STAT 578 - Advanced Bayesian Modeling - Fall 2019 Assignment 2

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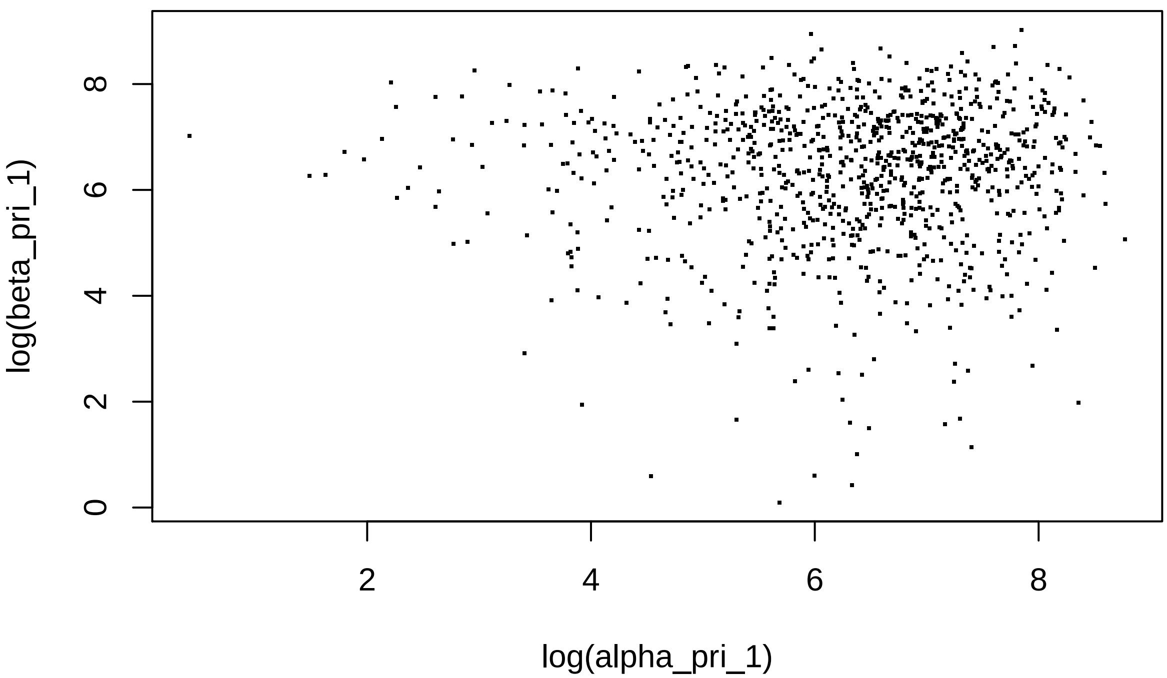
Solution for Problem 1

(a)

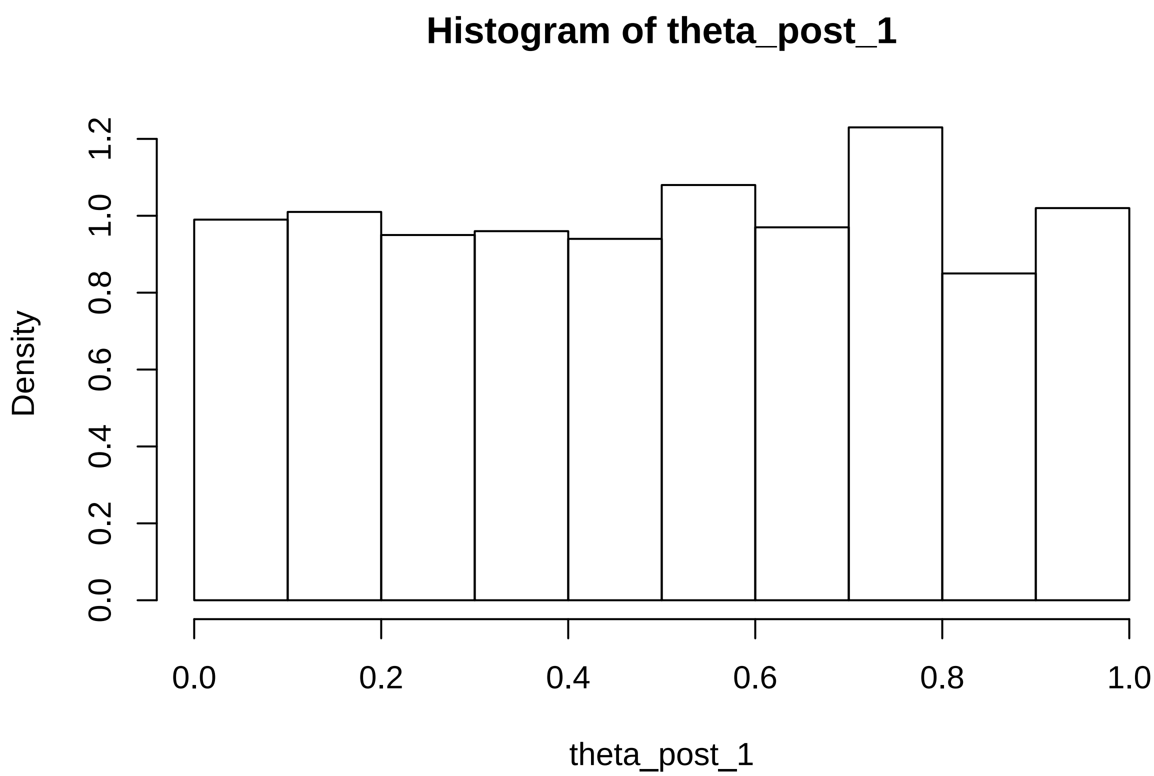
- (i)

```
set.seed(816)
alpha_pri_1 = rexp(1000, rate = 0.001)
beta_pri_1 = rexp(1000, rate = 0.001)
plot(log(alpha_pri_1), log(beta_pri_1), pch=".", cex=2)
```



- (ii)

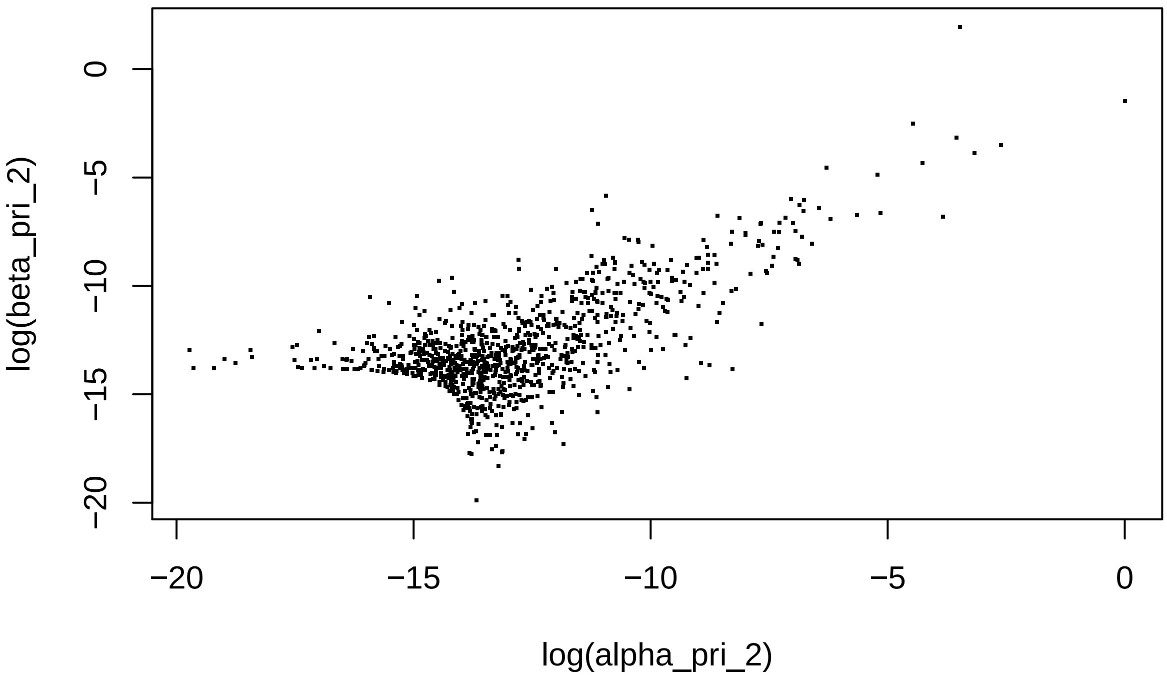
```
theta_post_1 = rbeta(1000, alpha_pri_1, beta_pri_1)
hist(theta_post_1, freq=FALSE)
```



(b)

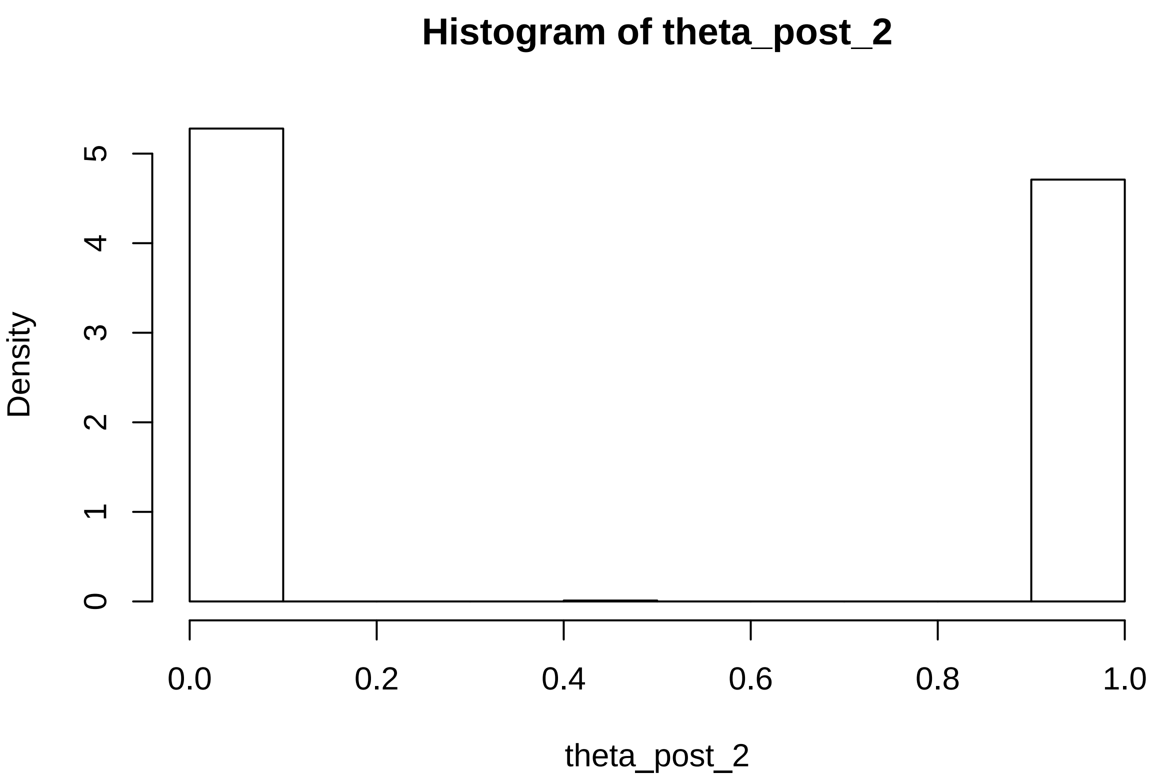
(i)

```
phi_1 = runif(1000, 0, 1)
phi_2 = runif(1000, 0, 1000)
alpha_pri_2 = phi_1 / (phi_2 ^ 2)
beta_pri_2 = (1 - phi_1) / (phi_2 ^ 2)
plot(log(alpha_pri_2), log(beta_pri_2),pch=".", cex=2)
```



(ii)

```
theta_post_2 = rbeta(1000, alpha_pri_2, beta_pri_2)
hist(theta_post_2, freq=FALSE, xlim=c(0,1))
```



Solution for Problem 2

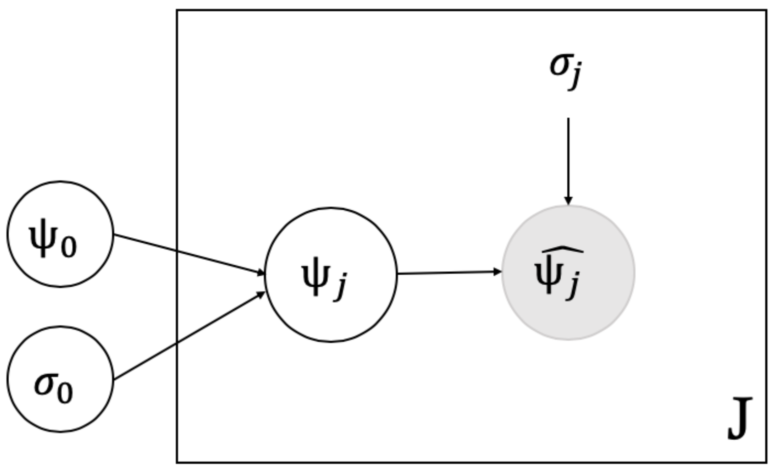
(a)

The improper prior densities of hyperpriors are:

$$p(\psi_0) \propto 1 \ for \ \psi_0 \in (-\infty, \infty)$$

$$p(\sigma_0) \propto 1 \ for \ \sigma_0 \ \in (0, \infty)$$

(b)



(c)

JAGS Model Definition:

```
model {
  for (j in 1:length(psihat)) {
    psihat[j] ~ dnorm(psi[j], 1 / (sigma[j] ^ 2))
    psi[j] ~ dnorm(psi0, 1 / (sigma0 ^ 2))
  }

  psi0 ~ dnorm(0, 1 / (1000 ^ 2))
  sigma0 ~ dunif(0, 1000)

  sigmasq0 <- sigma0 ^ 2
}
```

```
library(rjags)

df = read.table("data.txt", header=TRUE)[, 2:3]

model_1 = jags.model("asgn2template.bug", df)
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
```

```
## Graph information:
##    Observed stochastic nodes: 12
##    Unobserved stochastic nodes: 14
##    Total graph size: 70
##
## Initializing model
```

(d)

```r
update(model_1, 10000)
x_1 = coda.samples(model_1, c("psi0","sigmasq0"), n.iter=10000)
```

(1) Numerical summary of posterior densities for ψ_0,

```r
psi0 = as.matrix(x_1)[,"psi0"]
summary(psi0)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -0.4630  0.1260  0.2082  0.2148  0.2942  1.0592
```

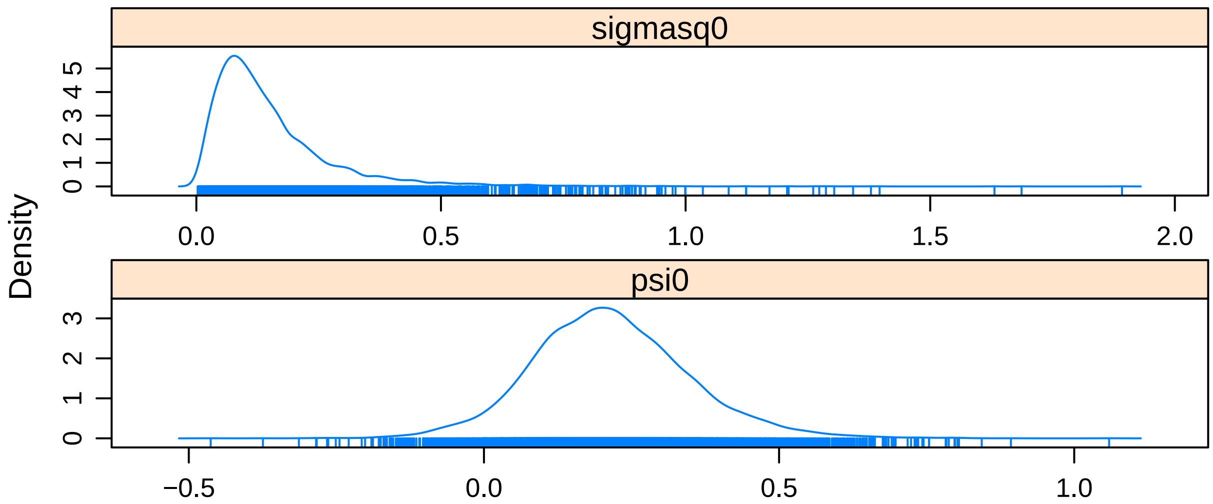
and σ_0^2,

```r
sigmasq0 = as.matrix(x_1)[,"sigmasq0"]
summary(sigmasq0)
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## 0.002943 0.070158 0.117139 0.152184 0.190534 1.892023
```

(2) Graphical estimates of posterior densities for ψ_0 and σ_0^2,

```r
require(lattice)

densityplot(x_1[,c("psi0","sigmasq0")])
```



(5)

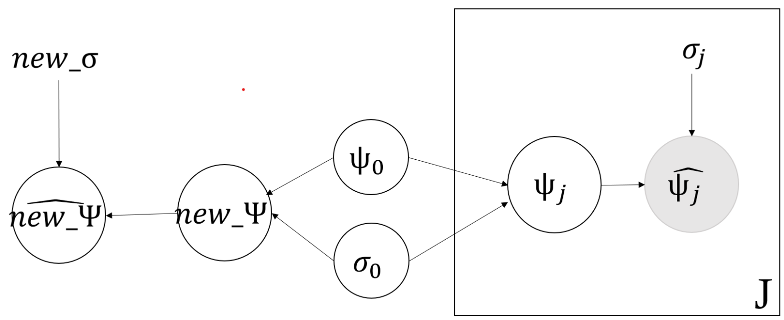
```r
psi0_mean = mean(psi0)
psi0_sd = sd(psi0)
psi0_95 = quantile(psi0, c(0.025, 0.975))

sigmasq0_mean = mean(sigmasq0)
```

```r
sigmasq0_sd = sd(sigmasq0)
sigmasq0_95 = quantile(sigmasq0, c(0.025, 0.975))
```

- Posterior expected values for ψ_0: 0.214781
- Posterior standard deviations for ψ_0: 0.1336794
- 95% central posterior intervals for ψ_0: (-0.0361837, 0.4987024)
- Posterior expected values for σ_0^2: 0.1521838
- Posterior standard deviations for σ_0^2: 0.132623
- 95% central posterior intervals for σ_0^2: (0.0194971, 0.5008974)

(e)

(i)



(ii) JAGS Model Definition:

```
model {
  for (j in 1:length(psihat)) {
    psihat[j] ~ dnorm(psi[j], 1 / (sigma[j] ^ 2))
    psi[j] ~ dnorm(psi0, 1 / (sigma0 ^ 2))
  }

  psi0 ~ dnorm(0, 1 / (1000 ^ 2))
  sigma0 ~ dunif(0, 1000)

  psihat_new ~ dnorm(psi_new, 1 / (sigma_new^2))
  psi_new ~ dnorm(psi0, 1 / (sigma0 ^ 2))
  psihat_new_ind <- psihat_new >= 2 * psi_new
}
```

```r
model_new = jags.model("asgn2template_new.bug", c(as.list(df), sigma_new = 0.25))
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 12
##    Unobserved stochastic nodes: 16
##    Total graph size: 77
##
## Initializing model
```

```
update(model_new, 10000)
x_new = coda.samples(model_new, c("psihat_new","psihat_new_ind"), n.iter=10000)
```

(iii)

```
psihat_new = as.matrix(x_new)[, "psihat_new"]
psihat_new_ind = as.matrix(x_new)[, "psihat_new_ind"]

psihat_new_mean = mean(psihat_new)
psihat_new_sd = sd(psihat_new)
psihat_new_95 = quantile(psihat_new, c(0.025, 0.975))

psihat_new_ind_mean = mean(psihat_new_ind)
```

- Posterior mean forthe estimated log-odds ratio: 0.2135174
- Posterior standard deviations for the estimated log-odds ratio: 0.4741045
- 95% central posterior intervals for the estimated log-odds ratio: (-0.7110292, 1.1734974)

(iv)

Posterior predictive probability that the new estimated log-odds ratio will be at least twice its standard error: 0.315