ADVANCED BAYESIAN MODELING

ROBUST INFERENCE:
MODELING WITH THE *t* DISTRIBUTION

When the ideal model would be a normal distribution, but the data have outliers, a (location-scale) t distribution is a viable alternative.

Using the t requires a way to handle the degrees of freedom parameter $\nu > 0$. Smaller values give long tails, larger values give short tails.

Fixed ν

A Bayesian model for a t-distributed sample:

$$
\begin{aligned}
y_1, \ldots, y_n \mid \mu, \sigma^2 &\sim \text{iid } t_\nu(\mu, \sigma^2) \\
p(\mu) &\propto 1 \qquad\qquad -\infty < \mu < \infty \\
p(\sigma^2) &\propto (\sigma^2)^{-1} \qquad\qquad \sigma^2 > 0
\end{aligned}
$$

ν is chosen arbitrarily, in this case.

Note: Using same improper joint prior as for a normal sample.

Provided $\nu \geq 1$ and all $n > 1$ values y_i are distinct, the posterior is proper.

(Otherwise, be careful!)

Generally, we consider only $\nu > 1$, to retain interpretation of the mean (and to avoid possible issues with propriety and computation).

Random ν

To avoid specifying ν, we can give it a prior.

Because t_ν converges to normal as $\nu \to \infty$, instead consider the reciprocal $1/\nu$:

$$0 \;<\; 1/\nu \;<\; 1$$

The lower bound represents the normal and the upper bound the Cauchy.

BDA3 suggests

$$1/\nu \;\sim\; \mathrm{U}(0, 1)$$

The resulting Bayesian model for a t-distributed sample:

$$
\begin{aligned}
y_1, \ldots, y_n \mid \mu, \sigma^2, \nu &\sim \text{iid } t_\nu(\mu, \sigma^2) \\
p(\mu) &\propto 1 \qquad\qquad -\infty < \mu < \infty \\
p(\sigma^2) &\propto (\sigma^2)^{-1} \qquad\qquad \sigma^2 > 0 \\
1/\nu &\sim \mathrm{U}(0, 1)
\end{aligned}
$$

Remarks

- Can extend to (fixed effect) linear regression: Replace normally distributed observations with t distributed observations.
- Can use the t distribution in the prior portion of the hierarchy – see schools example in BDA3, Sec. 17.4.
- There are robust alternatives to binomial and Poisson models (BDA3, Sec. 17.2), some of which use the t distribution.