Sequential Pattern and Sequential Pattern Mining

Sequence Databases & Sequential Patterns

- Sequential pattern mining has broad applications
  - Customer shopping sequences
    - Purchase a laptop first, then a digital camera, and then a smartphone, within 6 months
  - Medical treatments, natural disasters (e.g., earthquakes), science & engineering processes, stocks and markets, ...
  - Weblog click streams, calling patterns, …
  - Software engineering: Program execution sequences, …
  - Biological sequences: DNA, protein, …
- Transaction DB, sequence DB vs. time-series DB
- Gapped vs. non-gapped sequential patterns
  - Shopping sequences, clicking streams vs. biological sequences

Sequential Pattern and Sequential Pattern Mining

- Sequential pattern mining: Given a set of sequences, find the complete set of *frequent* subsequences (i.e., satisfying the min_sup threshold)

A *sequence database*

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

A *sequence:* < (ef) (ab) (df) c b >

- An element may contain a set of *items* (also called *events*)
- Items within an element are unordered and we list them alphabetically

<a(bc)dc> is a *subsequence* of <a(abc)(ac)d(cf)>

- Given *support threshold* *min_sup* = 2, <(ab)c> is a *sequential pattern*

Sequential Pattern Mining Algorithms

- Algorithm requirement: Efficient, scalable, finding complete set, incorporating various kinds of user-specific constraints
- The Apriori property still holds: If a subsequence s_1 is infrequent, none of s_1's super-sequences can be frequent
- Representative algorithms
  - GSP (Generalized Sequential Patterns): Srikant & Agrawal @ EDBT'96)
  - Vertical format-based mining: SPADE (Zaki@Machine Leanining'00)
  - Pattern-growth methods: PrefixSpan (Pei, et al. @TKDE'04)
- Mining closed sequential patterns: CloSpan (Yan, et al. @SDM'03)
- Constraint-based sequential pattern mining

GSP: Apriori-Based Sequential Pattern Mining

GSP: Apriori-Based Sequential Pattern Mining

- Initial candidates: All singleton sequences
  - <a>, <b>, <c>, <d>, <e>, <f>, <g>, <h>
- Scan DB once, count support for each candidate
- Generate length-2 candidate sequences

min_sup = 2

Cand.	sup
<a>	3
<b>	5
<c>	4
<d>	3
<e>	3
<f>	2
~~<g>~~	1
~~<h>~~	1

	<a>	<b>	<c>	<d>	<e>	<f>
<a>	<aa>	<ab>	<ac>	<ad>	<ae>	<af>
<b>	<ba>	<bb>	<bc>	<bd>	<be>	<bf>
<c>	<ca>	<cb>	<cc>	<cd>	<ce>	<cf>
<d>	<da>	<db>	<dc>	<dd>	<de>	<df>
<e>	<ea>	<eb>	<ec>	<ed>	<ee>	<ef>
<f>	<fa>	<fb>	<fc>	<fd>	<fe>	<ff>

	<a>	<b>	<c>	<d>	<e>	<f>
<a>		<(ab)>	<(ac)>	<(ad)>	<(ae)>	<(af)>
<b>			<(bc)>	<(bd)>	<(be)>	<(bf)>
<c>				<(cd)>	<(ce)>	<(cf)>
<d>					<(de)>	<(df)>
<e>						<(ef)>
<f>						

SID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)>

- Length-2 candidates: 36 + 15= 51
- Without Apriori pruning: 8*8+8*7/2=92 candidates

GSP (Generalized Sequential Patterns): Srikant & Agrawal @ EDBT'96)

GSP Mining and Pruning

5th scan: 1 cand. 1 length-5 seq. pat.

4th scan: 8 cand. 7 length-4 seq. pat.

3rd scan: 46 cand. 20 length-3 seq. pat. 20 cand. not in DB at all

2nd scan: 51 cand. 19 length-2 seq. pat. 10 cand. not in DB at all

1st scan: 8 cand. 6 length-1 seq. pat.

<(bd)cba>

<abba> <(bd)bc> …

<abb> <aab> <aba> <baa> <bab> …

<aa> <ab> … <af> <ba> <bb> … <ff> <(ab)> … <(ef)>

<a> <b> <c> <d> <e> <f> <g> <h>

Candidates cannot pass min_sup threshold

Candidates not in DB

min_sup = 2

SID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)>

- Repeat (for each level (i.e., length-k))
  - Scan DB to find length-k frequent sequences
  - Generate length-(k+1) candidate sequences from length-k frequent sequences using Apriori
  - set k = k+1
- Until no frequent sequence or no candidate can be found

SPADE: Sequential Pattern Mining in Vertical Data Format

Sequential Pattern Mining in Vertical Data Format: The SPADE Algorithm

- A sequence database is mapped to: <SID, EID>
- Grow the subsequences (patterns) one item at a time by Apriori candidate generation

SID	Sequence
1	<a(abc)(ac)d(cf)>
2	<(ad)c(bc)(ae)>
3	<(ef)(ab)(df)cb>
4	<eg(af)cbc>

min_sup = 2

Ref: SPADE (Sequential PAttern Discovery using Equivalent Class) [M. Zaki 2001]

SID	EID	Items
1	1	a
1	2	abc
1	3	ac
1	4	d
1	5	cf
2	1	ad
2	2	c
2	3	bc
2	4	ae
3	1	ef
3	2	ab
3	3	df
3	4	c
3	5	b
4	1	e
4	2	g
4	3	af
4	4	c
4	5	b
4	6	c

a		b		...
SID	EID	SID	EID	...
1	1	1	2	
1	2	2	3	
1	3	3	2	
2	1	3	5	
2	4	4	5	
3	2			
4	3			

ab			ba			...
SID	EID (a)	EID(b)	SID	EID (b)	EID(a)	...
1	1	2	1	2	3	
2	1	3	2	3	4	
3	2	5				
4	3	5				

aba			...
SID	EID (a)	EID(b)	EID(a)	...
1	1	2	3	
2	1	3	4	

PrefixSpan: Sequential Pattern Mining by Pattern-Growth

PrefixSpan: A Pattern-Growth Approach

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

Prefix	Suffix (Projection)
<a>	<(abc)(ac)d(cf)>
<aa>	<(_bc)(ac)d(cf)>
<ab>	<(_c)(ac)d(cf)>

- Prefix and suffix
  - Given <a(abc)(ac)d(cf)>
  - Prefixes: <a>, <aa>, <a(ab)>, <a(abc)>, …
  - Suffix: Prefixes-based projection

- PrefixSpan Mining: Prefix Projections
  - Step 1: Find length-1 sequential patterns
    - <a>, <b>, <c>, <d>, <e>, <f>
  - Step 2: Divide search space and mine each projected DB
    - <a>-projected DB,
    - <b>-projected DB,
    - …
    - <f>-projected DB, …

PrefixSpan (Prefix-projected Sequential pattern mining) Pei, et al. @TKDE'04

PrefixSpan: Mining Prefix-Projected DBs

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

Length-1 sequential patterns
<a>, <b>, <c>, <d>, <e>, <f>

prefix <a>

<a>-projected DB

<(abc)(ac)d(cf)>
<(_d)c(bc)(ae)>
<(_b)(df)cb>
<(_f)cbc>

Length-2 sequential patterns
<aa>, <ab>, <(ab)>, <ac>, <ad>, <af>

prefix <b>

<b>-projected DB

prefix <c>, …, <f>

…

… …

prefix <aa>

<aa>-projected DB

..

prefix <af>

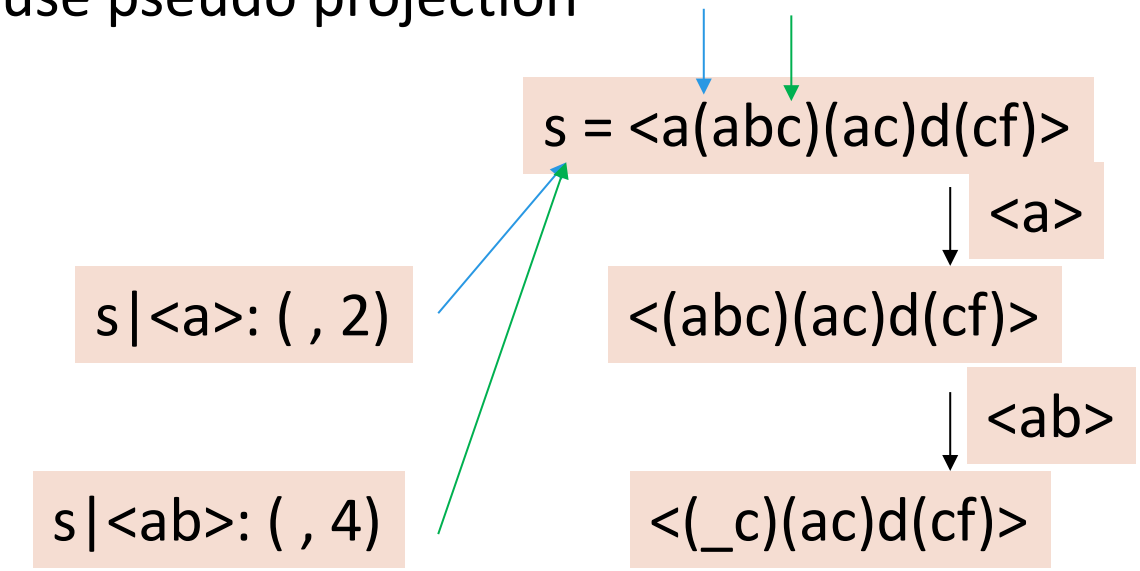
<af>-projected DB

Major strength of PrefixSpan:

- No candidate subseqs. to be generated
- Projected DBs keep shrinking

Implementation Consideration: Pseudo-Projection vs. Physical Projection

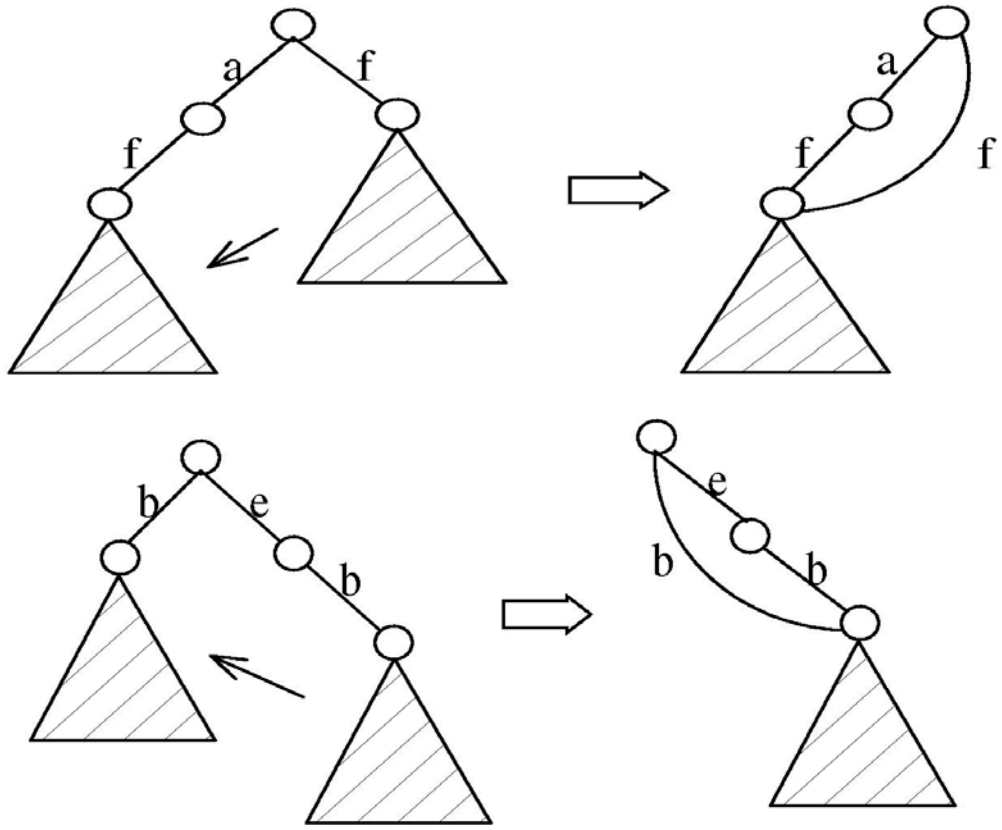
- Major cost of PrefixSpan: Constructing projected DBs
  - Suffixes largely repeating in recursive projected DBs
- When DB can be held in main memory, use pseudo projection
  - No physically copying suffixes
  - Pointer to the sequence
  - Offset of the suffix
- But if it does not fit in memory
  - Physical projection
- Suggested approach:
  - Integration of physical and pseudo-projection
  - Swapping to pseudo-projection when the data fits in memory



CloSpan: Mining Closed Sequential Patterns

CloSpan: Mining Closed Sequential Patterns

- A closed sequential pattern *s*: There exists no superpattern *s'* such that *s'* ⊃ *s*, and *s'* and *s* have the same support
- Which ones are closed? <abc>: 20, <abcd>:20, <abcde>: 15
- Why directly mine closed sequential patterns?
  - Reduce # of (redundant) patterns
  - Attain the same expressive power
- Property P_1: If $s \supset s_1$, s is closed iff two project DBs have the same size
- Explore *Backward Subpattern* and *Backward Superpattern* pruning to prune redundant search space
- Greatly enhances efficiency (Yan, et al., SDM'03)



Summary

Summary: Sequential Pattern Mining

- Concepts of Sequential Pattern Mining
- Sequential Pattern Mining Algorithms
  - GSP (Generalized Sequential Patterns)
  - Vertical Format-Based Mining: SPADE
  - Pattern-Growth Methods: PrefixSpan
- Mining Closed Sequential Patterns: CloSpan

Recommended Readings

- R. Srikant and R. Agrawal, “Mining sequential patterns: Generalizations and performance improvements”, EDBT’96
- M. Zaki, “SPADE: An Efficient Algorithm for Mining Frequent Sequences”, Machine Learning, 2001
- J. Pei, J. Han, B. Mortazavi-Asl, J. Wang, H. Pinto, Q. Chen, U. Dayal, and M.-C. Hsu, "Mining Sequential Patterns by Pattern-Growth: The PrefixSpan Approach", IEEE TKDE, 16(10), 2004
- X. Yan, J. Han, and R. Afshar, “CloSpan: Mining Closed Sequential Patterns in Large Datasets”, SDM'03