THE UNIVERSITY of EDINBURGH

Computer Security

INFR10067

Fall 2025

Cryptography

Cryptographic hash functions and MACs

Markulf Kohlweiss
School of Informatics
University of Edinburgh

Introduction

Encryption $\Rightarrow$ confidentiality against eavesdropping

What about authenticity and integrity against an active attacker?
$\longrightarrow$ cryptographic hash functions and Message authentication codes
$\longrightarrow$ this lecture

Building block for constructing authenticated encryption schemes, bitcoin proof of work, Merkle trees, protecting passwords, and much more.

One-way functions (OWFs)

A OWF is a function that is easy to compute but hard to invert:

Definition (One-way)

A function f is a one-way function if for all y there is no efficient algorithm which can compute x such that $f(x) = y$

Constant functions ARE NOT OWFs

any x is such that $f(x) = c$

The successor function in $\mathbb{N}$ IS NOT a OWF

given $succ(n)$ it is easy to retrieve $n = succ(n) - 1$

Multiplication of large primes IS a OWF:

integer factorisation is a hard problem - given $p \times q$ (where p and q are primes) it is hard to retrieve p and q

Collision-resistant functions (CRFs)

A function is a CRF if it is hard to find two messages that get mapped to the same value threw this function

Definition (Collision resistance)

A function f is collision resistant if there is no efficient algorithm that can find two messages m_1 and m_2 such that $f(m_1) = f(m_2)$

Constant functions ARE NOT CRFs
for all m_1 and m_2, $f(m_1) = f(m_2)$

The successor function in $\mathbb{N}$ IS a CRF
the predecessor of a positive integer is unique

Multiplication of large primes IS a CRF:
every positive integer has a unique prime factorisation

Cryptographic hash functions

A cryptographic hash function takes messages of arbitrary length and returns a fixed-size bit string such that any change to the data will (with very high probability) change the corresponding hash value.

Definition (Cryptographic hash function)

A cryptographic hash function $H : \mathcal{M} \to \mathcal{T}$ is a function that satisfies the following 4 properties:

- $|\mathcal{M}| >> |\mathcal{T}|$
- it is easy to compute the hash value for any given message
- it is hard to retrieve a message from it hashed value (OWF)
- it is hard to find two different messages with the same hash value (CRF)

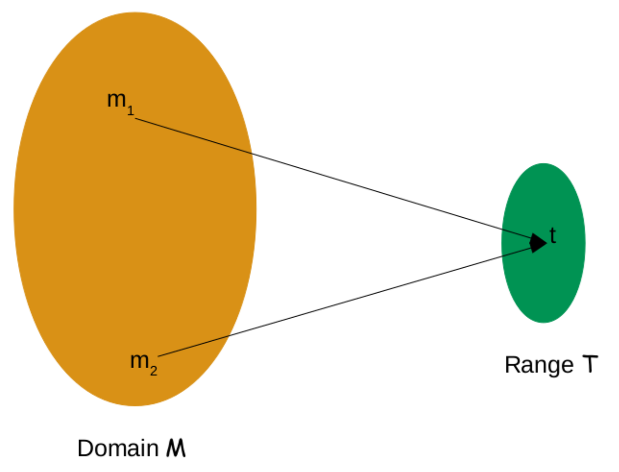
Examples: MD4, MD5, SHA-1, RIPEMD160, SHA-256, SHA-512, SHA-3...

$\rightsquigarrow$In new projects use SHA-256 or SHA-512 or SHA-3

Cryptographic hash functions: applications

- **Commitments -** Allow a participant to commit to a value v by publishing the hash $H(v)$ of this value, but revealing v only later. Ex: electronic voting protocols, digital signatures, ...
- **File integrity -** Hashes are sometimes posted along with files on "read-only" spaces to allow verification of integrity of the files. Ex: SHA-256 is used to authenticate Debian GNU/Linux software packages
- **Password verification -** Instead of storing passwords in cleartext, only the hash digest of each password is stored. To authenticate a user, the password presented by the user is hashed and compared with the stored hash.
- **Key derivation -** Derive new keys or passwords from a single, secure key or password.
- **Building block of other crypto primitives -** Used to build MACs, block ciphers, PRG, ...

Collisions are unavoidable



The domain being much larger than the range, collisions necessarily exist

The birthday attack - attack on all schemes

Theorem

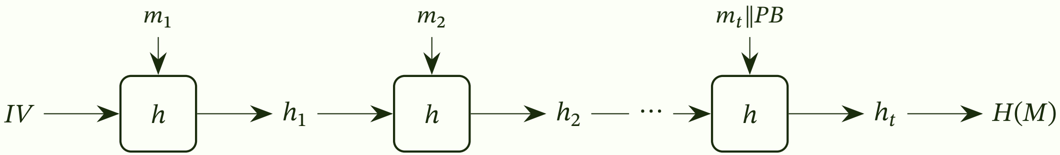
Let $H : \mathcal{M} \to \{0,1\}^n$ *be a cryptographic hash function (*$|\mathcal{M}| >> 2^n$*)*
Generic algorithm to find a collision in time $O(2^{n/2})$ *hashes:*

1. *Choose* $2^{n/2}$ *random messages in* $\mathcal{M}$*:* $m_1, \dots, m_{2^{n/2}}$
2. *For* $i = 1, \dots, 2^{n/2}$ *compute* $t_i = H(m_i)$
3. *If there exists a collision (*$\exists i, j.\ t_i = t_j$*)*
   then return (m_i, m_j)
   else go back to 1

Birthday paradox Let $r_1, \dots, r_\ell \in \{1, \dots, N\}$ be independent variables. For $\ell = 1.2 \times \sqrt{N}$,
$Pr(\exists i \neq j.\ r_i = r_j) \geq \frac{1}{2}$

$\Rightarrow$ the expected number of iteration is 2
$\Rightarrow$ running time $O(2^{n/2})$

The Merkle-Damgard construction



Merkle–Damgård construction

- Compression function: $h : \mathcal{T} \times \mathcal{X} \to \mathcal{T}$
- PB: $1000 \ldots 0||$mes-len (add extra block if needed) (different variants!)

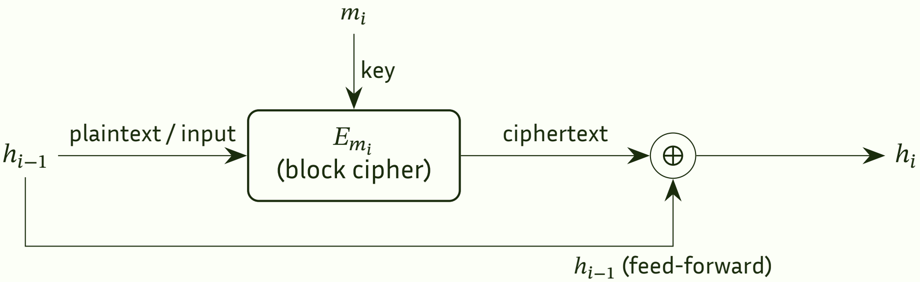
Theorem

Let H be built using the MD construction to the compression function h. If H admits a collision, so does h.

Example of MD constructions: MD5, SHA-1, SHA-2, ...

Compression functions from block ciphers

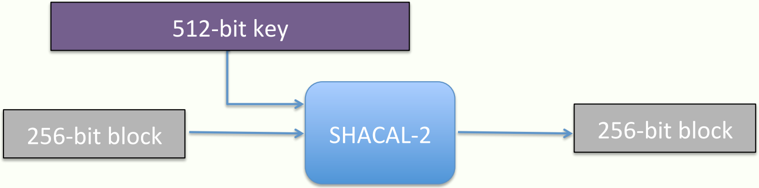
Let $E : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$ be a block cipher



Davies–Meyer: $h_i = E_{m_i}(h_{i-1}) \oplus h_{i-1}$

Example of cryptographic hash function: SHA-256

- Structure: Merkle-Damgard
- Compression function: Davies-Meyer
- Bloc cipher: SHACAL-2



THE UNIVERSITY of EDINBURGH

Computer Security

INFR10067

Fall 2025

Cryptography

Message Authentication Codes (MACs)

Encryption is not always enough

e=E(K_E,Transfer 100 € on Bob's account) $\longrightarrow$

Royal Bank of Scotland

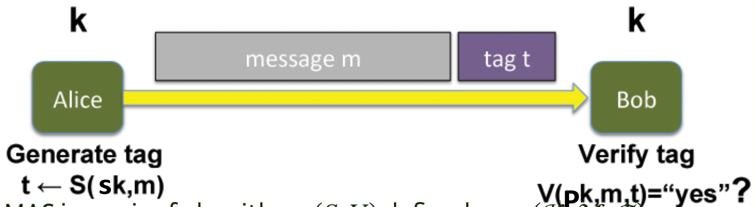
What if the encryption scheme E is the OTP - $e = K_E \oplus$ Transfer 100 € on Bob's account?

$\xrightarrow{e}$

$e\oplus$0...0Bob0...0$\oplus$0...0Eve0...0 $\longrightarrow$ =E(K_E,Transfer 100 € on Eve's account)

Royal Bank of Scotland

Goal: message integrity



A MAC is a pair of algorithms (S, V) defined over $(\mathcal{K}, \mathcal{M}, \mathcal{T})$:

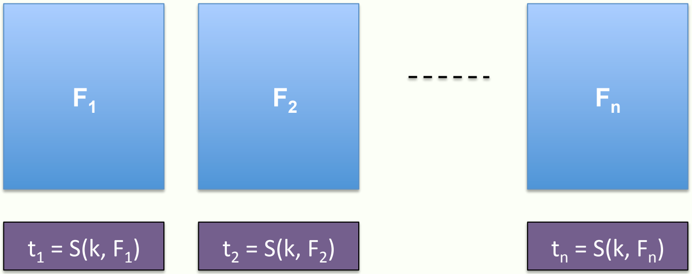
- $S : \mathcal{K} \times \mathcal{M} \to \mathcal{T}$
- $V : \mathcal{K} \times \mathcal{M} \times \mathcal{T} \to \{\top, \bot\}$
- Consistency: $V(k, m, S(k, m)) = T$

Unforgeability

It is hard to computer a valid pair $(m, S(k, m))$ without knowing k

File system protection

- At installation time



k derived from user password

- To check for virus file tampering/alteration:
  - reboot to clean OS
  - supply password
  - any file modification will be detected

Block ciphers and message integrity

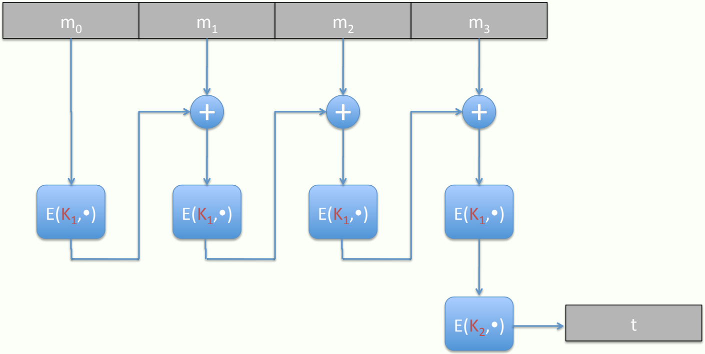
Let (E, D) be a block cipher. We build a MAC (S, V) using (E, D) as follows:

- $S(k, m) = E(k, m)$
- $V(k, m, t) =$ if $m = D(k, t)$
  then return $\top$
  else return $\bot$

But: block ciphers can usually process only 128 or 256 bits

Our goal now: construct MACs for long messages

ECBC-MAC

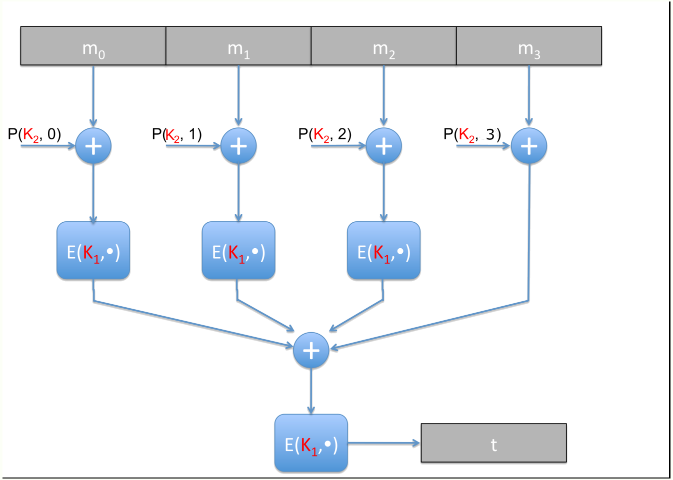


- $E \, : \, \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$ a block cipher
- $ECBC\text{-}MAC \, : \, \mathcal{K}^2 \times \{0,1\}^* \to \{0,1\}^n$

$\to$ the last encryption is crucial to avoid forgeries!!

Ex: 802.11i uses AES based ECBC-MAC

PMAC



- $E: \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$ a block cipher
- $P: \mathcal{K} \times \mathbb{N} \to \{0,1\}^n$ an easy to compute function
- $PMAC: \mathcal{K}^2 \times \{0,1\}^* \to \{0,1\}^n$

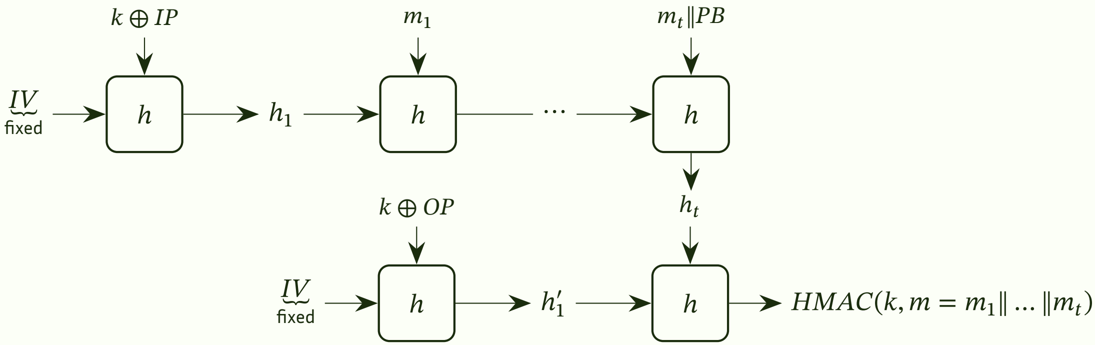
See also: https://www.youtube.com/watch?v=gZiBYDX9Fpo

HMAC

MAC built from cryptographic hash functions

$$HMAC(k,m) = H(k \oplus OP || H(k \oplus IP || m))$$

IP, OP: publicly known padding constants



HMAC construction

Naive MAC from Hashing

Simplest way to build a MAC from a hash function, prepend the key:

$$MAC(k, m) = H(k \| m)$$

This is not generally secure, but works for SHA3/Keccak as it prevents length extension attack.

Source: https://keccak.team/keccak_strengths.html

See also: https://crypto.stackexchange.com/questions/1070/why-is-hk-mathbin-vert-x-not-a-secure-mac-construction

Computer Security

INFR10067

Fall 2025

Cryptography

Authenticated encryption

Plain encryption is malleable

- The decryption algorithm never fails
- Changing one bit of the i^{th} block of the ciphertext
  - CBC decryption: will affect last blocks after the i^{th} of the plaintext
  - ECB decryption: will only the i^{th} block of the plaintext
  - CTR decryption: will only affect one bit of the i^{th} block of the plaintext

Decryption should fail if a ciphertext was not computed using the key

Goal

Simultaneously provide data confidentiality, integrity and authenticity
$\rightsquigarrow$ decryption combined with integrity verification in one step

Encrypt-then-MAC

1. Always compute the MACs on the ciphertext, never on the plaintext
2. Use two different keys, one for encryption (K_E) and one for the MAC (K_M)

Encryption

1. $C \leftarrow E_{AES}(K_E, M)$
2. $T \leftarrow HMAC\text{-}SHA(K_M, C)$
3. return $C||T$

Decryption

1. if $T = HMAC\text{-}SHA(K_M, C)$
2. then return $D_{AES}(K_E, C)$
3. else return $\perp$

Do not:

- Encrypt-and-MAC: $E_{AES}(K_E, M)||HMAC\text{-}SHA(K_M, M)$
- MAC-then-Encrypt: $E_{AES}(K_E, M||HMAC\text{-}SHA(K_M, M))$

AES GCM

Galois Counter Mode

Combines

1. Galois field based One-time MAC for authentication
2. AES based Counter Mode for encryption

- Trick: One-time MAC is encrypted too
  ⇒ secure for many messages
- Widely adopted for its performance
- Many good implementations of this mode