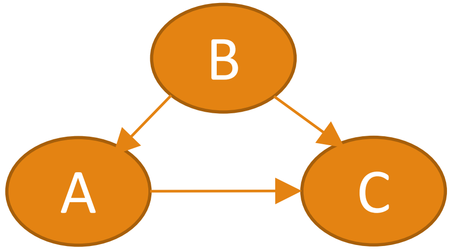
Bayesian Networks

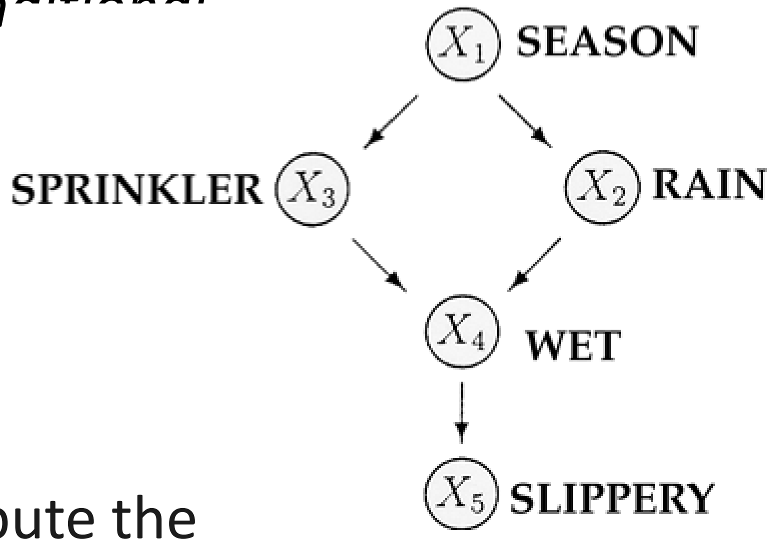
From Naïve Bayes to Bayesian Network

- A naïve Bayes classifier assumes that the value of a particular feature is independent of the value of any other feature, given the class variable
  - This assumption is often too simple to model the real world well
- Bayesian network (or Bayes network, belief network, Bayesian model, or probabilistic directed acyclic graphical model) is a probabilistic **graphical model**
  - Represented by a set of *random variables* and *their conditional dependencies* via a *directed acyclic graph* (DAG)



$p(A,B,C) = p(B) \cdot p(A|B) \cdot p(C|A,B)$

directed **acyclic** graphical model

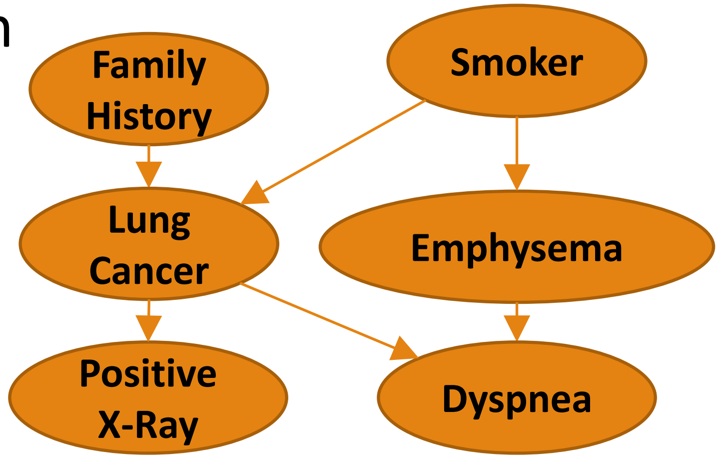


  - Ex. Given symptoms, the network can be used to compute the probabilities of the presence of various diseases

Bayesian Network

- **Bayesian network** (or **Bayesian belief network, probabilistic network**):
  - Allows *class conditional independencies* between *subsets* of variables
  - Represents the joint distribution compactly in a factorized way
  - Can be described by a generative process
- Two components:
  - A *directed acyclic graph* (called a structure)
    - The **nodes** represent random variables, observed or hidden
    - The **edges** represent direct dependency between variables
  - A set of *conditional probability tables* (CPTs)
    - CPTs are attached to the nodes

Nodes: random variables   Links: dependency



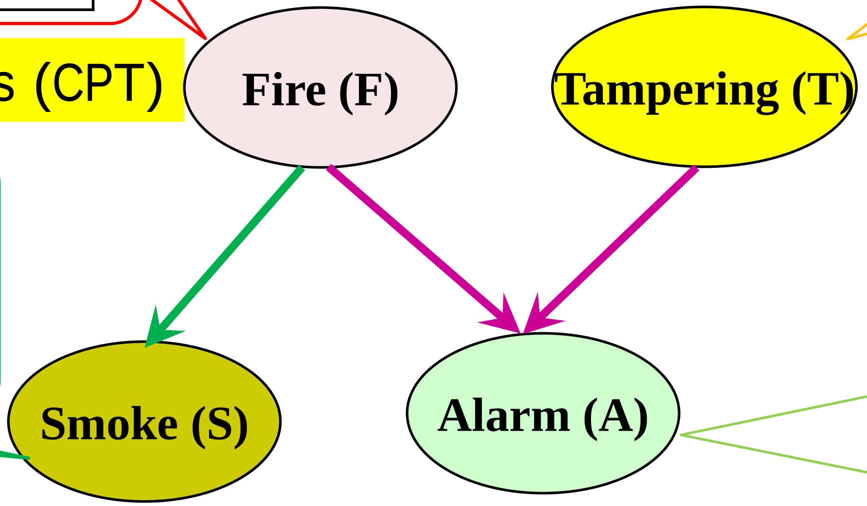
	FH, S	*FH, ~S*	*~FH, S*	*~FH, ~S*
LC	0.8	0.5	0.7	0.1
~LC	0.2	0.5	0.3	0.9

Representing Joint Distribution with Bayesian Network

Conditional **P**robability **T**ables (CPT)

Fire=1	Fire=0
0.02	0.98

Tampering=1	Tampering=0
0.1	0.9



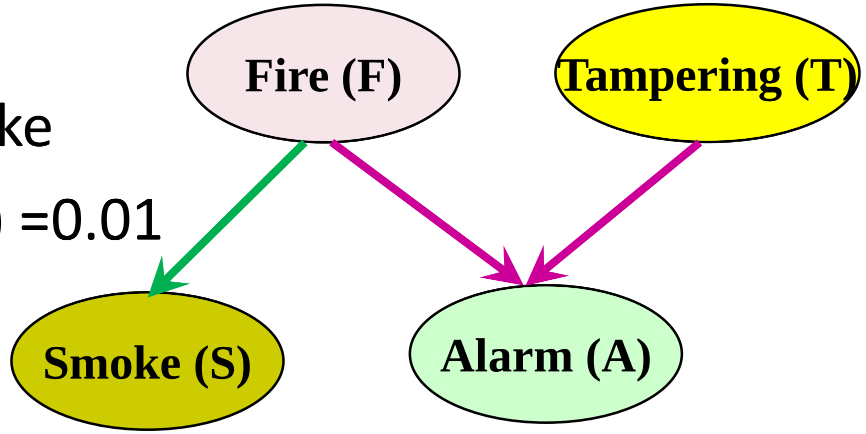
	Smoke=1	Smoke=0
Fire=1	0.9	0.1
Fire=0	0.01	0.99

	Alarm=1	Alarm=0
F=1,T=1	0.5	0.5
F=1,T=0	0.99	0.01
F=0,T=1	0.85	0.15
F=0,T=0	0.0001	0.9999

From the Bayesian network we can read the factorization of the joint distribution: $p(F, S, A, T) = p(F) \cdot p(T) \cdot p(S|F) \cdot p(A|F, T)$

In general, we have the chain rule for Bayesian networks: $p(X) = \prod_k p(x_k|\mathrm{Parents}(x_k))$

Bayesian Network: An Example

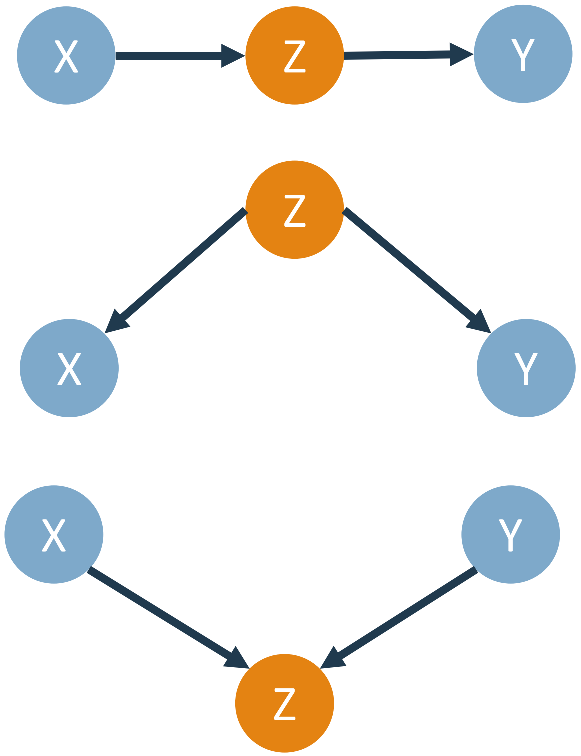


- Causal Reasoning:
  - The value of variable Fire influences variable Smoke
    - If F=1 → p(S=1|F=1) =0.9; if F=0 → p(S=1|F=0) =0.01
- Evidential Reasoning:
  - The value of variable Smoke also influences Fire
    - If S=1 → p(F=1|S=1) = $\frac{p(S=1|F=1)*p(F=1)}{p(S=1)}$ = 0.647; if S=0, p(F=1|S=0)=0.002
- Intercausal Reasoning:
  - The value of variable Fire does not influence Tampering
    - p(T|F) = p(T), F and T are independent
  - However, observing Alarm makes Fire and Tampering coupled
    - p(T|F, A) = $\frac{p(T,A,F)}{p(A|F)p(F)} = \frac{p(A|F,T)p(F)p(T)}{p(F)\sum_T p(A,T|F)} = \frac{p(A|F,T)p(T)}{\sum_T p(A|F,T)p(T)}$
    - p(T=1|F=1, A=1) = 0.053; p(T=1|F=0, A=1) ≈ 1.000

Dependencies in Bayesian Network

- Markovian assumption
  - Each variable becomes independent of its non-effects once its direct causes are known

- Consider the local structure of three variables
  - Cascade
    - Given the middle node Z, X, and Y are independent
  - Common parent
    - Given the parent node Z, X, and Y are independent
  - V-structure (common child):
    - Given the child node Z, X, and Y are coupled
      - Also called X explains away Y w.r.t. Z



Three Major Tasks on Bayesian Network

- **Representation (Construction)**
  - How can we construct a Bayesian network so that the probability distribution reflects real-world phenomenon?
- **Inference**
  - Given a Bayesian network, how can we answer questions about events?
  - Marginal inference: What is the probability of a variable after summing all other variables?
  - MAP inference: What is the most likely assignment to the unobserved variables?
- **Learning (Training)**
  - How can we fit a Bayesian network to data?
  - In some cases, the network structure needs to be learned as well

How Are Bayesian Networks Constructed?

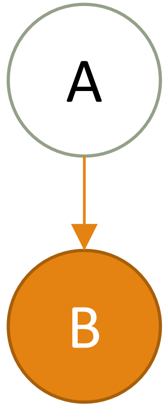
- **Subjective construction**
  - Human identification of (direct) causal structure
  - People are quite good at identifying direct causes from a given set of variables & whether the set contains all relevant direct causes
- **Synthesis from other specifications**
  - E.g., from a formal system design: Block diagrams & information flow
- **Learning from data** (e.g., from medical records or student admission records)
  - Learn parameters given its structure or learn both structure and parameters
  - Maximum likelihood principle: Favors Bayesian networks that maximize the probability of observing the given data set

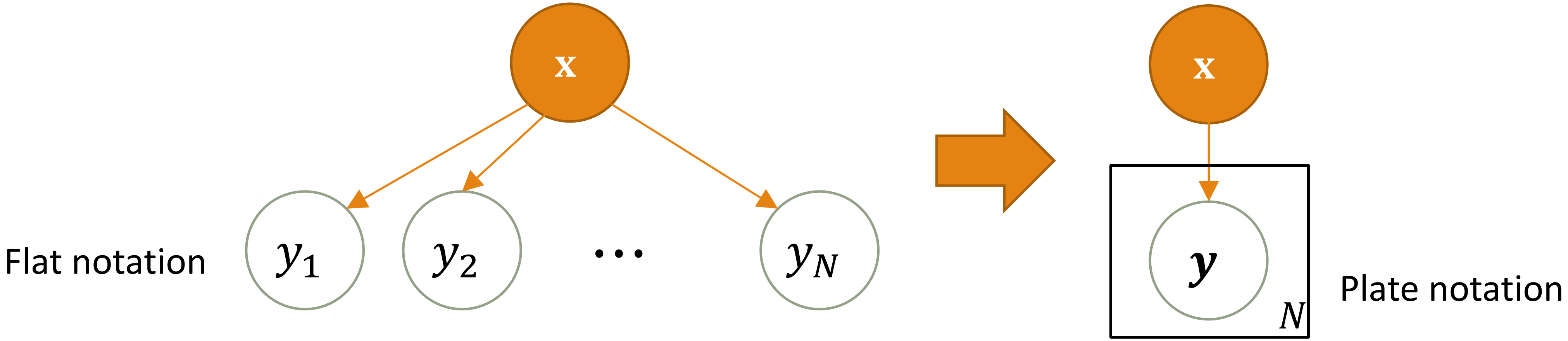
Training Bayesian Networks: Several Scenarios

- Scenario 1: Given both the network structure and all variables observable
  - *Compute only the CPT entries*
- Scenario 2: Network structure known, some variables hidden
  - Use *gradient descent* (a greedy hill-climbing method), i.e., search for a solution along the steepest descent of a criterion function
- Scenario 3: Network structure unknown, all variables observable
  - Search through the model space to *reconstruct network topology*
- Scenario 4: Unknown structure, all hidden variables
  - No good algorithms known for this purpose
- D. Heckerman. A Tutorial on Learning with Bayesian Networks. In *Learning in Graphical Models,* M. Jordan, ed. MIT Press, 1999

Probabilistic Graphic Model: Plate Notation

- How to represent variables and their dependencies concisely in a graphical model?
- Node (or **circle**) representing variables
  - A solid (or shaded) circle means the corresponding variable is *observed*; otherwise it is *hidden*
- **Directed edge** representing dependency among variables:
  - A Directed Acyclic Graphical (DAG) model
- Using plate notation instead of flat notation
  - The variables in the same plate share the same **conditional probability table**





An Example of Plate Notation

Flat notation

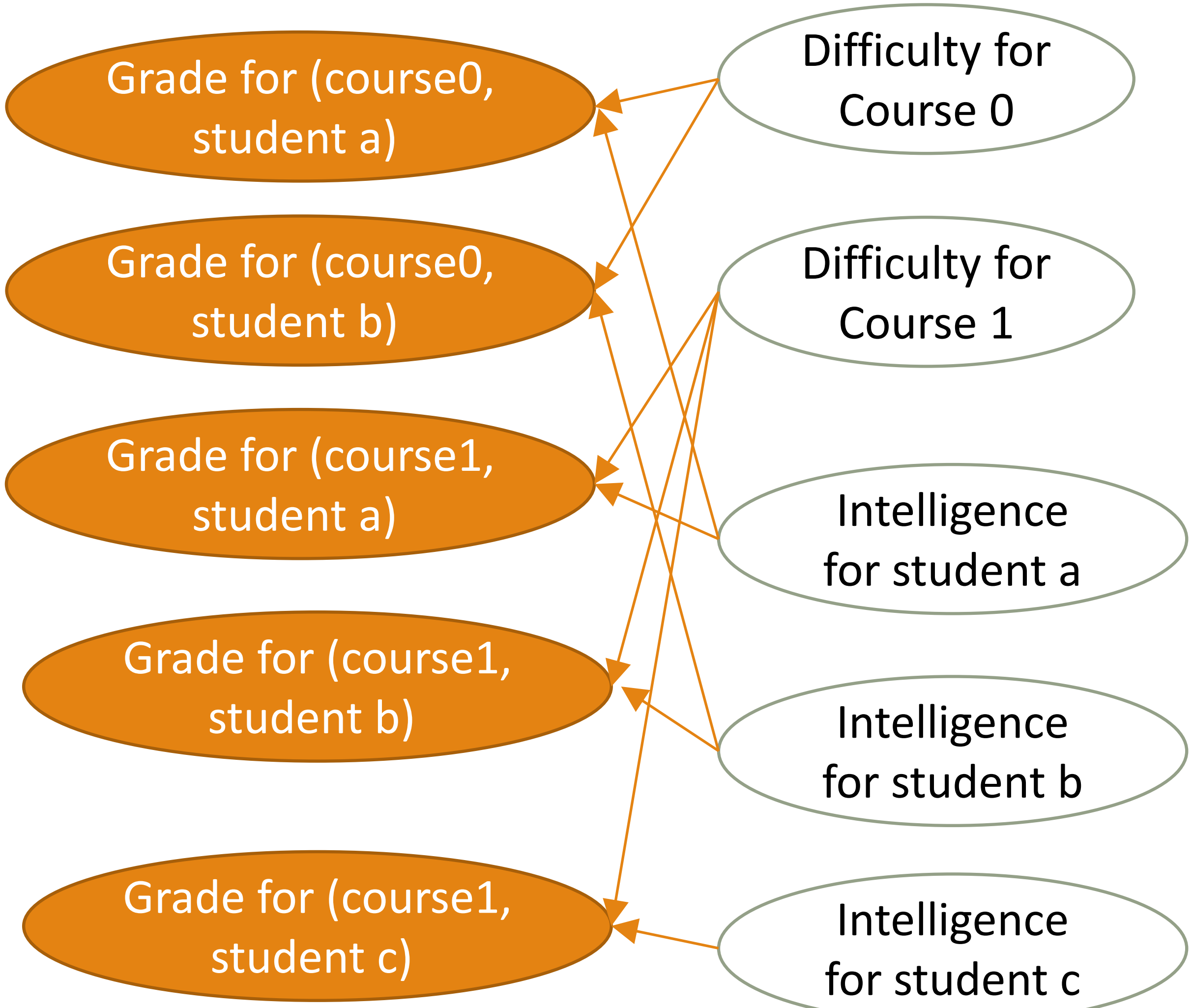
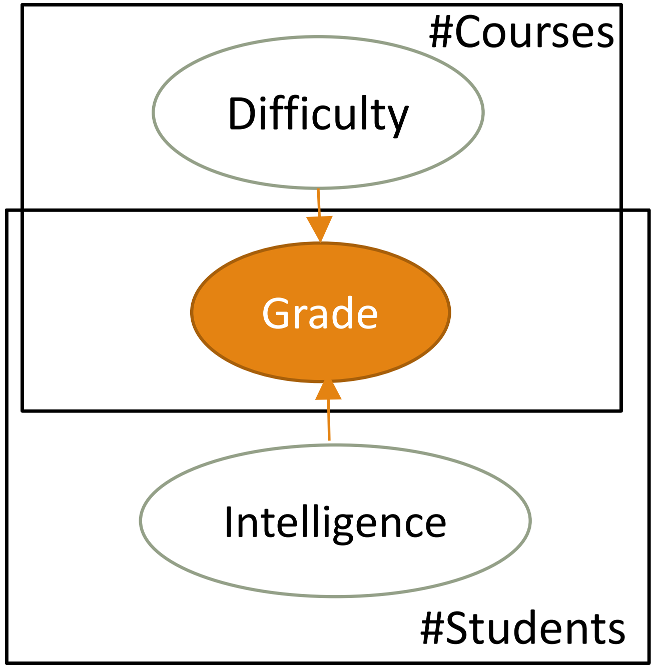


Plate notation



Summary

Summary

- Bayes Classifier

- Bayesian Networks

Recommended Readings

- Heckerman, D. (1999). A tutorial on learning with Bayesian networks. *Learning in Graphical Models, M. Jordan ed.* Retrieved from https://www.microsoft.com/en-us/research/publication/a-tutorial-on-learning-with-bayesian-networks/?from=http%3A%2F%2Fresearch.microsoft.com%2Fen-us%2Fum%2Fpeople%2Fheckerman%2Ftutorial.pdf
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References

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Appendix: Detailed Computation of Some Probabilities on Slide #16 “Bayesian Network: An Example”

$$
\begin{aligned}
s(F=1|S=1) &= \frac{p(S=1,F=1)}{p(S=1)} \\
&= \frac{p(S=1|F=1)\times p(F=1)}{\sum_f p(S=1,F)} \\
&= \frac{p(S=1|F=1)\times p(F=1)}{p(S=1|F=1)p(F=1)+p(S=1|F=0)p(F=0)} \\
&= \frac{0.9\times 0.02}{0.9\times 0.02+0.01\times 0.98} \\
&= 0.647
\end{aligned}
$$

$$
\begin{aligned}
p(T=1|F=1,A=1) &= \frac{p(T=1,F=1,A=1)}{p(A=1|F=1)p(F=1)} \\
&= \frac{p(A=1|F=1,T=1)p(F=1)p(T=1)}{p(A=1|F=1)p(F=1)} \\
&= \frac{p(A=1|F=1,T=1)p(T=1)}{\sum_T p(A=1,T|F=1)} \\
&= \frac{0.5\times 0.1}{0.5\times 0.1+0.99\times 0.9} \\
&= 0.053
\end{aligned}
$$

$$
\begin{aligned}
p(T=1|F=0,A=1) &= \frac{p(A=1|T=1,F=0)p(T=1)}{\sum_T p(A=1,T|F=0)} \\
&= \frac{0.85\times 0.1}{0.85\times 0.1+0.001\times 0.9} \\
&= 0.9989 \sim 1.000
\end{aligned}
$$