

ADVANCED BAYESIAN MODELING

BAYESIAN TOOLS FOR INFERENCE:
TOOLS FOR ESTIMATION AND TESTING

Classical (non-Bayesian) inference tools include point estimates, standard errors, confidence intervals, hypothesis tests, and point predictions and intervals.

Bayesian inference has corresponding tools:

- ▶ Posterior point estimates
- ▶ Measures of posterior spread
- ▶ Posterior (credible) intervals
- ▶ Posterior probabilities
- ▶ Posterior predictive distributions

Posterior Estimation

Single-number summaries of the posterior distribution of parameter θ include:

- ▶ The posterior mean (if it exists)

$$E(\theta \mid y)$$

- ▶ The posterior median
- ▶ The posterior mode (if well-defined)

In statistics, the posterior mean is the most popular.

For your dog-walking service, with flat $U(0, 1)$ prior on

θ = true proportion of potential clients

the posterior was $\text{Beta}(\alpha = 3, \beta = 19)$.

Then (BDA3, Table A.1)

- ▶ The posterior mean is

$$E(\theta \mid y = 2) = \frac{3}{3 + 19} \approx 0.136$$

- ▶ The posterior mode is

$$\frac{3 - 1}{3 + 19 - 2} \approx 0.100$$

When not available analytically, the posterior mean and median can be approximated by simulation.

In R, function `rbeta` simulates from a beta distribution:

```
> posterior.samples <- rbeta(1000, 3, 19) # 1000 independent samples
```

```
> mean(posterior.samples)
```

```
[1] 0.1370276
```

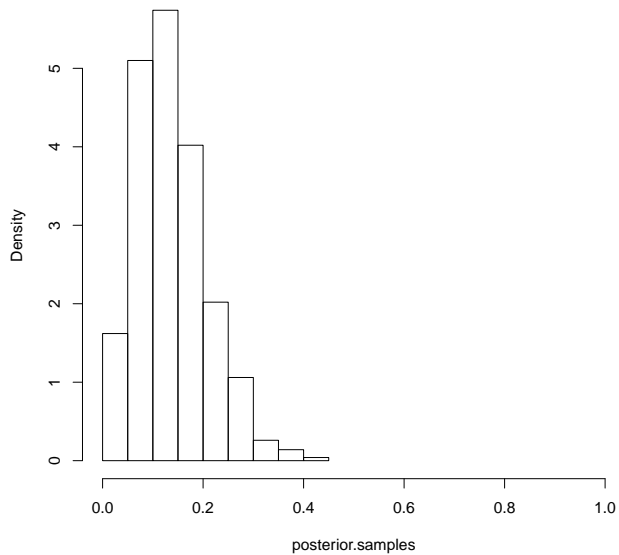
```
> median(posterior.samples)
```

```
[1] 0.128049
```

A histogram (just to verify):

```
> hist(posterior.samples, freq=FALSE, xlim=c(0,1))
```

Histogram of posterior.samples



Approximating a posterior mode generally requires optimization:

```
> optimize(dbeta, c(0,1), 3, 19, maximum=TRUE)
```

```
$maximum
```

```
[1] 0.1000202
```

```
$objective
```

```
[1] 5.988776
```


Posterior Spread

To measure uncertainty, could use the posterior standard deviation (if it exists):

$$\sqrt{\text{var}(\theta \mid y)}$$

For example,

```
> sd(posterior.samples) # approx. posterior std. dev.  
[1] 0.07048138
```

approximates the true posterior standard deviation (BDA3, Table A.1):

$$\sqrt{\frac{3 \cdot 19}{(3 + 19)^2 (3 + 19 + 1)}} \approx 0.0716$$

Posterior Intervals

Instead of posterior location and spread, consider a range of values ...

A **posterior (credible) interval** is one that contains θ with specified posterior probability.

Could be defined in several ways ...

Most commonly, the $100(1 - \alpha)\%$ posterior interval is the range between the $\alpha/2$ and $1 - \alpha/2$ posterior quantiles.

This is called a **central** posterior interval.

Central posterior intervals are easily approximated by simulation.

E.g., a 95% central posterior interval:

```
> quantile(posterior.samples, c(0.05/2, 1-0.05/2))  
      2.5%      97.5%  
0.02887084 0.29349264
```

This approximates the true interval:

```
> qbeta(c(0.05/2, 1-0.05/2), 3, 19)  
[1] 0.03048897 0.30377441
```

So, with 95% posterior probability, between about 3% and 30% of households in your community might be interested in a dog-walking service.

Posterior Probabilities

Suppose you believe your dog-walking service would be successful if at least 15% of households in the community were interested.

How probable is that?

You could use the posterior probability

$$\Pr(\theta \geq 0.15 \mid y = 2)$$

Posterior probabilities are easily approximated by simulation:

```
> mean(posterior.samples >= 0.15)
[1] 0.377
```

This approximates the exact value

```
> 1 - pbeta(0.15, 3, 19)
[1] 0.3704955
```

So your dog-walking service has (at least) a 37% chance of success.

This bears similarity to a classical hypothesis test:

$$H_0 : \theta \leq 0.15 \qquad H_a : \theta > 0.15$$

The posterior probability $\Pr(H_0 \mid y)$ is sometimes used like a classical p -value.

However, there are more formal Bayesian approaches (later).