Introducción a la Inteligencia Artificial
Clase 7

.UBA fiuba
FACULTAD DE INGENIERÍA

Gaussian Mixture Models GMM tMM

$$p(x/\bar{\mu},\bar{\bar{\Sigma}}) = \sum_i \pi_i \mathcal{N}(x/\mu_i,\sigma_i^2)$$

Formulación

$f(x) \propto \sum g(x)$

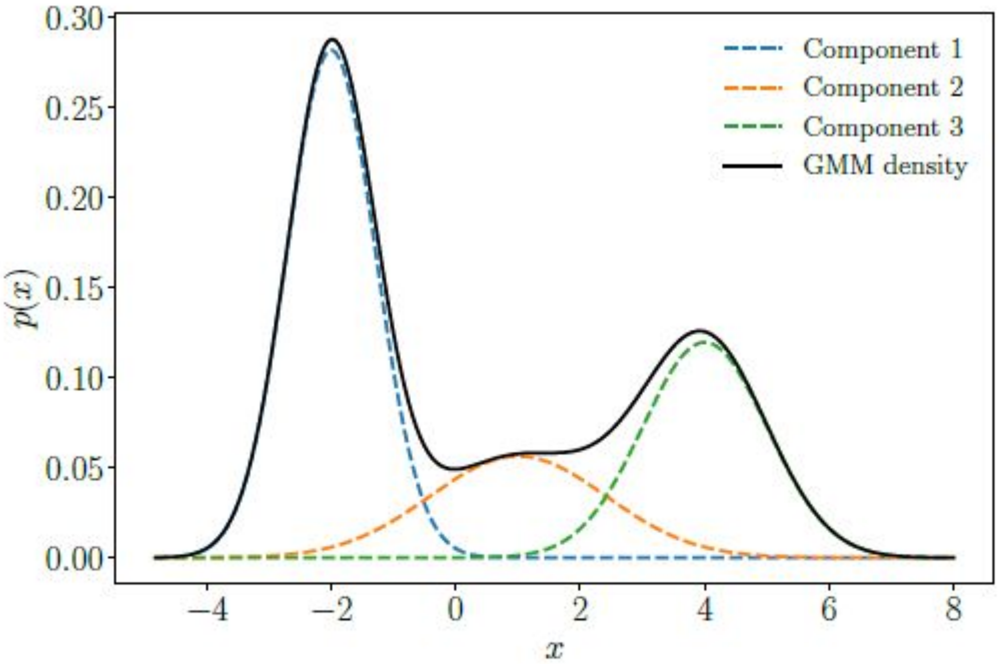
parametros:

$\vec{\mu}$: medias de las comp

$\bar{\bar{\Sigma}}$: matriz de cov.

π_i: peso de la partición

K: cant. de particiones



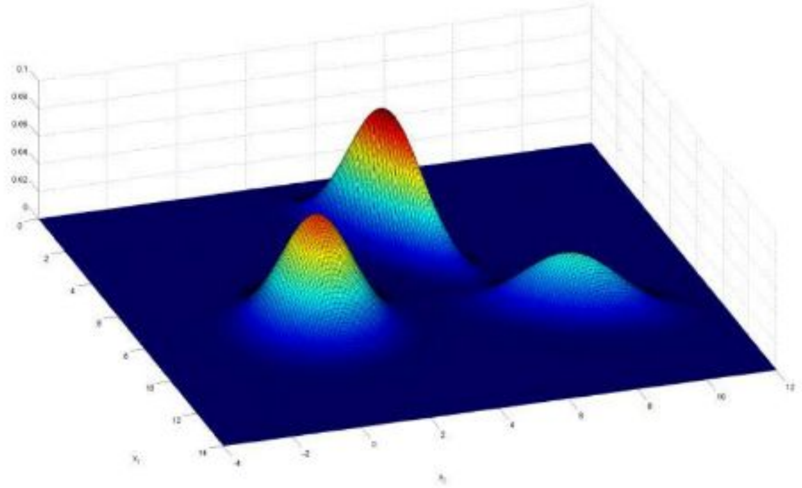
$$p(x\,|\,\boldsymbol{\theta}) = 0.5\mathcal{N}(x\,|\,-2,\tfrac{1}{2}) + 0.2\mathcal{N}(x\,|\,1,2) + 0.3\mathcal{N}(x\,|\,4,1)$$

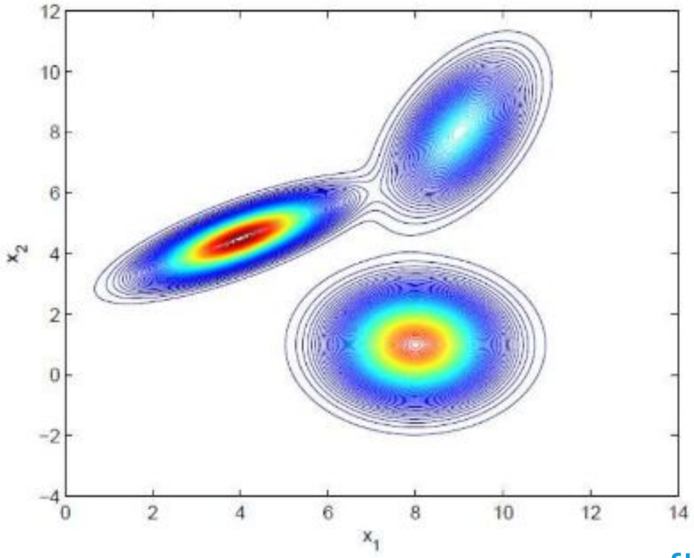
Gaussian Mixture Models

Formulación

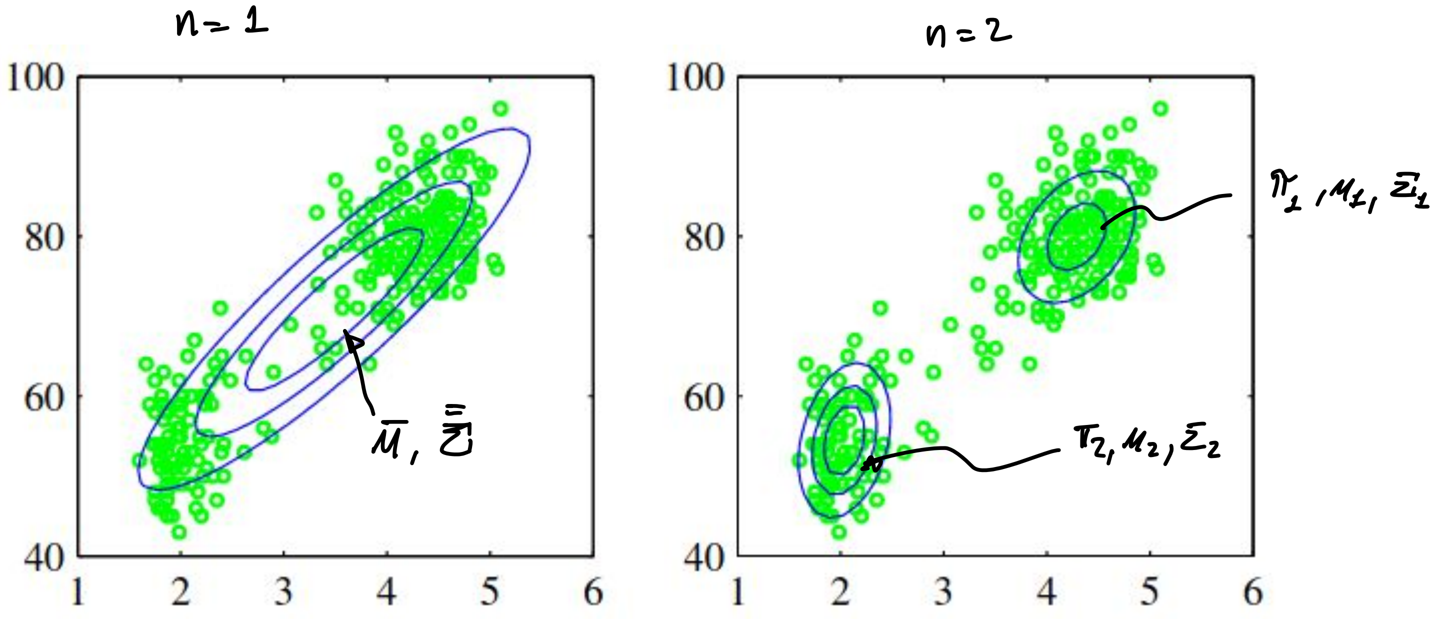
$\sum \pi_i = 1$, puedo forzar la forma de Σ_i

$$p(x) = \underbrace{0.3}_{\pi_1} \mathcal{N}\left(x \,\middle|\, \underbrace{\begin{pmatrix} 4 \\ 4.5 \end{pmatrix}}_{\mu_1}, \underbrace{\begin{pmatrix} 1.2 & 0.6 \\ 0.6 & 0.5 \end{pmatrix}}_{\Sigma_1}\right) + \underbrace{0.5}_{\pi_2} \mathcal{N}\left(x \,\middle|\, \underbrace{\begin{pmatrix} 8 \\ 1 \end{pmatrix}}_{\mu_2}, \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\Sigma_2}\right) + \underbrace{0.2}_{\pi_3} \mathcal{N}\left(x \,\middle|\, \underbrace{\begin{pmatrix} 9 \\ 8 \end{pmatrix}}_{\mu_3}, \underbrace{\begin{pmatrix} 0.6 & 0.5 \\ 0.5 & 1.5 \end{pmatrix}}_{\Sigma_3}\right)$$



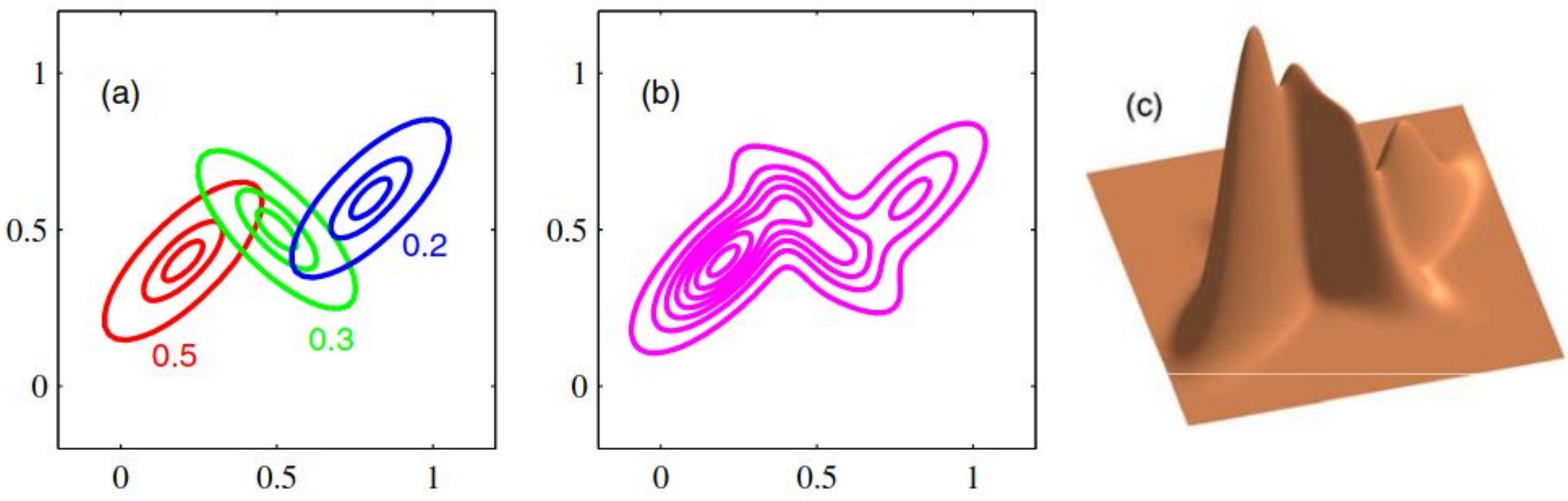


Gaussian Mixture Models: Estudio de fenómenos naturales



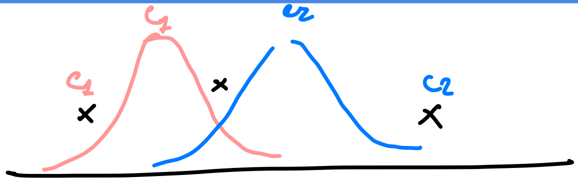
"Old Faithful" dataset. 272 mediciones de erupciones del "Old Faithful" geyser en el Parque Nacional Yellowstone. El eje horizontal representa la duración de una erupción (medida en minutos) y el vertical el tiempo hasta la próxima erupción.

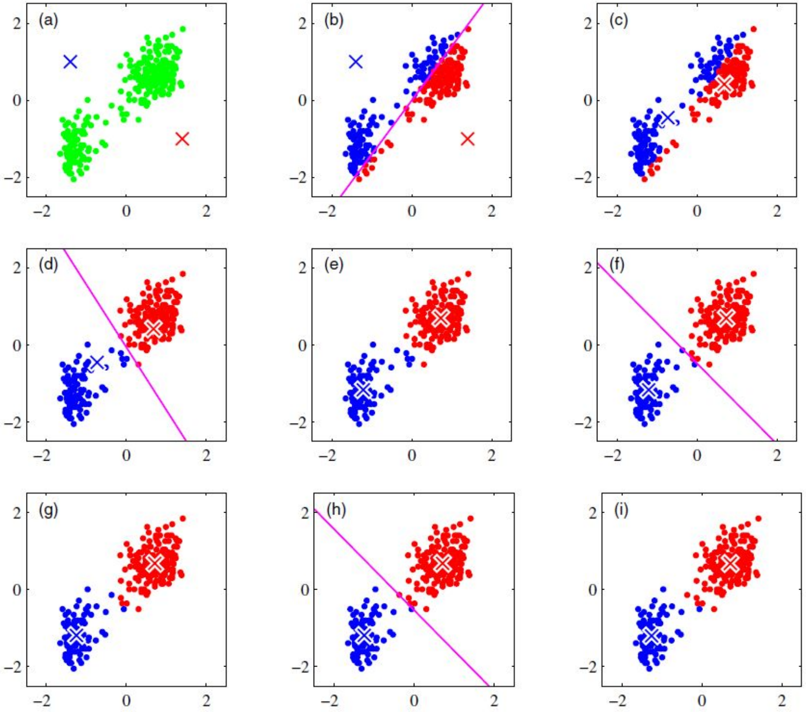
Gaussian Mixture Models: Clustering



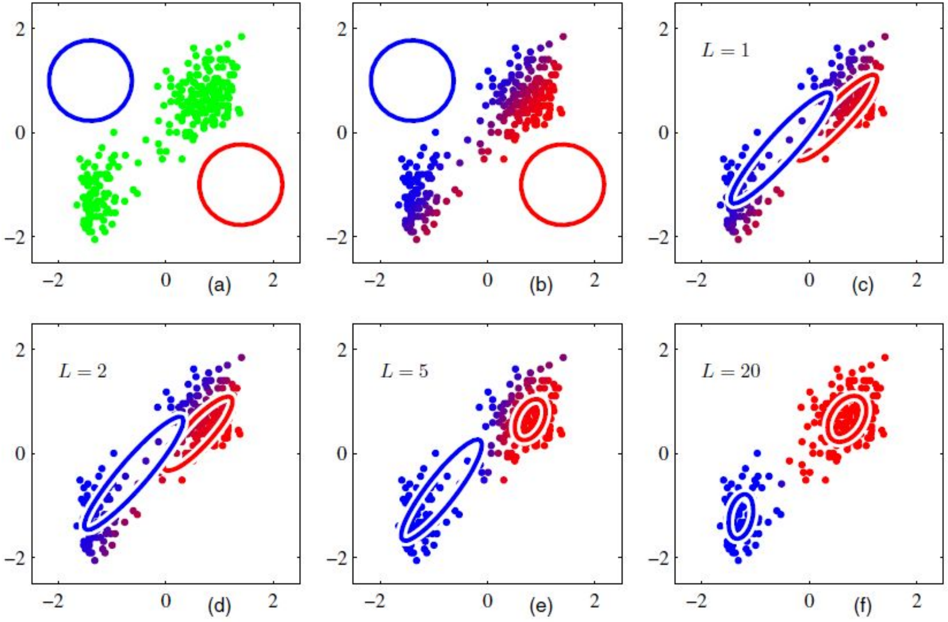
Ejemplo de Gaussian Mixture. En la imagen (a) se muestran las tres distribuciones subyacentes indicando con colores sus variables latentes. En la imagen (b) las curvas de nivel de la distribución conjunta y en la (c) la densidad.

Gaussian Mixture Models: Clustering



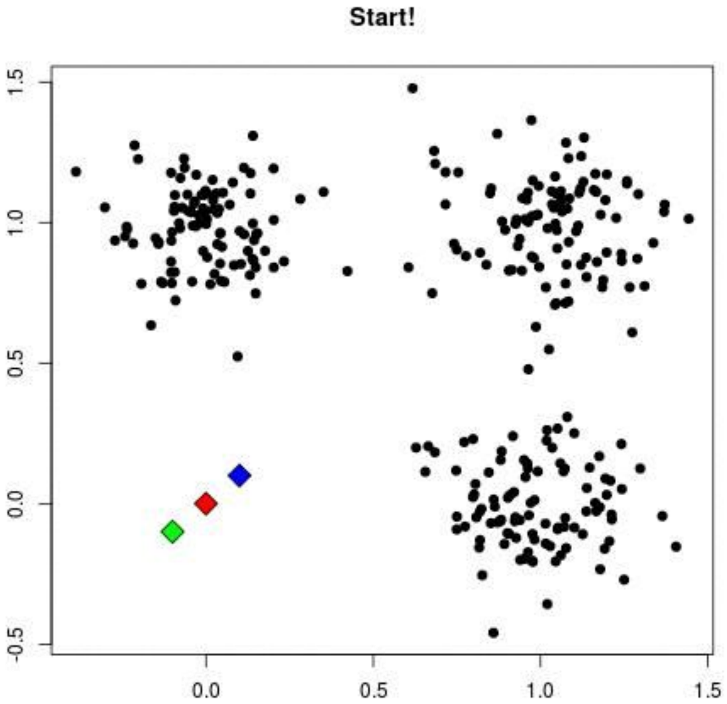


KMeans

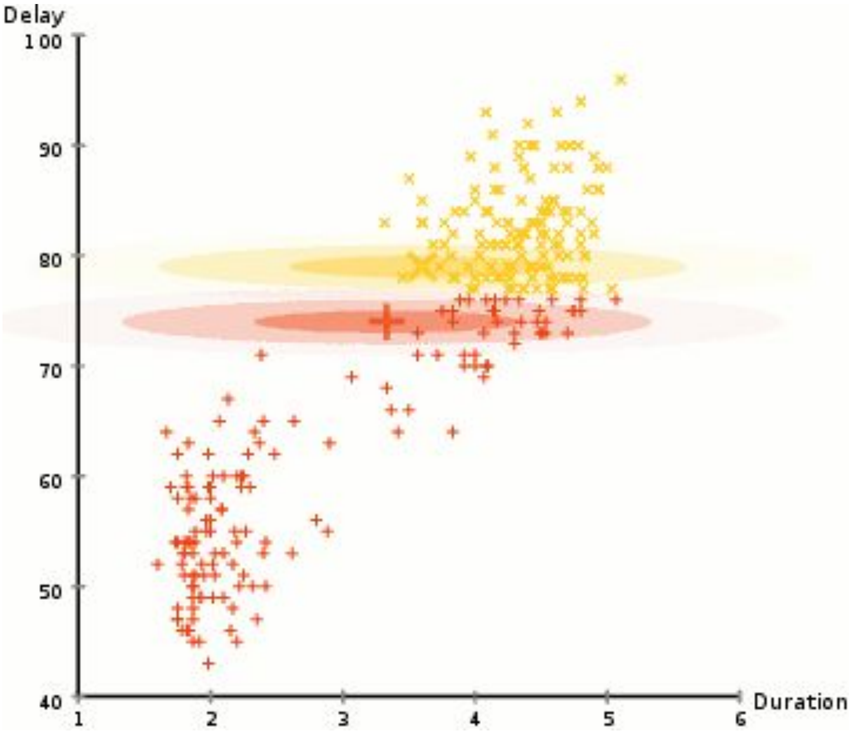


GMM

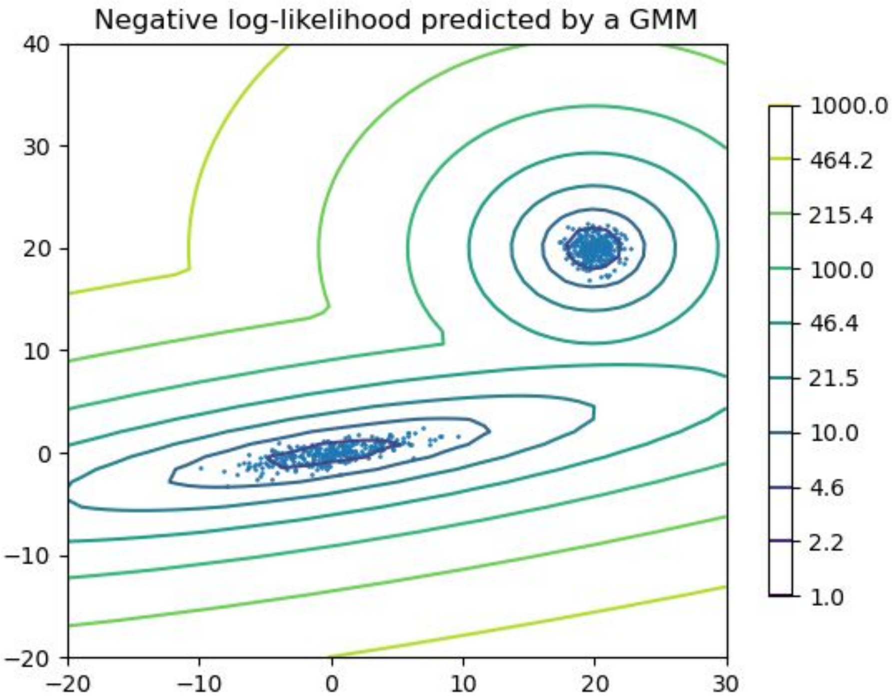
Gaussian Mixture Models - kMeans



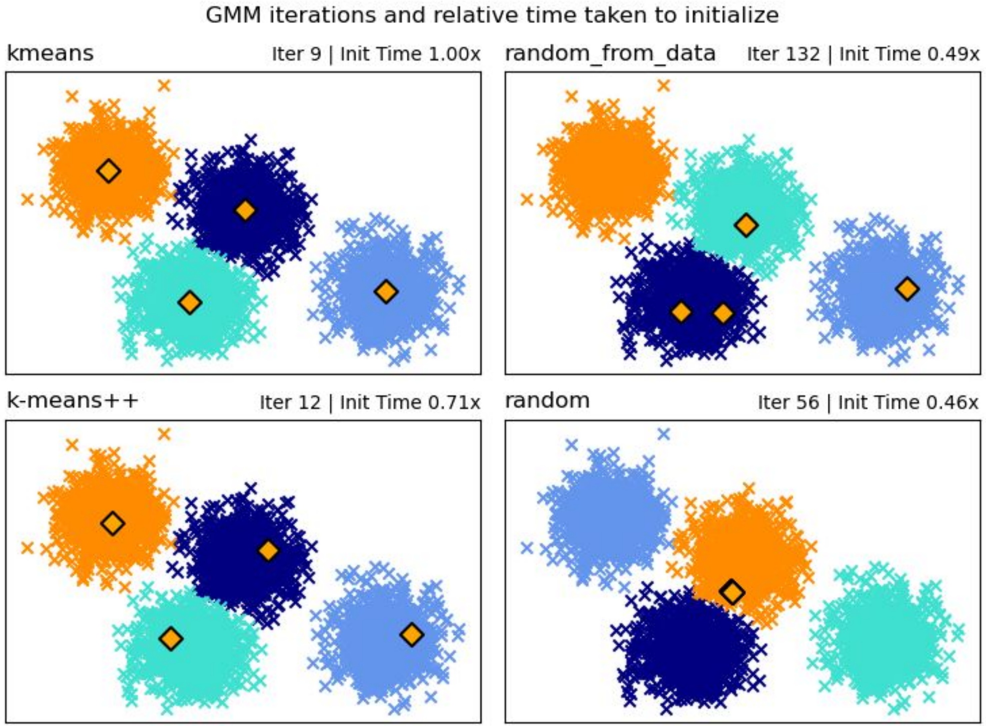
Gaussian Mixture Models: Clustering



Gaussian Mixture Models: Detección de anomalías



Gaussian Mixture Models: Inicialización



Formulación

$$p(\boldsymbol{x}) = \sum_{k=1}^{K} \pi_k p_k(\boldsymbol{x})$$

$$0 \leqslant \pi_k \leqslant 1\,, \quad \sum_{k=1}^{K} \pi_k = 1\,,$$

Mixture Models - General

$$p(\boldsymbol{x} \,|\, \boldsymbol{\theta}) = \sum_{k=1}^{K} \pi_k \mathcal{N}\big(\boldsymbol{x} \,|\, \boldsymbol{\mu}_k,\, \boldsymbol{\Sigma}_k\big)$$

$$0 \leqslant \pi_k \leqslant 1\,, \quad \sum_{k=1}^{K} \pi_k = 1\,,$$

$$\boldsymbol{\theta} := \{\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k, \pi_k : \; k = 1, \ldots, K\}$$

Gaussian Mixture Models

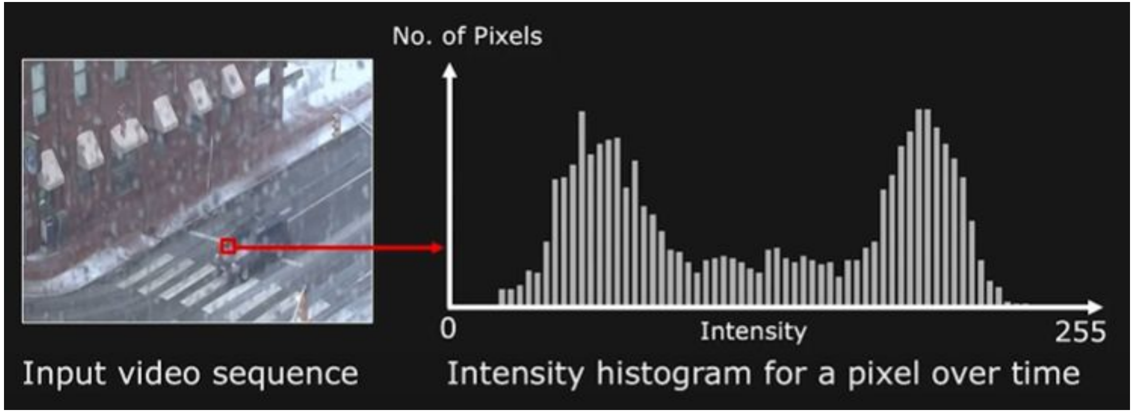
GMM se optimiza por metodo EM

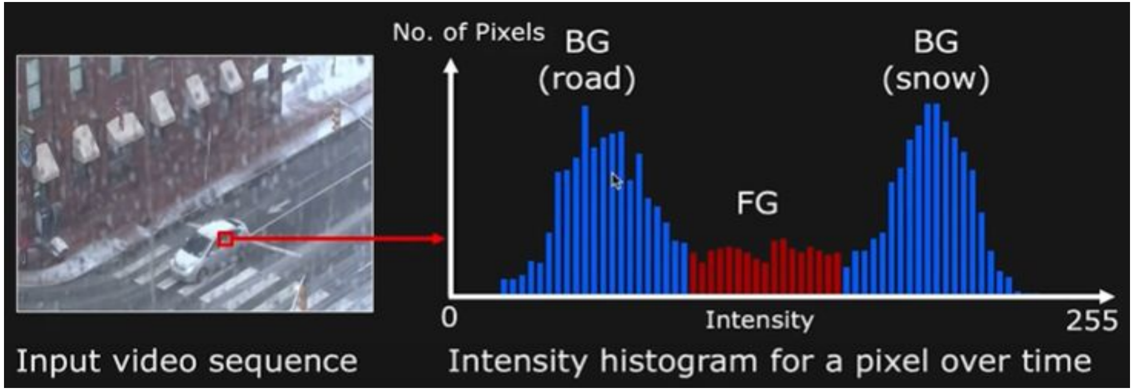
Expectation Maximization.

Gaussian Mixture Models - Object Tracking

pixel (R, G, B) → (255, 0, 0)

pixel (0 – 255)





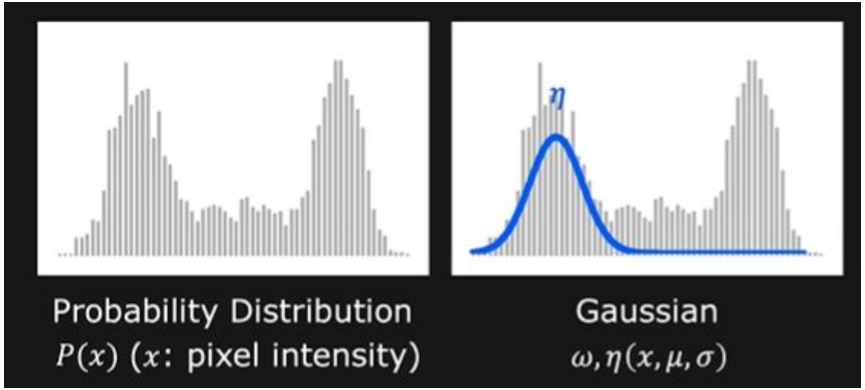
Gaussian Mixture Models - Object Tracking

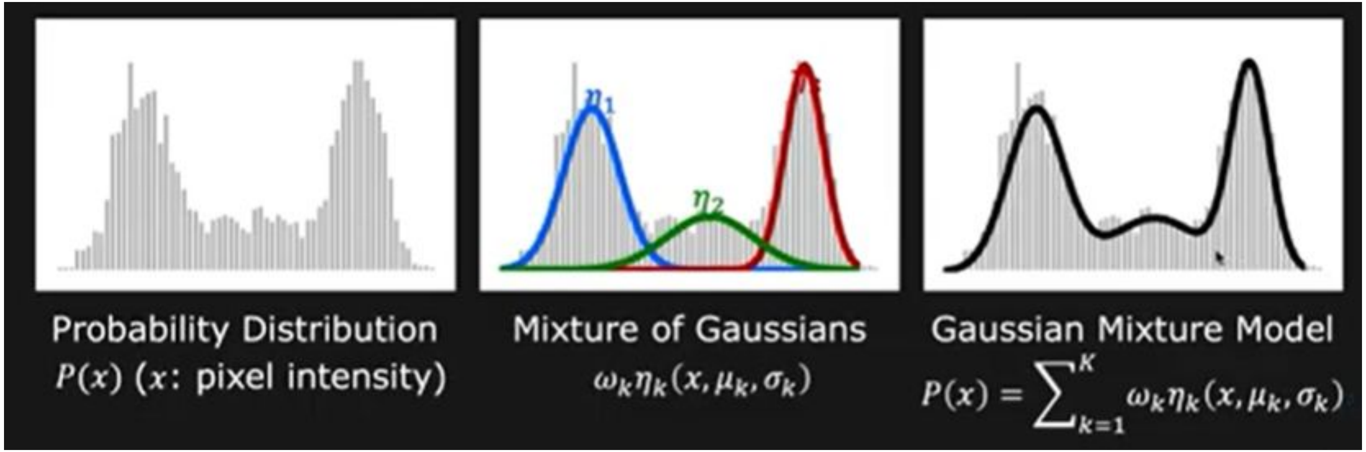
$$\boxed{P(\mathbf{X}) \cong \sum_{k=1}^{K} \omega_k \eta_k(\mathbf{X}, \boldsymbol{\mu}_k, \Sigma_k)} \quad \text{such that} \quad \sum_{k=1}^{K} \omega_k = 1$$

where: $$\boxed{\eta(\mathbf{X}, \boldsymbol{\mu}, \Sigma) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(\mathbf{X}-\boldsymbol{\mu})^T(\Sigma)^{-1}(\mathbf{X}-\boldsymbol{\mu})}}$$

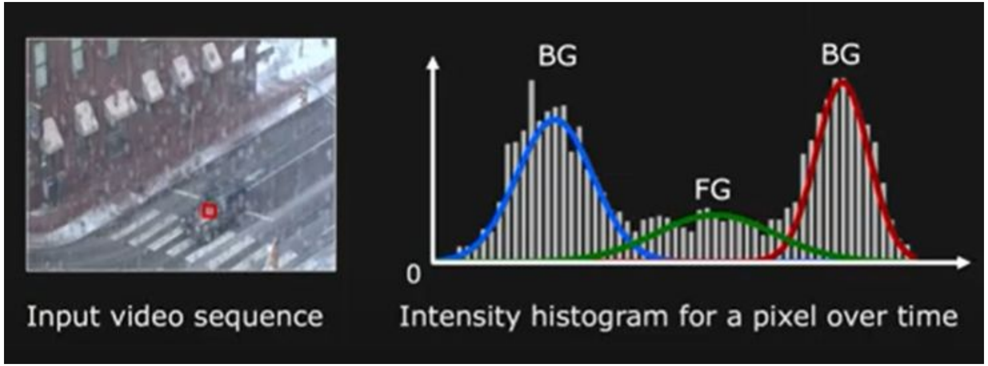
Mean $\boldsymbol{\mu} = \begin{bmatrix} \mu_r \\ \mu_g \\ \mu_b \end{bmatrix}$ Covariance matrix $\Sigma = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$ (can be a full matrix)

Gaussian Mixture Models - Object Tracking





Gaussian Mixture Models - Object Tracking



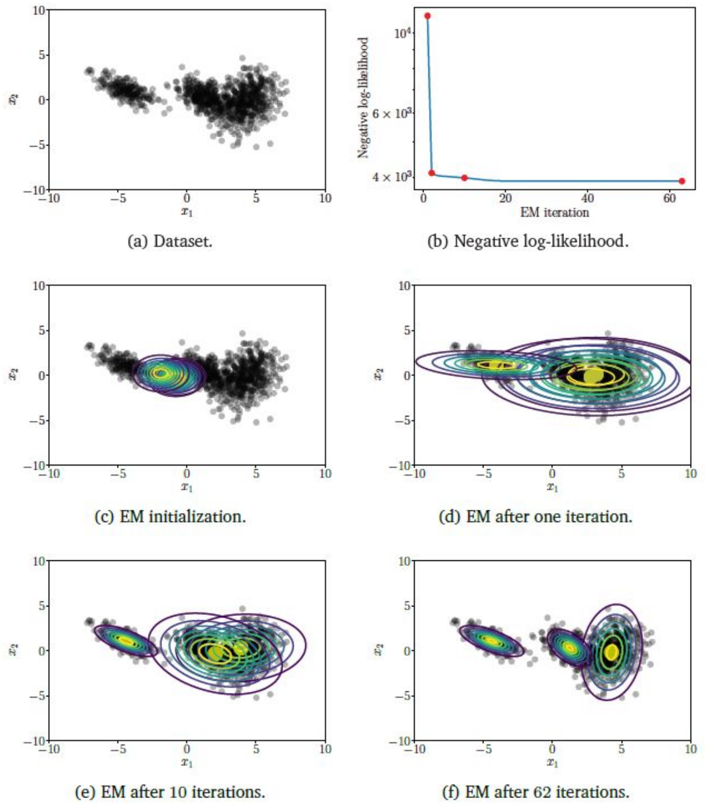
$\uparrow \dfrac{\omega}{\sigma}$ **Background** $\dfrac{\pi}{\|\Sigma\|^2}$

esto depende de un threshold

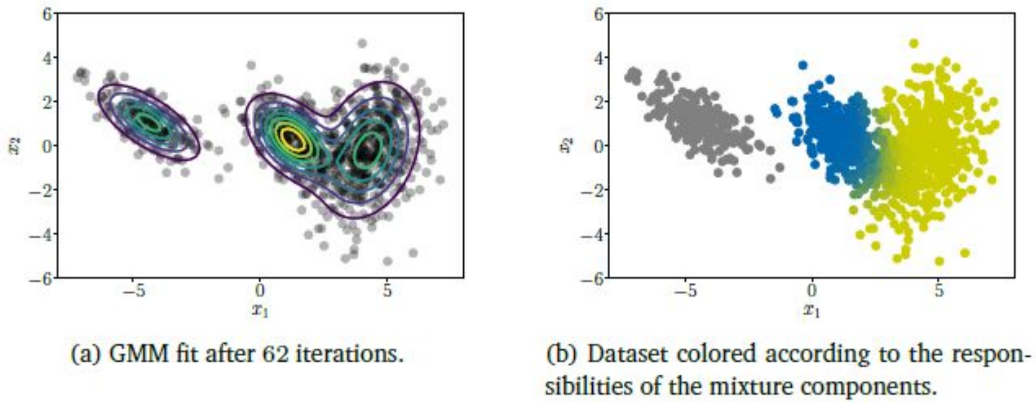
$\downarrow \dfrac{\omega}{\sigma}$ **Foreground**

GMM y EM - JAMBOARD

Gaussian Mixture Models - Teoría



(a) Dataset. (b) Negative log-likelihood. (c) EM initialization. (d) EM after one iteration. (e) EM after 10 iterations. (f) EM after 62 iterations.



(a) GMM fit after 62 iterations. (b) Dataset colored according to the responsibilities of the mixture components.

Notebooks

Bibliografía

- Mathematics for Machine Learning | Deisenroth, Faisal, Ong
- Pattern Recognition and Machine Learning | Bishop
- Gaussian Mixture Model | John McGonagle, Geoff Pilling, Andrei Dobre
- Expectation-Maximization Algorithms | Stanford CS229: Machine Learning
- First Principles of Computer Vision| Computer Science Department, School of Engineering and Applied Sciences, Columbia University