STAT 578: Advanced Bayesian Modeling

Week 7 – Lesson 2

Gibbs Sampling

Fall 2019

Gibbs Sampler in Practice

Data Example

Recall the Flint water data:

$$y_i \;=\; \text{logarithm of first-draw lead level (ppb), for observation } i$$

Sample statistics:

$$n \;=\; 271 \qquad \bar{y} \;\approx\; 1.40 \qquad s^2 \;\approx\; 1.684$$

Similar to conjugate case (from a previous lesson), let:

$$\mu_0 \;=\; 1.10 \qquad \tau_0^2 \;=\; \sigma_0^2 \;=\; 1.17 \qquad \nu_0 \;=\; 1$$

For implementation in R:

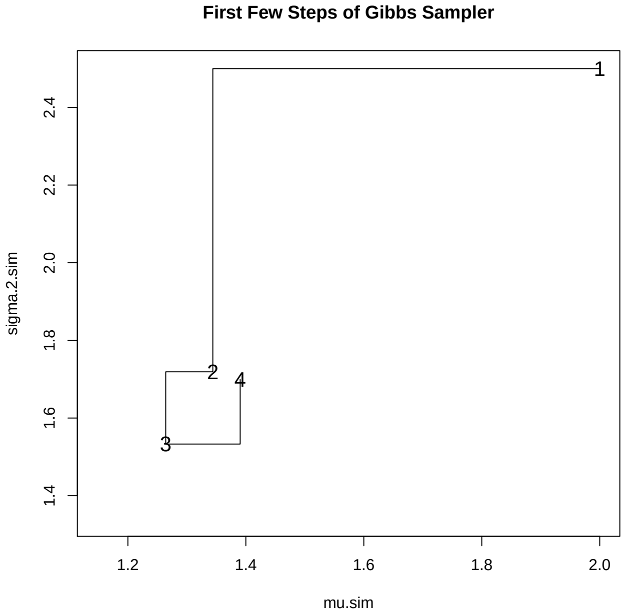
```
> n <- 271
> ybar <- 1.40
> s.2 <- 1.684

> mu0 <- 1.10
> tau.2.0 <- sigma.2.0 <- 1.17
> nu0 <- 1
```

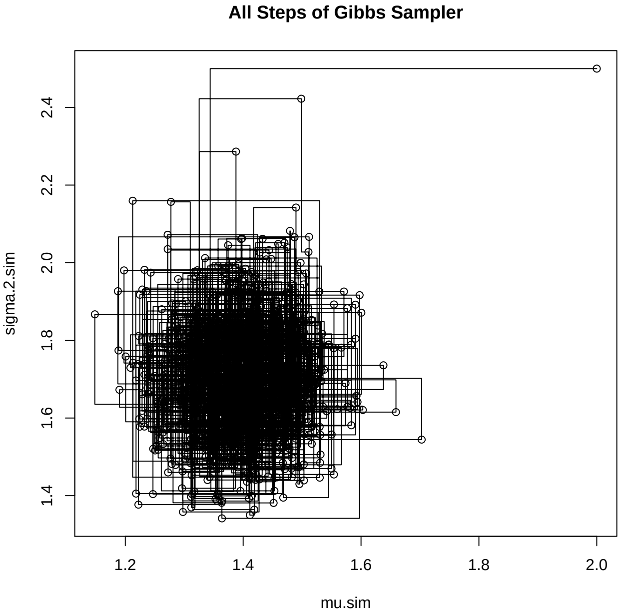
Define some convenience functions:

```
> mun <- function(sigma.2){
+   (mu0/tau.2.0 + n*ybar/sigma.2) / (1/tau.2.0 + n/sigma.2)
+ }

> tau.2.n <- function(sigma.2){
+   1 / (1/tau.2.0 + n/sigma.2)
+ }

> sigma.2.n <- function(mu){
+   (nu0*sigma.2.0 + (n-1)*s.2 + n*(mu-ybar)^2) / (nu0 + n)
+ }
```

Now run Gibbs sampler:

```
> n.sim <- 1000
> mu.sim <- numeric(n.sim)
> sigma.2.sim <- numeric(n.sim)

> mu.sim[1] <- 2          # starting value
> sigma.2.sim[1] <- 2.5  # starting value

> for(t in 2:n.sim){
+   mu.sim[t] <- rnorm(1, mun(sigma.2.sim[t-1]),
+                      sqrt(tau.2.n(sigma.2.sim[t-1])))
+
+   sigma.2.sim[t] <- 1 / rgamma(1, (nu0+n)/2,
+                                 (nu0+n)*sigma.2.n(mu.sim[t])/2)
+ }
```

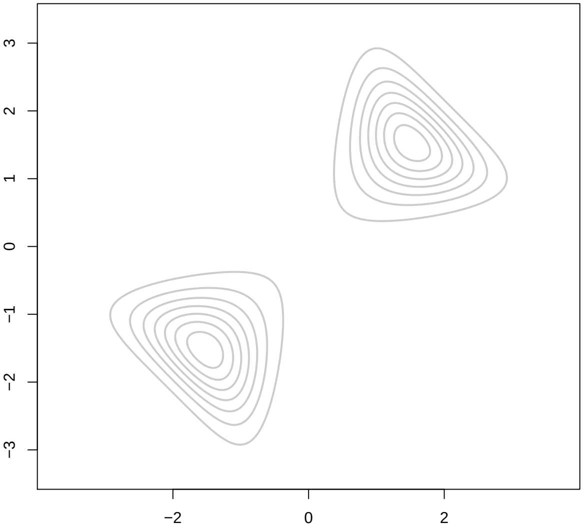

First Few Steps of Gibbs Sampler
1
2
3
4
sigma.2.sim
mu.sim
1.4
1.6
1.8
2.0
2.2
2.4
1.2
1.4
1.6
1.8
2.0

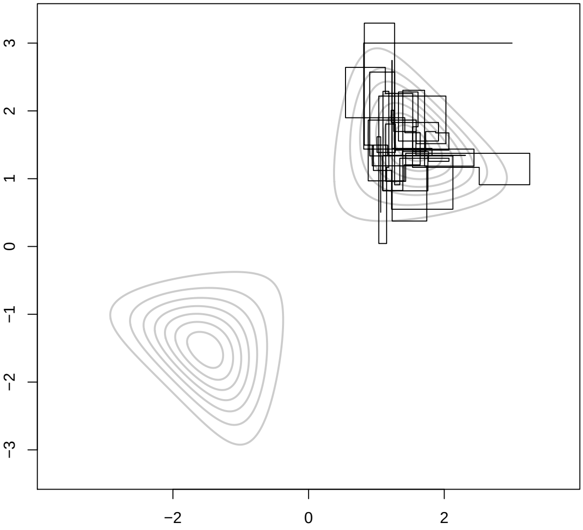

All Steps of Gibbs Sampler
sigma.2.sim
mu.sim
1.4
1.6
1.8
2.0
2.2
2.4
1.2
1.4
1.6
1.8
2.0

Note: Gibbs sampler is very efficient in this case because μ and σ^2 have almost no posterior correlation.

Gibbs sampler efficiency can be much lower when parameters have high posterior correlation – see BDA3, Fig. 11.2 for an example.

A Gibbs sampler can have especially serious problems when the posterior has more than one mode ...


−2
0
2
3
2
1
0
−1
−2
−3


3
2
1
0
−1
−2
−3
−2
0
2

Gibbs samplers can also be applied in hierarchical models.

For an example, see BDA3, Sec. 11.6.