360.243 Numerical Simulation and Scientific Computing II (VU 3,0) 2022S

EXERCISE 4

Group 12

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Introduction

The aim of this exercise is to model the steady state heat conduction

$$-\frac{\partial}{\partial x_i}q_i + Q = 0$$

for a square domain using the constitutive law

$$q_i = -k_{ij}\frac{\partial}{\partial x_j}T$$

The general FEM approach is to obtain the local stiffness matrices for each element, assemble them into a global stiffness matrix, assemble the load vector and solve the system. Due to the given boundary conditions (partly given temperature and partly given flux), the system needs to be split into two parts and solved respectively.

Our group specific input values are:

L = 0.01M, hz = 0.001m, k = 401 W/mk, c = 20, T(y = L) = 293K, q(y = 0) = 1000000W/m^2

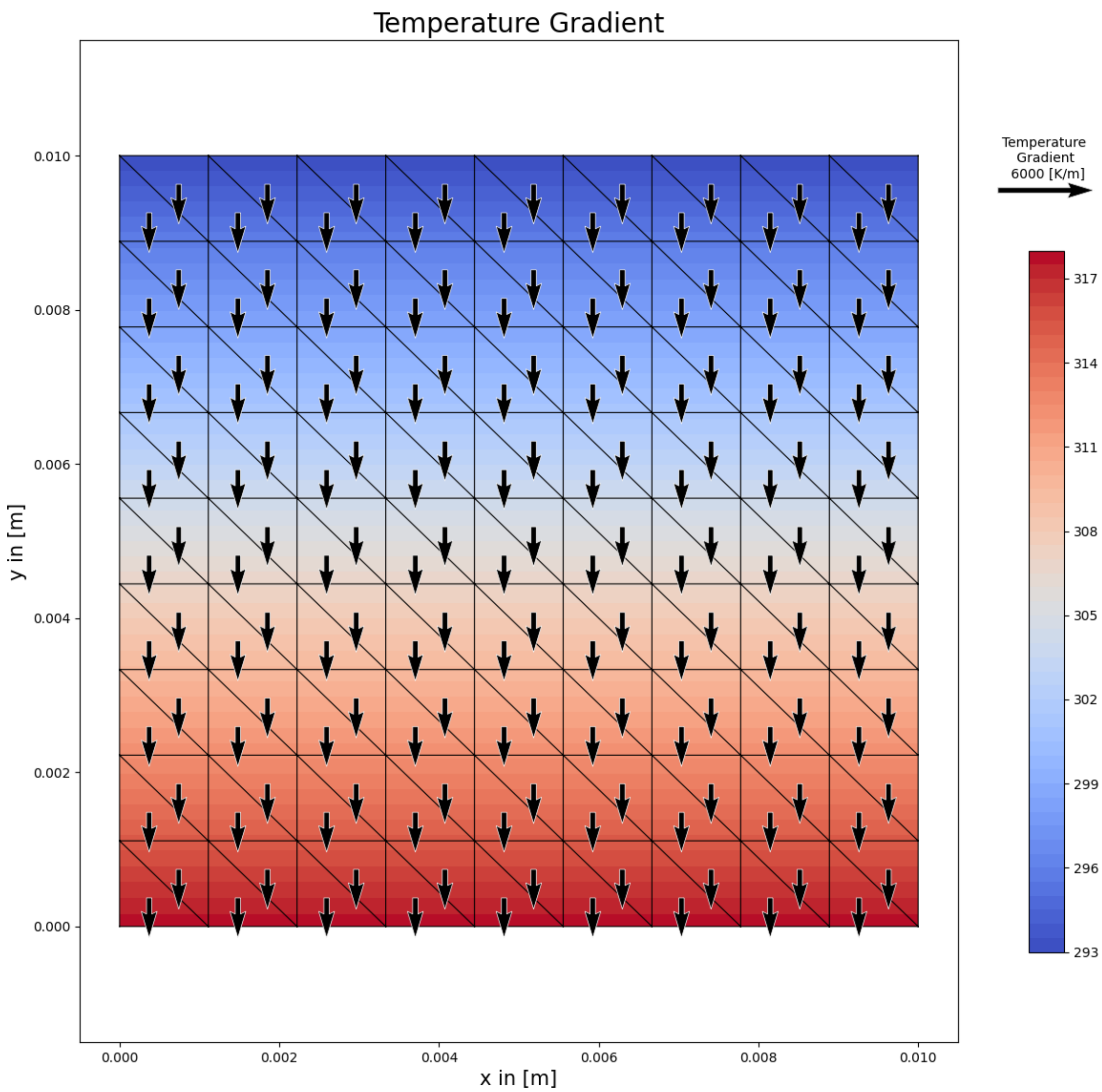
Solution basic variation

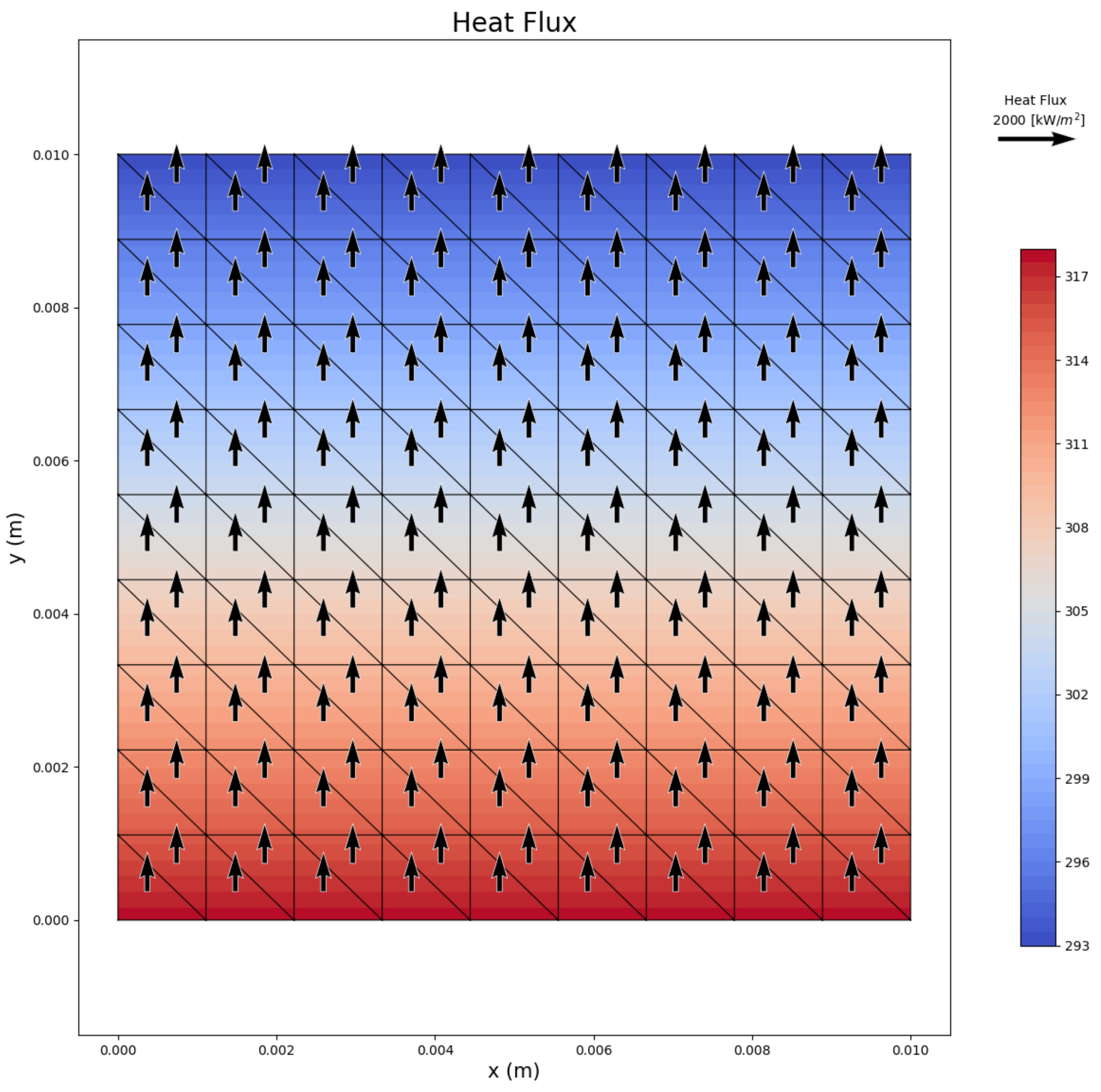
In all plots, the color scheme represents the temperature distribution in K.

The temperature gradient in each element equals the overall temperature gradient ΔT/Δy. In our case this equals to -2494 K/m.

The flux remains constant (up to rounding error) in the entire domain and it equals to the given flux of 1000000 W/m^2

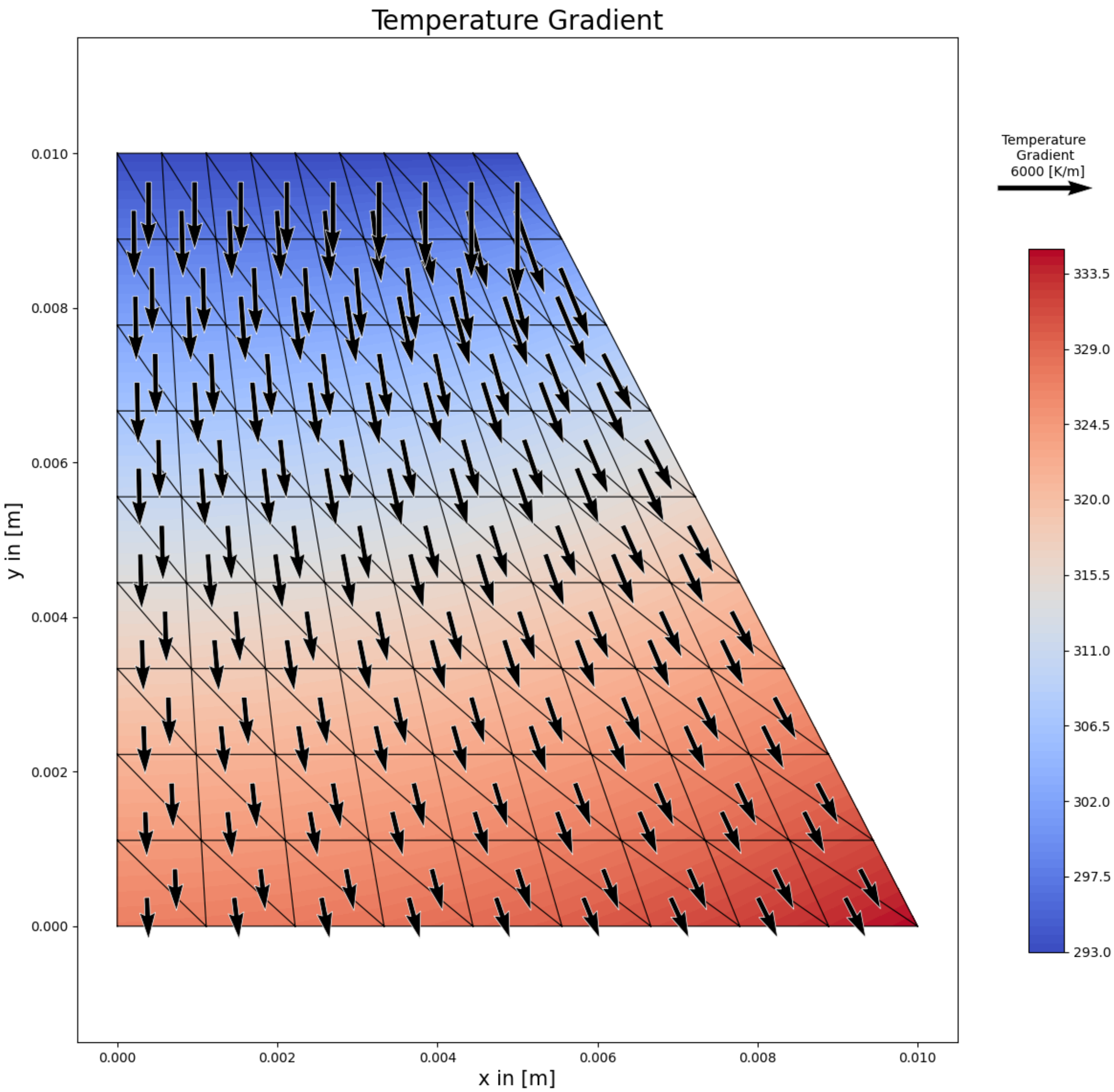
Using the constitutive law, one obtains the conductivity k, which is the negative ratio between the heat flux and the temperature gradient. In our case this is 401 W/mK and remains constant over the whole domain.


Temperature Gradient
Temperature
Gradient
6000 [K/m]
317
314
311
308
305
302
299
296
293
0.010
0.008
0.006
0.004
0.002
0.000
y in [m]
0.000
0.002
0.004
0.006
0.008
0.010
x in [m]

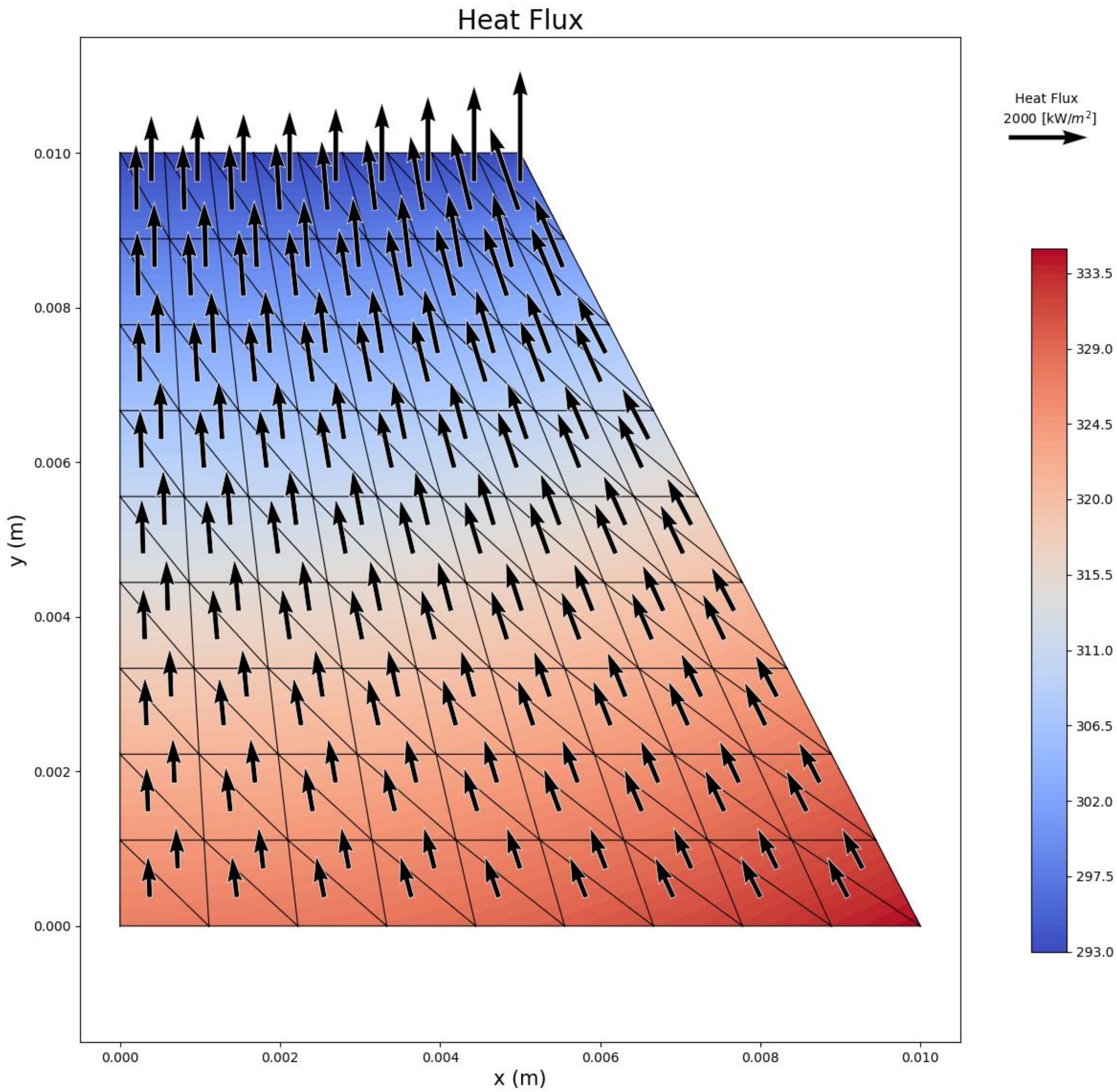


Solution V1

In this case, the upper boundary is the half length of the lower (input) boundary. As to be expected the flux/element and temperature gradient increase towards the upper boundary. This happens due to the energy conservation law.

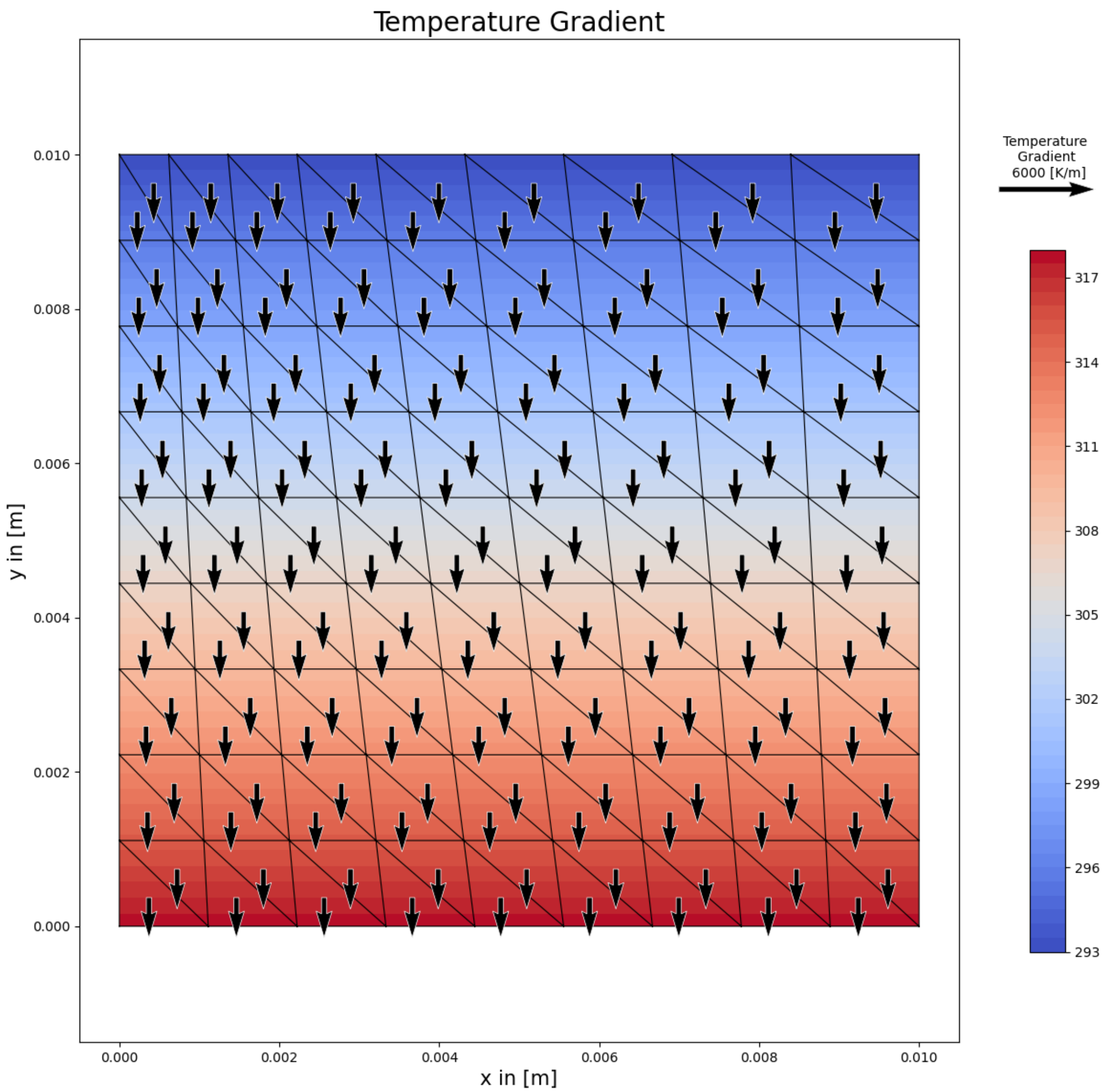


Temperature Gradient
y in [m]
x in [m]
0.010
0.008
0.006
0.004
0.002
0.000
0.000
0.002
0.004
0.006
0.008
0.010
Temperature
Gradient
6000 [K/m]
333.5
329.0
324.5
320.0
315.5
311.0
306.5
302.0
297.5
293.0

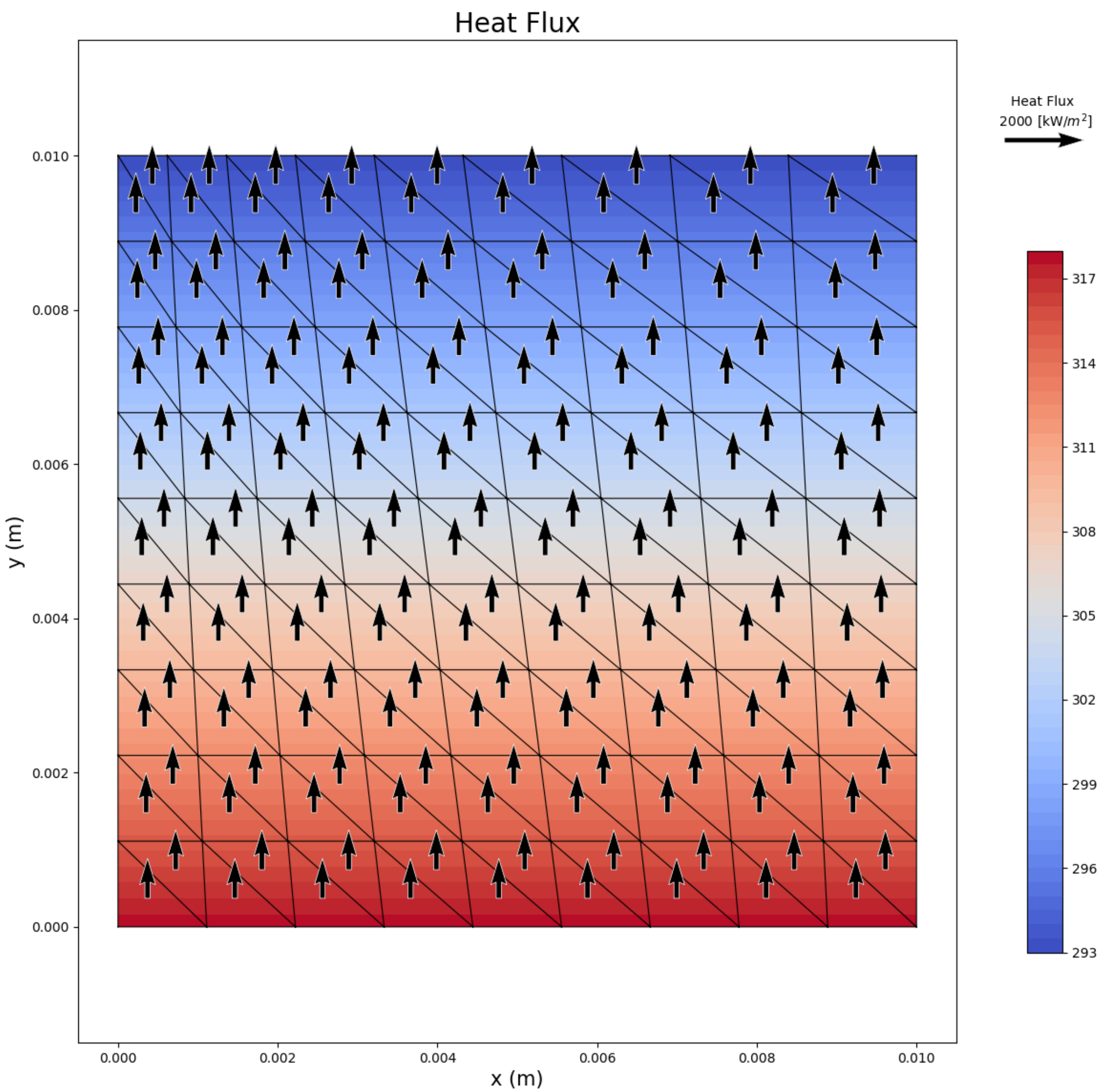


Solution V2

The domain in version 2 is the same as the original basic square domain. The problem at hand is the same as the original and therefore the physical properties should therefore remain the same. This has to be ensured by the FEM implementation. FEM element transformations are by definition interpolation equivalent. This ensures constant solutions between a regular mesh and a skewed mesh. We observe this in our implementation.

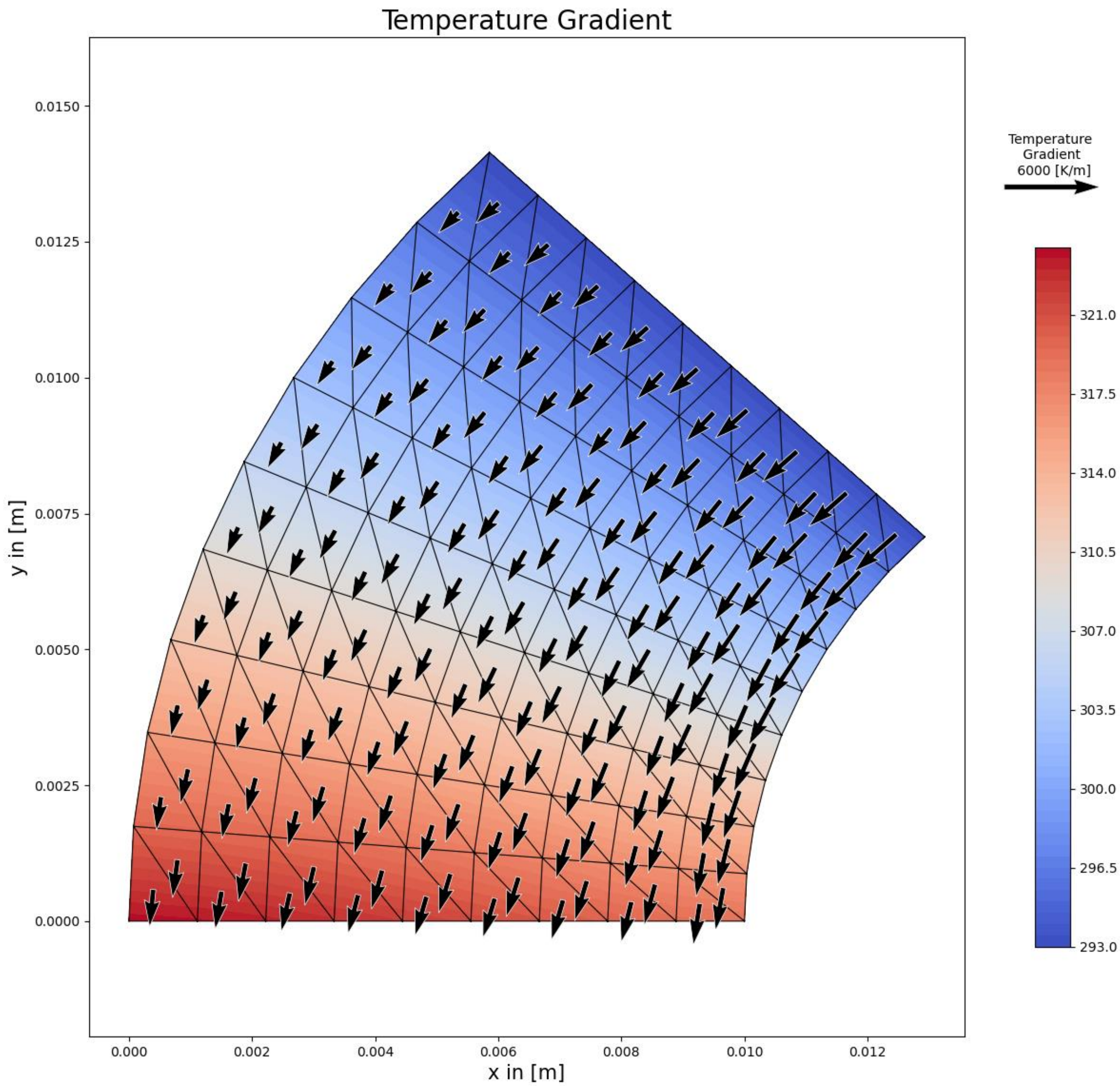


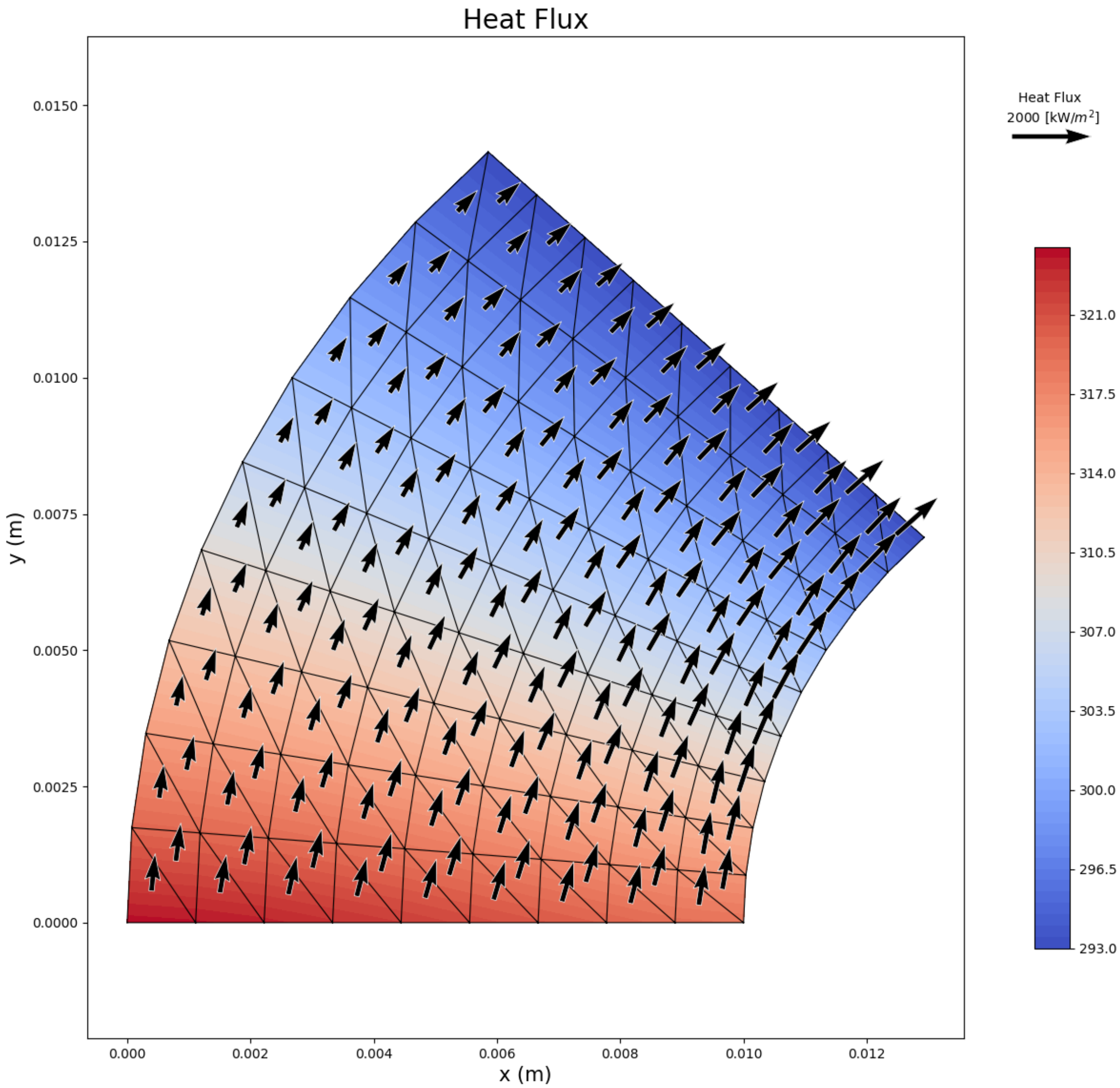
Temperature Gradient
y in [m]
x in [m]
0.000
0.002
0.004
0.006
0.008
0.010
Temperature
Gradient
6000 [K/m]
317
314
311
308
305
302
299
296
293



Solution V3

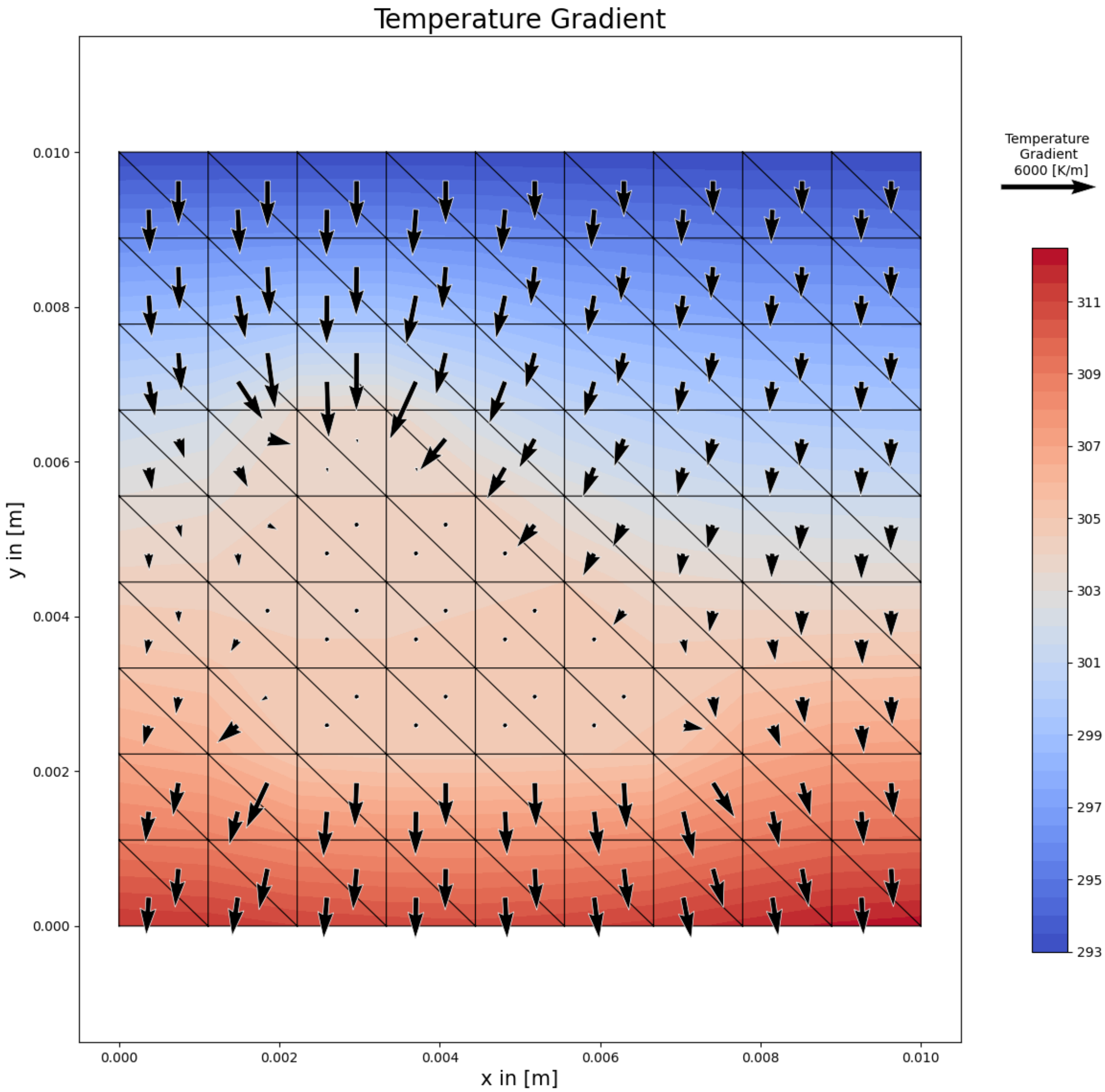
Because of the curvature of the domain, the overall length on the inner (right side in our plot) boundary is much shorter than the outer one. Therefore the gradients and fluxes on this side are much higher than on the left side. The overall transported heat remains the same, as expected. The gradual change in the gradient can be explained by the homogeneous heat conductivity and geometry of our closed system. The heat escapes via "the shortest way" due to the principal of minimal potential energy.

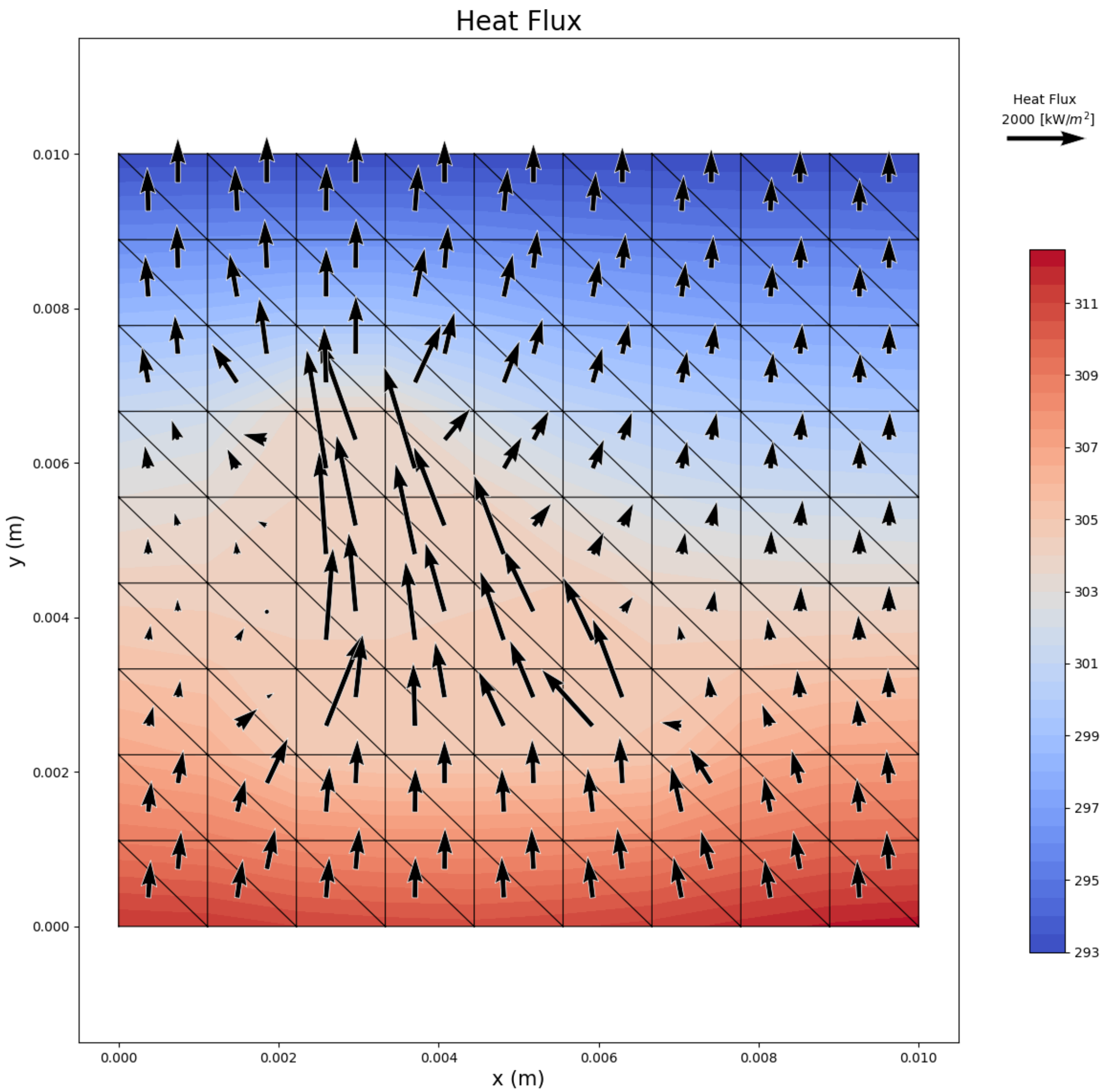

Temperature Gradient
y in [m]
x in [m]
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
0.000
0.002
0.004
0.006
0.008
0.010
0.012
Temperature Gradient 6000 [K/m]
321.0
317.5
314.0
310.5
307.0
303.5
300.0
296.5
293.0



Solution V4.1

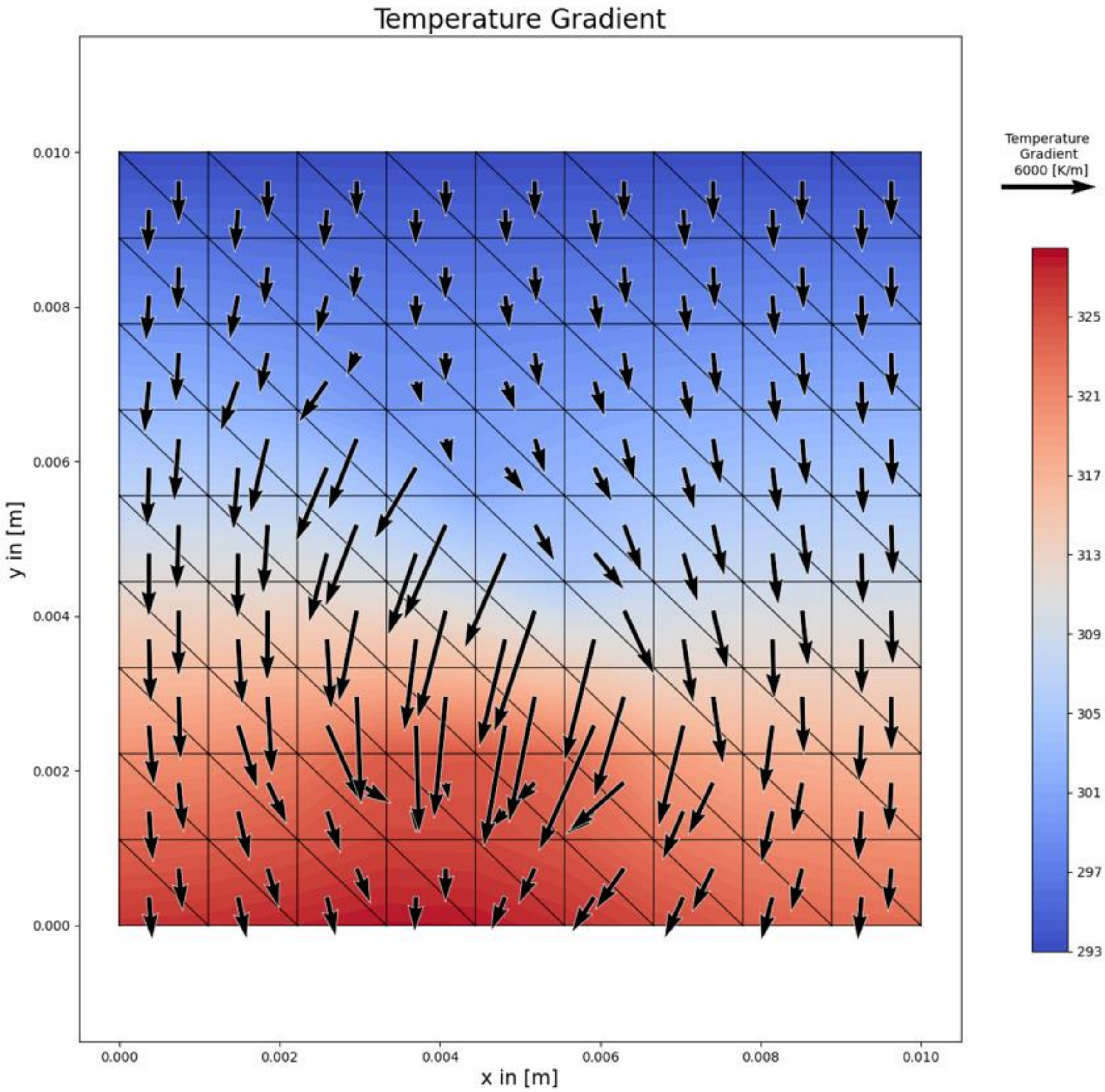
Within the modified region, the temperature is higher than in the original domain. This is due to the upscaled heat conductivity k, with k_{mod} = kc. Also the heat flux in this region is multiplied by c. Due to the high heat conductivity, neighboring elements (within the modified region) experience only a very limited temperature change. Therefore, also the gradient remains very small.

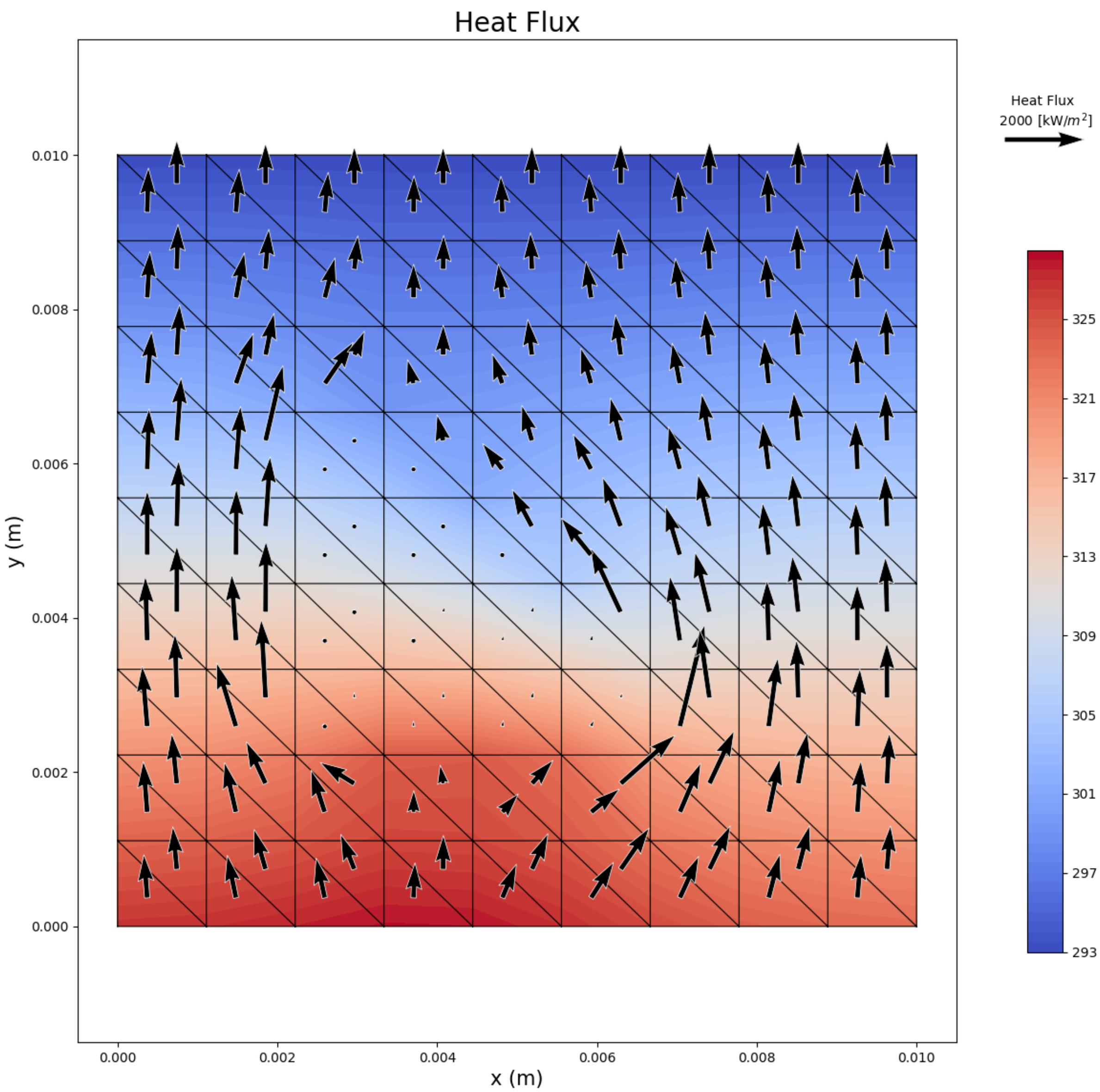

Temperature Gradient
Temperature
Gradient
6000 [K/m]
y in [m]
x in [m]
0.000
0.002
0.004
0.006
0.008
0.010
311
309
307
305
303
301
299
297
295
293



Solution V4.2

Within the modified region, the temperature is lower than in the original domain. This is due to the downscaled heat conductivity k, with $k_{mod} = k/c$. Also the heat flux in this region is divided by c. Due to the low heat conductivity, neighboring elements (within the modified region) experience a very high temperature change. Therefore, also the gradient gets extremely big.


Temperature Gradient
Temperature
Gradient
6000 [K/m]
325
321
317
313
309
305
301
297
293
0.010
0.008
0.006
0.004
0.002
0.000
y in [m]
0.000
0.002
0.004
0.006
0.008
0.010
x in [m]


Heat Flux
Heat Flux
2000 [kW/m^2]
y (m)
x (m)
0.000
0.002
0.004
0.006
0.008
0.010
0.000
0.002
0.004
0.006
0.008
0.010
325
321
317
313
309
305
301
297
293