

CR11 — Mathematical methods for image synthesis

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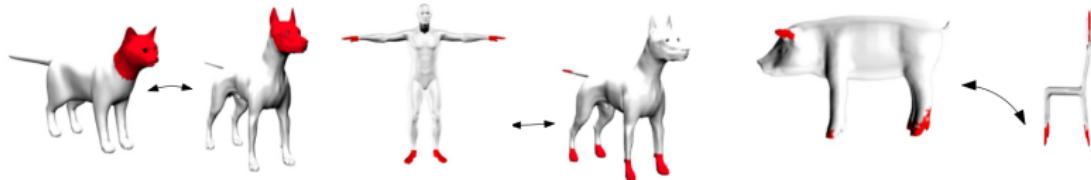
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Stable Region Correspondences Between Non-Isometric Shapes

V. Ganapathi-Subramanian, B. Thibert, M. Ovsjanikov, L. Guibas

Computer Graphics Forum. Vol. 35. No. 5. 2016.



We are given two shapes as triangulated meshes.

Vertex sets $S^{(1)} = \{p_1, \dots, p_{d_1}\}$ and $S^{(2)} = \{q_1, \dots, q_{d_2}\}$.

Feature functions $f^{(1)}$ and $f^{(2)}$ (e.g. Gaussian curvature).

$$f^{(1)} : S^{(1)} \mapsto \mathbb{R} \quad \text{and} \quad f^{(2)} : S^{(2)} \mapsto \mathbb{R}$$



Even if values are different, the rank of a value is useful!

We define two permutations $r^{(1)}$ and $r^{(2)}$ of \mathfrak{S}_n :

$r_i^{(1)} = j$ if $f^{(1)}(p_j)$ is the i^{th} value in sorted order.

$r_i^{(2)} = j$ if $f^{(2)}(q_j)$ is the i^{th} value in sorted order.

Let K be an integer that divides d_1 and d_2 . For $1 \leq k \leq K$:

$$C_k^{(1)} = \left\{ r_i^{(1)} \mid (k-1)\frac{d_1}{K} \leq i \leq k\frac{d_1}{K} \right\}$$

$$C_k^{(2)} = \left\{ r_i^{(2)} \mid (k-1)\frac{d_2}{K} \leq i \leq k\frac{d_2}{K} \right\}$$

Now we can define the affinity matrix!

$$W = \sum_{k=1}^K \mathbb{1}_{C_k^{(2)}} \cdot \mathbb{1}_{C_k^{(1)}}^T = \left(\begin{array}{ccc|c|c|c} 1 & \dots & 1 & (0) & (0) & r_1^{(2)} \\ \vdots & \vdots & & & & \\ 1 & \dots & 1 & 1 & \dots & r_2^{(2)} \\ \hline (0) & & & \vdots & \vdots & \\ & & & 1 & \dots & \\ \hline (0) & & & (0) & & \\ & & & \vdots & \vdots & \\ & & & 1 & \dots & r_{d_2}^{(2)} \\ \hline \end{array} \right) \underbrace{\begin{array}{c} r_1^{(1)} \\ r_2^{(1)} \\ \vdots \\ r_{d_1}^{(1)} \end{array}}_{r^{(1)}}$$

Coefficient i,j is 1 if $f^{(1)}(p_j)$ and $f^{(2)}(q_i)$ have similar ranks...

Now we consider N different features!

→ The affinity matrix is the sum of each matrix.

We need to "normalize" the matrix

→ Multiply each coefficient by $K/(Nd_1d_2)$.

Remark that $(1 \dots 1)^T$ is an eigenvector of $W^T W$.

$$\rightarrow W \cdot (1 \dots 1)^T = (1/d_1)(1 \dots 1)^T$$

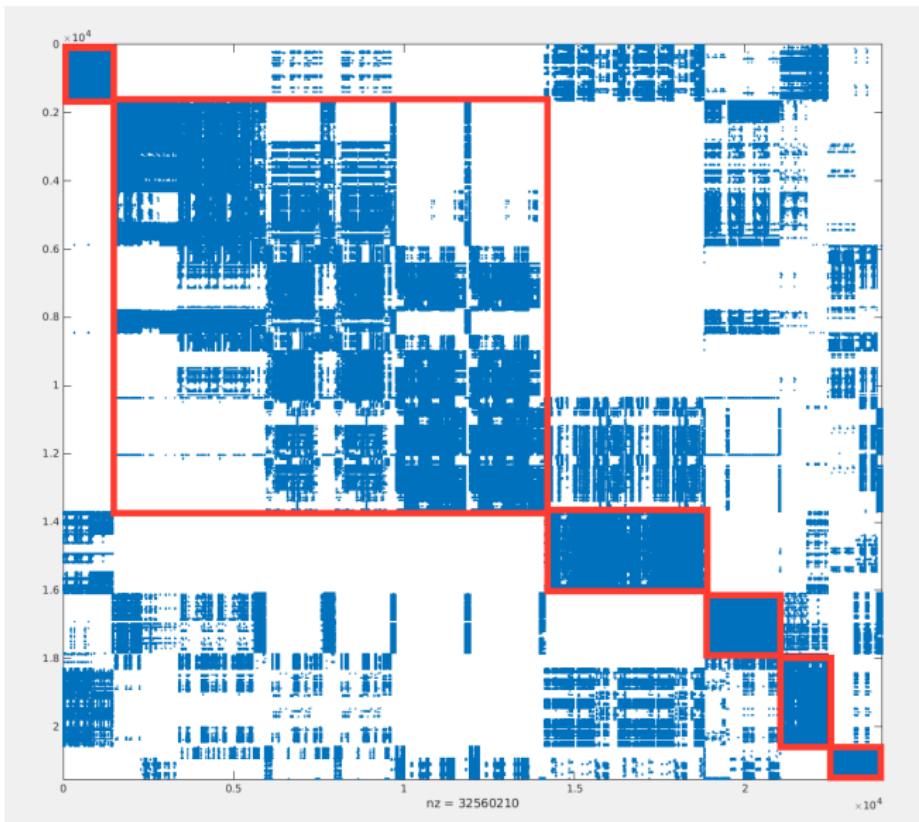
$$\rightarrow W^T \cdot (1 \dots 1)^T = (1/d_2)(1 \dots 1)^T$$

Let $\|M\|$ be the sum of the absolute value of all coefficients of M .

Let $M_{I,J}$ be the submatrix of M on lines I and columns J .

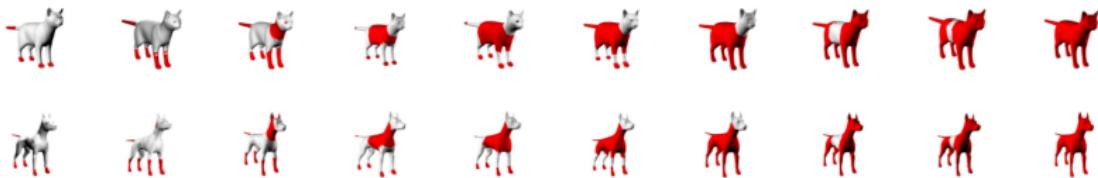
We are now ready to define a stable pair of size (n, m) .

Subset $\Omega^{(1)} \times \Omega^{(2)} \subseteq S^{(1)} \times S^{(2)}$ with $|\Omega^{(1)}| = n$ and $|\Omega^{(2)}| = m$,
such that $\|W_{\Omega^{(2)}, \Omega^{(1)}}\|$ is maximal.



Several algorithms are given to efficiently compute stable pairs.

The procedure is stable when adding noise (random i.i.d. features)

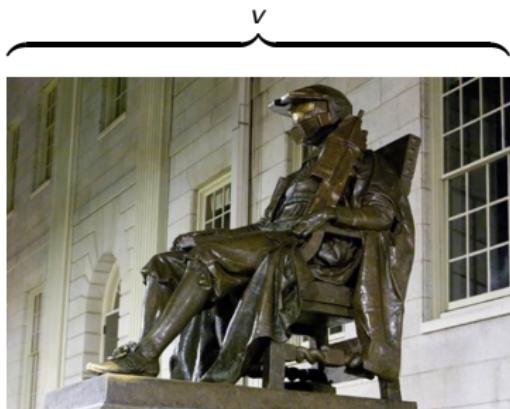




\times



$+$



$=$



Idea : "Shift" the colors, but try keep the shapes.

Let \tilde{u} such that $\tilde{u} = v$ on $\partial\Omega$

Minimize $\mathcal{L}(\tilde{u}) = \int_{\Omega} |\nabla(\tilde{u} - u)|^2$

We deduce a Poisson equation $\nabla^2 \tilde{u} = 0$ in Ω

We discretize this equation, with a discrete laplacian operator.

It's a sparse linear system, we use the conjugate gradient method.



Initial image



After 1 iteration



After 10 iteration



After 100 iteration



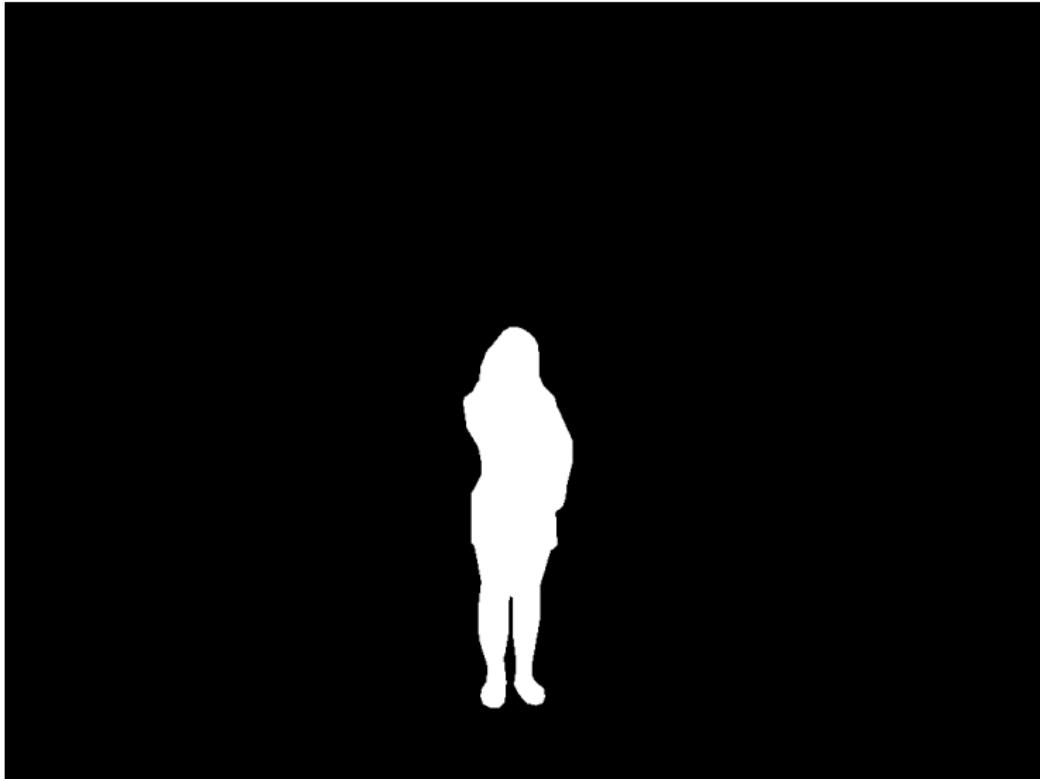
After 1000 iteration



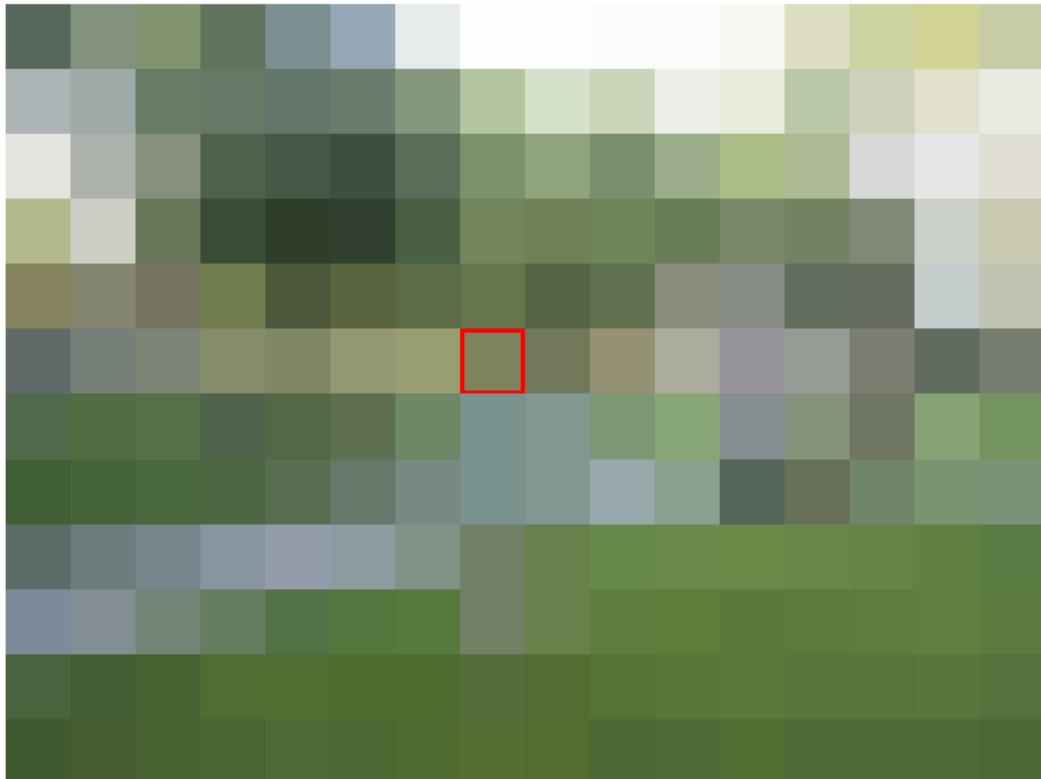
After 10000 iteration



Goal : remove one occupant



Goal : remove one occupant



We want to change the value of this pixel.



Look at it's neighborhood.



Find the pixel with the most similar neighborhood.



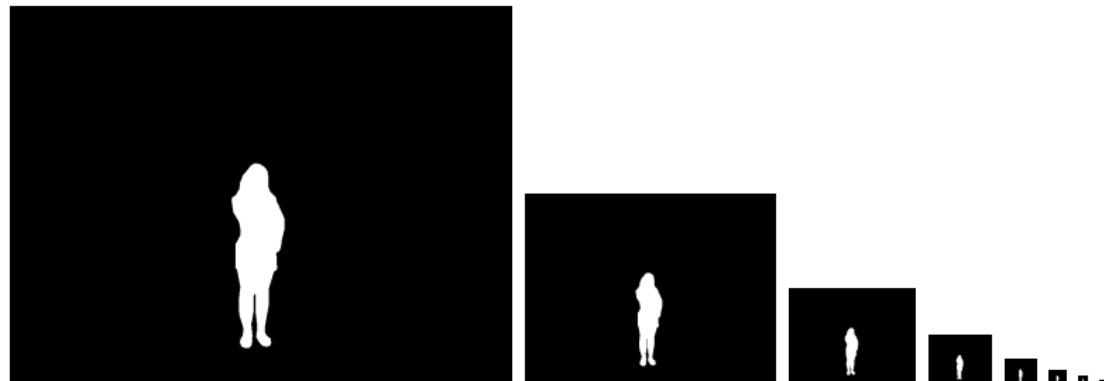
Replace the value of the pixel.

Multiscale approach :



- ① Recursively divide image size by two
- ② Use smaller solution to deduce a "best guess"
- ③ Compute solution

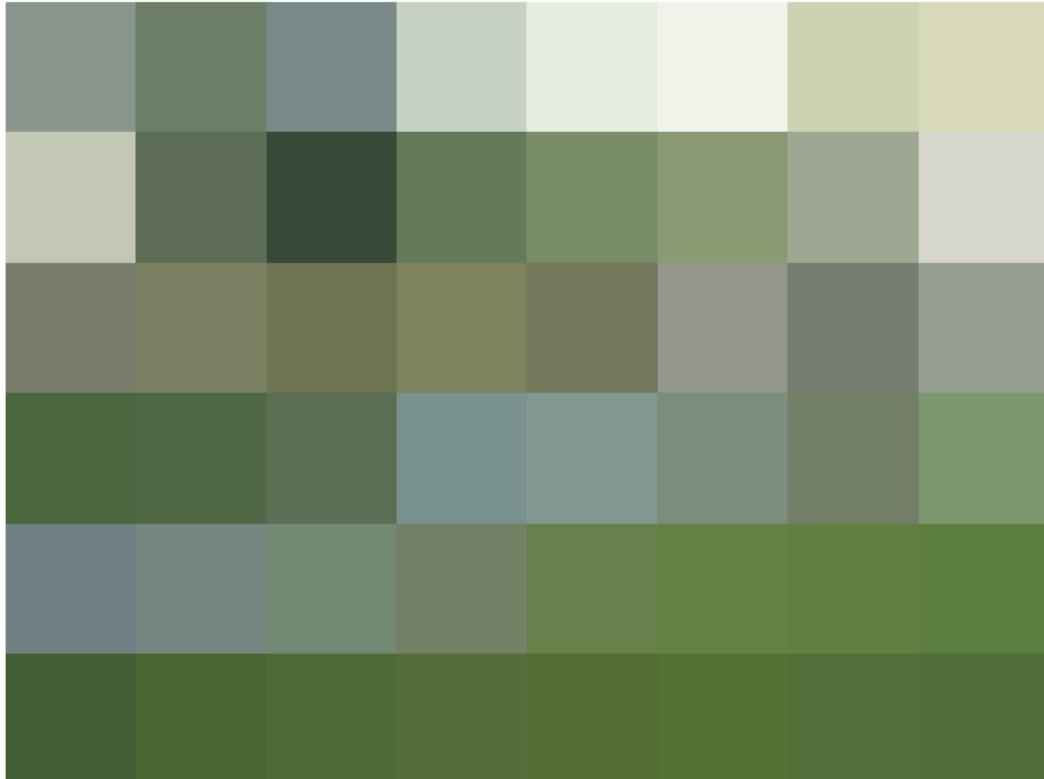
Multiscale approach :



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Stable Region Correspondences
Poisson Image Editing
Image Inpainting
Seam Carving

Goal
Basic approach
Using a "hint"



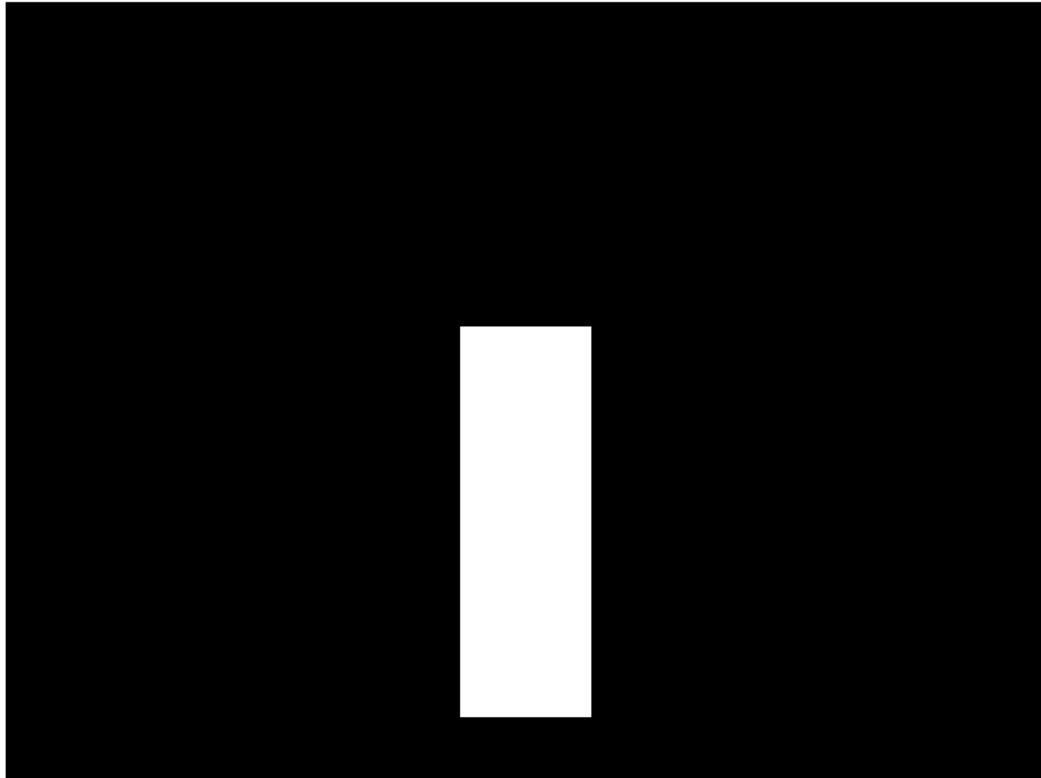
Base case

Image of size 6x8



Input

Image of size 12x16



Mask

Image of size 12x16



Guess

Image of size 12x16



Output

Image of size 12x16

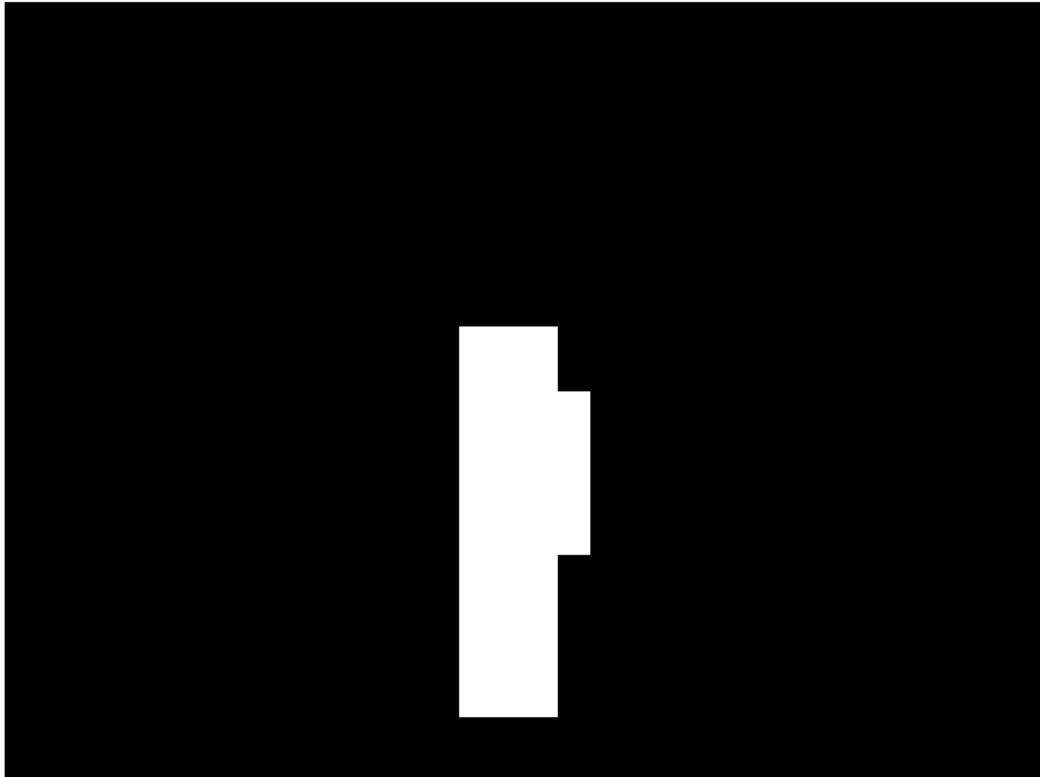
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Input

Image of size 24x32



Mask

Image of size 24x32

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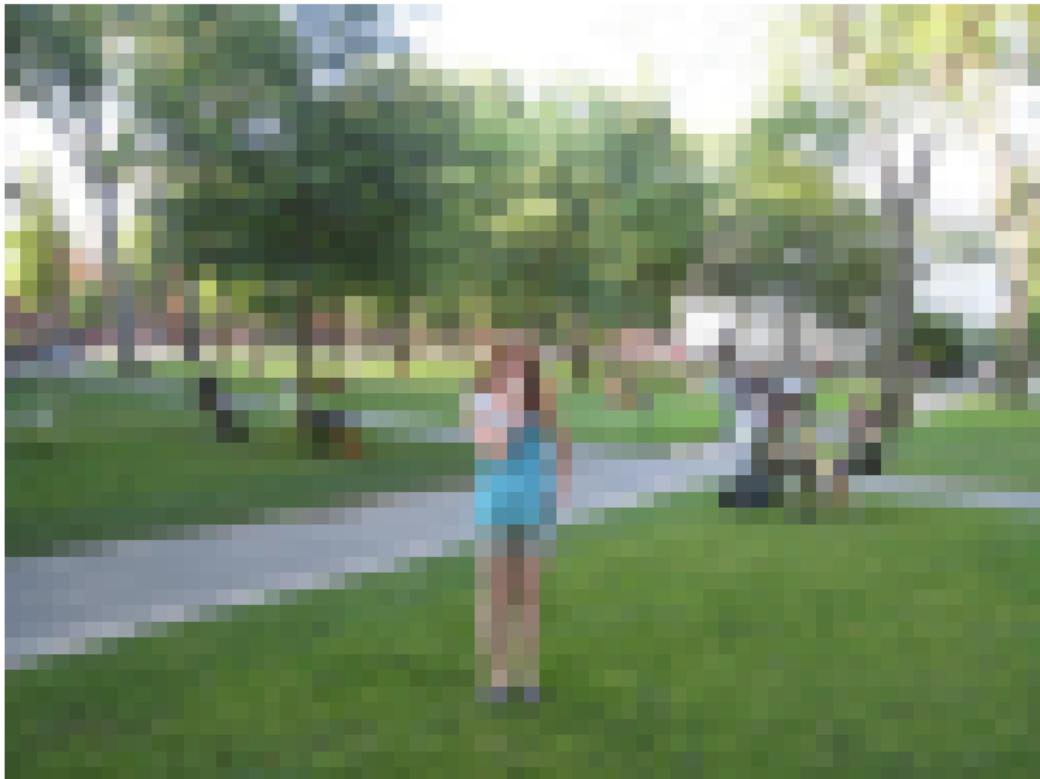
Guess

Image of size 24x32



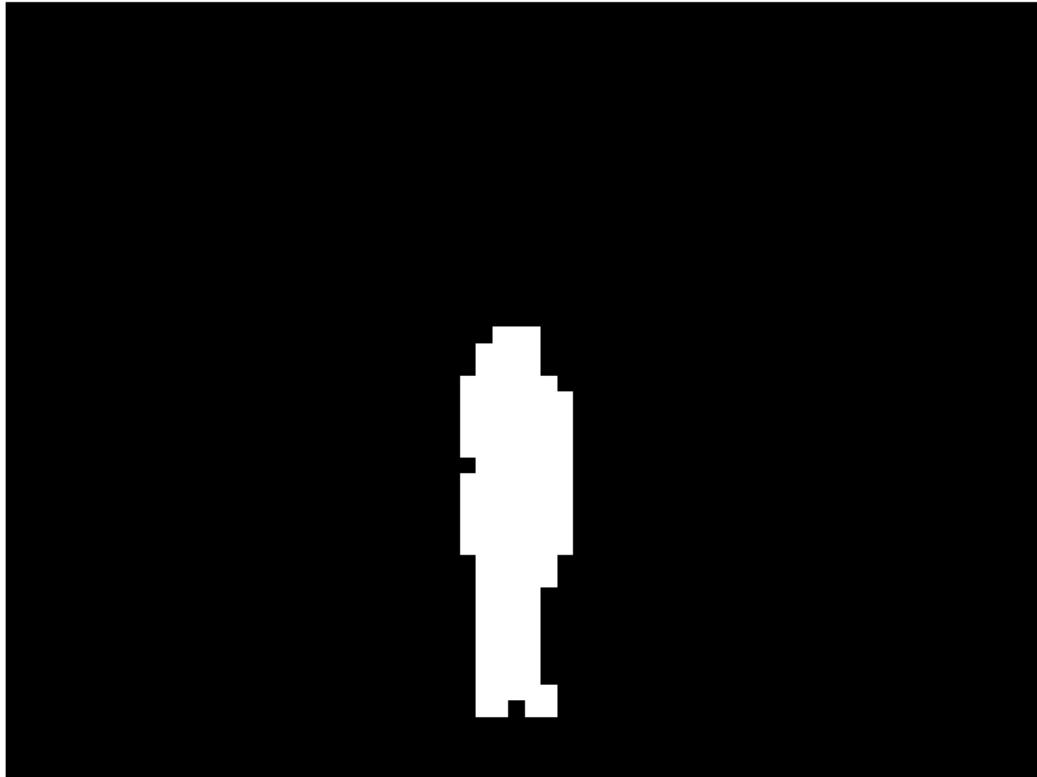
Output

Image of size 24x32



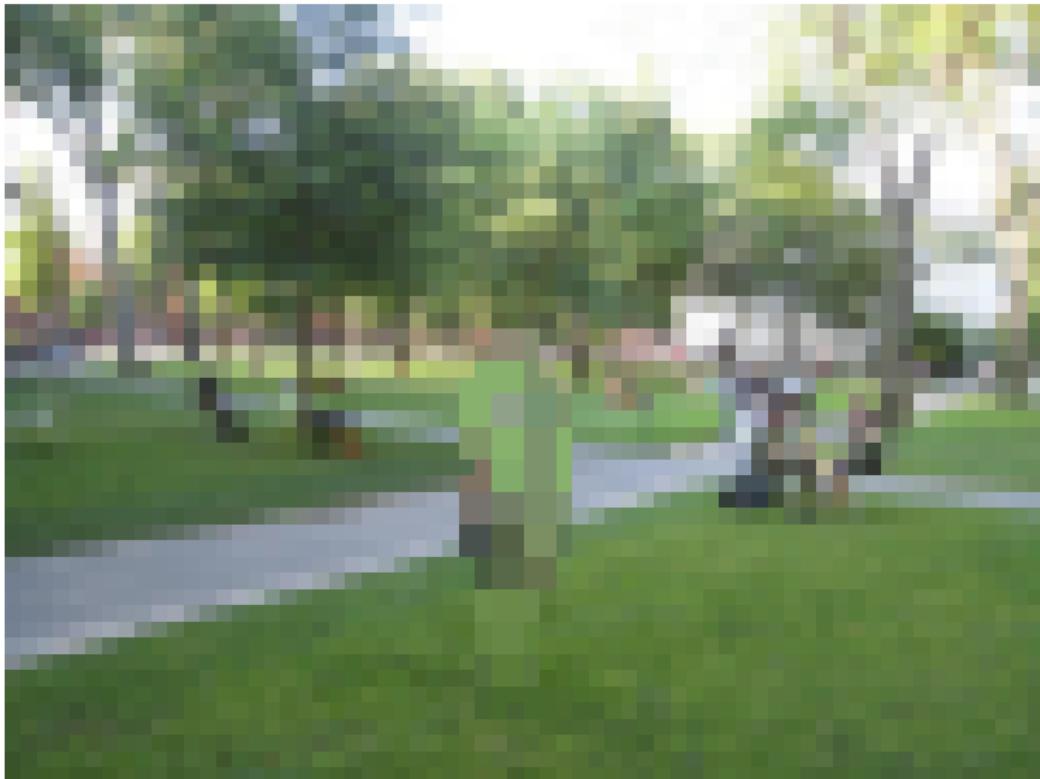
Input

Image of size 48x64



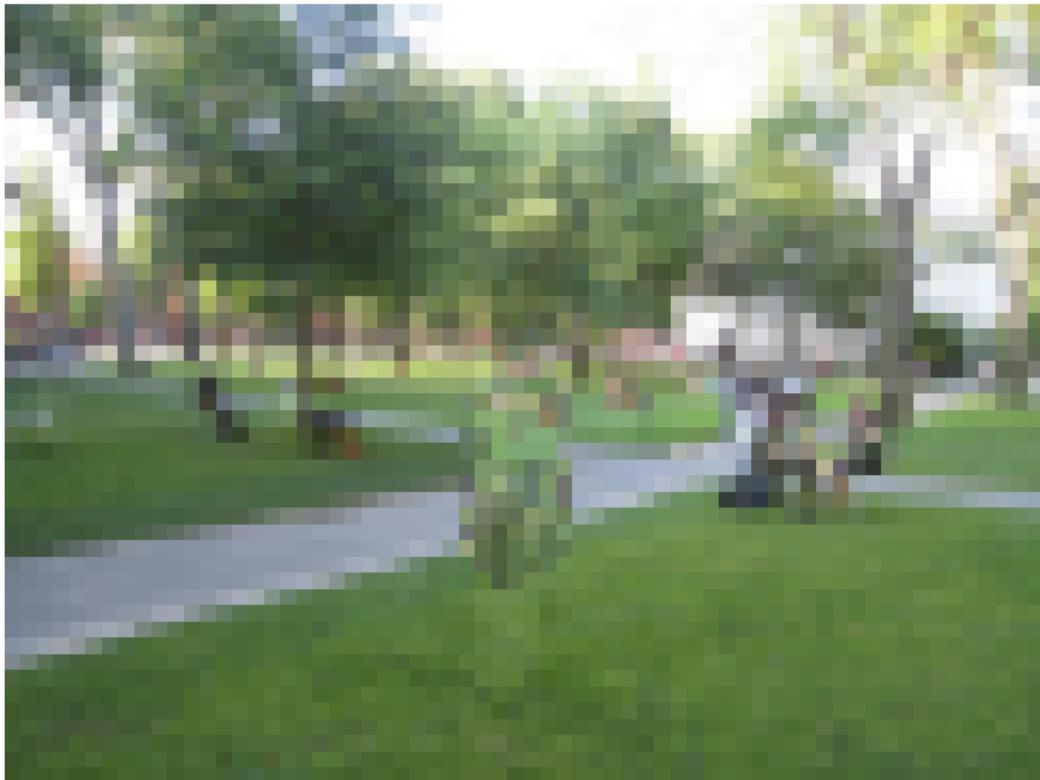
Mask

Image of size 48x64



Guess

Image of size 48x64



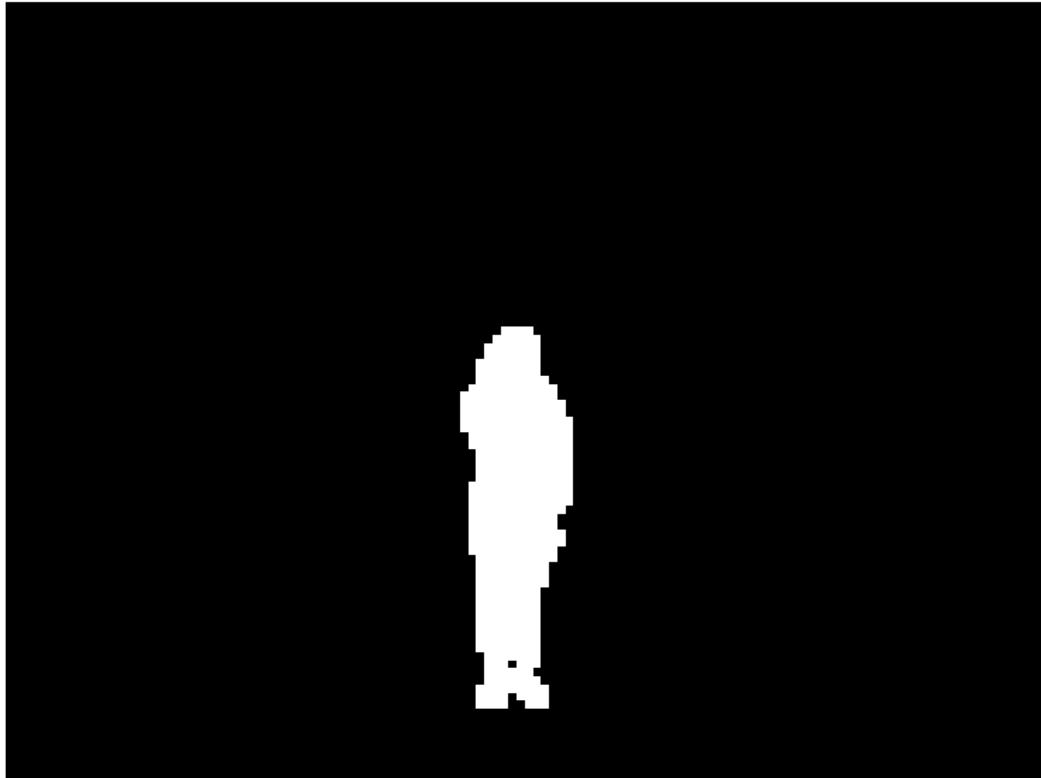
Output

Image of size 48x64



Input

Image of size 96x128



Mask

Image of size 96x128



Guess

Image of size 96x128



Output

Image of size 96x128



Input

Image of size 192x256



Mask

Image of size 192x256



Guess

Image of size 192x256



Output

Image of size 192x256

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Input

Image of size 384x512



Mask

Image of size 384x512



Guess

Image of size 384x512

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Output

Image of size 384x512

Stable Region Correspondences
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Input

Image of size 768x1024



Mask

Image of size 768x1024

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Guess

Image of size 768x1024

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Output

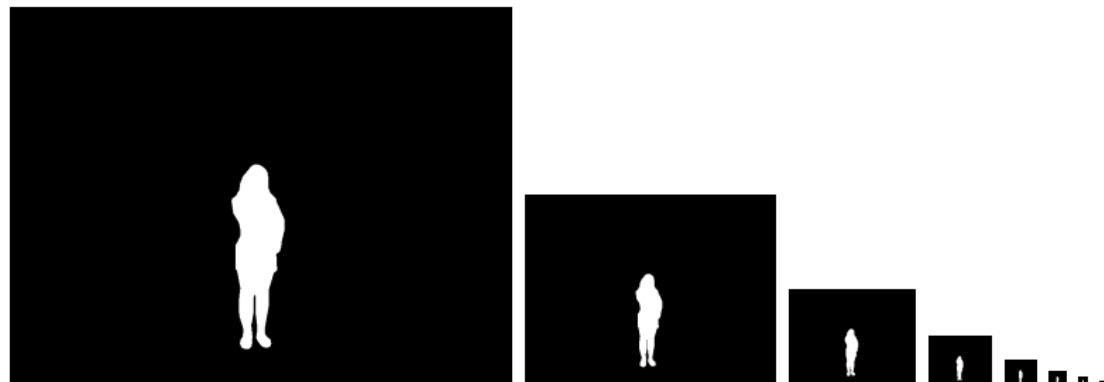
Image of size 768x1024

Multiscale approach :



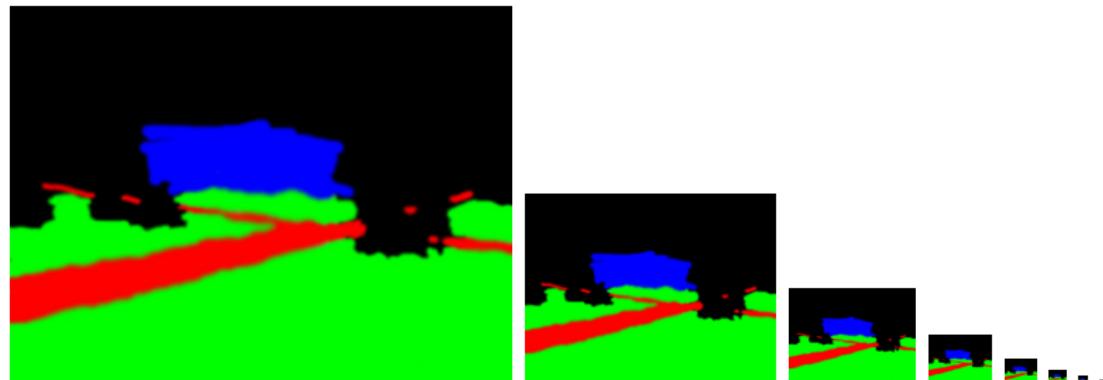
- ➊ Recursively divide image size by two
- ➋ Use smaller solution to deduce a "best guess"
- ➌ Compute solution **using a "hint"**

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- ➊ Recursively divide image size by two
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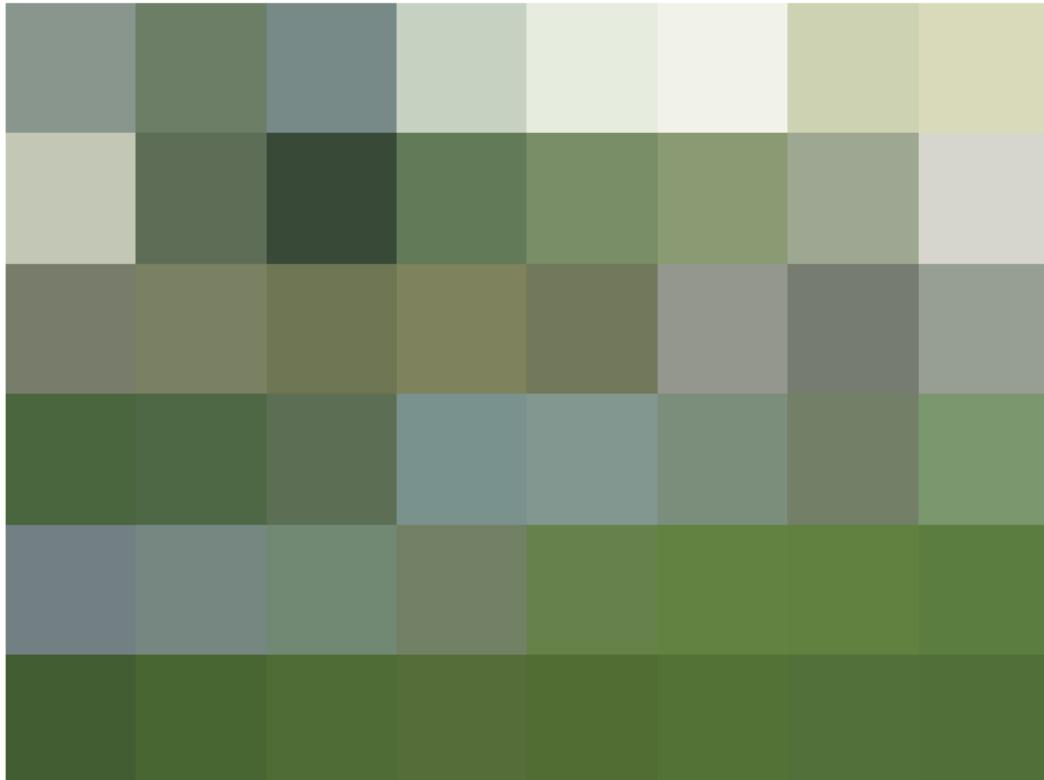
Multiscale approach :



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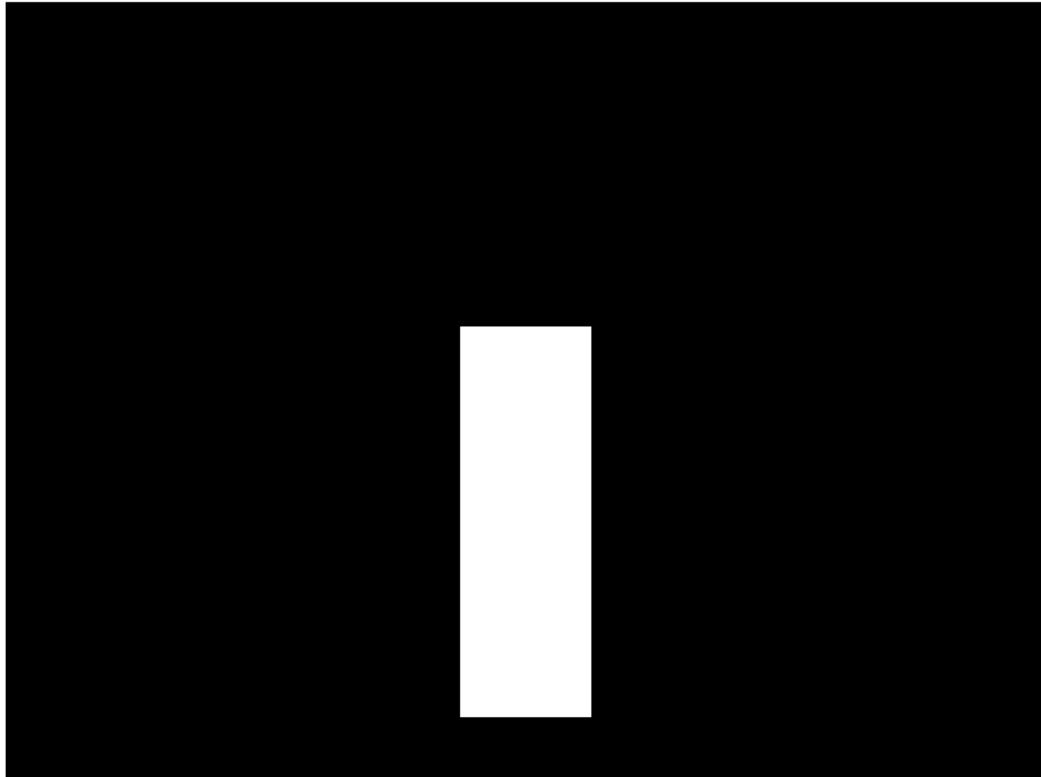
Base case

Image of size 6x8



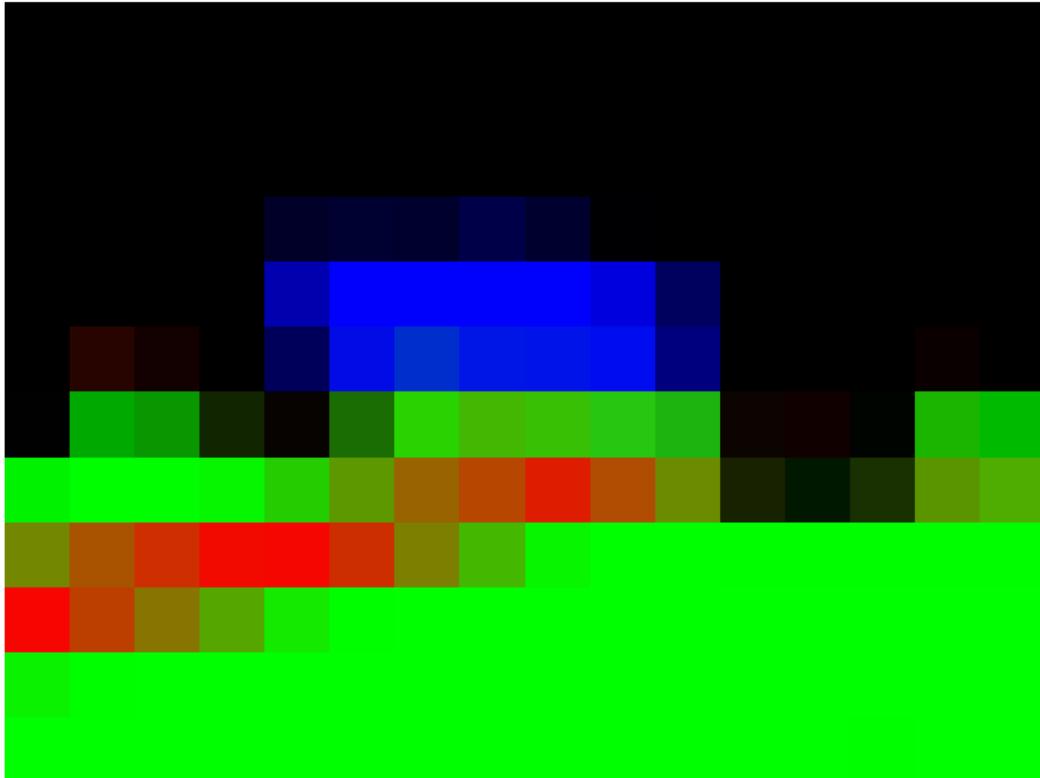
Input

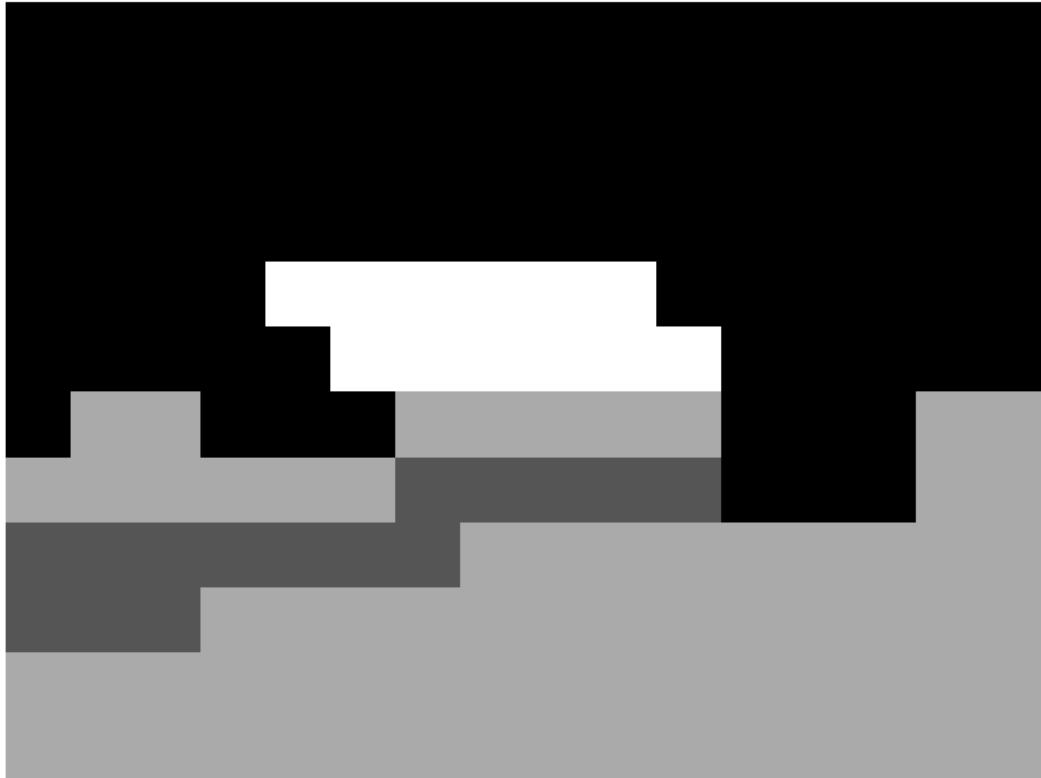
Image of size 12x16



Mask

Image of size 12x16





Hint clustering

Image of size 12x16



Guess

Image of size 12x16



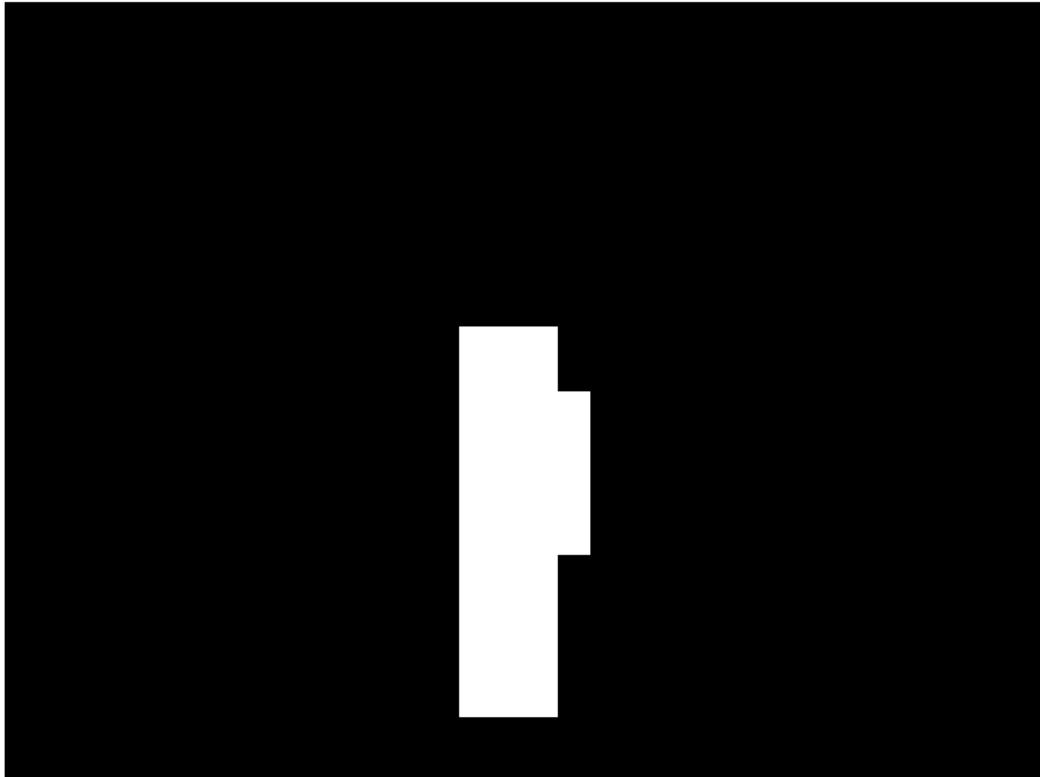
Output

Image of size 12x16



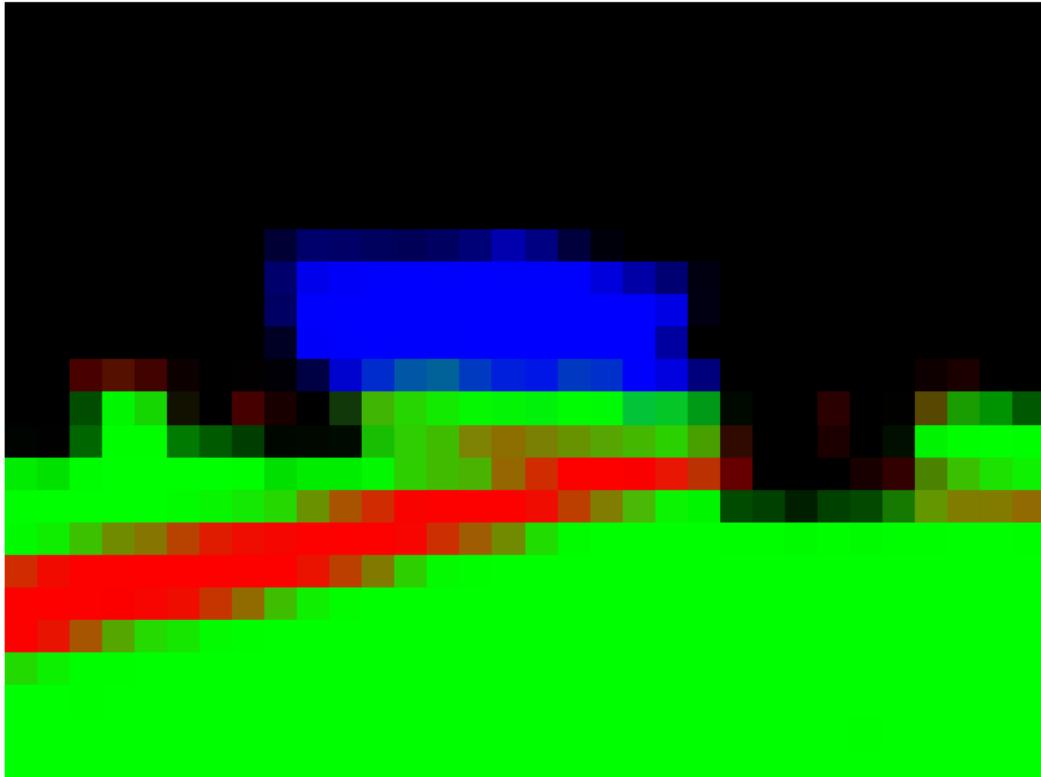
Input

Image of size 24x32



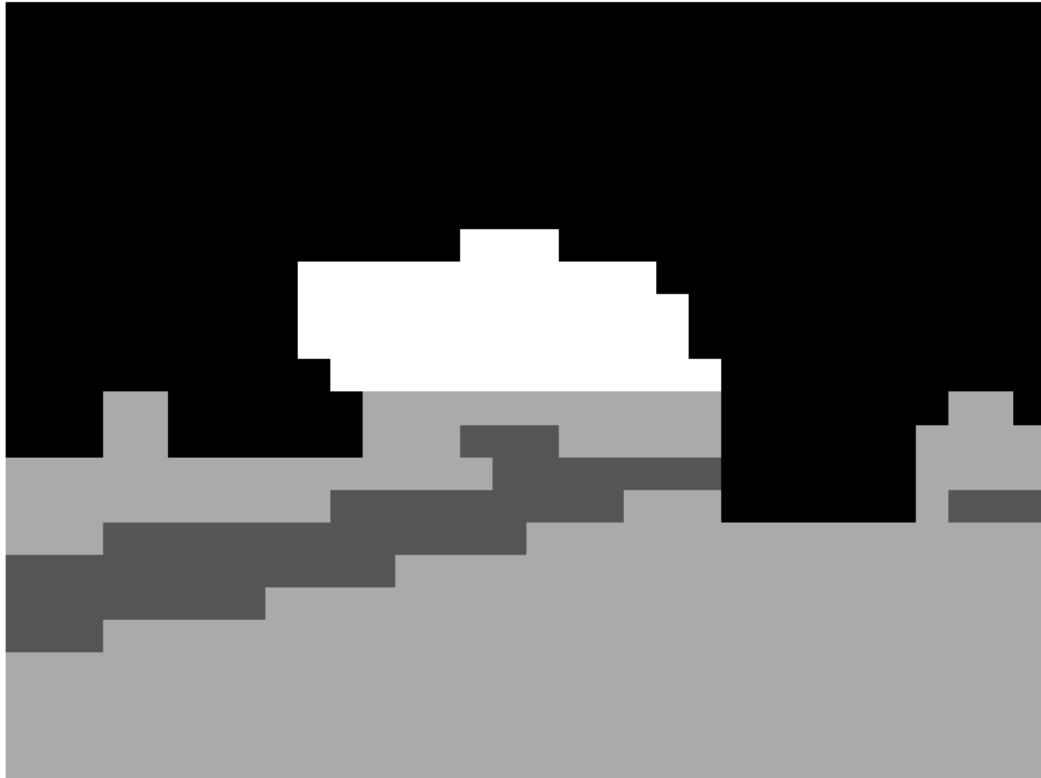
Mask

Image of size 24x32



Hint

Image of size 24x32



Hint clustering

Image of size 24x32



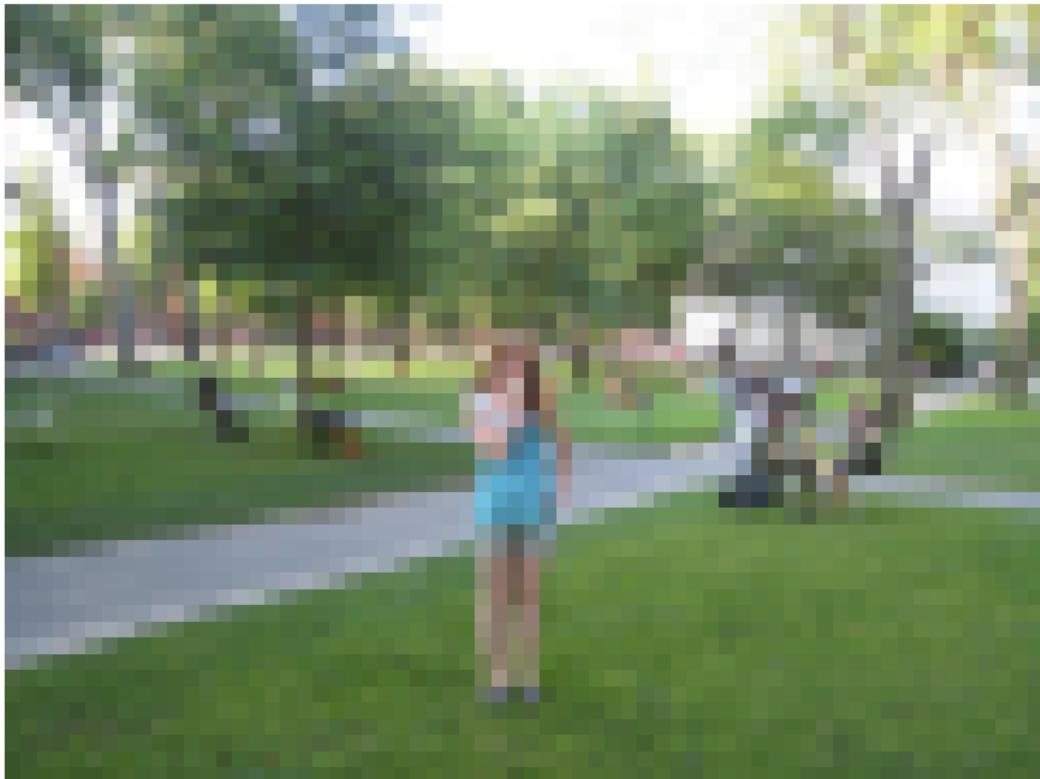
Guess

Image of size 24x32



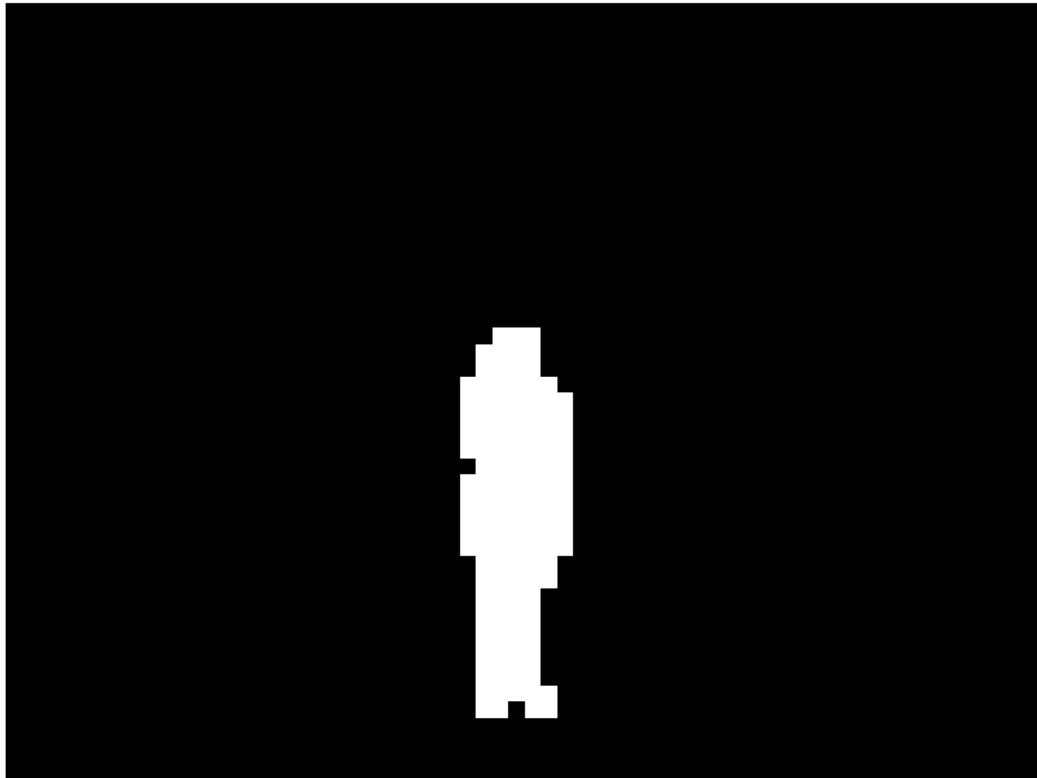
Output

Image of size 24x32



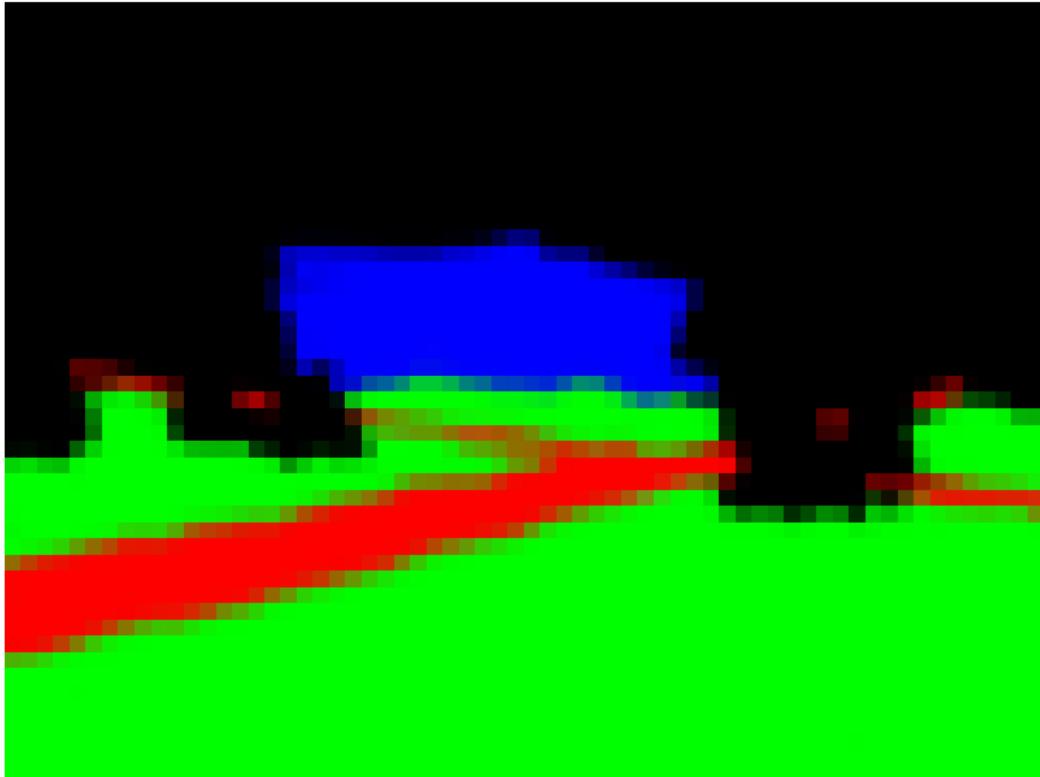
Input

Image of size 48x64



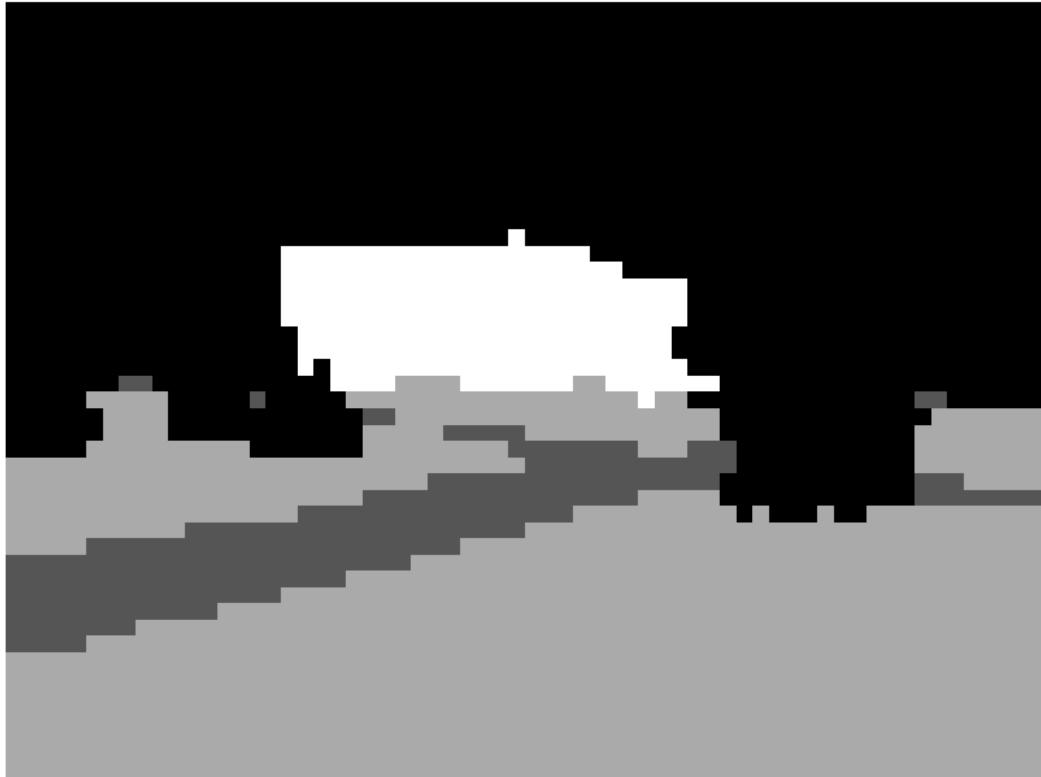
Mask

Image of size 48x64



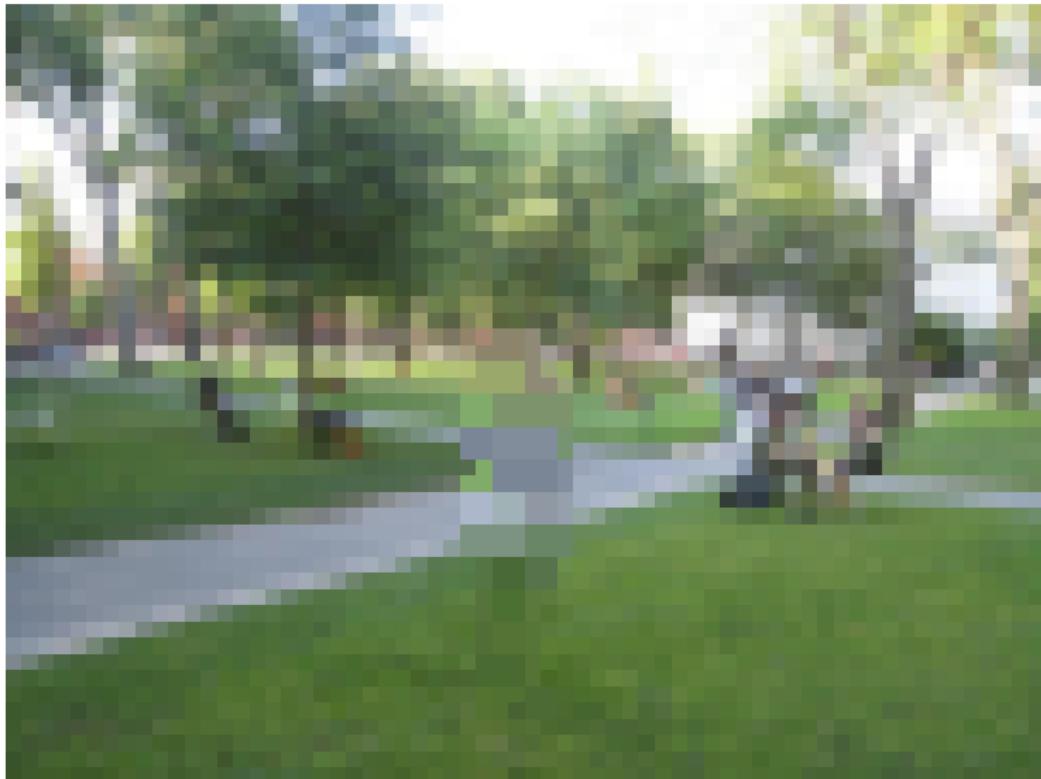
Hint

Image of size 48x64



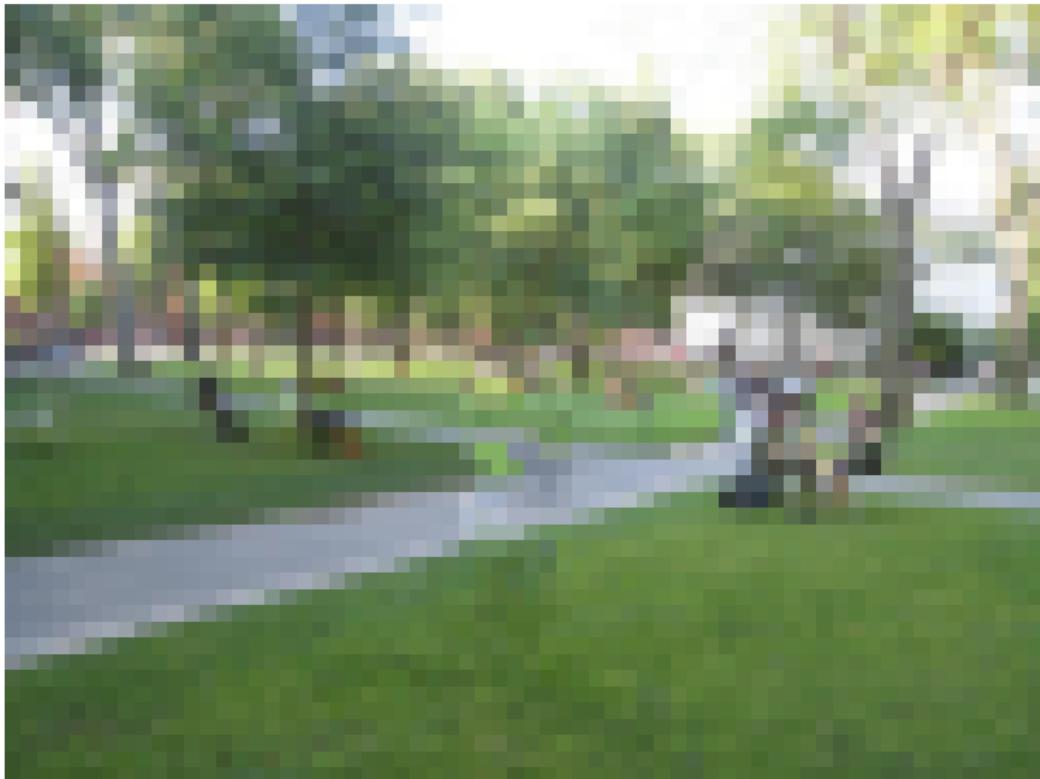
Hint clustering

Image of size 48x64



Guess

Image of size 48x64



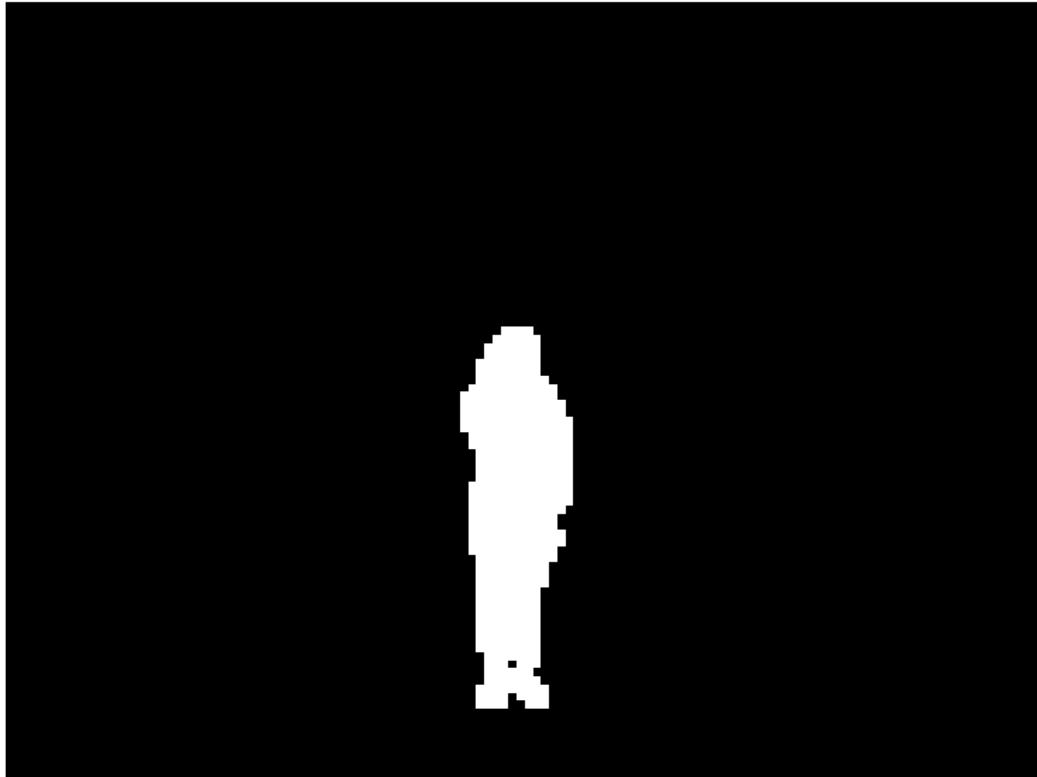
Output

Image of size 48x64



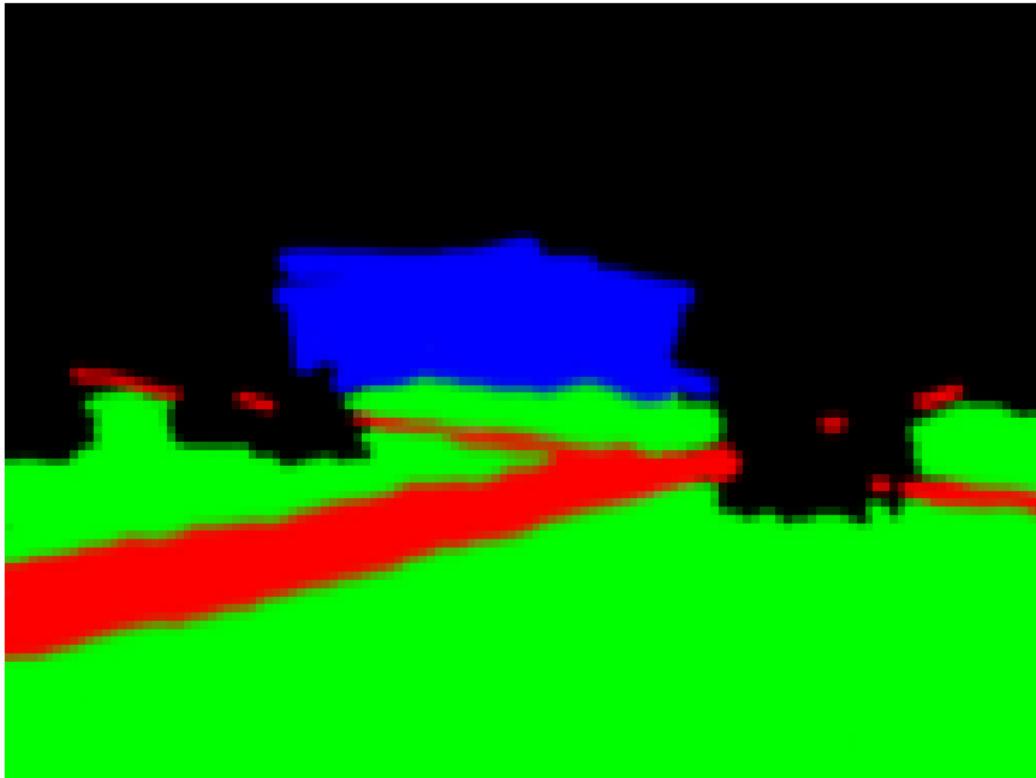
Input

Image of size 96x128



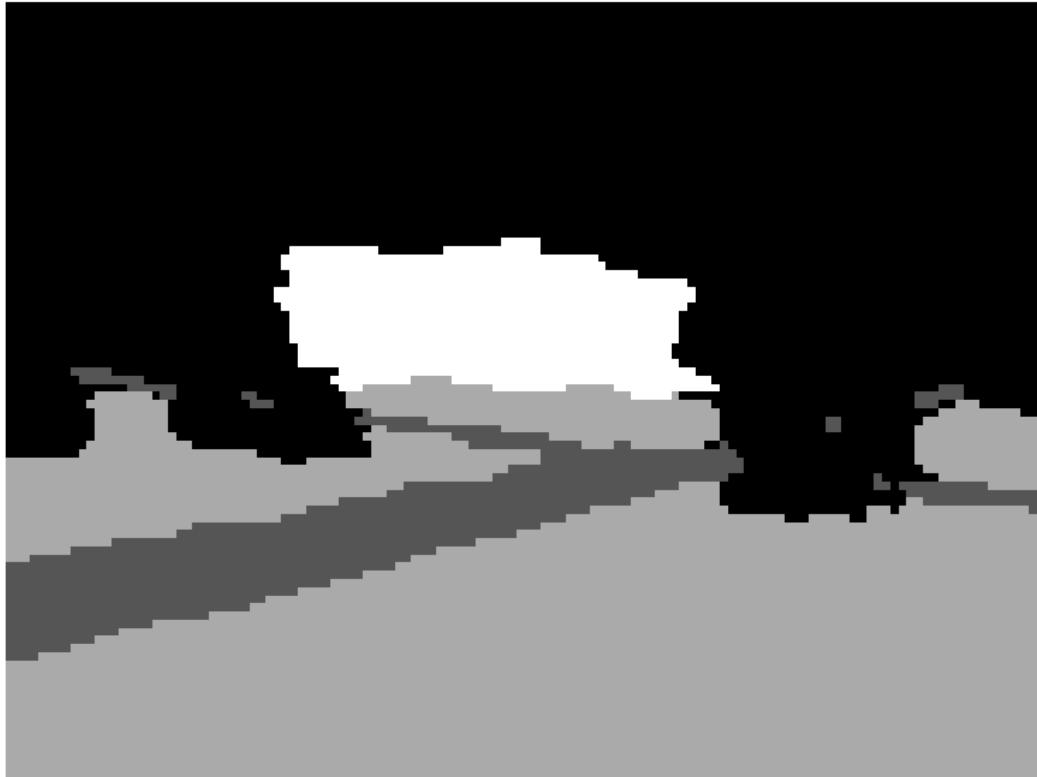
Mask

Image of size 96x128



Hint

Image of size 96x128



Hint clustering

Image of size 96x128



Guess

Image of size 96x128



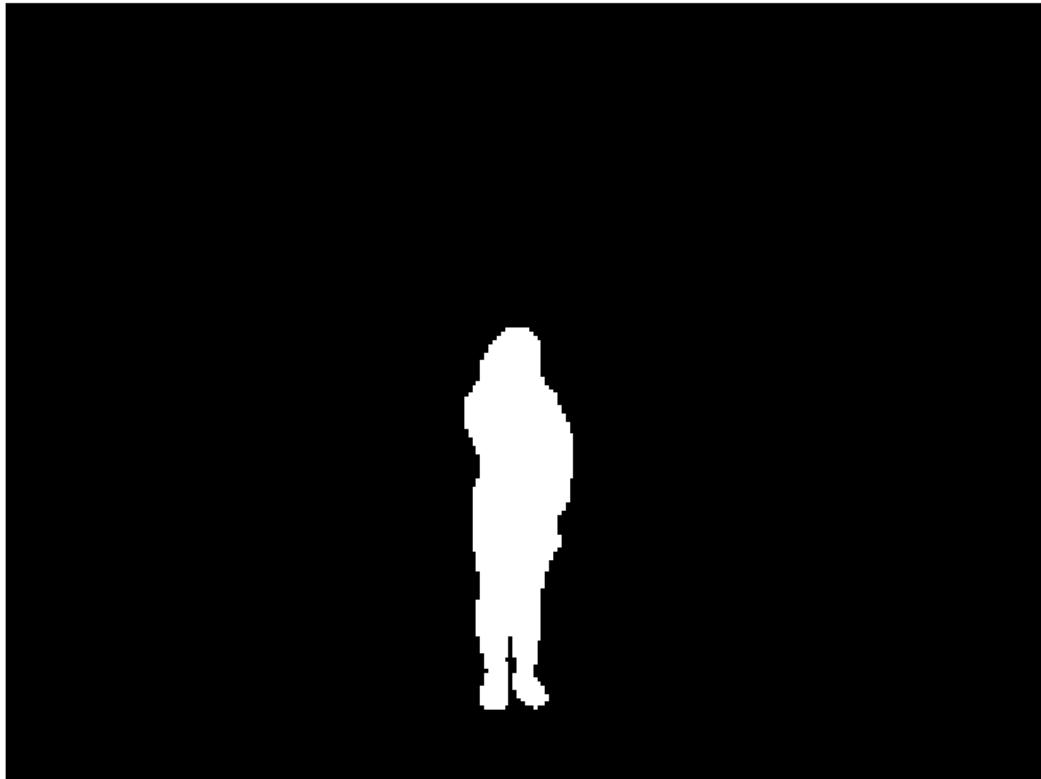
Output

Image of size 96x128



Input

Image of size 192x256

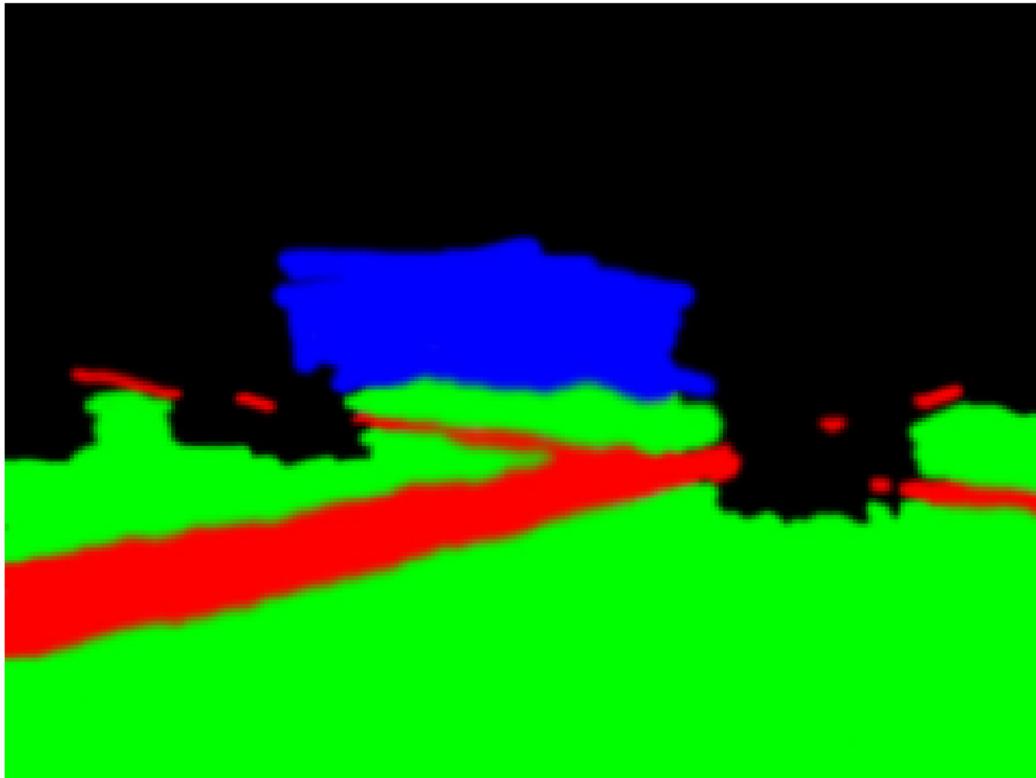


Mask

Image of size 192x256

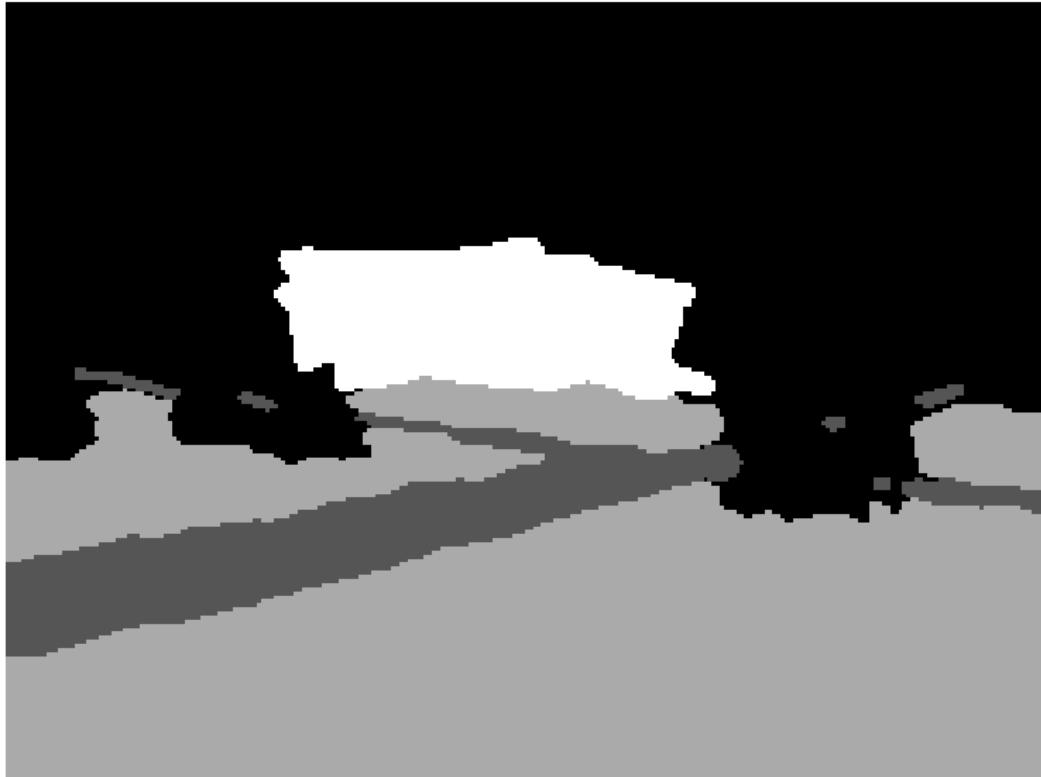
Stable Region Correspondences
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Seam Carving

Goal
Basic approach
Using a "hint"



Hint

Image of size 192x256



Hint clustering

Image of size 192x256



Guess

Image of size 192x256



Output

Image of size 192x256

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Input

Image of size 384x512

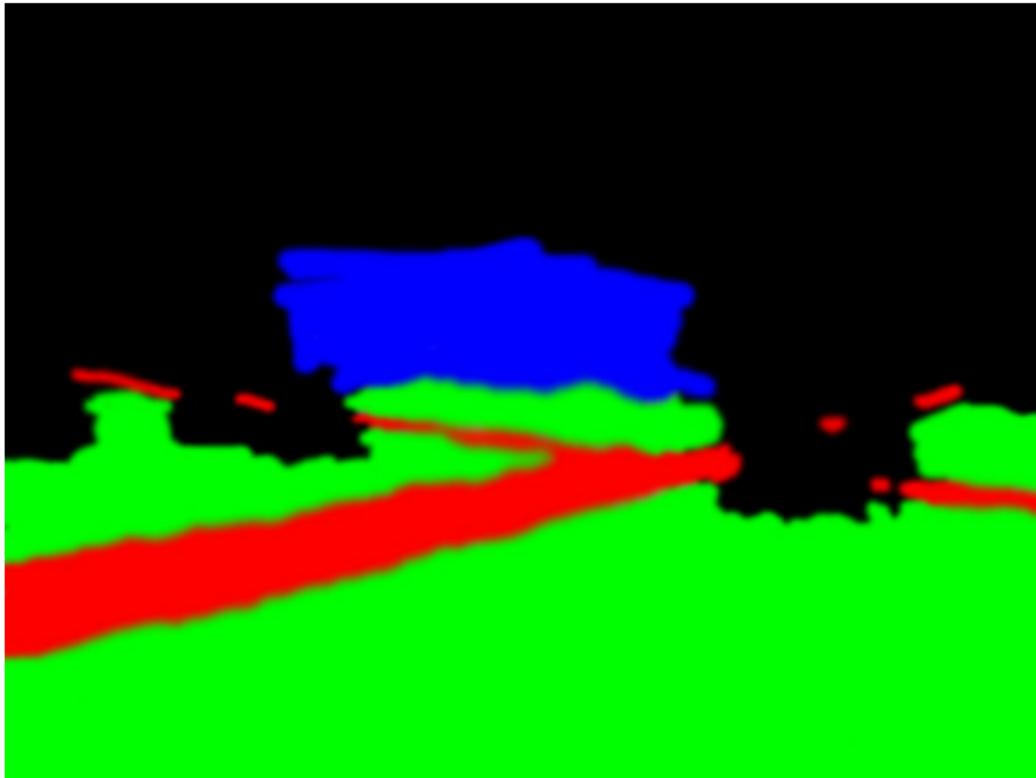


Mask

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Input

Image of size 768x1024

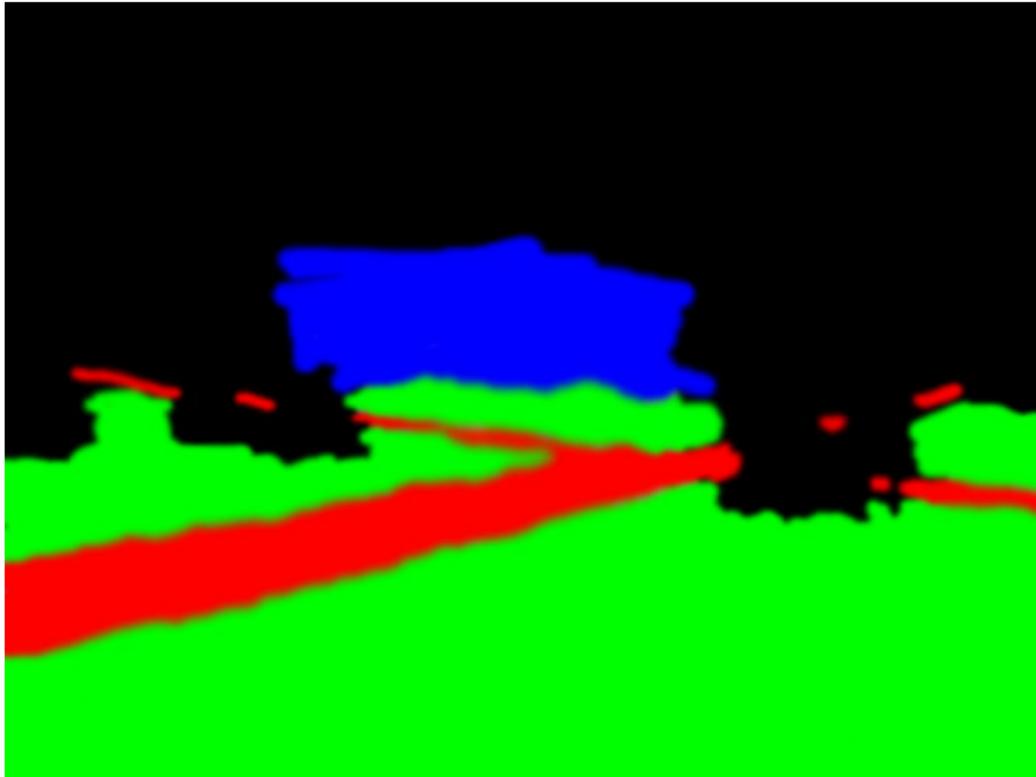


Mask

Image of size 768x1024

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Hint clustering

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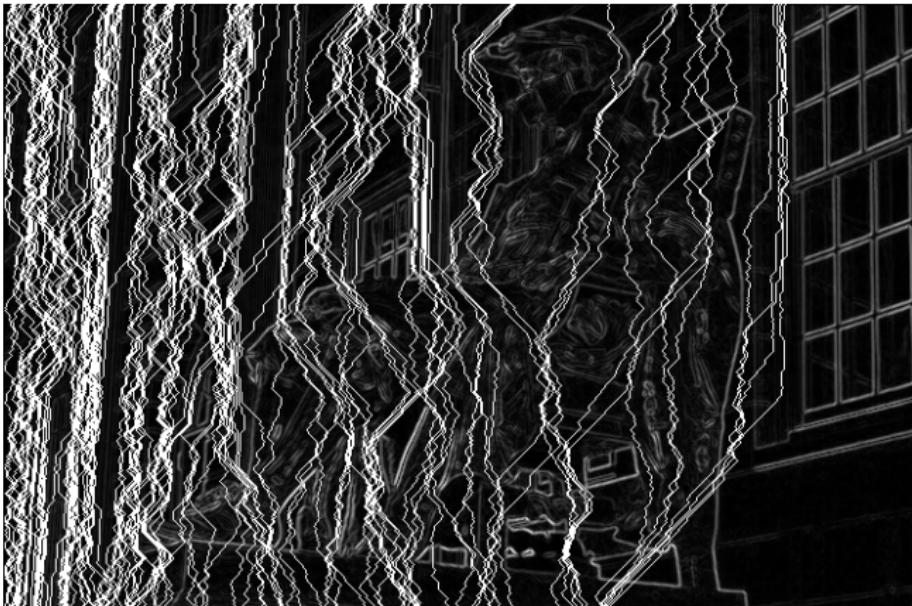


Output

Image of size 768x1024



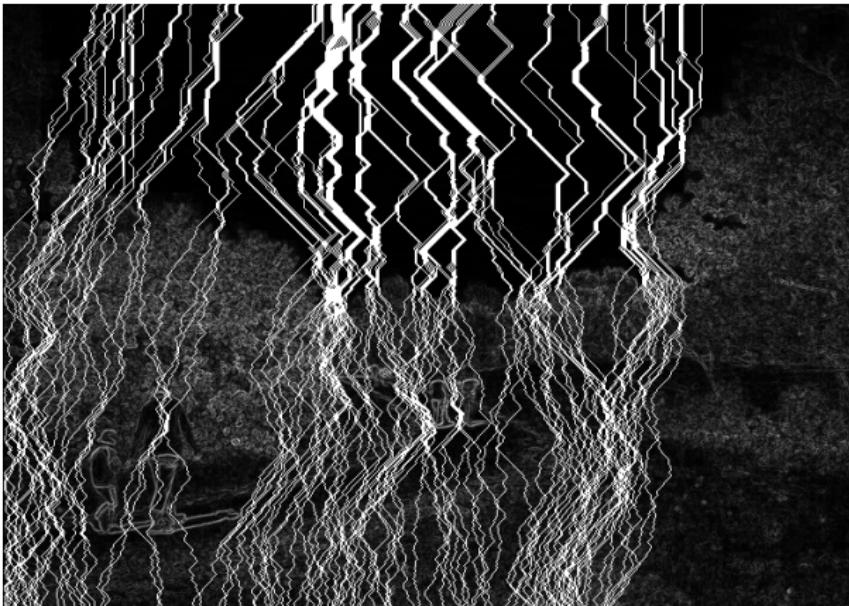






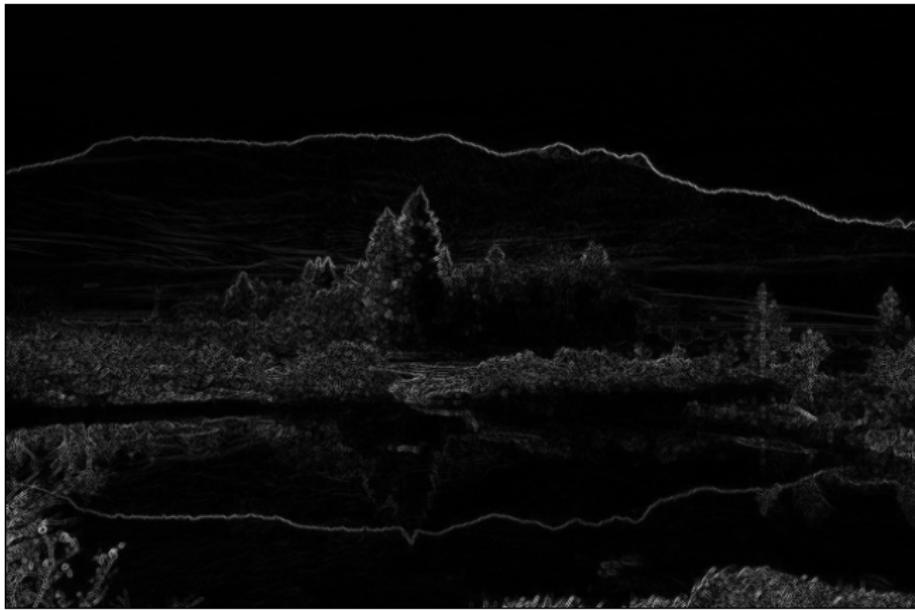


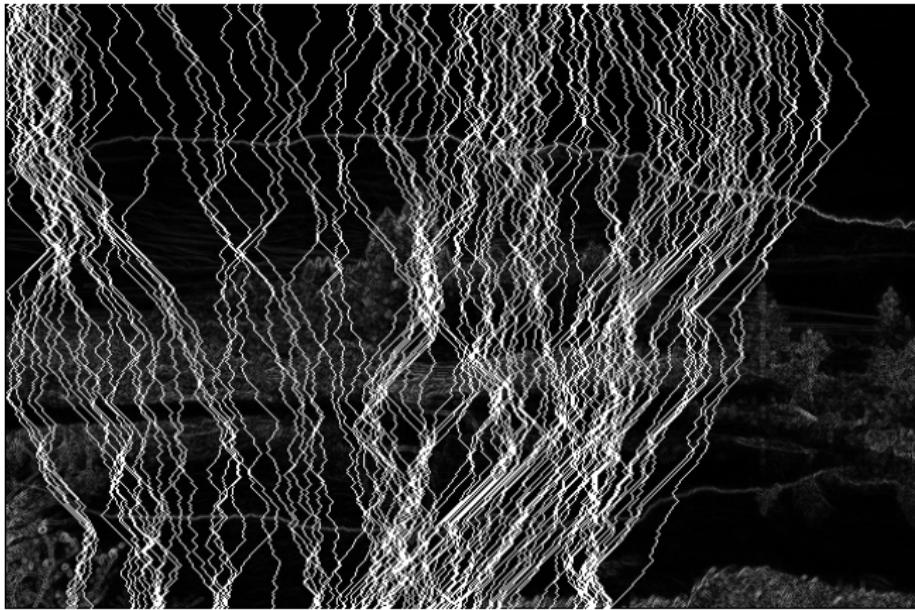














Thank you for your attention!