Öv9 (2)

Öv9

Bestäm en ortonormal bas för

$$V = \text{span}\left\{ \underset{\vec{u}_1}{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}}, \underset{\vec{u}_2}{\begin{bmatrix}2\\0\\1\\1\end{bmatrix}}, \underset{\vec{u}_3}{\begin{bmatrix}1\\1\\0\\1\end{bmatrix}} \right\}$$

(1) $\bar{v}_1 = \dfrac{\bar{u}_1}{|\bar{u}_1|} = \bar{u}_1 = (1,0,0,0)$, $U_1 = \{v_1\}$

(2) $\bar{v}_2 = \dfrac{\text{Perp}_{U_1}(\bar{u}_2)}{|\text{Perp}_{U_1}(\bar{u}_2)|}$, $\text{Perp}_{U_1}(\bar{u}_2) = \bar{u}_2 - \text{Proj}_{U_1}(\bar{u}_2)$

$\text{Proj}_{U_1}(\bar{u}_2) = (\bar{u}_2 \cdot \bar{v}_1) \cdot \bar{v}_1$

$= (2,0,1,1)\cdot(1,0,0,0)\cdot\bar{v}_1 = 2 \cdot \bar{v}_1 = (2,0,0,0)$.

$\text{Perp}_{U_1}(\bar{u}_2) = (2,0,1,1) - (2,0,0,0) = (0,0,1,1)$

$\bar{v}_2 = \dfrac{(0,0,1,1)}{\sqrt{0^2+0^2+1^2+1^2}} = \dfrac{1}{\sqrt{2}}(0,0,1,1)$.

$\bar{U}_2 = \{\bar{v}_1, \bar{v}_2\}$

(3) $\bar{v}_3 = \dfrac{\text{Perp}_{U_2}(\bar{u}_3)}{|\text{Perp}_{U_2}(\bar{u}_3)|}$, $\text{Perp}_{U_2}(\bar{u}_3) = \bar{u}_3 - \text{Proj}_{U_2}(\bar{u}_3)$

$\text{Proj}_{U_2}(\bar{u}_3) = (\bar{u}_3 \cdot \bar{v}_1)\cdot\bar{v}_1 + (\bar{u}_3 \cdot \bar{v}_2)\cdot\bar{v}_2$

Övn 4. ①

③ $\text{Proj}_{V_2}\bar{u}_3 = (\bar{u}_3 \cdot \bar{v}_1)\cdot\bar{v}_1 + (\bar{u}_3\cdot\bar{v}_2)\cdot\bar{v}_2$

$= (1,1,0,1)\cdot(1,0,0,0)\cdot\bar{v}_1 + (1,1,0,1)\cdot\frac{1}{\sqrt{2}}(0,0,1,1)\cdot\bar{v}_2$

$= 1\cdot\bar{v}_1 + \frac{1}{\sqrt{2}}\bar{v}_2 = (1,0,0,0) + \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}(0,0,1,1)$

$= (1,0,\frac{1}{2},\frac{1}{2})$

$\text{Perp}_{V_2}(\bar{u}_3) = \bar{u}_3 - \text{Proj}_{V_2}(\bar{u}_3) =$

$= (1,1,0,1) - (1,0,\frac{1}{2},\frac{1}{2}) = (0,1,-\frac{1}{2},\frac{1}{2})$

Normera: $\dfrac{\text{Perp}_{V_2}(\bar{u}_3)}{|\text{Perp}_{V_2}(\bar{u}_3)|} = \dfrac{(0,1,-\frac{1}{2},\frac{1}{2})}{\sqrt{0^2+1^2+(-\frac{1}{2})^2+(\frac{1}{2})^2}} = \dfrac{1}{\sqrt{1+\frac{1}{2}}}(0,1,-\frac{1}{2},\frac{1}{2})$

$= \dfrac{1}{\sqrt{\frac{3}{2}}}(0,1,-\frac{1}{2},\frac{1}{2}) = \sqrt{\frac{2}{3}}(0,1,-\frac{1}{2},\frac{1}{2})$

$= \sqrt{\frac{2}{3}}\cdot\frac{2}{2}(0,1,-\frac{1}{2},\frac{1}{2}) = \sqrt{\frac{2}{3\cdot 2\cdot 2}}(0,2,-1,1)$

$= \frac{1}{\sqrt{6}}(0,2,-1,1).$

Svar: en ON-Bas för V Blir då

$$V_{ON} = \left\{ \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \frac{1}{\sqrt{6}}\begin{bmatrix}0\\2\\-1\\1\end{bmatrix} \right\}.$$