## Introduction to Theoretical Computer Science

## Exercise Sheet: Week 3

Here are a few routine exercises.

- 1. Consider the language  $L = \{ww \mid w \in \{a, b\}^*\}$ . Show that it is not regular.
- 2. Is the language  $L' = \{ww \mid w \in \{a, b\}^* \land |w| < 4\}$  regular? Why or why not?
- 3. We saw in lectures that the language L is not context-free. Show that, by contrast, its complement is context-free.

The following questions/comments are intended as prompts for discussion. Of course, you can ask/discuss about anything. Some of these topics we've touched on in discussion in lectures – this is an opportunity to think about them a bit more.

- 4. Suppose we augmented a finite automaton with an additional feature: a single mutable variable. On each transition, it may read from or write to this variable.
  - a) If the variable is of type  $\Sigma$  (that is, it can contain one alphabet symbol), what class of languages can such automata recognise?
  - b) If the variable is of type N, what class of languages can they recognise?
- 5. A queue automaton is like a pushdown automaton, but with a queue (fifo) instead of a stack (lifo). These can recognise any Turing-recognisable language. Now suppose we instead defined a pushdown automaton with an additional stack. What class of languages can these 2-stack PDAs recognise?