

Exercise Sheet: Week 5

- (1) Let *Even* be the decision problem $(\mathbb{N}, \{n : n \text{ is even}\})$ and *Odd* be $(\mathbb{N}, \{n : n \text{ is odd}\})$. Give m-reductions between the two problems.
- (2) We remarked that (computable) predicates are (computable) functions. A function $f : \mathbb{N} \rightarrow \mathbb{N}$ is also a predicate: its *characteristic predicate* $\chi_f(x, y)$ is true iff $y = f(x)$. Show that f is computable iff χ_f is computable.
- (3) Suppose that Q_1 and Q_2 are semidecidable queries over the same domain D . Show that $Q_1 \cup Q_2$ and $Q_1 \cap Q_2$ are semidecidable.
- (4) Suppose that L is a decidable language over some alphabet Σ . Show that the language L^* is decidable.
- (5) Following on from question 2: Given a computable predicate $\psi(x, y)$, is it decidable whether ψ is the characteristic predicate of some function f ?