Övn ③ Beräkna projektionen av vektorn $\bar{r} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ på delrummet

V i uppgift ②.

Vi fick ON-bas $V_{ON} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{6}} \begin{bmatrix} 0 \\ 2 \\ -1 \\ 1 \end{bmatrix} \right\}$

$\bar{v}_1$, $\bar{v}_2$, $\bar{v}_3$

Sökt: $\text{Proj}_{V_{ON}}(\bar{r}) =$

$$\text{Proj}_{V_{ON}}(\bar{r}) = (\bar{r} \cdot \bar{v}_1)\bar{v}_1 + (\bar{r} \cdot \bar{v}_2)\bar{v}_2 + (\bar{r} \cdot \bar{v}_3)\bar{v}_3$$

$$= \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \bar{v}_1 + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \bar{v}_2 + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \frac{1}{\sqrt{6}} \begin{bmatrix} 0 \\ 2 \\ -1 \\ 1 \end{bmatrix} \bar{v}_3 =$$

$$= 1 \cdot \bar{v}_1 + 0 \cdot \bar{v}_2 + \frac{2}{\sqrt{6}} \cdot \bar{v}_3 =$$

$$= (1,0,0,0) + \frac{2}{\sqrt{6}} \frac{1}{\sqrt{6}} (0,2,-1,1) = (1,$$

$$= (1,0,0,0) + \frac{1}{3}(0,2,-1,1) = \left(1, \frac{2}{3}, -\frac{1}{3}, \frac{1}{3}\right)$$

Svar: Projektionen blir $\begin{bmatrix} 1 \\ \frac{2}{3} \\ -\frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$