

OVIO (6) W^\perp f.s. $\left(\begin{array}{ccccc|c} -\bar{v}_1 & & & & & 1 \\ & -\bar{v}_2 & & & & 0 \end{array} \right)$

$$W^\perp = \left(\begin{array}{ccccc|c} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{10}} & \frac{2}{\sqrt{10}} & \frac{2}{\sqrt{10}} & 0 & \frac{1}{\sqrt{10}} & 0 \end{array} \right) \begin{array}{l} -\sqrt{2} \\ -\sqrt{10} \end{array} \sim \left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \end{array} \right) \begin{array}{l} 0 \\ 4 \cdot \frac{1}{2} \end{array} \quad (-1)$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \end{array} \right) \Rightarrow \begin{cases} x_1 = x_5 \\ x_2 = x_3 - x_5 \\ x_3 = s, x_4 = t, \\ x_5 = u, s, t, u \in \mathbb{R} \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ s \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ t \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ u \end{bmatrix}$$

$$W^\perp = \text{span} \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\} \quad \bar{v}_1 = \frac{\bar{u}_1}{|\bar{u}_1|} = (0, 1, 1, 0, 0) \quad U_1 = \{\bar{v}_1\}$$

$$\bar{v}_2 = \frac{\text{Perp}_{U_1}(\bar{u}_2)}{|\text{Perp}_{U_1}(\bar{u}_2)|}$$

$$\text{Perp}_{U_1}(\bar{u}_2) = \bar{u}_2 - \text{Proj}_{U_1}(\bar{u}_2)$$

$$= (0, 1, 1, 0, 0) - ((\bar{u}_2 \cdot \bar{v}_1) \cdot \bar{v}_1) = (0, 1, 1, 0, 0) - 0$$

$$|\text{Perp}_{U_1}(\bar{u}_2)| = \sqrt{2} \Rightarrow \bar{v}_2 = \frac{1}{\sqrt{2}} (0, 1, 1, 0, 0) \quad U_2 = \{\bar{v}_1, \bar{v}_2\}$$

$$\bar{v}_3 = \frac{\text{Perp}_{U_2}(\bar{u}_3)}{|\text{Perp}_{U_2}(\bar{u}_3)|} = \frac{\bar{u}_3 - ((\bar{u}_3 \cdot \bar{v}_1) \cdot \bar{v}_1 + (\bar{u}_3 \cdot \bar{v}_2) \bar{v}_2)}{|\text{Perp}_{U_2}(\bar{u}_3)|} =$$

$$\frac{(1, -1, 0, 1, 1) + \frac{1}{2} (0, 1, 1, 0, 0)}{|\text{Perp}_{U_2}(\bar{u}_3)|} = \frac{(1, -\frac{1}{2}, \frac{1}{2}, 0, 1)}{\sqrt{1 + \frac{1}{4} + \frac{1}{4} + 1}} = \frac{1}{\sqrt{10}} (2, 1, 1, 0, 2)$$

$$W_{\text{ON}}^\perp = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{v}_3, \frac{1}{\sqrt{10}} \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 2 \end{bmatrix} \right\}$$

Dim. Sets $\dim(W) + \dim(W^\perp) = \dim(\mathbb{R}^5)$
 $2 + 3 = 5$ ✓