ADVANCED BAYESIAN MODELING

USING MCMC IN PRACTICE:
AUTOCORRELATION AND MIXING

Autocorrelation

Consider a time-invariant Markov chain that has reached stationarity.

Let ψ be a scalar function on the chain's state space (e.g., an element of θ) that has a (posterior) variance.

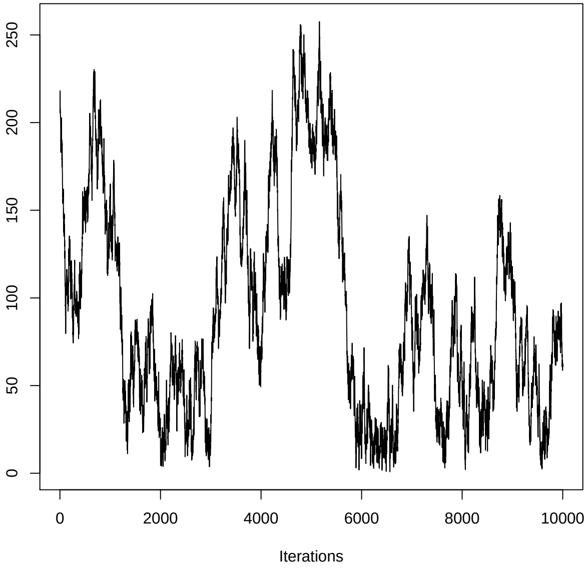
The successive values of ψ form a stationary *time series*.

For ψ, the **autocorrelation at lag** t is the correlation between two samples of ψ that are t iterations apart:

$$\rho_t \quad = \quad \frac{\mathrm{cov}(\psi_i, \psi_{i+t})}{\sqrt{\mathrm{var}(\psi_i)\,\mathrm{var}(\psi_{i+t})}}$$

A chain with high autocorrelations often moves through the sampling region very slowly. (This is true even though the chain has reached stationarity.)

For example, a trace plot might look like ...


0
50
100
150
200
250
0
2000
4000
6000
8000
10000
Iterations

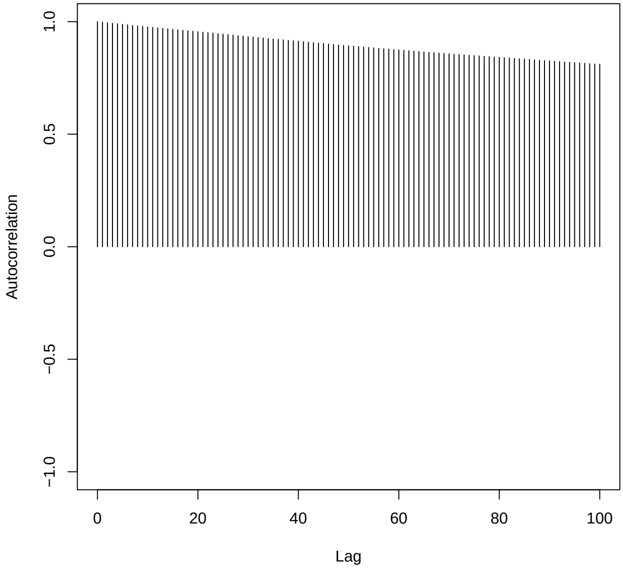
Autocorrelation Function

The **autocorrelation function** for ψ is its autocorrelation ρ_t versus $t = 0, 1, 2, \ldots$

It is often estimated by estimating ρ_t using the sample correlation of all pairs (ψ_i, ψ_{i+t}) available from the chain.

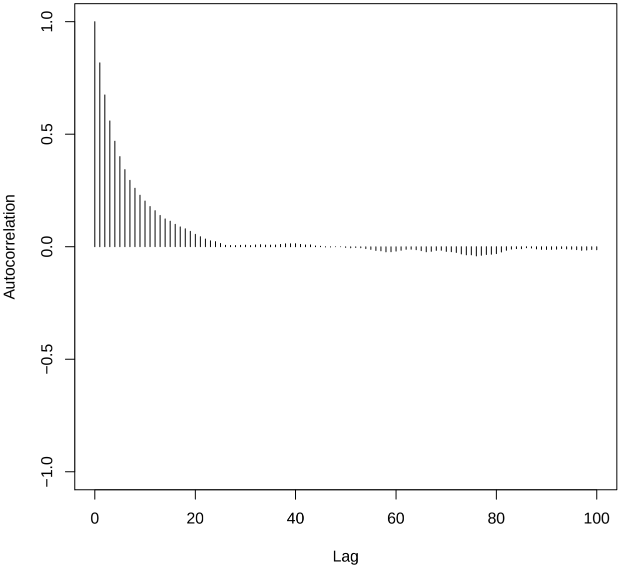
The estimated autocorrelation function can then be graphed.

For example, for the sequence from the trace plot shown earlier ...


1.0
0.5
0.0
−0.5
−1.0
Autocorrelation
0
20
40
60
80
100
Lag

That showed extremely high autocorrelations: They decayed very slowly with t.

We prefer autocorrelations that decay more quickly ...



1.0
0.5
0.0
−0.5
−1.0
Autocorrelation
0
20
40
60
80
100
Lag

Mixing

Mixing refers to the rate of decay of dependence between states of a chain that are t iterations apart, as $t \to \infty$.

The mixing is **slow** if the dependence is high even at long lags. Generally, this means at least some autocorrelations will decay slowly.

The mixing is **fast** if the dependence goes away quickly. This means that autocorrelations will decay quickly to zero as lag increases.

For Bayesian purposes, we prefer chains with fast mixing, since fewer iterations are required for a given accuracy (later).