Problem 7. 1

① $$\left(\sum_{i=1}^{9} i \cdot x_i\right) = Y$$

$$Y \equiv x_{10} \, [11]$$

$$Y + 10 \cdot x_0 \equiv 11 x_0 \, [11]$$

$$\Rightarrow Y + 10 x_0 \equiv 0 \, [11]$$

$$\cancel{\left[\begin{array}{l} 10 \equiv 1 \, [11] \\ \Rightarrow 10 x_0 \equiv x_0 \, [11] \end{array}\right.}$$

$$\cancel{Y + 10 x_0 \equiv x_0 \, [11]}$$

useless

② To compute A mod 11, ~~we do~~

$$\sum_{i=1}^{10} i x_i \equiv 0 \; [11]$$

$$\sum_{i=1}^{10} i x_i - \sum_{i=1}^{10} i y_i = i(x_i - y_i)$$

$$\equiv$$

$k \in \{0, \dots, 10\}$

$$\sum_{i=1}^{10} i y_i \equiv k \; [11]$$

$$\sum_{i=1}^{10} i y_i - \sum_{i=1}^{10} i x_i \equiv k \; [11]$$

$$= \underset{1,2 \dots, 9}{i}\underbrace{(y_i - x_i)}_{1,2,3,\dots,9} \equiv k \; [11]$$

| this can not be in this set

↓ (which is a requirement to be 0 because they need 11 and they are only 1, 2, 3 ... 9)

11, 22, 33, 44, 55, 66, 77, 88

(3) $\sum_{i=1}^{10} i y_i \equiv k \; [11]$

$$\sum_{i=1}^{10} i y_i - \sum_{i=1}^{10} i x_i \equiv k \; [11]$$

$$i(x_{i+1} - x_i) + (i+1)(x_i - x_{i+1}) \equiv k \; [11]$$

$$\begin{array}{ll} \overset{i}{\downarrow}\,\overset{i+1}{\downarrow} & \\ 1\,3\,4 & x \\ \rightarrow 1\,4\,3 & y \end{array}$$

$$= \cancel{i x_{i+1}} - \cancel{i x_i} + \cancel{i x_i} - \cancel{i x_{i+1}} + x_i - x_{i+1} \equiv k \; [11]$$

$$= \underbrace{x_i - x_{i+1}}_{} \equiv k \; [11]$$

can never be 11

⇒ has to be 0 to be ≡ 0 [11]

Problem 7.2

$$\varphi(10) = 4 \cdot 5^0 \cdot 1 \cdot 2^0 = 4$$

$$347^{348} = 347^{4k} \equiv 1\,[10]$$

$$347^{348} \equiv k\,[5 \cdot 2]$$

$$347^{348} \equiv k'\,[5]$$

$\varphi(5) = 4$ 347 co-prime 5

$\varphi(2) = 1$ 347 co-prime 2

$$\Rightarrow 347^{348} \equiv 1\,[5]$$

$$347^{348} \equiv 1\,[2]$$

$M_1 = 2^{-1} = 3$ (modulo land 5)

$\Pi_2 = S^{-1} = 1$ (modulo bnd 2)

$(3 \cdot 2 \cdot$

Problem 7.4

$a = 4$

$m = 8$

1, 2, 3

$\Rightarrow$ 4 n'est pas inversible mod 8.

if a is not invertible mod m, then
$pgcd(a, m) > 1 = d$

be invertible, $\exists x$ s.t.:

$$ax \equiv 1 \, [m]$$

$$\Rightarrow ax - 1 = mk$$

$$\Rightarrow ax - mk = 1$$

$$\begin{bmatrix} \gcd(a, m) > 1 \\ = d \end{bmatrix}$$

$$a = kd$$
$$m = k'd$$

b is all the factors that are in m but not in "a"

$$a \cdot b = mq$$

$$kd \cdot b = k'dq$$

$$= k'd = k'dq$$

$\Rightarrow$ multiple b (no matter what we choose for a)

Problem 7.3

$$\left[\;\begin{cases} y = [1]_7 \\ x = [0]_7 \end{cases} \qquad \begin{cases} y = [2]_7 \\ x = [1]_7 \end{cases}\right.$$

$$[2]_7 x + [5]_7 y - [2]_7 x - [4]_7 y$$

$$= [1]_7 y = [3]_7 \Rightarrow y = [3]_7$$

$$[2]_7 x + [5]_7 [3]_7 = [5]_7$$

$$\Rightarrow [2]_7 x + [1]_7 = [5]_7$$ mult. inverse of 2 mod 7 is

$$\Rightarrow x = [4]_7 [4]_7 = [16]_7 = [2]_7$$

$$1 = 7\cdot a + 2\cdot \boxed{b}$$

		u $\parallel$ $\tilde{v}$	v $\parallel$ $\tilde{u}-q\tilde{v}$
$\gcd(7,2)$	$7 = 2\cdot 3 + 1$ ($\to q$)	1	3
$\gcd(2,1)$	$2 = 2\cdot 1 + 0$ ($\to q$)	0	1
$\gcd(1,0) = 1$		1	0

$$1\cdot u + 0\cdot v = 1$$

u: has to be 1

v: v can be anything (0)

$$347^n \equiv 1\,[10]$$

Problem 7.5

The square of an even number is an even number

$$(2k)^2 = \boxed{2(2k^2)}$$

$$(2k+1)^2 = 4k^2 + 4k + 1$$
$$= \underbrace{4(k^2+k)}\ + 1$$

$$[4(k^2+k)+1]_{100} \equiv [11]_{100}$$

$$\Rightarrow \underbrace{[4]_{100}[k]_{100}[k+1]_{100} \equiv [10]_{100}}$$

$\Rightarrow$ impossible?

$$[4k^2+4k+1]_4 \equiv [1]_4 \neq$$
$$[\ldots 11]_4 = [3]_4$$

on cherche $[x^2]_{10} = [1]_{10}$

$\downarrow$

1 ou 9

maintenant on cherche

$$[x^2]_{100} = [11]_{100}$$

on a plus que ces possibilités :

11	$(11)^2 = 121$	19	$(20)^2 - 19 - 20 = -39 = 61$
21	$(20)^2 = 0 + 21 + 20 = 41$	29	$= (30)^2 - 29 - 30 = 41$
31	$(30)^2 = 0 + 61 = 61$	39	
41	81	49	$= 21$
51	1	59	$= 1$
61	21	69	$= 81$
71	41	79	$= 61$
81	61	89	$= 41$
91	81	99	$= 21$

$\Rightarrow$ aucun nb qui finit avec 11 n'est un carré

Problem 7.6

$174 \cdot 3 = 522$

	u	v
$549 = 174 \cdot 2 + 144$	5	$-4-10 = -14$
$174 = 144 \cdot 1 + 30$	-4	5
$144 = 30 \cdot 4 + 24$	1	-4
$30 = 24 \cdot 1 + 6$	0	1
$24 = 6 \cdot 4 + 0$	1	0

$\gcd(6,0) = 6$

$6 \cdot u + 0 \cdot v = 6$

$27 \cdot 6 = 162$

$\gcd(a,b)$	$a = bq + r$	q	$\tilde{u}$ ($\tilde{v}$)	$\tilde{v}$ ($\tilde{u} - q\tilde{v}$)
$(549, 174)$	$549 = 174 \cdot 3 + 27$	3	13	-41
$(174, 27)$	$174 = 27 \cdot 6 + 12$	6	-2	13
$(27, 12)$	$27 = 12 \cdot 2 + 3$	2	1	-2
$(12, 3)$	$12 = 3 \cdot 4 + 0$	4	0	1
$(3, 0)$ $= 3$			1	0

$$3 \cdot u + 0 \cdot v = 3$$

$$\gcd(a,m)=1$$

$$au+mv=1$$

$$\Leftrightarrow$$

$$ax\equiv 1\ [m]$$

$$ax-mk=1$$

1

$$\left[\sum_{i=1}^{9} i x_i\right] 10 + x_{10} \equiv ? \ [11]$$

$$-x_{10} + x_{10} \equiv 0 \ [11]$$

$3 \to 4$

-1

$$\sum_{i=1}^{10} i x_i \equiv 0 \ [11]$$

$$\sum_{i=1}^{10} i x_i - i(x_i - y_i) \equiv ? \ [11]$$

$$i(x_i - y_i) \equiv 0 \ (11)$$

$$\Rightarrow x_i - y_i \equiv 0 \ [11]$$

$\boxed{7.2}$

$347 -$

348

47

$(-3)^{348}$

$[(-3)\cdot(-3)]^{347}$

$(9)^{347}$

$[-1]^{347}$

1

$\boxed{7.3}$

$$[2]_7 x + [5]_7 y = [5]_7$$

$$[2]_7 x + [5]_7 y = [5]_7$$

$$- \quad [2]_7 x - [4]_7 y = [2]_7$$

$$[1] y = [3]_7$$

$$y = [3]_7$$

$$[1] x + [3 \cdot 2]_7 = [1]_7$$

$$\Rightarrow \quad [1] x = [-5]_7 = [2]_7$$

$$\int_1^2 \int_{1/x}^{x} x + \frac{1}{y} \, dy \, dx$$

$$= \int_1^2 \Big[xy + \ln(y) \Big]_{1/x}^{x} \, dx$$

$$= \int_1^2 \underbrace{x^2 + \ln(x) - 1 - \ln\left(\frac{1}{x}\right)}_{x^2 + 2\ln(x) - 1} \, dx$$

$$= \left[\frac{1}{3} x^3 + 2x\ln(x) - x \right]$$

$$\int_{1/2}^{2} \int_{2}^{y}$$

$$+ \int_{1/2}^{2} \int_{1/2y}^{1}$$

$$1 \geqslant x > 1/y$$

$$1 < x < 2$$

$$\boxed{1 \leqslant \frac{1}{x} < y} < x \leqslant 2$$

$$1 < xy < x^2$$

$$\boxed{\frac{1}{y}} < x < \frac{x^2}{y} \leqslant \frac{4}{y}$$

$$\Leftrightarrow 1 < xy < x^2$$

$$\Leftrightarrow 1 < xy < x$$

$$\Leftrightarrow \frac{1}{y} < x < \frac{x^2}{y}$$

$$\Leftrightarrow \frac{1}{y} < x < \frac{4}{y}$$

$$\int_1^2 \int_{1/y}^{4/y}$$

$\gcd(a, m) \neq 1$

$a = p_1^{k_1} p_2^{k_2} \dots$

$m = p_1^{k_1'} p_2^{k_2'} \dots$

Soit $b = p_1$

$m = 8$

$a = 4$

$2^2 \qquad 2^3 \qquad k = \dfrac{m}{\gcd(a, m)}$

$ak = m$

$$(2k)^2 = 4k^2$$

$$(2k+1)^2 = 4k^2+4k+1$$
$$= 4(k^2+k)+1$$
$$\%4 = 1$$

4
8
12
16
.....

$\boxed{111\ldots08}+\boxed{3}$

$$\begin{array}{r} {}^{1}\,{}^{1} \\ 348 \\ +\ 174 \\ \hline 522 \\ 27 \end{array} \qquad \begin{array}{r} {}^{1} \\ 174 \\ +\ 174 \\ \hline 348 \end{array} \qquad \begin{array}{r} {}^{4} \\ 27 \\ \cdot\ \ 6 \\ \hline 162 \\ \boxed{14} \end{array}$$

$\gcd(a,b)$	$a=bq+r$	q	$u=\tilde{v}$	$v=\tilde{u}-q\tilde{v}$
$\gcd(549,174)$	$549=174\cdot 3+27$	3	-13	41
$\gcd(174,27)$	$174=27\cdot 6+14$	6	-2	13
$\gcd(27,14)$	$27=14\cdot 1+13$	1	-1	2
$\gcd(14,13)$	$14=13\cdot 1+1$	1	1	-1
$\gcd(13,1)$	$13=1\cdot 13+0$	13	0	1
$\gcd(1,0)$	$1=0\,0+1$	0	1	0

$au+bv$

$1\cdot u+0\cdot 0=1$

$1-$

$-1-6\cdot 2$

-13

$2-3(-13)$

$=2+39=$

$2 \cdot 2 \cdot 7$

$\varphi(28) = 2 \cdot 6$
$= 12$

$31 \equiv 3 [28]$

$31^{120} \cdot 31^{3}$

$31^{3} \bmod 28$

$3^{3} = 27$

$3 \cdot 41$

$(-3)^{123}$

$\left((-3)^{3}\right)^{41}$

-1

$(-1)^{41} = \boxed{-1}$

$$\cancel{7-9}+4-3+\cancel{2}$$

$$=1$$

Question 2

$$au + bv = 1$$

gcd(a,b)	a = bq + r	q
gcd(70,51)	$70 = 51 \cdot 1 + 19$	1
gcd(51,19)	$51 =$	

$$51 \cdot 5$$
$$- 19 \cdot 8$$

$$\begin{array}{r} 51 \\ - \; 5 \\ \hline 255 \end{array}$$

$$\begin{array}{r} 51 \\ - \; 3 \\ \hline 153 \end{array}$$

$$\begin{array}{r} 7 \\ 19 \\ - \; 8 \\ \hline 152 \end{array}$$

$$[10]_{56}x = [38]_{56}$$

Inverse de 10 mod 56

$$[150]_{56}$$

$$= 38$$

56
112
168

$24x = [4]_{13}$

$24 \equiv 24$
$48 \equiv 3$

1 $24 \equiv 24\ (45)$
2 $24 \cdot 2 \equiv 3\ (45)$
3 $24 \cdot 2 + 24 \equiv 27\ (45)$
4 $\quad '' + 24 \equiv 6\ (45)$
5 $\equiv 30$
6 9
7 33
8 12
9 36
10 15
11 39
18

42

21

0

24

$$24x = [4]_{45}$$

$$24x = 45k + 4$$

$$\boxed{24x \text{ finit } \cancel{\text{par } 9} \text{ ou par } 4}$$

4

Problem 8.1

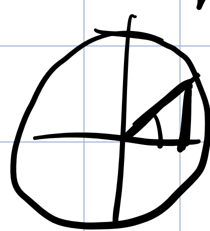
(a) $(\mathbb{Z}, \cdot)$

identity: $[1]$
inverse: does not exist

$(\mathbb{R}^n, +)$

identity:

$\tan\left(\frac{opp}{adj}\right)$ $\frac{y}{x}$



$$\frac{\frac{1}{2}}{\frac{1}{2}}$$

$r\cos\varphi \quad r^2 \sin^2\varphi \quad r^2$

$$0 \leq \frac{y}{x} \leq 1$$

$$0 \leq \varphi \leq \frac{\pi}{4}$$

$$\int_0^{\pi/4}\int_0^2 r^6 \cos\varphi \sin^2\varphi \, dr\, d\varphi$$

$$= \int_0^{\pi/4} \left[\frac{1}{7} r^7 \cos\varphi \sin^2\varphi\right]_{r_0}^{2} d\varphi$$

$$= \int_0^{\pi/4} \frac{2^7}{7} \cos\varphi \sin^2\varphi \, d\varphi$$

$$= \left[\frac{2^7}{7}\frac{1}{3}\sin^3\varphi\right]_0^{\pi/4}$$

$$= \frac{2^7}{21}\left(\frac{\sqrt{2}}{2}\right)^3 \qquad 8\cdot$$

$$= \frac{2^7 \cdot 2\sqrt{2}}{2^3 \cdot 3 \cdot 7} = \frac{2^8 \cdot \sqrt{2}}{2^3}$$

$r(x, y, z) = z - f(\bar{r}_0)$

✓

$(\mathbb{R}^n, +)$ id $= 0$

inv $= -r$

$(\mathbb{R}^n, \cdot)$ no inverse of v

(d)

$$\left(e^{i\theta}\right)^n = 1$$

$$\theta = 2k\pi$$

$$e^{i\theta}$$

	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

$2^4 \quad \boxed{1}$

$\cdot$	1	i	-1	$-i$
1	1	i	-1	$-i$
i	i	-1	$-i$	1
-1	-1	$-i$	1	i
$-i$	$-i$	1	i	-1

$\boxed{i}^4 \quad \boxed{1}$

$$\left(e^{i\theta}\right)^4$$

$$4i\theta = 2k\pi i$$

$$\Leftrightarrow \theta = \frac{1}{2}k\pi$$

b)

	1	S
1	1	S
S	S	1

	1	2
1	1	2
2	2	1

c)

	00	01	10	11	
00	00	01	10	11	1
01	10	00	11	10	2
10	10	11	00	01	2
11	11	10	01	00	2

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

0 1 2 4 7 8 11

0 1 2 3 4 5 6

1

$3^2 \cdot 3$

$3^2(1+10+100+\dots)$

si on multiplie par

$(1+10+100+\dots)\cdot 3$

c'est bon

$$\begin{array}{r} 130 \\ +130 \\ \hline 1160 \\ +160 \\ \hline 320 \end{array}$$

320

333

346

359

372

385

$GCD(a,b)$	$a=bq+r$	q	$u=\tilde{v}$	$v=\tilde{u}-q\tilde{v}$
$GCD(380,13)$	$380=13\cdot 29+3$	29		
$GCD(13,3)$				
$GCD(3,1)$				
$GCD(1,0)$				

$$\varphi(29) = 28$$

$$\boxed{3^{280}} \cdot \boxed{3^{251}}$$

2

(a) $22x = 19 \cdot 63$

$$\begin{array}{r} -63 \\ +10 \\ \hline -53 \\ +9 \\ \hline -44 \end{array}$$

$-40 \cancel{-4}$

$$\begin{array}{r} -40 \\ +100 \\ \hline 60 \end{array} \qquad +32$$

$$\begin{array}{r} 56 \\ +32 \\ \hline 88 \end{array}$$

$$\begin{array}{r} 56 \\ -35 \\ \hline 21 \end{array}$$

$22 \cdot \boxed{4} = 88$

(b) $99x = [21]_{100}$

- $x = [21]_{100}$
- $79 =$

$\boxed{9.2}$

$$\varphi(m) = (p-1)(q-1)$$

a $m = p \cdot q$

$$ed = k\,\varphi(m) + 1$$

$$\Leftrightarrow ed + k\,\varphi(m) = 1$$

$$\gcd(e, \phi(m)) = 1$$

$$\operatorname{lcm}(28, 40)$$

$$2 \cdot 2 \cdot 7 \qquad 2 \cdot 2 \cdot 2 \cdot 5$$

$$\boxed{2^3 \cdot 7 \cdot 5}$$

$\gcd(e,$

000 010 0

1 a

F_8

p^k

0 1 a b ...

$\mathbb{Z}/p\mathbb{Z}$

0 1

$\mathbb{Z}/2\mathbb{Z}$

$+$ 011

+	$\boxed{000}$	010	011
000			
010			

$\{2, 2,$

	0	1	a	b	c

$a+a = a(1+1) = 0$

$1+1=0$

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$

8 $\quad (3^2)^2$

①

$$\begin{pmatrix} 6 & 2 \\ 2 & 5 \\ 5 & 10 \end{pmatrix}$$

$5 \cdot 4 = 20$

$2 \cdot 4 = 8$

$$\begin{pmatrix} 6 & 0 \\ 2 & 0 \\ 0 & 1 \end{pmatrix}$$

30

13

26

$$\rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

2

$3-2=1$

$\boxed{13^2}$

$v_1 = 6a + 2b$

$v_2 = 2a + 8b$

$\boxed{v_3 = 5a + 10b}$

$3 \cdot 5 = 15$

$3v_2 = 6a + 2b$

$3v_2 - v_1 = 0$

$\boxed{v_1 = 3v_2}$

	0	1	x	y
0	0	0	0	0
1	0	1	x	y
x	0	x	y	1
y	0	y	1	x

$$6-3=3$$

$\boxed{11.5}$

$$\frac{n!}{(n-p)!\,p!}$$

1) $(x \pm y)^n$

$$= \sum_{p=0}^{n} \binom{n}{p} x^{n-p} y^p$$

$$\frac{n!}{(n-p)!\,p!}$$ → ça contient n donc ⊙

2) $\boxed{x^{\text{card}} = 1}$

f_g a g éléments mais pas groupe comm. ⊙

$$x^{(\text{card}-1)} = 1$$

$x^q = x$

x^p $\qquad \phi(p) = p-1$

x

	1	a	b
1		$(x)^k$	

$(x)^{pk} = x \Leftrightarrow (x^p)^k = x$

$\Leftrightarrow (x)^k = x$

$\frac{q}{k}$ Comme k doit diviser q

$$x+y$$
$$= x-y$$

$+$	0	1	x	y
0	0	1	x	y
1	1	0	y	x
x	x	y	0	1
y	y	x	1	0

$$x \xrightarrow{+y} 1$$

$$1 \xrightarrow{+y} x$$

$$f(a,b)$$

$$+ \frac{\partial f}{\partial x}(a,b)(x-a)$$

$$+ \frac{\partial f}{\partial y}(a,b)(y-b)$$

$$+ \frac{1}{2}\left(\frac{\partial^2 f}{\partial x^2}(a,b)(x-a)^2 \right.$$

$$+ 2\,\frac{\partial^2 f}{\partial x \partial y}(a,b)(x-a)(y-b)$$

$$\left. + \frac{\partial^2 f}{\partial y^2}(a,b)(y-b)^2 \right)$$

1

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 12 & 4 & 2 & 6 \\ 0 & 1 & 3 & 4 & 10 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 4 & 2 & 6 \\ 0 & 1 & 3 & 4 & 10 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 5 & 3 & 7 \\ 0 & -1 & 4 & 2 & 6 \\ 0 & 1 & 3 & 4 & 10 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 5 & 3 & 7 \\ 0 & 0 & 7 & 6 & 3 \\ 0 & 1 & 3 & 4 & 10 \end{pmatrix} \qquad 16 \equiv 3$$

$$\to \begin{pmatrix} 1 & 0 & 5 & 3 & 7 \\ 0 & 1 & 3 & 4 & 10 \\ 0 & 0 & 7 & 6 & 3 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 5 & 3 & 7 \\ 0 & 1 & 3 & 4 & 10 \\ 0 & 0 & 1 & -1 & 6 \end{pmatrix} \begin{array}{l} \cdot(-1)\cdot 3 \\ \cdot(-1)\cdot 5 \end{array}$$

$$\to \begin{pmatrix} 1 & 0 & 5 & 3 & 7 \\ 0 & 1 & 3 & 4 & 10 \\ 0 & 0 & -3 & 3 & -5 \end{pmatrix}$$

$30 = 4$

-30

-4

$$\to \begin{pmatrix} 1 & 0 & 5 & 3 & 7 \\ 0 & 1 & 0 & 7 & 5 \\ 0 & 0 & -5 & 5 & -4 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 0 & 8 & 3 \\ 0 & 1 & 0 & 7 & 5 \\ 0 & 0 & 1 & 12 & 6 \end{pmatrix}$$

$$\underbrace{}_{I_n} \quad \underbrace{}_{-P}$$

$-P^T$

$$H = \left(\begin{array}{ccc|cc} -8 & -7 & -12 & 1 & 0 \\ -3 & -5 & -6 & 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|cc} 5 & 6 & 1 & 1 & 0 \\ 10 & 8 & 7 & 0 & 1 \end{array}\right)$$

3 col lin indep de H

4 a

5×2

$$\begin{pmatrix} 5 & 10 \\ 6 & 8 \\ & 7 \end{pmatrix}$$

$$1\times 5$$

$$\begin{pmatrix} 1 & 7 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 7 & 11 & 12 & 10 \end{pmatrix}$$

$$1\times 2$$

$$= \begin{pmatrix} 7\cdot 6 + 11 + 12 & 7\cdot 8 + 11\cdot 7 + 10 \end{pmatrix}$$

$$= \begin{pmatrix} 42 - 2 - 1 & 56 + (-2)\cdot 7 - 3 \end{pmatrix}$$

$$= \begin{pmatrix} 39 & 56 - 17 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 39 \end{pmatrix}$$

5×2

$$\begin{pmatrix} 5 & 10 \\ 6 & 8 \\ 1 & 7 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

1×5

$$\begin{pmatrix} 8 & 12 & 11 & 12 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \cdot 5 - 6 - 2 - 1 \\ 10 \cdot 8 - 8 - 2 \cdot 7 + 10 \end{pmatrix}$$

$$= \begin{pmatrix} 45 - 9 & 98 - 14 - 3 \end{pmatrix}$$

$$= (36$$

$5 \quad 10$

$$(0, 7, 8, 12, 10) \begin{pmatrix} 6 & 8 \\ 1 & 7 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= (7 \cdot 6 + 8 - 1$$

$$42 + 7$$

$$= \begin{pmatrix} 49 & \ldots \end{pmatrix}$$

$$(5 \quad 2)$$

b $\binom{5}{1} \cdot 12$

c pa +

d