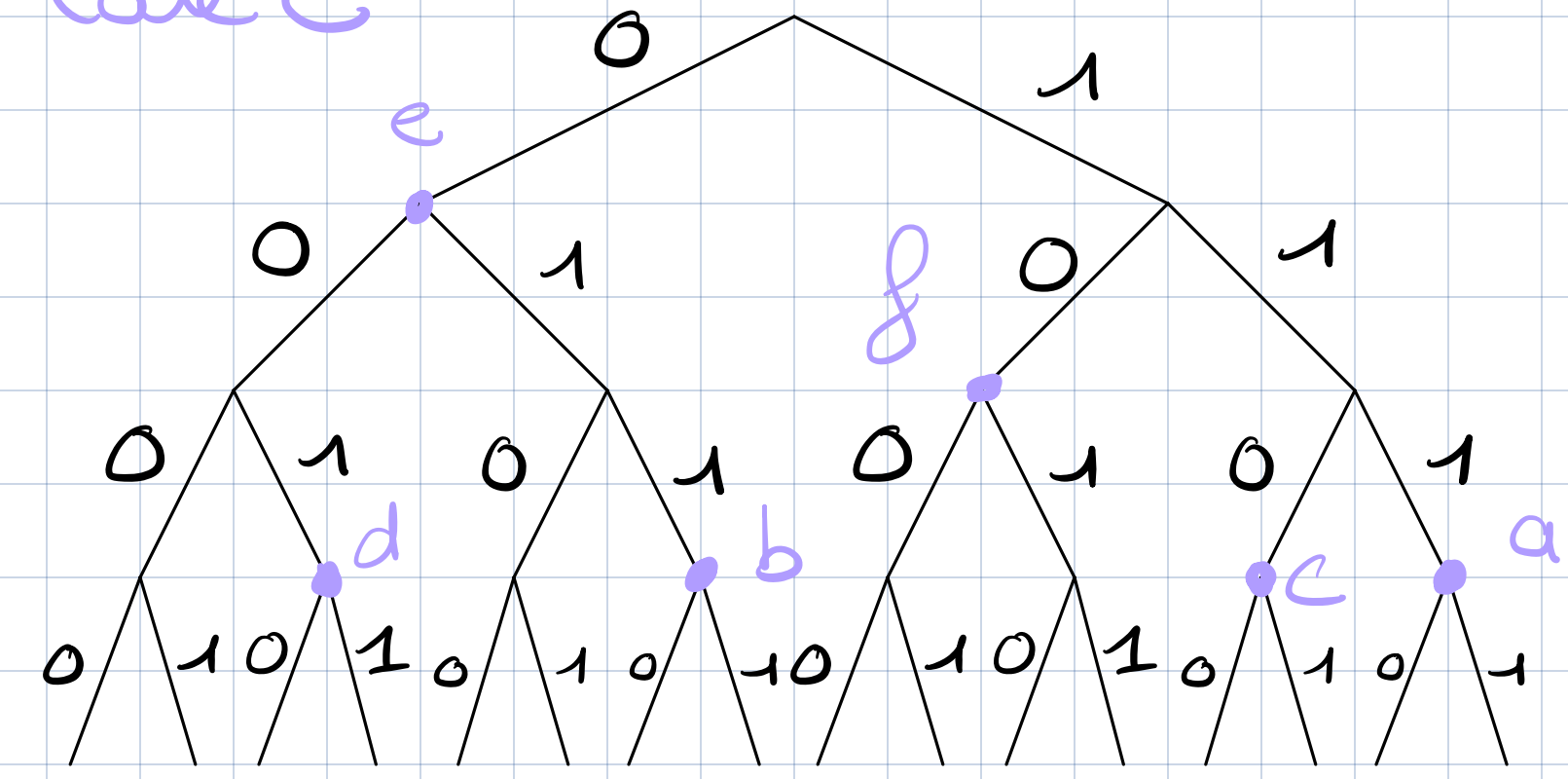
Code A



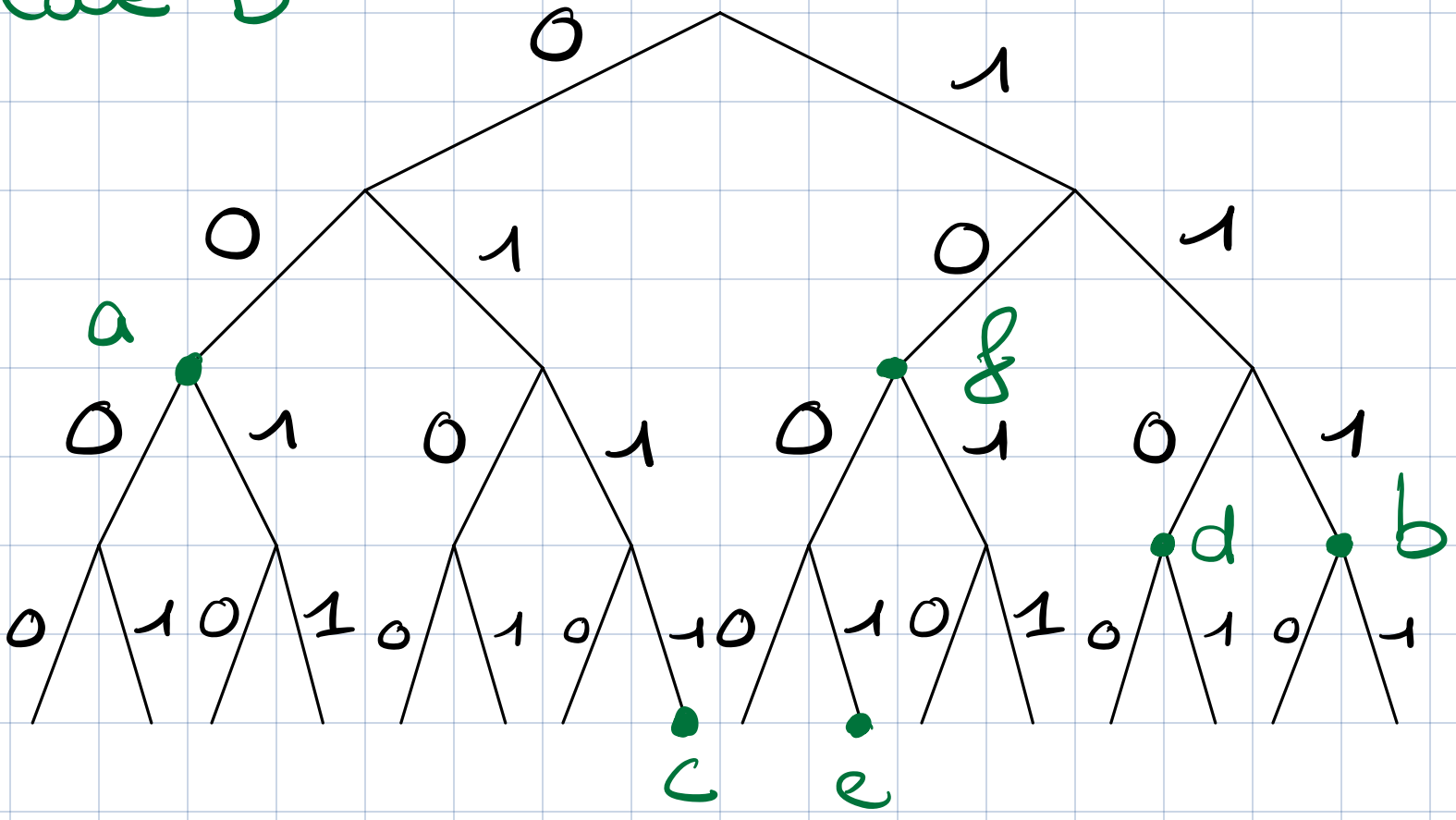
Code B



Code C



Code D



(3) Code A ✓

$$2^{-4} + 2^{-4} + 2^{-2} + 2^{-3} + 2^{-2} + 2^{-3} = \frac{7}{8} \leq 1$$

satisfies.

Code B ✓

$$3^{-3} + 3^{-3} + 3^{-2} + 3^{-3} + 3^{-1} + 3^{-2} = \frac{23}{27} \leq 1$$

Code C ✗

$2^{-3} \cdot 4 + 2^{-1} + 2^{-2} = \frac{5}{4} > 1$

Code D ✓

$2 \cdot 2^{-2} + 2^{-3} \cdot 2 + 2^{-4} \cdot 2$

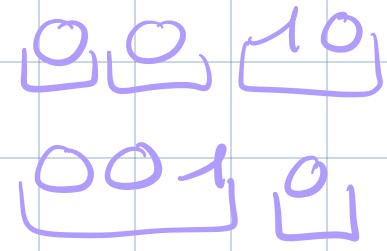
$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$

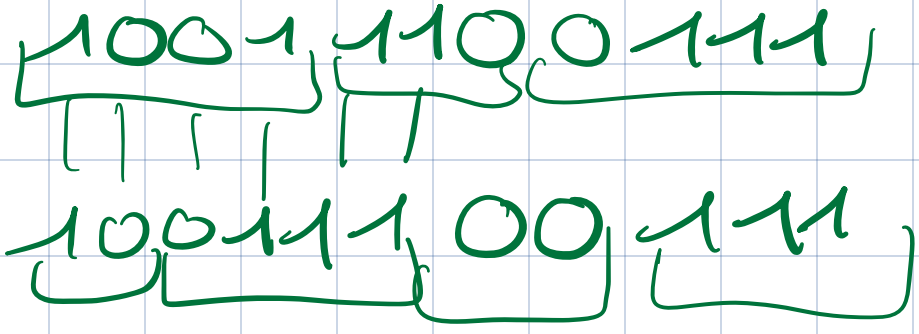
④ $\neg$[kraft] $\Rightarrow$ $\neg$[uniq. dec.]

kraft $\Rightarrow$ ∃ instantaneous w. same length BUT not necessarily the same.

⑤ A is reverse prefix-free

B is prefix free





⑥ A is not ($10 \neq 1001$)

B is.

C is not. ($0 \neq 001$)

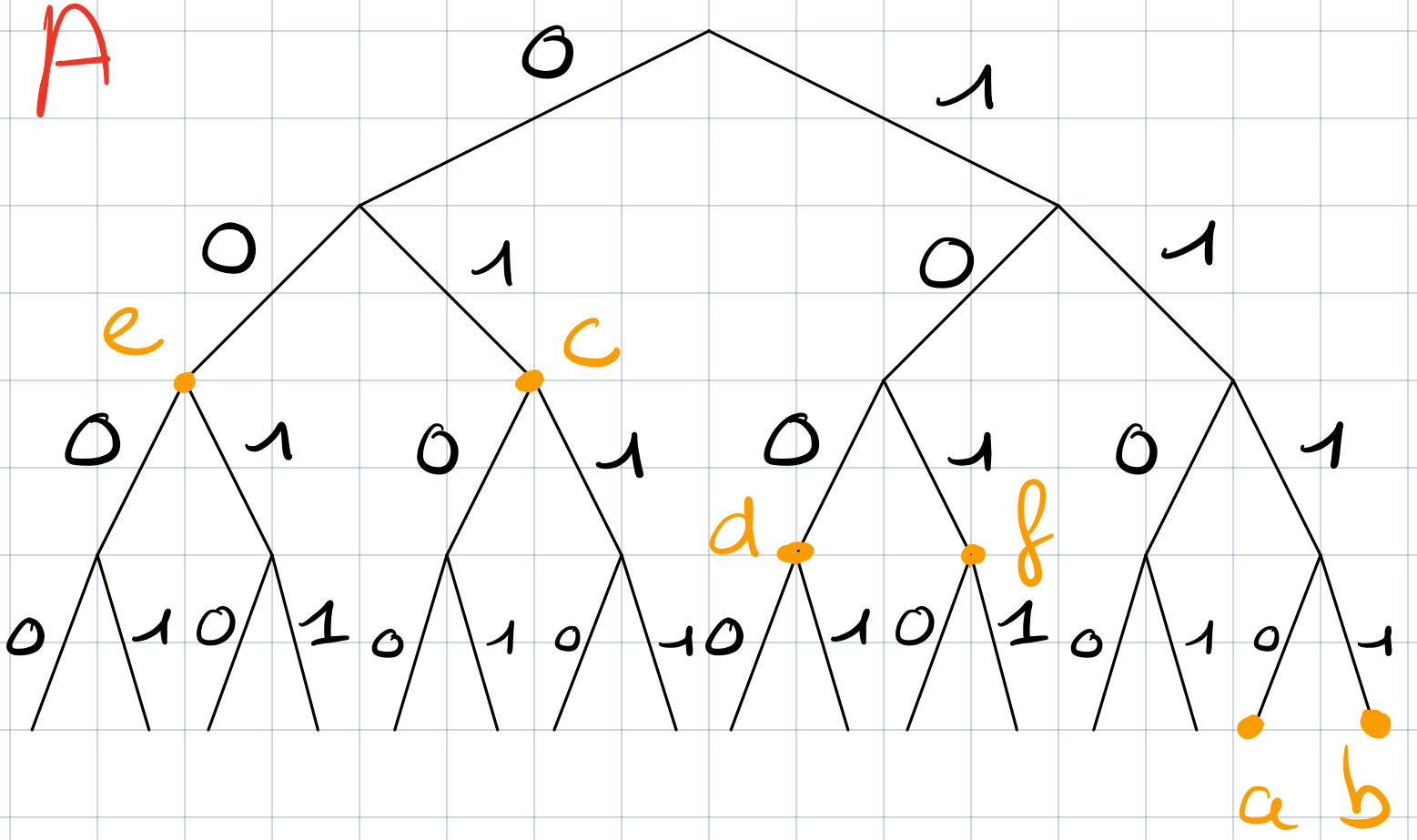
D is not.

(7) diff between complete tree?

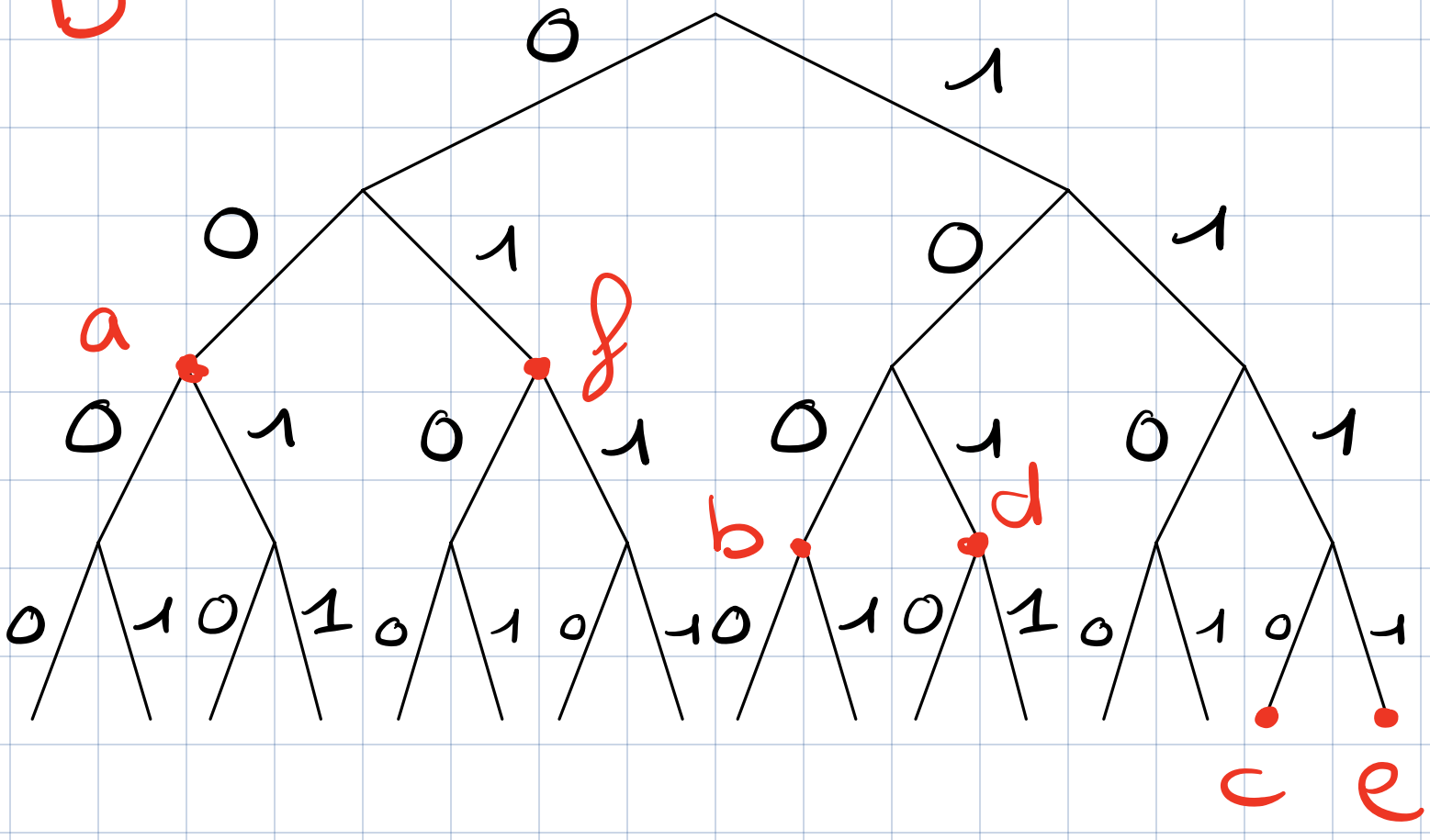
(8)

	A	D
a	1110	00
b	1111	100
c	01	1110
d	100	101
e	00	1111
f	101	01

A



D



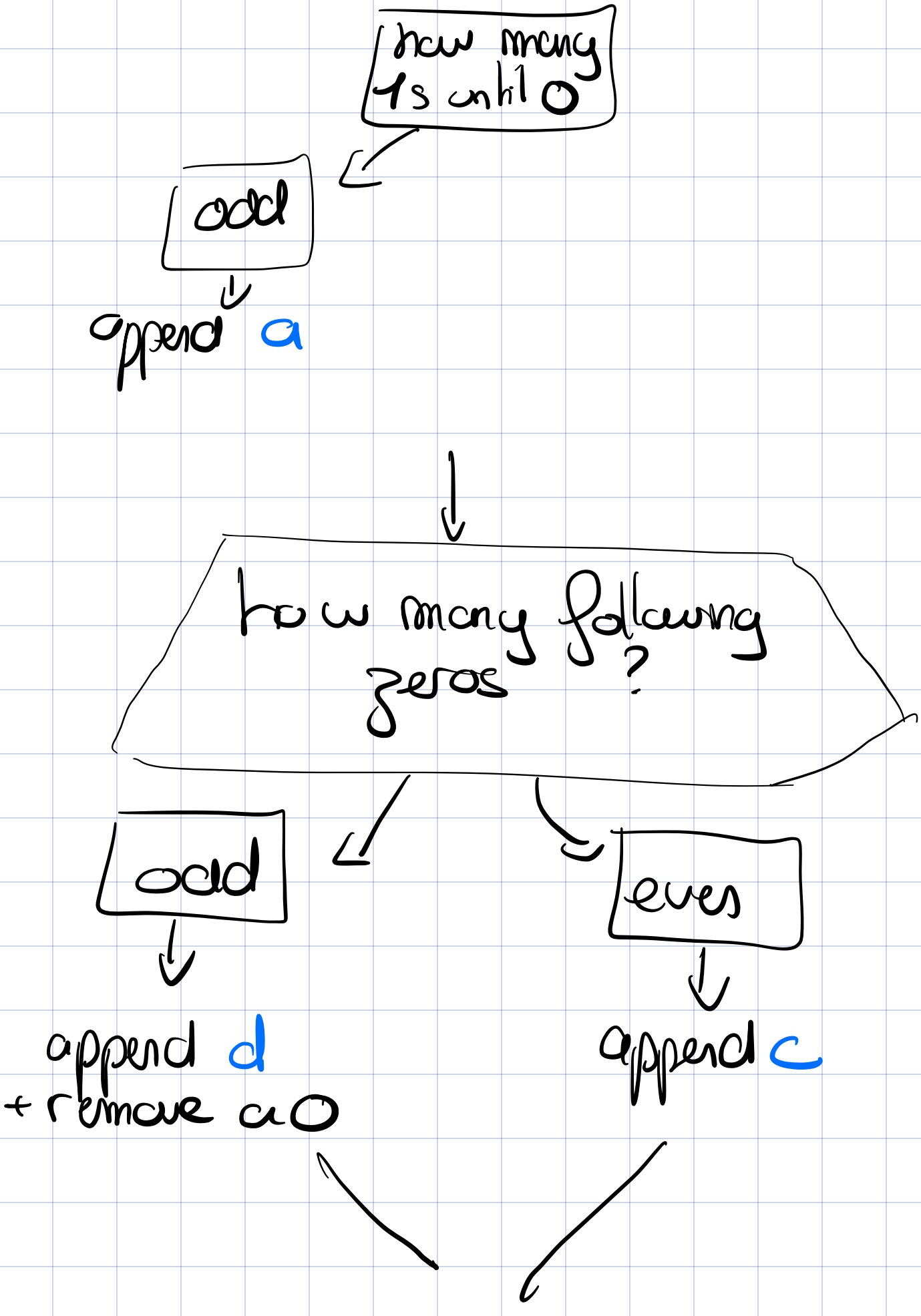
Problem 2.3

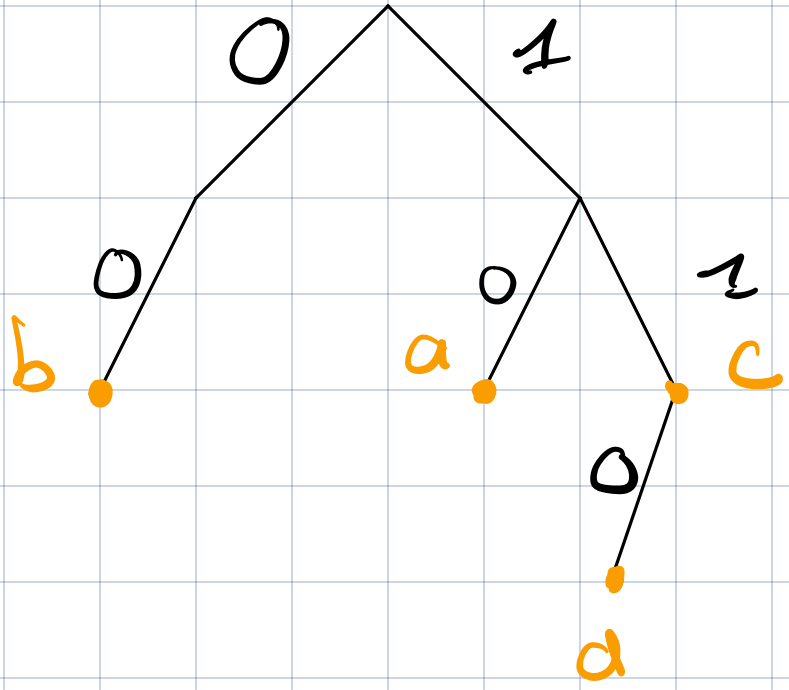
① $c(11) \neq d(110)$
$a(01) \neq a(011)$

② procedure decode (code) {

- when you get a one → read the number of following 0s.
if it is odd

prof said: "well it's up to you, you can draw a simple tree to decode or use a complex algorithm — this course is not about coding".

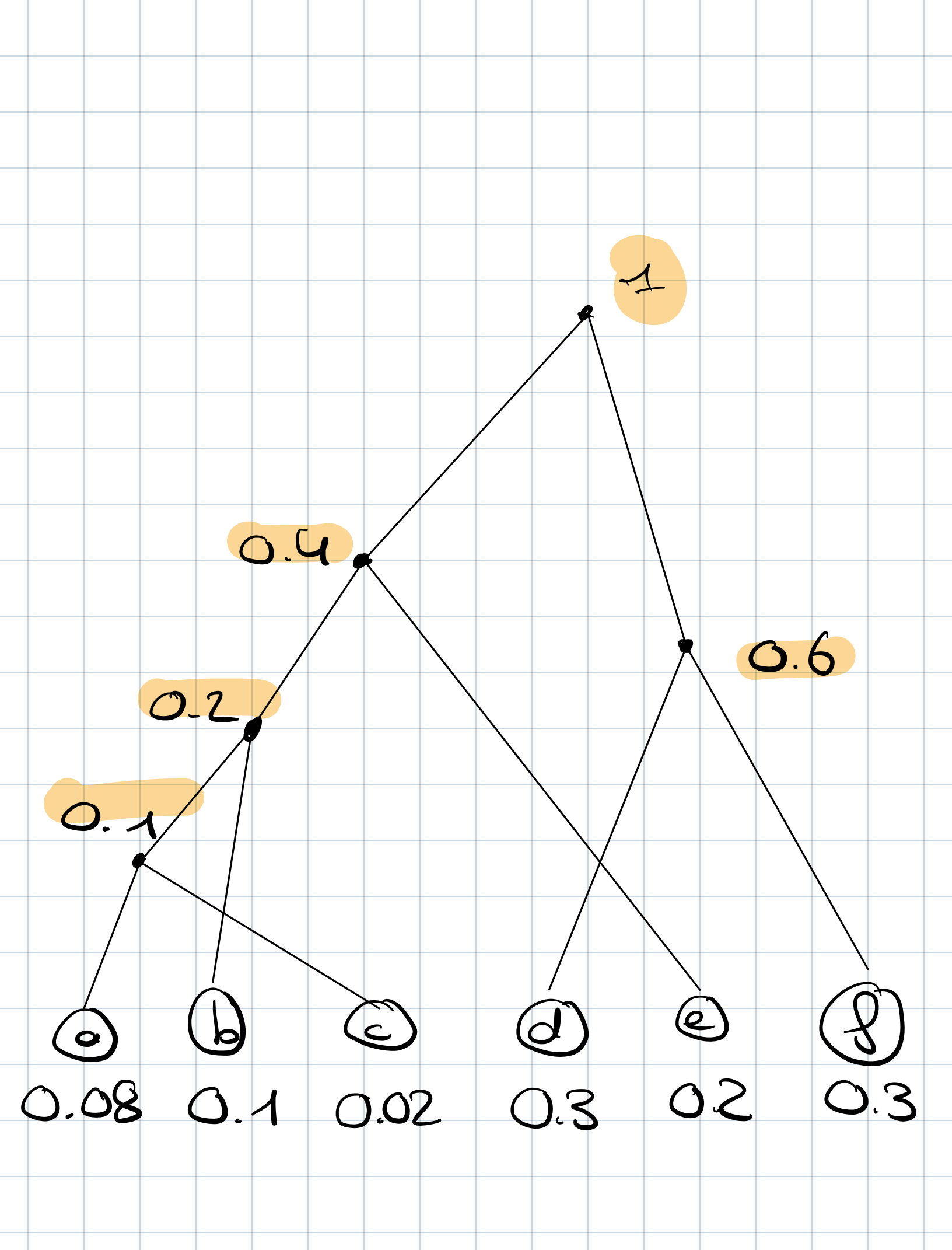

how many 1s until 0
odd
append a
how many following zeros?
odd
append d
+ remove a0
even
append c



11 →

Exercise 2.4

Source symbol	Probability	Length $\lceil -\log_2 p(s) \rceil$
a	0.08	4
b	0.1	4
c	0.02	6
d	0.3	2
e	0.2	3
f	0.3	2


1
0.4
0.6
0.2
0.1
a
b
c
d
e
f
0.08
0.1
0.02
0.3
0.2
0.3

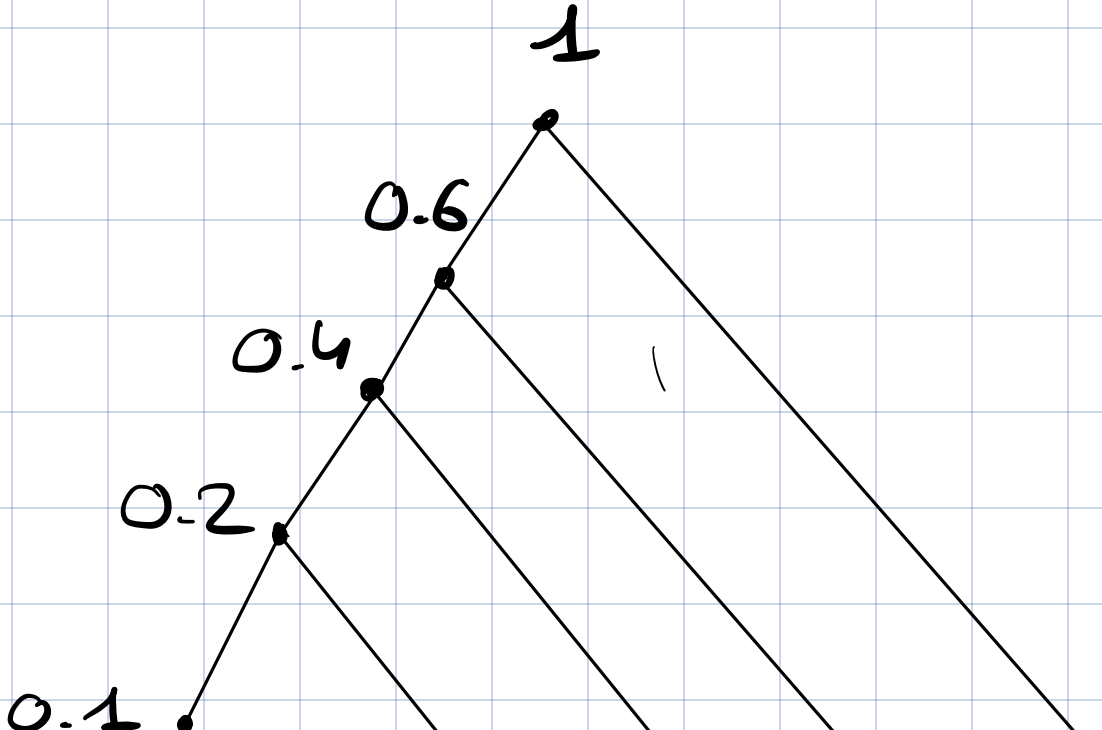
$$L(S,T) = 4 \cdot 0.08 + 0.1 \cdot 4 + 6 \cdot 0.02 + 2 \cdot 0.3 + 3 \cdot 0.2 + 2 \cdot 0.3$$

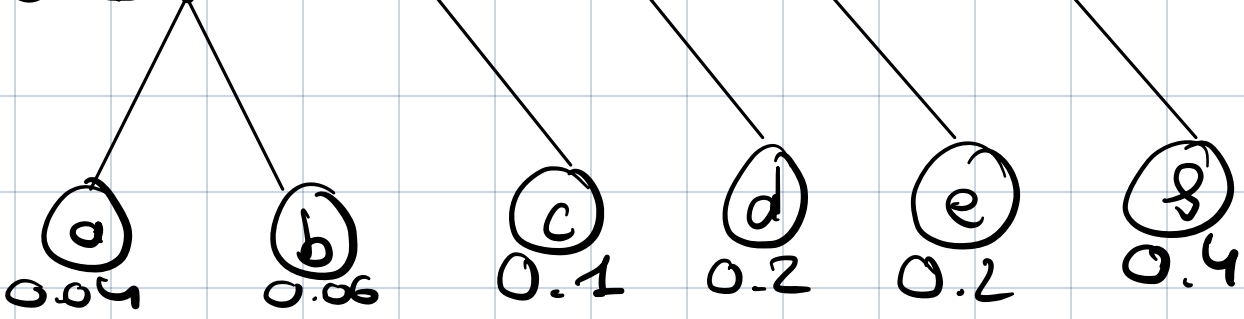
$$= 2.64 \text{ (Shannon)}$$

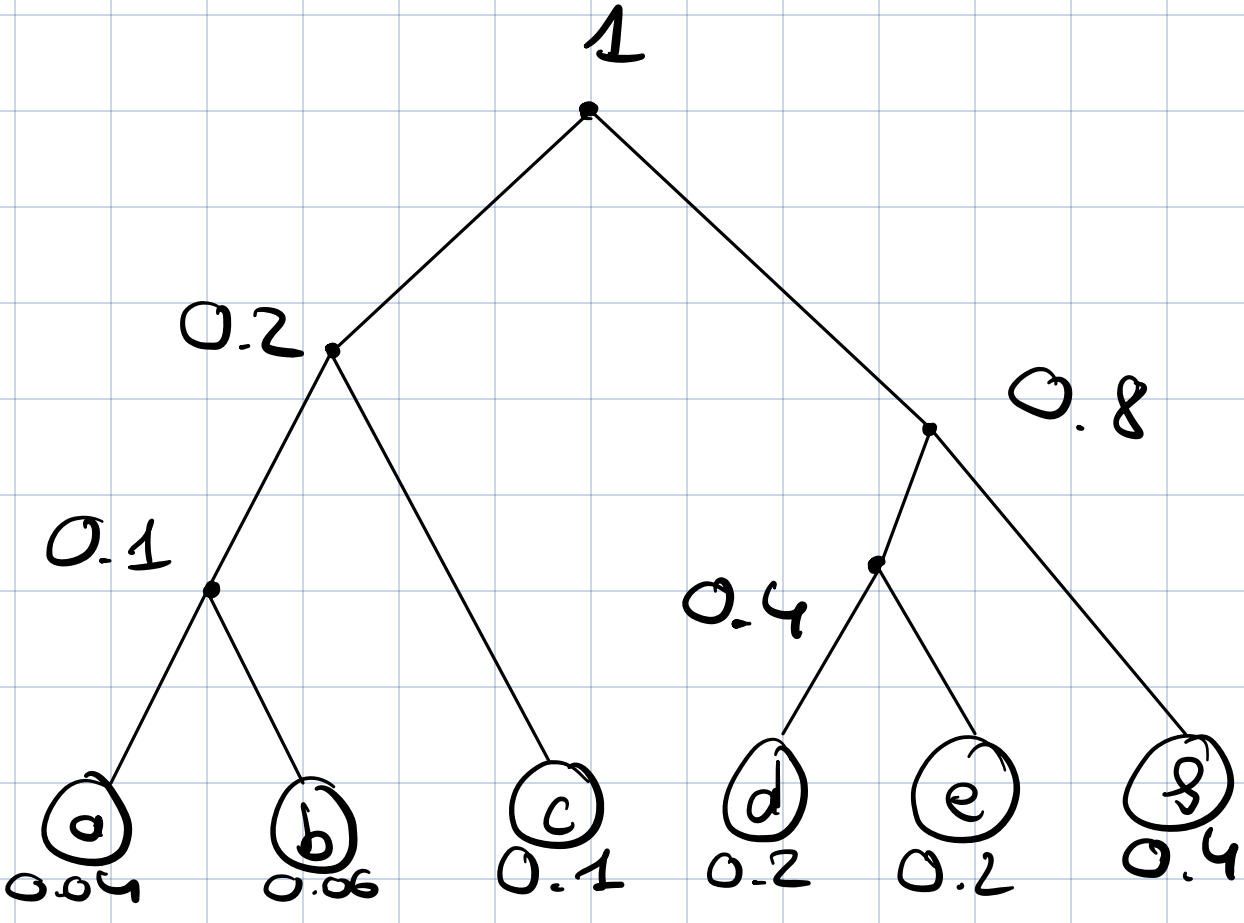
$$L(S,T) = 2.3 = \text{sum of nodes}$$

2

Source Symbol	Probability
a	0.04
b	0.06
c	0.1
d	0.2
e	0.2
f	0.4







(3) Let's assume they have different lengths then one of them is not optimal.