Viscous dissipation in simulations of stratified rotating convection

Simon Lance[1], Matthew Browning[1], Laura Currie[2]

[1] University of Exeter; [2] Durham University

sl484@exeter.ac.uk

Why you should care about viscous dissipation

All stars convect, and in a steady state these convective flows must be maintained against viscous and Ohmic dissipation. Previous studies have shown both theoretically [1] and experimentally [2,3] that the magnitude of this dissipation, E, is bounded by the ratio of the layer depth to the thermal scale height.

$$E = \frac{\Phi}{L} = \int_V \tau_{ij}\frac{\partial u_i}{\partial x_j} + \cancel{\frac{j^2}{\sigma}}\, dV \qquad E \leq \frac{d}{H_T}$$

No magnetism

- Φ = Total dissipative heating
- L = Luminosity
- $\tau_{i,j}$ = Viscous stress tensor
- u = Velocity
- j = Electric current = 0
- σ = Conductivity
- d = Fluid depth
- H_T = Thermal scale height

In an unstratified system this ratio is small it follows that this dissipation is negligible, however in system with significant thermal stratification (such as stellar convection) it is possible to achieve $E > 1$. That is, the dissipation can exceed the luminosity supplied to the layer.

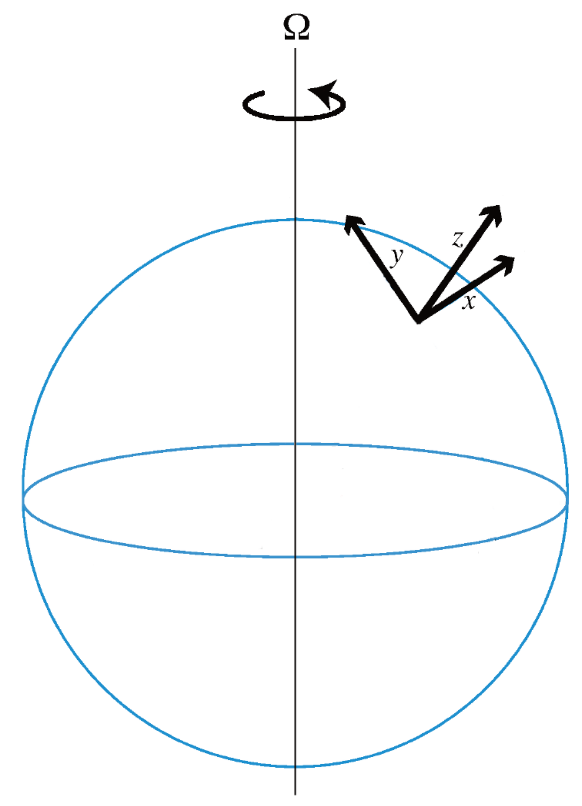

Figure 1 – A schematic view of the location of our cartesian layer on a rotating sphere.

Simulation setup

Utilising the open source python framework Dedalus [4], we consider stratified cartesian layers on a rotating sphere. Our layer is described by a series of non-dimensional numbers:

Number	Representation
Rayleigh (Ra)	Ratio of buoyancy driving to dissipation effects
Critical Rayleigh (Ra_c)	Rayleigh number required for convective onset
Taylor (Ta)	Ratio of rotational to viscous forces
Prandtl (Pr)	Ratio of viscous to thermal diffusivities (set as as 1)
Rossby (Ro_c)	Ratio of inertial to rotational forces
N_ρ	Number of density scale heights across our layer

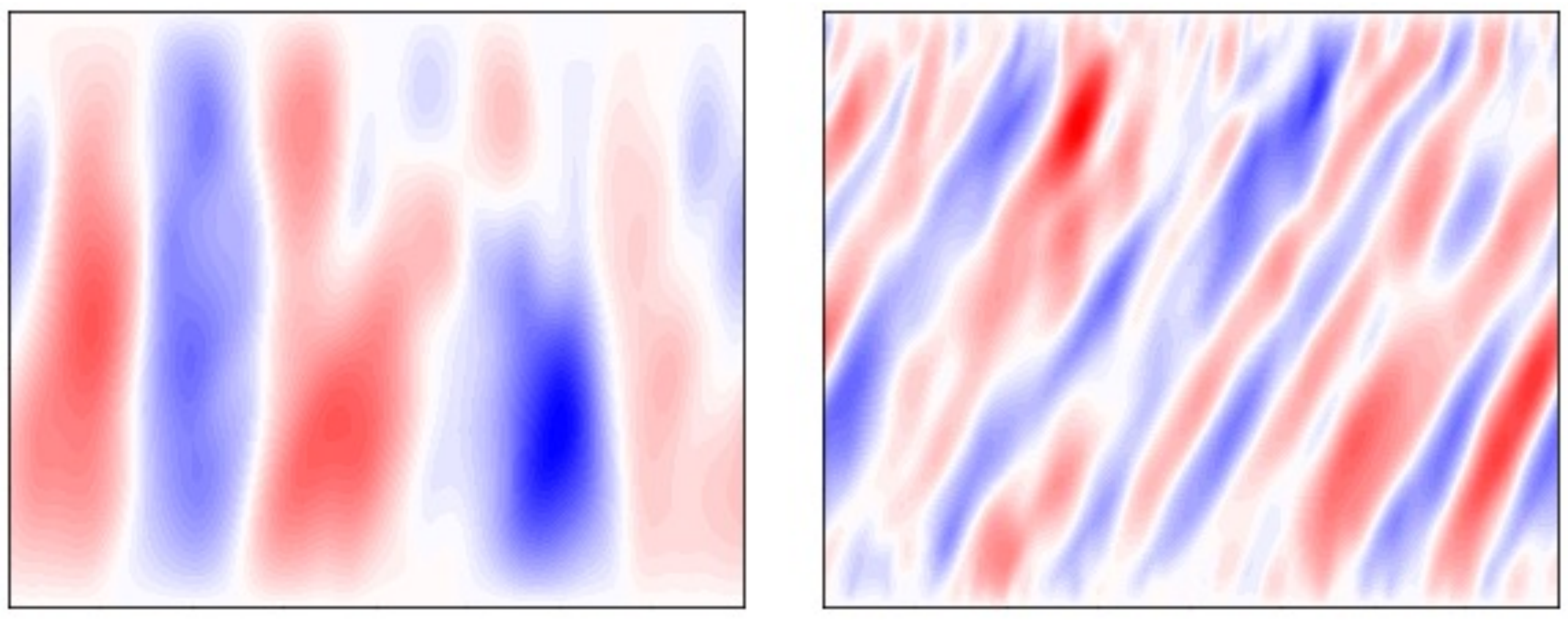

Figure 2 - Two vertical velocity profiles of non-rotationally (left) and rotationally influenced (right) convection. Both cases have a stratification of N_ρ= 0.5, Ra/Ra_c = 10^2 and Ro_c = 4.57 and 0.59 for (a) and (b) respectively. In the rotational influenced case the convective motions align themselves with the axis of rotation in Taylor-Proudman like convective cells.

Non-rotating limit holds for the rotational case!

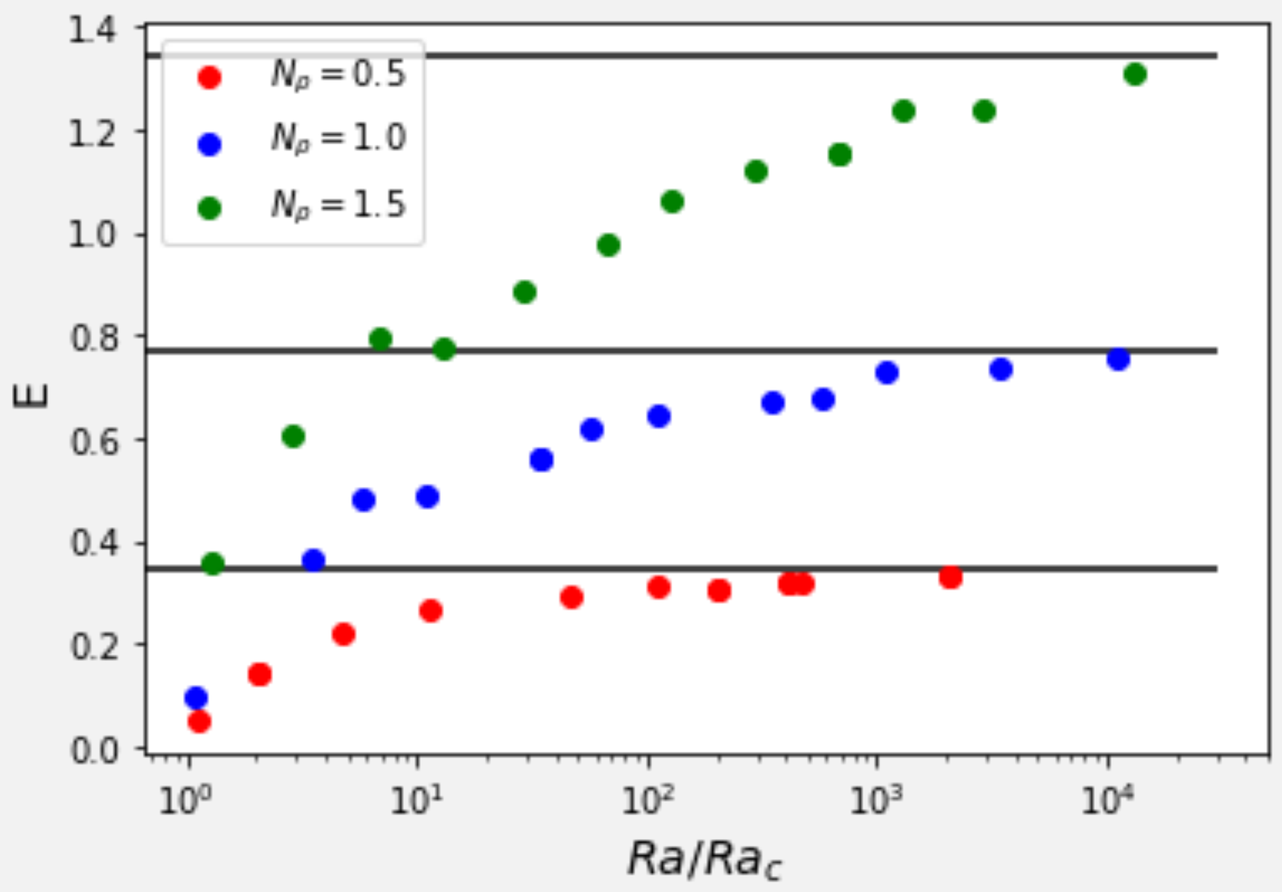

Figure 3 – The values of E for a series of simulations at three different stratifications for increasing supercriticality.

$$E \leq \frac{d}{\widetilde{H}_T}$$

$$\widetilde{H}_T = \frac{H_{T,0}H_{T,d}}{H_{T,z^*}}$$

Eq1. The adjusted upper bound (see [3]) and the modified thermal scale height

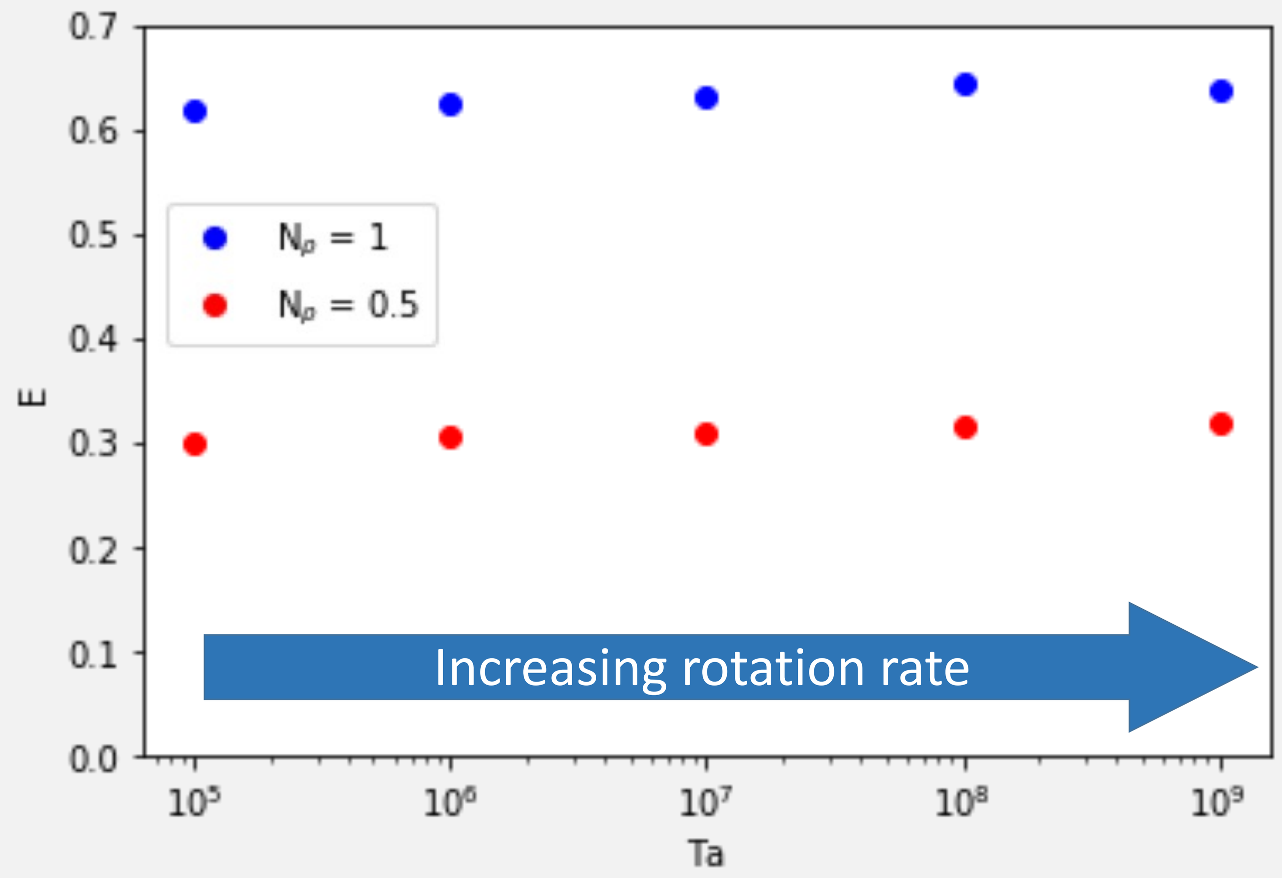

Figure 4 – The values of E for a series of simulations at fixed supercriticality with increasing rotational influence at two different degrees of stratification.

Much like in the non-rotating case the value of E is bounded by the ratio of the layer depth and a modified thermal scale height $\widetilde{H}_T$ and for sufficiently supercritical convection the value of E asymptotically tends towards this modified upper bound (see Eq1).

As can be seen in figure 2 the influence of rotation is clearly present in our system. As for how it interacts with the dissipation, its influence is isolated to how it inhibits the onset of convective flows ($Ra_c \sim Ta^{2/3}$). Thus to achieve the sufficient supercriticality required to reach these upper bounds we require higher Ra, more turbulent convection.

[1] Hewitt, J., Mckenzie, D., & Weiss, N. (1975). Dissipative heating in convective flows. Journal of Fluid Mechanics

[2] Jarvis, G., & Mckenzie, D. (1980). Convection in a compressible fluid with infinite Prandtl number. Journal of Fluid Mechanics

[3] Currie, L., & Browning, M. (2017). The Magnitude of Viscous Dissipation in Strongly Stratified Two-dimensional Convection. The Astrophysical Journal Letters