Question 1

(1) No, the maximal entropy of a random variable is when it is uniformly distributed.

(2) $H(Y, X_1) = H(Y) + H(X_1 / Y)$

$\downarrow$

$X_1 = 0 \quad X_2 = 1$
$X_1 = 1 \quad X_2 = 0$
$X_1 = 0 \quad X_2 = 0$
$X_1 = 1 \quad X_2 = 1$

are X_1 and Y indep?
Yes. because X_2 is uniformely distributed

(3) $H(X_1, X_2) = \frac{1}{4} \log(4) \cdot 4 = 2$

Yes

4 False.

Question 2

① Yes, they are independent (I I D)

② False, they are indep.

③ Yes, they are indep.

④ $\lim_{n \to \infty} H(S_n) = H(S_1)$

$$= \frac{1}{30}\log(30)\cdot 5$$

$$+ \frac{5}{6}\log\left(\frac{6}{5}\right)$$

$$= \frac{1}{6}\log(30) + \frac{5}{6}\log\left(\frac{6}{5}\right)$$

$$= \frac{1}{6}\log(6.5) + \frac{5}{6}\log\left(\frac{6}{5}\right)$$

False.. ?

$$= \frac{1}{6}\log(6)$$

5 Yes, probabilities are always the same.

$$\boxed{\lim_{n\to\infty} H(S_n) \text{ exists}}$$

$$\lim_{n\to\infty} \frac{H(S_1 \dots S_n)}{n}$$

6 Yes, because stationary $\Rightarrow$ regular?

Question 3

① False, the key is repeated !

②

③ False.

④

0 1 0 0 1 0

XOR 0 1 0 1 0 1

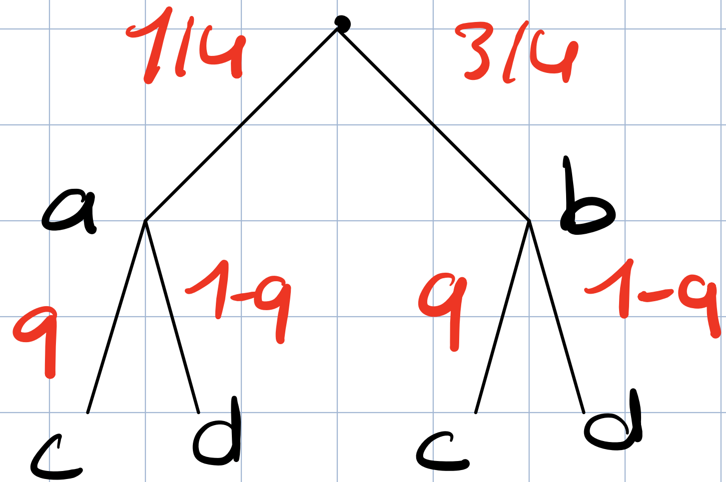
1 1 1 1 1 1

XOR 0 1 0 0 1 0

1 0 1 1 0 1

Question 4

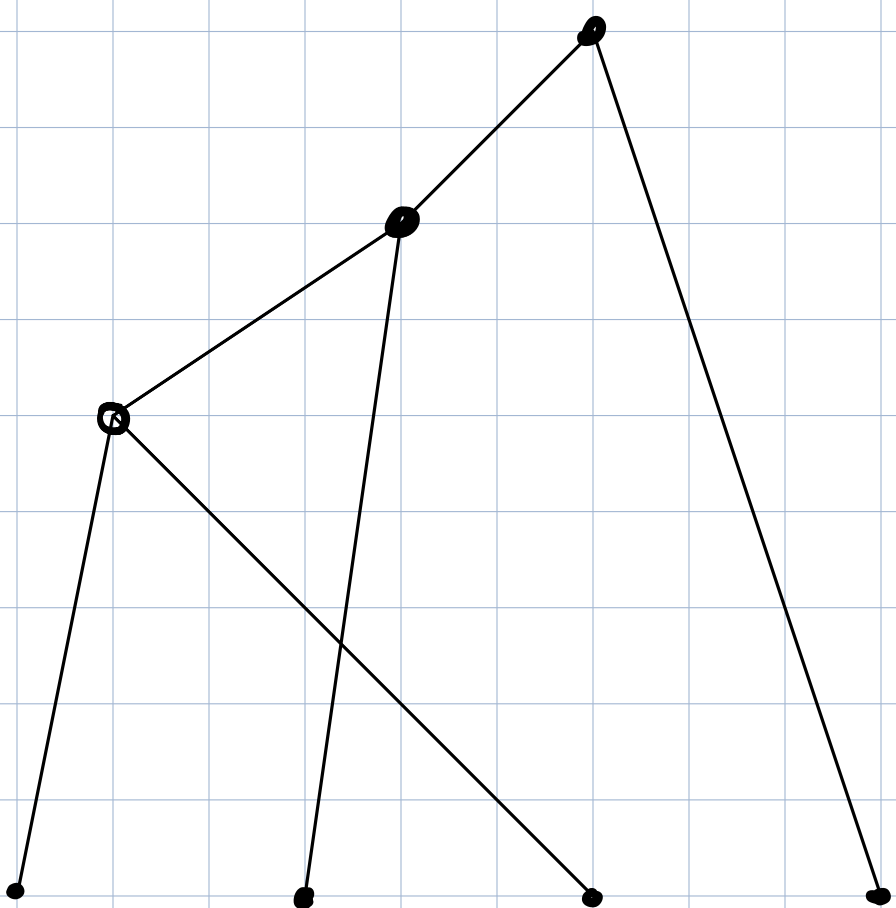
$H(S_1 S_2)$



$$2 \cdot \frac{1}{4}(q + 1 - q) + \frac{3}{4}(q + 1 - q) \cdot 2$$

$$= \frac{1}{2} + \frac{6}{4} = \frac{1}{2} + \frac{3}{2} = 2$$

TRUE



$\frac{q}{4}$ $\frac{1-q}{4}$ $\frac{3q}{4}$ $\frac{3-3q}{4}$

ac ad bc bd

$\frac{1}{8}^{-}$ $\frac{3}{16}^{+}$ $\frac{3}{16}^{-}$ $\frac{9}{16}^{+}$

Question 5

$$H(Y_1 \dots Y_n)$$

$$H(Y_i \mid Y_1 \dots Y_{i-1})$$

$$Y_i : \begin{cases} 0 & \text{si } Y_{i-1} = 0 \text{ et } X_{i-1} = 0 \\ 1 & \text{si } Y_{i-1} = 1 \text{ et } X_{i-1} = 0 \\ 1 & \text{si } Y_{i-1} = 0 \text{ et } X_{i-1} = 1 \\ 0 & \text{si } Y_{i-1} = 1 \text{ et } X_{i-1} = 1 \end{cases}$$

$$p(X_{i-1} = 0) = \frac{3}{4}$$
$$p(X_{i-1} = 1) = \frac{1}{4}$$

$$p(Y_2 = 1) = \frac{3}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} = \frac{4}{8} + \frac{1}{8}$$

$$\begin{array}{cccc} 1 & 0 & 0 & 1 \end{array} = \frac{5}{8}$$

$$p(Y_3=1) = \frac{5}{8}\cdot\frac{3}{4} + \frac{3}{8}\cdot\frac{1}{4}$$

$$= \frac{15}{24} + \frac{3}{24} = \frac{18}{24} = \frac{3}{4}$$

$$p(Y_4=1) =$$

$$H(Y_i \mid \_\_) = \left[\boxed{\frac{1}{3}}\log(3) + \frac{3}{4}\log\left(\frac{4}{3}\right)\right] n$$

$$H(Y_n) + H(Y_1$$

$$H(Y_1 \dots Y_n)$$

$$H(Y_1 \dots Y_n)$$

$$= H(Y_n \mid Y_1 \dots Y_{n-1}) + H(Y_1 \dots Y_{n-1})$$

$$\frac{1}{3}\log(3) + \frac{3}{4}\log\left(\frac{4}{3}\right) \qquad +$$

$$= \dots$$

$$= \underline{\quad} + H(Y_1)$$

$$(n-1)\left(\frac{1}{4}\log(4) + \frac{3}{4}\log\left(\frac{4}{3}\right)\right) + 1$$

$$= n\left(\frac{1}{2} + \frac{6}{4} - \frac{3}{4}\log(3)\right)$$

$$- \frac{1}{2} - \frac{6}{4} - \frac{3}{4}\log(3) + 1$$

$$= n\left(2 - \frac{3}{4}\right.$$